Lecture 07

Ensemble Methods

STAT 479: Machine Learning, Fall 2018

Sebastian Raschka

http://stat.wisc.edu/~sraschka/teaching/stat479-fs2018/

Announcements First

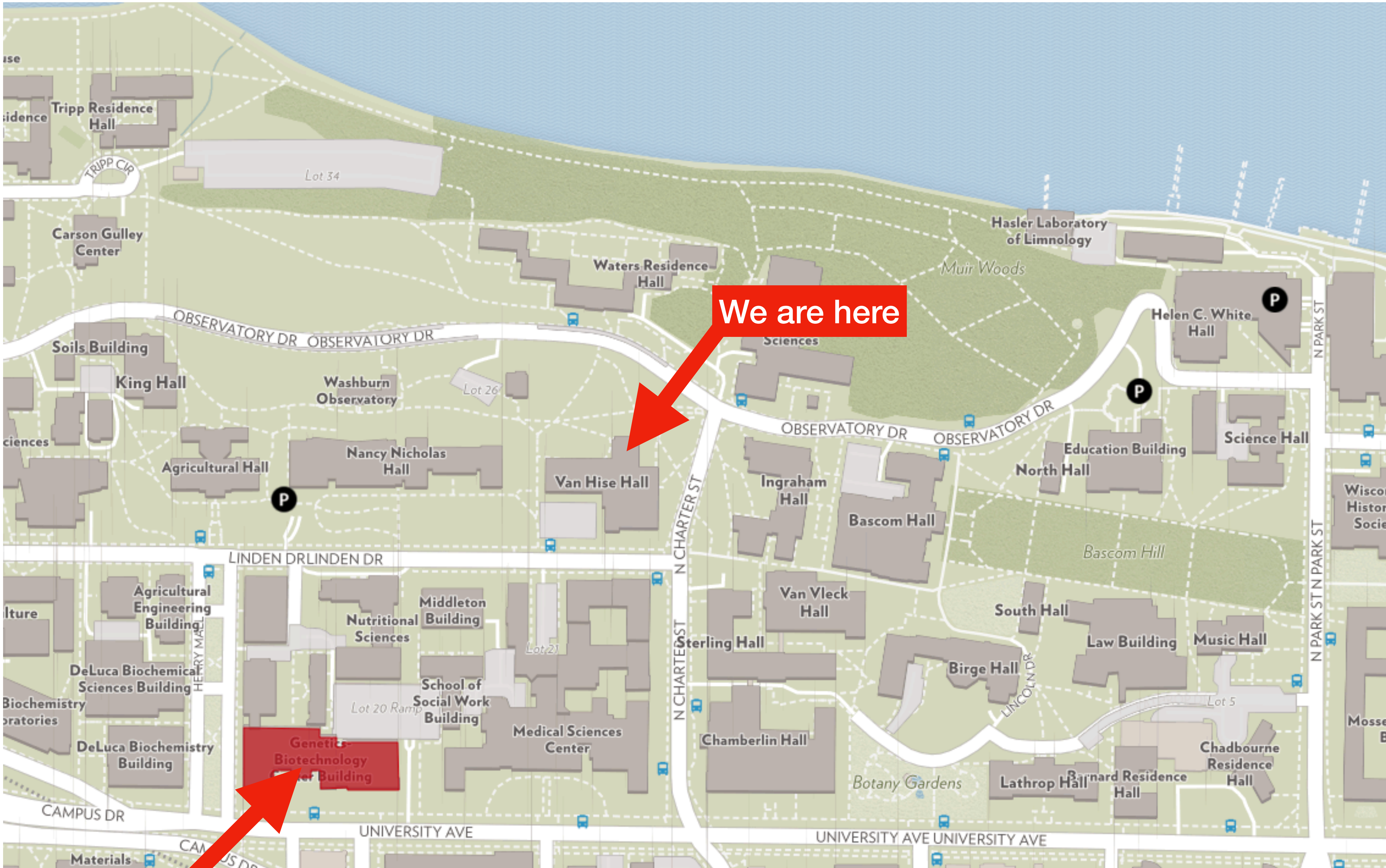
[DataSci] Machine Learning Series
Scheduled: Thursday Oct 10, 6:30 PM
Location: Genetics-Biotech Center 1441

Soft-biometric characteristics include a person's age, gender, race, and health status. As many Deep Learning-centric applications are developed in recent years, the automatic extraction of soft biometric attributes can happen without the user's agreement, thereby raising several privacy concerns. <u>This talk will introduce how to extract soft-biometric attributes from facial images, as well as how to conceal soft-biometric information for enhancing privacy.</u>

Don't worry if you do not have programming experience with Python! Dr. Rashka will also give a tutorial introducing PyTorch and how we can use it to train a simple gender classifier and ordinal regression model for estimating the apparent age from face images.

Best,

Lareina Liu
UW Data Science Club
.Data



Genetics-Biotech Center 1441

Example Exam Question

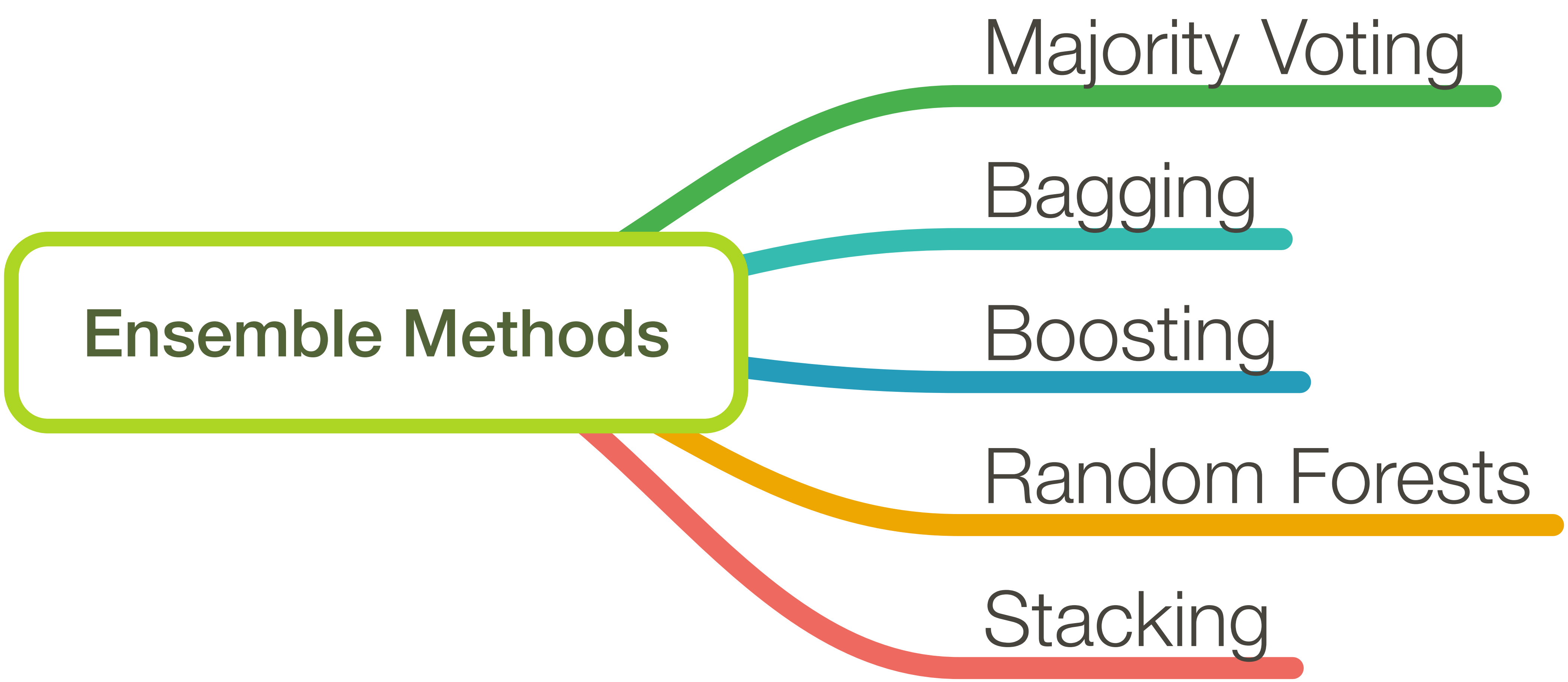
(6 points) Does the (computational) time complexity of a k-Nearest Neighbor classifier grow linearly, quadratically, or exponentially with the number of samples in the training dataset? Explain your answer in 1-2 sentences.

Example Exam Question

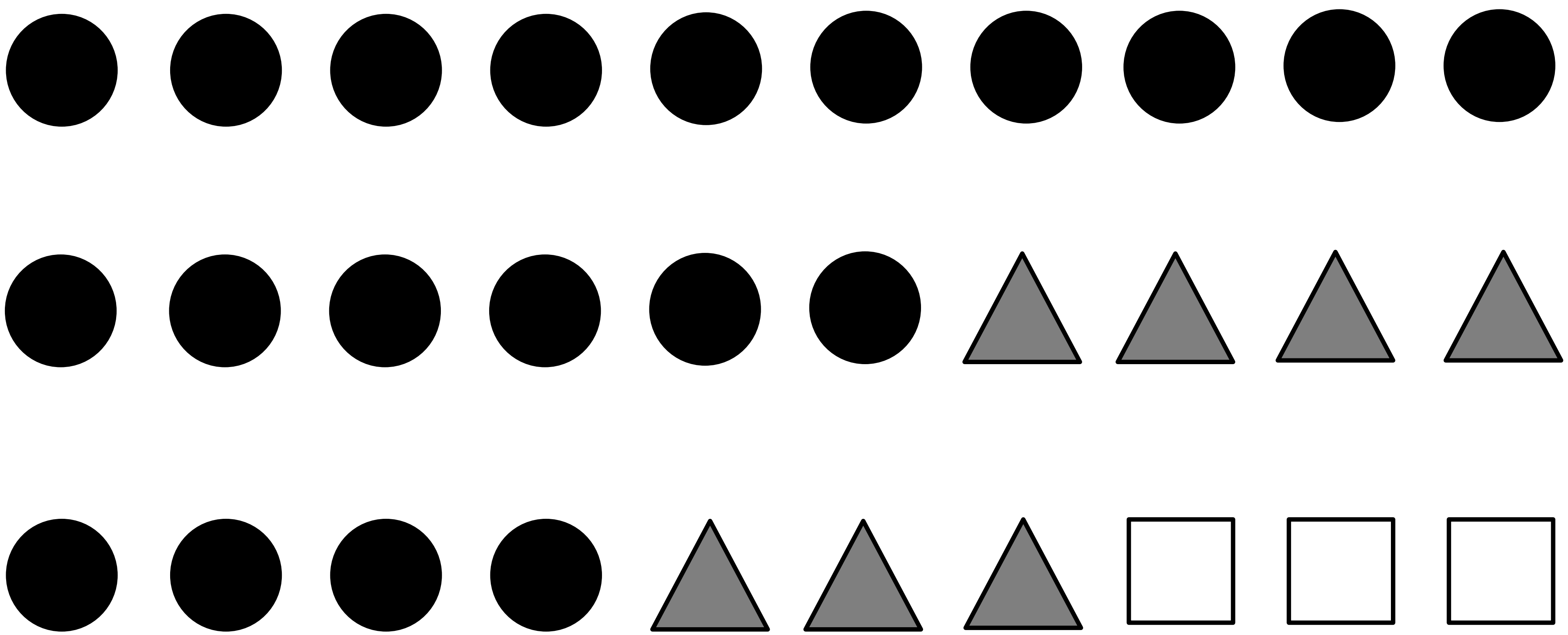
(6 points) Can you represent the following boolean function with a decision tree? If you answer "no," explain why in 1-2 sentences. Otherwise, draw a decision tree that separates the data records perfectly.

x_1	x_2	$f(x_1, x_2)$
1	1	0
0	0	0
1	0	1
0	1	0

Lecture Overview



Majority Voting

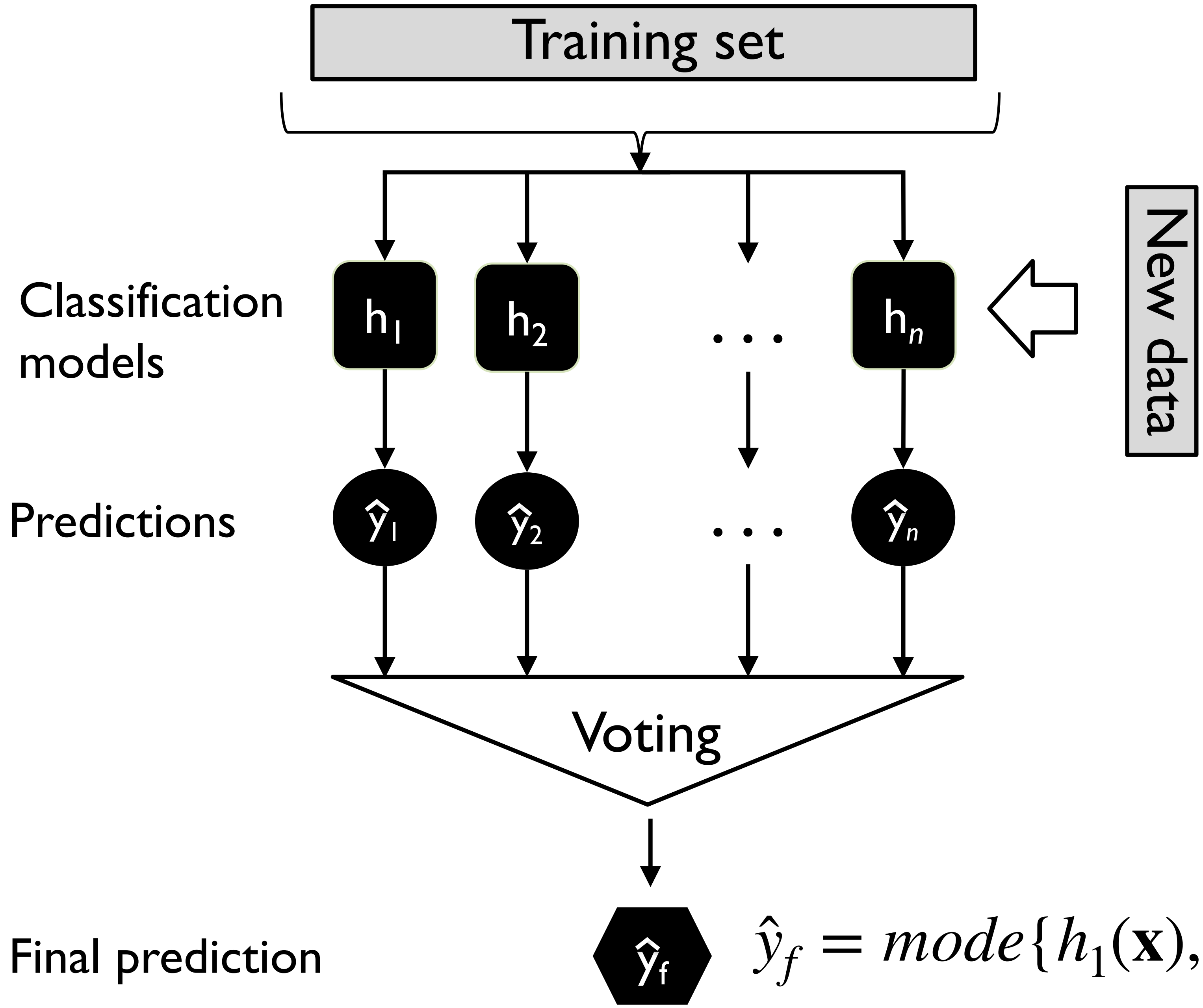


Unanimity

Majority

Plurality

Majority Vote Classifier



$$\hat{y}_f = mode\{h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots h_n(\mathbf{x})\}$$

where $h_i(\mathbf{x}) = \hat{y}_i$

Why Majority Vote?

- assume *n* independent classifiers with a base error rate ϵ
- here, independent means that the errors are uncorrelated
- assume a binary classification task
- assume the error rate is better than random guessing (i.e., lower than 0.5 for binary classification)

$$\forall \epsilon_i \in \{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}, \epsilon_i < 0.5$$

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$$\forall \epsilon_i \in \{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}, \epsilon_i < 0.5$$

The probability that we make a wrong prediction via the ensemble if *k* classifiers predict the same class label

$$P(k) = \binom{n}{k} \epsilon^k (1 - \epsilon)^{n-k} \qquad k > \lceil n/2 \rceil$$

(Probability mass func. of a binomial distr.)

Why Majority Vote?

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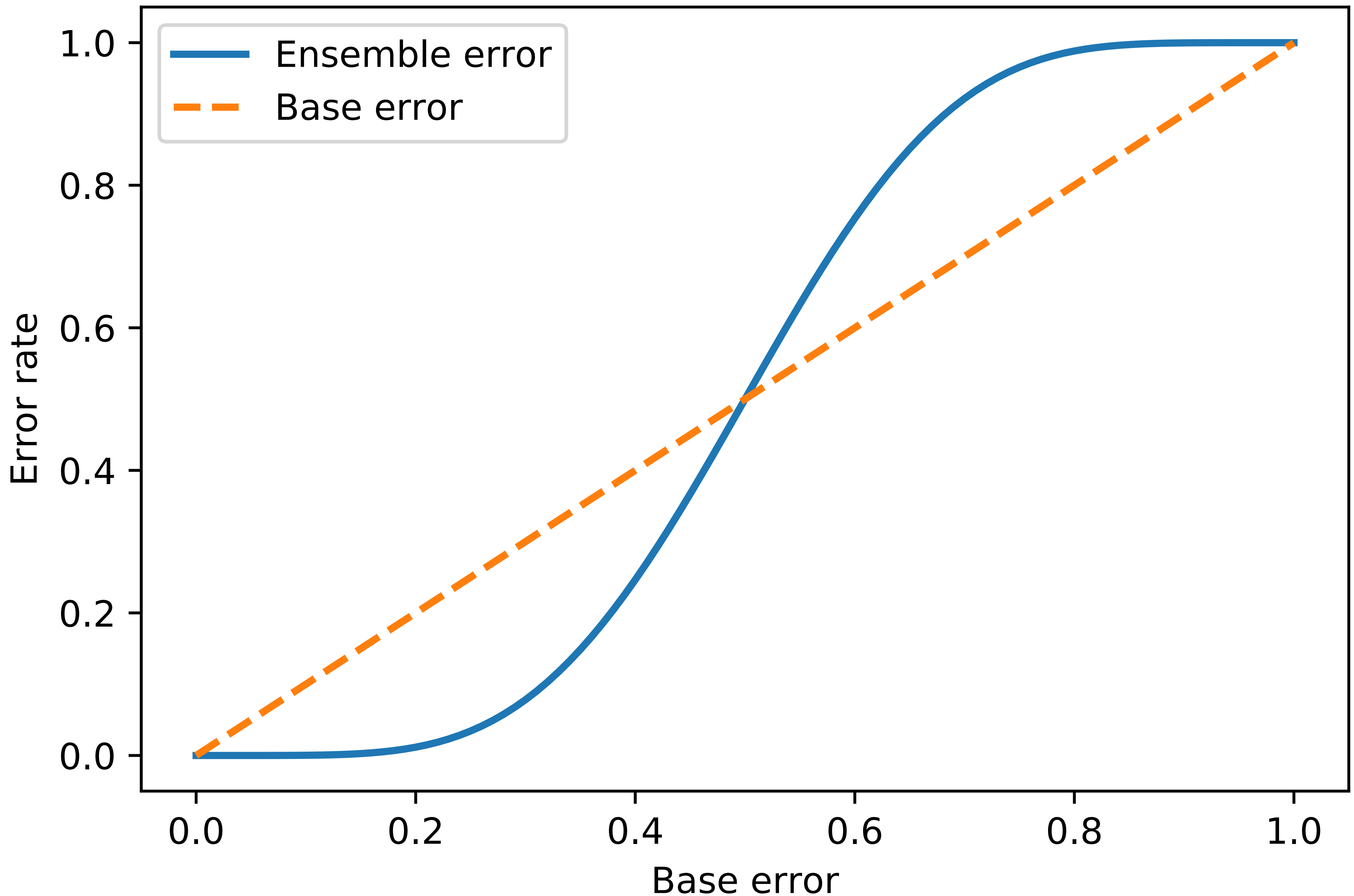
Ensemble error:

$$\epsilon_{ens} = \sum_{k}^{n} \binom{n}{k} \epsilon^k (1 - \epsilon)^{n-k}$$

$$\epsilon_{ens} = \sum_{k=6}^{11} \binom{11}{k} 0.25^k (1 - 0.25)^{11-k} = 0.034$$

$$\epsilon_{ens} = \sum_{k}^{n} \binom{n}{k} \epsilon^k (1-\epsilon)^{n-k}$$



"Soft" Voting

$$\hat{y} = \arg\max_j \sum_{i=1}^{n} w_i p_{i,j}$$

$p_{i,j}$: predicted class membership probability of the *i*th classifier for class label *j*

w_j : optional weighting parameter, default $w_i = 1/n, \forall w_i \in \{w_1, \ldots, w_n\}$

"Soft" Voting

Use only for well-calibrated classifiers!

$$\hat{y} = \arg\max_{j} \sum_{i=1}^{n} w_i p_{i,j}$$

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w_j : optional weighting parameter, default $w_i = 1/n, \forall w_i \in \{w_1, \ldots, w_n\}$

"Soft" Voting

$$\hat{y} = \arg\max_j \sum_{i=1}^{n} w_i p_{i,j}$$

Binary classification example

$$j \in \{0,1\} \qquad h_i (i \in \{1,2,3\})$$

$$h_1(\mathbf{x}) \rightarrow [0.9, 0.1]$$
$$h_2(\mathbf{x}) \rightarrow [0.8, 0.2]$$
$$h_3(\mathbf{x}) \rightarrow [0.4, 0.6]$$

"Soft" Voting $\hat{y} = \arg\max_j \sum_{i=1}^{n} w_i p_{i,j}$

Binary classification example

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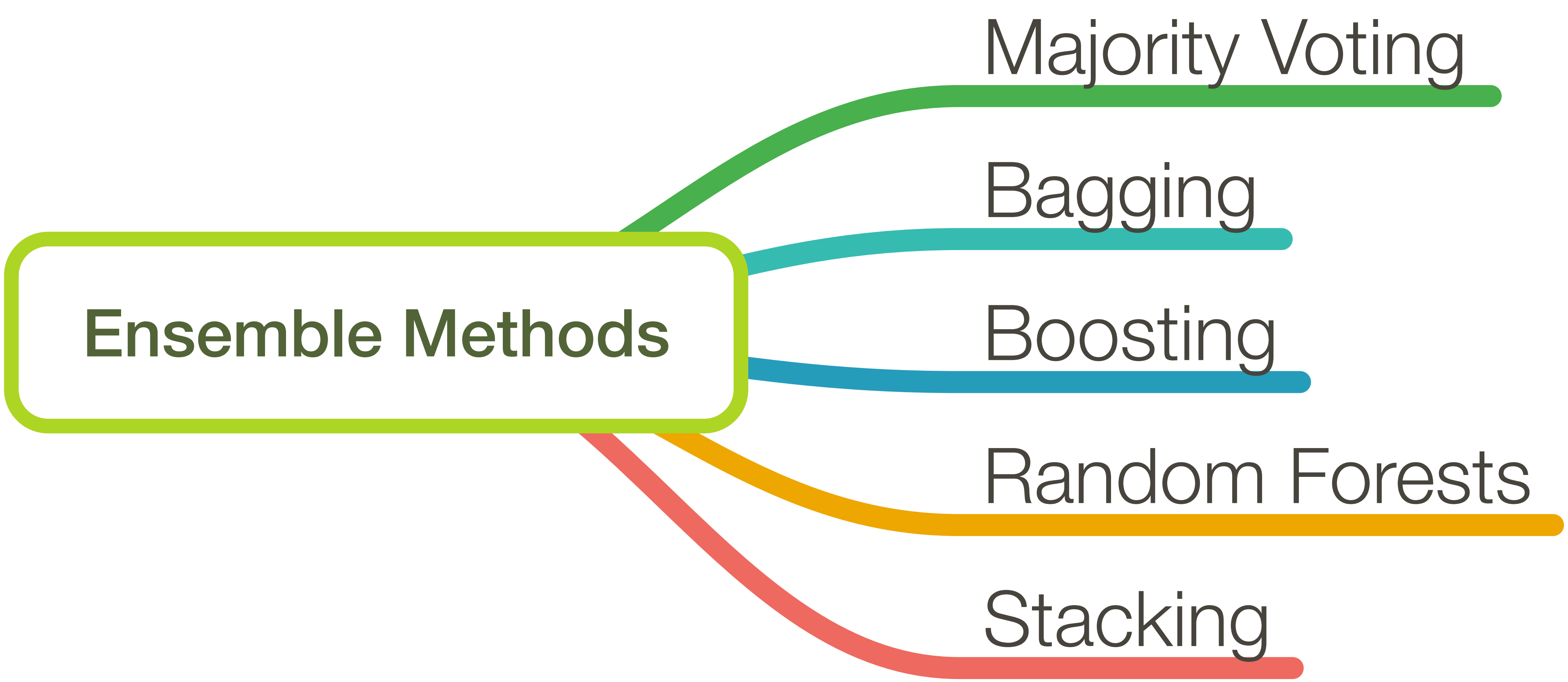
$$h_2(\mathbf{x}) \rightarrow [0.8, 0.2]$$
$$h_3(\mathbf{x}) \rightarrow [0.4, 0.6]$$

$$p(j = 0 \mid \mathbf{x}) = 0.2 \cdot 0.9 + 0.2 \cdot 0.8 + 0.6 \cdot 0.4 = 0.58$$

$$p(j = 1 \mid \mathbf{x}) = 0.2 \cdot 0.1 + 0.2 \cdot 0.2 + 0.6 \cdot 0.6 = 0.42$$

$$\hat{y} = \arg\max_j \left\{ p(j = 0 \mid \mathbf{x}), p(j = 1 \mid \mathbf{x}) \right\}$$

Overview


Ensemble Methods
Majority Voting
Bagging
Boosting
Random Forests
Stacking

Bagging

(Bootstrap Aggregating)

Breiman, L. (1996). Bagging predictors. *Machine learning*, *24*(2), 123-140.

Bagging

(Bootstrap Aggregating)

Algorithm 1 Bagging

Let n be the number of bootstrap samples

for i=1 to n **do**

  Draw bootstrap sample of size m, $\mathcal{D}_i$

  Train base classifier h_i on $\mathcal{D}_i$

$\hat{y} = mode\{h_1(\mathbf{x}), ..., h_n(\mathbf{x})\}$

Bootstrap Sampling

Bootstrap Sampling

$$P(\textbf{not chosen}) = \left(1 - \frac{1}{n}\right)^n,$$

$$\frac{1}{e} \approx 0.368, \quad n \to \infty.$$

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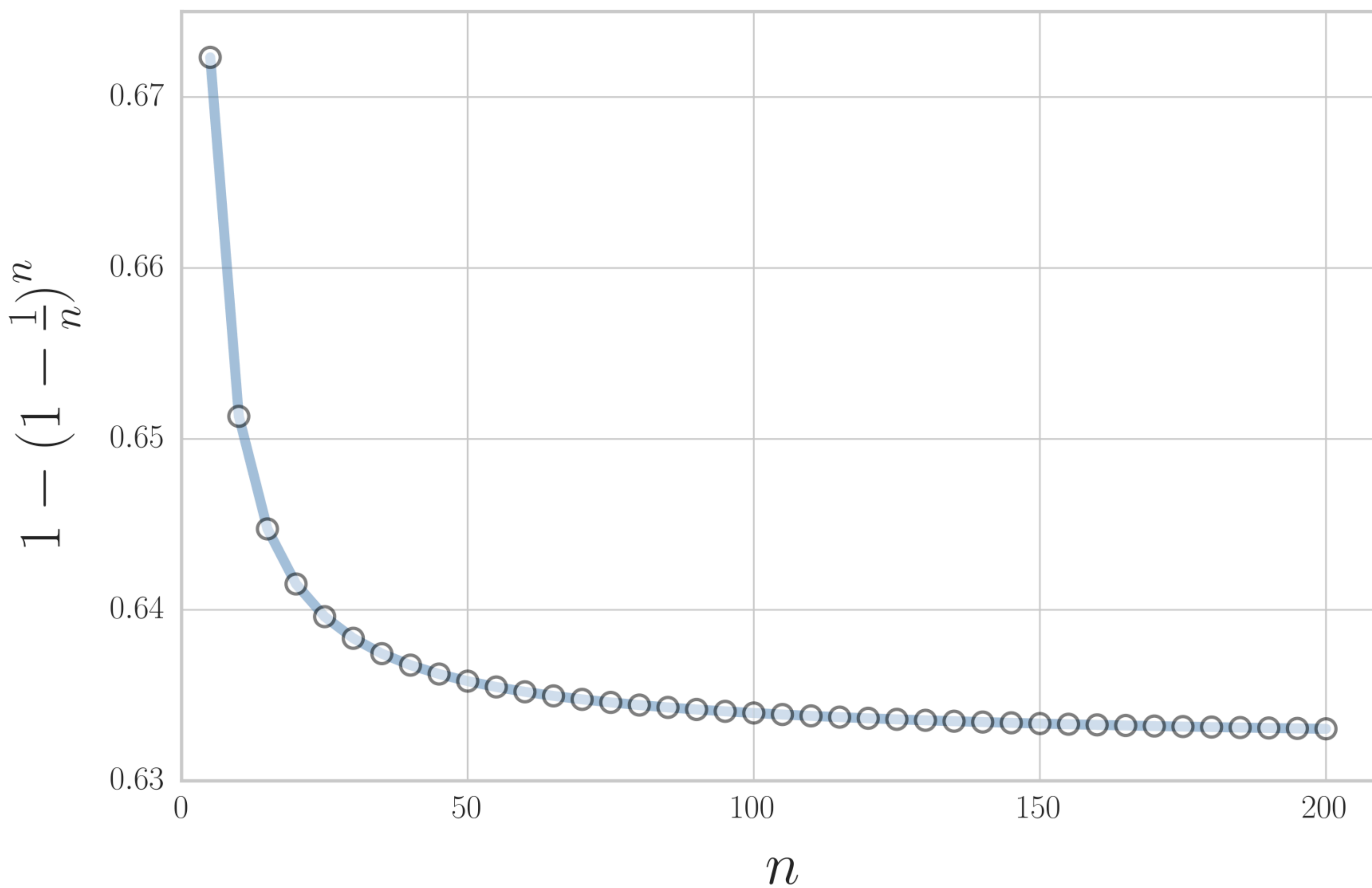
$$P(\textbf{chosen}) = 1 - \left(1 - \frac{1}{n}\right)^n \approx 0.632$$

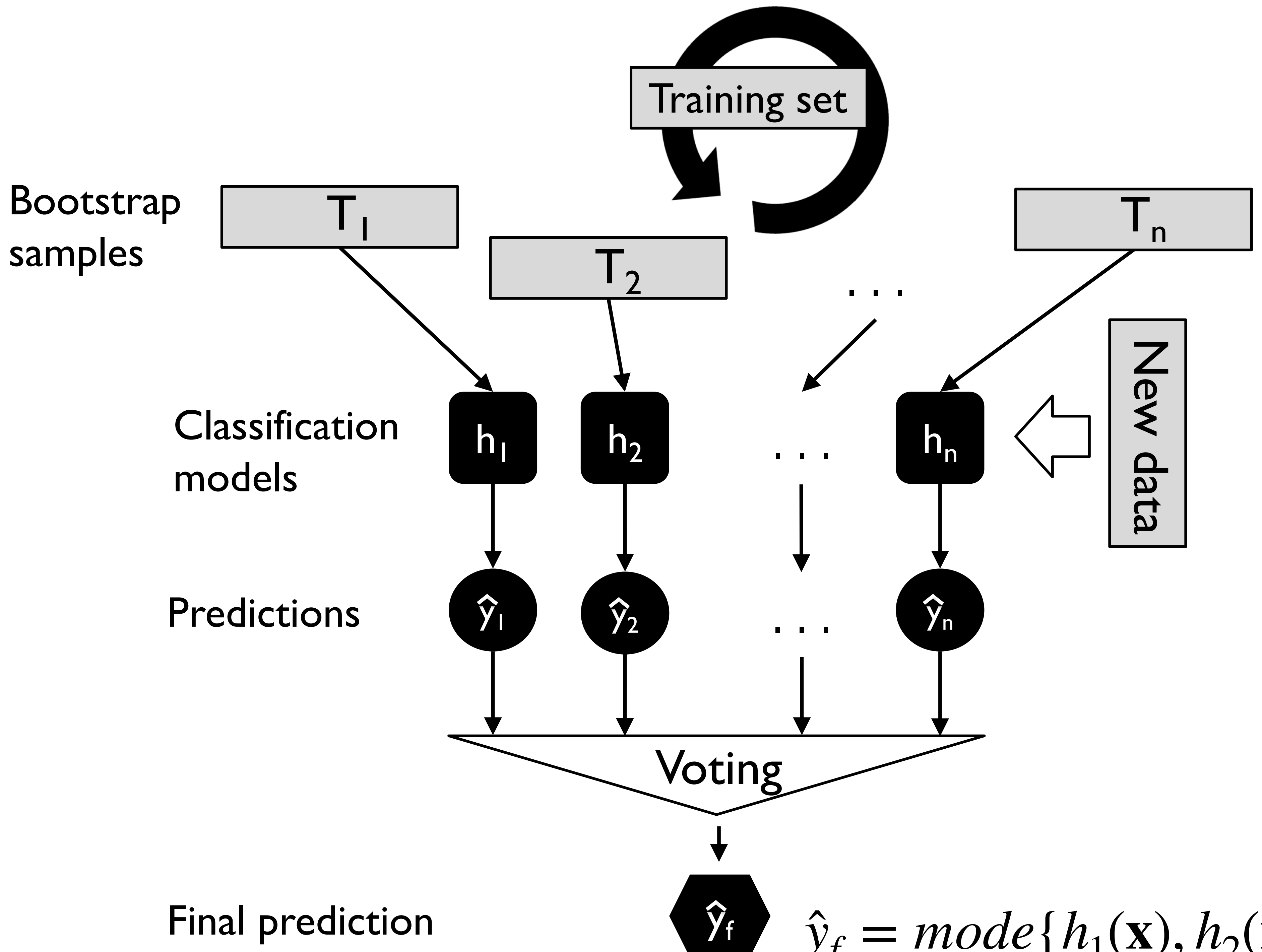


Bootstrap Sampling

Training example indices	Bagging round 1	Bagging round 2	...
1	2	7	...
2	2	3	...
3	1	2	...
4	3	1	...
5	7	1	...
6	2	7	...
7	4	7	...

h_1 h_2 h_n

Bagging Classifier



$$\hat{y}_f = mode\{h_1(\mathbf{x}), h_2(\mathbf{x}), \dots h_n(\mathbf{x})\}$$

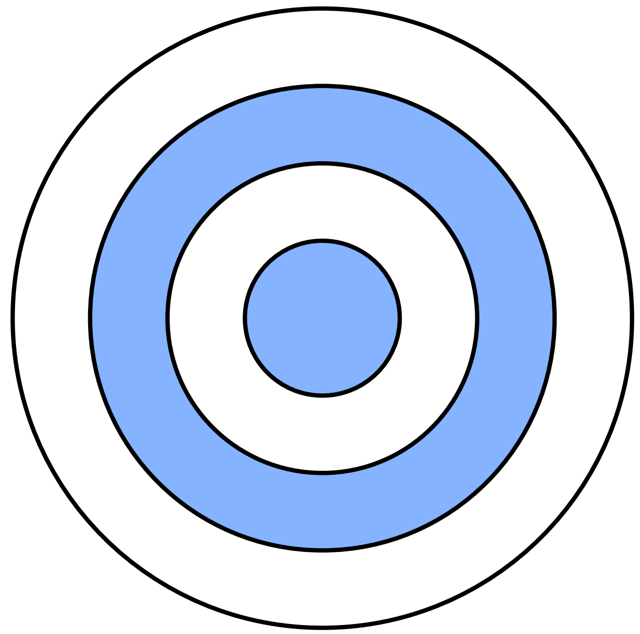
where $h_i(\mathbf{x}) = \hat{y}_i$

Bias-Variance Decomposition

Loss = Bias + Variance + Noise

(more technical details in next lecture on model evaluation)

Bias-Variance Intuition

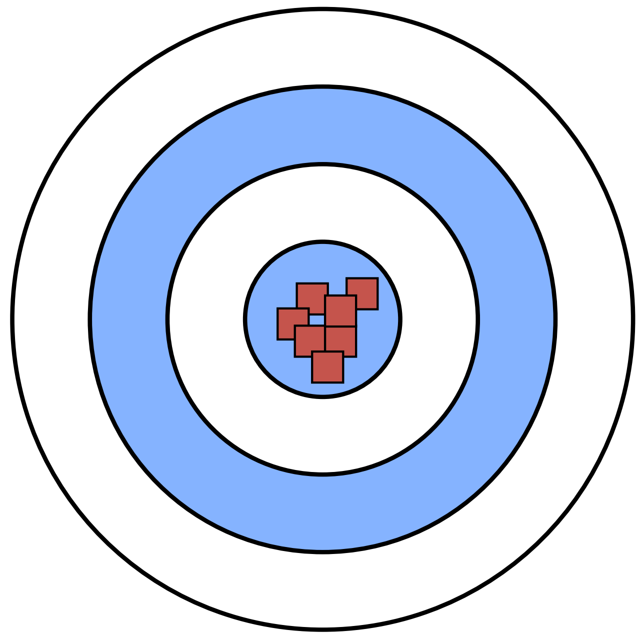
	Low Variance (Precise)	**High Variance** (Not Precise)
Low Bias (Accurate)		
High Bias (Not Accurate)		

Bias-Variance Intuition

Low Variance
(Precise)

High Variance
(Not Precise)

Low Bias
(Accurate)



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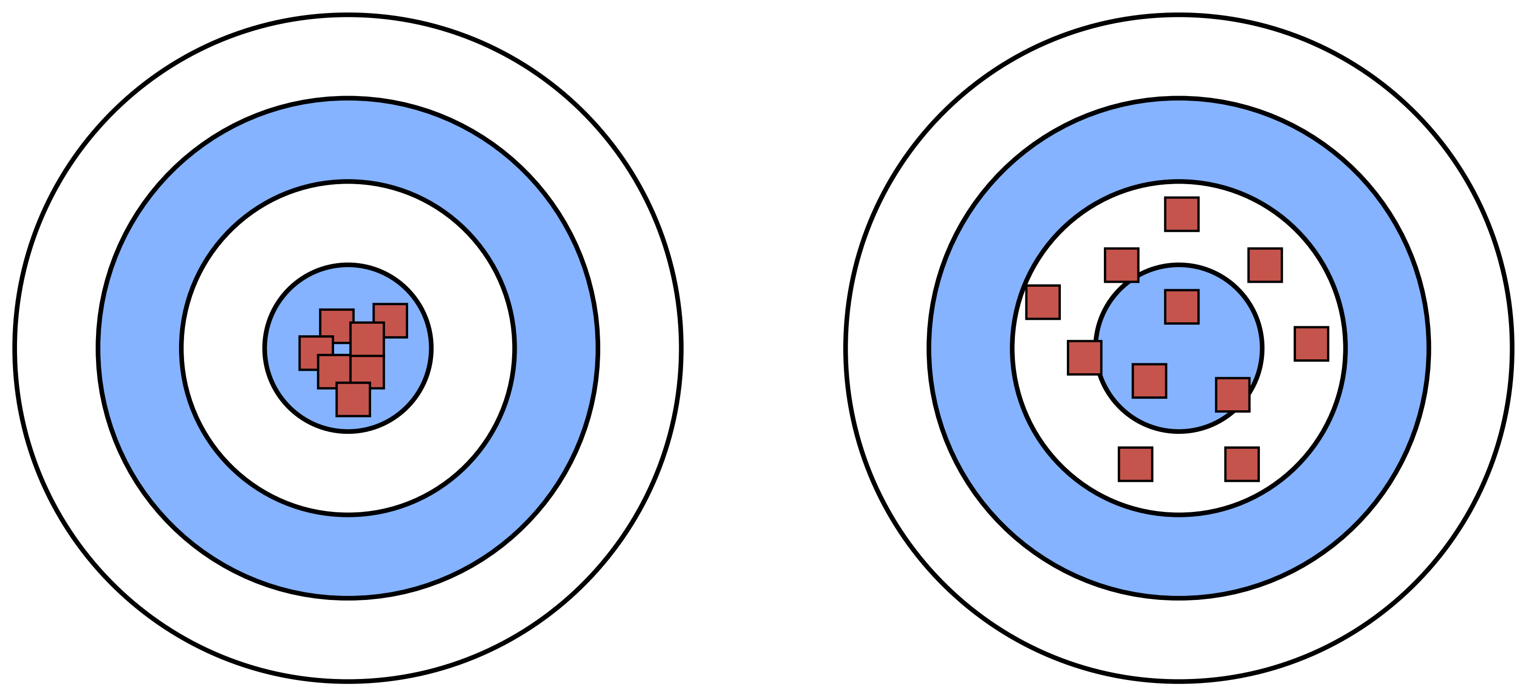

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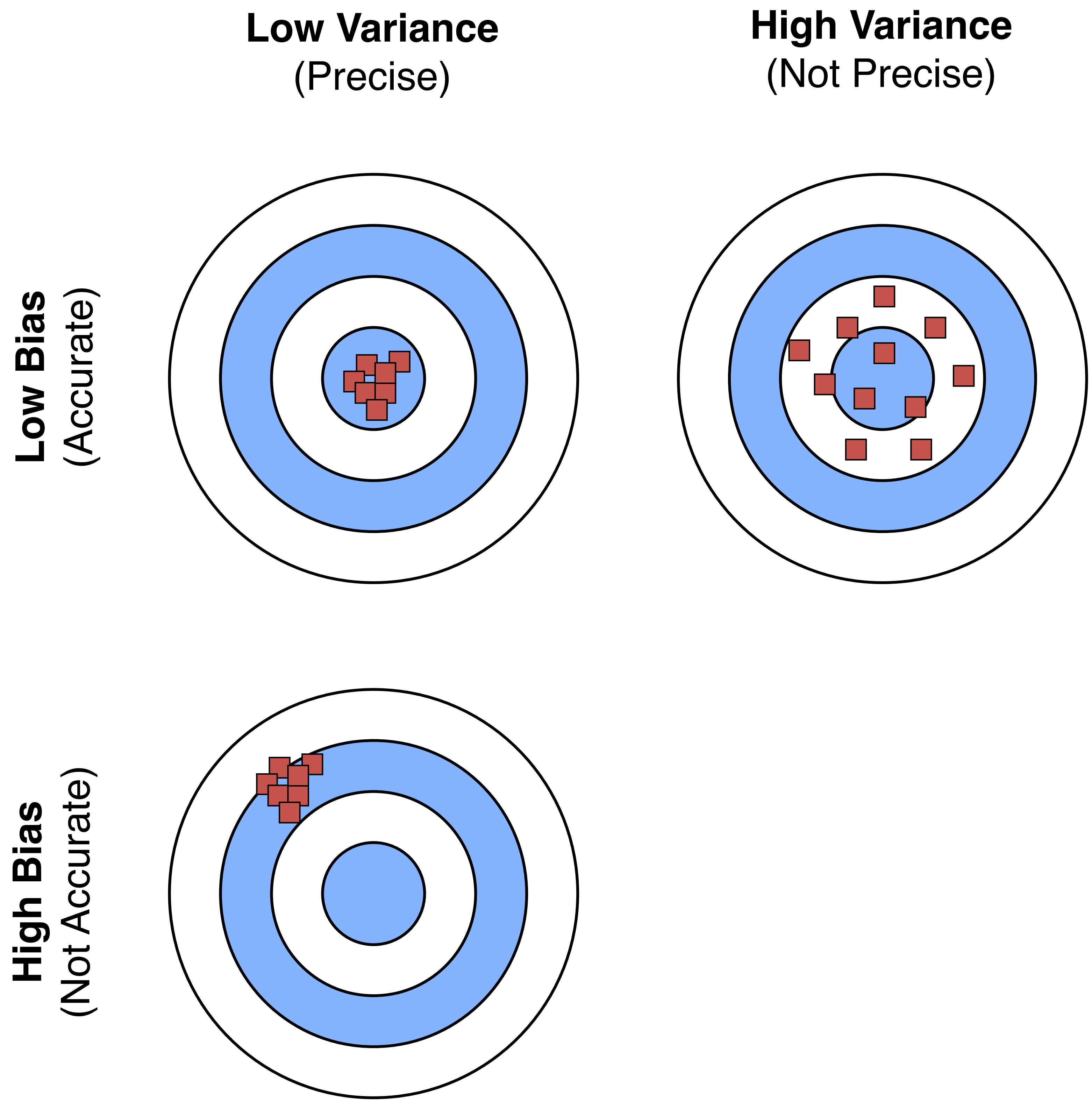

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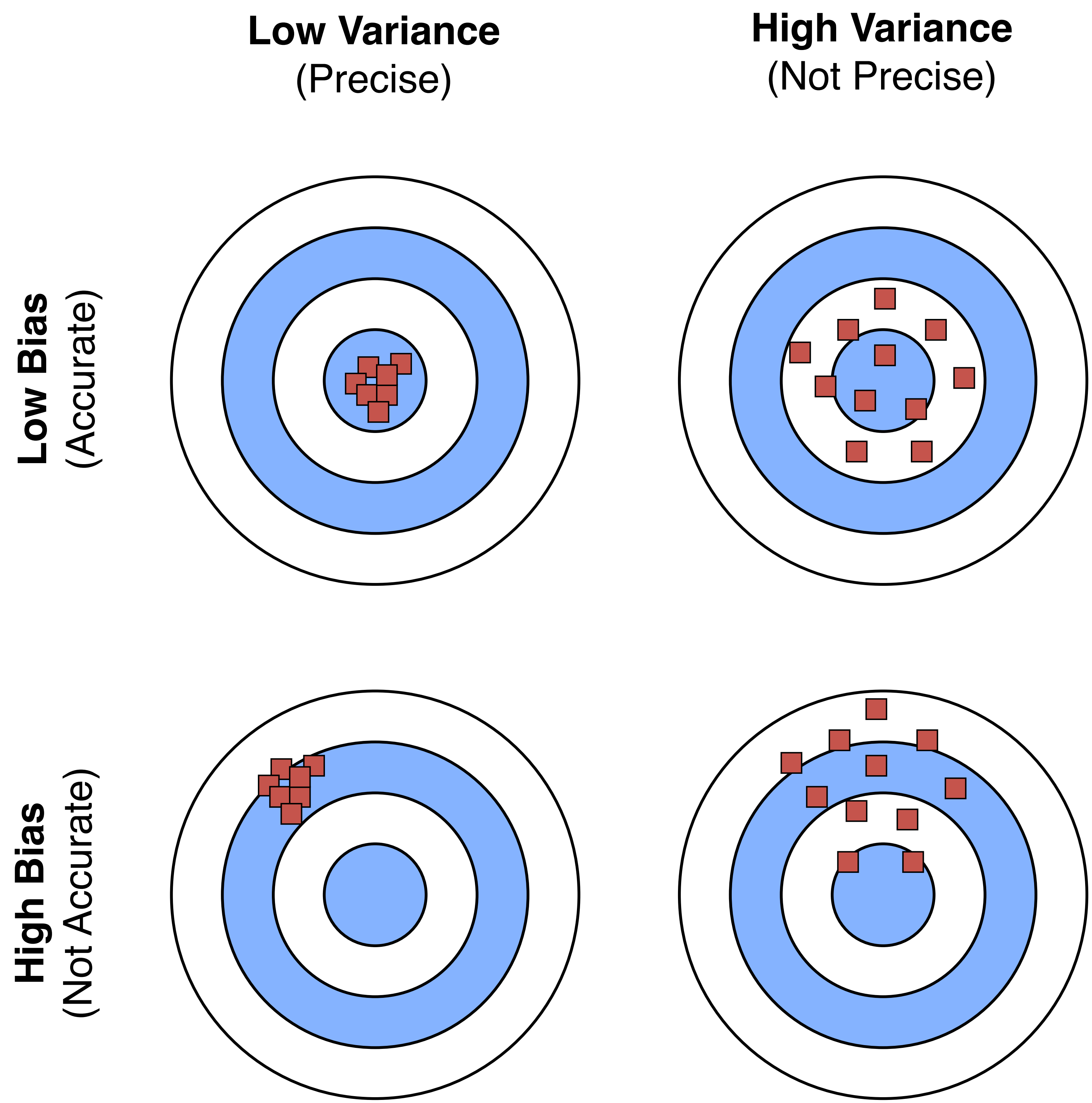
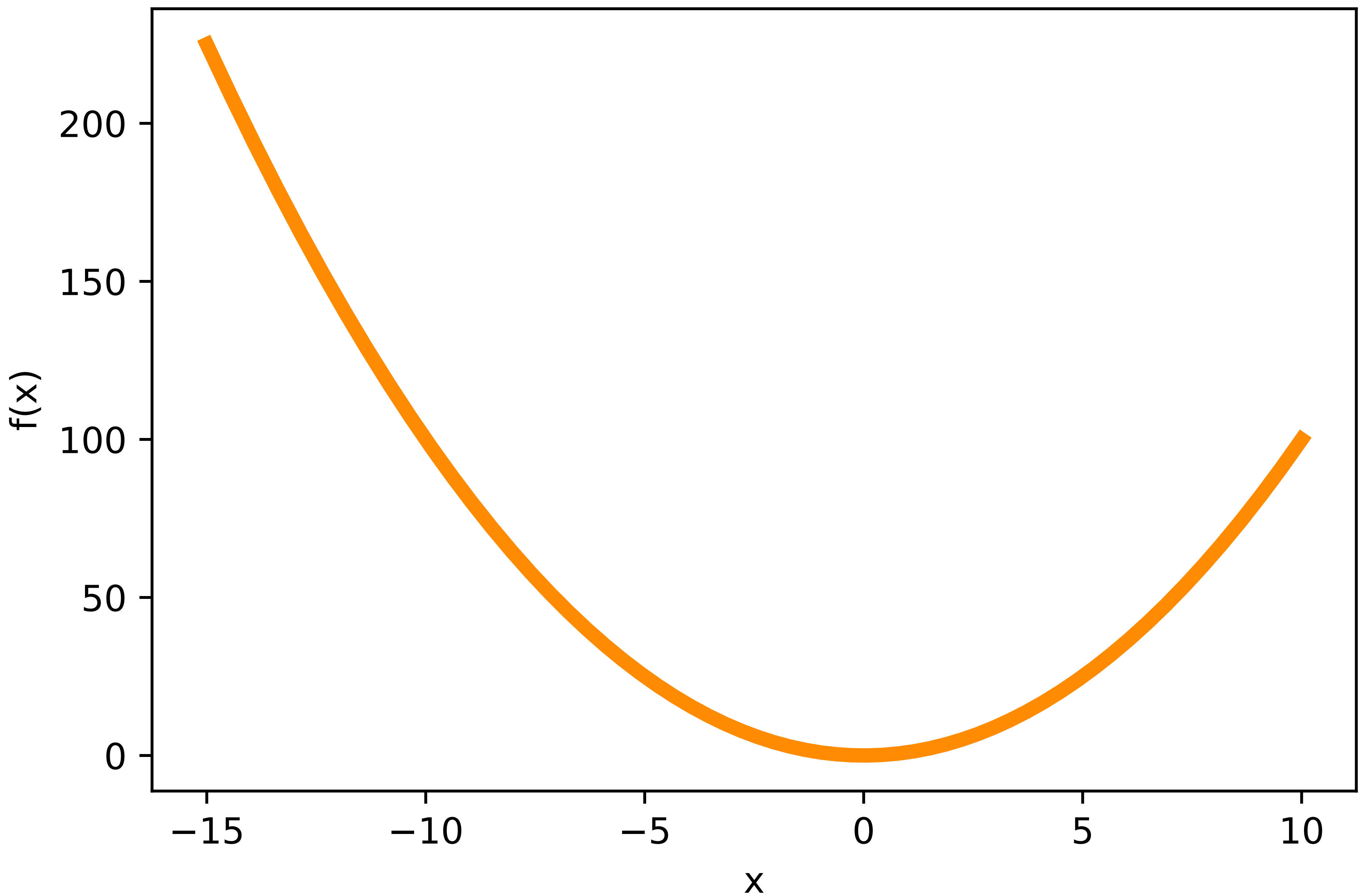
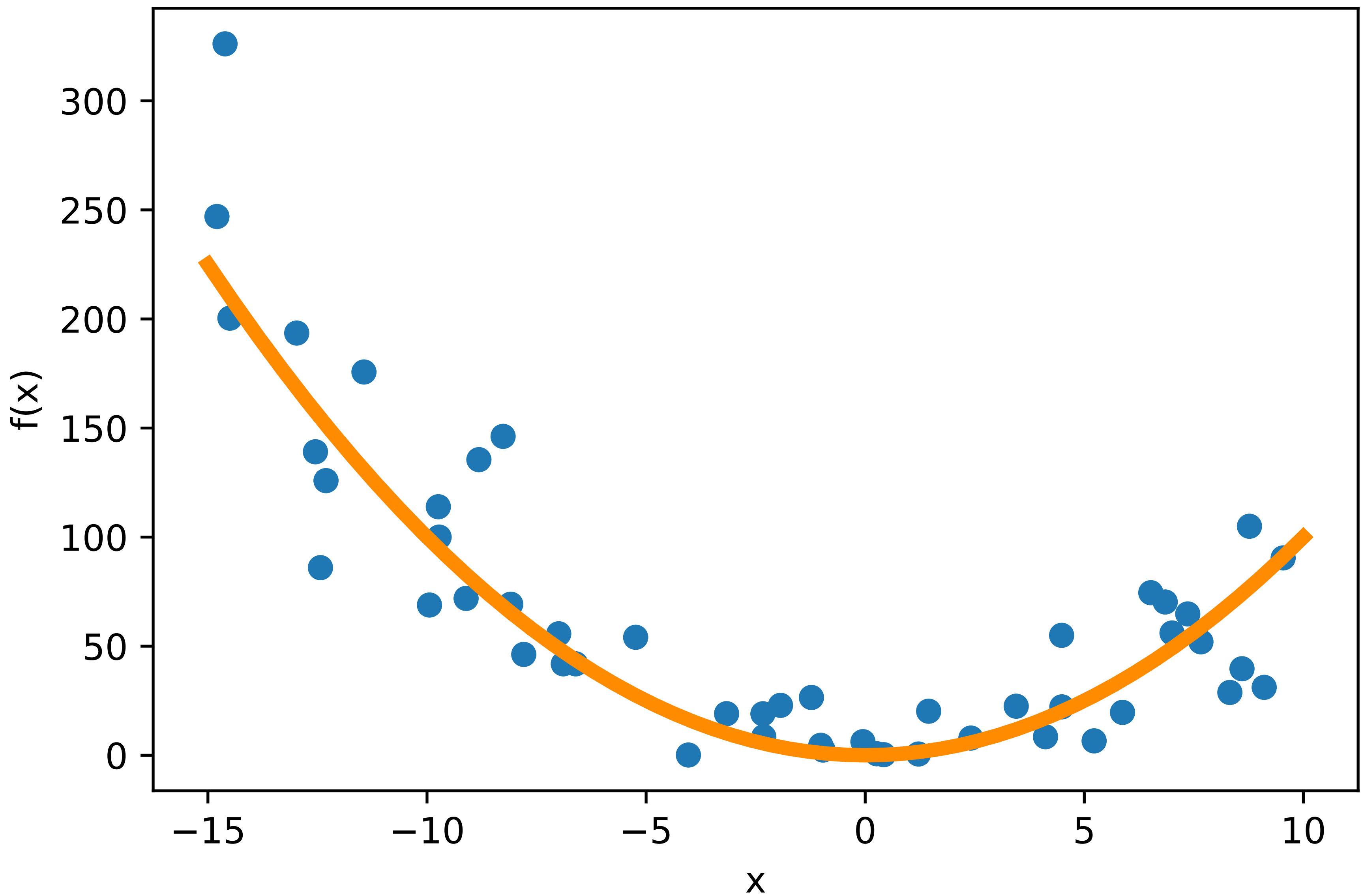


Bias and Variance Example



where f(x) is some true (target) function

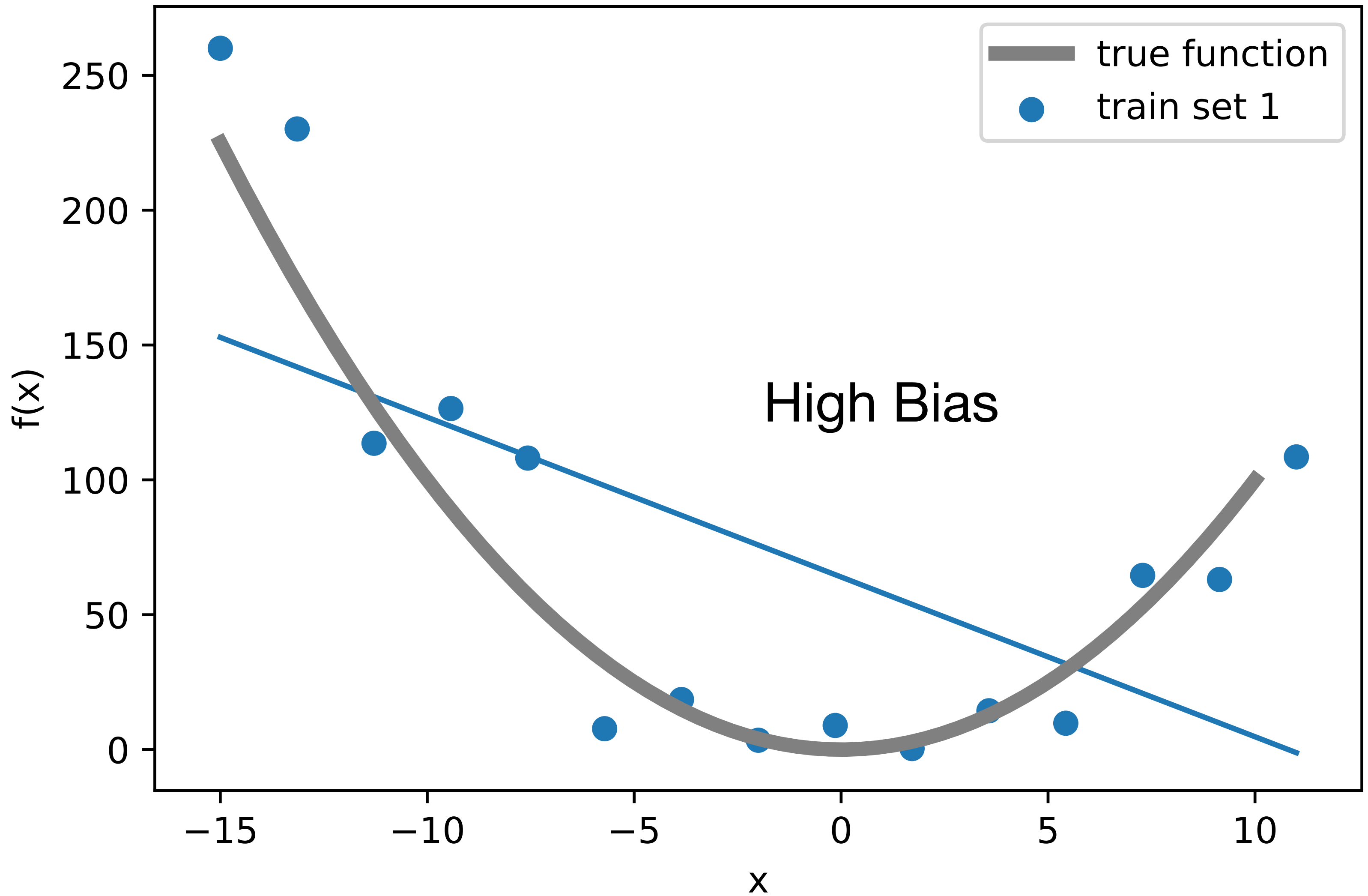
Bias and Variance Example



where f(x) is some true (target) function

the blue dots are a training dataset;
here, I added some random Gaussian noise

Bias and Variance Example

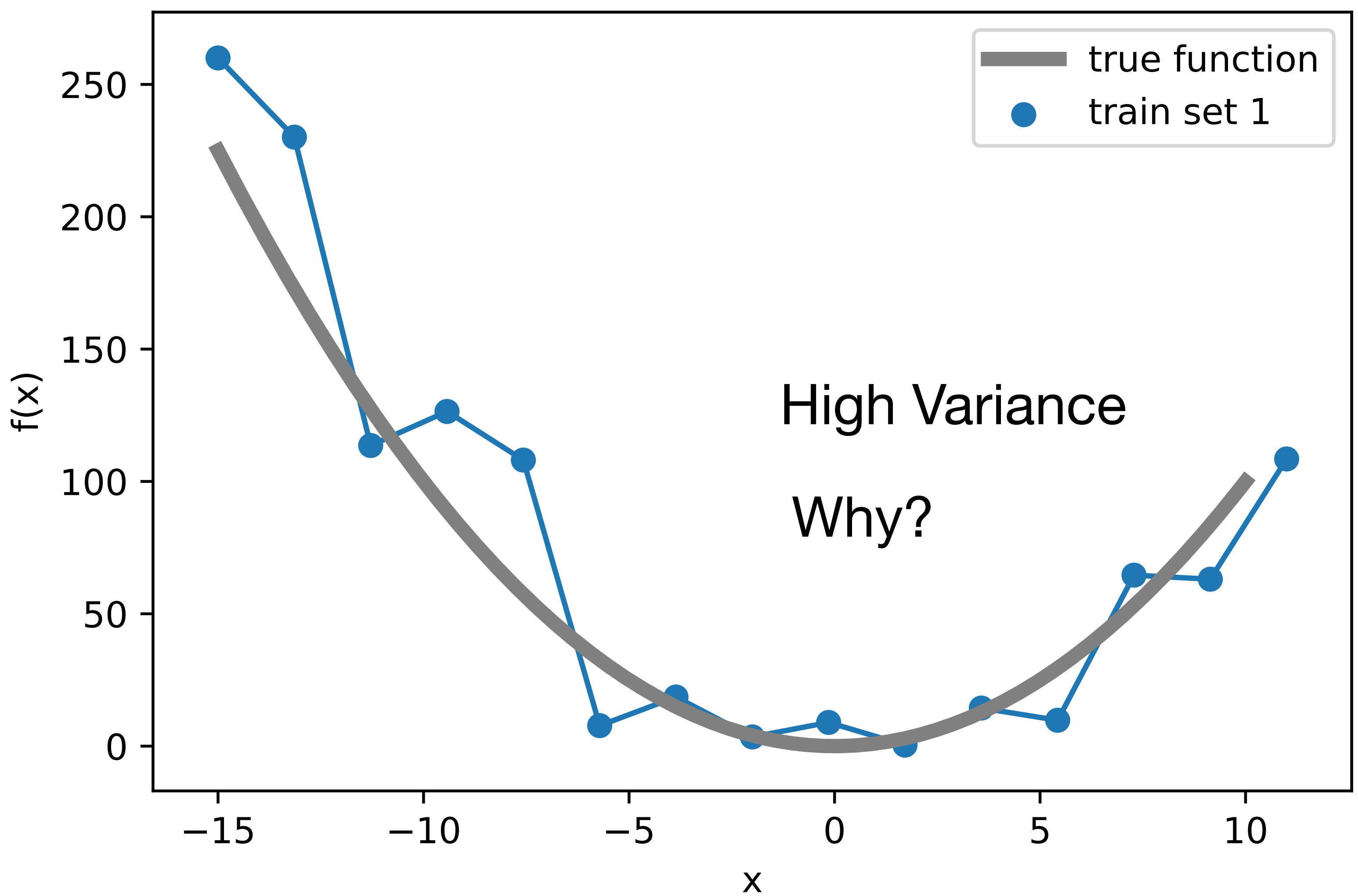


where f(x) is some true (target) function

the blue dots are a training dataset;
here, I added some random Gaussian noise

here, suppose I fit a simple linear model (linear regression)
or a decision tree stump

Bias and Variance Example

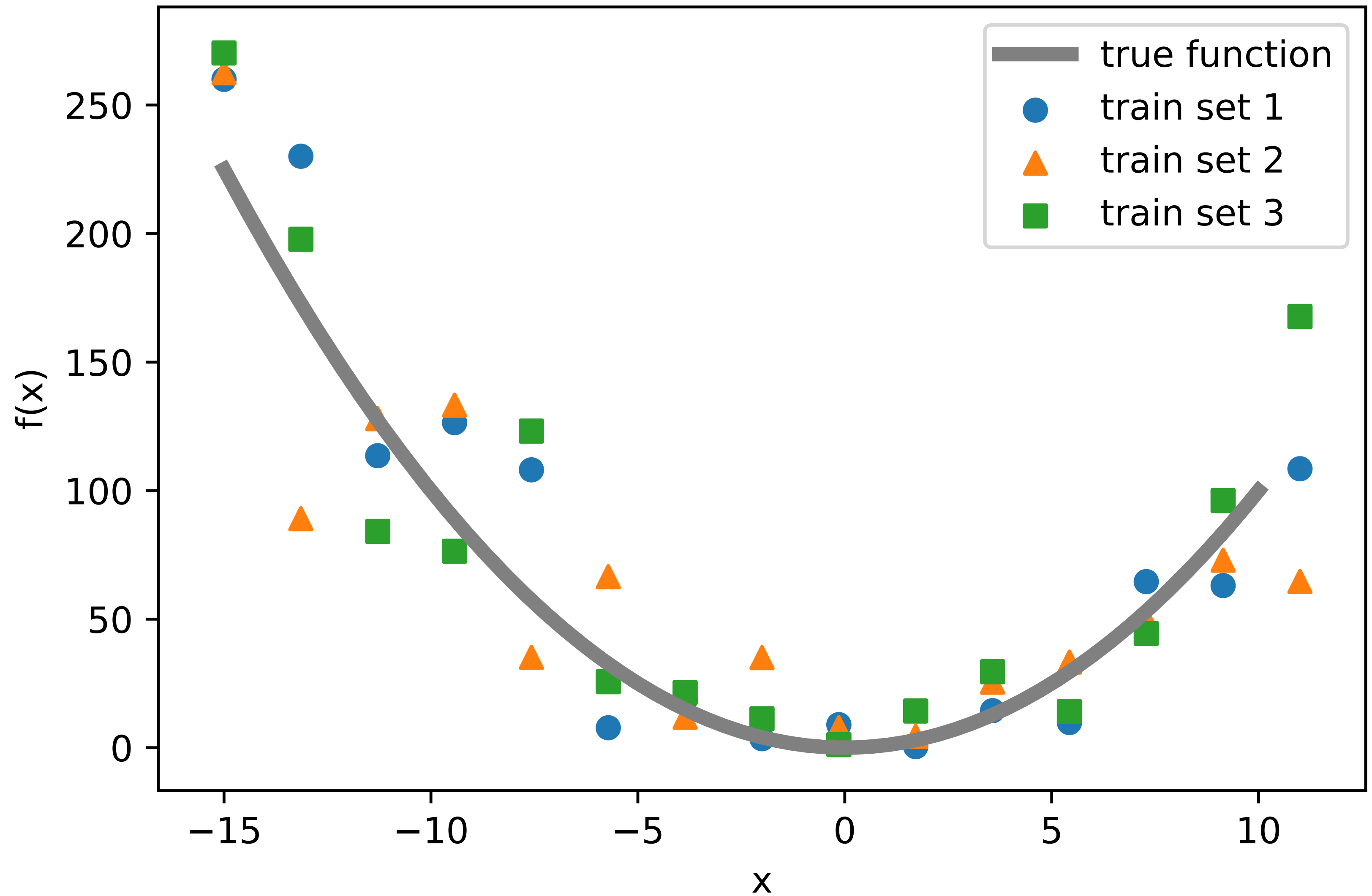


where f(x) is some true (target) function

the blue dots are a training dataset;
here, I added some random Gaussian noise

here, suppose I fit an unpruned decision tree

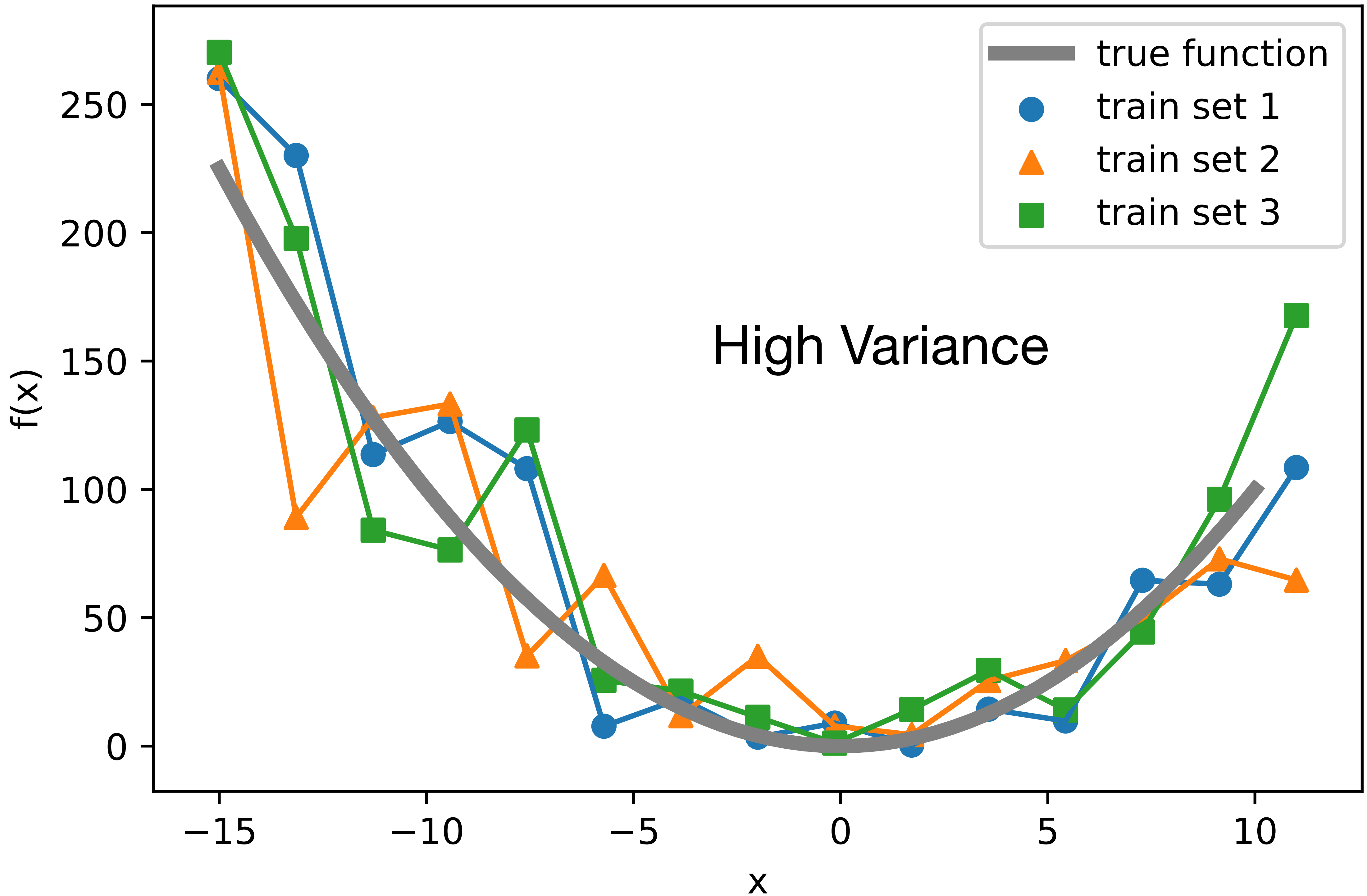
Bias and Variance Example



where f(x) is some true (target) function

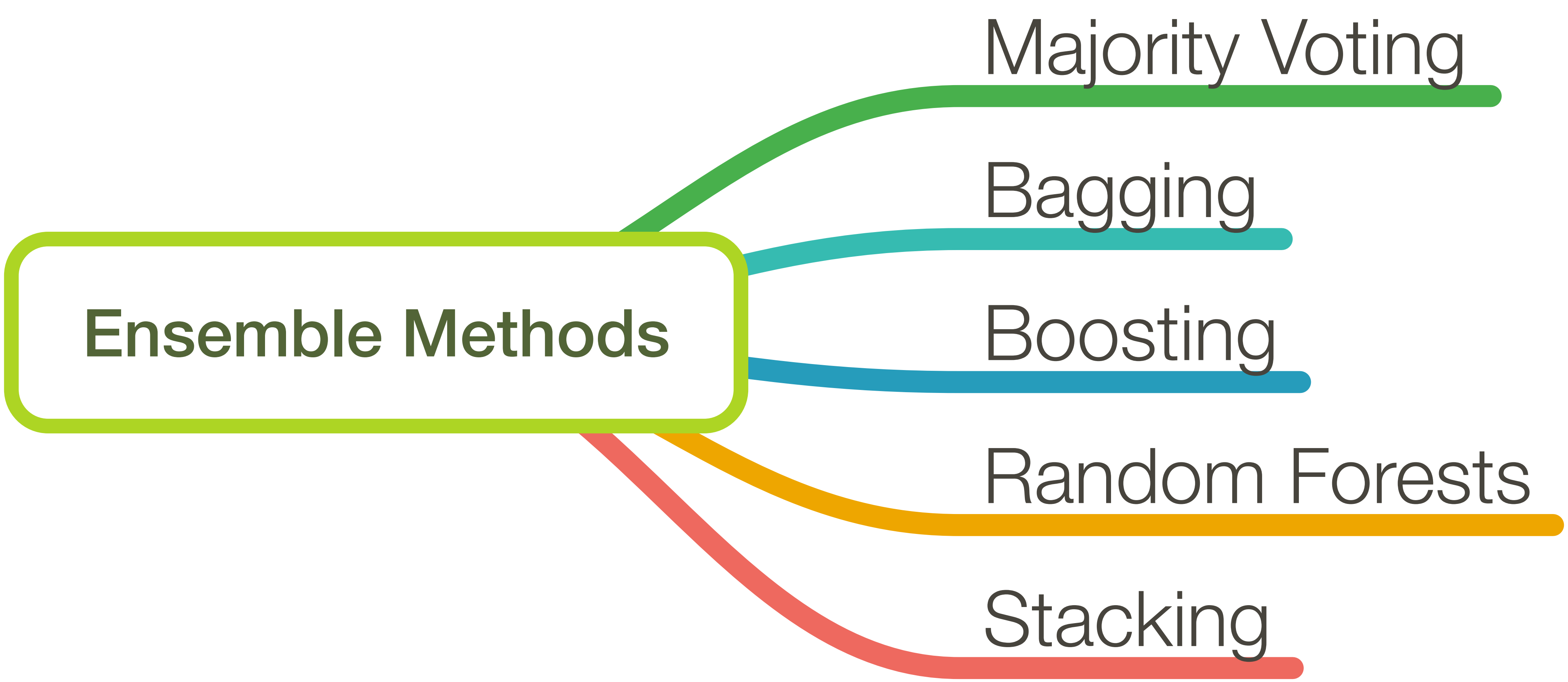
suppose we have multiple training sets

Bias and Variance Example



So, why does bagging work/what does it do?

Overview


Ensemble Methods
Majority Voting
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Boosting

Adaptive Boosting

e.g., AdaBoost

Freund, Y., & Schapire, R. E. (1997). A decision-theoretic generalization of on-line learning and an application to boosting. *Journal of computer and system sciences*, *55*(1), 119-139.

Gradient Boosting

e.g., LightGBM, XGBoost, scikit-learn's GradientBoostingClassifier

Friedman, J. H. (2001). Greedy function approximation: a gradient boosting machine. *Annals of statistics*, 1189-1232.

Chen, T., & Guestrin, C. (2016). Xgboost: A scalable tree boosting system. In *Proceedings of the 22nd ACM SIGKDD International conference on knowledge discovery and data mining* (pp. 785-794). ACM.

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Differ mainly in terms of how
- weights are updated
- classifiers are combined

Gradient Boosting

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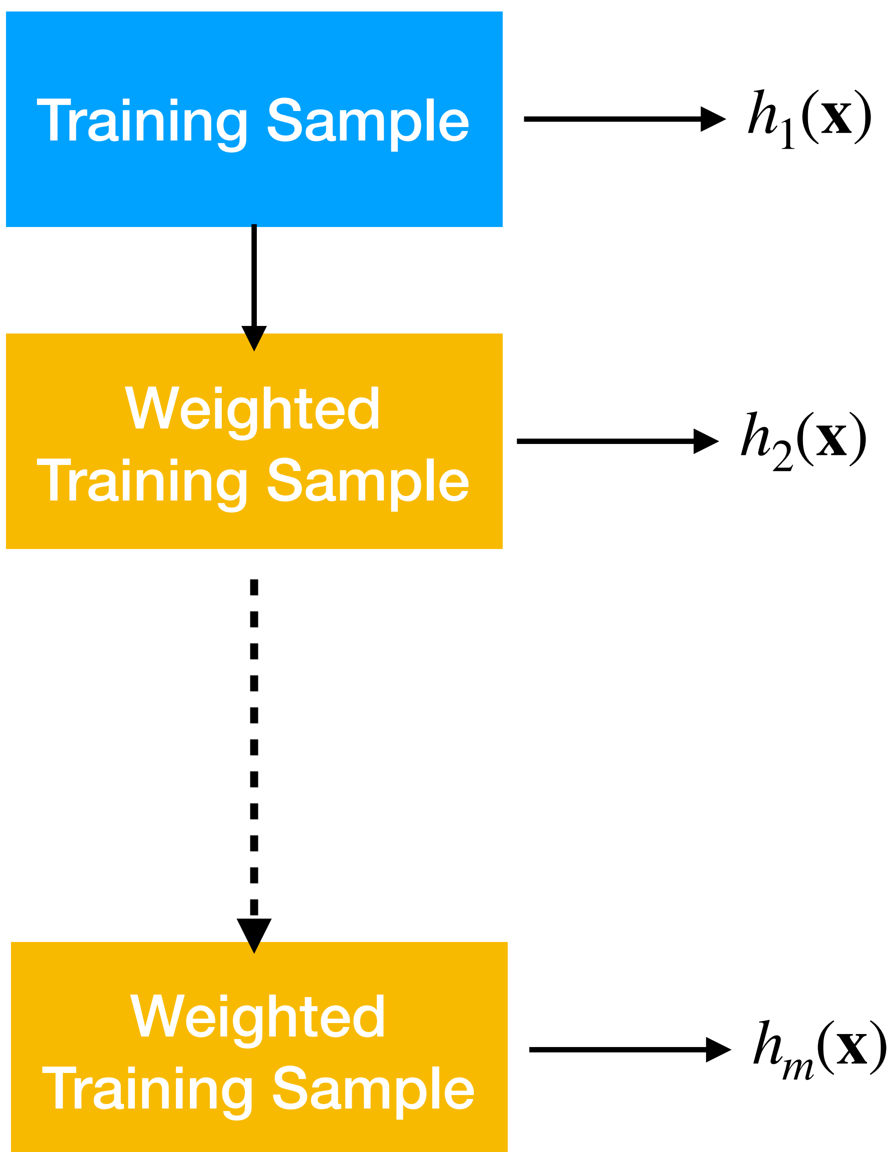
General Boosting



$$h_m(\mathbf{x}) = sign\left(\sum_{j=1}^{m} w_j \, h_j(\mathbf{x})\right) \quad \text{for} \quad h(\mathbf{x}) \in \{-1,1\}$$

or $$h_m(\mathbf{x}) = \arg\max_i \left(\sum_{j=1}^{m} w_j \, \mathbf{1}[h_j(\mathbf{x}) = i]\right) \quad \text{for} \quad h(\mathbf{x}) = i, \qquad i \in \{1,\ldots,n\}$$

General Boosting

- Initialize a weight vector with uniform weights

- Loop:
  - Apply weak learner* to weighted training examples (instead of orig. training set, may draw bootstrap samples with weighted probability)

  - Increase weight for misclassified examples

- (Weighted) majority voting on trained classifiers

* a learner slightly better than random guessing

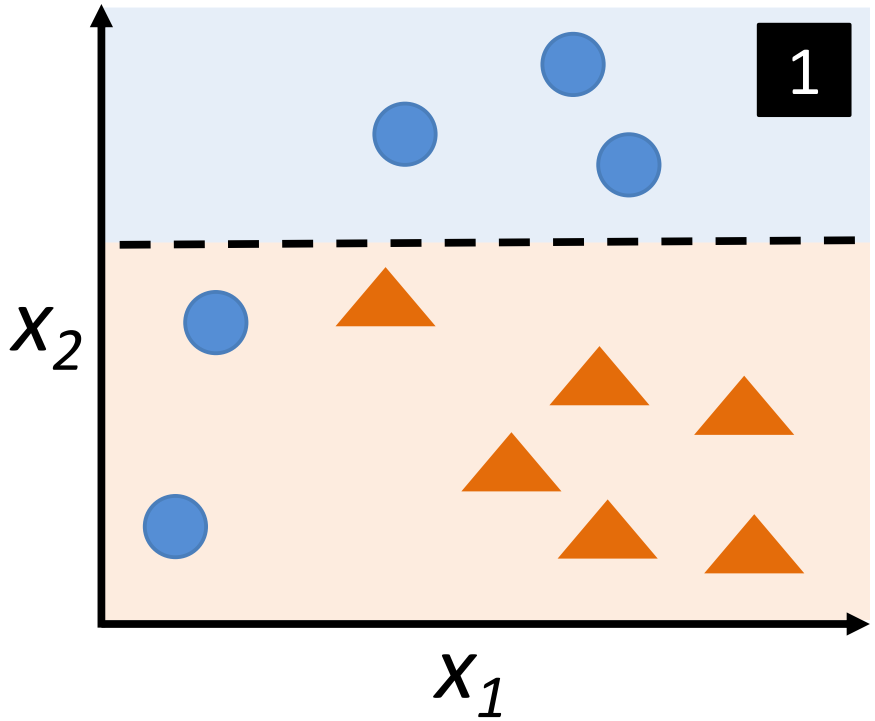
AdaBoost

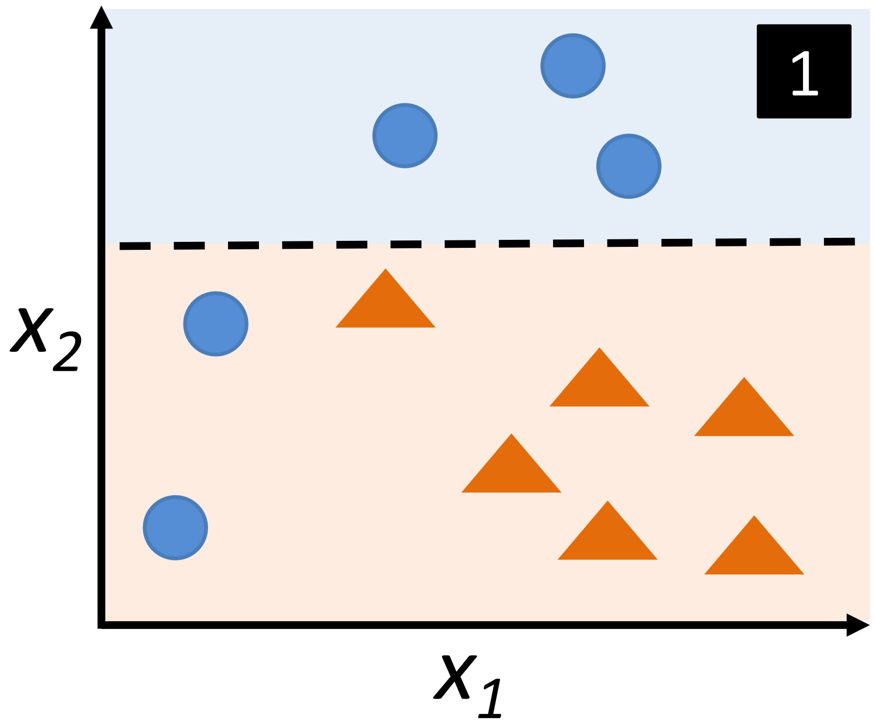
Algorithm 1 AdaBoost

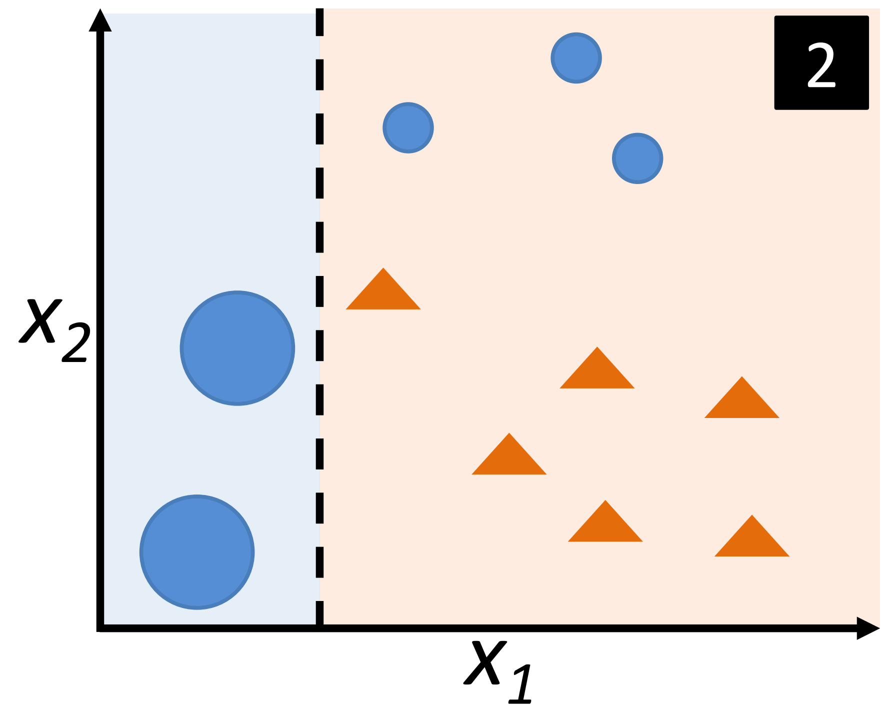
Initialize k: the number of AdaBoost rounds
Initialize $\mathcal{D}$: the training dataset, $\mathcal{D} = \{\langle \mathbf{x}^{[1]}, y^{[1]} \rangle, ..., \mathbf{x}^{[n]}, y^{[n]} \rangle\}$
Initialize $w_1(i) = 1/n, \quad i = 1, ..., n, \ \mathbf{w}_1 \in \mathbb{R}^n$

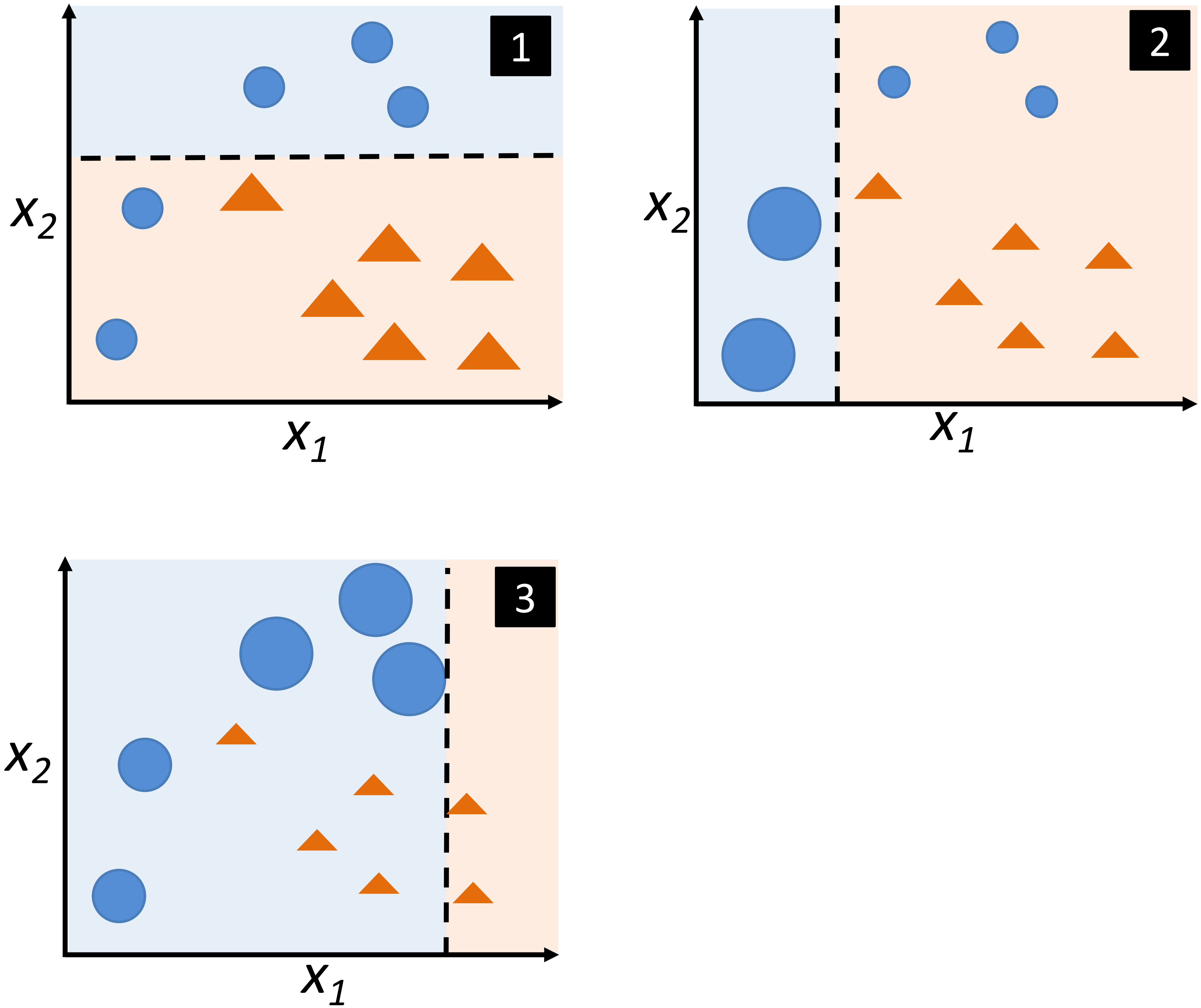
for r=1 to k **do**
    For all $i : \mathbf{w}_r(i) := w_r(i) / \sum_i w_r(i)$ [normalize weights]
    $h_r := FitWeakLearner(\mathcal{D}, \mathbf{w}_r)$
    $\epsilon_r := \sum_i w_r(i) \, \mathbf{1}(h_r(i) \neq y_i)$ [compute error]
    if $\epsilon_r > 1/2$ then stop
    $\alpha_r := \frac{1}{2} \log[(1 - \epsilon_r)/\epsilon_r]$ [small if error is large and vice versa]
    $w_{r+1}(i) := w_r(i) \times \begin{cases} \mathrm{e}^{-\alpha_r} & \text{if } h_r(\mathbf{x}^{[i]}) = y^{[i]} \\ \mathrm{e}^{\alpha_r} & \text{if } h_r(\mathbf{x}^{[i]}) \neq y^{[i]} \end{cases}$
Predict: $h_k(\mathbf{x}) = \arg\max_j \sum_r \alpha_r \mathbf{1}[h_r(\mathbf{x}) = j]$

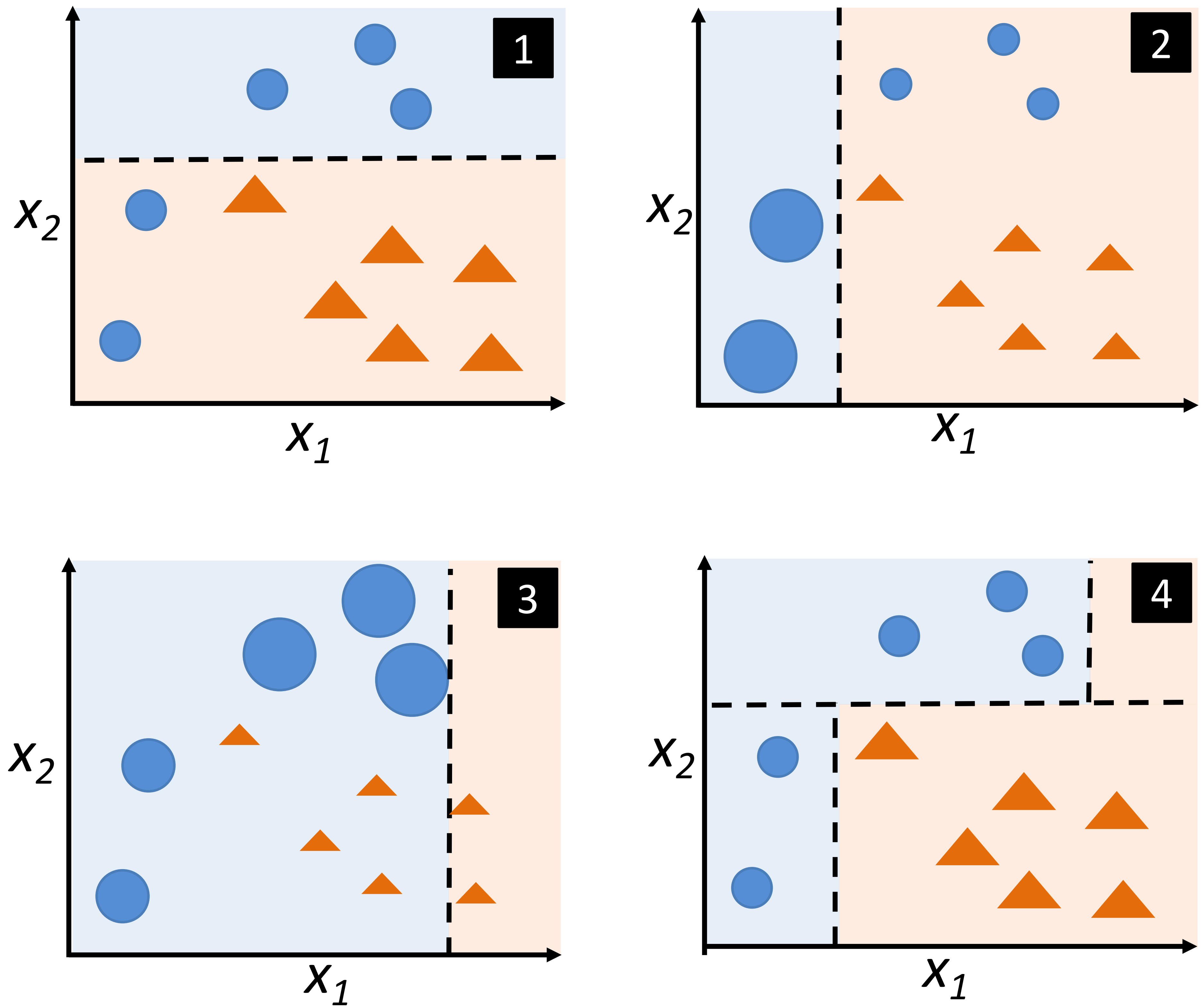
Decision Tree Stumps


1
x_2
x_1


1
x_2
x_1


2
x_2
x_1


1
x2
x1
2
x2
x1
3
x2
x1


1
2
3
4
x_1
x_2

Gradient Boosting

Gradient Boosting

Gradient boosting is somewhat similar to AdaBoost:

- trees are fit sequentially to improve error of previous trees
- boost weak learners to a strong learner

The way how the trees are fit sequentially differs in AdaBoost and Gradient Boosting, though ...

Gradient Boosting -- Conceptual Overview

- **Step 1:** Construct a base tree (just the root node)
- **Step 2:** Build next tree based on errors of the previous tree
- **Step 3:** Combine tree from step 1 with trees from step 2

Gradient Boosting -- Conceptual Overview --> A Regression-based Example

x1# Rooms	x2=City	x3=Age	y=Price
5	Boston	30	$1.5 x 10^6
10	Madison	20	$0.5 x 10^6
6	Lansing	20	$0.25 x 10^6
5	Waunakee	10	$0.1 x 10^6

- **<u>Step 1:</u>** Construct a base tree (just the root node)

$$\hat{y}_1 = \frac{1}{n}\sum_{i=1}^{n} y^{(i)} = 0.5875$$

Gradient Boosting -- Conceptual Overview --> A Regression-based Example

- **Step 2:** Build next tree based on errors of the previous tree

First, compute (pseudo) residuals: $r_1 = y - \hat{y}_1$

x1#	x2=City	x3=Age	y=Price	r=Res
5	Boston	30	\$1.5 x 10^6	1.5 - 0.5875 = 0.9125
10	Madison	20	\$0.5 x 10^6	0.5 - 0.5875 = -0.0875
6	Lansing	20	\$0.25 x 10^6	0.25 - 0.5875 = -0.3375
5	Waunake	10	\$0.1 x 10^6	0.1 - 0.5875 = -0.4875

Gradient Boosting -- Conceptual Overview --> A Regression-based Example

- **<u>Step 2:</u>** Build next tree based on errors of the previous tree

Then, create a tree based on x1, ..., x3 to fit the residuals

x1#	x2=City	x3=Age	y=Price	r=Res
5	Boston	30	$1.5 x 10^6	1.5 - 0.5875 = 0.9125
10	Madison	20	$0.5 x 10^6	0.5 - 0.5875 = -0.0875
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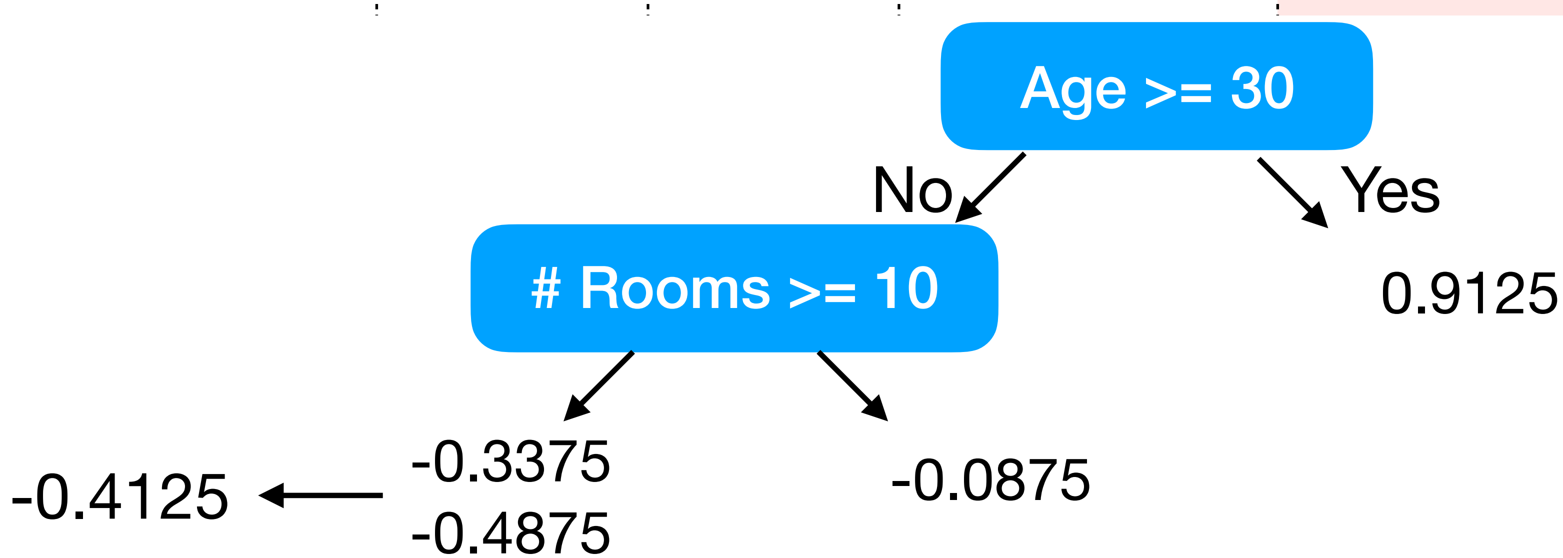

Gradient Boosting -- Conceptual Overview
--> A Regression-based Example

- **<u>Step 3:</u>** Combine tree from step 1 with trees from step 2

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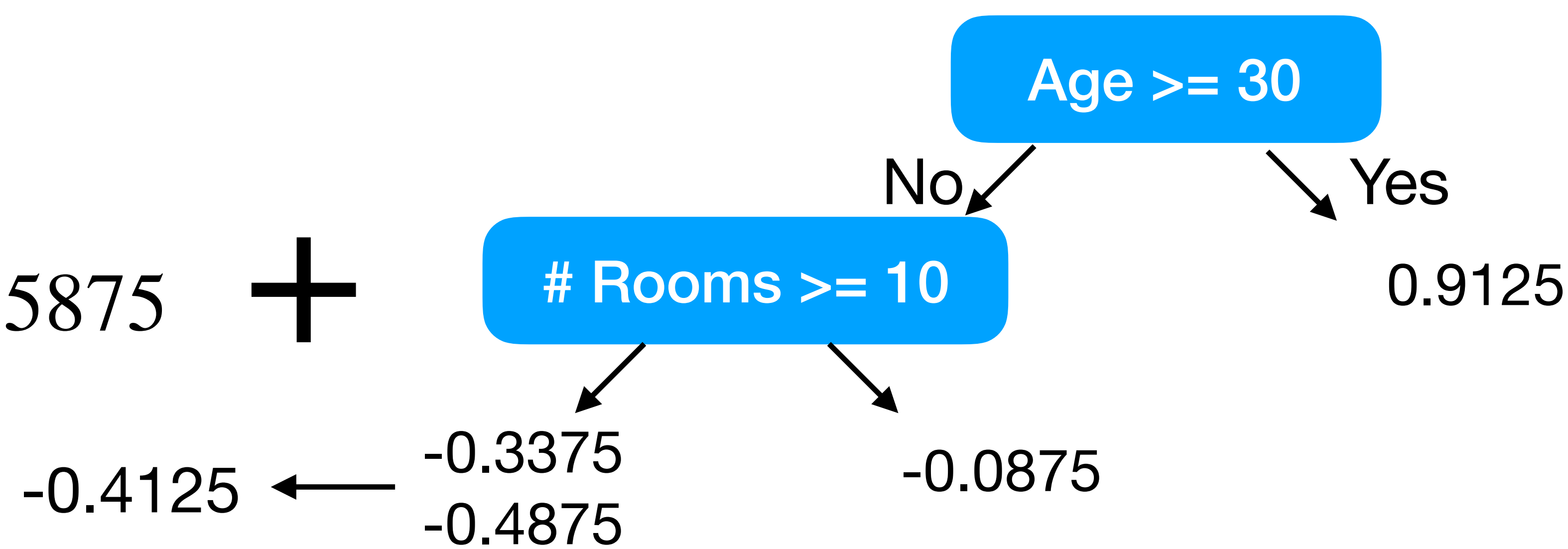
$$\hat{y}_1 = \frac{1}{n}\sum_{i=1}^{n} y^{(i)} = 0.5875 \quad +$$



Gradient Boosting -- Conceptual Overview
--> A Regression-based Example

- **<u>Step 3:</u>** Combine tree from step 1 with trees from step 2

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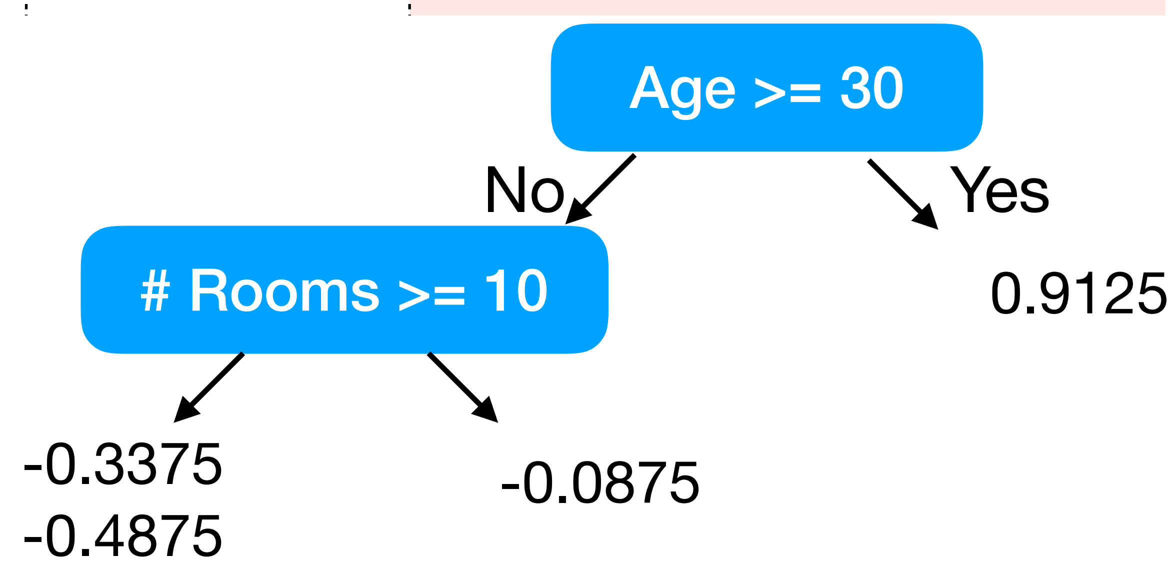
E.g., predict Lansing →

$$\hat{y}_1 = \frac{1}{n} \sum_{i=1}^{n} y^{(i)} = 0.5875 \quad +$$



-0.4125 ←

E.g., predict Lansing

$$0.5875 + \alpha \times (-0.4125)$$

where α learning rate between 0 and 1 (if $\alpha = 1$, low bias but high variance)

Gradient Boosting -- Algorithm Overview

Step 0: Input data $\{\langle \mathbf{x}^{(i)}, y^{(i)} \rangle\}_{i=1}^{n}$

Differentiable Loss function $L\big(y^{(i)}, h(\mathbf{x}^{(i)})\big)$

Step 1: Initialize model $h_0(\mathbf{x}) = \underset{\hat{y}}{\operatorname{argmin}} \sum_{i=1}^{n} L\big(y^{(i)}, \hat{y}\big)$

Step 2: for $t = 1$ to T

A. Compute pseudo residual $r_{i,t} = -\left[\dfrac{\partial L(y^{(i)}, h(\mathbf{x}^{(i)}))}{\partial h(\mathbf{x}^{(i)})}\right]_{h(\mathbf{x})=h_{t-1}(\mathbf{x})}$

for $i = 1$ to n

B. Fit tree to $r_{i,t}$ values, and create terminal nodes $R_{j,t}$ for $j = 1,\ldots,J_t$

...

Gradient Boosting -- Algorithm Overview

Step 2: for $t = 1$ to T

A. Compute pseudo residual $r_{i,t} = -\left[\frac{\partial L(y^{(i)}, h(\mathbf{x}^{(i)}))}{\partial h(\mathbf{x}^{(i)})}\right]_{h(\mathbf{x})=h_{t-1}(\mathbf{x})}$

for $i = 1$ to n

B. Fit tree to $r_{i,t}$ values, and create terminal nodes $R_{j,t}$ for $j = 1,\ldots,J_t$

C. for $j = 1,\ldots,J_t$, compute

$$\hat{y}_{j,t} = \underset{\hat{y}}{\operatorname{argmin}} \sum_{\mathbf{x}^{(i)} \in R_{i,j}} L\left(y^{(i)}, h_{t-1}(\mathbf{x}^{(i)}) + \hat{y}\right)$$

D. Update $h_t(\mathbf{x}) = h_{t-1}(\mathbf{x}) + \alpha \sum_{j=1}^{J_t} \hat{y}_{j,t} \, \mathbb{I}\left(\mathbf{x} \in R_{j,t}\right)$

Step 3: Return $h_t(\mathbf{x})$

Gradient Boosting -- Algorithm Overview Discussion

Step 0: Input data $\{\langle \mathbf{x}^{(i)}, y^{(i)} \rangle\}_{i=1}^{n}$

Differentiable Loss function $L\left(y^{(i)}, h(\mathbf{x}^{(i)})\right)$

E.g., Sum-squared error in regression

$$SSE' = \frac{1}{2}\left(y^{(i)} - h(\mathbf{x}^{(i)})\right)^2$$

$$\frac{\partial}{\partial h(\mathbf{x}^{(i)})} \frac{1}{2}\left(y^{(i)} - h(\mathbf{x}^{(i)})\right)^2 \quad \text{[chain rule]}$$

$$= 2 \times \frac{1}{2}\left(y^{(i)} - h(\mathbf{x}^{(i)})\right) \times (0 - 1) = -\left(y^{(i)} - h(\mathbf{x}^{(i)})\right)$$

[neg. residual]

Gradient Boosting -- Algorithm Overview Discussion

Step 1: Initialize model $h_0(\mathbf{x}) = \underset{\hat{y}}{\operatorname{argmin}} \sum_{i=1}^{n} L\left(y^{(i)}, \hat{y}\right)$

pred. target

turns out to be the average (in regression)

$$\frac{1}{n}\sum_{i=1}^{n} y^{(i)}$$

Gradient Boosting -- Algorithm Overview Discussion

Loop to make T trees (e.g., T=100)

Step 2: for $t = 1$ to T

A. Compute pseudo residual $r_{i,t} = -\left[\frac{\partial L(y^{(i)}, h(\mathbf{x}^{(i)}))}{\partial h(\mathbf{x}^{(i)})}\right]_{h(\mathbf{x})=h_{t-1}(\mathbf{x})}$

for $i = 1$ to n

pseudo residual of the t-th tree and i-th example

Derivative of the loss function

Gradient Boosting -- Algorithm Overview Discussion

Loop to make T trees (e.g., T=100)

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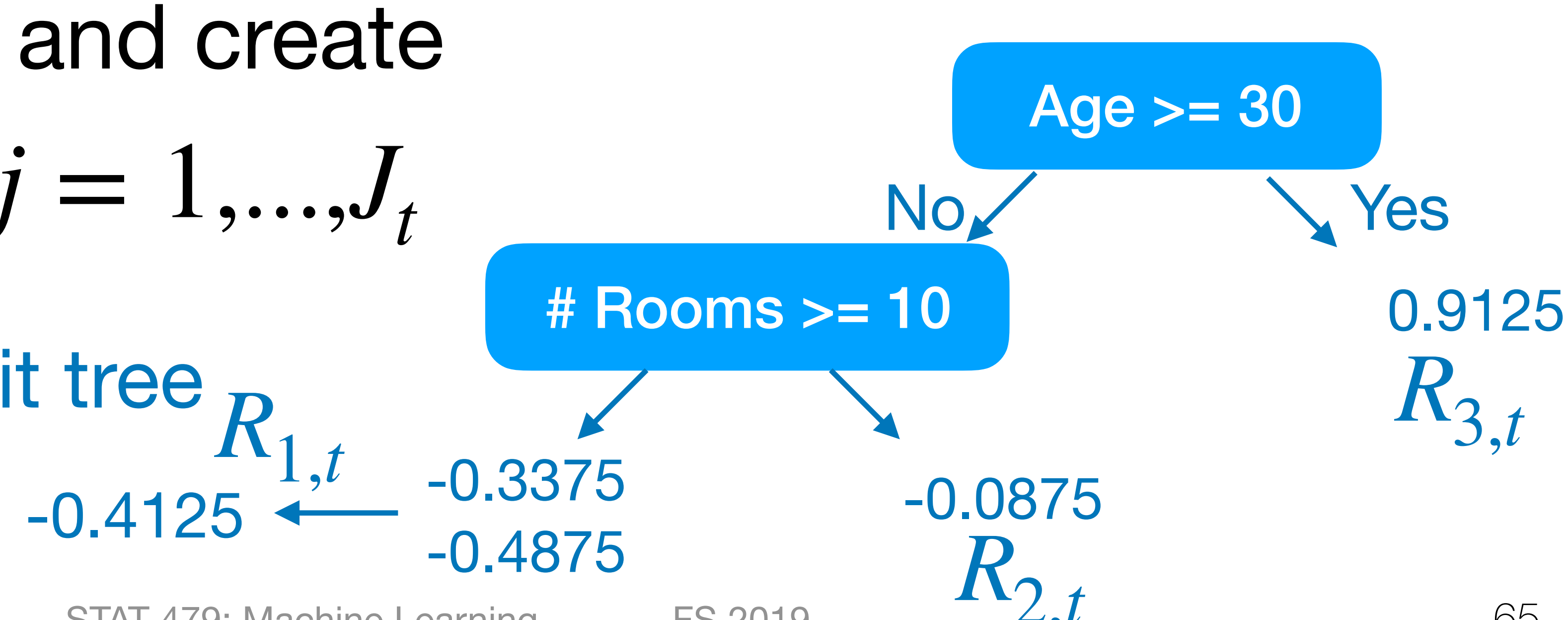
for $i = 1$ to n

pseudo residual of the t-th tree and i-th example

Derivative of the loss function

B. Fit tree to $r_{i,t}$ values, and create terminal nodes $R_{j,t}$ for $j = 1,\ldots,J_t$

Use features in dataset to fit tree



Gradient Boosting -- Algorithm Overview Discussion

Step 2: for $t = 1$ to T

A. Compute pseudo residual $r_{i,t} = -\left[\frac{\partial L(y^{(i)}, h(\mathbf{x}^{(i)}))}{\partial h(\mathbf{x}^{(i)})}\right]_{h(\mathbf{x})=h_{t-1}(\mathbf{x})}$

for $i = 1$ to n

B. Fit tree to $r_{i,t}$ values, and create terminal nodes $R_{j,t}$ for $j = 1,\ldots,J_t$

C. for $j = 1,\ldots,J_t$, compute

$$\hat{y}_{j,t} = \underset{\hat{y}}{\operatorname{argmin}} \sum_{\mathbf{x}^{(i)} \in R_{i,j}} L\left(y^{(i)}, h_{t-1}(\mathbf{x}^{(i)}) + \hat{y}\right)$$

Compute the residual for each leaf node

Only consider examples at that leaf node

Like step 1 but add previous prediction

Gradient Boosting -- Algorithm Overview Discussion

Step 2: for $t = 1$ to T

A. Compute pseudo residual $r_{i,t} = -\left[\dfrac{\partial L(y^{(i)}, h(\mathbf{x}^{(i)}))}{\partial h(\mathbf{x}^{(i)})}\right]_{h(\mathbf{x})=h_{t-1}(\mathbf{x})}$

for $i = 1$ to n

B. Fit tree to $r_{i,t}$ values, and create terminal nodes $R_{j,t}$ for $j = 1,\ldots,J_t$

C. for $j = 1,\ldots,J_t$, compute

$$\hat{y}_{j,t} = \underset{\hat{y}}{\operatorname{argmin}} \sum_{\mathbf{x}^{(i)} \in R_{i,j}} L\left(y^{(i)}, h_{t-1}(\mathbf{x}^{(i)}) + \hat{y}\right)$$

D. Update $h_t(\mathbf{x}) = h_{t-1}(\mathbf{x}) + \alpha \sum_{j=1}^{J_t} \hat{y}_{j,t} \, \mathbb{I}\left(\mathbf{x} \in R_{j,t}\right)$

learning rate between 0 and 1 (usually 0.1)

Summation just in case examples end up in multiple nodes

Gradient Boosting -- Algorithm Overview Discussion

For prediction, combine all *T* trees, e.g.,

$$h_0(\mathbf{x}) = \underset{\hat{y}}{\operatorname{argmin}} \sum_{i=1}^{n} L\big(y^{(i)}, \hat{y}\big)$$

$$+\alpha \; \hat{y}_{j,t=1} \quad = \underset{\hat{y}}{\operatorname{argmin}} \sum_{\mathbf{x}^{(i)} \in R_{i,j}} L\big(y^{(i)}, h_{(t=1)-1}(\mathbf{x}^{(i)}) + \hat{y}\big)$$

...

$$+\alpha \; \hat{y}_{j,T} \quad = \underset{\hat{y}}{\operatorname{argmin}} \sum_{\mathbf{x}^{(i)} \in R_{i,j}} L\big(y^{(i)}, h_{T-1}(\mathbf{x}^{(i)}) + \hat{y}\big)$$

Gradient Boosting -- Algorithm Overview Discussion

For prediction, combine all *T* trees, e.g.,

$$h_0(\mathbf{x}) = \underset{\hat{y}}{\operatorname{argmin}} \sum_{i=1}^{n} L\left(y^{(i)}, \hat{y}\right)$$

$$+\alpha \ \hat{y}_{j,t=1}$$

The idea is that we decrease the pseudo residuals by a small amount at each step

$$\ldots$$

$$+\alpha \ \hat{y}_{j,T}$$

Gradient Boosting -- Classification

Replace "average" of h_0 by *log(odds)*

probability of an event

$$odds = \frac{p}{1-p}$$

$$p = \frac{odds}{1 + odds} = \frac{e^{log(odds)}}{1 + e^{log(odds)}}$$

Gradient Boosting -- Classification

Replace "average" of h_0 by *log(odds)*

Pseudo residual becomes $(y - p)$

Prediction (*log(odds)* & *residual*) are computed via the following transform:

$$\frac{\sum_i residual^{(i)}}{\sum_i p_{t-1}^{(i)} \times (1 - p_{t-1}^{(i)})}$$

Gradient Boosting -- Classification

Replace "average" of h_0 by *log(odds)*

Pseudo residual becomes $(y - p)$

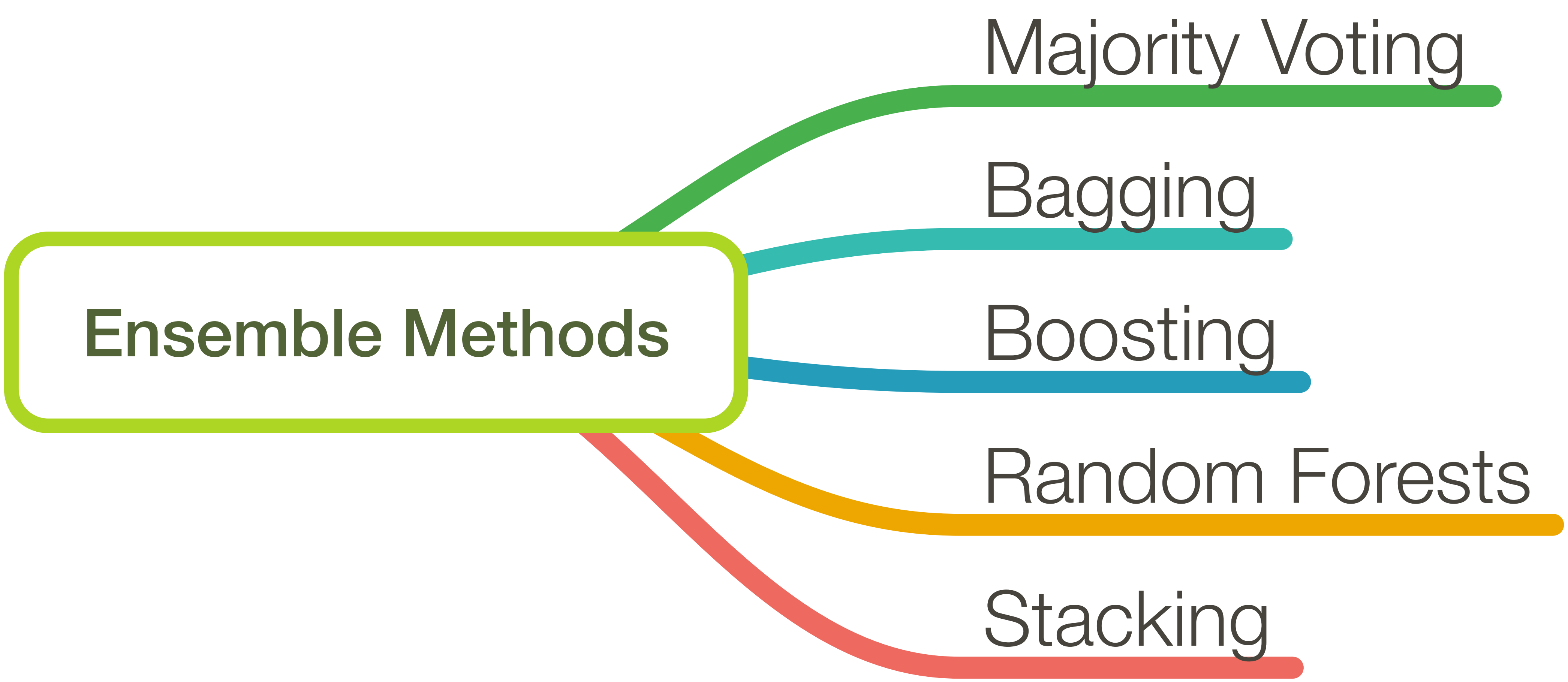
Prediction (*log(odds)* & *residual*) are computed via the following transform:

$$\frac{\sum_i residual^{(i)}}{\sum_i p_{t-1}^{(i)} \times (1 - p_{t-1}^{(i)})}$$

Loss function becomes neg. log likelihood

$$L(y^{(i)}, h(\mathbf{x})) = -\sum_{i=1}^{n} y^{(i)} \times \log(p) + (1 - y^{(i)}) \times \log(1 - p)$$

Overview


Ensemble Methods
Majority Voting
Bagging
Boosting
Random Forests
Stacking

Random Forests

Random Forests

= Bagging w. trees + random feature subsets

Random Feature Subset for each Tree or Node?

Tin Kam Ho used the "**random subspace method,**" where each tree got a random subset of features.

"Our method relies on an autonomous, pseudo-random procedure to select a small number of dimensions from a given feature space …"

- Ho, Tin Kam. "The random subspace method for constructing decision forests." IEEE transactions on pattern analysis and machine intelligence 20.8 (1998): 832-844.

"Trademark" random forest:

"… random forest with random features is formed by selecting at random, at each node, a small group of input variables to split on."

- **Breiman**, Leo. "Random Forests" Machine learning 45.1 (2001): 5-32.

Random Feature Subset for each Tree or Node?

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"Trademark" random forest:

"… random forest with random feature[…] random, at each node, a small group of input variab[…]

- Breiman, Leo. "Random Forests" Ma[…]2.

num features $= \log_2 m + 1$

where m is the number of input features

In contrast to the original publication
[Breiman, “Random Forests”, Machine Learning, 45(1), 5-32, 2001]
the scikit-learn implementation combines classifiers by averaging their probabilistic prediction, instead of letting each classifier vote for a single class.

"Soft Voting"

Will discuss Random Forests and feature importance in *Feature Selection* lecture

(Loose) Upper Bound for the Generalization Error

Breiman, “Random Forests”, Machine Learning, 45(1), 5-32, 2001

$$\textbf{PE} \leq \frac{\bar{\rho} \cdot (1 - s^2)}{s^2}$$

$\bar{\rho}$: Average correlation among trees

s : "Strength" of the ensemble

Extremely Randomized Trees (ExtraTrees)

Geurts, P., Ernst, D., & Wehenkel, L. (2006). Extremely randomized trees. *Machine learning*, *63*(1), 3-42.

Random Forest random components:

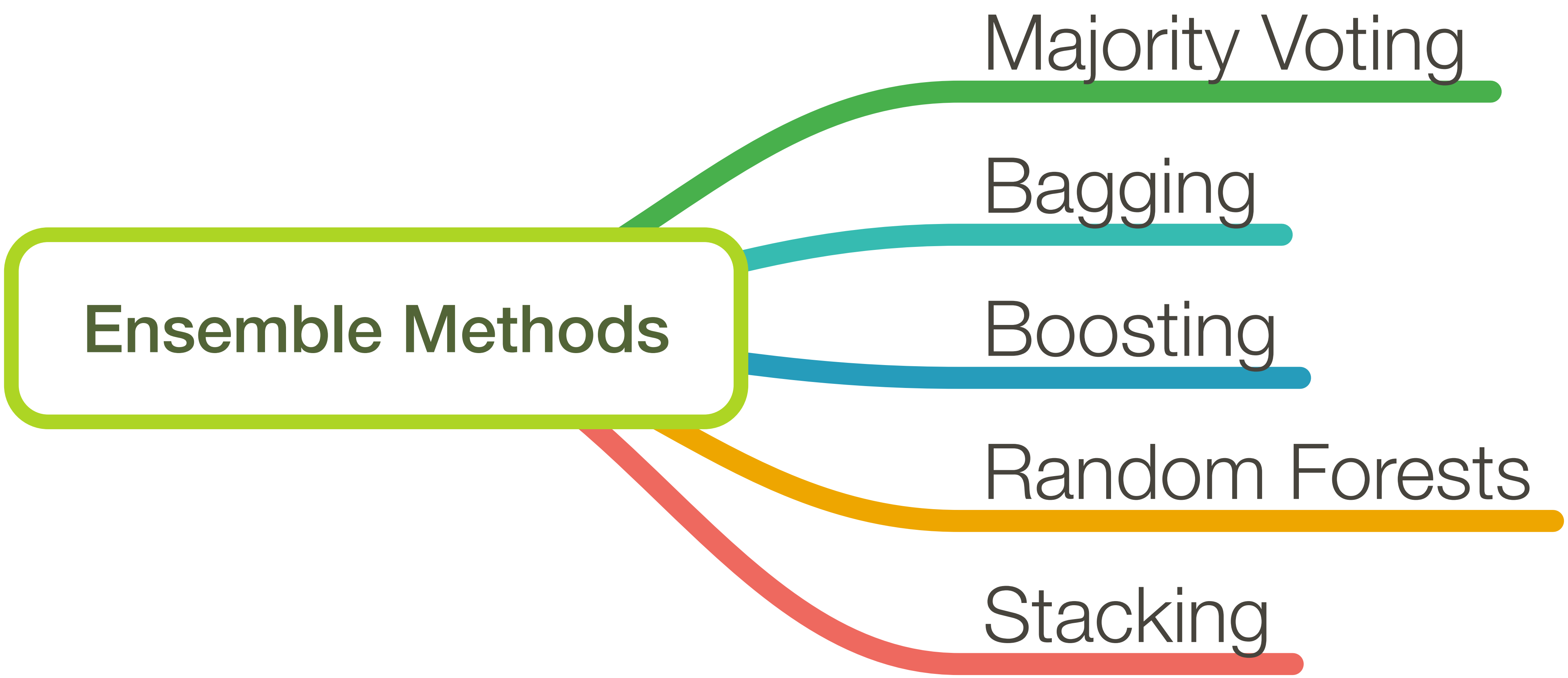
1) ______________

2) ______________

ExtraTrees algorithm adds one more random component

3) ______________

Overview


Ensemble Methods
Majority Voting
Bagging
Boosting
Random Forests
Stacking

Stacking

Stacking Algorithm

Wolpert, David H. "Stacked generalization." Neural networks 5.2 (1992): 241-259.

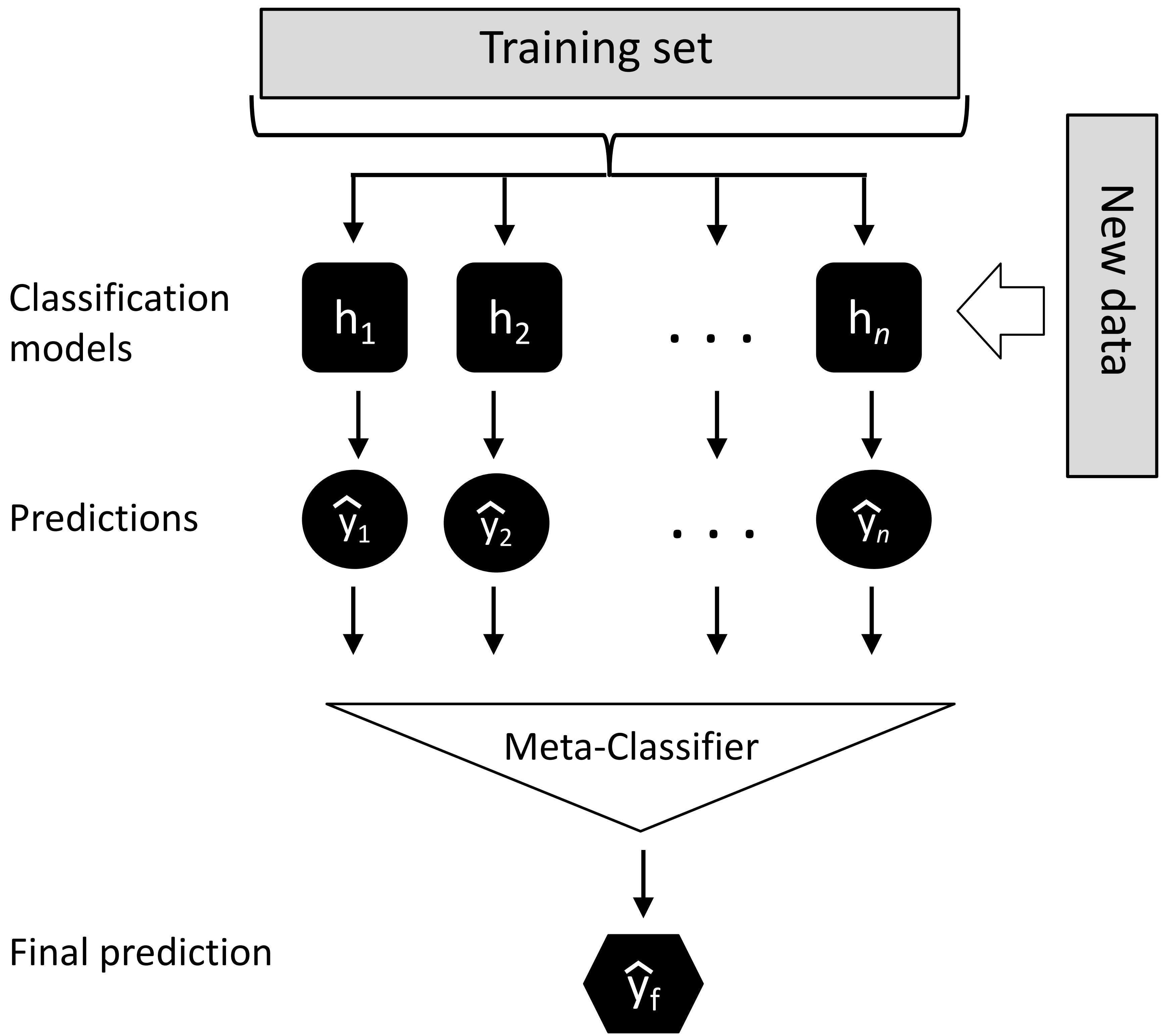
Algorithm 19.7 Stacking

Input: Training data $\mathcal{D} = \{\mathbf{x}_i, y_i\}_{i=1}^{m}$ ($\mathbf{x}_i \in \mathbb{R}^n$, $y_i \in \mathcal{Y}$)
Output: An ensemble classifier H

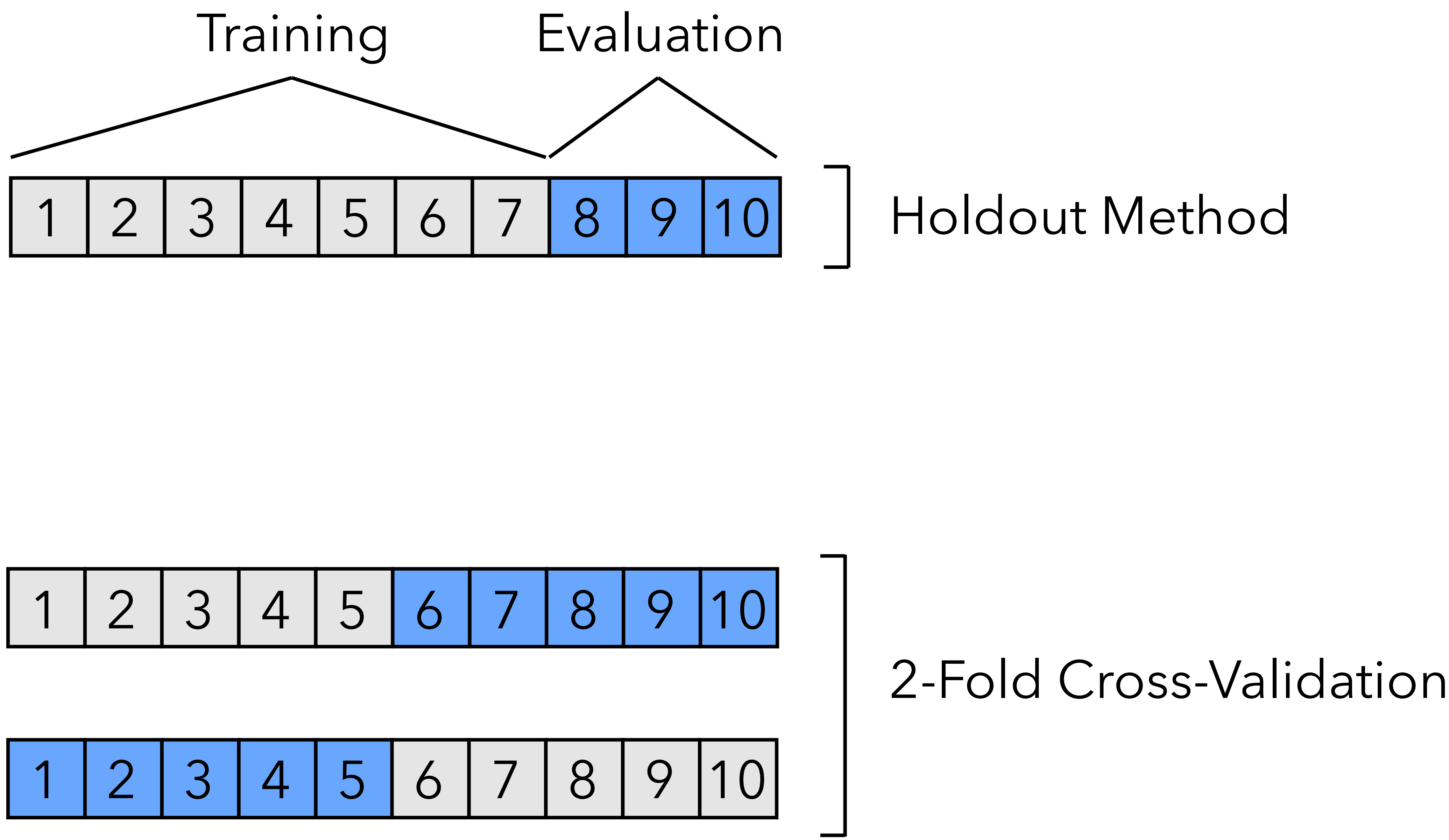
Step 1: Learn first-level classifiers
for $t \leftarrow 1$ to T **do**
    Learn a base classifier h_t based on $\mathcal{D}$
end for
Step 2: Construct new data sets from $\mathcal{D}$
for $i \leftarrow 1$ to m **do**
    Construct a new data set that contains $\{\mathbf{x}'_i, y_i\}$, where $\mathbf{x}'_i = \{h_1(\mathbf{x}_i), h_2(\mathbf{x}_i), \ldots, h_T(\mathbf{x}_i)\}$
end for
Step 3: Learn a second-level classifier
Learn a new classifier h' based on the newly constructed data set
return $H(\mathbf{x}) = h'(h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_T(\mathbf{x}))$

Tang, J., S. Alelyani, and H. Liu. "Data Classification: Algorithms and Applications." Data Mining and Knowledge Discovery Series, CRC Press (2015): pp. 498-500.

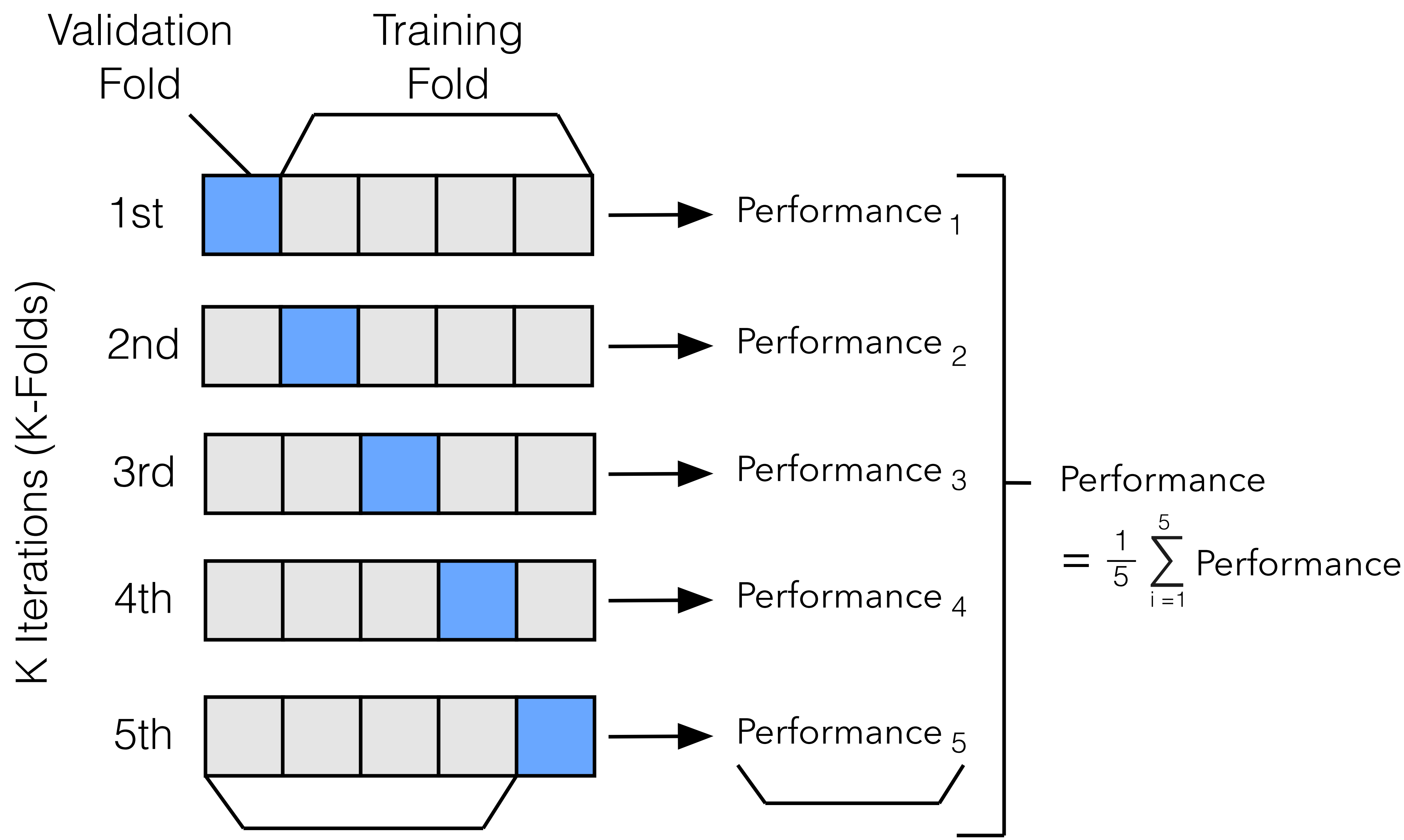
Stacking Algorithm

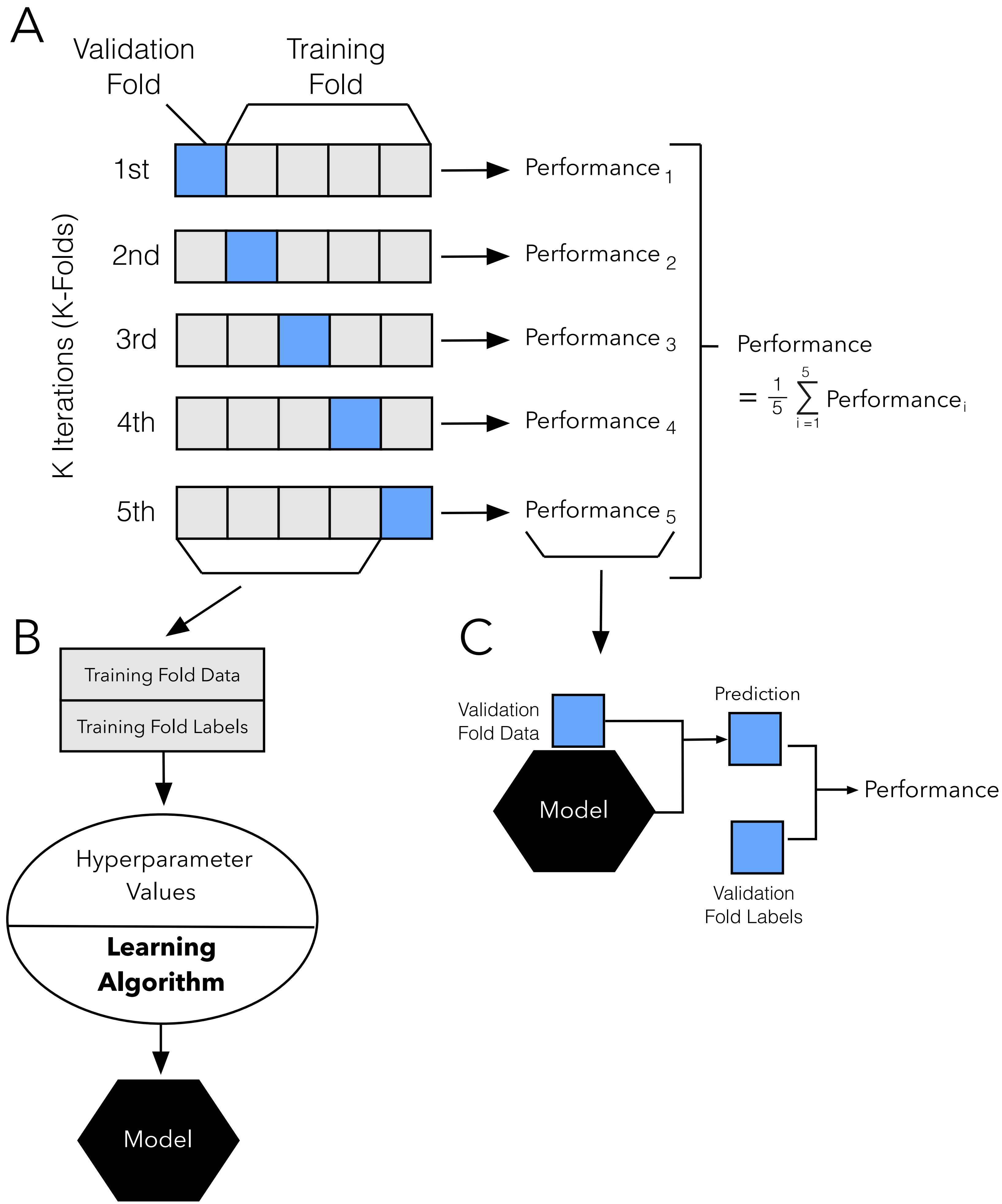


Cross-Validation



k-fold Cross-Validation




A
Validation Fold
Training Fold
K Iterations (K-Folds)
1st
2nd
3rd
4th
5th
Performance 1
Performance 2
Performance 3
Performance 4
Performance 5
Performance
$= \frac{1}{5} \sum_{i=1}^{5} \text{Performance}_i$
B
Training Fold Data
Training Fold Labels
Hyperparameter Values
Learning Algorithm
Model
C
Validation Fold Data
Model
Prediction
Performance
Validation Fold Labels

Stacking Algorithm with Cross-Validation

Wolpert, David H. "Stacked generalization." Neural networks 5.2 (1992): 241-259.

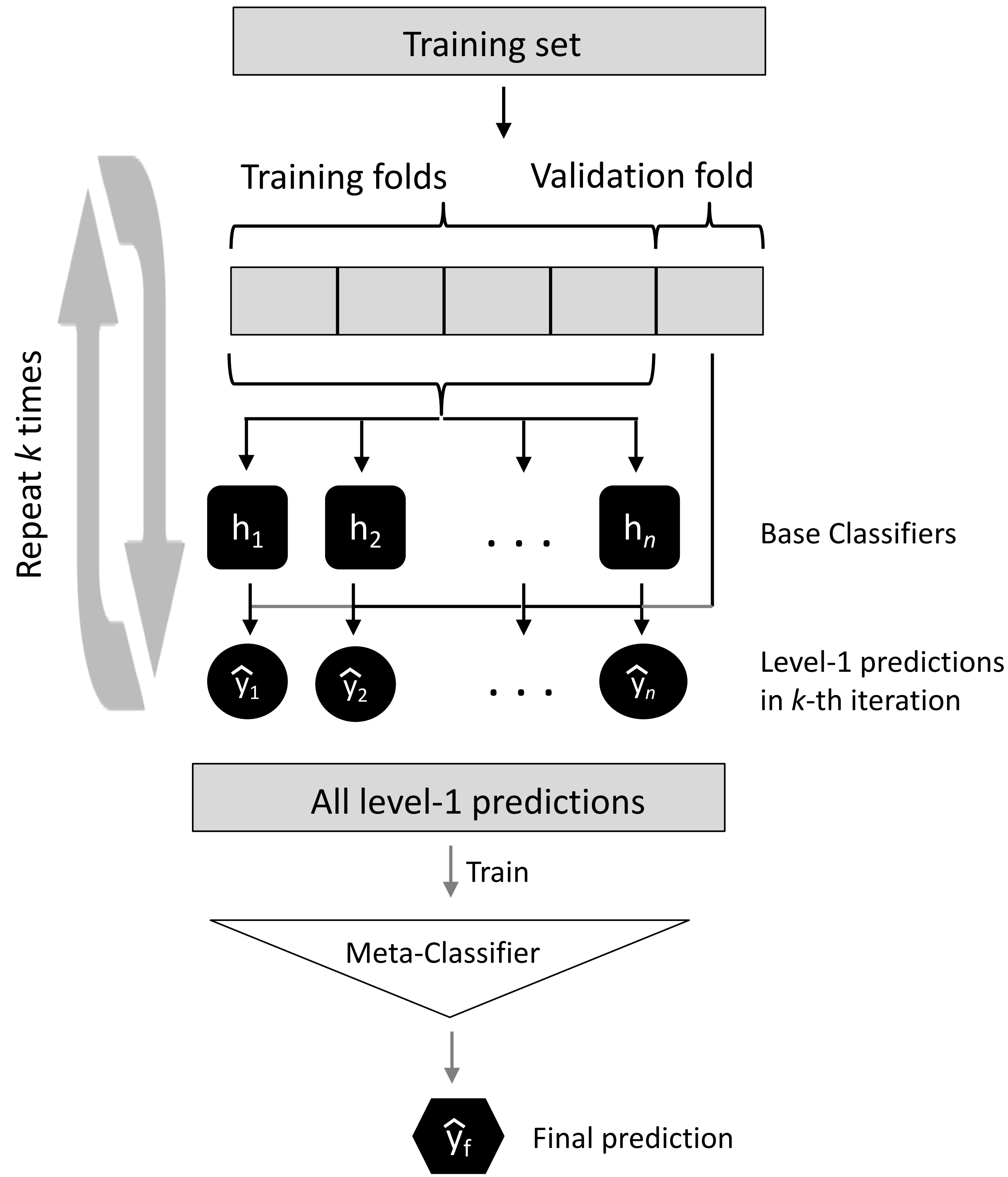
Algorithm 19.8 Stacking with K-fold Cross Validation

Input: Training data $\mathcal{D} = \{\mathbf{x}_i, y_i\}_{i=1}^m$ ($\mathbf{x}_i \in \mathbb{R}^n$, $y_i \in \mathcal{Y}$)
Output: An ensemble classifier H

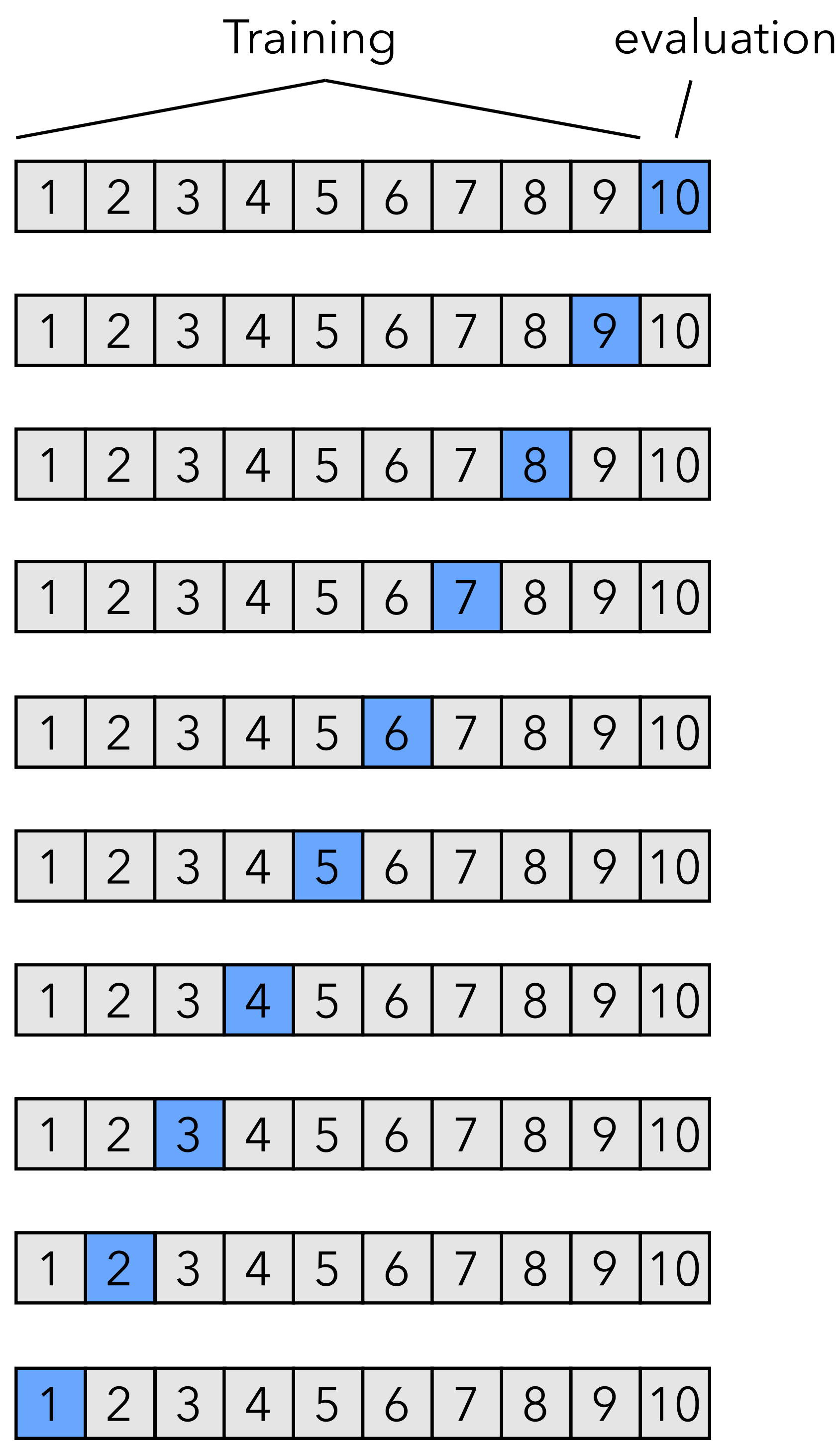
Step 1: Adopt cross validation approach in preparing a training set for second-level classifier
Randomly split $\mathcal{D}$ into K equal-size subsets: $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_K\}$
for $k \leftarrow 1$ to K **do**
    Step 1.1: Learn first-level classifiers
    for $t \leftarrow 1$ to T **do**
        Learn a classifier h_{kt} from $\mathcal{D} \setminus \mathcal{D}_k$
    end for
    Step 1.2: Construct a training set for second-level classifier
    for $\mathbf{x}_i \in \mathcal{D}_k$ **do**
        Get a record $\{\mathbf{x}'_i, y_i\}$, where $\mathbf{x}'_i = \{h_{k1}(\mathbf{x}_i), h_{k2}(\mathbf{x}_i), \ldots, h_{kT}(\mathbf{x}_i)\}$
    end for
end for
Step 2: Learn a second-level classifier
Learn a new classifier h' from the collection of $\{\mathbf{x}'_i, y_i\}$
Step 3: Re-learn first-level classifiers
for $t \leftarrow 1$ to T **do**
    Learn a classifier h_t based on $\mathcal{D}$
end for
return $H(\mathbf{x}) = h'(h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_T(\mathbf{x}))$

Tang, J., S. Alelyani, and H. Liu. "Data Classification: Algorithms and Applications." Data Mining and Knowledge Discovery Series, CRC Press (2015): pp. 498-500.

Stacking Algorithm with Cross-Validation



Leave-One-Out CV



Demos

http://rasbt.github.io/mlxtend/user_guide/classifier/EnsembleVoteClassifier/

http://scikit-learn.org/stable/modules/generated/sklearn.ensemble.VotingClassifier.html

http://scikit-learn.org/stable/auto_examples/ensemble/plot_bias_variance.html#sphx-glr-auto-examples-ensemble-plot-bias-variance-py

http://scikit-learn.org/stable/auto_examples/ensemble/plot_adaboost_hastie_10_2.html#sphx-glr-auto-examples-ensemble-plot-adaboost-hastie-10-2-py

https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.GradientBoostingClassifier.html

http://rasbt.github.io/mlxtend/user_guide/classifier/StackingClassifier/

http://rasbt.github.io/mlxtend/user_guide/classifier/StackingCVClassifier/

Reading Assignments

Python Machine Learning, 2nd Ed., Ch07