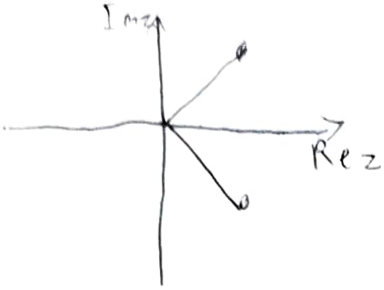
Superconspect

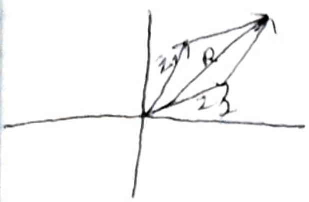
12.1.1.6



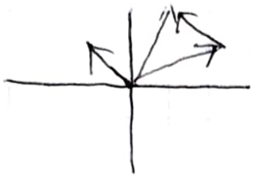
$z = 2 + 2i$

$\bar{z} = 2 - 2i$

12.5.2,2



$R = z_1 + z_2$



$z_1 - z_2$

15.2.3

$|z| = 5$

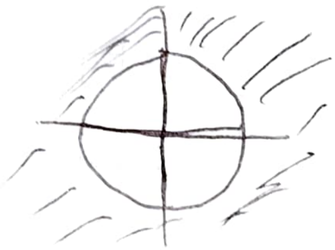
$|x + yi| = 7$

$\sqrt{x^2 + y^2} = 7$

$x^2 + y^2 = 7^2$

$|z| \geq 2$

$x^2 + y^2 \geq 2^2$



если $\geq$

borders fill

outside

если $\leq$

borders fill

inside

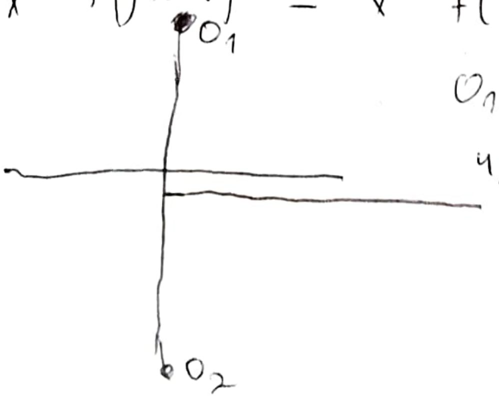
если $<$

inside

borders пунктир

$|z - 4i| = |z + 5i|$

$x^2 + (y - 4)^2 = x^2 + (y + 5)$

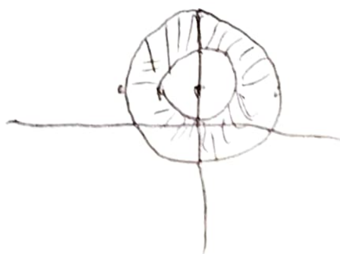


$O_1 O_2 = 9$

4,5

$1 \leq |z - i| \leq 2$

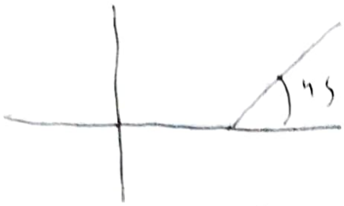
$|z - i| \leq 2 \qquad |z - i| \geq 1$



$x^2 + (y - 1)^2 \geq 1$

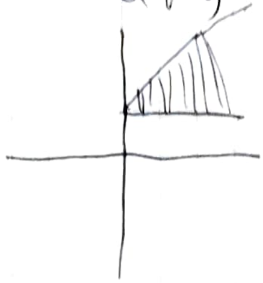
$\arg(z-z_0)=\alpha$

$\arg(z-3)=\frac{\pi}{4}$

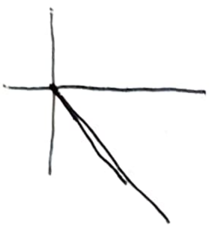


$0<\arg(z-i)\leq\frac{\pi}{4}$

$\arg(z-i)>0$



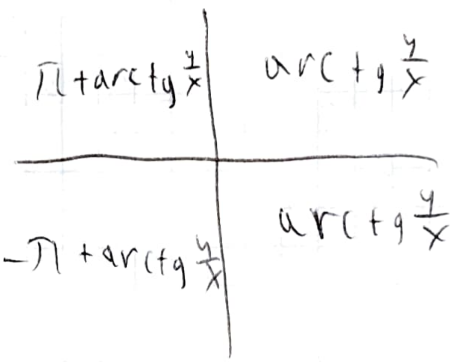
$\arg(z-1)=-\frac{\pi}{4}$



12.1.2.3

$z=a+bi=r(\cos\varphi+i\sin\varphi)$

$\varphi=\arg z=\operatorname{arctg}\frac{b}{a}$



$z_1\cdot z_2=r_1\cdot r_2(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2))$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2))$

$z^n=(a+bi)^n=r^n(\cos(n\varphi)+i\sin(n\varphi))$

$\sqrt[n]{z}=\sqrt[n]{r}\left(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n}\right)$

$\pi +$ ||

$-\pi +$ |||

$\cos 3x = 4\cos^3 x - 3\cos x$

$(\cos x + i\sin x)^3 = \cos 3x + i\sin 3x$

$\cos^3 x + 3\cos^2 x\, i\sin x + 3\cos x\, i^2\sin^2 x + i^3\sin^3 x$

$\cos^3 x + 3\cos^2 x\, i\sin x - 3\cos x\sin^2 x - i\sin^3 x =$
$(\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x)$

$\cos 3x = \cos^3 x - 3\cos x\sin^2 x$

$\cos^3 x - 3\cos x(1-\cos^2 x) =$
$= \cos^3 x - 3\cos x + 3\cos^3 x =$
$= 4\cos^3 x - 3\cos x$

12.1.17

$re^{i\varphi} = r(\cos\varphi + i\sin\varphi)$

$z_1 \cdot z_2 = r_1 \cdot r_2\, e^{(\varphi_1+\varphi_2) i}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{(\varphi_1-\varphi_2) i}$

$z^n = (re^{i\varphi})^n = r^n e^{i\varphi n}$

$\frac{\sqrt{3}}{3} = \frac{\pi}{6}$ $\sqrt{3} = \frac{\pi}{3}$

$1 = \frac{\pi}{4}$

$\cancel{\frac{\sqrt{3}}{2} = \frac{\pi}{3}}$

12.5.3.3 Туынды қолдану арқылы геометриялық және физикалық мазмұнды есептер

Задачи макс минимума (Геометрия)

1. S – находим оптимизирующую величину
   ex. S
2. Выбираем независимую переменную x и y
   x – сторона прямоугольника
   y выражаем через x
   $$P = 2(x+y) = 80$$
   $$y = 40 - x$$
3. Составляем функцию
   $$S(x) = x(40-x) = 40x - x^2$$
4. Туындылайтын и теңестіреміз 0-ге
   $$S'(x) = 40 - 2x = 0$$
   $$x = 20$$
5. Қоямыз орнына
   $$S'(20) = 800$$
   20x20 максимум площадь

Задачи физ содержания

1. Какую физ величину найти (a, v, S?)
   Запиши функцию. Например $v(t)$ – ?
   $$x(t) = 20x^2 + 12$$
2. $v = S'$
   $a = v'$
   $\omega = \varphi'$
   $I = q'$
   $P = A'$
3. Туындылайсың
   $$x'(t) = 40x$$
   $$v(t) = 40x$$
   $$v(1) = 40\ м/с$$

$F = ma$ ($S = m v'(t)$)

Равнодействующая

12,5 Күрделі емес жағдайларда есеп шығару бойынша дифференциалдық теңдеулер

Популяция өсуі

$$\frac{\partial N}{\partial t} = kN$$

$$N(t) = Ce^{kt}$$

Радиация ыдырау

$$\frac{\partial M}{\partial t} = -KM$$

$$M(t) = C e^{-Kt}$$

Жылу алмасу

$$\frac{\partial T}{\partial t} = -K(T - T_0)$$

$$T(t) = T_0 + \overbrace{(\underbrace{T}_{T_б} - T_0)}^{C} e^{-Kt}$$

T – зат температурасы

T_0 – орта $T_б = T(0)$

$$\frac{1}{3}\pi r^2 h$$

Сұйықтың ағуы (цилиндр)

$$\frac{\partial V}{\partial t} = -K\sqrt{h}$$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial h} \cdot \frac{\partial h}{\partial t}$$

$$K\sqrt{h} = \frac{\partial V}{\partial h} \cdot \frac{\partial h}{\partial t}$$

$$\cancel{\frac{\partial V}{\partial h} = \pi r}$$

$$V = \pi r^2 h$$

$$V'(h) = \pi r^2 = \frac{\partial V}{\partial h}$$

$$K\sqrt{h} = \pi r^2 \cdot \frac{\partial h}{\partial t}$$

$$\frac{\partial h}{\partial t} = \frac{K\sqrt{h}}{\pi r^2}$$

$$\int \frac{\partial h}{\sqrt{h}} = \int \frac{K}{\pi r^2} \partial t$$

$$2h^{\frac{1}{2}} = \frac{K}{\pi r^2} t + C$$

$$h = \frac{(A t + C)^2}{4}$$

Сұйықтың ағуы конус $V = \frac{1}{3}\pi r^2 h$ $V(h) = \frac{1}{3}\pi r^2$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial h} \cdot \frac{\partial h}{\partial t}$$

$$K\sqrt{h} = \frac{1}{3}\pi r^2 \cdot \frac{\partial h}{\partial t}$$

$$\frac{K\sqrt{h}}{\frac{1}{3}\pi r^2} = \frac{\partial h}{\partial t}$$

$$\frac{K}{\frac{1}{3}\pi r^2} \partial t = \frac{\partial h}{\sqrt{h}}$$

$$\frac{3K}{\pi r^2} t = 2h^{\frac{1}{2}}$$

$$h = \frac{\left(\frac{3K}{\pi r^2} + C\right)^2}{2}$$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial h} \cdot \frac{\partial h}{\partial t}$$

$$\cancel{r = Kh}$$

$$h = Kr$$

$$r = Kh$$

$$-C\sqrt{h} = \pi K^2 h^2 \cdot \frac{\partial h}{\partial t}$$

$$\frac{\partial h}{\partial t} = \frac{-C\sqrt{h}}{\pi K^2 h^2}$$

$$h^{\frac{3}{2}} \partial h = \frac{-C}{\pi K^2} \cdot \partial t$$

$$\frac{h^{\frac{5}{2}}}{2,5} = \frac{-C_1}{\pi K^2} t + C_2$$

Қозғалыс

$F = ma = m\frac{\partial v}{\partial t}$

F(t) = кез келген күш функциясы
например mg

$m\frac{\partial V}{\partial t} = F(t)$

$\frac{\partial V}{\partial t} = \frac{mg}{m}$

$\partial V = g\,\partial t$

$v(t) = 9{,}81\,t$

$\frac{\partial S}{\partial t} = \frac{\partial S}{\partial r} \cdot \frac{\partial r}{\partial t}$

$2\pi r$

Гармоникалық тербелістер

$ma = -kx$

$a = \frac{\partial^2 x}{\partial t^2}$

$\frac{\partial^2 x}{\partial t^2} + \omega^2 x = 0$

$x = C_1 \cos\omega t + C_2 \sin\omega t$

$C_1 = A\sin\varphi \qquad C_2 = A\cos\varphi$

$x = A\sin\varphi\cos\omega t + A\cos\varphi\sin\omega t$

$x = A\sin(\omega t + \varphi)$

Амплитуда

$\sqrt{C_1^2 + C_2^2}$

$\operatorname{tg}\varphi = \frac{C_1}{C_2}$

$V = \frac{1}{3}\pi r^2 h$

$V_{cy} = \frac{1}{3}\pi x^2 \cdot y$

$\frac{r}{x} = \frac{h}{y}$ $\quad \frac{1}{3}\pi r^2 \cdot \frac{yr}{x}$

$x = \frac{yr}{h}$

$h = \frac{yr}{x}$

$\cancel{dT = -K(T-T_0)}$

$$\frac{dT}{T-T_0} = -K\,dt$$

$\ln|T-T_0| + \ln|C| = -Kt$

$e^{-Kt} = (T-T_0)C$

$T(t) = T_0 + Ce^{-Kt}$

1λ $\quad h$

x $\quad 2x$

$r = Kh$

$r^2 = K^2h^2$

$$\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt}$$

$$\frac{d^2x}{dt^2} + 4K\frac{dx}{dt} + 3K^2x = 0$$

$\frac{1}{3}\pi K^2 h^3$

$\lambda^2 + 4K\lambda + 3K^2$ $\quad -C\sqrt{h} = \pi K^2 h^2 \cdot \frac{dh}{dt}$

$D = 16K^2 - 4\cdot 3K^2 = 4K^2$

$\lambda_1 = -K$

$\lambda_2 = -3K$

$\cancel{d^{\frac{3}{2}}}\, dh = \frac{-C}{\pi K^2}\cdot dt$

$$\frac{d^{\frac{5}{2}}}{2{,}5} = -\frac{C}{\pi K^2}t$$

$d^{\frac{5}{2}} = At \cdot 2{,}5$

d

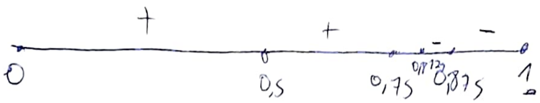
Бином

$(a+b)^n = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + C_n^2 a^{n-2} b^2 + C_n^k a^{n-k} b^k$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\cdot 2}x^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^3 + \frac{n(n-1)(n-2)(n-3)}{4!}x^4$

Хорды

$\cancel{f_1} = x^3 - 7x + 5 = 0$ $\varepsilon = 0{,}01$ $[0;1]$



1. $f(0) = 5 > 0$, $f(1) = -1 < 0$
2. $x_1 = \frac{1+0}{2} = 0{,}5$ $f(0{,}5) = 1{,}625 > 0$
3. $x_2 = \frac{0{,}5+1}{2} = 0{,}171875 > 0$
4. $x_3 = \frac{0{,}75+1}{2} = 0{,}875$ $f(0{,}875) = -0{,}455078 < 0$
5. $x_4 = \frac{0{,}75+0{,}875}{2} = 0{,}8125 = f(0{,}8125) = -0{,}1511 < 0$
6. $x_5 = \frac{0{,}75+0{,}8125}{2} = 0{,}78125$ $f(0{,}78125) = 0{,}0079 > 0$
7. $x_6 = \frac{0{,}78125+0{,}8125}{2} = \cancel{0{,}0717}$ $0{,}7969$

$|x_6 - x_5| = 0{,}01 =$

$x = 0{,}79$

$C_n^k = \frac{n!}{(n-k)!\,k!}$

Итерация

$e = 0{,}01$ $[-2;-1]$

$x^3 - 2x + 2 = 0$

$\cancel{f(x)}$

$x^3 = 2x - 2$

$x = \sqrt[3]{2x-2}$

$|f'(x)| = \left|\frac{2}{3(2x-2)^3}\right| < 1$

значит другая формула

$x_0 = \frac{-2-1}{2} = -1{,}5$

$x_1 = \sqrt[3]{2(-1{,}5)-2} = -1{,}7099$

$x_2 = \sqrt[3]{2x_1 - 2} = -1{,}7565$

$x_3 = \sqrt[3]{2x_2 - 2} = -1{,}7665$

$|x_3 - x_2| = 0{,}01$

$x_4 = \sqrt[3]{2x_3 - 2} = -1{,}769$

$|x_4 - x_3| \leq 0{,}01$ корень $-1{,}76$

12.4.3.1 Үлестірім параметрінің [illegible] нүктелік бағасы

$$S^2 = \frac{n}{n-1} D_T$$

$$D_T = \frac{\Sigma x_i^2}{n} - \bar{x}^2$$

$$S^2 = \frac{n}{n+1}\left(\frac{\Sigma x^2}{n} - \bar{x}^2\right)$$

$$\bar{x} = \frac{4\cdot 5 + 9\cdot 10 + 3\cdot 20 + 6\cdot 25}{50} = 5{,}2$$

$$D_T = \frac{4^2\cdot 5 + 9^2\cdot 10 \ldots}{50} - 5{,}2^2 = 3{,}16$$

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$\left[\bar{x} - 2\frac{\sigma}{\sqrt{n}} ; \bar{x} + 2\frac{\sigma}{\sqrt{n}}\right]$$