Programming Assignment #2 - CS325

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Introduction

In this writeup, we will both assess time complexity and prove three algorithms designed to solve the Maximum Subarray problem.

According to Wikipedia:

'In computer science, the maximum subarray problem is the task of finding the contiguous subarray within a one-dimensional array of numbers (containing at least one positive number) which has the largest sum. For example, for the sequence of values 2, 1, 3, 4, 1, 2, 1, 5, 4; the contiguous subarray with the largest sum is 4, 1, 2, 1, with sum 6.'

Pseudocode

Brute Force:

```
def brute_force(x):
  max_sum = 0
  l = len(x)
  for i in xrange(l):
    new_sum = 0
    for j in range(i,l):
      new_sum += x[j]
      if new_sum > max_sum: max_sum = new_sum
  return max_sum
```

Divide and Conquer:

```
def div_and_conq(x):
  max_sum = 0
  def inner(x,max_sum):

    if (len(x) <= 1): # base case
      if sum(x) > max_sum:
        max_sum = sum(x)
      return max_sum

    mid = int(len(x)/2) # split into two halves
    l = x[:mid]
    r = x[mid:]

    max_sum = max(max_sum, sum(l)) # is left half max?
    max_sum = max(max_sum, sum(r)) # is right half max?

    # does max consists of suffix+prefix?
    suffix_sum = 0
    max_suffix_sum = 0
    for i in range(mid-1,-1,-1):
      suffix_sum += x[i]
      max_suffix_sum = max(max_suffix_sum, suffix_sum)
    prefix_sum = 0
```

```
        max_prefix_sum = 0
        for i in range(mid,len(x)):
            prefix_sum += x[i]
            max_prefix_sum = max(max_prefix_sum, prefix_sum)
        max_sum = max(max_sum, max_suffix_sum + max_prefix_sum)

        # recursive calls
        ret = inner(l,max_sum)
        if (ret != None):
            max_sum = max(ret,max_sum)
        ret = inner(r,max_sum)
        if (ret != None):
            max_sum = max(ret,max_sum)

        return max_sum # end of inner function

    return inner(x,max_sum)
```

Dynamic Programming:

```
def dynamic_prog(x):
    this_sub_arr_sum = 0
    max_sum = 0
    for i in x:
        if this_sub_arr_sum + i > 0:
            this_sub_arr_sum = this_sub_arr_sum + i
        else:
            this_sub_arr_sum = 0
        if this_sub_arr_sum > max_sum:
            max_sum = this_sub_arr_sum

    return max_sum
```

Correctness Proofs

Divide and Conquer:

Dynamic Programming:

Asymptotic Analysis of Run Time

Brute Force:

Divide and Conquer:

Dynamic Programming:

Empirical Testing of Correctness

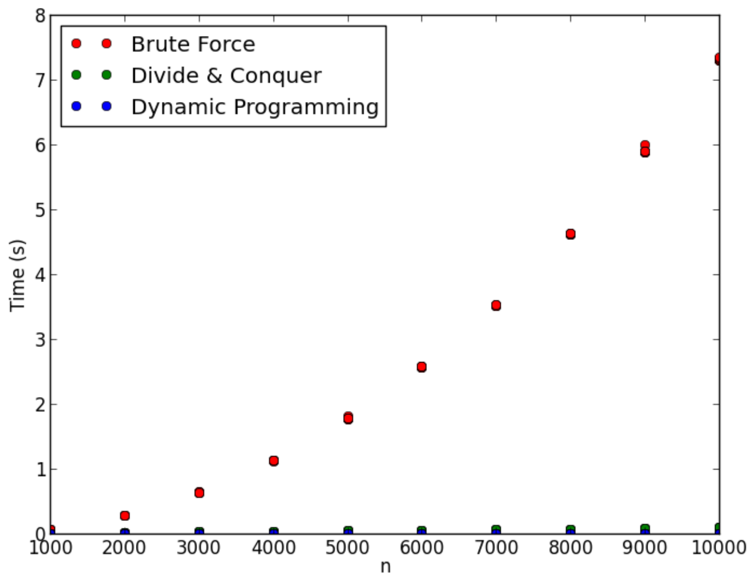
Correctness of the three algorithms was verified using the provided file of test cases verify_2.txt. See the function 'test_correctness' in main.py for details.

Here is the output from the final text input file (name.txt):

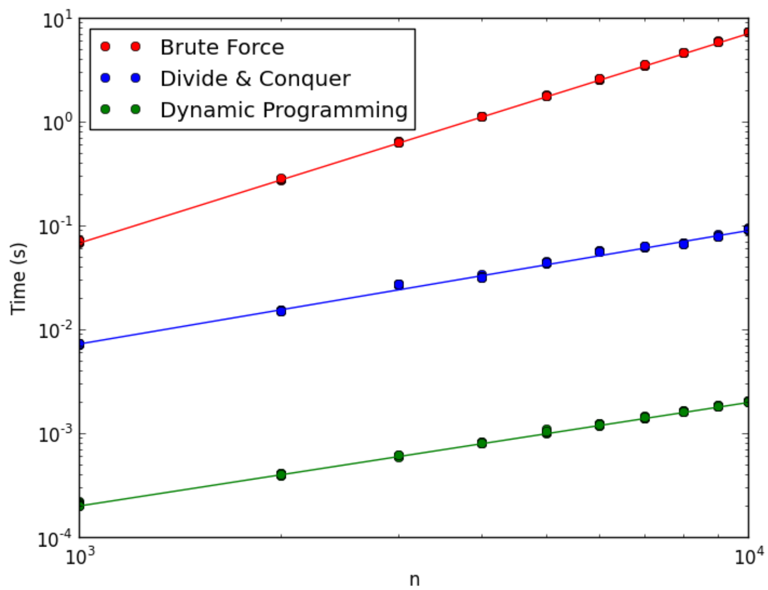
- OUTPUT HERE (TODO)

Empirical Analysis of Run Time

Linear Plot:



Log-log Plot:



Slope of lines in log-log plot:

The equation for the best fit line on the log-log plot (calculated using numpy.polyfit()) has the following form:

$f(n) = e^{\text{y-intercept}} * n^{\text{slope}}$

Brute Force:

- slope: 2.0177140011

Divide & Conquer:

- slope: 1.09279503259

Dynamic Programming:

- slope: .996887276493

Performance Comparison

'In your report, present and compare the empirical run time results of the three different algorithms. Provide a discussion of the comparative benefits and drawbacks of different algorithms.'