

# Optimal Aircraft Design Decisions under Uncertainty via Robust Signomial Programming

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Aircraft design benefits ~~greatly~~ from optimization under uncertainty, since design feasibility and performance can have large sensitivities to uncertain parameters. ~~The traditional~~ *ad-hoc* ~~mathematically non-rigorous~~ methods of capturing uncertainty do not adequately explain the trade-offs between feasibility and optimality, and require prior engineering knowledge which may not be available for novel aerospace vehicle concepts. This paper proposes a solution method for engineering design optimization problems under uncertainty using robust signomial programs (RSPs). The proposed RSP formulation leverages an existing approximate robust geometric programming (RGP) formulation ~~developed by Saab~~, *not standard to cite names in an abstract* and extends it by allowing difference-of-log-convex constraints that appear in many design problems. Signomial programs have demonstrated potential in the solution of multidisciplinary non-convex optimization problems such as aircraft design, and the formulation and solution of robust signomial programs (RSPs) allows for conceptual engineering design that captures parametric uncertainty. The paper details ~~the~~ *the* method ~~which is~~ based on solving a sequence of RGPs, where each RGP is a local approximation of the RSP. Then it explores the trade-off between robustness and optimality rigorously by implementing RSPs on a simple aircraft design problem, and demonstrates the effect of robustness requirements on aircraft design decisions.

## Nomenclature

CEG	Convex Engineering Group
GP	geometric program
LHS	left hand side
MDO	multidisciplinary design optimization
NLP	nonlinear program
SP	signomial program
RGP	robust geometric program
RHS	right hand side
RO	robust optimization
RSP	robust signomial program
SO	stochastic optimization

## I. Introduction

Aircraft design exists in a niche of design problems where "failure is not an option"<sup>a</sup>. This is remarkable since aircraft design problems are rife with uncertainty about technological capabilities, environmental factors, manufacturing quality and the future state of markets and regulatory agencies. Optimization under

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<sup>a</sup>Quoting Gene Kranz, the mission director of Apollo 13.

~~uncertainty for aircraft presents low hanging fruit, since~~ The program risk of aircraft design problems is high and the goal of these methods is to be able to provide designs that are robust to realizations of uncertainty in the real world. *opt. under uncertainty*

Zang et al. [1] succinctly describe the categories of benefits for optimization under uncertainty for aircraft. These are the following:

- *Confidence in analysis tools will increase.* The uptake of new design tools in the aerospace industry has been low due to heavy reliance on legacy design methods and prior experience when faced with risky design propositions, even in the design of novel configurations where understanding of the design tradespaces is lacking. Robustness will increase confidence in analysis tools because of its ability to better capture the effects of technological uncertainty on the potential benefits of new configurations.
- *Design cycle time, cost, and risk will be reduced.* Design cycle costs as well as the engineering hours per aircraft have been increasing [2] precipitously since the 1950's. Aircraft design and development is costly, so the ability to handle uncertainty in the conceptual design process is critical for the long-term success of an aircraft, helping reduce the program risk.
- *System performance will increase while ensuring that reliability requirements are met.* The effectiveness of an aircraft depends heavily on its ability to deliver on performance, which is dependent on assumptions about the current technological environment and the ability to produce vehicles of a certain quality.
- *Designs will be more robust.* The ability to provide designs with feasibility and performance guarantees will mean that designs and products will be more robust to uncertainties in manufacturing quality, environmental factors, technology level and markets, and better able to handle off-nominal operating conditions.

*Unclear whether these are your interpretation of Zang et al., or a direct quote. Also, it's taking a long time to get to the point - this feels a bit too general.*

*too many colloquial sayings. Try to cut.*  
In economics, the idea that risk is related to profit is well understood and leveraged. In aerospace engineering however we often forget that there is no such thing as a free lunch and that the consequence of risk aversity necessarily is performance that is left on the table. Considering that conceptual design in the aerospace industry hedges against program risk, the Robust Optimization (RO) frameworks proposed in this paper will give aerospace engineers the ability to rigorously trade robustness and the performance penalties that result from it.

### I.A. Approaches to optimization under uncertainty

Faced with the challenge of ~~developing general nonlinear programs~~ *finding designs* that can ~~incorporate~~ *handle* uncertainty, the aerospace field has developed a number of mathematically non-rigorous methods to design under uncertainty. Oftentimes, aerospace engineers will implement *margins* in the design process to account for uncertainties in parameters that a design's feasibility may be sensitive to, such as material properties or maximum lift coefficient. Another traditional method of adding robustness is through *multi-mission design* [3], which ensures that the aircraft is able to handle multiple kinds of missions in the presence of no uncertainty. This is a type of *finitely adaptive* optimization geared to ensure objective performance in off-nominal operations.

~~The weaknesses of these non-rigorous methods are many. They provide no quantitative measures of robustness or reliability [1]. They rely on the expertise of an experienced engineer to guide the design process, without explicit knowledge of the trade-off between robustness and optimality [4]. This is a dangerous proposition especially in the conceptual design phase of new configurations, since prior information and expertise is not available. In these scenarios, it is especially important to go back to fundamental physics and use rigorous mathematics to explore the design space [3]. Furthermore, they are often too conservative, ruling out potentially beneficial technologies and configurations due to the inability to adequately trade off performance and risk.~~

There are two rigorous approaches to solving design optimization problems under uncertainty, which are Stochastic Optimization (SO) and RO. Stochastic optimization<sup>b</sup> deals with probability distributions of uncertain parameters by propagating these uncertainties through the physics of a design problem to ensure

<sup>b</sup>Note that stochastic optimization is an overloaded term, and exists in three contexts in the literature. The first is the solution of deterministic problems with stochastic search space exploration. The second is the solution of simulations, often partial differential equations, with uncertain parameters. The final is the solution of design optimization problems with stochastic parameters. We explore the third category.