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Project 1 Report

Approach - Given Data

To solve the problem, we defined the unknown variables as the quantity of each bond purchased. Our objective is to minimize the amount that we have to spend today to exactly meet our liabilities in the future years. According to the nature of coupon bonds, we will be paid the amount of the coupon before the bond matures and at maturity, the coupon and face value will be disbursed. While accounting for the cash flows, we have to also consider the expenses incurred by purchasing the bonds in years after year 0. The price of these bonds must be paid from the cash flows during the year. To summarize:

1. Between the start time and maturity of a bond, it pays the coupon.
2. At maturity, the bond pays the face value and the coupon
3. If a bond starts in year after year 0 we will pay for the price of the bond by cash flows in that year, assuming that the bond needs to be purchased.
4. Our objective function will not have any weights on the bonds not purchased in year 0 as they are purchased by the cash flows in the corresponding year

```
obj = np.array([0]*len(df2))
for p in range(len(df2)):
    if(df2['StartTime'][p]==0):
        obj[p] = df2['Price'][p]

B = np.array(df["Liability"])

sense = np.array(['>']*len(df))
bond_Model = gp.Model()
bond_ModX = bond_Model.addMVar(len(df2))
bond_ModCon = bond_Model.addMConstrs(A, bond_ModX, sense, B)
bond_Model.setMObjective(None,obj,0,sense=gp.GRB.MINIMIZE)

bond_Model.optimize()
print("Optimal Value:",bond_Model.objVal)
```

Our code assigns the liabilities.csv to df and bonds.csv to df2. Calling the find_cf function and passing these files as dataframes to the function gives us the results. The result is also in the form of two dataframes. One dataframe describes the bond and the number of bonds purchased while the other one displays the shadow price and the upper and lower bound of the constraints.

Results - Given Data

From the given data, the optimal mixture of bonds and forwards we found using the given bonds and liabilities was:

	Bond	Number of Bonds
0	1	6522.49
1	2	0.00
2	3	12848.62
3	4	0.00
4	5	15298.32
5	6	15680.78
6	7	0.00
7	8	12308.01
8	9	0.00
9	10	12415.73
10	11	10408.99
11	12	9345.79
12	13	0.00

The most we buy is Bond 6 and the least we buy is Bond 1. From the optimal mixture, we see that we are not purchasing Bond 2, 4, 7, 9 and 13. Note that we are not purchasing any of the forwards. Based on the purchase today and the constraint we have given, the cash inflow will cover the future expenditures.

We have drawn two plots.

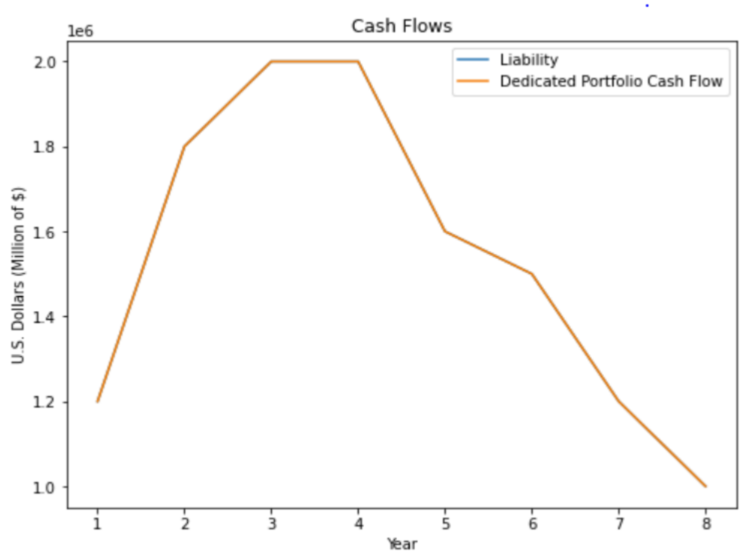

Figure 1: In this figure, we compare the expected cash flow resulting from liabilities and that from our dedicated portfolio. We only see one line in the plot since we are exactly matching the liabilities with the cash flow in the year. The liability is highest in year 3 and 4.

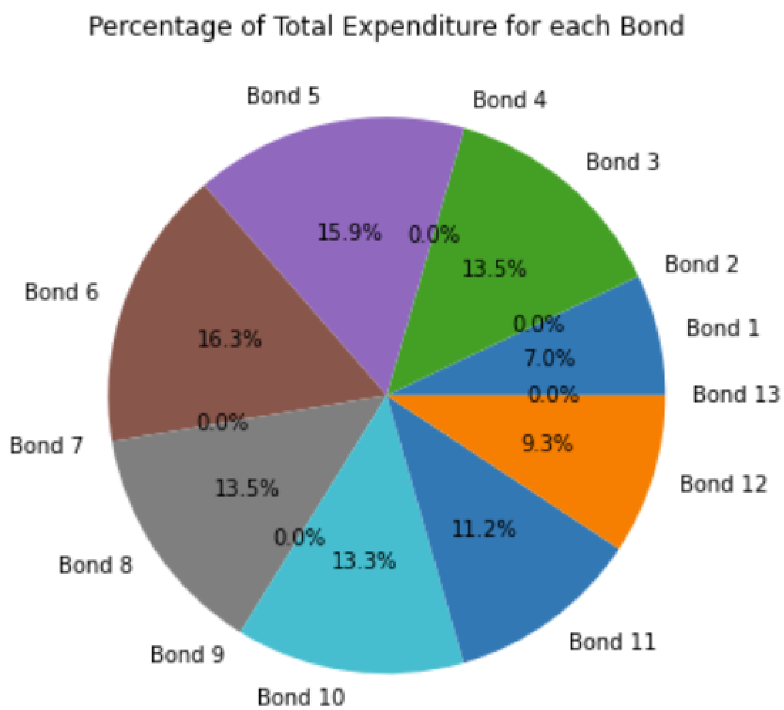

Figure 2: This pie chart indicates how much money will be allocated to each bond. The amount of money taken to purchase Bond 6 will be largest, and simultaneously Bond 6 will take the largest number. One notable thing is that here the bonds which do not take any portion of the expenditure are purely the result from the selection during optimization. It does mean that it happens because not choosing those bonds can allow the portfolio to minimize the number of bonds. This is not because the bonds we put here are forward bonds. So except for the zero-value bonds, Bond 1 takes the least place among the bonds which will be bought.

Sensitivity Analysis - Given Data

	Shadow Price	Lower Bound	Upper Bound	Liabities
0	0.971429	5.151384e+05	inf	1200000
1	0.923671	4.701682e+05	2.205234e+07	1800000
2	0.909876	4.319224e+05	3.106210e+07	2000000
3	0.834424	3.691993e+05	2.089037e+07	2000000
4	0.653628	2.584273e+05	1.075133e+07	1600000
5	0.617183	1.591014e+05	1.261887e+07	1500000
6	0.530350	6.542056e+04	1.197295e+07	1200000
7	0.522580	-1.164153e-10	1.582050e+07	1000000

This data frame represents the sensitivity of the objective value to the constraints. A dollar increase in the liabilities in the first year increases the objective value by approximately 97 cents. This change, also known as the shadow price, is bounded by the lower limit of $515,138 while having no upper limit bound. Similarly, a dollar increase in liabilities in the second year will increase the objective value by approximately 92 cents.

Approach - WSJ

We used selenium in python to extract data from the WSJ link provided. The data was extracted on 10/06/2021. Using pandas in Jupyter Notebook we formatted the data to make it compatible with the function. Once the data was formatted, we simply called the function to get the value for our objectives and the quantity of each bond to buy along with the sensitivity to the liabilities.

```
wd = webdriver.Chrome('chromedriver',options=chrome_options)
columns = ['Date','Coupon','Bid Price','Ask Price']
df = pd.DataFrame(columns = columns)

wd = webdriver.Chrome('chromedriver',options=chrome_options)
url = "https://www.wsj.com/market-data/bonds/treasuries"
wd.get(url)

rows=1+len(wd.find_elements_by_xpath('//*[@id="root"]/div/div/div/div[2]/div/div/div[4]/div/table/tbody/tr'))
cols = len(wd.find_elements_by_xpath('//*[@id="root"]/div/div/div/div[2]/div/div/div[4]/div/table/tbody/tr[1]/td'))
path = '//*[@id="root"]/div/div/div/div[2]/div/div/div[4]/div/table/tbody/tr'
for r in range(1, rows):
       for p in range(1, cols-1):
            value = wd.find_element_by_xpath(path+"[" + str(r) + "]/td["+str(p)+']').text
            df.loc[r-1,columns[p-1]] = value
```

Results - WSJ

From the scraped data, the optimal mixture of bonds we found using the given liabilities was:

	Bond	Number of Bonds
0	1	0.000000
1	2	0.000000
2	3	6649.949565
3	4	0.000000
4	5	0.000000
5	6	13132.070909
6	7	15952.825341
7	8	0.000000
8	9	0.000000
9	10	16012.648436
10	11	0.000000
11	12	13113.518016
12	13	0.000000
13	14	12998.680482
14	15	0.000000
15	16	10827.346362
16	17	0.000000
17	18	9422.850412

We are purchasing the largest number of bonds for Bond 10 and the smallest number of bonds for Bond 3. From the optimal mixture, we see that we are not purchasing Bond 1, 2, 4, 5, 8, 9, 11, 13, 15, and 17. Based on the purchase today and the constraint we have given, the cash inflow will cover the future expenditures.

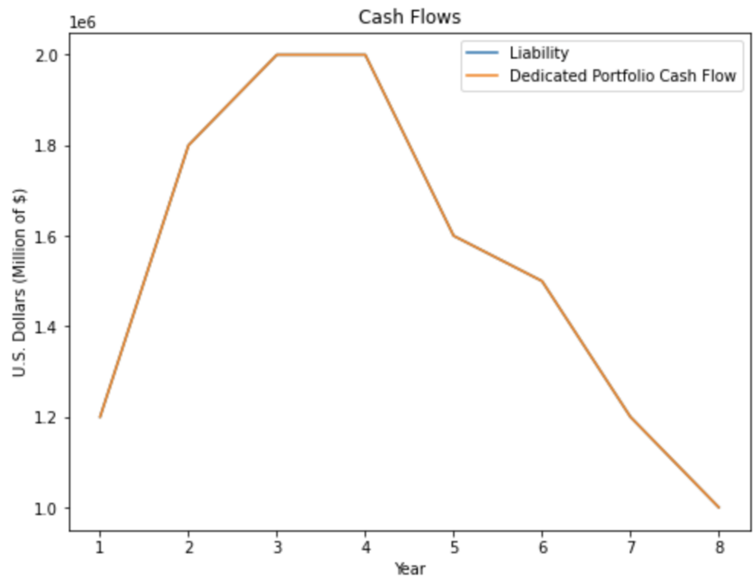

Figure 3: This line plot illustrates the cash flows, from year 1 to 8, of both the liabilities and the dedicated portfolio. The line for the liability cash flows and the line for the portfolio cash flows are overlapping because we set a constraint of matching their cash flows. Moreover, the cash flows of the dedicated portfolio built using the WSJ data is the same as the cash flows of the dedicated portfolio built using the given data are the same because we set a constraint of matching them with the same liability cash flows.

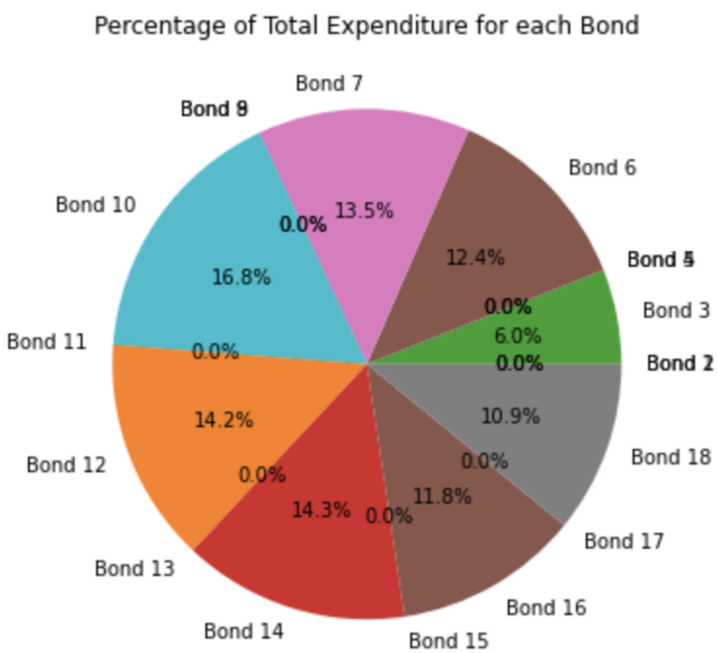

Figure 4: This pie chart illustrates the percentage of total expenditure for each bond. The chart shows 0.0% for Bonds 1, 2, 4, 5, 8, 9, 11, 13, 15, and 17. Bond 10 takes up the largest proportion of the portfolio.

Sensitivity - WSJ

This data frame represents the sensitivity of the objective value to the constraints, but here we used the scraped data from the web on 6th, October. A dollar increase in the liabilities in the first year increases the objective value by approximately 99 cents. This change, also known as the shadow price, is bounded by the lower limit of $486,793 while having no upper limit bound. Similarly, a dollar increase in liabilities in the second year will increase the objective value by approximately 99 cents, although here there is a lower limit of $404,717 and upper limit of $13,924,520.

	Shadow Price	Lower Bound	Upper Bound	Liabities
0	0.988345	486792.909121	inf	1200000
1	0.986568	404717.465941	1.392452e+07	1800000
2	0.978923	398735.156438	2.048331e+08	2000000
3	0.960864	288648.198443	1.382425e+07	2000000
4	0.942162	200131.951838	1.445613e+07	1600000
5	0.921622	117265.363768	1.598017e+07	1500000
6	0.902543	57714.958775	1.890694e+07	1200000
7	0.886490	-0.000000	1.787399e+07	1000000