20 Hooke's Law

Theory

A spiral spring subject to extension by an applied load conforms to Hooke's law. Hooke's law states that the stress is proportional to the strain. If a graph is drawn after the initial loading, where some force is required to separate the turns of the spring which are pressed against each other, a straight line is obtained from which the extension per load (n) can be obtained.

If, now a mass M is attached to the spring, and the spring is extended by a further distance x, a restoring force of $(x/n)g$ is obtained. The spring on being released executes vertical oscillations, and the equation of motion of the mass is

$$\ddot{x} + \frac{g}{Mn}x = 0$$

The motion is thus simple harmonic and the periodic time T is

$$T = 2\pi\sqrt{\frac{Mn}{g}}$$

The above analysis assumes the spring to be weightless. The load M must be increased by an amount m_e equal to the effective mass of the spring. Hence the periodic time is

$$T = 2\pi\sqrt{\frac{(M + m_e)n}{g}}$$

The Experiment

Experiment 1: Verification of Hooke's Law and determination of extension per unit load.

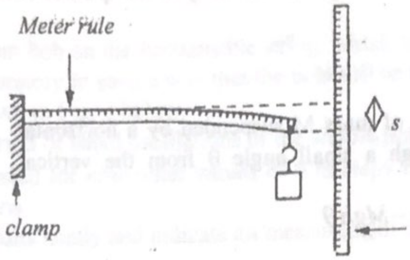
a) Measure the lengths of the springs provided. Clamp one of the springs in addition to the meter scale in such a way that the pointer on the spring moves lightly over the latter. Hook the weight hanger to the spring and note the extension, *e*. Add loads, *m*, to the weight hanger in steps of 20 grams up to 150 grams and note the corresponding extensions. Take your readings when you are unloading the spring. Tabulate your results. By plotting a graph of extension against load, determine the extension per unit load.

b) Repeat the experiment above with the two springs connected in *series* and then in *parallel*. In each case, determine the *effective spring constants*.

Experiment 2: Determination of the acceleration due to gravity and effective mass m_e of a spring.

c) With the weight hanger alone attached to the spring, set the system to vibrate vertically by exciting it with a small additional displacement. Measure the periodic time, *T*, by timing at least 20 vibrations. Repeat the experiment by attaching weights in steps of 20 grams up to 150 grams. By plotting a suitable graph determine g and m_e.

M3 - Determination of Young's Modulus – The Cantilever Experiment



Procedure

Affix a load of mass M =150 g to one free end of the metre rule. Clamp the free end of the metre rule firmly to the edge of the bench by means of a G clamp or otherwise, with a definite length $l = 95$ cm projecting from it. Note that the affixed load will cause only a small depression, *s*. Set the metre rule with the affixed load into vertical vibrations by exciting it through a small vertical displacement. Measure the periodic time, *T*, for 20 oscillations. Measure the width, *b*, and thickness, *d*, of the metre rule.

Repeat the experiment by reducing the definite length, *l*, in steps of 5cm to a minimum length of 65 cm.

Theory

The depression *s* due to a load *W* (=Mg) at the end of a cantilever of length *l* is

$$s = \frac{Wl^3}{3IE}$$

This strain brings into focus the external stresses which produce a restoring force equal to W. If the acceleration of the load is $\ddot{s}$ when the cantilever is displaced to produce vertical oscillations, then

$$\ddot{s} + \frac{3IE}{Ml^3} \cdot s = 0$$

Hence the motion is simple harmonic and the periodic time, T, is

$$T = 2\pi\sqrt{\frac{Ml^3}{3IE}}$$

For a beam of rectangular cross-section, $I = \frac{bd^3}{12}$ where *b* and *d* are as defined above.

Calculations and Discussions

- By plotting a graph of l^3 against T^2, determine the Young's Modulus of the metre rule.
- Is your *experimental* value close to the *theoretical* value? Is it possible to discuss some of the factors that might have contributed to the difference in the two values?
- By studying carefully the given theory, can you suggest any other method that can be used to determine the Young's Modulus of this material ?

T^2