SPECIFICATION OF THE PROJECT "ORBITS IN COXETER GROUPS"

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1. Input

The input to your program consists of

- a string of length 2 of the form `X`n, where `X` is a letter and n a number
- A vector v of length n

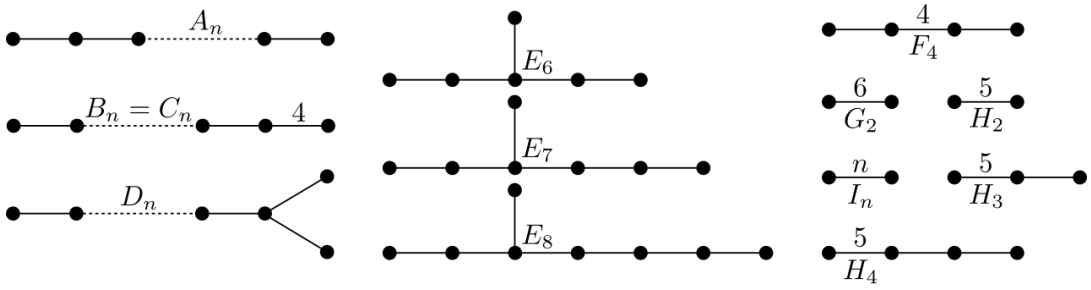
You should read this data from a file that contains the string on the first line and the vector on the second.

Your code for processing this data should be in a separate function that is called from the function responsible for reading the input. This also makes it possible to test your program from the testsuite.

2. Validation

- The input string must refer to a Coxeter arrangement of hyperplanes, i.e., be one of

`A`n`, B`n`, D`n`, E6, E7, E8, F4, G2, H3, H4, I`n`.`



- The vector v should really have length n

3. Processing

(1) First, you must determine a set of reflecting linear hyperplanes corresponding to the selected Coxeter group. These should be represented by their normal vectors, and these in turn should be hard-coded in your program.

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For example, a standard set of normal vectors for the Coxeter arrangement of type A_n is formed by the rows of the matrix

$$\begin{pmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ \dots & & & & & & \\ 0 & 0 & 0 & 0 & \dots & -1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \end{pmatrix},$$

and for type B_n by the rows of the matrix

$$\begin{pmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ \dots & & & & & & \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}.$$

Notice that the matrix for A_n has size $n \times (n+1)$, so it specifies n normal vectors in $\mathbb{R}^{n+1}$. This is the only Coxeter group for which this happens, all the others (for example the group B_n in the second example) are generated by n vectors in $\mathbb{R}^n$.

Also, you should check that the relation

$$\cos \frac{\pi}{p_{i,j}} \;=\; -\frac{\langle w_i,\, w_j \rangle}{\|w_i\| \|w_j\|}$$

holds for any pair of vectors w_i, w_j of A_n and B_n, where $\frac{\pi}{p_{i,j}}$ is the angle between the hyperplanes with normal vectors w_i and w_j, and $p_{i,j}$ is encoded by the Coxeter-Dynkin diagram: Two nodes not connected by an edge have $p_{i,j} = 2$, an undecorated edge between two nodes represents $p_{i,j} = 3$, and in all other cases the edge is labeled with $p_{i,j}$.

Can you find representative matrices for other Coxeter diagrams?

4. Output

By default, you should output the size of the orbit of v that is obtained by repeatedly reflecting v in the hyperplanes given by the w_i. Via a flag on the command line, you should be able to specify whether to additionally output the orbit itself.