Day 8 (27/01/26)

- 2 vectors LI $\Rightarrow$ can represent any vector in the whole space where vectors lie

Check: If 3 vectors together will it always project in 3D? (as to 2 vectors 2D?)

NOTE: dimension = no. of independent directions

- $x = \alpha_0 \psi_0 + \alpha_1 \psi_1$

What is α_0 and α_1?

$\Rightarrow \quad x \cdot \psi_0 = \alpha_0 \psi_0 \psi_0 + \alpha_1 \psi_1 \cdot \psi_0$

$x \cdot \psi_0 = \alpha_0 \quad = \|x\| \cos(90-\theta)$

$= \|x\| \sin\theta$

Similarly multiply / ~~do~~ inner product by ψ_1, to find α_1

- positive semidefinite $\equiv$ positive definite

Q Can an orthogonal function be represented as linear summation other orthogonal ~~vectors~~ functions?

- pNorm — if $p=2$: standard Norm (L2 - norm)

- regularisation — helps in constraining the output to a certain range; that is helpful in prevention of over/under shoot; thus penalizing

H.W

plot norm function as function of p

$$\|x\|_p = \left(\sum_{0}^{N-1} |x_n|^p \right)^{1/p}$$

finding λ ← (experimentally ~~issue~~ figured out)

regularizing loss function : penalizing large weights, to keep model simple

Day 3 (29/01/26)

Q What you found interesting?

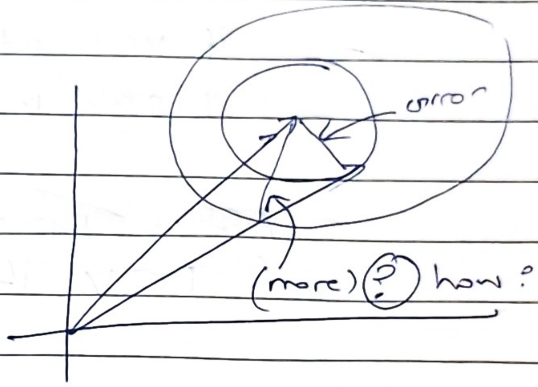
- Polynomial series approximation

  $\exists\, p(x)$ s.t. $|f(x) - p(x)| < \epsilon$

  but what degree/order of $p(x)$ — not defined

- Higher derivatives capture the global picture of data (i.e. movement etc)

- Subspace, Basis $(1, t)$ — ?
  ↳ Hilbertz : orthogonal
  Riesz : non orthogonal



- Legendre polynomials — orthogonal to each other (of various degree)
  → oscillation req for orthogonality

- Oscillation ≡ frequency

Learning ↓

- Signals → Represent in known signals → i.e basis

  What advantage?

  (i) estimation — analysis {computation} — simplified
  (ii) Comparison betn signals reduced
  ↳ as α_k's only there
  (iii) α_k → feature of signal → helps in storage
  (iv) transmission easy as less now
  (v) compression easy

$\cos^2\theta + s^2 = 1$

$k^2\cos^2\theta +$

NOTE:

a) $\langle x, y \rangle = \|x\| \, \|y\| \cos(\theta_x - \theta_y)$

b) basis - unit vectors

c) vector space = set of vectors V - Commutativity, associativity, distributivity (holds)

- Subset ≡ subspace
  if vector addn holds : $\forall x, y \in S, \; x+y \in S$
  if scalar multiplication holds : $\forall x \in S, \alpha \in \mathbb{C}$ (or $\mathbb{R}$) $\alpha x \in S$

- Affine subspace
  $T \subset V$ if $\exists x \in V$ and $S \subset V$, (subspace)
  $t \in T \Rightarrow t = x + s \;, \; s \in S$

- Span : (Span of a set of vectors S) is
  set of all finite linear combinations of vectors in S.

$$\left\{ \mathrm{Span}(S) = \sum_{k=0}^{N-1} \alpha_k \psi_k \;\middle|\; \alpha_k \in \mathbb{C} \text{ (or } \mathbb{R}), \; \psi_k \in S \text{ and } N \in \mathbb{N} \right\}$$

vectors → subspace → span
→ Linear independence → Dimension

Set of vectors ψ_k is LI if :

$$\left\{ \sum_{k=0}^{N-1} \alpha_k \psi_k = 0 \text{ iff } \alpha_k = 0 \; \forall k \right\}$$

NOTE: Infinite set of vectors is LI if every finite subset is LI !!

$\langle \alpha x, y \rangle = \underline{\underline{\alpha}}(x, y)$ $\quad \{ \langle x, y \rangle^* = y \cdot x^*$

$(\alpha \cdot x_1, \alpha x_2)$ $\quad \langle y, x \rangle^* = x^* y \}$

$\Rightarrow (\alpha^* x^* + y_0 + \alpha^* x_*^* y_*) \Rightarrow \alpha^* (x_0^* y_0 + x_1^* y_1)$

- vector space V has dimension N if LI set is of size N and any N+1 or more elements $\equiv$ LD

  if no such finite N exists $\Rightarrow$ V is infinite dimensional

- Inner product satisfies:
  - (a) distributivity : $\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
  - (b) Linearity in the first argument : $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$
  - (c) Hermitian symmetry : $\langle x, y \rangle^* = \langle y, x \rangle$

    $\cancel{\hookrightarrow \text{ex. } \langle x,y \rangle = x_0 y_0^*}$

    $\cancel{(x_0 y_0^*)^* = x_0^* y_0}$ $\quad \cancel{y x_0^*}$

    ex. (of violation)

    if $\langle x, y \rangle = x_0 y_0^*$

    $\langle x, y \rangle^* = (x_0 y_0^*)^*$

    $= x_0^* y_0^*$ not equal !!

    $\langle y, x \rangle = y_0 x_0^*$
  - (d) positive definiteness : $\langle x, x \rangle \geq 0$ and $\langle x, x \rangle = 0$ iff $x = 0$

- Orthonormal = Orthogonal and $\underset{\|x\|^2}{\underline{\underline{\langle x, x \rangle}}} = 1 \quad \forall x \in S$

  referred for a set of vectors S

- Norm : 3 properties — a) Positive definiteness

  $\|x\| \geq 0$, and $\|x\| = 0$ iff $x = 0$

  b) Positive scalability

  $\|\alpha x\| = |\alpha| \|x\|$

  c) Triangle inequality :

  $\|x + y\| \leq \|x\| + \|y\|$

  $\hookrightarrow$ Equality iff $y = \alpha x$

- Convergent seq of vectors in a normed vector space

A seq of vectors $x_0, x_1, \ldots$ is said to converge to $v \in V$

when

$$\lim_{k \to \infty} \|v - x_k\| = 0$$

(or)

given any $\epsilon > 0$, $\exists K_\epsilon$ such that,

$$\|v - x_k\| < \epsilon \quad \forall k > K_\epsilon$$

- Cauchy sequence of vectors

vectors get close to each other

A seq of vectors $x_0, x_1, \ldots$ in a normed vector space is called Cauchy seq,

when given any $\epsilon > 0$, there exists a K_ϵ s.t.,

$$\|x_k - x_m\| < \epsilon \quad \forall k, m > K_\epsilon$$

- Completeness and Hilbert Space

A space is complete if:

Every Cauchy seq in that space actually converges to a vector inside the same space

$\mathbb{Q}$ is not complete as $\sqrt{2} \notin \mathbb{Q}$ but $\sqrt{2} \in \mathbb{R}$ $\Rightarrow$ Complete ($\mathbb{R}$)

A complete inner product space is called a ~~Hilbert~~ Hilbert Space

a vector space where an inner product is defined

classmate

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(Text Data) - (related pb)

Day 10 (02/02/26)

Case: Sentiment Classifier
→ approached by bag of words
assignment on this

- if Seq of no ⇒ Linear Algebra easily
- Q ◦ but what if seq of (text ?), how to convert to nos ?
- different languages: (a) Natural language (human)
  (b) formal "
  (c) programming "
- sentiment classifier
  (i) 2 sentiments : happy or sad
  (ii) 7 sentiments : (a research paper)

- twitter dataset - binary

CHECK

Why twitter are restricted ?
Why input lengths restricted for comments etc sometimes ?

- Text to Number - Classification problem
  How to do it ?

T1 → Bag of words
{ (Word : count of occurrence) }
key value

problem : (i) context is lost (relationship amongst word lost)
(ii) ordering of words lost

i) meaning of underlying words not learnt

not a matter ! for spam email Classification ⇒ bag of words representation good

Sarcasm detection
→ text and emoji
→ CSE dept
→ Project going on

~~What~~ is Apache license?
→ ~~How~~ does one apply for it ?
→ ~~open~~ source ?
→ ~~How~~ do people apply for license?

NOTE: Stop words : are filler words
example. The, on, '.'
To remove them : make a list of it
(to) { already made : Stopwords English (EN) Github }
[~~most~~ all of them ~~cannot~~ may not be stopword in every situation - so check before using otherwise - Classifier ~~may~~ not work

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NOTE: depending on situation, case may be sensitive

BAG of words
(i) integer representation
(ii) if size huge ⇒ Sparsity may be there (as lots of 0s)

Q What if a new word comes?
↳ assign it to unknown : the new word ~~(?)~~ (make a column called unknown and if word ∉ BoW increase cnt of unknown).
↳ grow your vocabulary

prev used for search engines

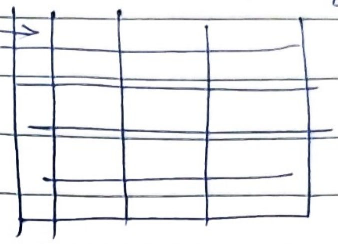
T2 → BoW + TF-IDF ~~less freq~~
- More importance/weightage to ^ unique words per doc/seq
- Common words has 0 importance
- pb: (i) no word order considered
  (ii) Computational cost a little high
- usage: (iii) good for search ranking

Case 2: Next word prediction
(i) req names-data set
↳ Indian names dataset : pip install Indian-Names
↳ what bias included?
(i) Diverse country
(ii) regional difference

(ii) What next? — (i) Probabilistic model
(ii) since only one char, scan through the list of names and find
(M1)
(iii) Combining both : Check pb(alphabet/dataset) for alphabet ∈ [~~A-Z,~~ a-z] and get highest one
(iv) (M2) Compute conditional pb — i.e given prev char,

What could be next char ?
↑ find probability

° Bigram matrix



(each cell gives a cnt
↓
get pb by considering a row)

(# Generative model to create new names)

° each row a probability

° What distribution can be applied ?

(multinomial distribution)

(i) a – z rows
a – z columns

curr char | next char

ii) Check all names which has curr char next char count ☰
iii) for each row → sum up all nos
iv) take pb for each row !!

° Name creation Model

In this situation how to verify ?

(Limitations)
NOTE: n-grams matrix – might become sparse

Q how to quantify if the models predicted correctedly ?
(or if it's a valid ones)

Q how to use for next sentence ?

What could be next char ?
↳ find probability

∘ Bigram matrix

(# Generative model to
Create new names)

∘ each row a
probability

∘ What distribution can
be applied ?

(multinomial distribution)

(each cell
gives a cnt
get pb by
Considering
a row)

(i) a–z rows
a–z columns

next char

curr
char

ii) Check all names which
has curr char next char count ≡

iii) for each row → sum up all nos

iv) take pb for each row !!

∘ Name creation
Model

In this
situation
how to
verify ?

(Limitations)

NOTE: n-grams matrix – might become sparse

Q how to quantify if the models predicted correctly ?
(or if it's a valid one)

Q how to use for next sentence ?

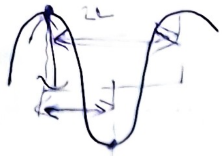
Day 11 (03/02/26)

(# Signal Representation)

∘ Are signals periodic in nature ? (Yes) ex. EEG

∘ Can we use non-polynomials as basis ?
↳ ex. sin waves

NOTE: sum of sine waves ≡ sinewaves → periodic



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- Polynomial representation

Pros {
→ high degree ⇒ difficulty in visualisation
→ Can capture local aspect of data
→ helps in predicting for ~~unknown~~ new data points : extrapolation

Cons {
→ if oscillation req ⇒ use high degree polynomial
⇒ but unwanted oscillations may come
→ Sensitive to outliers

Q Can we use fourier series representation?

$(\sin(A+B) = \sin A\cos B + \sin B\cos A)$

$$f(x) = \sum_{m=0}^{\infty} A_m \sin\left(\frac{\pi m x}{L} + \phi_m\right)$$

origin: heat distribution
periodic

find out by experimental

$$= \underbrace{\frac{a_0}{2}}_{\text{for } m=0} + \sum_{m=1}^{\infty(M)} \underbrace{a_m \cos\left(\frac{\pi m x}{L}\right)}_{\text{(harmonic of base freq)}} + b_m \sin\left(\frac{\pi m x}{L}\right)$$

← for $m > 1$

NOTE: $\sin \omega x$
(ω how fast it oscillates)

NOTE: Multiplication of 2 periodic signals ⇒ inner product over a period

- We can compute a_0, a_m, b_m by using the property that basis vectors are orthogonal !

- fourier series converge for large m (infinite)

- Signals and functions same (in our context)

→ i.e convergence

- intrapolation can be done easily for fourier ~~trans~~ series transformation of a signal but extrapolation difficult!

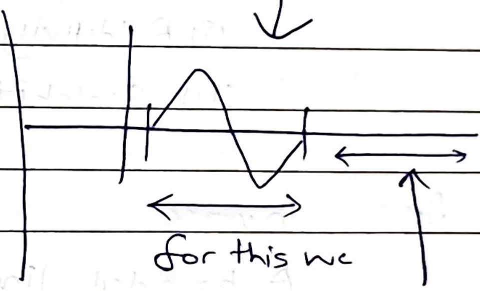
→ (regardless of whether differential or not)

→ (like close convergence not complete)

→ and convergence/ representation only for certain period

- fourier : if signal oscillatory

else → polynomial representation



for this we can

(we can't say anything about this)

~~$f: \mathbb{Z} \to \mathbb{R}$~~

HILBERT SPACE

examples: $\mathbb{C}^N$ (finite dimensional complex vectors)

$\ell^2(\mathbb{Z})$ (square summable sequences)

$L^2(\mathbb{R})$ (square integrable functions)

Def 1 Linear Operator (from H_0 to H_1)

A function $A: H_0 \to H_1$ is Linear operator when for all x, y in H_0 and α in $\mathbb{C}$ (or $\mathbb{R}$), the following hold:

(i) Additivity: $A(x+y) = Ax + Ay$

(ii) Scalability: $A(\alpha x) = \alpha(Ax)$

Def 2 Inverse

A bounded linear operator $A: H_0 \to H_1$ is called invertible if there exists a bounded linear operator $B: H_1 \to H_0$ s.t,

$$B(Ax) = x \quad \forall x \in H_0 \quad \text{and}$$
$$A(By) = y \quad \forall y \in H_1$$

Def 3 Unitary Operator

A bounded linear operator $A: H_0 \to H_1$ is called unitary when,

(i) it is invertible, and

(ii) it preserves inner products

$$\langle Ax, Ay \rangle_{H_1} = \langle x, y \rangle_{H_0} \quad \forall x, y \text{ in } H_0$$

Def 4 Eigen vector of a linear operator

An eigen vector of a linear operator $A : H \to H$ is a non zero vector $v \in H$ such that,

$$Av = \lambda v \qquad \{\text{i.e just scaling}\}$$

for some $\lambda \in \mathbb{C}$.

The constant λ is the corresponding eigen value and (λ, v) is called an eigen pair

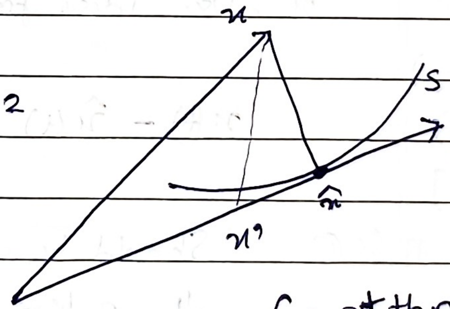
NOTE: Linear operators considered orthogonal projection operators

Approximations

- $\hat{x} = \arg\min\limits_{s \in S} \|x - s\|$ $\qquad \|\cdot\|$ is L2 norm

x → resides in H

s → resides in S



$$\|x - s\|^2 = \|x - \hat{x}\|^2 + \|\hat{x} - s\|^2$$

since,

$$x - s = x - \hat{x} + \hat{x} - s$$

NOTE: $\hat{x}$ is projection of x on S

$$\Rightarrow \boxed{(x - \hat{x}) \perp S} \; *$$

for orthogonal vectors a and b,

$\|a+b\|^2 = \|a\|^2 + \|b\|^2$

(as inner product is 0 ⇒ no cross term)

NOTE: $\hat{x} \in S$

$s \in S$

$$\Rightarrow \hat{x} - s \in S \quad \Rightarrow (\hat{x} - s) \geq 0$$

$$\Rightarrow \|x - s\|^2 \geq \|x - \hat{x}\|^2.$$

by applying this (if $s = \hat{x}$)

for $s \neq \hat{x}$ where $\hat{x}$ is projection

$$\|\hat{x} - s\|^2 > 0$$

$$\Rightarrow \quad \|x - s\|^2 = \|x - \hat{x}\|^2 + \text{positive no!}$$

This Implies the $\arg\min_{s} \|x - s\|$ satisfies for

$\hat{x}$ //

→ Orthogonality, Pythagoras, Projection theorem

NOTE: if L_2 norm is not considered, then the point may change !! ← i.e $\hat{x}$ / orthogonal projection may not be the min ~~ps~~ norm one

*

example. Consider the function $x(t) = \cos\left(\frac{3\pi t}{2}\right)$ in the Hilbert Space $L^2([0,1])$. Find the degree-1 polynomial closest to x. Use ideas from orthogonal projection

$L^2([a,b])$

$L^2(\mathbb{R})$

$L^2(\Omega)$

Codomain: $\mathbb{R}$ or $\mathbb{C}$

$$x(t) - \hat{x}(t) = \cos\left(\frac{3\pi t}{2}\right) - (a_0 + a_1 t)$$

Should be orthogonal to the entire subspace of degree 1 polynomials

a set of vectors / functions whose linear combinations give the whole space (span it) and that are LI

the basis vector for this $\{1, t\}$

$$0 = \langle x(t) - \hat{x}(t), 1 \rangle = \int_0^1 \left(\cos\left(\tfrac{3}{2}\pi t\right) - (a_0 + a_1 t)\right) \cdot 1 \, dt$$

take t as well

and solve both eqns //

Bases

Defn The set of vectors $\Phi = \{\varphi_k\}_{k \in K} \subset V$ where

K is finite or countably infinite

is called basis for a normed vector space V when,

(i) It is complete in V, meaning,

$\forall x \in V$, there is a seq $\alpha \in \mathbb{C}^K$ such that,

$$x = \sum_{k \in K} \alpha_k \varphi_k \quad \text{——} \; (1)$$

and

(ii) for any $x \in V$, the seq α that satisfies is unique

Riez basis

Defn set of vectors $\Phi = \{\varphi_k\}_{k \in K} \subset H$ is Riez bases

for Hilbert space H when,

NOTE: largest λ_{min} and smallest λ_{max} $\Rightarrow$ optimal stability constants for Φ

(i) it is a basis for H

(ii) $\exists$ stability constants λ_{min} and λ_{max},

s.t $0 < \lambda_{min} \leq \lambda_{max} < \infty$

and $\forall x \in H$, $x = \sum_{k \in K} \alpha_k \varphi_k$ satisfies

$$\lambda_{min} \|x\|^2 \leq \sum_{k \in K} \|\alpha_k\|^2 \leq \lambda_{max} \|x\|^2$$

$\to \|\phi_k\| = 1$

$\to$ if orthogonal + normalised

$\hookrightarrow \langle \phi_i, \phi_j \rangle = 0$

NOTE: orthonormal basis, $\forall i \neq j$

$$\Rightarrow \sum_{k \in K} \|\alpha_k\|^2 = \|x\|^2$$

but, Riesz bases is like an orthonormal basis but possibly tilted / scaled i.e with bounds!!

Synthesis operator $\to$ coefficients $\to$ vector

Def^n Basis synthesis operator

Given a Riesz basis $\{\varphi_k\}$ $k \in K$ for a Hilbert space H, the synthesis operator associated with it is,

$$\Phi : \ell^2(K) \to H \quad \text{with} \quad \Phi(\alpha) = \sum_{k \in K} \alpha_k \varphi_k$$

it is a set of all sq:

$\alpha = (\alpha_k)_{k \in K}$ such that,

$$\sum_{k \in K} |\alpha_k|^2 < \infty$$

Analysis Operator $\to$ vector $\to$ coefficients

Def^n Basis Analysis Operator

Given a Riesz basis $\{\varphi_k\}_{k \in K}$ for a Hilbert space H, the analysis operator associated with it is,

$$\Phi^* : H \to \ell^2(K) \quad \text{with} \quad (\Phi^*(x))_k = \langle x, \varphi_k \rangle, \; k \in K$$

x belongs to H

Analysis - Synthesis

Def^n Orthonormal Basis Expansions

let $\Phi = \{\varphi_k\}_{k \in K}$ be an orthonormal basis for a Hilbert Space H. ~~The unique exa~~

(i) The unique expansion with respect to Φ of any x in H has expansion coefficients

$$\alpha_k = \langle x, \varphi_k \rangle \text{ for } k \in K, \text{ or}$$

$$\alpha = \Phi^*(x)$$

(ii) Synthesis with these coefficients yields

$$x = \sum_{k \in K} \langle x, \varphi_k \rangle \varphi_k$$

$$= \Phi(\alpha) = \Phi(\Phi^*(x))$$

Parseval Equalities

Def^n let Φ be orthonormal basis for Hilbert Space H, expansion with coefficients satisfies the Parseval equality,

$$\|x\|^2 = \sum_{k \in K} |\langle x, \varphi_k \rangle|^2$$

$$= \|\Phi^*(x)\|^2 = \|\alpha\|^2$$

NOTE: in complex Hilbert space, $\langle x,y \rangle \neq \langle y,x \rangle$

if real H.S

$\Rightarrow \langle x,y \rangle^* = \langle x,y \rangle$

but $\langle x,y \rangle = \langle y,x \rangle^*$

and generalised Parseval equality.

$$\langle x,y \rangle = \sum_{k \in K} \langle x, \varphi_k \rangle \langle y, \varphi_k \rangle^*$$

$$= \langle \phi^*(x), \phi^*(y) \rangle$$

$$= \langle \alpha, \beta \rangle$$

Legendre Polynomials : special set of polynomials

Def^n $\quad L_k(t) = \frac{1}{2^k k!} \frac{d^k}{dt^k} (t^2-1)^k, \quad k \in \mathbb{N}$

are orthogonal on $[-1,1]$

inner product for functions — $\int_{-1}^{1} L_i(t) L_j(t)\, dt = 0$

$\hookrightarrow (i \neq j)$

NOTE: $\|x - \hat{x}\|^2 = \int_a^b \cancel{f(x)^2} \cancel{dt} (x(t) - \hat{x}(t))^2 dt$

classmate

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Day 12 (09/02/26)

Q. What does it mean to compress the signal?

- Save storage
- as no plots stored : functional representation of signal

Q. Why fit a model to a signal representation?

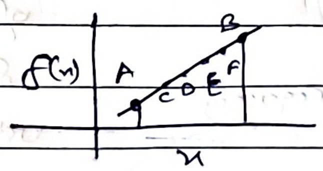
(i) extrapolation

(ii) Compress :- helps in storage

→ If database has more points/values, can we compress it? — is the meaning

\# params should be less

Example.



- instead of storing all x and corresponding $f(x)$
- just store A and B and $f(x) = mx + c$

parameters

↑

first that need to be estimated

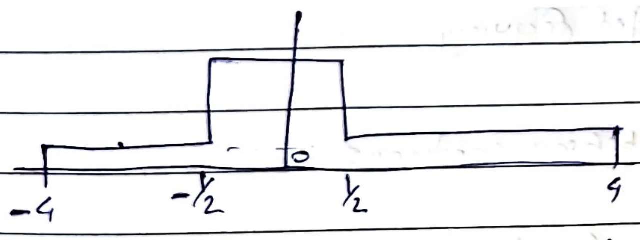
NOTE: Removal of noise is known as filtering

Today: Application of Fourier Series

Generalisation of Fourier Series for non periodic signals

$$\Pi(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| \geq 0 \end{cases}$$

Called Hat signals or ~~R~~ec function or Indicator f^n



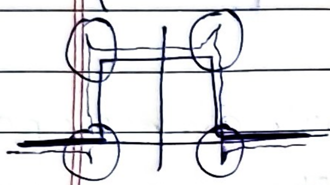
→ not periodic f^n

how to represent in fourier series? $T = \infty$

(but for us : only in that interval) → (-4, 4) ⟸ (assume periodic in finite interval)

NOTE: our interest is fitting fourier series for the given interval

- Sharp discontinuity - Shooting : error
  ↳ Gibbs phenomenon : over / under shooting (: bounded)
  ↳ not infinite
  When fourier tries to approximate exactly the signal
  In fourier
  •(but in polynomial: no control)



- $f(x) = \sum_{\boxed{k=-\infty}}^{\infty} c_k e^{imx\pi k/L}$: compressed form of fourier series using euler formula

(has sinusoidal variation)

↳ no negative frequency → it's just a representation as using euler formula ↓ which is using sine and cos

Fundamental period = T

frequency can be : $\frac{1}{T}, \frac{2}{T}, \ldots$

NOTE: C_k found using orthogonality of function idea!

C_0 ← (constant also called direct value)

Similarly : j doesn't exist in real - it helps in simplify stuffs

- Spectrum of function f(x)
  : plot b/w c(k) and frequency

- as T ↑ → spectrum becomes continuous

- Sync function ← $\frac{1}{\pi n} \sin\left(\frac{\pi n}{T}\right)$

for aperiodic signal

NOTE: $\underline{C_k}$ → represented by $\hat{f}(s)$

C_k: for periodic signal

$\hat{f}$: represent C; generally complex

s: represent k

Periodic signal – Fourier series
Aperiodic " – " transform

Signal $f(t)$ → $\boxed{F.T}$ → continuous representation ($\hat{f}(s)$)

possible? – yes (IFT)

NOTE: it's a choice to deal with either $f(t)$ or $\hat{f}(s)$

- feature extraction: can be done on both // ← here we are preserving everything about both!

Q. let's say $f(n)$

$g(n) = (f(n))^2$

↳ can we get $f(n)$? (NO)

- as $f(n)$ can be +ve/-ve

NOTE: C_k → represented by $\hat{f}(s)$ ← for aperiodic signal

$\overline{}$ ↑ for periodic signal

represent C

represent k

generally complex

Periodic Signal — Fourier series → discrete values

Aperiodic " — " transform → continous fn ↳ needs to be discretized

Signal $f(t)$ → $\boxed{\text{F.T}}$ → Continous representation $(\hat{f}(s))$

possible? — yes (IFT)

NOTE: it's a choice to deal with either $f(t)$ or $\hat{f}(s)$

- feature extraction: Can be done on both // ← here we are preserving everything about Both!

Q. let's say $f(n)$

$g(n) = (f(n))^2$

↳ Can we get $f(n)$? (NO)

- as $f(n)$ can be +ve/-ve

Day 13 (10/02/26)

- inner product between two functions — Cross corelation

- for fourier domain we use f instead of (t) as it is frequency domain

~~does~~ Continous ensure periodic in nature? (No)

- if signal is smooth (Cosine) → fourier transform is sparse (this is compression) frequency domain

$f(t) \xrightarrow{①} \hat{f}(t)$

time domain

$\hat{f}(t) \xrightarrow{② \text{ modify}} \hat{f}'(t)$

$\boxed{f'(t)} \longleftarrow \hat{f}'(t)$

Q. how does original function get modified ?

=> to get a more useful (or new) signal

NOTE: #Convolution

$$h(u) = \int_{-\infty}^{\infty} g(u-x)\, f(x)$$

$\boxed{u = t + x}$

↳ f is convolved with g

- g is first flipped i.e $g(-x)$
- then it's shifted by u (move u by diff to get diff area) (↗ in simulation)
- and so for every value of g afterwards it is multiplied / inner product with f

our ear listens in frequency domain smtg (FACT)

(Check!) ~~(f)~~

helps in: ← (i) spectrum enhancement (ii) noise removal (iii) feature extraction !!

- Convolving results in another function
  ↳ in time domain, it is complex but in freq domain, it's just $F(t) \cdot F(g)$

(Check!) ∘ Convolution demo page

- convolution of 2 rect function = ~~?~~ (Triangle)
  (i) symmetric
  (as area overlapping inc gradually, then peaks when fully overlapped, and then it decreases)

- of 2 triangular function = (Smoother function)
  ↓ (if repeated) - convolving
  (similar to guassian)

(it is simply the function that assigns how much importance each neighbouring sample gets when forming out)

$$\text{Mean } y(n) = \frac{1}{2k+1} \sum_{p=-k}^{k} x[n+p] + \underset{(0)}{e[n+p]}$$

← ~~waiting~~ weighting function is 1

- Moving Average
→ take a window of 2k+1
→ find mean for that
→ Shift by 1 sample and repeat

→ helps in

⇓

helps to remove out noise from time series → avg value of noise is 0

↳ expectation of noise!!

NOTE: how to define or make g(u−x) will be discussed later!

Polynomial = disCrete
f. series = ''
f. transform = Continous
↳ inverse possible

(it is simply the function that assigns how much importance each neighbouring sample gets when forming out)

classmate

Date
Page

$$\text{Mean } y(n) = \frac{1}{2k+1} \sum_{p=-k}^{k} x[n+p] + e[n+p]$$

$e[n+p] \leftarrow (0)$

$x[n+p] \leftarrow$ ~~Weighting~~ ~~Waiting~~ Weighting function is 1

- Moving Average
  - → take a window of 2k+1
  - → find mean for that
  - → Shift by 1 sample and repeat

→ helps in

⇓

helps to remove out noise from time series → avg value of noise is 0

expectation of noise !!

NOTE: how to define or make $g(u-x)$ will be discussed later !

Day 14 (13/02/26) # Application of F.S

- If T unknown, let it equal to be the interval of the whole signal

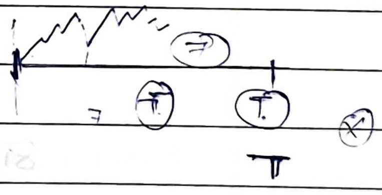
Polynomial = discrete

f. series = ||

f. transform = Continous

↳ Inverse possible

- Covid cases
  - ↳ what is the oscillation part?
  - where is it coming from?
  - If observing oscillation, ask the frequency !



→ Some reasoning for peak at 0 as DC — fp more !

→ we see peaks at 7th day — because the results are compiled on the first day after weekend !

- 

NOTE: we move along different points in y and plot the increments in x

Obtained from continous signal
by sampling

(Nature is smooth)

Discrete Signal

Signals may not be continous

a) Sampling - measuring the value of some quantity

b) Sampling period - time taken between 2 samples
↳ but only ~~for~~ if uniform

c) inverse of it is sampling frequency

d) how to choose the sampling rate {Nyquist rate}

→ sensor measuring variation of density as function of time

→ no idea betn t and t+T : Signal working

→ multiple signals can pass through the same red dots — when we use ML to fit a model and it picks only 1 : Aliasing

→ increasing sampling rate can match the underlying true signals — after interpolation

→ rate of change considered as sinusoidal frequency

$$\boxed{f_s > 2 f_{max}}$$: Nyquist rate

f_{max} from continous signal

Signal Continous	$\xrightarrow{\text{find max } f}$	get f_s and obtain discrete signal

NOTE: Vid — ~~pixel~~ camera is not able to take frame at the rotors. Sampling rate
{helicopter one} (similar to fan)