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Simulation methods in Medical Engineering, HL2008

Mathematical and numerical backgrounds

Parameter study of two ODE-problems

Computer Lab 2, Boundary Value Problem: Heat Conduction

Consider a long pipe of length L with with small cylindrical cross section. In the pipe there is a fluid heated by an electric coil. The heat is spreading along the pipe and the temperature $T(z)$ at steady state is determined by the diffusion-convection ODE:

$$-\frac{d}{dz}(\kappa\frac{dT}{dz}) + v\rho C\frac{dT}{dz} = Q(z) \qquad (*)$$

where all parameters are constant: κ is the heat conduction coefficient, v is the fluid velocity in the z-direction through the pipe, ρ is the fluid density and C is the heat capacity of the fluid. The driving function $Q(z)$, modeling the electric coil, is defined as

$$Q(z) = \begin{cases} 0, & \text{if } 0 \leq z < a \\ Q_0 \cdot \sin\left(\frac{z-a}{b-a}\pi\right), & \text{if } a \leq z \leq b \\ 0, & \text{if } b < z \leq L \end{cases}$$

At $z = 0$ the fluid has the temperature T_0 giving the boundary condition:

$$T(0) = T_0$$

At $z = L$ the boundary condition is

$$T(L) = T_{out}$$

Use the following values of the parameters in the problem: $L = 10$, $a = 1$, $b = 3$, $Q_0 = 50$, $\kappa = 0.5$, $\rho = 1$, $C = 1$, $T_{out} = 300$, $T_0 = 400$ and $v = 0, 0.1, 0.5, 1$. The case $v = 0$ corresponds to no convection, only diffusion.

Discretize the z-interval $[0, L]$ with constant stepsize and use a gridpoint-numbering where $z_0 = 0$ and $z_{N+1} = L$.

Discretize the ODE and insert the boundary condition values. A tridiagonal linear system of algebraic equations $A\mathbf{u} = \mathbf{b}$ is obtained. The vector $\mathbf{u}$ contains the T-values at the grid points.

Write a MATLAB-program to set up and the linear system of equations. Plot of the solution $T(z)$ for $v = 0$, $N = 9, 19, 39, 79$ in the same graph. Note the convergence of the curves in the graph.

Use $N = 79$ (the smallest stepsize) to solve the problem for $v = 0, 0.1, 0.5, 1$. Use `subplot`-command to obtain the four graphs in the same figure.