Exam 2013, questions and answers

Principles of Electrical Engineering (Concordia University)

Elec-275: Winter 2013 Final Exam Solution

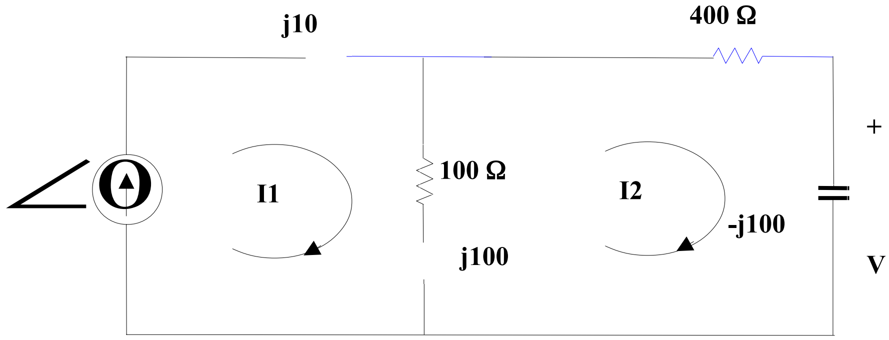
1. (a) For the time-domain circuit of Fig. 1, draw its phasor domain circuit. [Designate I_1, I_2, and V as the phasors of $i_1(t)$, $i_2(t)$, and $v(t)$ respectively]. Draw this phasor circuit.

   (b) Using **mesh analysis** on this phasor circuit, determine I_2 and V. Use the meshes shown.

   (c) Then write the time domain expressions of $i_2(t)$ and $v(t)$.

Solution:

(a) Phasor circuit:



(b) $I_1 = 0.5 \angle 0$

KVL: $500\, I_2 - (100+j100)\, I_1 = 0$; or $500\, I_2 = (100+j100) \cdot 0.5 \angle 0 = 50+j50$.

or $\mathbf{I_2} = 0{,}1+j0.1 = 0.1414 \angle 45$; $\mathbf{V_2} = -j100\, I_2 = 14.14 \angle -45$.

(c) $\boldsymbol{i_2(t)} = 0.1414 \cos(1000t+45)$ amps; $\boldsymbol{v_2(t)} = 14.14 \cos(10000t-45)$ volts.

2. Using **nodal analysis** in the phasor circuit of Fig.2,

   (a) determine the voltages V_2, V_3, and the current I ;

   (b) draw the phasor diagrams (plot of phasors in the complex plane) of V_1, V_2, V_3, and I.

Solution:

(a) KCL @ V_2: $\dfrac{V_2 - 10\angle 0}{10} + \dfrac{V_2}{j20} + \dfrac{V_2 - V_3}{-j10} = 0$; or $(-1+j2)V_2 + 2V_3 = j20$ ... (1)

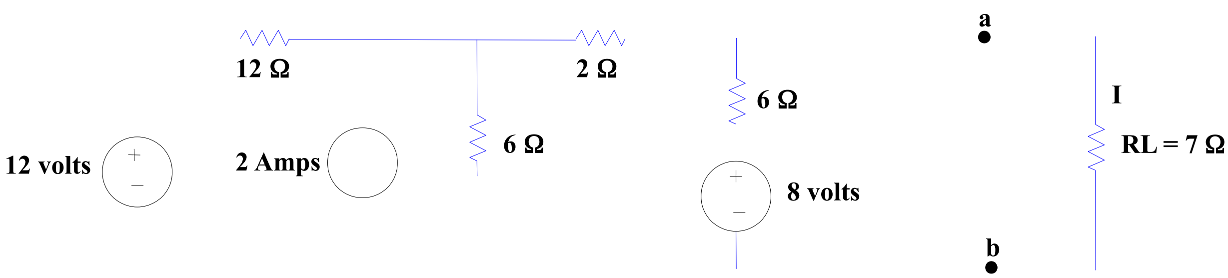
KCL @ V_3: $\dfrac{V_3}{j10} + \dfrac{V_3}{5} + \dfrac{V_3 - V_2}{-j10} - 2\angle 30 = 0$; or $V_2 + j2V_3 = -10+j17.32$ ...(2)

From (1) and (2): $\mathbf{V_2} = 6.33 \angle 41.6$; $\mathbf{V_3} = 5.3 \angle 24.2$; $\mathbf{I} = V_2/(j20) = 0.3165 \angle -48.4$

(b) Phasor diagrams may now be drawn.

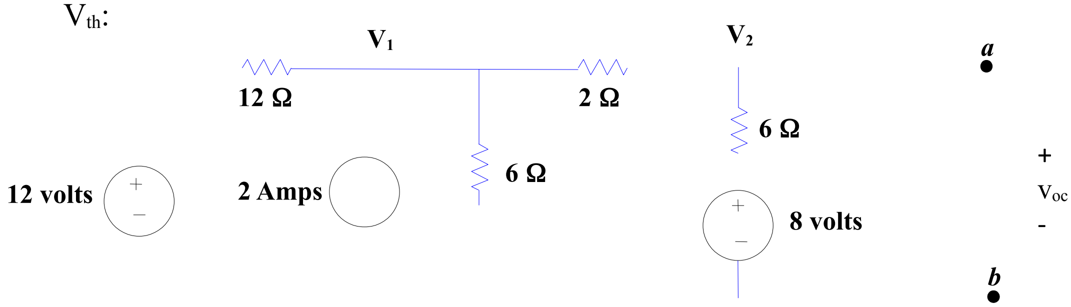
3. (a) Replace the circuit to the left of ***a - b*** of Fig. 3 by its **Thevenin** equivalent. Draw this equivalent circuit.

(b) Using this equivalent circuits, determine the current **I** through the load resistor R_L.



Solution:

V_{th}:

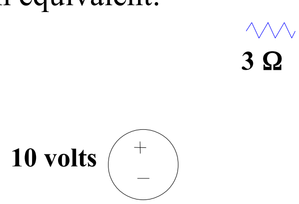


KCL @ V_1: $$\frac{V_1-12}{12}-2+\frac{V_1}{6}+\frac{V_1-V_2}{2}=0$$; or $9V_1 - 6V_2 = 36$ ... (1)

KCL @ V_2 : $$\frac{V_2-V_1}{2}+\frac{V_2-8}{6}=0$$; or $-3V_1 + 4V_2 = 8$ ... (2).

From (1) and (2), $V_2 = \mathbf{V_{th}} = 10$ volts.

$\mathbf{R_{th}} = 3\ \Omega$. Thevenin equivalent:



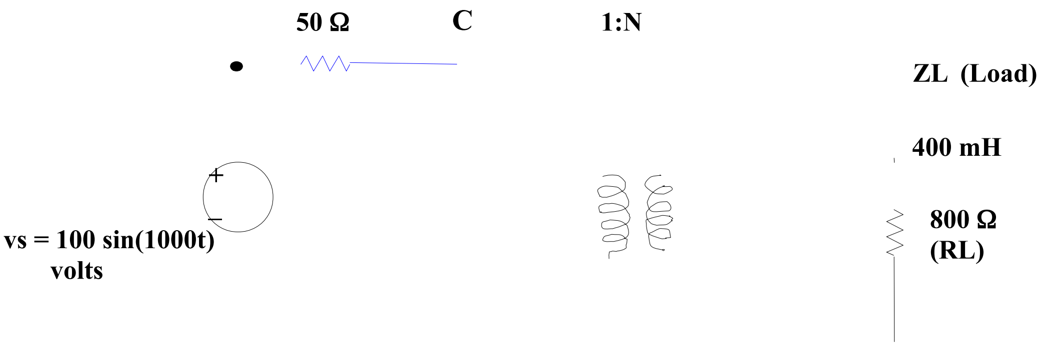
(b) $\mathbf{I} = 1$ A.

4. An ideal transformer with a turns ratio of **N** in Fig. 4 is used to match the load $\mathbf{Z_L}$ for maximum power transfer. For that purpose, determine:

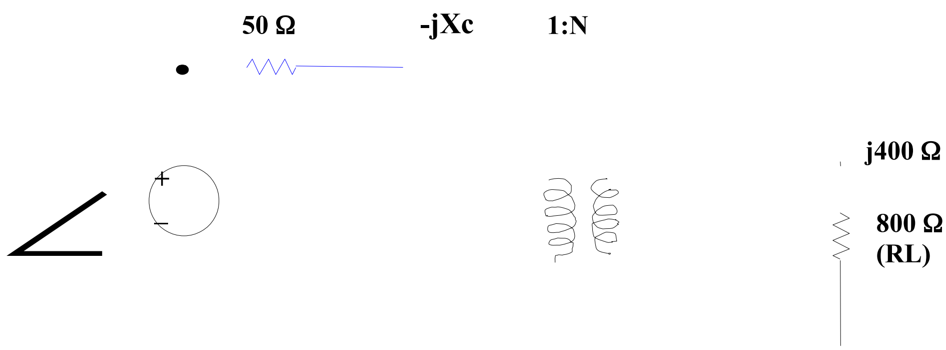
(a) the transformer turns ratio;

(b) the value of the capacitor **C**;

(c) the power absorbed by the load**.**



Solution:



(a): $N^2 = \frac{800}{50} = 16$; **N** = 4.

(b) $X_c = \frac{400}{16} = 25 = \frac{1}{1000 \times C}$; **C** = 40 μF.

(c) **P** = $\frac{(\frac{50}{\sqrt{2}})^2}{50} = 25$ watts.

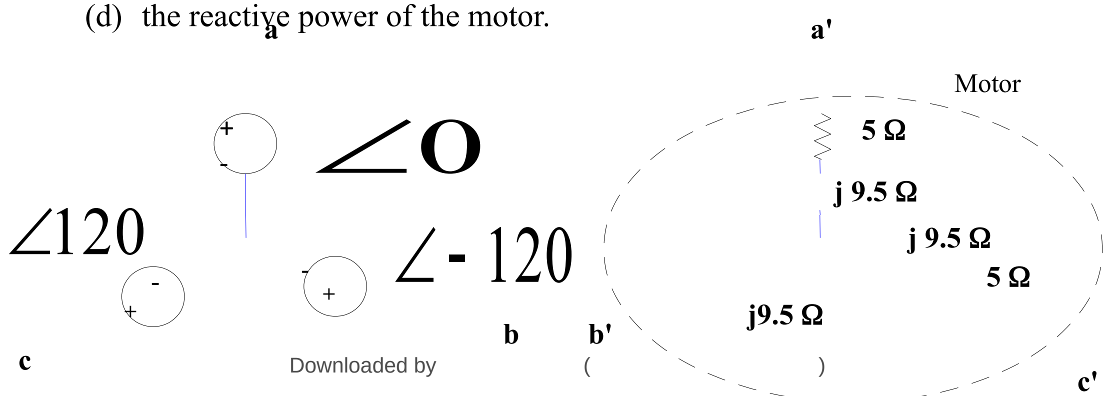
5. A three-phase 60 Hz power supply is connected to a three-phase motor as shown in Fig. 5. Find:

   (a) the power factor

   (b) the apparent power of the motor

   (c) the real power of the motor

   (d) the reactive power of the motor.



Solution:

Using single phase circuit: $Z_L = 5+j9.5 = 10.735 \angle 62.24$

(a) **P.F.** = cos 62.24 = 0.4657.

(b) S = $\frac{V_{rms}^2}{Z^{¿}} = \frac{120^2}{10.735 \angle -62.24} = 1341.4 \angle 62.24$ = 624.7+j 1187

Total power = 3 S = 4024.2 $\angle$ 62.24

Apparent power = 4024.2 VA

(c) **Real power**= 1874.34 W

(d) **Reactive power** = 3561 VAR.

6. For the magnetic circuit of Fig.6:

- Air gap cross sectional area = 2 cm ¿ 2 cm (for both gaps)
- Air gap lengths:

  l_{g1} = 2 mm
  l_{g2} = 4 mm

- Neglect the reluctance of the magnetic metallic structure (compared to those of the air gaps), as well as the fringing effect.
- The magnetizing coil has 100 turns and carries a current of 0.5 amps.

(a) Determine, for each air gap:

(i) the reluctance **R**;
(ii) the flux **ϕ.**

(b) Find
(i) the flux density **B** for air gap-1 only;
(ii) the field intensity **H** for air gap-1 only.

(c) Find the equivalent reluctance seen by the magnetomotive force **NI**.

Solution:

A = 4 ¿ 10^{-4} m^2.

(a) Air gap 1: (i) $\mathbf{R_1} = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}}=3.98\times10^{6}$ A-turns/Wb;

(ii) $\boldsymbol{\phi_1} = \dfrac{NI}{R_1}=\dfrac{50}{3.98\times10^{6}}=12.566\times10^{-6}$ Wb.

Air gap 2: (i) $\mathbf{R_2}$ = 7.98 ¿ 10^6 A-turns/Wb;

(ii) $\boldsymbol{\phi_2}$ = 6.283 ¿ 10^{-6} Wb.

(b) (i) $\mathbf{B} = \dfrac{\varphi_1}{A}=\dfrac{12.566\times10^{-6}}{4\times10^{-4}}=0.0314$ Wb/m^2

(ii) $\mathbf{H} = \dfrac{B}{\mu_0}=\dfrac{0.0314}{4\pi\times10^{-7}}=25\times10^{3}$ A-turn/m.

(c) $R_{eq} = R_1 \| R_2$ = 2.65 A-turn/Wb.

7. (a) I_f = 240/120 = 2 A.

At no-load: I_a= 6- 2 = 4A. E_b = 240- 0.4 ¿ 4 = 238.4 = $(K_a\phi)\omega_m$

= $K_a\phi$ ¿ $\dfrac{2000}{60}\times2\pi$; or $\mathbf{K_a\phi}$= $\dfrac{238.4}{2000\times2\pi}\times60$ = 1.138.

(b) At full-load: I_a = 50 -2 = 48 amps; $\mathbf{E_b}$ = 240 - 48 ¿ 0.4 = 220.8 volts.

(c) Speed = ω_m = $\dfrac{220.8}{1.138}\times\dfrac{60}{2\pi}=1852.35$ rpm.

(d) Torque = $(K_a\phi)\, I_a$ = 1.138 ¿ 48 = 54.637 N-m

(e) Power= Torque ¿ speed = 54.637 ¿ $\dfrac{1852.35}{60}\times2\pi$ = 10,598.4 W = 14.2 HP.

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