Leonardo Farias Araujo

Calculo II
Prova P2

1-) a-) $\int x e^{x-1} dx$

$u = x$
$dv = e^{x-1} dx$
$du = dx$
$v = e^{x-1}$

$x e^{x-1} - \int e^{x-1} dx$
$x e^{x-1} - \int e^{x} dx$
$x e^{x-1} - e^{x}$
$x e^{x-1} - e^{x-1}$

$\boxed{x e^{x-1} - e^{x-1} + C, \ C \in \mathbb{R}}$

b-) $\int \theta \cos \theta \, d\theta$

$u = \theta$
$dv = \cos(\theta) d\theta$
$du = d\theta$
$v = \operatorname{sen}(\theta)$

$\theta \cdot \operatorname{sen}(\theta) - \int \operatorname{sen}(\theta) d\theta$
$\theta \cdot \operatorname{sen}(\theta) - (-\cos(\theta))$
$\theta \cdot \operatorname{sen}(\theta) + \cos(\theta)$

$\theta \cdot \operatorname{sen}(\theta) + \cos(\theta) + C, \ C \in \mathbb{R}$

2-) $\int_1^2 \frac{1}{x^2 \sqrt{9+4x^2}} dx = \frac{-5+2\sqrt{13}}{18}$

$\int_{\arctan(2/3)}^{\arctan(4/3)} \frac{2 \sec(u)}{9 \tan^2(u)} du = \frac{2}{9} \int_{\arctan(2/3)}^{\arctan(4/3)} \frac{\sec(u)}{\tan^2(u)} du$

$= \frac{2}{9} \int_{\arctan 2/3}^{\arctan(4/3)} \frac{\cos(u)}{\operatorname{sen}^2(u)} = \frac{2}{9} \int_{\frac{2\sqrt{13}}{13}}^{4/5} v^{-2} dv$

$= \frac{2}{9} \left[\frac{v^{-2+1}}{-2+1} \right]_{\frac{2\sqrt{13}}{13}}^{4/5} = -\frac{5}{4} + \frac{\sqrt{13}}{2} = \frac{2}{9} \left(-\frac{5}{4} + \frac{\sqrt{13}}{2} \right)$

$= \frac{-5+2\sqrt{13}}{18}$

3.) a) $f(x,y) = 3x^4 - 2y^3x + x^2 - 2$

$f_x \frac{\partial}{\partial x}(3x^4 - 2y^3x + x^2 - 2)$

$= \frac{\partial}{\partial x}(3x^4) - \frac{\partial}{\partial x}(2y^3x) + \frac{\partial}{\partial x}(2)$

$\frac{\partial}{\partial x}(3x^4) = 12x^3$ | $\frac{\partial}{\partial x}(x^2) = 2x$

$\frac{\partial}{\partial x}(2y^3x) = 2y^3$ | $\frac{\partial}{\partial x} 2 = 0$

$= \boxed{12x^3 + 2x - 2y^3}$

$f_y = \frac{\partial}{\partial y}(3x^4) - \frac{\partial}{\partial y}\underset{6xy^2}{(2y^3x)} + \frac{\partial}{\partial y}(x^2) - \frac{\partial}{\partial y}$

$= 0 - 6xy^2 + 0 - 0 \qquad = \boxed{-6xy^2}$

$f_{xx} = \frac{\partial}{\partial x}\, 12x^3 + 2x - 2y^3 = 36x^2 + 2\, 0 \qquad = \boxed{36x^2 + 2}$

$f_{yy} = \frac{\partial}{\partial y}(-6xy^2) = -6x \frac{\partial}{\partial y}(y^2) = -6x\,2y = \boxed{-12xy}$

2.) $f(x,y) = 5x^3y^3 - y \operatorname{sen} x + 2y^3$

$f_x \frac{\partial}{\partial x}\, 5x\, y^3 = 15x^2y^3$

$\frac{\partial}{\partial x} = y \qquad x = y \operatorname{sen} x$

tilibra

$\frac{\partial}{\partial x}\ 2y^3 = 0 \qquad = \boxed{15x^2y^3 - y\ \text{sen}\ x}$

$fy\ \frac{\partial}{\partial y} = 5x^3y^3 = 15x^3y^2$

$\frac{\partial}{\partial y} = y\ \text{sen}\ x = \text{sen}\ x$

$\frac{\partial}{\partial y} = 2y^3 = 6y^2$

$15x^3y^2 - \text{sen}\ x + 6y^2$

$fxx\ \frac{\partial}{\partial x}\ 15x^2y^3 - y\ \text{sen}\ x$

$\frac{\partial}{\partial x} = 15x^2y^3 = 30x^3$

$\frac{\partial}{\partial x} = y\ \text{sen}\ x = -y\ \text{sen}\ x \qquad = \underline{x = 30^3 + y\ \text{sen}\ x}$

4-) a-) $\int\int (x-y)^2\, dx\, dy$

$\int (x-y)^2\, dx = \frac{1}{3}(-y+x)^3 + C$

$= \int(\frac{1}{3}(-y+x)^3 + C)\, dy$

$= \int(\frac{1}{3}(-y+x)^3 + C)\, dy = Cy - \frac{1}{12}(x-y)^4 + C$

$= Cy - \frac{1}{12}(x-y)^4 + C$

b-) $\int_0^1 \int_0^x 6x(y+3)\, dy\, dx$

$\int_0^1 \int_0^x 6x(y+3)\, dy\, dx$

$\int_0^x 6x(y+3)\, dy = 6x(3x + \frac{x^2}{2})$

$= \int_0^1 6x(3x + \frac{x^2}{2})\, dx$ $\qquad \boxed{\frac{27}{4}}$

$\int_0^1 6x(3x + \frac{x^2}{2})\, dx = \frac{27}{4}$

5-) a-) $(-2, 2\pi/3)$

$R = x \qquad x = R\cos(\theta)$

$\theta = \frac{2\pi}{3} \qquad x = 2\cos\left(\frac{2\pi}{3}\right) = 1$

$y = R\,\text{sen}(\theta)$

$y = -2\,\text{sen}\left(\frac{2\pi}{3}\right) = -\sqrt{3} \qquad \boxed{(1, -\sqrt{3})}$

b-) $(4, 5\pi/8)$

$R = 4 \qquad x = 4\cos\left(\frac{5\pi}{8}\right)$

$\theta = \frac{5\pi}{8} \qquad 4\cos\left(\frac{5\pi}{8}\right) = -2\sqrt{2-\sqrt{2}}$

$x = -2\sqrt{2-\sqrt{2}}$

$y = 4\,\text{sen}\left(\frac{5\pi}{8}\right) = 4\,\text{sen}\left(\frac{5\pi}{8}\right) = 2\sqrt{2+\sqrt{2}}$

$\boxed{\left(-2\sqrt{2-\sqrt{2}}, 2\sqrt{2+\sqrt{2}}\right)}$

c-) $\left(3, \frac{13\pi}{4}\right) \qquad x = 3\cos\frac{13\pi}{4}$

$R = 3 \qquad 3\cos\left(\frac{13\pi}{4}\right) = \frac{-3}{\sqrt{2}} \qquad x = \frac{-3}{\sqrt{2}}$

$\theta = \frac{13\pi}{4} \qquad y = 3\,s\left(\frac{13\pi}{4}\right) = 3\,\text{sen}\left(\frac{13\pi}{4}\right) = \frac{-3}{\sqrt{2}}$

$\boxed{\left(\frac{-3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)}$

d) $(1, -1)$

$x = 1$

$y = -1$

$R = \sqrt{1^2 + (-1)^2}$

$R = \sqrt{2}$

$\theta = \left(\frac{1}{1}\right) + 2\pi$

$\theta = \frac{-\pi}{4} + 2\pi$

$\left(\sqrt{2}, \frac{-\pi}{4} + 2\pi\right)$