

# Homework 2

Math. 481a, Spring 2026

## SHOW ALL YOUR WORK

**IMPORTANT:** Please do all your work in space provided.

If needed, you can use backspaces. No additional sheets of paper will be accepted.

Check that your homework has a total of 6 pages—there are no blank pages.

**Problem 1.** (1+1+1+1 points)

Aitken's  $\Delta^2$  Method has the form

$$\hat{p}_n = \frac{p_{n+2}p_n - p_{n+1}^2}{p_{n+2} - 2p_{n+1} + p_n}$$

(a) Apply the above Aitken's  $\Delta^2$  Method to the functional iteration  $x_{n+1} = g(x_n)$  and express your result as a new functional iteration  $x = G(x)$ .

(b) Does the functional iteration for  $g(x) = x + x^2$  converge or diverge? Assume  $p_0 > 0$ . *Provide details!*

(c) Use the result of part (a) to find  $G(x)$  with  $g(x) = x + x^2$ .

(d) Does the functional iteration for  $G(x)$  converge or diverge? *Provide details!*

**Problem 2.** (2 points)

Use Theorem 2.1 (p. 51) to find a bound for the number of iterations needed to achieve an approximation with accuracy  $10^{-5}$  to the solution of  $x^3 + x - 4 = 0$  lying in the interval  $[1, 4]$ .

**Problem 3.** (2+1+1 points)

The following (incomplete) code in *Matlab* was proposed to use the bisection method in order to compute a root of the equation  $\tan x - x = 0$ .

```
% The bisection algorithm for finding
% a root of the equation tan(x)-x=0.
f=inline('tan(x)-x');
a=4.3; b=4.6; iter=0;
if f(a)*f(b)>0
error( 'f(a) and f(b) do not have opposite signs' )
else
    p = (a + b)/2;
    err = abs(f(p));
    while err > 0.01
        if f(a)*f(p)<0
            ....;
        else
            ....;
        end
        iter=iter+1;
        ....;
    err = abs(f(p));
    end
end
```

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(a) Fill in the lines indicated by the arrows in order to complete the above code.

**Note:** You are **NOT** allowed to change or remove any other lines of the code.

(b) How many iterations are needed for the above accuracy?

(c) What is the approximation of the root of  $\tan x - x = 0$  computed by the above (completed) code?

**Problem 4.** (1+1+1+1 points)

The following four methods are proposed to compute  $7^{1/5}$ . Rank them in order, based on their apparent speed of convergence, assuming  $p_0 = 1$ .

$$\begin{array}{ll} \text{(a)} & p_n = \left( \frac{7}{p_{n-1}} \right)^{1/4} \\ & \\ \text{(c)} & p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{5p_{n-1}^4} \end{array} \quad \begin{array}{ll} \text{(b)} & p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{p_{n-1}^2} \\ & \\ \text{(d)} & p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{12} \end{array}$$

**Problem 5.** (2 points)

Determine a function  $g$  and an interval  $[a, b]$  on which fixed-point iteration will converge to a positive solution of  $3x^2 - e^x = 0$ . Find the solution to within  $10^{-3}$ .

**Note:** Stop the algorithm when  $|p_N - p_{N-1}| < 10^{-3}$ .

**Problem 6.** (3 points)

Use Theorem 2.4 (p. 61) to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to  $\sqrt{2}$  whenever  $x_0 \in [1, 2]$ .

**Problem 7.** (3 points)

Use Newton's method to approximate, to within  $10^{-4}$ , the value of  $x$  that produces the point on the graph of  $y = x^2$  that is closest to  $(1, 0)$ .

**Hint:** Minimize  $[d(x)]^2$ , where  $d(x)$  represents the distance from  $(x, x^2)$  to  $(1, 0)$ .

**Note:** Stop the algorithm when  $|p_N - p_{N-1}| < 10^{-4}$ .