NPV to Gpkit

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Variables

FV : [$] Value of an investment at some future date
r: [1/time] Rate
C: [$] Cash flow, money used to make investment
PV: [$] Value of an investment at the present time
t: [time] Time

Problem Set Up

Let's assume that we want the Net Present Value (NPV) to be $10 Million. NPV can be expressed by:

$$\text{NPV} = \sum_{i=0}^{N} \text{PV}_i$$

.

Let's assume that the time periods of the payments are not equal but that the PV_i are equal and that there are 3 payments. PV can be expressed by

$$\text{PV} = \text{C}e^{-rt}$$

.

With these assumptions we can get rid of the i subscript and claim that

$$\text{NPV} = 3\text{PV}$$

Now the problem becomes solving for the length of each time period such that each PV is equal. Let's assume that the same payment, C, is made at each payment period and that C is given. This allows us to write

$$\text{C} = \text{PV}e^{r\Delta t_i}$$

Let's assume that t_0 is the time of evaulation of the NPV and that t_i is when every payment is made and every PV evaluated. This means that

$$\begin{bmatrix} \Delta t_1 = t_1 - t_0 \\ \Delta t_2 = t_2 - t_1 \\ \Delta t_3 = t_3 - t_2 \end{bmatrix}$$

GP Set Up

This is now set up for a GP optimization problem.

1. Variables

$$
\begin{aligned}
&NPV \leftarrow 10e6[\$] \\
&C \leftarrow 4e6[\$] \\
&r \leftarrow 0.1[1/\text{year}] \\
&PV: \text{ Free Variable}[\$] \\
&t_{1,2,3}: \text{ Free Variable}[\text{years}] \\
&\Delta t_{1,2,3}: \text{ Free Variable}[\text{years}]
\end{aligned}
$$

2. Objective

$$\text{minimize} : t_3$$

3. Constraints

$$
\begin{aligned}
&NPV \leq 3PV \\
&C \geq PV\left(1 + rt_i + \tfrac{rt_i^2}{2!} + \tfrac{rt_i^3}{3!}\right) \\
&t_1 \geq \Delta t_1 \\
&t_2 \geq \Delta t_2 + t_1 \\
&t_3 \geq \Delta t_3 + t_3
\end{aligned}
$$