331-NO- Answers-Finals-Review

Introduction to Formal Methods for Software Engineering (Concordia University)

CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE
AND SOFTWARE ENGINEERING

SOEN 331
Formal Methods for Software Engineering

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<u>SAMPLE FINAL EXAMINATION</u>

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PROPOSITIONAL LOGIC

1. Among the statements below, identify which one is a *proposition*.
   (a) What a nice car!
   (b) I think 7 is greater than 5.
   (c) Please lower the volume of that music!
   (d) Canada does not stretch from the Atlantic to the Pacific.
   (e) a $>$ 10.

2. For propositions p and q when is the sentence "q if p" *false*?
   (a) When the premise is true and the conclusion is false.
   (b) When the premise is false and the conclusion is true.
   (c) When the premise and the conclusion have different truth values.
   (d) When the premise and the conclusion are both false.
   (e) None of the above

3. A set of sentences is *consistent* iff
   (a) It is possible that every sentence in the set is true.
   (b) Every sentence in the set implies at least one other sentence in the set.
   (c) The set forms a valid argument.

(d) The set forms a sound argument.

(e) None of the above.

4. For propositions p and q, the statement $p \to q$ is *logically equivalent* to

   (a) $\neg p \to \neg q$.

   (b) $q \to p$.

   (c) $p \leftrightarrow q$.

   (d) $\neg q \to \neg p$.

5. For propositions p and q, the inverse of $p \to q$ is logically equivalent to

   (a) $\neg q \to p$.

   (b) $p \to \neg q$.

   (c) $\neg q \to \neg p$.

   (d) $q \to p$.

   (e) $p \wedge \neg q$.

6. For propositions p and q, the converse of $p \to q$ is logically equivalent to

   (a) $p \to \neg q$.

   (b) $q \to p$.

   (c) $\neg(p \leftrightarrow q)$.

   (d) $\neg q \to \neg p$.

   (e) $\neg(q \to p)$.

7. Let variable p denote "an exception is thrown" and variable q denote "a loop will repeat forever." The following sentence "Unless an exception is thrown, this loop will repeat forever." translates into formal logic as

   (a) $\neg q \to p$.

   (b) $\neg q \to \neg p$.

   (c) $q \to \neg p$.

   (d) $p \wedge q$.

   (e) $p \leftrightarrow q$.

8. *Being a natural number greater than 1 that has no positive divisors other than 1 and itself* (p) is a criterion for *being a prime number* (q). This can be written as

   (a) $p \to q$.

   (b) $q \to p$.

   (c) $\neg p \to \neg q$.

   (d) $\neg q \to \neg p$.

(e) $p \leftrightarrow q$.

9. Consider the following statements: $p \to q, q \vdash p$.

   (a) This is a validating pattern, called "modus ponens."
   (b) This is a non-validating pattern called "affirming the consequent" (or "converse error").
   (c) This is a validating pattern called "hypothetical syllogism."
   (d) This is a validating pattern called "modus tollens."
   (e) This is a non-validating pattern called "denying the antecedent" (or "inverse error").

10. Consider the following statement: $p \to q, \neg p \vdash \neg q$.

    (a) This is a validating pattern, called "modus ponens."
    (b) This is a non-validating pattern called "affirming the consequent" (or "converse error").
    (c) This is a validating pattern called "modus tollens."
    (d) This is a non-validating pattern called "denying the antecedent" (or "inverse error").
    (e) This is a validating pattern called "denying the antecedent" (or "inverse error").

11. From the sentences below, indicate all instances of *incorrect* use of terminology:

    (i) Bill's statement is unsound.
    (ii) Karl presented a valid argument.
    (iii) Philip's argument is consistent.
    (iv) Jasmine's statement is valid.
    (v) Misha's conclusion is sound.
    (vi) Mary's argument is sound.

    (a) Sentences iii, and iv.
    (b) Sentences iii, iv, and v.
    (c) Sentences i, iii, iv, and v.
    (d) Sentences i, iv, and v.
    (e) Sentences i, iii, iv, v, and vi.

12. Let ϕ, ψ, χ, and τ be (well-formed) formulas and $\bullet$ be any connective. Determine which of the expressions below is *not* a formula:

    (a) $\neg\neg\neg\phi$.
    (b) $\phi \bullet \neg\psi$.
    (c) $\neg\phi \bullet \psi \bullet \neg\chi \bullet \tau$.
    (d) $\phi \wedge \neg\phi$.

(e) $\phi \neg \bullet \psi$.

13. For propositions p and q, the statement $p \rightarrow q$ is a

    (a) Tautology.
    (b) Contradiction.
    (c) Contingency.
    (d) Not a valid formula.
    (e) None of the above.

14. For propositions p and q, the statement $p \leftrightarrow q$

    (a) Implies that p and q are both true.
    (b) Implies that p and q cannot both be false.
    (c) Is true when p and q have the same truth value and is false otherwise.
    (d) Is true when p and q have a different truth value and is false otherwise.
    (e) This is not a proposition.

15. For propositions r and s, r is a *criterion* for s iff:

    (a) $(r \rightarrow s) \wedge (\neg r \rightarrow \neg s)$.
    (b) $(r \rightarrow s) \vee (\neg r \rightarrow \neg s)$.
    (c) $(r \rightarrow s) \wedge (\neg s \rightarrow \neg r)$.
    (d) $(r \rightarrow s) \vee (\neg s \rightarrow \neg r)$.
    (e) $r \vee (\neg r \wedge s)$.

16. You are shown a set of four cards placed on a table, each of which has a number on one side and a colored patch on the other side. The visible faces of the cards show **2**, **9**, and the colors **white**, and **black**. Which card(s) must you turn over in order to test the truth of the proposition that **"If a card shows an odd number on one face, then its opposite face is black"**?

    (a) 2 and white.
    (b) 9 and white.
    (c) 2 and black.
    (d) 2 or black.
    (e) 9 and black.

17. Consider the following set of sentences that represent the requirements of a multi-threaded system for two threads t_1 and t_2:

    - $\neg[(t_1\ active) \wedge (t_2\ active)]$.
    - $(t_1\ active) \oplus (t_2\ active)$.
    - $(t_1\ active) \rightarrow (t_2\ active)$.

The statements (requirements) constitute the following:

(a) A consistent set.

(b) An inconsistent set.

(c) A valid but not sound argument.

(d) An invalid argument.

(e) A sound argument

18. For propositions p and q, the inverse of $p \to q$ is logically equivalent to

(a) $\neg q \to p$.

(b) $p \to \neg q$.

(c) $\neg q \to \neg p$.

(d) $q \to p$.

(e) $p \wedge \neg q$.

19. For propositions p and q, the converse of $p \to q$ is logically equivalent to

(a) $\neg p \to \neg q$.

(b) $q \to p$.

(c) $\neg(p \leftrightarrow q)$.

(d) $\neg q \to \neg p$.

(e) $\neg(q \to p)$.

20. Consider the following argument:

- If one is the Prime Minister of Canada, then one must be 35 years of age or older.
- Justin Trudeau is 35 years of age or older.
- <u>Thus</u>, Justin Trudeau is the Prime Minister of Canada.

(a) The argument must be accepted because the conclusion is true.

(b) The argument must be accepted because both the conclusion and all premises are true.

(c) The argument must be accepted because it is valid with true premises.

(d) The argument is invalid by *inverse error*.

(e) The argument is invalid by *converse error*.

21. An argument *cannot* be

(a) Valid with false conclusion.

(b) Invalid with a true conclusion.

(c) Valid, with true premises and sound.

(d) Unsound with true conclusion.

(e) Sound with false premises.

PREDICATE LOGIC

22. In “Wish You Were Here”, Pink Floyd sing: “We’re just two lost souls / Swimming in a fish bowl [...]” If we take the sentence “some people are lost souls”, and we let $P(x)$ to denote ”x is a person” and $Q(x)$ to denote “x is a lost soul”, then the sentence translates to

    (a) $\forall x(P(x) \rightarrow Q(x))$.
    (b) $\exists x(P(x) \rightarrow Q(x))$.
    (c) $\forall x(P(x) \land Q(x))$.
    (d) $\exists x(P(x) \land Q(x))$.
    (e) $\forall xP(x) \land \exists xQ(x)$.

23. Consider the predicate $loves(x, y)$ denoting “x loves y” in the domain of people. Chose one among the following statements:

    (a) If $\exists y \forall xP(x, y)$ is true, then $\forall x \exists yP(x, y)$ is also true.
    (b) If $\exists y \forall xP(x, y)$ is false, then $\forall x \exists yP(x, y)$ is also false.
    (c) If $\forall x \exists yP(x, y)$ is true, then $\exists y \forall x\, loves(x, y)$ is also true.
    (d) If $\exists y \forall xP(x, y)$ is true, then $\forall x \exists yP(x, y)$ is false.
    (e) If $\forall x \exists yP(x, y)$ is false, then $\exists y \forall xP(x, y)$ is true.

24. The formula $\forall y\ \exists x\ likes(x, y)$ translates to

    (a) “There is someone who likes everyone.”
    (b) “There is someone who is liked by everybody.”
    (c) “There is someone who likes someone.”
    (d) “Everyone likes someone.”
    (e) “Everyone is liked by someone.”

This paragraph refers to Questions 25 - 30: In the domain of all people, consider the predicate “being honest.”

25. For the statement “All people are honest”, identify a **contradictory** statement from the ones below:

    (a) Some people are not honest.
    (b) No people are honest.
    (c) Some people are honest.
    (d) All people cheat.

26. For the statement “Some people are honest”, identify its corresponding **superaltern** statement from the ones below:

(a) No people are honest.
(b) Some people are not honest.
(c) All people are cheat.
(d) No people cheat.

27. For the statement "No people are honest", identify a **contradictory** statement from the ones below:

    (a) No people cheat.
    (b) Some people are not honest.
    (c) Some people are honest.
    (d) All people are honest.

28. For the statement "No people are honest", identify its corresponding **subaltern** statement from the ones below:

    (a) No people cheat.
    (b) Some people are not honest.
    (c) Some people are honest.
    (d) All people are honest.

29. For the statement "All people are honest", identify a **contrary** statement from the ones below:

    (a) Some people are honest.
    (b) Some people are not honest.
    (c) No people cheat.
    (d) No people are honest.

30. For the statement "Some people are honest", identify a **subcontrary** statement from the ones below:

    (a) No people are honest.
    (b) Some people are not honest.
    (c) All people are honest.
    (d) All people cheat.

This paragraph refers to Questions 31 - 34 : Let $P(x)$ denote the statement "x is a person" and $Q(x)$ denote the statement "x cheats."

(i) $\forall\, x\,[P(x) \rightarrow Q(x)]$. (**A**)
(ii) $\forall\, x\,[P(x) \rightarrow \neg Q(x)]$. (**E**)
(iii) $\exists\, x\,[P(x) \wedge Q(x)]$. (**I**)

(iv) $\exists\, x\, [P(x) \wedge \neg Q(x)]$. (**O**)

31. Indicate any and all pairs of statements that are *contradictories*:
    (a) (i, iii) and (ii, iv).
    (b) (i, iv) and (ii, iii).
    (c) (iii, iv).
    (d) (i, ii).

32. Indicate any and all pairs of statements that are *contraries*:
    (a) (i, ii).
    (b) (iii, iv).
    (c) (i, iii) and (ii, iv).
    (d) (i, iv) and (ii, iii).
    (e) (i, iii).

33. Indicate any and all pairs of statements that are *subcontraries*:
    (a) (i, ii).
    (b) (i, iii).
    (c) (ii, iv).
    (d) (iii, iv).

34. Indicate any and all pairs of (superaltern, subaltern) statements:
    (a) (i, iv) and (ii, iii).
    (b) (iv, i) and (iii, ii).
    (c) (i, iii) and (ii, iv).
    (d) (iii, i) and (iv, ii).

BINARY RELATIONS

35. Consider the binary relation "likes" (over the set of all people). Select one among the following statements that describe the relation:
    (a) It is a partial order relation.
    (b) it is an equivalence relation.
    (c) It is antisymmetric.
    (d) It is symmetric but not antisymmetric.
    (e) It is neither asymmetric nor antisymmetric.

36. Consider the binary relation "is a subset of: $\subseteq$" (over the set of natural numbers). Select one among the following statements that describe the relation:

(a) It is reflexive and symmetric.
(b) It is both irreflexive and antisymmetric.
(c) It is neither irreflexive nor antisymmetric.
(d) It is an equivalence relation.
(e) It is a partial order.

37. Chose one among the following statements about binary relations:

    (a) Since a binary relation can be both asymmetric and antisymmetric, as well as neither of the two, then we can conclude that *asymmetry* $\leftrightarrow$ *antisymmetry*.
    (b) Antisymmetry implies symmetry.
    (c) Asymmetry implies antisymmetry.
    (d) A binary relation cannot be neither asymmetric nor antisymmetric.
    (e) A binary relation cannot be both asymmetric and antisymmetric.

38. Consider the binary relation "is grandson of". Select one among the following statements that best describes the relation:

    (a) It is antisymmetric
    (b) It is both asymmetric and antisymmetric.
    (c) It is both irreflexive and symmetric.
    (d) It is antireflexive and transitive.

39. Consider the sentences below:

    (i) Not symmetric implies asymmetric.
    (ii) A relation can be both symmetric and antisymmetric.
    (iii) A relation can be neither symmetric, nor asymmetric.
    (iv) A relation cannot be both asymmetric and antisymmetric.
    (v) Asymmetry does not imply antisymmetry.
    (vi) A relation can be neither asymmetric nor antisymmetric.

    Select one group with all correct statements:

    (a) i, ii, vi.
    (b) iii, iv, vi.
    (c) ii, iv, v.
    (d) i, v, vi.
    (e) ii, iii, vi.

40. Consider the binary relation "is a factor of" (over the set of natural numbers). Select one among the following statements that describe the relation:

(a) It is both asymmetric and antisymmetric.

(b) It is both reflexive and irreflexive.

(c) It is neither symmetric nor antisymmetric.

(d) It is an equivalence relation.

(e) It is a partial order.

FUNCTIONS

41. For the sets $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$, a set of mappings is defined as follows: $\{1 \mapsto d, 2 \mapsto d, 3 \mapsto c\}$. This set defines

(a) A function that is total, injective but not surjective.

(b) A function that is total, not injective, and not surjective.

(c) A function that is total, not injective but surjective.

(d) A function that is partial, not injective, not surjective.

(e) Not a function.

42. Given a function $f : A \to B$, in order to prove that it is **injective** (one-to-one), we would have to show that

(a) $\forall\, a, b \in A : a \neq b \to f(a) \neq f(b)$.

(b) $\forall\, a, b \in A : a \neq b \to f(a) = f(b)$.

(c) $\exists\, a, b \in A : a \neq b \wedge f(a) = f(b)$.

(d) $\exists\, a, b \in A : a \neq b \wedge f(a) \neq f(b)$.

43. Given a function $f : A \to B$, in order to prove that it is **surjective** (onto), we would have to show that

(a) $\forall\, b \in B, \exists\, a \in A : f(a) = b$.

(b) $\exists\, b \in B, \forall\, a \in A : f(a) \neq b$.

(c) $\exists\, a, b \in A : a \neq b \wedge f(a) = f(b)$.

(d) $\exists\, a, b \in A : a \neq b \wedge f(a) \neq f(b)$.

44. For a given function $f : A \to B$, in order to prove that is *not* an onto, we would have to show which one of the following:

(a) $\exists\, a \in A, \forall\, b \in B : f(a) \neq b$.

(b) $\exists\, a, b \in A : a \neq b \wedge f(a) = f(b)$.

(c) $\exists\, b \in B, \forall\, a \in A : f(a) \neq b$.

(d) $\forall\, a, b \in A : a \neq b \to f(a) = f(b)$.

(e) That the function has an inverse.

RELATIONAL CALCULUS

This paragraph refers to Questions 45 - 49: Consider the following relation:

$$car : Model \leftrightarrow Make$$

where

$$
\begin{array}{l}
car = \\
\quad \{ \\
\quad\quad corolla \mapsto toyota, \\
\quad\quad cherokee \mapsto jeep, \\
\quad\quad comanche \mapsto jeep, \\
\quad\quad liberty \mapsto jeep, \\
\quad\quad sentra \mapsto nissan, \\
\quad\quad m3 \mapsto bmw, \\
\quad\quad x3 \mapsto bmw, \\
\quad\quad z4 \mapsto bmw \\
\quad \}
\end{array}
$$

45. The expression $\{cherokee, m3\} \lhd car$ would produce

(a)

$$
\begin{array}{l}
\{ \\
\quad corolla \mapsto toyota, \\
\quad comanche \mapsto jeep, \\
\quad liberty \mapsto jeep, \\
\quad sentra \mapsto nissan, \\
\quad x3 \mapsto bmw, \\
\quad z4 \mapsto bmw \\
\}
\end{array}
$$

(b)

$$
\begin{array}{l}
car' = \\
\{ \\
\quad corolla \mapsto toyota, \\
\quad comanche \mapsto jeep, \\
\quad liberty \mapsto jeep, \\
\quad sentra \mapsto nissan, \\
\quad x3 \mapsto bmw, \\
\quad z4 \mapsto bmw \\
\}
\end{array}
$$

(c) $\{cherokee \mapsto jeep, m3 \mapsto bmw\}$

(d) $car' = \{cherokee \mapsto jeep, m3 \mapsto bmw\}$

(e) $\{jeep, bmw\}$

46. The expression $car \rhd \{bmw\}$ would produce

(a) $car' = \{m3 \mapsto bmw, x3 \mapsto bmw, z4 \mapsto bmw\}$

(b) $\{m3 \mapsto bmw, x3 \mapsto bmw, z4 \mapsto bmw\}$

(c) $\{m3, x3, z4\}$

(d)

$$\begin{aligned}&\{\\&\quad corolla \mapsto toyota,\\&\quad cherokee \mapsto jeep,\\&\quad comanche \mapsto jeep,\\&\quad liberty \mapsto jeep,\\&\quad sentra \mapsto nissan\\&\}\end{aligned}$$

(e)

$$\begin{aligned}&car' =\\&\quad \{\\&\quad\quad corolla \mapsto toyota,\\&\quad\quad cherokee \mapsto jeep,\\&\quad\quad comanche \mapsto jeep,\\&\quad\quad liberty \mapsto jeep,\\&\quad\quad sentra \mapsto nissan\\&\quad \}\end{aligned}$$

47. The expression $\{corolla, comanche, sentra, x3, z4\} \lhd car$ would produce

(a) $car' = \{cherokee \mapsto jeep, liberty \mapsto jeep, m3 \mapsto bmw\}$

(b) $car' = \{corolla \mapsto toyota, comanche \mapsto jeep, sentra \mapsto nissan,$
$x3 \mapsto bmw, z4 \mapsto bmw\}$

(c) $\{cherokee \mapsto jeep, liberty \mapsto jeep, m3 \mapsto bmw\}$

(d) $\{corolla \mapsto toyota, comanche \mapsto jeep, sentra \mapsto nissan, x3 \mapsto bmw, z4 \mapsto bmw\}$

(e) $\{toyota, jeep, nissan, bmw, bmw\}$

48. The expression $car \trianglerighteq \{toyota, nissan, bmw\}$ would produce

(a) $car' = \{cherokee \mapsto jeep, comanche \mapsto jeep, liberty \mapsto jeep\}$

(b) $car' = \{corolla \mapsto toyota, sentra \mapsto nissan, m3 \mapsto bmw, x3 \mapsto bmw, z4 \mapsto bmw\}$

(c) $\{corolla \mapsto toyota, sentra \mapsto nissan, m3 \mapsto bmw, x3 \mapsto bmw, z4 \mapsto bmw\}$

(d) $\{cherokee \mapsto jeep, comanche \mapsto jeep, liberty \mapsto jeep\}$

(e) $\{corolla, sentra, m3, x3, z4\}$

49. The expression $car \oplus \{corolla \mapsto lexus, m3 \mapsto porsche, liberty \mapsto ford\}$ would produce

(a) $\{corolla \mapsto lexus, m3 \mapsto porsche, liberty \mapsto ford\}$

(b) $car' = \{corolla \mapsto lexus, m3 \mapsto porsche, liberty \mapsto ford\}$

(c) $\{corolla, m3, liberty\}$

(d)

$$\begin{aligned}
\{ \\
& corolla \mapsto toyota, \\
& corolla \mapsto lexus, \\
& cherokee \mapsto jeep, \\
& comanche \mapsto jeep, \\
& liberty \mapsto jeep, \\
& liberty \mapsto ford, \\
& sentra \mapsto nissan, \\
& m3 \mapsto bmw, \\
& m3 \mapsto porsche, \\
& x3 \mapsto bmw, \\
& z4 \mapsto bmw \\
\}
\end{aligned}$$

(e)

$$\begin{aligned}
\{ \\
& corolla \mapsto lexus, \\
& cherokee \mapsto jeep, \\
& comanche \mapsto jeep, \\
& liberty \mapsto ford, \\
& sentra \mapsto nissan, \\
& m3 \mapsto porsche, \\
& x3 \mapsto bmw, \\
& z4 \mapsto bmw \\
\}
\end{aligned}$$

correct

This paragraph refers to Questions 50 - 55: Consider the following relation:

$$phone : Name \leftrightarrow Phone$$

where

$$\begin{aligned}
& phone = \\
& \{ \\
& \quad aki \mapsto 4019, \\
& \quad philip \mapsto 4107, \\
& \quad doug \mapsto 4107, \\
& \quad doug \mapsto 4136, \\
& \quad philip \mapsto 0113, \\
& \quad frank \mapsto 0110, \\
& \quad frank \mapsto 6190 \\
& \}
\end{aligned}$$

50. Select one from the following:

    (a) The relation can be expressed as an injective function.
    (b) The relation can be expressed as a surjective function.
    (c) The relation can be expressed as a bijective function.
    (d) The relation can be expressed as a partial function.
    (e) The relation cannot be represented as a function.

51. The expression $\{doug, philip\} \lhd phone$ would produce

    (a)

$$\begin{array}{l} \{ \\ \quad aki \mapsto 4019, \\ \quad frank \mapsto 0110, \\ \quad frank \mapsto 6190 \\ \} \end{array}$$

    (b)

$$\begin{array}{l} \{ \\ \quad doug \mapsto 4107, \\ \quad philip \mapsto 0113 \\ \} \end{array}$$

    (c) $\{4107, 4136, 0113\}$

    (d)

$$\begin{array}{l} \{ \\ \quad philip \mapsto 4107, \\ \quad doug \mapsto 4107, \\ \quad doug \mapsto 4136, \\ \quad philip \mapsto 0113 \\ \} \end{array}$$

    (e)

$$\begin{array}{l} phone = \\ \quad \{ \\ \quad\quad philip \mapsto 4107, \\ \quad\quad doug \mapsto 4107, \\ \quad\quad doug \mapsto 4136, \\ \quad\quad philip \mapsto 0113 \\ \quad \} \end{array}$$

52. The expression $phone \rhd \{4019, 0113, 0110, 6190\}$ would produce

(a)

$$\{ aki \mapsto 4019, philip \mapsto 0113, frank \mapsto 0110, frank \mapsto 6190 \}$$

(b)

$$\{ philip \mapsto 4107, doug \mapsto 4107, doug \mapsto 4136 \}$$

(c) $\{aki, philip, frank\}$

(d)

$$phone = \{ aki \mapsto 4019, philip \mapsto 0113, frank \mapsto 0110, frank \mapsto 6190 \}$$

(e)

$$phone' = \{ aki \mapsto 4019, philip \mapsto 0113, frank \mapsto 0110, frank \mapsto 6190 \}$$

53. The expression $\{philip, frank\} \lhd phone$ would produce

(a)

$$\{ aki \mapsto 4019, doug \mapsto 4107, doug \mapsto 4136 \}$$

(b)

$$\{ \quad philip \mapsto 4107, \quad philip \mapsto 0113, \quad frank \mapsto 0110, \quad frank \mapsto 6190 \quad \}$$

(c) $\{4107, 0113, 0110, 6190\}$

(d)

$$phone' = \{ \quad aki \mapsto 4019, \quad doug \mapsto 4107, \quad doug \mapsto 4136 \quad \}$$

(e)

$$\{ \quad philip \mapsto 0113, \quad frank \mapsto 0110 \quad \}$$

54. The expression $phone \rhd \{4107, 4136, 0113\}$ would produce

(a)

$$\{ \quad philip \mapsto 4107, \quad doug \mapsto 4107, \quad doug \mapsto 4136, \quad philip \mapsto 0113 \quad \}$$

(b) $\{philip, doug\}$

(c) $\{philip, doug, doug, philip\}$

(d) $phone' = \{philip, doug, doug, philip\}$

(e)

$$\{ \quad aki \mapsto 4019, \quad frank \mapsto 0110, \quad frank \mapsto 6190 \quad \}$$

55. The expression $phone \oplus \{doug \mapsto 5374, philip \mapsto 3000, frank \mapsto 2001\}$ would produce

(a)

$$\{ aki \mapsto 4019, philip \mapsto 3000, doug \mapsto 5374, frank \mapsto 2001 \}$$

(b)

$$\{ aki \mapsto 4019, philip \mapsto 4107, philip \mapsto 3000, doug \mapsto 4107, doug \mapsto 5374, doug \mapsto 4136, philip \mapsto 0113, frank \mapsto 0110, frank \mapsto 6190, frank \mapsto 2001 \}$$

(c) $\{doug \mapsto 5374, philip \mapsto 3000, frank \mapsto 2001\}$

(d) $\{doug \mapsto 5374, philip \mapsto 3000, frank \mapsto 2001, doug \mapsto 4107\}$

(e)

$$\{ aki \mapsto 4019, philip \mapsto 4107, doug \mapsto 4107, doug \mapsto 4136, philip \mapsto 0113, frank \mapsto 0110, frank \mapsto 6190 \}$$

56. Consider a map which can be annotated with locations. Upon instantiation, a map object contains no annotations. A location is a pair of description and its corresponding point. Even though descriptions are unique, locations may share a point on the map. This is captured by function $location : Description \rightarrow\!\!\!\!\!+ Point$. An operation can guarantee the addition of a new location on the map provided the corresponding description is not already annotated in the map. The operation would be associated with which pair of assertions, for input parameters $description? : Description$, and $point? : Point$:

(a) `pre`: $description? \notin location$
`post`: $location' = location \cup \{description? \mapsto point?\}$

(b) `pre`: $description? \notin \operatorname{ran} location$
`post`: $location' = location \cup \{description? \mapsto point?\}$

(c) `pre`: $description? \notin \operatorname{dom} location$
`post`: $location' = location \cup \{description? \mapsto point?\}$

(d) `pre`: $true$
`post`: $location' = location \oplus \{description? \mapsto point?\}$

57. Consider basic types $[LicencePlate, ParkingSpot]$ and a multi-storey long term parking garage. A vehicle that enters the garage is assigned to a single spot, and a spot can only accommodate a single vehicle. The parking management keeps a log of the cars that are parked. If we were to model the assignment of vehicles to parking spots by a function, then that function would be

(a) Total injective.

(b) Total surjective.

(c) Total bijective.

(d) Partial injective.

(e) Partial surjective.

58. Consider the specification of a system that serves as a scheduler of threads identified by their unique integer id. The threads are blocked over a critical section, and the scheduler follows a First-In-First-Out protocol.

Thread Scheduler

$elements : seq\,\mathbb{N}$
$size : \mathbb{N}$

$size \geq 0$

The schemata for operations *Enqueue* and *Dequeue* are partially given below:

Enqueue

$\Delta\, ThreadScheduler$
$thread? : \mathbb{N}$

...
$size' = size + 1$

Dequeue

$\Delta\, ThreadScheduler$
$thread! : \mathbb{N}$

$elements \neq \langle\rangle$
...
$size' = size - 1$

Assuming that we have decided to deploy the Queue ADT to implement the Scheduler with variable *elements* serving as the underlying data structure whose head will be treated as the front of the queue. For the schemas above, choose appropriate missing **postconditions**:

(a) (`Enqueue`) $elements' = \langle elements \rangle ^\frown \langle thread? \rangle$
(`Dequeue`) $thread! = head(elements), elements' = tail(elements)$

(b) (`Enqueue`) $elements' = elements ^\frown \langle thread? \rangle$
(`Dequeue`) $thread! = head(elements), elements' = tail(elements)$

(c) (`Enqueue`) $elements' = elements ^\frown thread?$
(`Dequeue`) $thread! = head\langle elements \rangle, elements' = tail(elements)$

(d) (`Enqueue`) $elements' = elements ^\frown \langle thread? \rangle$
(`Dequeue`) $thread! = head(elements), elements = tail(elements)$

(e) (`Enqueue`) $elements' = \langle thread? \rangle ^\frown elements$
(`Dequeue`) $thread! = last(elements), elements' = tail(elements)$

TEMPORAL LOGIC

59. For a formula ϕ, the *invariance* property is given by

(a) $\Diamond\phi$.

(b) $\Box\Diamond\phi$.

(c) $\bigcirc\phi$.

(d) $\Diamond\Box\phi$.

(e) $\Box\phi$.

60. For a formula ϕ, the *guarantee* property is given by

(a) $\Diamond\phi$.

(b) $\Box\Diamond\phi$.

(c) $\bigcirc\phi$.

(d) $\Diamond\Box\phi$.

(e) $\Box\phi$.

61. For a formula ϕ, the *recurrence* property is given by

(a) $\Diamond\phi$.

(b) $\Box\Diamond\phi$.

(c) $\Diamond\Box\phi$.

(d) $\bigcirc\phi$.

(e) $\Box\phi$.

62. For a formula ϕ, the *stability* property is given by

(a) $\Diamond\phi$.

(b) $\Box\Diamond\phi$.

(c) $\Diamond\Box\phi$.

(d) $\bigcirc\phi$.

(e) $\Box\phi$.

63. For well-formed formulas ϕ and ψ, the expression $\phi\,\mathcal{W}\,\psi$ is logically equivalent to

(a) $(\phi\,\mathcal{U}\,\psi) \wedge \Box\phi$

(b) $(\phi\,\mathcal{U}\,\psi) \vee \Box\psi$

(c) $(\phi\,\mathcal{U}\,\psi) \oplus \Box\phi$

(d) $(\psi\,\mathcal{R}\,\phi) \vee \Box\psi$

(e) $(\psi\,\mathcal{R}\,\phi) \wedge \Box\phi$

64. Consider the sentence "It is always the case that the system remains at state active until 08:00 or it remains active unless it is disabled." For propositions a denoting "active", t denoting "time becomes 08:00" and d denoting "system disabled", select the appropriate temporal formula:

(a) $\Box(a\mathcal{U}t \ \vee \ a\mathcal{W}d)$.

(b) $\Box(a\mathcal{U}t \ \vee \ a\mathcal{U}d)$.

(c) $\Box(a\mathcal{W}t \ \vee \ a\mathcal{W}d)$.

(d) $\Box(t\mathcal{R}a \ \vee \ d\mathcal{R}a)$.

(e) $\Box(t\mathcal{R}a \ \vee \ a\mathcal{W}d)$.

65. Consider the following set of sentences that represent the requirements of a multi-threaded system for two threads t_1 and t_2:

- $\Box\neg[(t_1\ active) \wedge (t_2\ active)]$.
- $\Box[(t_1\ active) \oplus (t_2\ active)]$.
- $\Box[(t_1\ active) \rightarrow \bigcirc(t_2\ active)]$.
- $\Box[(t_2\ active) \rightarrow \bigcirc(t_1\ active)]$.

The statements (requirements) constitute the following:

(a) A consistent (satisfiable) set.

(b) An inconsistent (unsatisfiable) set.

(c) A valid but not sound argument.

(d) An invalid argument.

(e) A sound argument

66. The behavior of a program is expressed by the following pattern:

$$\Box \begin{bmatrix} \textbf{start} \rightarrow a \oplus b \\ \\ a \rightarrow \bigcirc^3 d \\ \\ a \rightarrow \bigcirc(d\,\mathcal{R}\,f) \\ \\ b \rightarrow \bigcirc^3 c \\ \\ b \rightarrow \bigcirc(d\,\mathcal{U}\,c) \\ \\ c \rightarrow \bigcirc b \\ \\ d \rightarrow e \\ \\ e \rightarrow \bigcirc m \end{bmatrix}$$

Select one among the following observations about the program behavior:

(a) The program can terminate upon the occurrence of the sequence $\langle\, b, (d \wedge e), (d \wedge e \wedge m), (c \wedge m)\,\rangle$.

(b) The program can terminate upon the occurrence of the sequence $\langle\, a, f, f, (d \wedge e \wedge f), m\,\rangle$.

(c) The program can terminate upon the occurrence of the sequence $\langle\, a, f, (d \wedge e \wedge f), f, m\,\rangle$.

(d) The program can never terminate, as the sequence $\langle\, b, (d \wedge e), (d \wedge e \wedge m), (c \wedge m)\,\rangle$ executes indefinitely.

(e) The program can never terminate, as the sequence $\langle\, b, (d \wedge e \wedge m), (d \wedge e), (c \wedge m)\,\rangle$ executes indefinitely.

67. The behavior of a program is expressed by the following pattern:

$$\Box \begin{bmatrix} \textbf{start} \rightarrow \phi \\ \\ \phi \rightarrow \bigcirc(\chi \wedge \psi \wedge \omega) \\ \\ \omega \rightarrow \bigcirc\tau \\ \\ (\chi \wedge \psi \wedge \omega) \rightarrow \bigcirc\upsilon \\ \\ (\tau \wedge \upsilon) \rightarrow \bigcirc\phi \end{bmatrix}$$

The behavior indicates the following:

(a) The program will terminate upon the occurrence of the sequence $\langle\, \phi, (\chi \wedge \psi \wedge \omega), (\tau \wedge \upsilon)\,\rangle$.

(b) The program will terminate upon the occurrence of the sequence $\langle\, \phi, (\chi \wedge \psi \wedge \omega), (\tau \wedge \upsilon), \phi\,\rangle$.

(c) The program will indefinitely reproduce the sequence $\langle\, (\chi \wedge \psi \wedge \omega), (\tau \wedge \upsilon), \phi\,\rangle$.

(d) The program will indefinitely reproduce the sequence $\langle\, \phi, (\chi \wedge \psi \wedge \omega), (\tau \wedge \upsilon), \phi\,\rangle$.

(e) The program will indefinitely reproduce the sequence $\langle\, \phi, (\chi \wedge \psi \wedge \omega), (\tau \wedge \upsilon)\,\rangle$.

END OF EXAMINATION.
