Unit - 2

Electromagnetic Theory

Basic laws of electrostatic & magnetostatic

Gauss law of electrostatic.

1) → Total Electric flux enclosed by closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed by that surface. i.e.

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\boxed{\oint \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}}$$

2) Gauss law of magnetostatic

→ The magnetic flux entering in a closed surface is always equal to the magnetic flux leaving the surface of same volume. i.e. net flux to a closed surface must be zero.

$$\boxed{\begin{array}{c} \phi_B = 0 \\ \text{or} \\ \oint \vec{B}\cdot ds = 0 \end{array}}$$

3) Faraday's law for electromagnetic Induction

→ whene... circu... in t... Indu... to ... ma...

integral form

...4)

→ Whenever the magnetic flux link with a circuit is change and emf is induced in the circuit. in magnitude of induced emf is directly proportional to negative rate of variation of magnetic flux link with a circuit.

$$e = -\frac{d\phi_B}{dt}$$

or, $$e = -\frac{\partial \phi_B}{\partial t}$$

or, $$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\oint \vec{B}\cdot d\vec{s}}$$

integral form

4) Ampere's circuital law

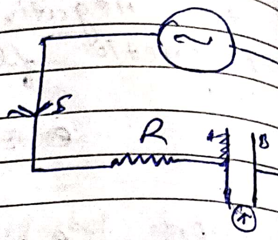
Acc. to Ampere's law the line integral of magnetic field be along the closed curve is equal to μ_0 times the net current to the area bounded by the curve.

$$\boxed{\begin{gathered}\oint \vec{B}\cdot d\vec{l} = \mu_0 I \\ \oint H\cdot dl = I\end{gathered}}$$

Displacement Current :

Conduction Current

$i_c = \frac{\partial q}{\partial t}$



displacement current Density $D = \frac{q}{A} \Rightarrow q = AD$

(electric displi)

$i_c = \frac{\partial (AD)}{\partial t}$

$$\boxed{i_c = A\frac{\partial D}{\partial t}}$$

i_d : $\boxed{i_d = A\frac{\partial D}{\partial t}}$

Maxewell proved that changing E.F in vaccum or in dielectric also produces a magnetic field. So a changing E.F is equivalent to a current which flows as long as electric field is changing and produces the same magnetic effect as a ordinary conduction current. This is known as displacement current.

$$\boxed{J_d = \frac{i_d}{A} = \frac{\partial D}{\partial t}}$$

Note :-

The accumulation of charges on condenser plates produces a potential difference between the two plates of the capacitor. The potential Difference is responsible for the flow of current between the capacitor plate. This is known as displacement Current.

Maxwell Equations:

In medium

$D = \epsilon E$

$B = \mu H$

$J_c = \sigma E$

$J_v = \rho V$

In space

$D = \epsilon_0 E$

$B = \mu_0 H$

$J_c = 0$

Standard meaning of used symbols:

$D \rightarrow$ displacement vector or electric flux density.

$E \rightarrow$ Electric field intensity.

$B \rightarrow$ Magnetic flux density.

$H \rightarrow$ Magnetic field intensity.

$\mu \rightarrow$ Permeability of the medium.

$\mu_r = \frac{\mu}{\mu_0} \Rightarrow \mu = \mu_r \mu_0$

$\epsilon \rightarrow$ Permeability of the medium

$\epsilon_0 \rightarrow$ " " space.

$\sigma \rightarrow$ Conductivity of medium.

$\rho \rightarrow$ Vol^me^ charge density.

$J_c \rightarrow$ Conduction current density.

$J \rightarrow$ Conventional current density.
$V \rightarrow$ volume.

Maxwell's first eq^n :

Acc to the Gauss law of electrostatic.

$$\phi_E = \frac{q}{\epsilon}$$

$$\downarrow$$

$$\oint \vec{E} \cdot ds = \frac{q}{\epsilon} \quad \text{—(1)}$$

$$q = \int_V \rho \, dv \quad \text{—(2)}$$

So eq (1) becomes

$$\boxed{\oint \vec{E} \cdot ds = \frac{1}{\epsilon} \int_V \rho \, dv} \rightarrow \text{Integral form.}$$

Using Gauss divergence theorem

$$\oint E \cdot ds = \int_V (\nabla \cdot E) \, dv. \qquad \left[\because \oint A \cdot ds = \int_V (\nabla \cdot A) \right.$$

Surface $\rightarrow$ volume

$$\Rightarrow \int_V (\nabla \cdot E) \, dv = \frac{1}{\epsilon} \int_V \rho \, dv$$

Comparing on both side

dot product divergence
cross " curl

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon}}$$

or $\boxed{div\ E = \frac{\rho}{\epsilon}}$

$$\nabla\cdot E = \frac{\rho}{\epsilon}$$

$$\nabla\cdot E\epsilon = \rho$$

$\boxed{\nabla\cdot D = \rho}$, $\because \boxed{D = \epsilon E}$

$\boxed{div\ D = \rho}$ → diff. form.

$\boxed{div\ \vec{D} = \rho}$

$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$

$\boxed{div\ \vec{E} = \frac{\rho}{\epsilon_0}}$

Maxwell's Second Equation:

Acc. to Gauss law of Magnetostatic.

$$\phi_B = 0$$

or, $\boxed{\oint \vec{B}\cdot ds = 0}$ (1) → Integral form of IInd eq

Using Gauss divergence theorem.

$$\oint \vec{B}\cdot ds = \int_v (\nabla\cdot B)\, dv$$

$$\Rightarrow \boxed{\int_v (\nabla\cdot B)\, dv = 0},$$

$$\Rightarrow \boxed{\nabla\cdot B = 0}$$

$\boxed{div\ B = 0}$ → Diff form of II eqn

Maxwell Third Equation

Acc. to Faraday's Law of Electromagnetic Induction $e = -\frac{d\phi_B}{dt}$ — ①

The Induced emf $e = \int E \cdot dl$

& $\phi_B = \oint B \cdot ds$

So, $$\boxed{\oint E \cdot dl = -\oint \frac{\partial B}{\partial t} \cdot ds} \rightarrow \text{Integral form}$$

Using Stoke's Theorem:

$$\oint E \cdot dl = \oint (\nabla \times E) \cdot ds.$$

$$\Rightarrow \oint (\nabla \times E) \cdot ds = -\oint \frac{\partial B}{\partial t} \cdot ds$$

$$\Rightarrow \boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

$$\boxed{\text{Curl } E = -\frac{\partial B}{\partial t}}$$

Diff form

Maxwell Fourth Eqn (Modified Form of Ampere's Law

Acc. to Ampere's law

$$\oint B \cdot dl = \mu_0 I \quad \text{--- (1)}$$

But acc. to Maxwell: $I = i_c + i_d$

where $i_c = \oint J_c \cdot ds = \oint J \cdot ds$

$i_d = \oint J_d \cdot ds$

Put the values of i_c & i_d in eq (1)

$$\oint \vec{B} \cdot dl = \mu_0 (i_c + i_d)$$

$$= \mu_0 \left[\oint J \cdot ds + \oint J_d \cdot ds \right]$$

$$\boxed{\oint \vec{B} \cdot dl = \mu_0 \oint (J + J_d)\, ds}$$

or

$$\boxed{\oint H \cdot dl = \oint (J + J_d)\, ds}$$

Integral form.

$$\boxed{\oint B \cdot dl = \mu_0 \oint \left(J + \frac{\partial D}{\partial t} \right) \cdot ds}$$

$$\boxed{J_d = \frac{\partial D}{\partial t}}$$

Using Stoke's th

$$\int B \cdot dl = \oint (\nabla \times B) \cdot ds.$$

$$\oint (\nabla \times B) \cdot ds = \mu_0 \oint \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

$$\boxed{(\nabla \times B) = \mu_0 \left(J + \frac{\partial D}{\partial t} \right)}$$ Diff form

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

$$\boxed{\text{curl } B = \mu_0 \left(J + \frac{\partial D}{\partial t} \right)}$$

Continuity Equation → Mathematical form law of conservation of charge.

Acc to the law of conservation of charges : the electric charges can be neither created nor destroyed.

Let us consider any region of volume 'V' bounded by a closed surface 'S' the current flow through the closed surface is :

$$\frac{dq}{dt} = i = \oint J \cdot ds \quad \text{---(1)}$$

Acc. to the conservation principle the current flow through a region must be balanced by rate of decrease of charge flowing through that region.

i.e.

$$i = \oint J \cdot ds = -\frac{dq}{dt} = -\int_v \frac{\partial \rho}{\partial t} dv$$

$$\left[\because q = \int_v \rho dv\right]$$

$$\Rightarrow \oint J \cdot ds + \int \frac{\partial \rho}{\partial t} dv = 0 \quad -(2)$$

Using Gauss Divergence theorem.

$$\oint J \cdot ds = \int_v (\nabla \cdot J) dv$$

So, Eq (2), becomes

$$\int_v (\nabla \cdot J) dv + \int_v \left(\frac{\partial \rho}{\partial t}\right) dv = 0$$

$$\text{So,} \quad \int_v \left(\nabla J + \frac{\partial \rho}{\partial t}\right) dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$$

↳ continuity Eq.

for Static field

$$\frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\nabla \cdot J = 0}$$

Another method for continuity Eq

from maxwell fourth eq

$$\nabla \times B = \mu_0 \left[J + \frac{\partial D}{\partial t} \right]$$

$$\nabla \times B = \mu_0 \left[J + \epsilon_0 \frac{\partial E}{\partial t} \right] \qquad [\because D = \epsilon_0 E] \quad \text{— (1)}$$

Taking divergence on both sides

$$\text{div.}(\nabla \times B) = \mu_0 \,\text{div}\, J + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \text{div} E.$$

$$\nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla \cdot E)$$

$\Rightarrow$ properties $\rightarrow$ $\boxed{\text{div} \cdot \text{Curl} = 0}$

$$0 = \mu_0 \nabla \cdot J + \mu_0 \cancel{\epsilon_0} \frac{\partial}{\partial t} (\rho / \cancel{\epsilon_0})$$

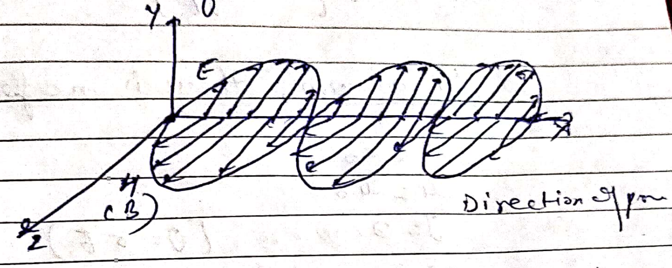
[From Maxw 1st eq $\nabla \cdot E = \frac{\rho}{\epsilon_0}$]

$$\Rightarrow \mu_0 \left[\nabla \cdot J + \frac{\partial \rho}{\partial t} \right] = 0$$

$$\Rightarrow \boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$$

Electromagnetic Waves (E.M Wave)

Electromagnetic wave is a wave of oscillation of electric and magnetic field in mutually perpendicular planes and these oscillations are perpendicular to the direction of propagation of wave.



Note :-

Acc. to Maxwell an oscillating moving charge continuously produces wave which are known as electromagnetic waves. because oscillating charge has an accelerating motion and a magnetic field is produced around the accelerating charge. As the velocity of the charge changes the magnetic field also changes with time. This produces an electric field.

E-Wave Equation In Vaccum (free Space)

→ free or vacuum.

Maxwell's equation are

i) $\nabla \cdot E = \frac{\rho}{\epsilon}$

ii) $\nabla \cdot B = 0$

iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

iv) $\nabla \times B = \mu\left[J + \frac{\partial D}{\partial t}\right]$

So in vacuum, there is no free charge

$\epsilon = \epsilon_0$

$\mu = \mu_0$

$J = 0$ $\quad [J = \sigma E)$

$\rho = 0$

So, Maxwell eq reduces to (in free space)

$(\vec{a}\cdot\vec{c})\vec{b} -$
$(\vec{a}\cdot\vec{b})\vec{c}$

i) $\nabla \cdot E = 0$

ii) $\nabla \cdot B = 0$ $\quad$ (non conductor $\mu \epsilon$

iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

iv) $\nabla \times B = \mu_0 \frac{\partial D}{\partial t} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$.

2

1) Wave Maxwell equation in terms of E

from Maxwell's Third eq^n.

$\nabla \times E = -\frac{\partial B}{\partial t}$

Scanned with CamScanner

Taking curl on both side.

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

Using vector identity $\vec{a} \times (\vec{b} \times \vec{c})$
$= (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = (\vec{\nabla} \cdot \vec{E})\vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla})\vec{E}$$
$$= -\frac{\partial}{\partial t}(\nabla \times B)$$

$$0 - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla \times B)$$ [$\because \nabla E = 0$ from Maxwell first eq^n].

$$-\nabla^2 E = +\frac{\partial}{\partial t} \cdot \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

(2) Wave eq^n in terms of D

We know $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

& $D = \epsilon_0 E \Rightarrow E = \frac{D}{\epsilon_0}$

So, $\nabla^2 \left[\frac{D}{\epsilon_0}\right] = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\left[\frac{D}{\epsilon_0}\right]$

Scanned with CamScanner

$$\Rightarrow \boxed{\nabla^2 D = \mu_0 \epsilon_0 \frac{\partial^2 D}{\partial t^2}}$$

③ Wave equation in terms of B

From maxwell's fourth equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Taking curl on both sides.

$$\nabla \times (\nabla \times B) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$(\cancel{\vec{\nabla} \cdot \vec{B}})_0 \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$-(\vec{\nabla}^2) \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t} \right)$$

$$+(\nabla^2) \vec{B} = + \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}}$$

④ Wave equation in terms of H.

$$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times (\nabla \times H) = \epsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$$

$$(\nabla \cdot H)\nabla - (\nabla \cdot \nabla)H = \epsilon_0 \frac{\partial}{\partial t}\left(-\mu_0 \frac{\partial H}{\partial t}\right)$$

$$+\nabla^2 H = +\mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}}$$

So finally

$$\boxed{\nabla^2 \begin{bmatrix} E \\ D \\ B \\ H \end{bmatrix} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} E \\ D \\ B \\ H \end{bmatrix}} \quad \text{--- (A)}$$

Generally any wave eqn.

$$\nabla^2 A = \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} \quad \text{--- (B)}$$

On comparing A & B

$$\frac{1}{v^2} = \mu_0 \epsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$v = 3 \times 10^8$ m/sec. (Hence EM travel with speed of light in vacuum).

v = speed in light

Transverse Nature Of E·M Wave

(Solution of Plane E·M wave equation)

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{--- (1)}$$

$$\nabla^2 B - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = 0 \quad \text{--- (2)}$$

Solution of these equation are.

$$\vec{E}(r,t) = \vec{E_0}\, e^{(i\vec{k}\cdot\vec{r} - i\omega t)} \quad \text{--- (3)}$$

$$\vec{B}(r,t) = \vec{B_0}\, e^{(i\vec{k}\cdot\vec{r} - i\omega t)} \quad \text{--- (4)}$$

So,

$$\vec{\nabla}\cdot\vec{E} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[\vec{E_0}\, e^{(i\vec{k}\cdot\vec{r} - i\omega t)}\right]$$

where $\vec{k}\cdot\vec{r} = (\hat{i}k_x + \hat{j}k_y + \hat{k}k_z)\cdot(\hat{i}x + \hat{j}y + \hat{k}z)$

propagation vector

$$= (k_x x + k_y y + k_z z)$$

$$\vec{\nabla}\cdot\vec{E} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})\, e^{i(k_x x + k_y y + k_z z) - i\omega t}\right]$$

$$= \frac{\partial}{\partial x}\left[E_{0x}\, e^{i(k_x x + k_y y + k_z z) - i\omega t}\right] +$$

$$\frac{\partial}{\partial y}\left[E_{0y}\, e^{i(k_x x + k_y y + k_z z) - i\omega t}\right] +$$

$$+\frac{\partial}{\partial z}\left[E_{0z}\, e^{i(k_x x + k_y y + k_z z) - i\omega t}\right]$$

$$= (i k_x) E_{0x} e^{(i)(k_x x + k_y y + k_z z) - i\omega t} + i k_y E_{0y} e^{(\ )} + i k_z E_{0z} e^{(\ )}$$

$$= i[k_x E_{0x} + k_y E_{0y} + k_z E_{0z}]\, e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

$$= i[\vec{K}\cdot\vec{E_0}]\, e^{i\vec{K}\cdot\vec{r} - i\omega t}$$

$$\vec{\nabla}\cdot\vec{E} = i[\vec{K}\cdot\vec{E}]$$

from Maxwells first eqn $\vec{\nabla}\cdot\vec{E} = 0$

$$\Rightarrow \quad i[\vec{K}\cdot\vec{E}] = 0$$

Similarly, $i[\vec{K}\cdot\vec{B}] = 0$

Hence $\vec{E}$ & $\vec{B}$ are $\perp$ to the direction of wave.

For Mutullay Perpendicular of Eletoric & Magnetic field:

$$\boxed{\begin{aligned} \vec{K}\times\vec{E} &= \omega\vec{B} \\ \vec{K}\times\vec{B} &= -\omega\mu_0\epsilon_0\vec{E} \end{aligned}}$$

$$\vec{B} = \frac{1}{\omega}(\vec{K}\times\vec{E})$$

$$\vec{B} = \frac{K}{\omega}(\hat{n}\times\vec{E}) \quad [\because \vec{K} = \hat{n}K]$$

$$\mu_0\vec{H} = \frac{K}{\omega}(\hat{n}\times\vec{E})$$

$$\vec{H}' = \frac{k}{\omega \mu_0} (\hat{n} \times \vec{E}')$$

$\left[\frac{\omega}{k} = c \Rightarrow \frac{1}{c} \right]$

$\left[c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right]$

$$\vec{H}' = \frac{1}{c\mu_0} (\hat{n} \times \vec{E}')$$

In terms of magnitude,

$$H = \frac{1}{c\mu_0} E$$

$$H = \frac{\sqrt{\mu_0 \epsilon_0}}{\mu_0} E \Rightarrow \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$= 376.72\,\Omega.$$

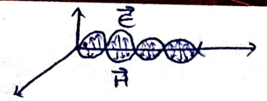
$$\boxed{\frac{E}{H} = \frac{E_0}{H_0} = Z = 376.72\,\Omega}$$ wave impedance factor) (of free space).

(Long day & Sus)

Poynting Vector & Poynting Theorem!

The rate of flow of wave energy through unit area of space normal to the direction of propagation of E.M. Wave. is represented by energy-flux ~~factor~~ vector $(\vec{P}')$. This energy flux vector is also called as Pointing vector.

The cross product of electric field (E) & magnetic field vector is



Known as Poynting vector.

$$\vec{P} = \vec{E} \times \vec{H}$$

SI unit of Point vector is = watt/m².

Poynting theorem

from Maxwell fourth eqn :

$$\nabla \times H = J + \epsilon \frac{\partial E}{\partial t} \quad \text{—①}$$

Taking dot product of E on both sides.

$$E\cdot(\nabla \times H) = E\cdot J + \epsilon E\cdot\frac{\partial}{\partial t}(E) \quad \text{—②}$$

Using vector identity :

$$\nabla\cdot(E\times H) = H\cdot(\nabla\times E) - E\cdot(\nabla\times H)$$

from this identity.

$$E\cdot(\nabla\times H) = H\cdot(\nabla\times E) - \nabla\cdot(E\times H)$$

Put this value in eq ②

$$H\cdot(\nabla\times E) - \nabla\cdot(E\times H) = E\cdot J + \epsilon E\cdot\frac{\partial}{\partial t}(E) \quad \text{—③}$$

from Maxwell 3rd Eqⁿ:

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

$$H\cdot\left[-\mu \frac{\partial H}{\partial t}\right] - \nabla\cdot(E\times H) = E\cdot J + \epsilon \frac{\partial}{\partial t}(E\cdot E) \quad \text{--- (9)}$$

$$\Rightarrow -\mu\left[H\cdot\frac{\partial H}{\partial t}\right] - \nabla\cdot(E\times H) = E\cdot J + \epsilon E\cdot\frac{\partial}{\partial t}(E)$$

$$\Rightarrow -\mu\left[H\cdot\frac{\partial H}{\partial t}\right] - \nabla\cdot(E\times H) = E\cdot J + \epsilon E\cdot\frac{\partial E}{\partial t}$$

We know $\frac{\partial}{\partial t}(H^2) = 2H\frac{\partial H}{\partial t}$

$$\Rightarrow H\cdot\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(H^2)$$

Similarly

$$E\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$$

$$-\mu\left(\frac{1}{2}\frac{\partial}{\partial t}H^2\right) - \nabla\cdot(E\times H) = E\cdot J + \epsilon\left[\frac{1}{2}\frac{\partial}{\partial t}E^2\right]$$

$$-\nabla\cdot(E\times H) = E\cdot J + \frac{\partial}{\partial t}\left[\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right]$$

$$\nabla\cdot(E\times H) = -E\cdot J - \frac{\partial}{\partial t}\left[\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right] \quad \text{--- (5)}$$

$$\int \nabla\cdot(E\times H)\,dv = -\int E\cdot J\,dv - \frac{\partial}{\partial t}\int\left(\frac{\epsilon E^2}{2}+\frac{\mu H^2}{2}\right)dv \quad -(6)$$

Using Gauss divergence theorem.

$$\int \nabla\cdot(E\times H)\,dv = \oint (E\times H)\,ds$$

$$\boxed{\underbrace{\oint (E\times H)\,ds}_{1} = -\underbrace{\int_v (E\cdot J)\,dv}_{2} - \underbrace{\frac{\partial}{\partial t}\int\left(\frac{\epsilon E^2}{2}+\frac{\mu H^2}{2}\right)dv}_{3}}$$

→ First term, from this equation first term represent the rate of flow of energy outward through the surface.

→ Second term, it represent Power dissipated (loss) in volume 'v'

$$E\cdot J = \frac{V}{d}\cdot\frac{I}{A} = \frac{VI}{Ad} = \text{Power loss per unit volume.}$$

→ Third term, in this "$-\frac{\partial}{\partial t}$" → represent the rate at which stored energy is decreasing.

→ $\frac{\mu H^2}{2}$ → energy ~~depe~~ density due to magnetic field.

$\frac{\epsilon E^2}{2}$ → energy density to E.F.

Scanned with CamScanner

Date
Page

Q. If the Earth receives 2 calorie $min^{-1} cm^{-2}$ Solar energy what are the amplitudes of electric and magnetic fields of radiation

Sol.

ENERGY DENSITY OF EM WAVE IN FREE SPACE

In a small volume energy of electric field:

$$U_E = \frac{1}{2} \epsilon_0 E^2 dv \quad \text{--- (1)}$$

in a small volume energy of M.F.

$$U_H = \frac{1}{2} \mu_0 H^2 dv \quad \text{--- (2)}$$

So, electric energy per unit volume that is electric field energy density.

$$U_E = \frac{1}{2} \epsilon_0 E^2 \quad \text{--- (3)}$$

Magnetic energy per unit volume.

$$U_H = \frac{1}{2} \mu_0 H^2 \quad \text{--- (4)}$$

So, Total energy density

$$U = U_E + U_B$$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2$$

We know $\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}}$

Date
Page

$$\frac{E^2}{H^2} = \frac{\mu_0}{\epsilon_0} \Rightarrow \mu_0 H^2 = \epsilon_0 E^2$$

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$$

$$\boxed{U = \epsilon_0 E^2}$$

Ratio of E.F & M.F energy density.

$$\frac{u_E}{u_B} = \frac{\frac{1}{2}\epsilon_0 E^2}{\frac{1}{2}\mu_0 H^2} = \frac{\epsilon_0 E^2}{\epsilon_0 E^2} \quad [\because \mu_0 H^2 = \epsilon_0 E^2]$$

$$\boxed{\frac{u_E}{u_B} = 1}$$

Hence, electric field energy density is equal to magnetic field energy density i.e. the energy associated with the electric field is equal to the energy associated with the magnetic field.

ENERGY & MOMENTUM of E-M WAVE :

Maxwell predicted that E-M wave transport linear momentum in the direction of propagation.

The momentum of a particle of mass 'm' moving with velocity 'v' is given by:-

$$\boxed{p = mv} \quad \text{--- (1)}$$

According to einstein mass-energy relation

$$U = mc^2 \quad \text{--- (2)}$$

$$\Rightarrow m = \frac{U}{c^2}$$

So, $\vec{p} = \frac{U}{c^2} \vec{v}$ — (3)

~~$U = \epsilon_0 E^2$~~.

The energy density in plane electromagnetic wave is give by

$$u = \epsilon_0 E^2 \quad \text{--- (4)}$$

So, the momentum density or momentum per unit volume associated with electro-magnetic wave is.

$$\vec{p} = \frac{u}{c^2} \vec{v} \quad \text{--- (5)}$$

If wave is propagating in x-axis, then

$$\vec{v} = c\hat{i}.$$

So, $\boxed{\vec{p} = \frac{u}{c^2} . c\hat{i} = \frac{u}{c} \hat{i}}$ — (6)

Scanned with CamScanner

By Poynting vector, $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$

But, $\vec{E}\times\vec{B} = \frac{E^2}{c}\hat{i}$.

$$\vec{S} = \frac{E^2}{\mu_0 c}\hat{i}$$

$$\left(u = \epsilon_0 E^2 \Rightarrow E^2 = \frac{u}{\epsilon_0} \right)$$

$$\vec{S} = \frac{u}{\mu_0 \epsilon_0 c}\hat{i} \quad \left[\because \frac{u}{\epsilon_0} = E^2\right]$$

We know $c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \Rightarrow \frac{1}{\mu_0\epsilon_0} = c^2$.

In plane density we know that.

$$\vec{S} = uc\hat{i} \Rightarrow u\hat{i} = \frac{\vec{S}}{c} \quad - (7)$$

Put the value in eq. (6) $\vec{p} = \frac{\vec{S}}{c^2}$.

$$\vec{p} = \frac{u}{c}\hat{i}$$

$$\vec{p} = \frac{\vec{S}}{c^2} = \frac{1}{\mu_0 c^2}(\vec{E}\times\vec{B})$$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0\epsilon_0}$$

$$\Rightarrow \mu_0 c^2 = \frac{1}{\epsilon_0}$$

$$\boxed{\vec{p} = \epsilon_0(\vec{E}\times\vec{B})} \longrightarrow (8)$$

$\because \vec{E} \times \vec{B} = \frac{E^2}{c}\hat{i}$

$\vec{p} = \frac{\epsilon_0 E^2}{c}\hat{i} \Rightarrow \vec{p} = \frac{u}{c}\hat{i} \Rightarrow \boxed{p = \frac{u}{c}}$ energy density, wave velocity

$\boxed{\text{Energy Density} = \text{momentum} \times \text{wave velocity}}$

Radiation Pressure & Energy Density:

When EM wave strikes on a surface its momentum is changes. The rate of change of momentum is equal to the force. This force acting on unit area of the surface applied a pressure i.e. called radiation pressure.

Let a E-M wave incident normally on a perfectly absorbing surface of area "A" for a time "t". If energy "U" is absorbed during this time, the momentum "p" delivered to the surface is given by Acc. to Maxwell.

$\boxed{p = \frac{U}{c}}$

If 'S' is the energy passing per unit area per unit time then

$U = SAt$ —②

coher wave in conc medium

So, $p = \frac{SAt}{c}$

$p = uAt$ — (3)

So, $F = \frac{p}{t} = \frac{uAt}{t} = uA$ — (4)

Radiation Pressure, $P_{rad} = \frac{F}{A}$

$P_{rad} = \frac{uA}{A}$

$\boxed{P_{rad} = u}$

The radiation pressure applied by a normally incident plane electromagnetic wave on a perfect absorber is equal to the energy density in that wave.

Skin Depth (Depth Of Penetration)

When EM wave travel in conducting medium

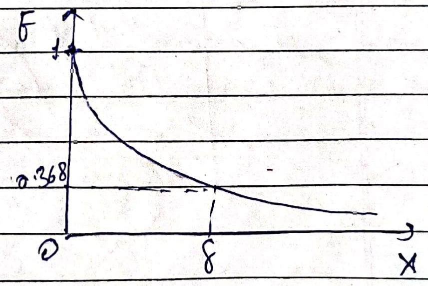
$e^{-\alpha x} = \frac{1}{e}$

$e^{-\alpha x} = e^{-1}$

$\alpha x = 1$

If $x = \delta$

$\Rightarrow \alpha \cdot \delta = 1 \Rightarrow \boxed{\delta = \frac{1}{\alpha}}$



Date
Page

$\therefore$ $\boxed{\text{Skin depth} = \frac{1}{\text{Attenuation const}}}$

-) Skin depth is defined as when the E.M wave travells in any medium so the value its initial electric field reduced to "1/e" times, this depth is known as skin depth or depth of penetration.

Conductor	Dielectric / Insulator
Skin depth. $\boxed{\delta = \sqrt{\frac{2}{\omega \mu \sigma}}}$	$\boxed{\delta = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}}$

Scanned with CamScanner