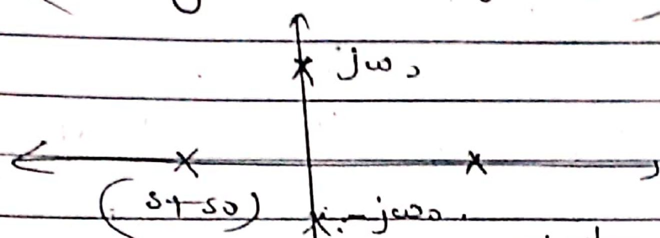
(unit - 3).

(Oscillators) -

(Sinusoidal, oscillator)

(Non-sinusoidal)

- Astable
- multip.
- multivibrator and Triangular wave generator



$\frac{K}{(s+s_0)} \longleftrightarrow K e^{-s_0 t}$ (stable)

$\frac{K}{(s-s_0)} \longleftrightarrow K e^{s_0 t}$

$\frac{K}{(s+j\omega_0)} \longleftrightarrow$ ——

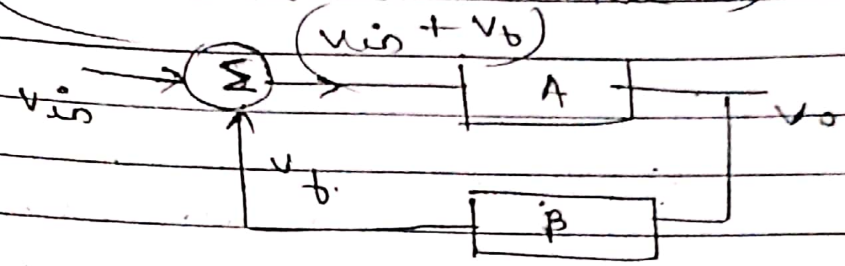
$\frac{K}{(s-j\omega_0)} \longleftrightarrow$

$\frac{K_1}{(s+j\omega_0)} + \frac{K_2}{(s-j\omega_0)}$ (don't have real part)

$e^{-j\omega_0 t} \qquad e^{j\omega_0 t}$

(To make system oscillator poles must be on imaginary axis)

Bark Hausen's criterion —



$$V_f = \beta V_o$$

$$V_o = A(V_{in} + V_f)$$
$$V_o = A[V_{in} + \beta V_o]$$

$$V_o(1 - A\beta) = A V_{in}$$
$$V_o = \frac{A V_{in}}{(1 - A\beta)} \quad \text{or} \quad \frac{V_o(s)}{V_i(s)} = \frac{A}{1 - A\beta}$$

if $(1 - A\beta) = 0$.

$$\frac{V_o}{V_{in}} \to \infty$$

$(V_o \to \text{finite})$

it means $V_{in} \to 0$.

It means V_o is finite when if V_i is '0'

$(A\beta = 1)$.
(loop gain)

$|A\beta| = 1$. $\angle A\beta = 0°$ or $360°$ or multiple of 2π.
(phase)

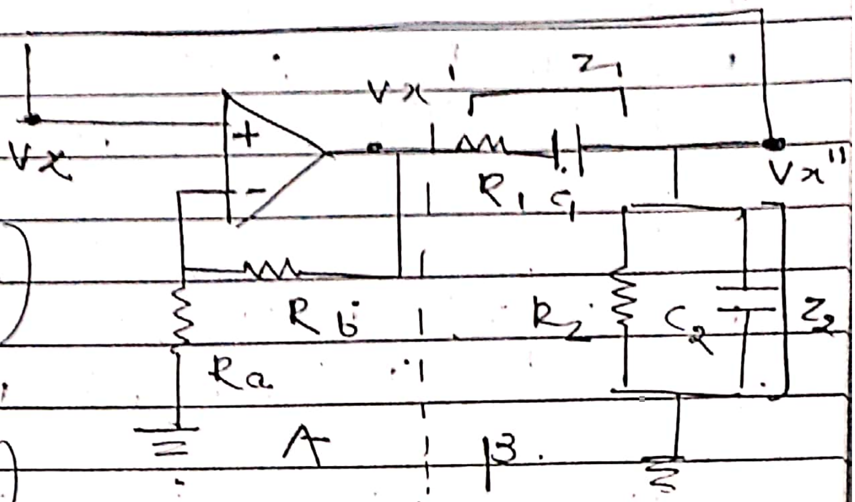
According to Bark Hausen's criteria then system will oscillate iff the loop gain is 1 . This means —

(i) The magnitude of the loop gain is '1'

(ii) The phase shift across the loop is ~~0°~~ '0' or multiple of 2π

Oscillators :-

(1) Wein Bridge Oscillator :-



$$V_x' = V_x\left(1 + \frac{R_b}{R_a}\right)$$

$$V_x'' = V_x'\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

$$Z_1 = R_1 + \frac{1}{sC_1} = \frac{R_1C_1s + 1}{sC_1}$$

$$Z_2 = \frac{R_2 \times \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{(1 + R_2C_2s)}$$

$$= V_x\left(1 + \frac{R_b}{R_a}\right)\left(\frac{R_2}{(1+R_2C_2s)\left[\left(\frac{R_1C_1s+1}{sC_1}\right) + \frac{R_2}{(1+R_2C_2s)}\right]}\right)$$

$$= V_x\left(1 + \frac{R_b}{R_a}\right)\frac{R_2}{\cancel{(1+R_2C_2s)}} \cdot \frac{sC_1\,\cancel{(1+R_2C_2s)}}{\left[(R_1C_1s+1)(R_2C_2s+1) + R_2C_1s\right]}$$

$$= V_x\left(1 + \frac{R_b}{R_a}\right)\frac{R_2C_1s}{\left[(R_1C_1s+1)(R_2C_2s+1) + R_2C_1s\right]}$$

$$= V_x\left(1 + \frac{R_b}{R_a}\right)\frac{(R_2C_1s)}{(R_2R_1C_1C_2s^2 + R_2C_2s + R_1C_1s + 1 + R_2C_1s)}$$

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$$\cancel{V_x} = \left(1 + \frac{R_b}{R_a}\right) \cancel{V_x} \frac{(R_2 C_1 s)}{R_1 R_2 C_1 C_2 s^2 + s\left[R_1 C_1 + R_2 C_2 + R_2 C_1\right] + 1}$$

$(s = j\omega)$

$$1 = \left(1 + \frac{R_b}{R_a}\right) \frac{j\omega R_2 C_1}{-R_1 R_2 C_1 C_2 \omega^2 + j\left[R_1 C_1 + R_2 C_2 + R_2 C_1\right]\omega + 1}$$

$$\therefore 1 = \left(1 + \frac{R_b}{R_a}\right) \frac{j\omega R_2 C_1}{(1 - \omega^2 R_1 R_2 C_1 C_2) + j\omega(R_1 C_1 + R_2 C_2 + R_2 C_1)}$$

To satisfy Bark Hausen's criteria (for phase shift) $(1 - \omega^2 R_1 R_2 C_1 C_2) = 0.$

(as $j\omega R_2 C_1$ leading by $\pi/2$ and $j\omega(R_1 C_1 + R_2 C_2 + R_2 C_1)$ is lagging by $\pi/2$)

$$\boxed{\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

(for magnitude)

$$1 = \left(1 + \frac{R_b}{R_a}\right) \frac{j\omega R_2 C_1}{j\omega(R_1 C_1 + R_2 C_2 + R_2 C_1)}$$

$$1 = \left(1 + \frac{R_b}{R_a}\right) \frac{(R_2 C_1)}{(R_1 C_1 + R_2 C_2 + R_2 C_1)}$$

$$R_1 C_1 + R_2 C_2 + R_2 C_1 = \left(1 + \frac{R_b}{R_a}\right) R_2 C_1$$

$$\left(\frac{R_1}{R_2} + \frac{C_2}{C_1} + 1\right) = \left(1 + \frac{R_b}{R_a}\right)$$

$$\boxed{\frac{R_b}{R_a} = \frac{R_1}{R_2} + \frac{C_2}{C_1}}$$

If $R_1 = R_2 = R$ and $C_1 = C_2 = C$

$$\boxed{\omega_0 = \frac{1}{RC}} \qquad f_0 = \frac{1}{2\pi RC}$$

$$\boxed{\frac{R_b}{R_a} = 2}$$

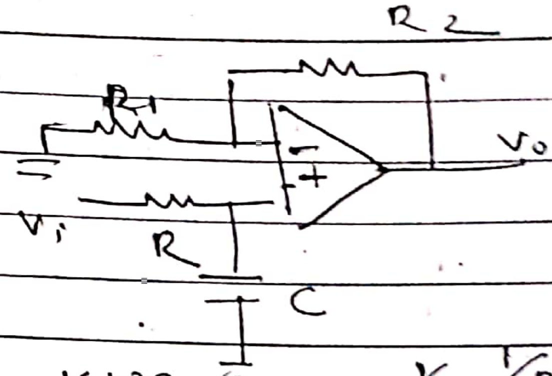
(i) To calculate the frequency of oscillation the phase shift criteria for Bark Hausen must be satisfy.

(ii) The gain contraint will be calculate is to be calculated by satisfying the magnitude criteria of Bark Hausen.

(RC phase shift oscillator)

Tutorial –

Numerical – 1



$$\frac{V_o(s)}{V_i(s)} = \frac{K\omega_c}{(s+\omega_c)} = \frac{K\,{}^1/_{RC}}{\left(s+{}^1/_{RC}\right)}$$

$f_c = 100$ Hz

dc gain, $k = 10$.

$$\therefore K = \left(1+\frac{R_2}{R_1}\right)$$

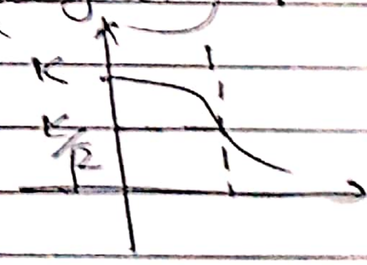
$$\Rightarrow \left(1+\frac{R_2}{R_1}\right) = 10$$

$$\frac{R_2}{R_1} = 10$$

$R_2 = 9R_1$

if $R_1 = 1k$ – (dc gain)

$(R_2 = 9k)$



$$f_c = \frac{1}{2\pi RC} = 100.$$

if $R = 1k$

$$C = \left(\frac{1}{2\pi \times 10^3 \times 100}\right) = \frac{10^{-5}}{2\times 3.14}$$

$= 0.16 \times 10^{-5}$

$= 1.6 \times 10^{-6}$

(2). cut off frequency - it is frequency at which gain is 3 dB down.

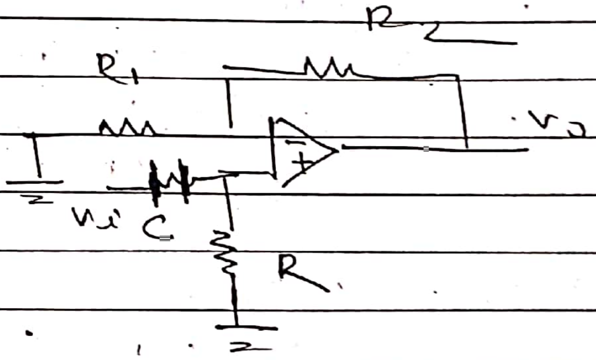
$$= 20\log\left(\frac{V_o}{V_i}\right)$$

$$= 20\log\left(\frac{1}{\sqrt{2}}\right)$$

$$= 20\left(0 - \frac{1}{2}\log 2\right)$$

$$= -3.01\ dB.$$

(2)



ac. gain = 10.

$$\frac{V_o(s)}{V_i(s)} = \frac{Ks}{(s+\omega c)}$$

$$= \frac{Ks}{\left(s+\frac{1}{RC}\right)}$$

$$K = \left(1+\frac{R_2}{R_1}\right) = 10$$

$$\frac{R_2}{R_1} = 9.$$

$R_2 = 9R_1$

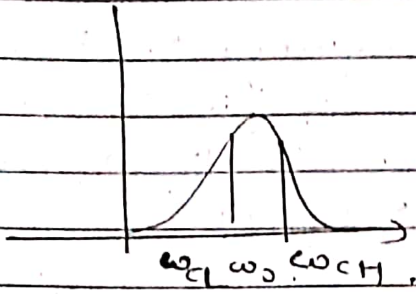
let $R_1 = 1K$.

$R_2 = 9K$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\times 3.14\times 1\times C}.$$

$$C = \frac{1}{2\times 100\times 3.14\times 10^3} = 1.6\,\mu F$$

(3)



f_{cL} f_0 f_{cH}

$f_0 = 100\,Hz$

$f_{cH} - f_{cL} = 50\,Hz$

$f_{cH} = 125\,Hz = \dfrac{1}{2\pi R_L C_L}$

$f_{cL} = 75\,Hz = \dfrac{1}{2\pi R_H C_H}$

$$H(s) = \frac{K\omega_0 s}{s^2 + \left(\frac{\omega_0 s}{Q}\right) + \omega_0^2}$$

$$= \frac{Kj\omega_0 \omega}{-\omega^2 + \left(\frac{\omega_0}{Q}\right)\omega + \omega_0^2}$$

$= KQ$

$KQ = 20$

$Q = \dfrac{f_0}{B.W} = \dfrac{100}{50} = 2$

$K \cdot 2 = 20$

$K = 10$ $\quad K'^2 = 10$

$K = 3.3$

$K' = \left(1 + \dfrac{R_2}{R_1}\right)$

$3.3 = 1 + \dfrac{R_2}{R_1}$

Design a band stop filter have having lower cutoff frequency 75 Hz higher cut off frequency 125 Hz. pass band gain 10.

$$\frac{V_o(s)}{V_i(s)} = -K\left[\frac{s^2 + 2\omega_L s + \omega_L \omega_H}{s^2 + (\omega_L + \omega_H)s + \omega_L \omega_H}\right]$$

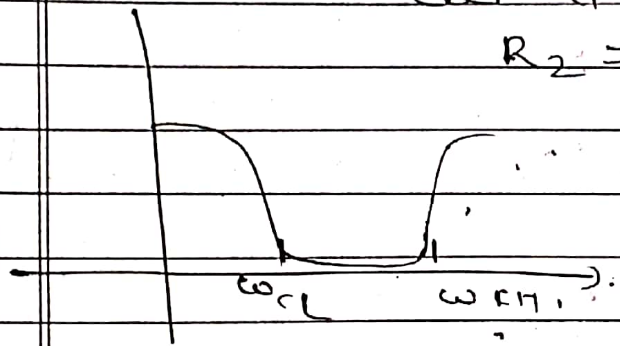
$$10 = K = \left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{R_2}{R_1} = 9$$

$$R_2 = 9R_1$$

let $R_1 = 1\,k\Omega$

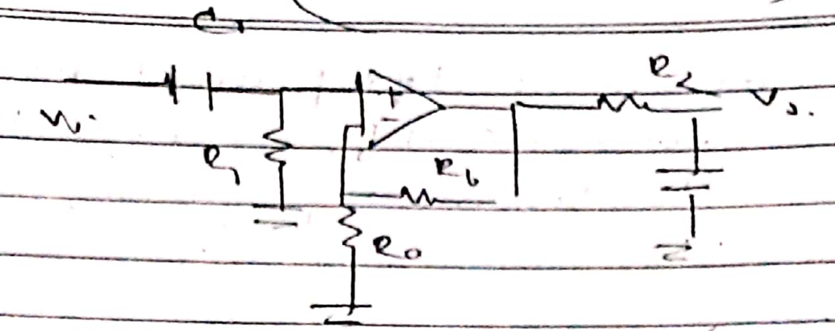
$R_2 = 9\,k\Omega$



$$75\ Hz = \frac{1}{2\pi \cancel{f_{cL}} R_L C_L}$$

$$125 = \frac{1}{2\pi \cancel{f_{cH}} R_H C_H}$$

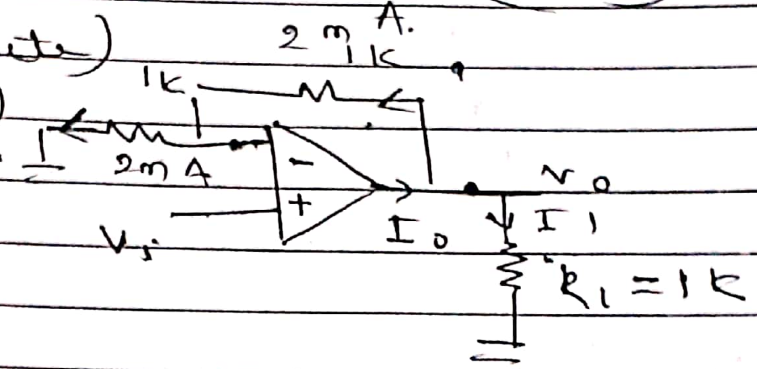
band pass filter



$$\left(1+\frac{R_b}{R_a}\right) = \left(1+\frac{R_2}{R_1}\right)^2$$

(Tute)

(Ques 1)



if $V_i = 2V$.

$I_0 = ?$

$$V_o = \left(1+\frac{R_2}{R_1}\right) V_i$$

$$V_o = (1+1) V_i$$

$$= 2 \times 2$$

$$= 4 \text{ Volts}$$

$$I_0 = \frac{V_o}{R_1}$$

$$= \frac{V_o}{1 \times 10^3}$$

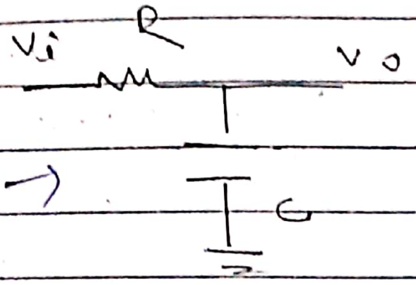
$$= 4 \times 10^{-3}$$

$= 4$ mA ans.

$$I_0 = (4+2)$$

$= 6$ mA ans.

(2) (RC phase shift oscillator) —



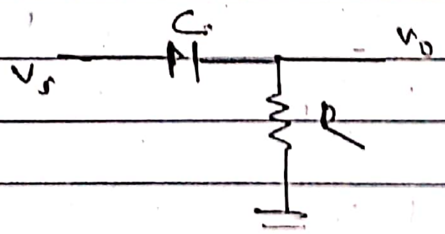
(Single RC Section) →

Analysing for Phase Shift & Gain

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{(1+sCR)}$$

$$H(j\omega) = \frac{1}{1+j\omega CR}$$

$$\boxed{\angle H(j\omega) = -\tan^{-1}(\omega CR)}$$



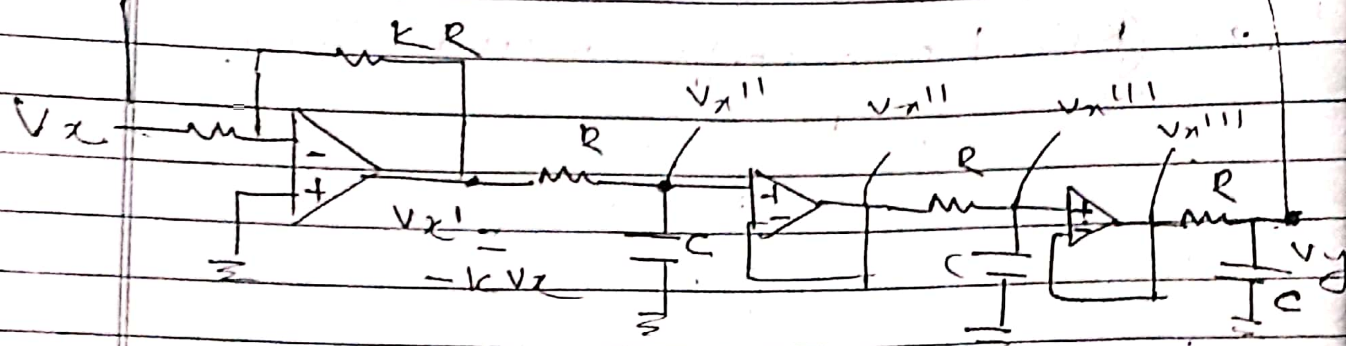
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{(R + 1/sC)} = \frac{sCR}{(sCR+1)}$$

$$H(j\omega) = \left(\frac{j\omega CR}{j\omega CR + 1}\right)$$

$$\angle H(j\omega) = \frac{\pi}{2} - \tan^{-1}(\omega CR)$$

Hence one RC section will provide maximum phase shift of 90° for $\omega \to 0$ & $\omega \to \infty$.
it means single RC section will always provide less than 90° phase shift.

RC Phase shift Oscillator :-



$$V_x'' = V_x'\left(\frac{1}{1+sCR}\right)$$

$$V_x''' = V_x''\,\frac{1}{(1+sCR)}$$

$$V_x''' = \frac{V_x'}{(1+sCR)^2}$$

$$V_y = \frac{V_x'''}{(1+sCR)} = \frac{V_x'}{(1+sCR)^3}$$

$$V_x = \frac{-kV_x}{(1+sCR)^3}$$

$$1 = \frac{-k}{1+s^3C^3R^3+3sCR+3s^2C^2R^2}$$

$$1 = A\beta$$

$$1 = \frac{-k}{1 - j\omega^3C^3R^3 + 3j\omega RC - 3\omega^2C^2R^2}$$

$$1 = \frac{-k}{(1-3\omega^2C^2R^2) + j(3\omega CR - \omega^3R^3C^3)}$$

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for satisfying phase shift criteria

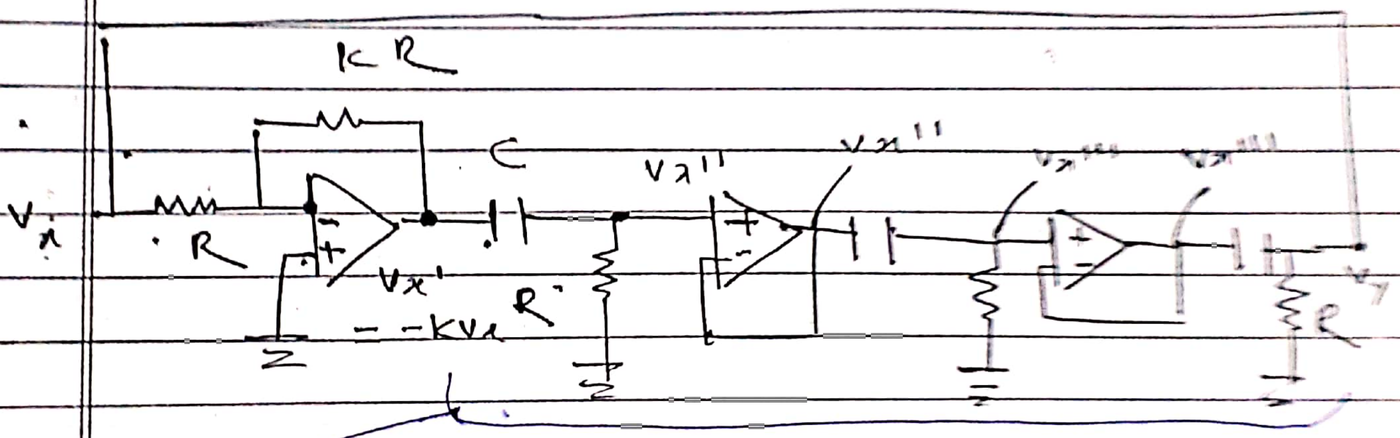
$$8\omega C R - \omega^3 R^3 C^3 = 0$$

freq. of oscillation → $\omega = \dfrac{\sqrt{3}}{RC}$

$$1 = \frac{-k}{(1 - 3 \times 3)}$$

Gain → $\boxed{k = 8}$

$$\omega_0 = \frac{\sqrt{3}}{RC} \qquad f_0 = \frac{\sqrt{3}}{2\pi RC}$$



in this feedback N/w, the position of R & C has been changed from previously used feedback N/w

$$\frac{\pi}{2} - \tan^{-1}(\omega C R) = 60$$

$$\tan^{-1}(\omega C R) = 30$$

$$\omega C R = \frac{1}{\sqrt{3}}$$

$$\omega = \frac{1}{\sqrt{3} RC}$$ ← freq of oscillation

$$F(j\omega) \; \frac{sCR}{(1 + sCR)} = \frac{j\omega RC}{1 + j\omega CR} = 1$$

$$|H(j\omega)| = \frac{1/\sqrt{3}}{\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}}$$

$$= \frac{1}{\sqrt{3}\times\sqrt{4}} \times \sqrt{3} = \frac{\sqrt{3}}{\sqrt{3}\,2} = \frac{1}{2}$$

$$K \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1$$

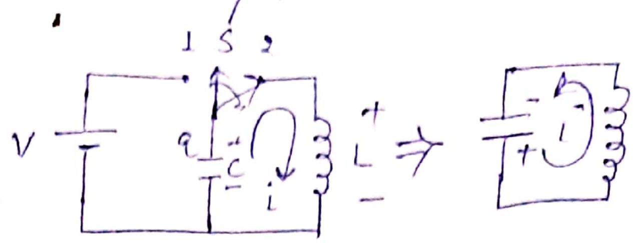
$$K \times \frac{1}{8} = 1$$

Gain → $\boxed{K = 8}$

LC oscillators:

(Electrostatic Energy ⇌ Magnetostatic energy)

Tank circuit:



→ first of all switch S is connected to ① (battery) ⇒ capacitor will be charged

→ After that switch is connected to ②
capacitor will start discharging, electrostatic energy $\left(\frac{q^2}{2C}\right)$ is started to convert in to Magnetostatic energy $\left(\frac{1}{2}Li^2\right)$

applying KVL.

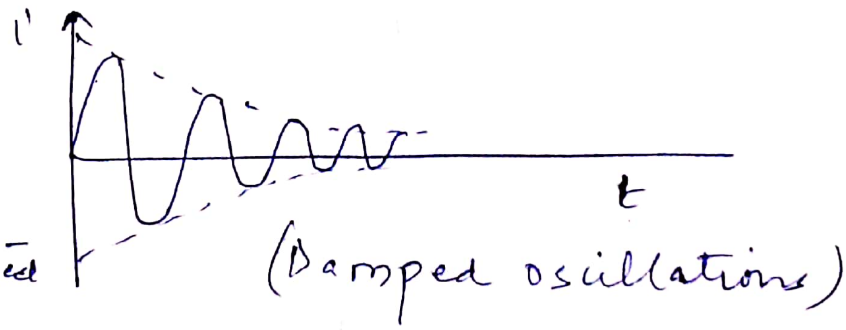
$$\frac{q}{C} - L\frac{di}{dt} = 0 \qquad i = -\frac{dq}{dt}$$

$$\frac{q}{C} - L\frac{d}{dt}\left[\frac{dq}{dt}\right] = 0 \Rightarrow L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

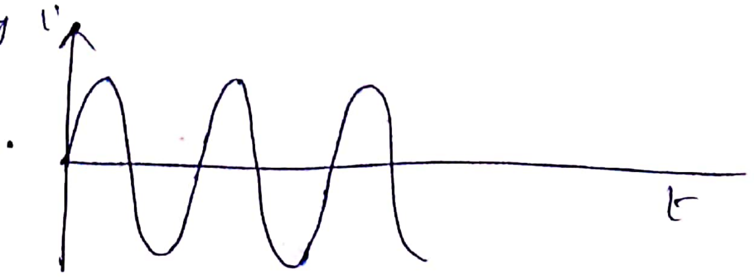
$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$ → this eqn will lead to sinusoidal oscillations (ideally)

→ But due to loses the oscillations generated will be damped in nature.

→ To get sustained oscillation, losses should be compensated by extra energy. by using amplifier

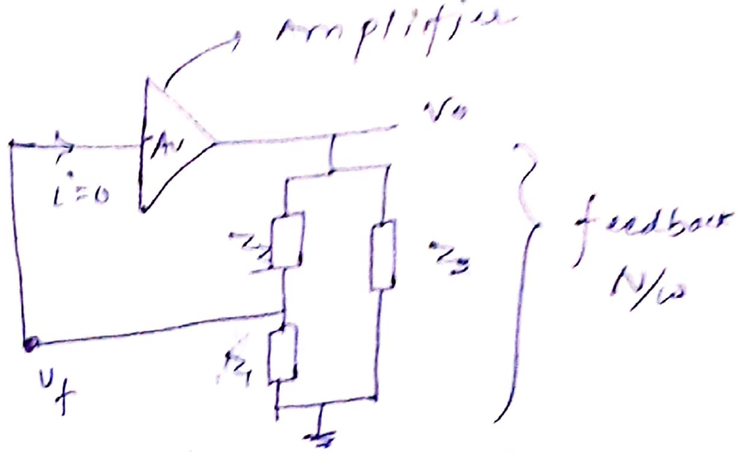


(Damped oscillations)



(Sustained oscillations)

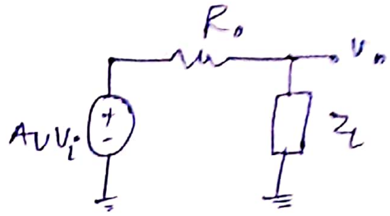
General configuration of LC oscillator:



for amplifier
i/p impedance $Z_i \to \infty$
o/p impedance $Z_o \to R_o$
Gain $\to A_v \angle(180°)$

as $Z_i \to \infty \Rightarrow i = 0 \Rightarrow Z_1$ & Z_3 are in series

for amplifier:



$$\left(Z_L = \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} \right.$$

$$v_o = A_V v_i \times \frac{Z_L}{R_o + Z_L}$$

forward Gain $A = \frac{v_o}{v_i} = A_V \frac{Z_L}{R_o + Z_L}$

$$\boxed{A = A_V \frac{Z_L}{R_o + Z_L}}$$

for feedback factor $\beta = \frac{v_f}{v_o} = \frac{Z_1}{Z_1 + Z_3}$

loop gain $A\beta = A_V \frac{Z_L}{R_o + Z_L} \cdot \frac{Z_1}{Z_1 + Z_3}$

$$A\beta = \frac{A_V \cdot \frac{Z_2(Z_1+Z_3)}{(Z_1+Z_2+Z_3)}}{R_o + \frac{Z_2(Z_1+Z_3)}{(Z_1+Z_2+Z_3)}} \cdot \frac{Z_1}{Z_1+Z_3}$$

$$A\beta = \frac{A_V \cdot Z_2 \cancel{(Z_1+Z_3)}}{R_o(Z_1+Z_2+Z_3) + Z_2(Z_1+Z_3)} \cdot \frac{Z_1}{\cancel{(Z_1+Z_3)}}$$

$$\boxed{A\beta = \frac{A_V \cdot Z_1 Z_2}{R_o(Z_1+Z_2+Z_3) + Z_2(Z_1+Z_3)}}$$

for all components used in feedback are Reactive in nature $\Rightarrow Z = jX$

if component is inductor $\Rightarrow X = \omega L$

if component is capacitor $\Rightarrow X = -1/\omega C$

Hence $Z_1 = jX_1$, $Z_2 = jX_2$ & $Z_3 = jX_3$

$\Rightarrow$ loop gain

$$A\beta = \frac{A_V \cdot jX_1 \cdot jX_2}{R_0(jX_1 + jX_2 + jX_3) + jX_2(jX_1 + jX_3)}$$

$$\boxed{A\beta = -\frac{A_V X_1 X_2}{jR_0[X_1 + X_2 + X_3] - X_2(X_1 + X_3)}}$$

for $180°$ phase shift

$jR_0(X_1 + X_2 + X_3) = 0$

$\boxed{X_1 + X_2 + X_3 = 0}$ $\quad (X_1 + X_3 = -X_2)$

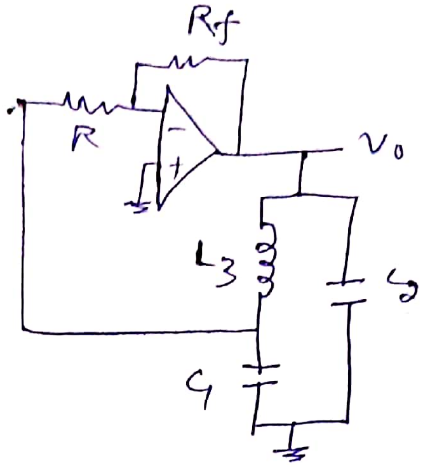
$$A\beta = -\frac{A_V X_1 \cancel{X_2}}{-\cancel{X_2}(X_1 + X_3)} = -\frac{A_V X_1}{X_2}$$

for $|A\beta| = 1 \Rightarrow A_V \frac{X_1}{X_2} = 1$

$$\Rightarrow \boxed{A_V = \frac{X_2}{X_1}}$$

① colpitt ~~Hartley~~ Oscillator :

for colpitt ~~Hartley~~ oscillator



$Z_1 \rightarrow$ capacitor $(X_1 = -\frac{1}{\omega C_1})$
$Z_2 \rightarrow$ capacitor $(X_2 = -\frac{1}{\omega C_2})$
$Z_3 \rightarrow$ Inductor $(X_3 = \omega L_3)$

for freq. of oscillation

$$X_1 + X_2 + X_3 = 0$$

$$-\frac{1}{\omega C_1} - \frac{1}{\omega C_2} + \omega L_3 = 0$$

$$\frac{1}{\omega}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = \omega L_3$$

$$\omega^2 = \frac{1}{L_3\left(\frac{C_1 C_2}{C_1 + C_2}\right)} = \frac{1}{L_3 C_{eq}} \quad \left(C_{eq} = \frac{C_1 C_2}{C_1 + C_2}\right)$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{L_3 C_{eq}}}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{L_3 C_{eq}}}}$$

for Gain

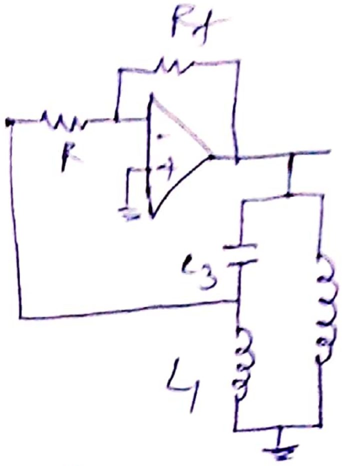
$$A_v = \frac{X_2}{X_1} = \frac{R_f}{R}$$

$$\frac{R_f}{R} = \frac{1/sC_2}{1/sC_1}$$

$$\boxed{\frac{R_f}{R} = \frac{C_1}{C_2}}$$

(2) Hartley ~~Colpitt~~ Oscillator :

for Hartley ~~Colpitt~~ oscillator



$Z_1 \rightarrow$ Inductor ($X_1 = \omega L_1$)

$Z_2 \rightarrow$ Inductor ($X_2 = \omega L_2$)

$Z_3 \rightarrow$ Capacitor ($X_3 = -\frac{1}{\omega C_3}$)

for freq. of oscillations

$$X_1 + X_2 + X_3 = 0$$

$$\omega L_1 + \omega L_2 - \frac{1}{\omega C_3} = 0$$

$$\omega (L_1 + L_2) = \frac{1}{\omega C_3}$$

$$\omega^2 = \frac{1}{(L_1+L_2)C_3} = \frac{1}{L_{eq} C_3} \quad (L_{eq} = L_1 + L_2)$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{L_{eq} C_3}}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi \sqrt{L_{eq} C_3}}}$$

for gain

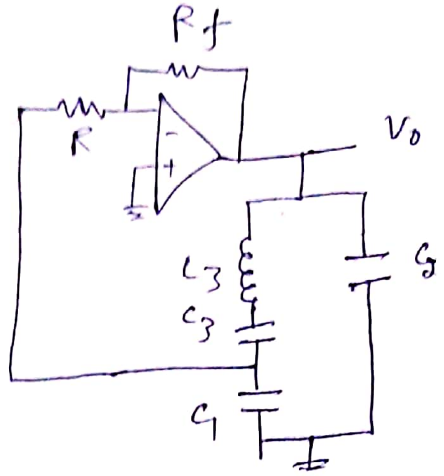
$$A_v = \frac{X_2}{X_1} = \frac{R_f}{R}$$

$$\frac{\omega L_2}{\omega L_1} = \frac{R_f}{R}$$

$$\boxed{\frac{R_f}{R} = \frac{L_2}{L_1}}$$

③ Clapp oscillator:

it is modified colpitt's oscillator



where

$X_1 \to -\frac{1}{\omega C_1}$

$X_2 \to -\frac{1}{\omega C_2}$

$X_3 \to \omega L_3 - \frac{1}{\omega C_3}$

for frequency of oscillations:

$$X_1 + X_2 + X_3 = 0$$

$$-\frac{1}{\omega C_1} - \frac{1}{\omega C_2} + \omega L_3 - \frac{1}{\omega C_3} = 0$$

$$\frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \omega L_3 \quad \left(\text{if } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{\omega C_{eq}} = \omega L_3$$

$$\Rightarrow \omega_0^2 = \frac{1}{L_3 C_{eq}}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{L_3 C_{eq}}}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{L_3 C_{eq}}}}$$

for Gain $A_V = \frac{X_2}{X_1} = \frac{R_f}{R}$

$$\frac{R_f}{R} = \frac{-1/\omega C_2}{-1/\omega C_1}$$

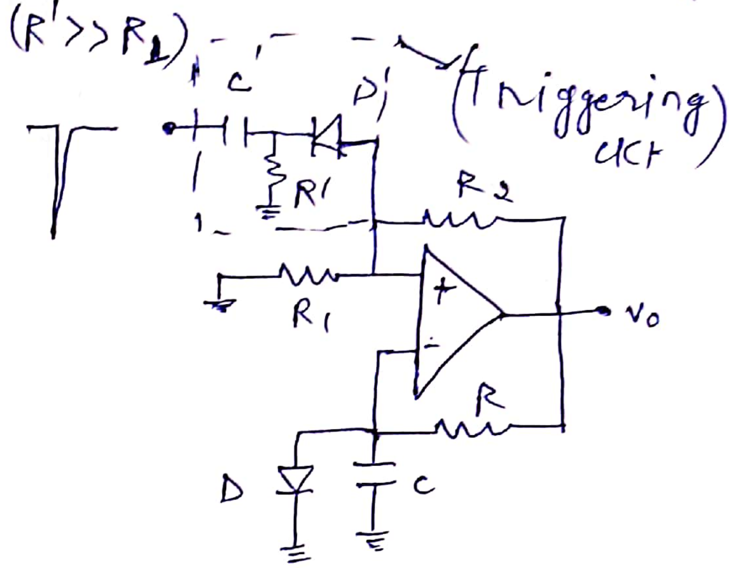
$$\boxed{\frac{R_f}{R} = \frac{C_1}{C_2}}$$

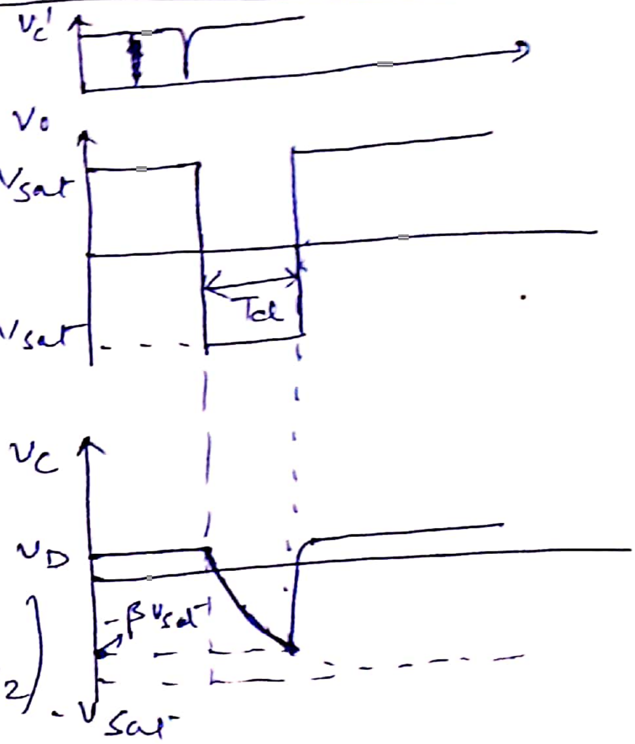
Non Sinusoidal Oscillators :-

1. Square wave Generator or Astable Multivibrator
2. Triangular wave generator
3. Monostable Multivibrator or pulse Generator
4. Bistable Multivibrator (or Schmitt Trigger)

[1, 2 & 4 ⇒ Already covered in unit-5]

3. Monostable Multivibrator or Pulse Generator :-





$$v_+ = \frac{R_1}{R_1+R_2} v_o = \beta v_o \quad \left(\beta = \frac{R_1}{R_1+R_2}\right)$$

As there is a +ve feedback

Hence $\quad V_o = +V_{sat}$ if $V_{id} > 0$ $(V_+ > V_-)$

$\quad = -V_{sat}$ if $V_{id} < 0$ $(V_+ < V_-)$

if $V_o = +V_{sat}$ ⇒ $V_+ = \frac{R_1}{R_1+R_2} V_{sat}$ or βV_{sat}

Diode is forward Biased, Hence the capacitor voltage is fixed to diode voltage (0.7 or V_D)

when a negative impulse is applied to Triggering circuit, D' will be forward biased and $V_+ < V_D$, such that $V_{id} < 0$

$\Rightarrow$ V_o will switch to $-V_{sat}$ from $+V_{sat}$ capacitor will start Discharging.

$V_+ = -\beta V_{sat}$

$$V_c(t) = V_f - (V_f - V_i)\, e^{-t/RC}$$

$V_f = -V_{sat}$, $V_i = V_D$

at $t = T_d$, $V_c(t) = -\beta V_{sat}$

$$-\beta V_{sat} = -V_{sat} - [-V_{sat} - V_D]\, e^{-T_d/RC}$$

$$(V_{sat} + V_D)\, e^{-T_d/RC} = V_{sat}(1-\beta)$$

$$T_d = RC \ln \frac{V_D + V_{sat}}{V_{sat}(1-\beta)}$$

$$T_d = RC \ln \frac{1 + V_D/V_{sat}}{1-\beta}$$

Pulse width $T = T_d$

$$\boxed{T = RC \ln \frac{1 + V_D/V_{sat}}{1-\beta}}$$

as $V_D << V_{sat}$ $\Rightarrow$ $V_D/V_{sat} << 1$

$\Rightarrow$ $\boxed{\text{Pulse width } T = RC \ln \frac{1}{1-\beta}}$