Leonardo Faria Araujo

$$\int_{-1}^{1} x^3 (x^2+3)^{10}\, dx$$

$$\int \frac{t^{11} - 3t^{10}}{2}\, dt = \frac{1}{2} \cdot \int t^{11} - 3t^{10}\, dt = \frac{1}{2} \cdot \left(\frac{t^{12}}{12} - \frac{3t^{11}}{11}\right)$$

$$\frac{1}{2} \cdot \left(\frac{(x^2+3)^{12}}{12} - \frac{3(x^2+3)^{11}}{11}\right) = \frac{(x^2+3)^{12}}{24} - \frac{3(x^2+3)^{11}}{22}$$

$$\left(\frac{(x^2+3)^{12}}{24} - \frac{3(x^2+3)^{11}}{22}\right)\Bigg|_{-1}^{1}$$

$$\frac{(1^2+3)^{12}}{24} - \frac{3(1^2+3)^{11}}{22} - \left(\frac{((-1)^2+3)^{12}}{24} - \frac{3((-1)^2+3)^{11}}{22}\right) = \boxed{0}$$

$$\int x^2 \cdot \ln x\, dx \quad \int \ln(x)\, x^2\, dx$$

$u = \ln(x)$

$dv = x^2\, dx$

$du = 1/x\, dx$

$v = \frac{x^3}{3}$

$$\ln(x) \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\, dx$$

$$\ln(x) \cdot \frac{x^3}{3} - \int \frac{x^2}{3}\, dx$$

$$\ln(x) \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \int x^2\, dx = \ln(x) \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3}$$

$$\frac{\ln(x)\, x^3}{3} - \frac{x^3}{9} = \frac{\ln(x) \cdot x^3}{3} - \frac{x^3}{9} + C,\ C \in \mathbb{R}$$

$$\boxed{\frac{\ln(x) \cdot x^3}{3} - \frac{x^3}{9} + C,\ C \in \mathbb{R}}$$

$\int x^2 e^x dx$

$u = x^2$ $\qquad x^2 e^x - \int e^x \cdot 2x \, dx$

$dv = e^x dx$ $\qquad x^2 e^x - 2 \cdot \int e^x x \, dx$

$du = 2x \, dx$ $\qquad x^2 e^x - 2(x e^x - \int e^x dx)$

$v = e^x$ $\qquad x^2 e^x - 2(x e^x - e^x) = x^2 e^x - 2x e^x + 2e^x$

$$\boxed{x^2 e^x - 2x e^x + 2e^x + C, C \in R}$$

$\int_0^{\ln} \int_0^{\ln 5} e^{2x-y} dx\, dy = \int_0^{\ln(5)} e^{2x-y} dx\, dy = dx \int -e^t dt$

$-dx \int e^t dt = -dx\, e^t = -dx\, e^{2x-y} = -dx\, e^{2x-y} \Big|_0^{\ln(5)}$

$-dx\, e^{2x-\ln(5)} - (-d)x\, e^{2x-0} = \frac{4 dx\, e^{2x}}{5}$

$\int_0^2 \int_0^1 (2x+y)^8 dx\, dy = \int \frac{t^8}{2} dt$

$= \frac{1}{2} \cdot \int t^8 dt = \frac{1}{2} \cdot \frac{t^9}{9} = \frac{1}{2} \frac{(2x+y)^9}{9} = \frac{(2x+y)^9}{18}$

$= \frac{(2x+y)^9}{19} \Big|_0^1 = \frac{(2 \cdot 1+y)^9}{18} - \frac{(2 \cdot 0+y)^9}{18} = \frac{(2+y)^9 - y^9}{18}$

$\int \frac{(2+y)^9 - y^9}{18} dy = \frac{1}{18} \cdot \int (2+y)^9 - y^9 dy$

$= \frac{1}{18} \cdot \left(\int (2+y)^9 dy - \int y^9 dy\right) = \frac{1}{18} \cdot \left(\frac{(2+y)^{10}}{10} - \frac{y^{10}}{10}\right)$

$= \frac{(2+y)^{10} - y^{10}}{180} = \frac{(2+y)^{10} - y^{10}}{180} \Big|_0^2 = \frac{(2+2)^{10} - 2^{10}}{180} - \frac{(2+0)^{10} - 0^{10}}{180}$

$$\boxed{\frac{261632}{45}}$$

tilibra