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Midterm ENGR 233

Applied Advanced Calculus (Concordia University)

Midterm I - Section QB

ENGR 233

Fall 2023

- This exam is worth 10% of your final grade.

- All seven questions are equally valued at 10 point each. The maximum mark is 60.

- **Solve any six questions or try all seven questions.** The best six marks will be recorded. Only faculty-approved calculators are allowed. No other electronic device is allowed. No documentation is allowed besides a formula sheet, that is provided.

- You cannot give nor receive help from anyone besides the instructor.

- Print your name and student ID legibly at the top of your exam booklet.

- You must explain what you are doing, and show your work. You will be graded on your work, not just on your answer. Make it clear and legible so we can follow it.

1. A moving particle has position function $\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$.

   (a) Find the object's velocity and acceleration functions $\vec{v}(t)$ and $\vec{a}(t)$.

   (b) Find the object's speed at time t.

   (c) Write down an integral that gives the arc length of the object's path between the points $(0,0,0)$ and $(-\pi, 0, \pi)$. Do not evaluate this integral.

   Show all your work.

2. The temperature at point (x, y, z) is $f(x, y, z) = xe^y - xz^2$ degrees Celsius.

    (a) If you are located at the point $(1, 0, 2)$, in what direction should you move to warm up as quickly as possible? Identify this direction with a unit vector $\vec{u}$.

    (b) If $\vec{u}$ is the unit vector you found in part (a), what is the directional derivative $D_{\vec{u}}f(1, 0, 2)$?

    Show all your work.

3. The following questions are about the plane $3(x - 8) + 7(y - 2) + (z + 14) = 0$. *Your solutions must include at least a sentence or two of justification in order to get full credit.*

    (a) Give an example of a normal vector for our plane.

    (b) Give an example of a plane parallel to our plane. (Your answer should be in the form of an equation.)

    (c) Describe the intersection of our plane with the plane $z = 24$. (Your answer should be in the form of an equation.)

4. For each of the following descriptions of the level curves, give an example of a surface that has those level curves. Your answer can be an equation or a word description.

    (a) The level curves are ellipses (that are not circles) with the same center.

    (b) The level curves are disconnected.

    (c) The level curves are parallel lines with irregular spacing.

5. (a) Find a unit vector in the direction of $\langle -1, 2, 5 \rangle$.

    (b) Decompose the vector $\langle 3, 1, 5 \rangle$ into a sum of two vectors, one that is parallel to $\langle -1, 2, 5 \rangle$, and one that is orthogonal to it.

    Show all your work.

6. Find the tangent plane to $z = 8e^{2y}\sin(4x)$ at $(\frac{\pi}{24}, 0, 4)$. Show all your work.

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7. Looking at the pictures of vector fields below, say which fields match the following description. Your answers should include all fields that match the description. No justification needed. Partial credit given based solely on the items selected.

    (a) Solenoidal fields.

    (b) The flow field of an incompressible fluid.

    (c) Irrotational fields.

A)

B)

C)

D)

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