Avaliação P1

Calculo II

Ciências da Computação

Leonardo Faria Araujo

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1-)

2-a) $\lim\limits_{x \to 3} (x^3 - 27)$ / $\lim\limits_{x \to 3} (x^2 - 9)$

$\lim\limits_{x \to 3} \left(\dfrac{\cancel{(x-3)} \cdot (x^2+3x+9)}{\cancel{(x-3)} \cdot (x+3)} \right)$

$\lim\limits_{x \to 3} = \left(\dfrac{x^2+3x+9}{x+3} \right) = \dfrac{3^2+3\cdot 3+9}{3+3} = \dfrac{9}{2}$

b-) $\lim\limits_{x \to 1} \left(\dfrac{x^2+x-2}{(x-1)^2} \right) = \left(\dfrac{x^2+2x-x-2}{(x-1)^2} \right) = \left(\dfrac{x\cdot(x+2) - (x+2)}{(x-1)^2} \right)$

$\lim\limits_{x \to 1} \left(\dfrac{(x+2) \cdot \cancel{(x-1)}}{\cancel{(x-1)}^2} \right) = \left(\dfrac{x+2}{x-1} \right)$

$\lim\limits_{x \to 1^-} \left(\dfrac{x+2}{x-1} \right) \quad -\infty$ A solução não existe

$\lim\limits_{x \to 1^+} \left(\dfrac{x+2}{x-1} \right) \quad +\infty$

3-)

4-) a-) $f'(x) = \frac{d}{dx}(5x^6 - 2x^3 + x^{-5})$

$f'(x) = \frac{d}{dx}(5x^6) + \frac{d}{dx}(-2x^3) + \frac{d}{dx}(x^{-5})$

$f'(x) = 5 \cdot 6x^5 - 2 \cdot 3x^2 - 5x^{-6}$

$f'(x) = \frac{30x^{11} - 6x^8 - 5}{x^6}$ $\quad f'(x) = \frac{30x^{11} - 6x^8 - 5}{x^6}$

b-) $f'(x) = \frac{d}{dx}\left(\frac{5x^2+1}{3x+2}\right)$

$f'(x) = \frac{\frac{d}{dx}(5x^2+1) \cdot (3x+2) - (5x^2+1) \cdot \frac{d}{dx}(3x+2)}{(3x+2)^2}$

$f'(x) = \frac{5 \cdot 2x \cdot (3x+2) - (5x^2+1) \cdot 3}{(3x+2)^2} = \frac{15x^2 + 20x - 3}{(3x+2)^2}$

$f'(x) = \frac{15x^2 + 20x - 3}{(3x+2)^2}$

tilibra

c-) $f'(x) = \frac{d}{dx}\left(\sqrt{x^2+1}\right)$

$f'(x) = \frac{d}{dy}\left(\sqrt{y}\right) \cdot \frac{d}{dx}\left(x^2+1\right)$

$f'(x) = \frac{1}{2\sqrt{y}} \cdot 2x = \frac{1}{2\sqrt{x^2+1}} \cdot 2x$

$f'(x) = \frac{x}{\sqrt{x^2+1}}$

d-) $f'(x) = \frac{d}{dx}\left((x^2-1).(x^4-1) + \frac{1}{\sqrt{x}}\right)$

$f'(x) = \frac{d}{dx}\left(x^6 - x^2 - x^4 + 1 + \frac{1}{\sqrt{x}}\right)$

$f'(x) = \frac{d}{dx}(x^6) - \frac{d}{dx}(x^2) - \frac{d}{dx}(x^4) + \frac{d}{dx}(1) + \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$

$f'(x) = 6x^5 - 2x - 4x^3 + 0 - \frac{1/2\sqrt{x}}{\sqrt{x}^2}$

$f'(x) = 6x^5 - 2x - 4x^3 - \frac{1}{2x\sqrt{x}}$

5-) a-) $f'(x) = \frac{d}{dx}(x^{-9}) = -9x^{-10} = \frac{-9}{x^{10}}$

b-) $f'(x) = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

6-)

7-) a-) $\lim_{x \to 4} \left(\frac{d/dx\,(x^4 - 256)}{d/dx\,(x^2 - 16)} \right) = \left(\frac{4x^3}{2x} \right) = 2x^2$

$2 \cdot 4^2 = 32$

b-) $\lim_{x \to 2} \left(\frac{e^x}{1} \right) = (e^x) = e^2$

c-) $\lim_{x \to +\infty} = \left(\frac{4x^3 + 2x}{e^x} \right) = \left(\frac{12x^2 + 2}{e^x} \right) = \left(\frac{24x}{e^x} \right) = \frac{24}{+\infty}$

8-) a-) $y = x^3 + 1$ $\quad x = y^3 + 1$ $\quad y^3 + 1 = x$ $\quad y^3 = x - 1$

$y = \sqrt[3]{x-1}$ $\quad f^{-1}(x) = \sqrt[3]{x-1}$

b-) $y = \sqrt[3]{3x-2}$ $\quad x = \sqrt[3]{3y-2}$ $\quad \sqrt[3]{3y-2} = x$

$3y - 2 = x^3$ $\quad 3y = x^3 + 2$ $\quad y = \frac{1}{3}x^3 + \frac{2}{3}$ $\quad f^{-1}(x) = \frac{1}{3}x^3 + \frac{2}{3}$

8-)

10-) a-) $-\frac{\sqrt{2}}{2}$ b-) $-\frac{\sqrt{2}}{2}$ c-) $-\frac{1}{2}$ d-) $\frac{\sqrt{3}}{2}$

e-) $\frac{\sqrt{3}}{2}$ f-) $\frac{1}{2}$ g-) $-\frac{1}{2}$ h-) $\frac{\sqrt{2}}{2}$

i-) $-\frac{1}{2}$ j-) $-\frac{\sqrt{3}}{2}$ k-) $-\frac{\sqrt{2}}{2}$ l-) $-\frac{\sqrt{3}}{2}$

11-) a-) $f'(x) = \frac{4}{3}x^{1/3} + 4x\frac{1}{3}x^{-2/3} = \frac{4x+4}{3\sqrt[3]{x^2}}$

b-) $f'(x) = 1\sqrt{4-x^2} + x \cdot \frac{1}{2\sqrt{4-x^2}} \cdot (-2x) = \frac{4-2x^2}{\sqrt{4-x^2}}$

12-) a) $\lim_{x \to +\infty} \left(2 + \frac{1}{x^2}\right) = 2+0 = 2$

b-) $\lim_{x \to +\infty} \left(\frac{x^2 \cdot (-2x + 3x^2)}{x^2 \cdot \left(5 - \frac{3}{x} - \frac{4}{x^2}\right)}\right) = \left(\frac{-2x + 3x^2}{5 - \frac{3}{x} - \frac{4}{x^2}}\right) = 5$

13-) a-)

b-)

14-)

15-) a-)

b-)

c-)

d-)

16-)

17) a-) $\log_3(5x-2) < \log_3(4)$, $x > 2/5$ $= 5x-2 < 4 = 5x < 4+2$
$= 5x < 6 = x < 6/5$, $x > 2/5$ $\left(\frac{2}{5}, \frac{6}{5}\right)$

b-) $\log_{0,3}(4x-3) < \log_{0,3}(5)$, $x > \frac{3}{4}$ $= 4x-3 > 5 = 4x > 5+3$
$= 4x > 8 = x > 2$, $x > 3/4$ $(2, +\infty)$

c-) $\log_2(2x^2-5x) \leq \log_2(3)$, $(-\infty, 0) \cup (5/2, +\infty)$
$2x^2-5x \leq 3 = 2x^2-5x-3 \leq 0 = x\cdot(2x+1)-3(2x+1) \leq 0$
$= (2x+1)\cdot(x-3) \leq 0$ $= [-1/2, 0) \cup (5/2, 3]$

d-) $\log_x (x^2-1) > \log_x (3x+9)$, $(-3,-1) \cup (1,+\infty)$
$x^2-1 < 3x+9 = x.(x+2)-5(x+2) < 0 = (x+2).(x-5) < 0 = (-2,-1) \cup (1,5)$

e-) $\log_{10}(x^2-x-2) < \log_{10}(x-4)$, $(4,+\infty)$
$x^2-x-2 < x-4 = x^2-2x+2=0 = x^2-2x+2 < 0, \Delta$ $x \in \emptyset$

18-)

19-)

20-) a-) $\log_2(2ab) - \log_2(c) = \log_2(2) + \log_2(a) + \log_2(b) - \log_2(c)$
$= 1 + \log_2(a) + \log_2(b) - \log_2(c)$

b-)