

Homework 2

Math. 481a, Spring 2026

SHOW ALL YOUR WORK

IMPORTANT: Please do all your work in space provided.

If needed, you can use backspaces. No additional sheets of paper will be accepted.

Check that your homework has a total of 6 pages—there are no blank pages.

Problem 1. (1+1+1+1 points)

Aitken's Δ^2 Method has the form

$$\hat{p}_n = \frac{p_{n+2}p_n - p_{n+1}^2}{p_{n+2} - 2p_{n+1} + p_n}$$

(a) Apply the above Aitken's Δ^2 Method to the functional iteration $x_{n+1} = g(x_n)$ and express your result as a new functional iteration $x = G(x)$.

(b) Does the functional iteration for $g(x) = x + x^2$ converge or diverge? Assume $p_0 > 0$. *Provide details!*

(c) Use the result of part (a) to find $G(x)$ with $g(x) = x + x^2$.

(d) Does the functional iteration for $G(x)$ converge or diverge? *Provide details!*

Problem 2. (2 points)

Use Theorem 2.1 (p. 51) to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-5} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1, 4]$.

Problem 3. (2+1+1 points)

The following (incomplete) code in *Matlab* was proposed to use the bisection method in order to compute a root of the equation $\tan x - x = 0$.

```
% The bisection algorithm for finding
% a root of the equation tan(x)-x=0.
f=inline('tan(x)-x');
a=4.3; b=4.6; iter=0;
if f(a)*f(b)>0
error( 'f(a) and f(b) do not have opposite signs' )
else
    p = (a + b)/2;
    err = abs(f(p));
    while err > 0.01
        if f(a)*f(p)<0
            .....;
        else
            .....;
        end
        iter=iter+1;
        .....;
    err = abs(f(p));
    end
end
```

(a) Fill in the lines indicated by the arrows in order to complete the above code.

Note: You are **NOT** allowed to change or remove any other lines of the code.

(b) How many iterations are needed for the above accuracy?

(c) What is the approximation of the root of $\tan x - x = 0$ computed by the above (completed) code?

Problem 4. (1+1+1+1 points)

The following four methods are proposed to compute $7^{1/5}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$.

$$(a) \quad p_n = \left(\frac{7}{p_{n-1}} \right)^{1/4}$$

$$(b) \quad p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{p_{n-1}^2}$$

$$(c) \quad p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{5p_{n-1}^4}$$

$$(d) \quad p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{12}$$

Problem 5. (2 points)

Determine a function g and an interval $[a, b]$ on which fixed-point iteration will converge to a positive solution of $3x^2 - e^x = 0$. Find the solution to within 10^{-3} .

Note: *Stop the algorithm when $|p_N - p_{N-1}| < 10^{-3}$.*

Problem 6. (3 points)

Use Theorem 2.4 (p. 61) to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{2}$ whenever $x_0 \in [1, 2]$.

Problem 7. (3 points)

Use Newton's method to approximate, to within 10^{-4} , the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

Hint: Minimize $[d(x)]^2$, where $d(x)$ represents the distance from (x, x^2) to $(1, 0)$.

Note: Stop the algorithm when $|p_N - p_{N-1}| < 10^{-4}$.