Recap S1:

Toward a variational estimation of the steady state in open quantum systems using Generalized Gibbs Ensemble

February 26, 2024

Open quantum systems

- Lindblad equation:

$$\dot{\rho} = \mathcal{L}\,\rho \equiv -i\,[H,\rho] + \sum_{k=1}^{N^2-1} \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right) \tag{1}$$

- Steady state:

$$\dot{\rho} \equiv 0$$

- thermalisation performed by Gibbs ensemble (GE):

$$\rho_{\mathrm{GE}} = \frac{\mathrm{e}^{-\beta \hat{H}}}{\mathrm{Tr}\left(\mathrm{e}^{-\beta \hat{H}}\right)} \tag{2}$$

- Von Neumann entropy

$$S(\rho) = -\mathrm{Tr}\left(\rho \ln \rho\right) \tag{3}$$

- definition GGE (Max Entropy Principle):

$$\rho_{\mathrm{GGE}} = \mathrm{argmax}_{\rho} \left\{ S(\rho) : \forall A \in \mathcal{C} : \mathrm{Tr}\left(A\,\rho\right) = \mathrm{Tr}\left(A\,\rho(0)\right) \right\} \tag{4}$$

- General form of GGE (= most general form of any steady state):

$$\rho_{\mathrm{GGE}} = \frac{\mathrm{e}^{-\sum\limits_{A \in \mathcal{C}} \lambda_A A}}{\mathrm{Tr}\left(\mathrm{e}^{-\sum\limits_{A \in \mathcal{C}} \lambda_A A}\right)} \tag{5}$$

Variationnal estimation of the steady state: principle

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1. Start from GGE ansatz with predefined conserved quantities:

$$\rho_{\mathrm{GGE}} = \frac{\mathrm{e}^{-\sum\limits_{A \in \mathcal{C}} \lambda_A A}}{\mathrm{Tr}\left(\mathrm{e}^{-\sum\limits_{A \in \mathcal{C}} \lambda_A A}\right)}$$

2. Vary the Lagrange multipliers $\{\lambda_A\}_{A \in \mathcal{C}}$ to minimize $||\, \dot{\rho}\, ||$

Application: the Damped harmonic oscillator

- Lindblad equation ($\bar{n} = (\exp \beta\omega - 1)^{-1}$):

$$\begin{aligned}\mathcal{L}_H \rho = &- i\, [H, \rho] \\ &+ \gamma(\bar{n}+1) \left(a\rho a^\dagger - \frac{1}{2} \left\{ a^\dagger a, \rho \right\} \right) \\ &+ \gamma\bar{n} \left(a^\dagger \rho a - \frac{1}{2} \left\{ a a^\dagger, \rho \right\} \right)\end{aligned} \tag{6}$$

- Different Hamiltonians (ϵ small) and ansätze:

$$\begin{aligned} H_0 &= a^\dagger a \\ H_1 &= a^\dagger a + \epsilon \left(a + a^\dagger \right) \\ H_2 &= a^\dagger a + \epsilon\, a^\dagger a^\dagger a a \end{aligned}$$

$$\rho_{\mathrm{GGE}\,1} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a}\right)}$$

$$\rho_{\mathrm{GGE}\,2} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}\right)}$$

Results

- Hamiltonian:

$$H_0 = a^\dagger a$$

- Ansatz:

$$\rho_{\mathrm{GGE}\,1} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a}\right)}$$

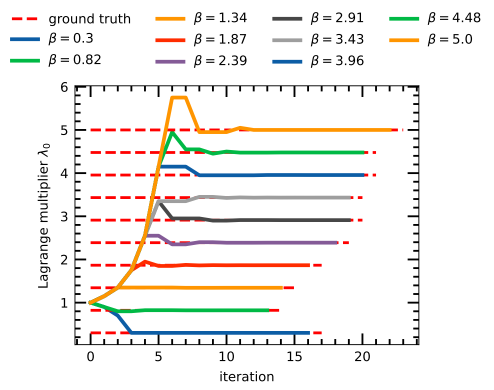


Figure: Optimization of the GGE: Lagrange multiplier as a function of the number of iterations.

Results - HO 2

$$H_0 = a^\dagger a$$

$$\rho_{\mathrm{GGE}\,2} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}\right)}$$

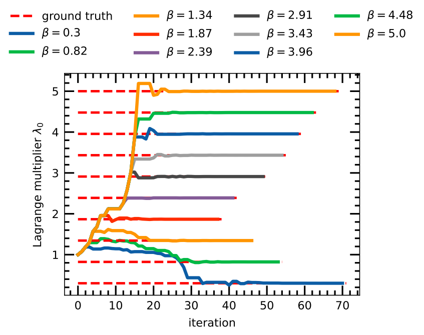


Figure: λ_0 as a function of the number of iterations.

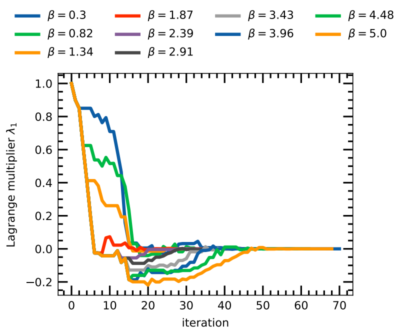
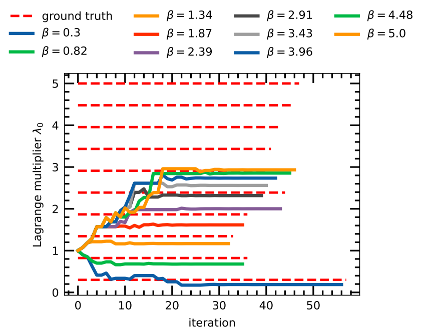


Figure: λ_1 as a function of the number of iterations.

$$H_1 = a^\dagger a + \epsilon \left(a + a^\dagger\right)$$

$$\rho_{\mathrm{GGE}\,2} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}\right)}$$



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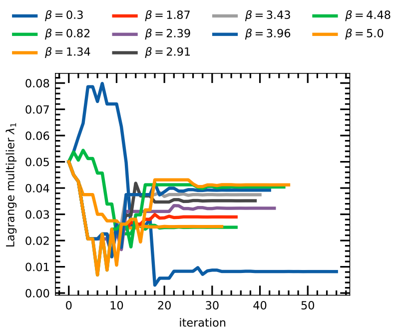


Figure: λ_1 as a function of the number of iterations.

Results - Kerr

$$H_2 = a^\dagger a + \epsilon\, a^\dagger a^\dagger a a$$

$$\rho_{\mathrm{GGE}\,2} = \frac{\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}}{\mathrm{Tr}\left(\mathrm{e}^{-\lambda_0\, a^\dagger a - \lambda_1\, a^\dagger a^\dagger a a}\right)}$$

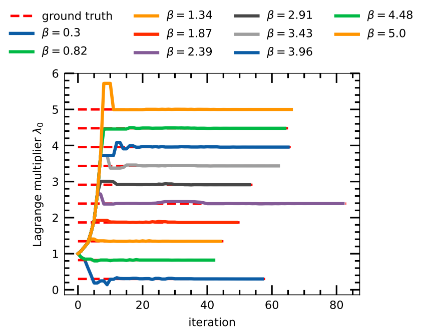
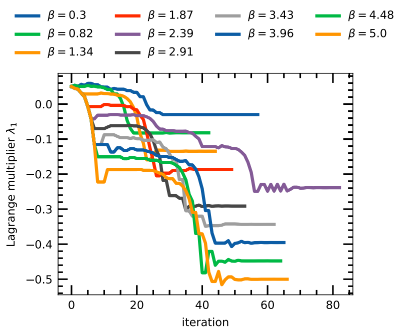


Figure: λ_0 as a function of the number of iterations.



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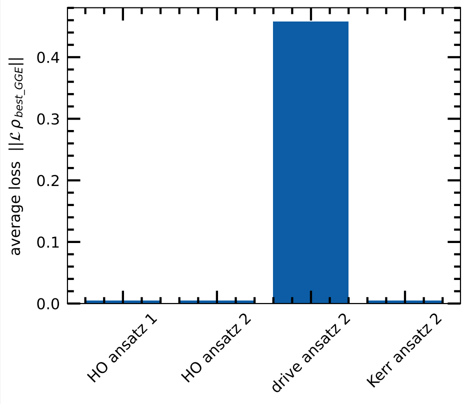

Figure: Comparison of the losses of the optimized GGEs in the various scenarios. The loss is the average value of $||\,\mathcal{L}\,\rho_{\mathrm{GGE}}\,||$ calculated across all true temperatures.

Outlook

Outlook

- Use a variational state less informed by the analytical solution (= more general way to express conserved quantities ?)
- Generalize the method in the multi-body case

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