ENGR213 MidTerm 1

Applied Ordinary Differential Equations (Concordia University)

ENGR 213 - APPLIED ADVANCED CALCULUS
TEST 1

This test has 5 problems, and each problem is worth 20 points. To receive full credit, you must show your work.

Problem 1. Find a solution to the differential equation

$$\frac{dy}{dt} = \frac{t+y}{1-t-y}.$$

Problem 2. Find an explicit solution to the Bernoulli differential equation

$$\frac{dy}{dx} + \frac{1}{5x}y = \frac{-1}{5}x\cot(x)y^6.$$

Problem 3. Use an appropriate integrating factor to make the differential equation

$$(x^3 - 2xy^2)\,dx - x^2y\,dy = 0$$

exact, then find an implicit solution to this differential equation.

Problem 4. A large tank is filled with 2000 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 12 gal/min. The well-mixed solution is pumped out at the same rate. Find a function that describes the amount of salt in the tank at time t. How much salt will be in the tank after a very long time?

Recall: The rate of change of the amount of salt in the tank is given by the difference between the rate of salt coming into the tank and the amount of salt leaving the tank.

Problem 5. A cylindrical tank that is 5m tall, with a radius of 2m is standing on end, and water is draining through a hole in the bottom whose radius is 3cm. If the tank was initially half full, at what time will the tank be completely drained?

Recall: The height of the water in the tank is governed by the differential equation

$$\frac{dh}{dt} = -\frac{A_h}{A_t}\sqrt{2gh}$$

where A_h is the area of the hole and A_t is the cross-sectional area of the tank.