Fall 2020 - Admission to advanced math courses

PART ONE

The next four questions are based on material that can be found in Chapter 1 of the MATH 147 course notes.

http://www.math.uwaterloo.ca/~beforres/M147/CourseNotes/Forrest_M147CN_F20.pdf

Please read the chapter and then select the best answer for each question.

a) To prove the statement "$\exists n \in \mathbb{N} : P(n)$" it suffices to give one example.

a) True
b) False
c) Not enough information.

b) To show that the statement "$\exists n \in \mathbb{N} : P(n)$ " is false, it suffices to give one example.

a) True
b) False
c) Not enough information.

c) The statement "I **always** lie" cannot be true.

a) True
b) False
c) Not enough information.

d) If $[(p \vee q) \Rightarrow r]$ is true and p is true, then r is true.

a) True
b) False
c) Not enough information

PART TWO

Note: The next 3 questions are based on the lecture titled Induction Part 1: The Principle of Mathematical Induction: **http://www.math.uwaterloo.ca/~beforres/M147/Lectures/Module1/INductionI.mp4**

Please watch the lecture and then answer each of the following questions:

a) Assume that $S \subseteq \mathbb{N}$ is such that if $k \in S$, then $k + 1 \in S$. Which of the following statements is always true.

a) $S = \mathbb{N}$.
b) If $5 \in S$, then 27 is in S.
c) Both of the above.
d) None of the above.

b) In the lecture on Induction, what is wrong with the proof that: "All dogs are shaggy." Choose the best answer:

1) Nothing is wrong, the proof is correct.
2) We fail to properly establish the base case $P(1)$ which in this case is false.
3) The argument used to show $P(k+1)$ assuming $P(k)$ is flawed.
4) None of the above.

c) Let $P(n)$ be the statement that in any collection of n real numbers $\{x_1, x_2, \ldots, x_n\}$ all of the values are equal. We will prove by induction that $P(n)$ is always true. The base case $P(1)$ is obviously true.

Next assume that $P(k)$ is true and that we have a collection of $k+1$ numbers

$$\{x_1, x_2, \ldots, x_k, x_{k+1}\}.$$

If we remove x_{k+1} from the collection, then we are left with a collection $\{x_1, x_2, \ldots, x_k\}$ of k numbers. Hence by $P(k)$ we have that

$$x_1 = x_2 = \cdots = x_k. \quad (*)$$

Next we remove x_1 from our original collection to get a new collection $\{x_2, \ldots, x_k, x_{k+1}\}$ of k numbers so again we have

$$x_2 = \cdots = x_k = x_{k+1}.$$

But then we have that

$$x_1 = x_2 = x_{k+1}$$

so in fact by $(*)$ we get that

$$x_1 = x_2 = \cdots = x_k = x_{k+1}$$

and $P(k+1)$ holds. As such, by induction $P(n)$ holds for all n.

Question: Is there something wrong with this proof or are all real numbers in fact equal? Choose the best answer:

1) Nothing is wrong, the proof is correct.
2) We fail to properly establish the base case $P(1)$ which in this case is false.
3) The argument used to show $P(k+1)$ assuming $P(k)$ is flawed.
4) None of the above.