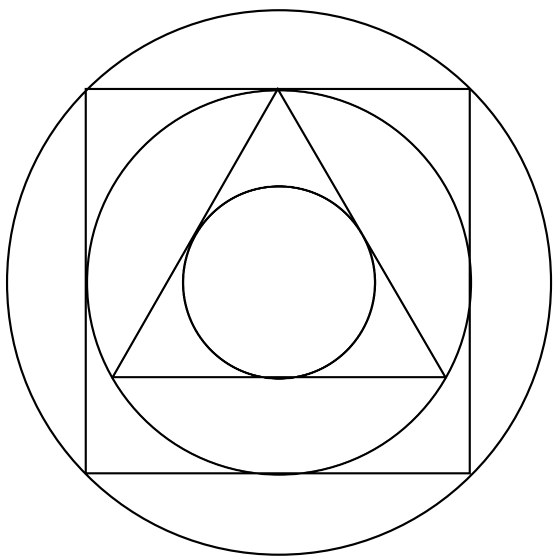
QUADRATURE OF THE CIRCLE

REVEALING

THE TRUE VALUE OF Pi



By

George R. Hull

QUADRATURE OF THE CIRCLE
Revealing
THE TRUE VALUE OF Pi

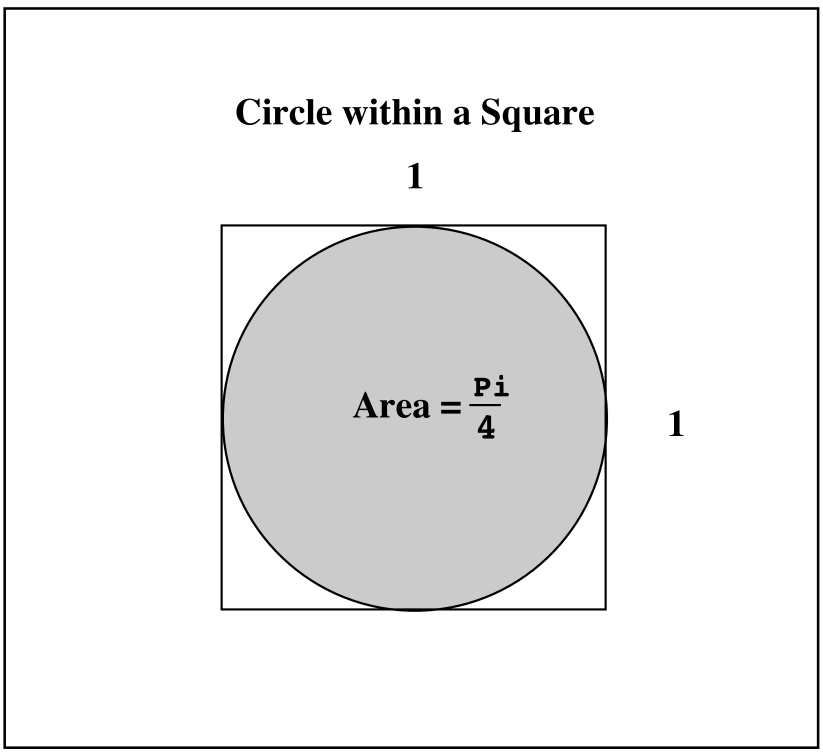
by
George R. Hull

This paper was inspired by and is dedicated to Mr. John A. Parker

Introduction Part 1

Since the days of ancient Greece men have sought the exact numerical ratio between circumference of the circle and it's diameter. Circumference is the distance around the outside edge of the circle. The diameter is the maximum straight line length which can be contained by the circle. This relationship is represented by the Greek letter Pi (sounded like pie) having the symbol π or Π.

It has long been recognized that a square and circle could be drawn with the circle fitted perfectly within the square. Under these conditions the diameter of the circle and the length of each side of the square are exactly equal. The area of the square could be easily calculated by multiplying the length of each side of the square. The area of the circle will be some exact fractional part of the area contained within the square. This fractional part has been known for centuries to be equal to Pi divided by four.



Finding the exact relationship between the area of the square and that of the circle has been called the problem of ' QUADRATURE OF THE CIRCLE'. This is a most famous problem that has attracted the attention of men for thousands of years and whose solution has escaped the grasp of most of these seekers.

Delta Spectrum Research, Inc. has collected and reviewed many classic and little known works of great men of the last century. Found during the research work on Sympathetic Vibratory Physics was a work written by Mr. John A. Parker published in 1874 under the title, *QUADRATURE OF THE CIRCLE.*

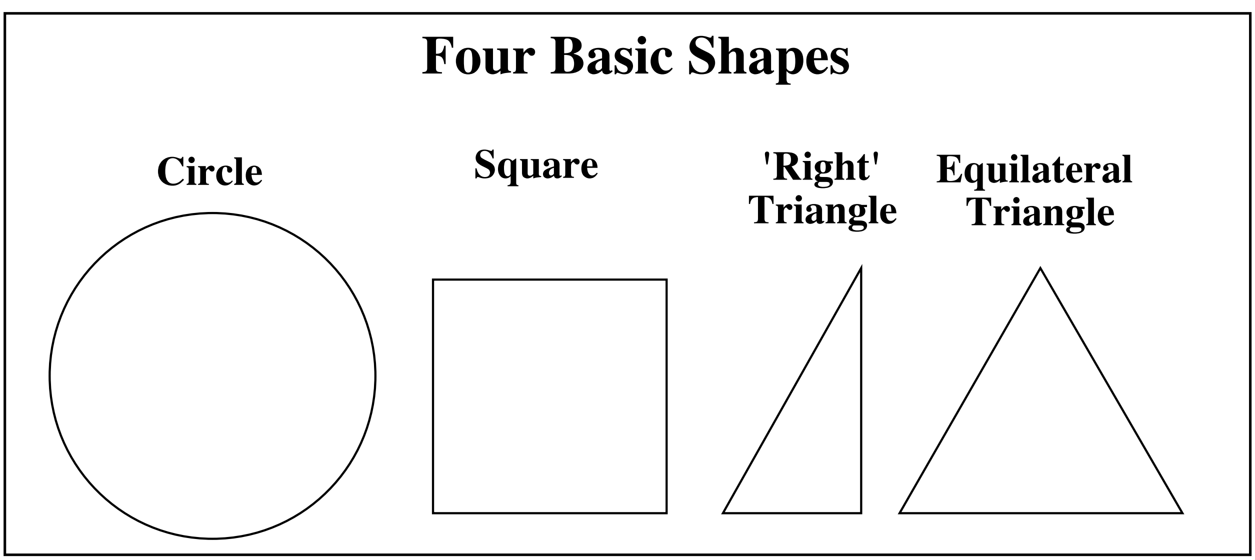
In *QUADRATURE OF THE CIRCLE Revealing THE TRUE VALUE OF Pi,* we explore and expand upon Mr. Parker's findings. In clear, up to date English the central principles of the circle are revealed which allow the true and correct value of Pi to be calculated. Several important characteristics of the the arc, straight line, circle, triangle and regular polygon which have not been given due recognition are explored. Methods for constructing each geometric shape are given and the great importance of the Pythagorean or 'right' Triangle described. General formulas are developed allowing any regular shape to be made equal to another by area, perimeter, or proportion.

Contained is the solution to 'the Quadrature Of The Circle' problem and the exact and correct value of Pi . Pi has the correct and exact value of 20,612 parts of circumference to 6,561 parts of diameter. This value when reduced to a decimal number is slightly larger (in the sixth decimal place) than the value obtained by trigonometric approximation. The larger value of Pi will be designated by the Greek capital letter Π and the smaller and commonly accepted value by the small greek letter π.

The polygon approximation method does not give an exact nor accurate value for Pi due to the inappropriate assumption that the difference between a curved line and a straight line can be made vanishingly small. Though a great many sides are utilized, and we will show the calculations for a polygon with 1,000,000 sides, the difference between a straight line and a curved line still remains. The circle is formed of a curved line circumference - it is not accurately represented by the many sided straight line polygon.

The importance of solving 'the Quadrature of the Circle' problem lies in finding relationships between the circular shape and shapes made of straight lines. Having found these concepts we can expand them to universal application.

This paper will illuminate the central principles of the circle, show the several important characteristics of the the arc, straight line, circle, triangle and regular polygon. We will show the solution to the Quadrature Of The Circle, the exact and correct value of Pi and seek to do so in such a



Introduction Part 2

When I began to write this paper I sought to present Mr. Parker's thoughts in the language that the average American reader could follow. The language has changed during the 117 years since Mr. Parker's book was published. In addition, Mr. Parker was a Geometer of very great skill with a rigorous style. His vocabulary and insistence that only geometry be used to prove his points, except when numerical example was need, made his work very difficult to follow. At certain critical points he seemed to make a leap to a new concept for which the reader had not been adequately prepared.
In this paper we will try not to make those same mistakes. I have sought to lay out the problem such that any interested reader, independent of how little or how much mathematical background he or she may previously have had, will be able to completely follow the arithmetical methods used.

To lessen the mathematical problem we will do much of the work using Geometric construction with drawing compass, straight edge ruler, and where required, a protractor. In this way I hope to build visual patterns that appeal to the right side of the brain. Following are the exact mathematics reduced to multiplicative patterns that allow us, and the left side of the brain, to accurately quantify the geometric patterns that have been introduced.

For those that have a background which includes mathematics, I hope that you will bear with me for I write also for you. For those that do not use mathematics regularly I hope that this paper is sufficiently complete.

The second reason for writing the paper in as complete a form as possible is to finally put right, once and for all, the issue of Quadrature of the Circle and the exact value of Pi.

In *The Prentice-Hall Encyclopedia Of Mathematics* seven pages are devoted to the history of Pi and attempts to solve the Quadrature of the Circle. These pages show that the efforts have been monumental and have occurred in every country of the world with known efforts existing in Babylonia in 2,000 BC.

The great inventor and mathematician Archimedes of Syracuse, Greece, living in about 212 BC has been credited with using an inscribed and circumscribed polygon having 96 sides to determine Pi. He established the value to be between 3 and $^{10}/_{71}$ to 3 and $^{10}/_{70}$. When we divide the fractions and add them to three we find: 3.1408 is less than Pi and 3.1428 is greater than Pi.

Many additional attempts to narrow this range have been undertaken. "In 1610 Ludolph van Ceulen, a German, computed Pi to 35 decimal places, using polygons having as many as 2 raised to the 62nd power - 4,611,686,018,427,387,904 sides - and spending much of his life doing it".

On pages 68 and 69 of *The Prentice-hall Encyclopedia Of Mathematics* the following sentences can be found, "Greek mathematicians tried to calculate the value of Pi from the area of the circle; they reduced this problem to quadrature or squaring of the circle - the attempt to construct a square of area exactly equal to the area of a circle." "It turns out that it is impossible with straightedge and compass alone to construct √Pi by any construction method. *Furthermore, it is impossible to calculate an exact value for Pi by any method at all*".

In this paper we will finally and conclusively show both the Quadrature of the Circle and the exact and correct value of Pi.

After 4,000 years of efforts it is fully time to put this issue to rest. By the time that you the reader have this paper in your hands I will have spent nearly 4 months of my life bring this solution to light. Mr. Parker developed his original process in 1851 and spent 30 years working on and presenting his findings.

The dear soul, Mr. Ceulen spent years of his life calculating Pi to 35 digits, probably to the dismay of his family and the snickers of his neighbors. The tragedy is that the polygon method of approximation and the true value of Pi begin to differ in the sixth decimal place; much of Mr. Ceulen's efforts may have been for not.

The Approach Utilized In This Paper

Chapter 1 This Introduction

Chapter 2 will introduce the circle, arc, wheel and concept of circumference.

Chapter 3 will introduce and show how to draw and divide the basic "Perfect" shapes known as the Equilateral Triangle, Square, Pentagon and Hexagon.

Chapter 4 will introduce measurement and calculation of area contained within the Square and 'right' triangle. Secondary concepts of Geometry, fractions, proportions, square root, squaring, and Trigonometry will be illustrated.

Chapter 5 will use the 'Right' Triangle to measure parameters of side length, perimeter, area and introduce a general formulae set for the measurement of the circle and all regular polygons.

Chapter 6 will connect all of the concepts introduced in the preceding chapters and solve the Quadrature of the Circle including calculation of the exact value of Pi

Chapter 7 will compare the true value of Pi to the approximated value in an in-depth examination.

Chapter 8 will develope an amazingly powerful method to describe and predict the relationships between three gravitating bodies such as the Moon, Earth and Sun.

Chapter 9 will summerize several interesting relationships that were uncovered in the preceding chapters.

The Appendix - includes several topics which support this work and the Bibliography.

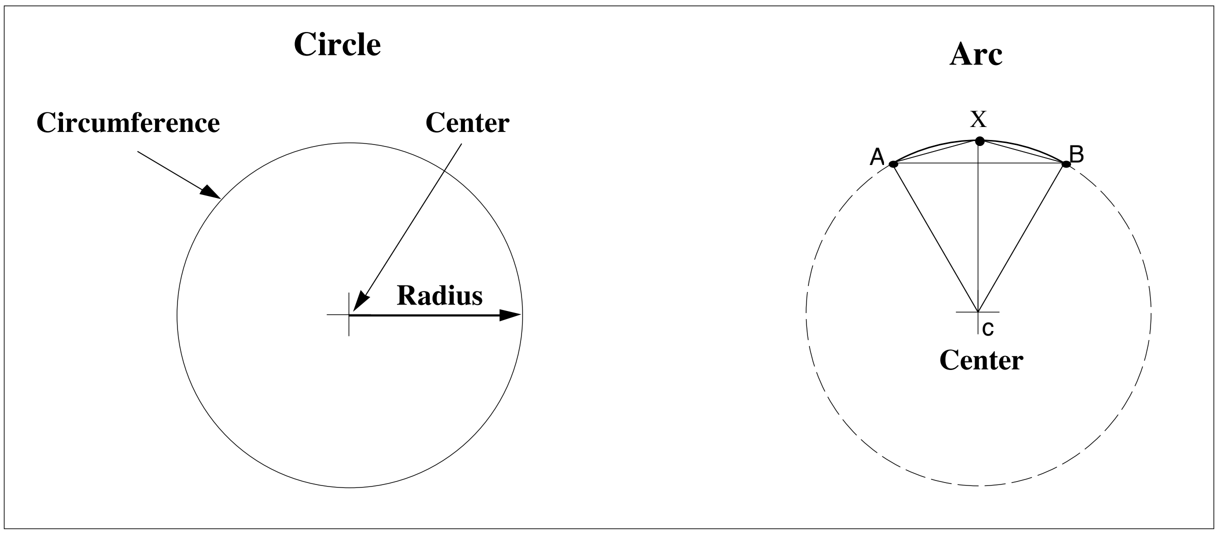
Good reading. Let us get this job *done* so that the correct relationships between the circle and Pi are established such that they will not be forgotten again.

I hope that, you the reader, will enjoy this work.

The Circular Shape

The Circular Shape

We begin our study of the circular shape by considering the way that real circles are made, their elements and their use by man and Nature. Man uses the circle in the form of wheels and gears to move his machines. Nature uses the circle to build atoms, planets and the orbital paths that keep all of these particles in exact placements one to the other. After generating a firm image in our mind we then relate the circle to mathematical processes.



By the Human Hand

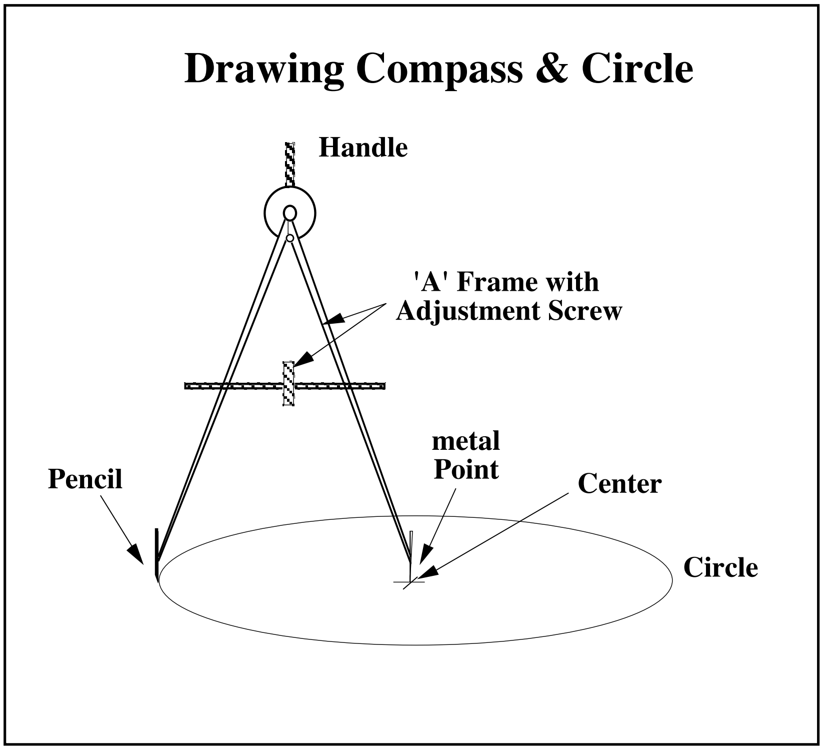
The circle is the most simple of all the shapes to make. It can be drawn on a sheet of paper by choosing a center and then making a line that is at all points a fixed distance from that center. This process is continued until the center point is completely enclosed by that line. Such a line is called a curved line. The resulting shape is called a circle. If only part of the circle was drawn then that shape would be called an arc. The distance from the center of the circle to the circumference line is called the radius. These concepts are illustrated in the drawing above.

The Arc - Curved Lines Differ From Straight Lines

Between curved lines and straight lines there is an essential difference in their method of construction and of their characteristic of length. In construction the arc is at all points equidistant from a central point (**c**). In construction the straight line (on a flat surface) is made by connecting two adjacent points with the least possible length.

Referring to the drawing labeled Arc we easily see that arc (**AXB**), connecting the two adjacent points **A** and **B** is longer than straight line (**AB**) connecting the same two points. Arcs (**AX**) and (**XB**) can be seen to be longer than lines (**AX**) and (**XB**). *The length of an arc connecting two points <u>will always be longer</u> than the combined length of any number of straight lines joining the same two points.*

The Drawing Compass

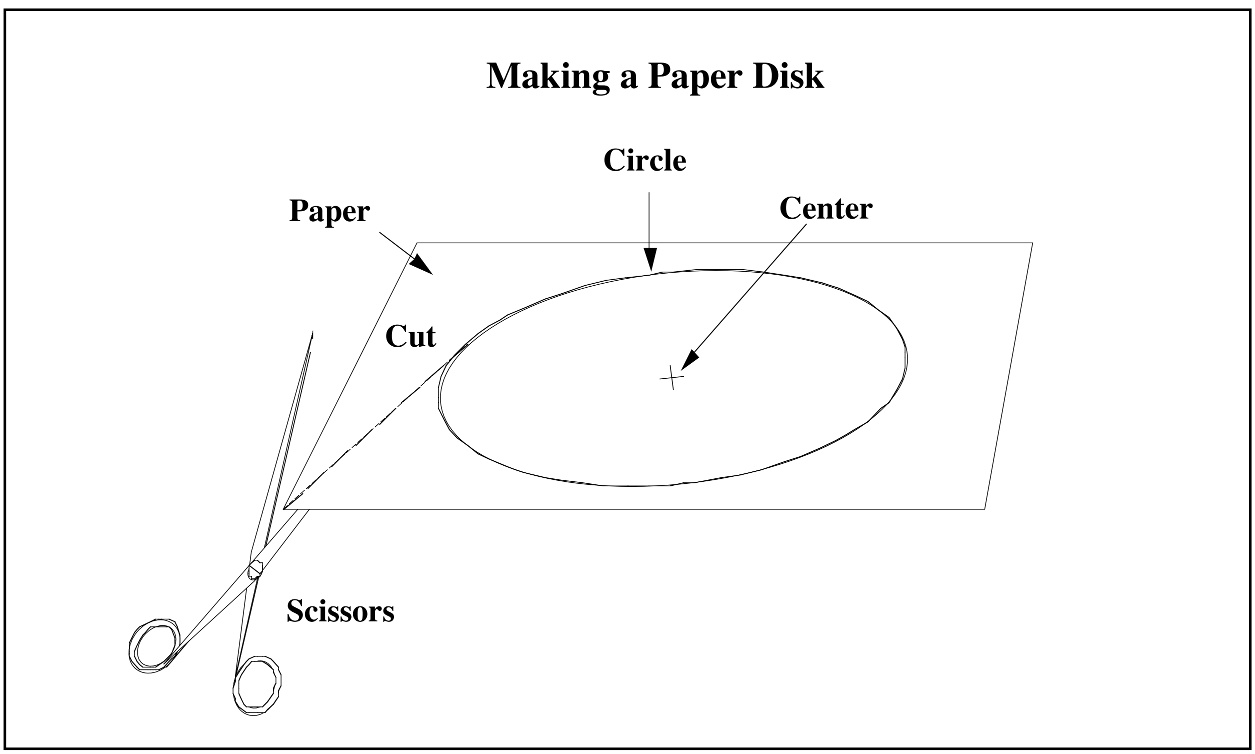


A tool called a drawing compass does a very good job of holding a pencil point exactly a fixed distance from a center point The drawing compass has a holder for the pencil point, a sharpened metal point to penetrate the paper, and a rigid frame in the shape of an 'A' which allows the pencil point and sharpened metal point to be held apart. An adjustment screw on the center bar of the 'A' frame allows the spacing between the pencil and metal points to be varied.

By holding the compass upright at the top of the 'A' frame and carefully moving the hand one can cause the pencil point to move around a center point thereby drawing a very accurate circle.

A Real Circular Shape - Making a Paper Disk

We can make a circle into something real by choosing a piece of material, say paper, larger than the intended size of the circle. We find a center point that is far enough from any edge of the



paper so tthat all parts of the circle will be on the paper. We use the drawing compass to draw the curving line of the circle. Carefully cut away all of the material that is farther away from the center than the outer edge of the curved line. The result is a round shape having a small hole in the center and an edge an equal distance from this center. This shape is called a disk.

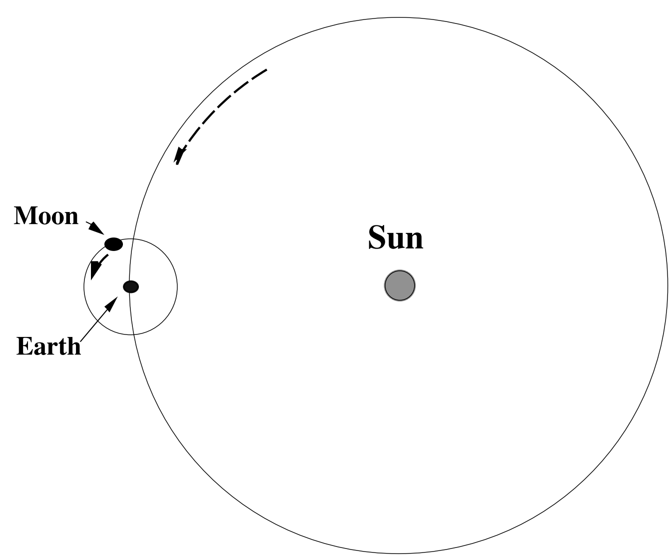
Planetary Orbits - The Circles of Nature

In space we see the best examples of natural circles and spheres. When we look at the moon when it is full, or at the sun during the day, each can be seen to have the apparent shape of a circle or disk. This shape and that of the planet earth that we stand upon are known to have an actual shape of a sphere. A sphere is a real shape having all of it's surface located a fixed distance from the center. When we see pictures of the earth taken by Astronauts standing on the moon then the earth's surface also looks like a disk. Rays of sun light cast shadows which help us to see both the earth and moon as having real shapes of a sphere.

After many years of observation and study astronomers have found that the moon revolves around the earth in a circular path and the earth revolves around the sun in a similar way. All the other planets of our solar system similarly revolve around the Sun, and for those that have moons, they too revolve around their central planet. Nature uses the circle as the most basic of all shapes. It is the center that is the starting point.

Sun, Earth and Moon Orbits

Not to Scale



The Wheel

Circumference is the distance around the outside boundary of a circle. The center of the circle is a constant distance from this boundary. If the outer boundary is made to rotate around a fixed center point called an axle - a wheel results. Wheels allow rotation about a point such that the axle can move in a straight line as the wheel rotates. The distance traveled by the axle as the wheel rotates is determined by the circumference of the circle forming the wheel.

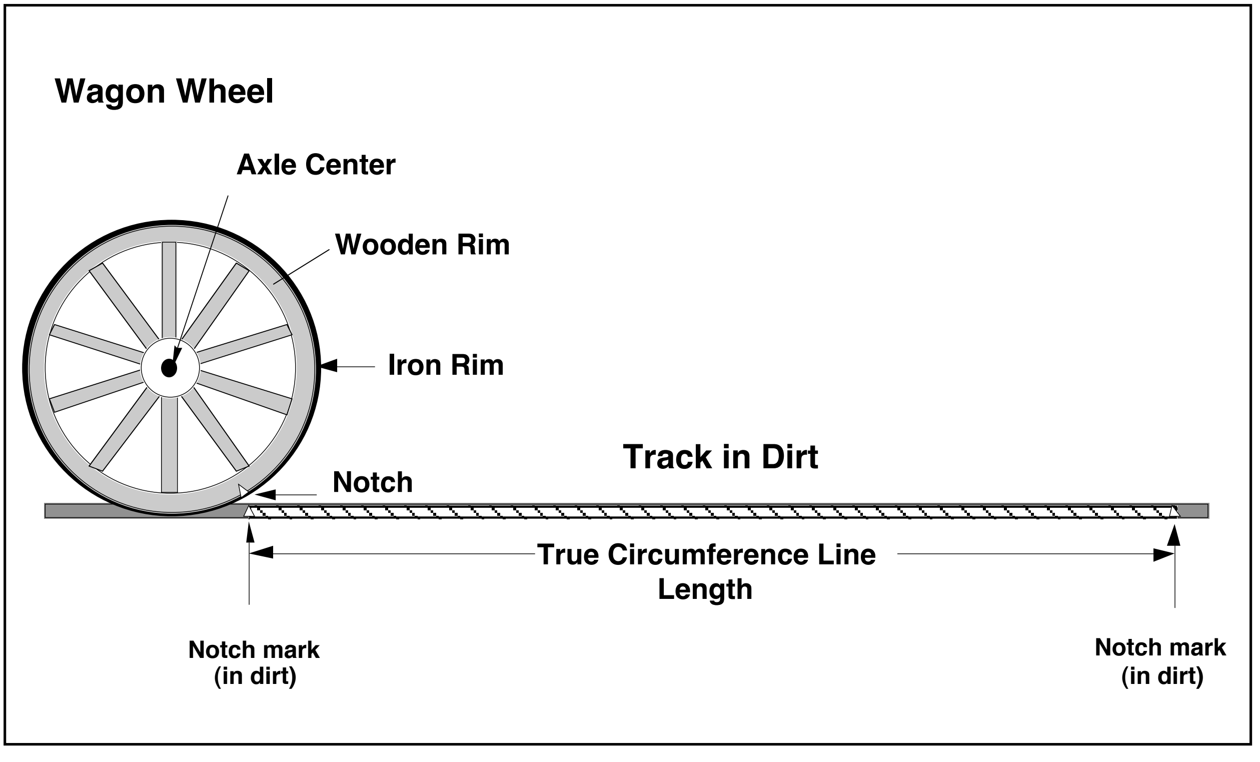
The Concept of the Wheel related to Circumference

Imagine an old stage coach wagon wheel as is illustrated on this page. The wheel has an axle, wooden spokes and outer wooden rim. Tightly fitted around and compressing the outer wooden rim is an iron rim.

The circumference of the wagon wheel is not the length of the iron band - no matter how thin it is made; it is, of its self, part of the wheel. Rather the surface of the outer most atoms of the iron rim, where iron embraces air, is the location where the line of circumference is to be measured.

As illustration, let a small notch exist in the iron rim of the wheel. Let the wheel roll over soft, fine dirt for a distance greater than that which causes one revolution of the wheel. Two notch marks will be visible in the dirt track made by that wheel. *The distance between these two marks is the true circumference line length.*

The line so formed lies outside of the wheel and is equal in length to a straight line which previously had the form of a closed (ends joined) curved line. If we measure the length of this line and divide that length by the diameter of the wheel we would have an approximate value for Pi. The exact number obtained would depend upon how accurately we measured each line length and the perfection with which the wheel was made.

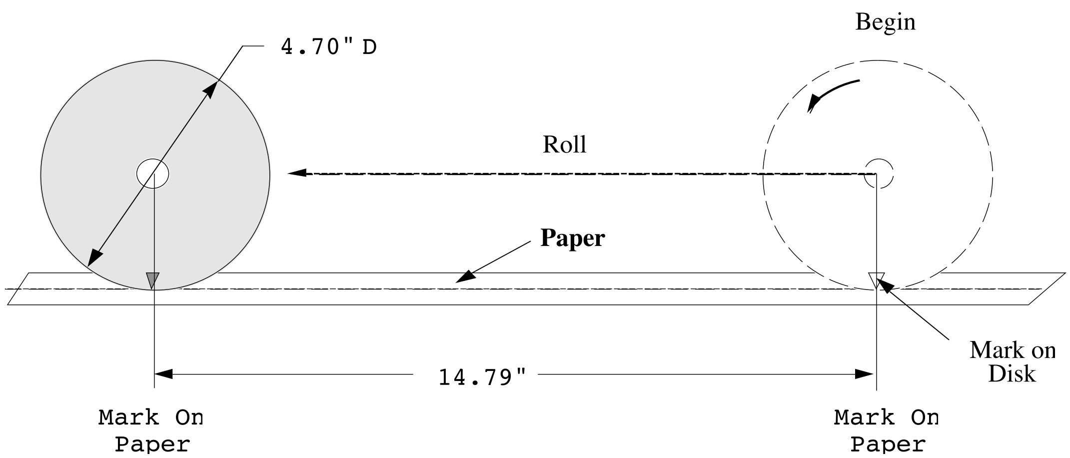


Measuring the Circumference of a 'CD' Disk

There is a simple experiment that will allow us to find an approximate value for Pi. Since wagon wheels are not very common in most homes or offices. I found that a plastic compact 'CD' disk was very accurately made and had a hole in the center which could serve as the location of an axle. See illustration on this page.

Your finger can be used as an axle and a piece of paper on a desk top as the track site. Mark a

Measuring The Circumference of a 'CD' Disk



Experimental Value for π

$$\pi = \frac{14.79 \text{ inches}}{4.70 \text{ inches}} = \mathbf{3.1468}$$

Accepted value of π = 3.141,592,654 +

$$\text{Error} = \frac{3.1468 - 3.141{,}592{,}654}{3.141{,}592{,}654} = 0.00166$$

Error = 0.00166 x 100%

Error = 0.166%

spot on the disk and on the piece of paper. Let the disk rotate about your finger as you move your hand in a straight line such that the disk rolls over the paper smoothly - neither leaving the paper or sslipping. Mark the paper where the spot on the 'CD' disk was again touching the paper.

I performed this experiment and measured the distance between the two marks on the piece of paper. The distance measured 14.79 inches. The disk diameter was measured to be 4.70 inches. Dividing 14.79 inches by 4.70 inches gives a resulting number of 3.1468. Notice that this number has no units of measure and is a ratio only.

Calculating Experimental Error

To calculate an experimental error (or difference between two things) we begin by subtracting the standard value from the experimental value and then divide by the standard value. We keep the sign of the subtraction process so that we know whether the experimental value was large or smaller than the standard value. A negative number means that the experimental value is smaller than the standard value.

The first 10 digits calculated to be equal to π by the common mathematical process are 3.141,592,654+. If I subtract the accepted value from my experimental value and divide the difference by the accepted value I calculate the measured value to be 0.00166 or about 0.166% larger than the accepted value. This difference is largely due to the limited accuracy of my steel ruler which is marked in 1/100 inch divisions.

Pi Is a Real Quantity

This simple technique gave an approximation of Pi to within .166% of the accepted value. You may want to perform this experiment yourself to confirm that Pi is a relationship between measurable qualities of the circle, disk or wheel. *Pi is not a mathematical abstraction. It has a real and fixed value which we and nature utilize continuously.*

Summary

In this section the circle, the arc, curved lines, planetary orbits, and the concept of circumference were introduced. The curved line was shown to be fundamentally different in method of construction from the straight line.

The paper disk and wheel were introduced to establish the concept of circumference as a line lying outside of the circle. The track of a wheel rotated about it's axis introduces an experimentally determined approximation of Pi.

In the next chapter we will draw and divide the circle in preparation for quantitative measurement.

Drawing The Perfect Shapes

Introduction

In the previous section we introduced the circle, arc, wheel, circumference and Pi as real and useful things. In this section we will draw several of the Perfect shapes. We will show how these shapes may be divided for purposes of analysis.

The perfect shapes consist of the Circle and all of the members of the Regular Polygon family. All of these shapes have a center, equal number of sides and equal angles between the sides. The angles that we will be working with are the ones formed between the center of the polygon and its vertices or sides.

We continue our journey

Drawing the Perfect Shapes

To make any of the perfect Shapes we need only a drawing compass, a protractor and a straight edge such as a ruler or drawing triangle. The drawing compass you have been introduced to. The protractor is a device usually made of plastic and shaped like a circle or half circle. Around the circumference of the protractor are marked degrees of angle. A center point and diameter line have been marked so that one can easily locate the degree angles relative to a given center point. The three drawing tools are illustrated on the following page.

If you don't have the three drawing tools mentioned you may want to purchase them so that you can make the circles and Perfect Shapes yourself. This is an enjoyable exercise and can be very illuminating.

It is possible to make all of the Shapes shown with only the drawing compass and a straight edge. The process is called Geometric Construction. We will be using Geometric construction techniques when we draw or divide our shapes. This is a very good basic exercise in measurement techniques.

Constructing shapes in a step by step fashion using rules and principles of construction is very much like constructing computer programs. If you are interested in Geometry or computer programming try these things for yourself.

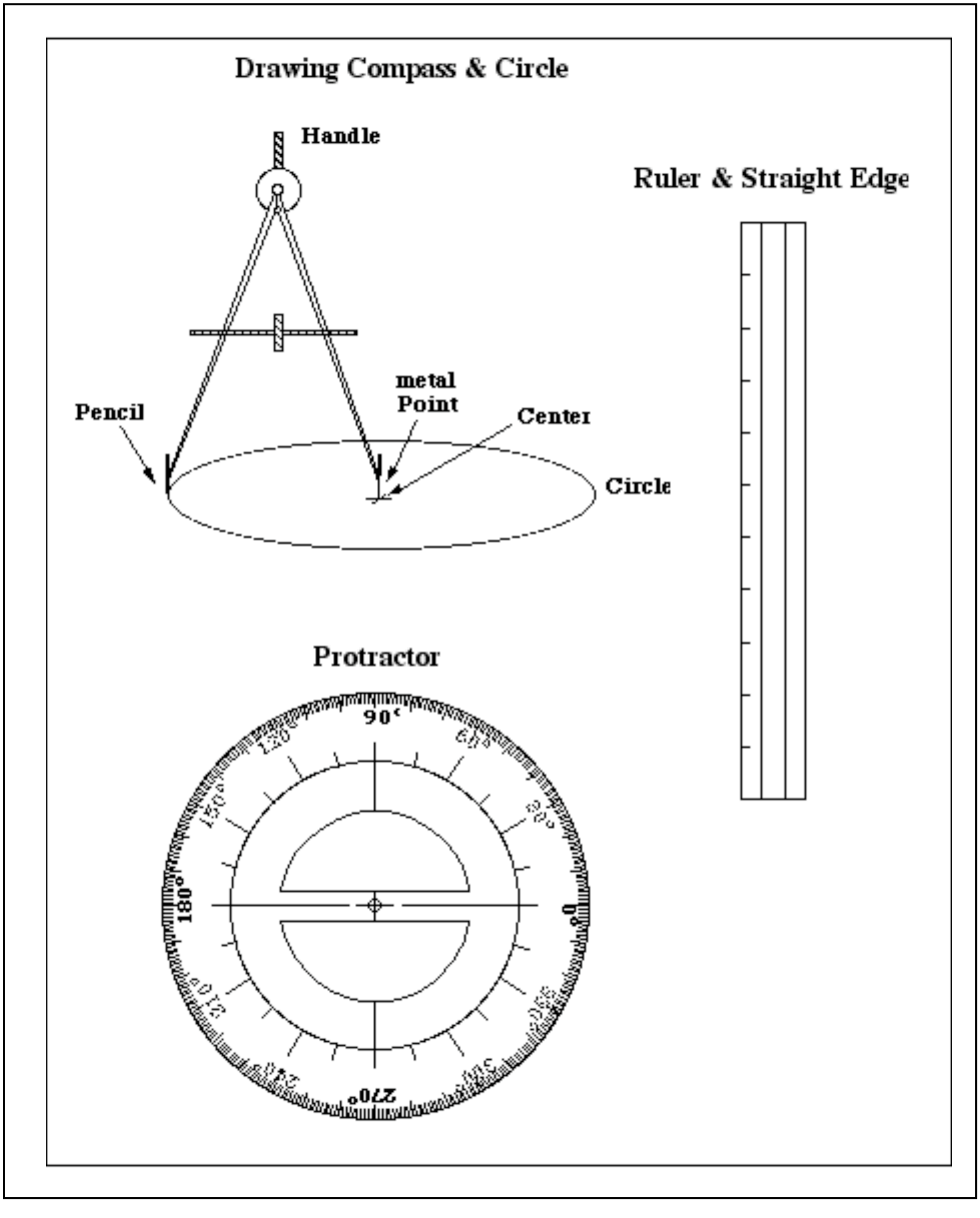
The Circle

The first perfect shape is the circle. It is constructed using the drawing compass and selecting a radius of any desired size. The process of construction is very simple: the metal point is placed against paper, the hand rotated and the circle is drawn.

The circle has a center and a circumference consisting of one continuous curved line containing 360 degrees of angle. The radius is the distance from the center of the circle to the line of circumference. The length of one complete circle of circumference divided by twice the length of radius is represented by the Greek letter **Pi**.

We will use the circle as the beginning shape whenever construction of any other regular shape is required. The perfect symmetry of the circle and its continuously curving line of circumference allow it to be drawn and divided into any size or shape desired.

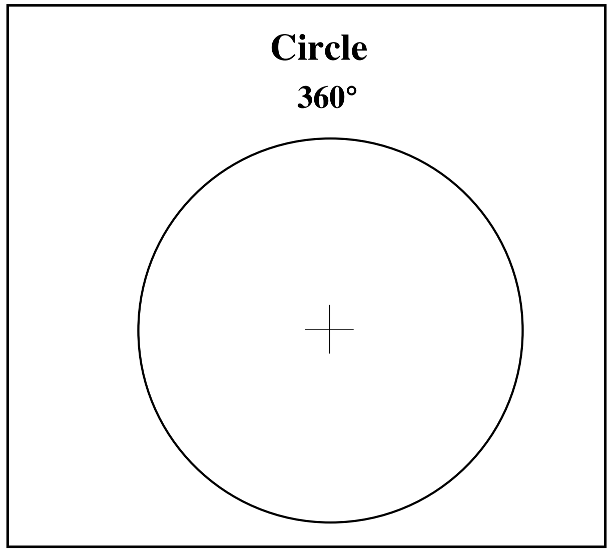
The Drawing Tools



The Circle has been Assigned 360 ° of Arc

The circle has no real divisions of its curved line circumference. It has been assigned 360 degrees of angle for 1 complete circular arc.

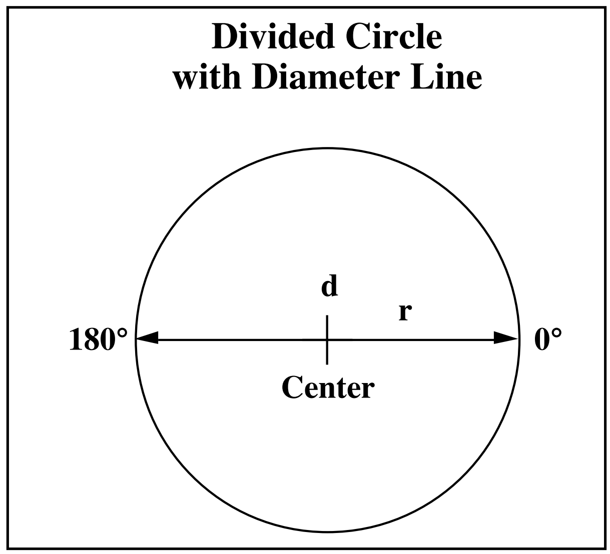
The drawing compass is used to draw the circle. A protractor may be used to mark or measure the angle between the center of circle and between two points on the circumference line. The Straight edge ruler may be used to draw a straight line connecting points on the circle.



Dividing the Circle

Draw a circle using the drawing compass.

Draw a straight line using our straight edged ruler passing through the center of the circle with ends terminating on the circumference line. This line is called a diameter line and we will refer to it by the using the letter 'd' made bold, **d**. It has a length of twice the radius line **r**, the radius line being the length from the center of the circle to *one* edge of the circumference line.



Label the point where the diameter and radius lines touch the right side of the circumference line, 0° degrees. The small zero ' ° ' placed above and to the right of a number designates it as being an angle of measurement in degrees of arc. We will label the other end of our diameter line 180° as the diameter line touches the circumference at a point half way round the circle. See the drawing at the right.

Bisecting an Angle

We now wish to divide the circle into additional halves again and do so with very high accuracy.

We can do this using our drawing compass. *The process we will be using is one of 'bisecting' our 180° angle into two exactly equal parts. At the same time we will exactly divide the diameter line into two parts.*

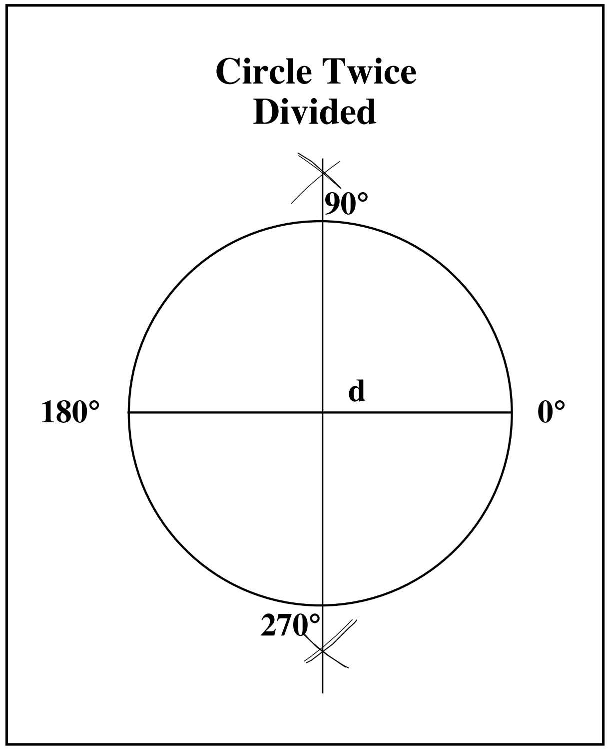
Striking the Arcs

Adjust the drawing compass to have a span between the pencil and metal points greater than the radius of the circle. Place the metal point at the intersection of the 0° radius line and the circumference line of the circle.

Draw a pair of short arc lines both above and below the center of the circle. This process is called 'striking an arc'.

Repeat the process placing the point of the drawing compass at the intersection of the 180° line and the circumference of the circle (do not change the span of the drawing compass).

Draw a second pair of short arc lines such that they cross the first pair.



Drawing the Dividing Line

Use a straight edged ruler to draw a straight line passing between the intersection of the upper and lower arc pairs. This line divides both the 180° angle and the diameter line into two exactly equal parts. This new line is at angle of exactly 90° to the diameter line. Such a line is said to be perpendicular. *Any time that an angle or line is bisected using this procedure the resulting line will be perpendicular to the line bisected or to a line connecting the two orginal points on the circle*.

Labeling the Divided Angles

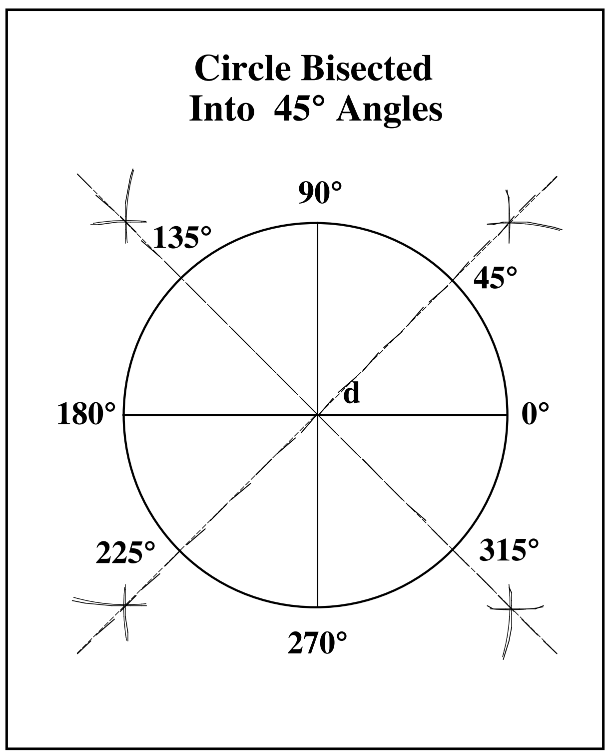
This new line exactly divides the 180° angle of the circle into two halves. The point where this line crosses the upper portion of the circle is at the 90° location. Wherethe line crosses the lower portion of the circle this is the 270° location (90° added to 180° equals 270°).

Finding the 45° Angles

We can use the same procedure of 'bisecting' to further divide our circle. To find the 45° angle use the 90° and 0° angles as placement points for the metal point of the drawing compass. Strike the intermediate arcs. Draw the dividing line using the crossing point of the two new arcs and the center of the circle. Proceed around the circle using each pair of angles which are 90° apart.

The circle has now been divided into additional angles of 45°, 135°, 225° and 315°.

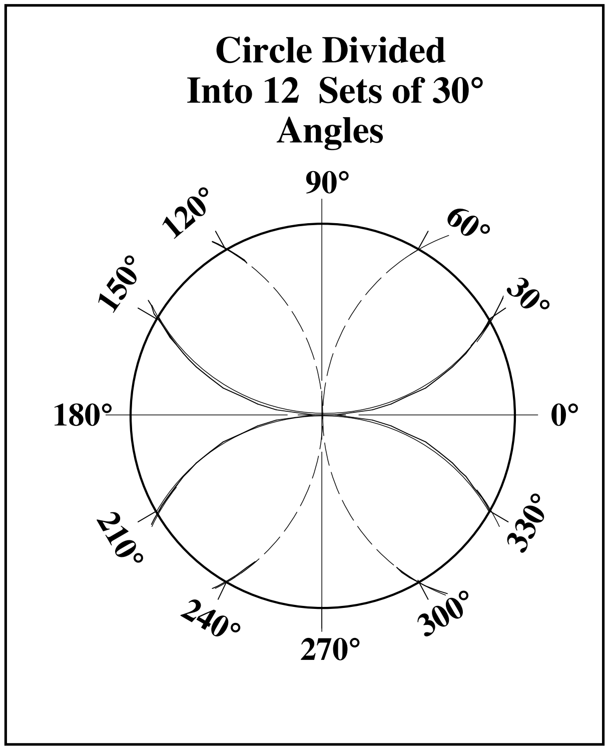
See drawing at the right.



Notice that the four new angles have a 90° relation ship one to another.

The Cord

The straight line distance between any two points located on the circumference of the circle is called a cord. There are an unlimited number of possible cords. The longest possible cord is the straight line length of the diameter. There is no minimum length as the circle has one continuously curving line.



Dividing The Circle Into Twelve 30° Angles

Adjust the drawing compass to have a span equal to the radius of the circle to be divided.

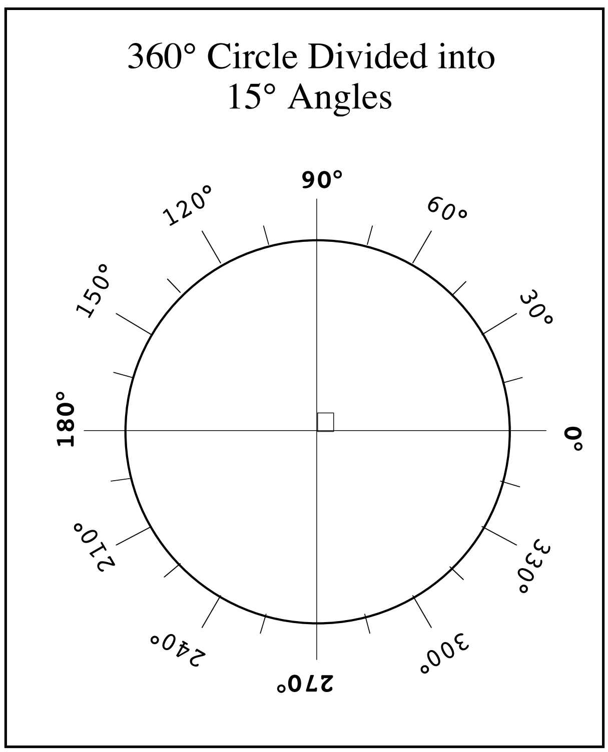
Place the metal point at the 0° mark on the circumference of the circle. Strike arcs on the circumference line on either side of the 0°/ 180° line. The arcs cross the circumference line at the 60° and 300° points (0° + 60° and 360° - 60°).

Proceed around the circle placing the metal point of the compass at the 90°, 180° and 270° locations on the circumference of the circle. At each location strike arcs crossing the circumference line at the two possible points. Each pair of arcs will cross the circumference line at 60° of angular measure from the compass point. By moving the compass point to each of the four 90° cardinal points a resultant spacing of 30° is produced between all points and the struck arcs.

Bisect Each of the 30° Angles

We bisect each 30° angle using the bisection technique employed previously. Label each point beginning at the 0° point at the right hand side adding 15° to each succeeding dividing arc.

The entire circle now has 24, accurately drawn, 15° angles equally spaced around the circumference line. These divisions are sufficient for our descriptive needs and have illustrated the important points of angular division, bisection and the perpendicular line.



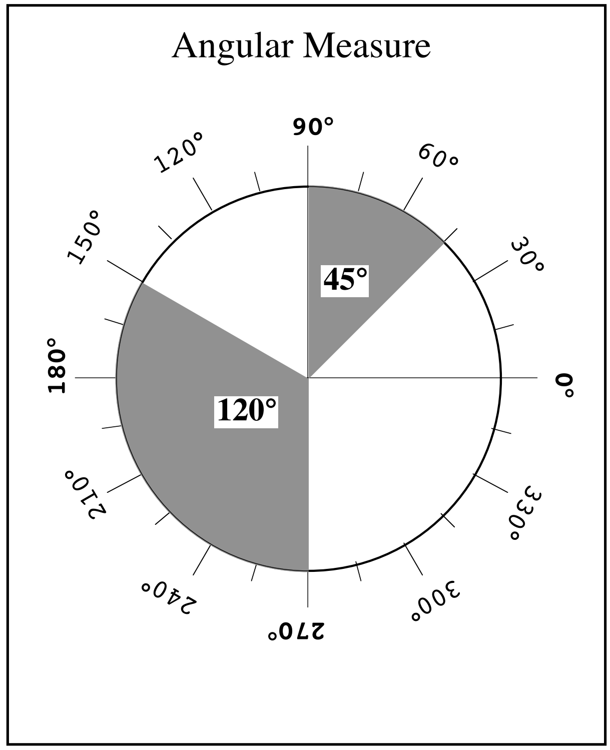
Perpendicular Lines and The 'right' Angle

The line connecting the center of the circle to the 90° angle mark forms a 90° angle to the line extending to the 0° mark. Two lines at 90° angle to one another are defined as being *perpendicular* to one another. Such a 90° angle is also called a 'right' angle.

A 90° angle can be represented by a small square box between the two perpendicular lines. Any two lines touching each other and having a 90° angle between them are said to be perpendicular.

Angular Measure

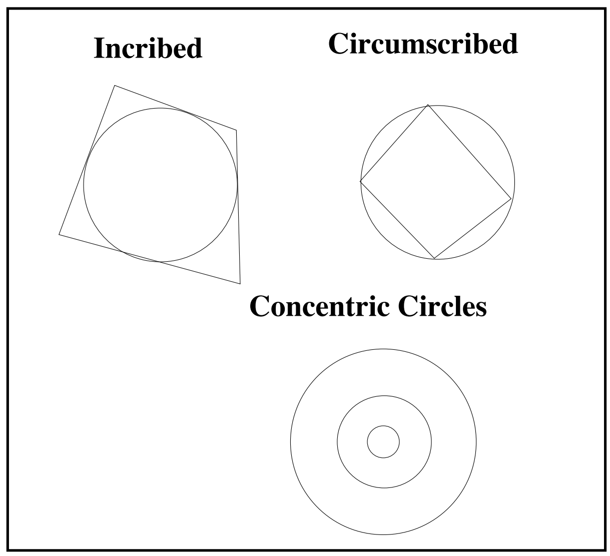
Any number of lines desired can be drawn from the center of a circle to its circumference. Each can have any desired angle to the reference line. The angular distance between any pair of lines is measured by subtracting the degree angle of one from the degree angle of the other. A protractor can be used to measure the angles or draw any angle desired. The small hole at the center of the protractor must align with the center of the circle and the diameter line of the circle.



Circles Combined with Other Shapes

An Inscribed Circle

If we have some shape and draw a circle inside this shape with the circumference of the circle just touching each side of the shape the circle is said to be Inscribed to that shape. The word inscribed is easy to remember because we draw the circle *inside* of the shape and the word 'scribe' means to draw or mark. Thus we '*draw inside*' or inscribe our circle.



A Circumscribed Circle

The opposite process is to circumscribe a circle about a shape using the minimum radius possible. In this case the circle encompasses or encloses the shape. Circumscribe means 'to draw around' as we did with the *circum*ference of a *circ*le.

The juncture points of all sides of a regular shape will be located on the circumference of our circumscribed circle.

Concentric Circles

Circles drawn using the same center point are said to be concentric to one another. Any number of circles or polygons may be drawn about one center.

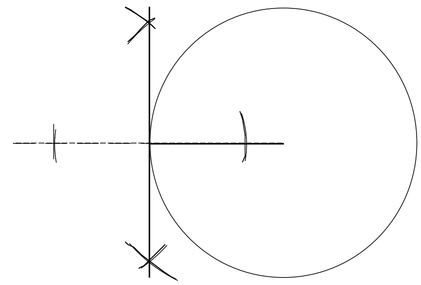
A Line Tangent to a Circle

If a line touches a circle at only one point it is said to be tangent to the circle. The process of drawing tangent lines or constructing a circle tangent to a line is a valuable process. We will show how this may be accomplished using only a drawing compass and straight edge.

Constructing a Line Tangent to a Circle

Draw a line from the center of the circle extending past the circumference of the circle. Construct another line perpendicular (at a 90° angle) to the point on the circumference of the circle where the line drawn from the center of the circle crosses the circumference line. The new line will be tangent to the circle.

A Line Tangent to a Circle



Making a Circumscribed Square

To make a circumscribed square we repeat the process of constructing a tangent line four times.

Select four extended radius lines at 90° angles to one another. Make a line perpendicular to each radius line at the point of intersection with the circumference line.

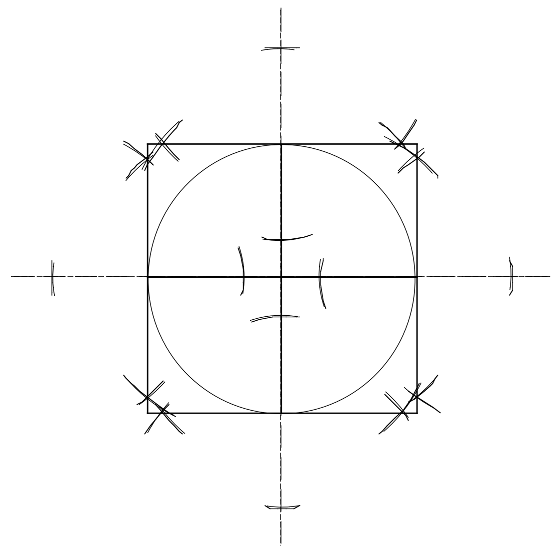
The four perpendicular lines are tangent to the circle and form a square circumscribing the circle.

The reverse of this process is also true. If we use the sides of a square as our starting point and bisect them, the resulting four lines will form the radii of the inscribed circle. In the process of generating the radii of the inscribed circle each side of the square has been divided into two equal parts.

We recognize the figure to the right as being our original circle within a square.

We also notice that the larger square has been divided into four equal and smaller squares. In the next chapter we will use the process of dividing a shape to produce a basic shape that is easily measured. That shape is the 'right' triangle.

Square Constructed About a Circle



On page 3.8 we begin to study the other regular shapes beyond that of the circle.

Drawing Members of The Polygon Family

The Equilateral Triangle and the Square are the first of two shapes in a family of shapes called Regular Polygons. A Regular Polygon has equal length sides (**s**) made of straight lines and equal angles between the sides. The word "Poly" means "many".

Regular Polygons may have any whole number of sides with a minimum number of three. The name of the polygon usually begins with the Greek or Latin word for the number of sides. For example the prefix word "tri" in Greek means three, "quad" four, "pent" five, "hex" six, "sept" seven, "oct" eight, etc.

To Draw any Regular Polygon

To draw any member of the regular polygon family we will begin with a circle and then inscribe the polygon within the circle. This approach is easily accomplished having many advantages which will be shown.

Follow the steps given below:

Step 1 Draw a circle of the size desired using the drawing compass. The center of the circle is also the center of the polygon.

Step 2 Divide 360° by the number of sides desired. The number calculated is the angle between each vertex point of the polygon (the point where two sides meet is called a vertex).

Step 3 Select a beginning point on the circumference of the circle drawn.

Step 4 Use a protractor to locate each angular point beginning with the first point chosen. Proceed around the circumference of the circle marking each vertex location until all vertex points have been located. There will be as many vertex points as there are sides.

Step 5 Draw a straight line between each pair of adjacent points. These lines form the sides of the Regular Polygon.

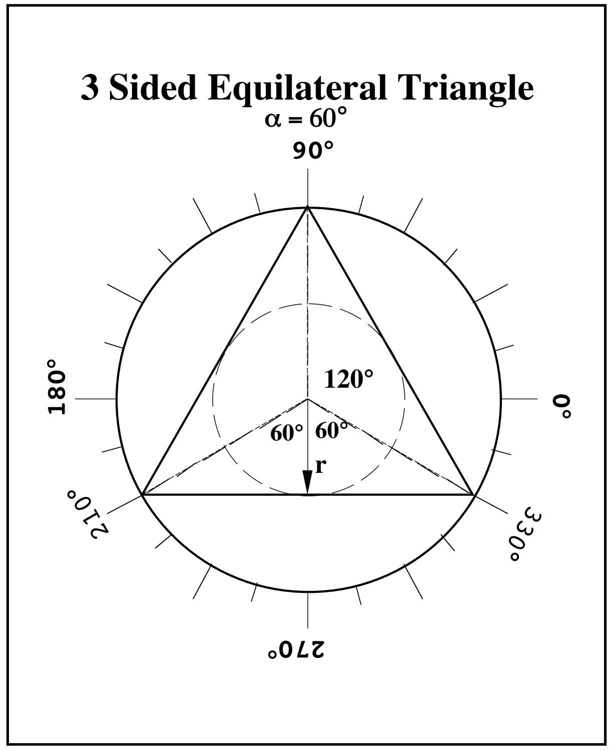
Constructing the Equilateral Triangle

The angular spacing between points is 360° divided by 3 equalling 120°. Choose a starting angle of 90° and proceed around the circle marking each angular point. Connect each pair of adjacent points using a straight line. This forms the sides of the Equilateral Triangle.

Refer to the Equilateral Triangle illustration on the next page.

Draw lines from each vertex to the center of the circumscribed circle. These lines delineate the interior angles of the Equilateral Triangle. The length of these lines is equal to the radius of the circumscribed circle.

Draw the inscribed circle. The radius is most easily located by drawing a line from the center of polygon toward a point on the circumference of the circle corresponding to an angle of one half of the size of the interior angle. This half angle is 60°.



Constructing the Square

Make a circle. Divide the number of sides desired - this number being four for the Square - into 360°. The angular spacing between points is 90°. Choose a starting angle of 45° and proceed around the circle marking each angular point. Connect Each pair of adjacent points using a straight line. This forms the sides of the Square.

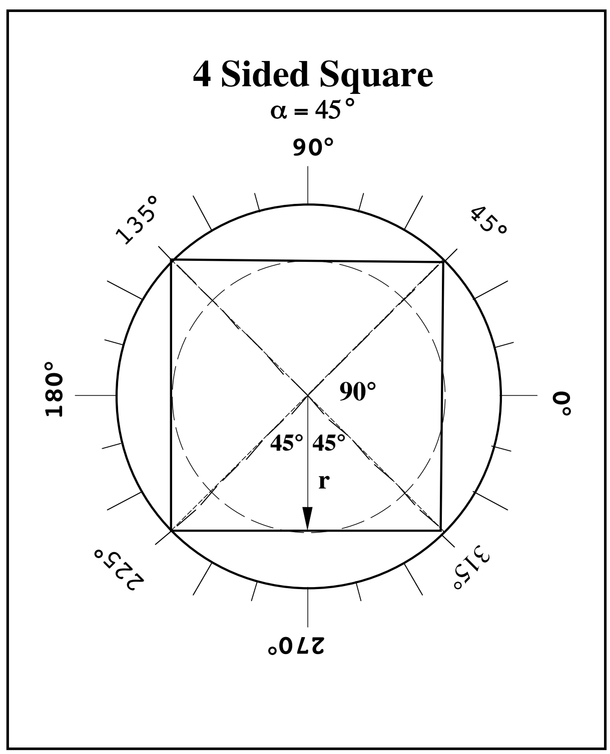
Refer to Illustration - Square.

To show relationships that we will develop in the next chapter please draw lines from each vertex to the center of the circumscribed circle.

The length of these lines is equal to the radius of the circumscribed circle.

Draw the inscribed circle. The radius is most easily located by drawing a line from the center of polygon toward a point on the circumference of the circle corresponding to an angle of one half of the size of the interior angle. The half angle for the Square is 45°.

The Square may be circumscribed and inscribed



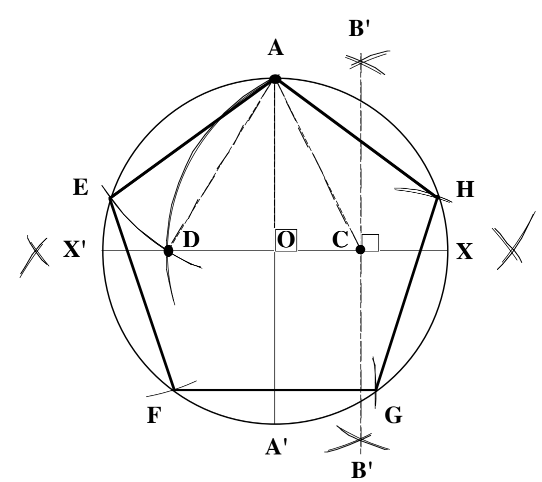
by the shape of the circle. The vertices of the square are located on the circumscribed circle and the midpoint of each side is tangent to the inscribed circle.

Drawing the Five Sided Polygon

To construct this shape we begin by drawing the circle. Next we draw a vertical diameter line. If we have a protractor we proceed around the circle circumference line marking points at 36° intervals. Begin at the top of the circle and connect every second point with a straight line. The resultant five sided shape is called a Pentagon. The inscribed circle and inscribed radius line are constructed as before.

Constructing The Pentagon

Constructed Pentagon



If we do not have a protractor having 36° and 72° angle markings we may still construct the Pentagon using a drawing compass and straight edge.

The procedure is as follows:

1. Draw a Circle using the drawing compass.

2. Draw a vertical diameter line AA' using a straight edge.

3. Construct a perpendicular diameter line using points A and A' (the same techniques as shown on page 3.3 and page 3.4). Draw perpendicular diameter line XX' passing through point O.

4. Bisect line OX by constructing perpendicular line BB'.

5. Place compass point at C and adjust span to equal length AC. Strike an arc from A through line OX'. Label the point of intersection D.

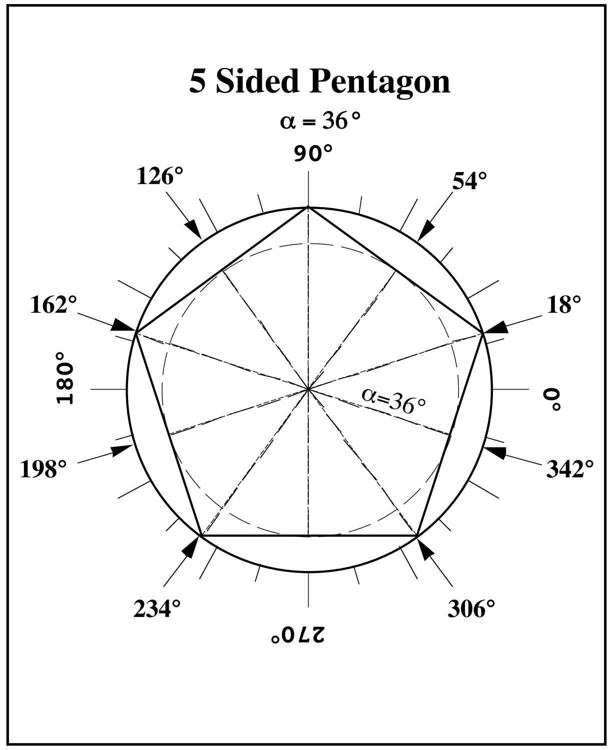
6. Place compass point at A and adjust span to equal length AD. Strike arc through D crossing the circumference line at point E. Label the point of intersection E. Point E marks a vertex. Length AD (and AE) are the length of one side of the pentagon.

7. Place the compass point at E using the compass span as set by distance AD; mark point F. Proceed around the circle marking the remaining vertex points G and H inturn.

8. Draw the connecting lines AE, EF, FG, GH and HA using the straight edge ruler. These lines form the 5 sides of the Pentagon.

Radius lines for the inscribed circle can be formed by drawing bisecting lines between each adjacent pair of vertices and the center of the circumscribed circle. The circumscribed radius lines are drawn from each vertex to the center.

The box at the right shows the Pentagon drawn with labeled angles, inscribed circle and radius lines. Using the technique on the previous page we are now able to generate 36° and 72° angles.

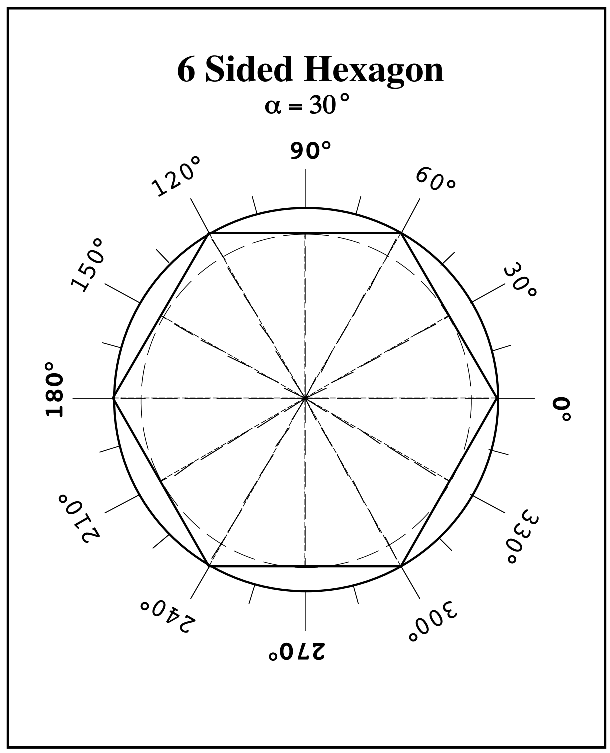


The 6 Sided Polygon

The six sided polygon is formed by connecting vertex points spaced 60° apart. These points are located using a protractor or the construction technique described on page 3.5.

The length of a side is equal to the radius of the circle. The vertex points may be located by choosing a starting point and sequentially marking the vertex points by stepping the compass around the circle as was done to produce the sides of the pentagon. An Equilateral Triangle can be formed by joining every second point marked on the circumference line.

The drawing for the Hexagon is given on this page without any additional explanation as the same procedures were used to generate it as were used to form the other figures.



Area and Measurement Using the 'Right' Triangle

Introduction

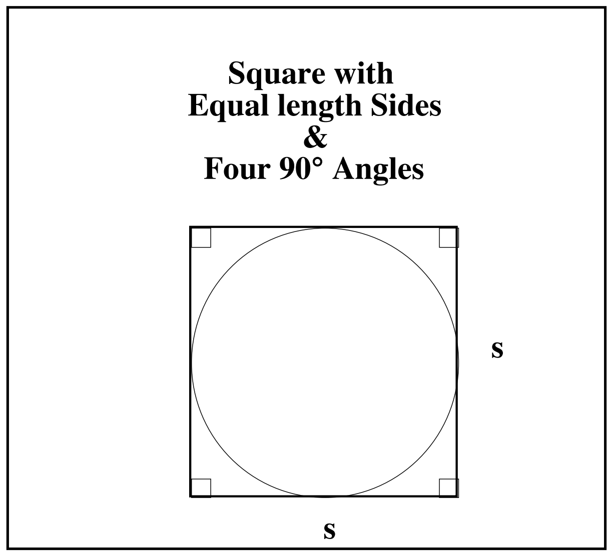
In this section we will describe the measures of a shape, how the area of a shape is measured using the square and introduce the highly versatile 'Right' Triangle as a measurement shape.

Introduced are several mathematical expressions which we will be using: the unit of measure, multiplication, proportion, the Pythagorean Theorem, squaring and square root of a number, and rearrangement of a formula to find one quantity when another is known.

We do this because this section is the heart of the mathematics needed to determine the numerical value of π after we have established all of the geometric relationships of shape and quantity needed to relate the square to the circle.

The Shape Of A Square Is Described By The Number, Length And Angle Of It's Sides

The shape of the Square consists of four sides having exactly the same length and four angles formed by each pair of sides having exactly 90° of angular measure.



The square shape contains four 90° angles. Each 90° or 'Right' angle can be represented by a small square drawn at the intersection of the two lines forming the 90°angle.

In this illustration we retained the inscribed circle to show that the length of the sides of the square are exactly equal and their length is equal to twice the radius of the inscribed circle

Measuring the Square

The size of a square is measured by the length of one of its sides **s**.

The Area of Square

The area of a square is found by locating one of the 90° angles, such as the lower right one, and multiplying the length of each side **s.**

Mathematically we can say that the area **A** of a Square is equal to the length of a side **s** squared.

$$\text{Area } \mathbf{A} = \mathbf{s}^2$$

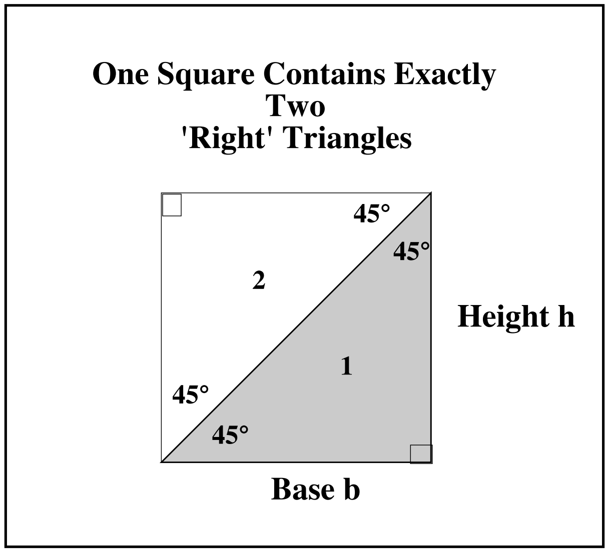
We denoted the process of squaring by writing a small numeral 2 to the right and above the number squared:

All quantities of area are represented by the capital letter 'A' made bold: **A**.

One Square Contains Two 'Right' Triangles

If the shape of the Square is divided into two pieces by drawing a line between the opposite corners two 'Right' Triangles are formed.

Both triangles formed have exactly the same shape and exactly one half each of the area of the square.



The 'Right' Triangle

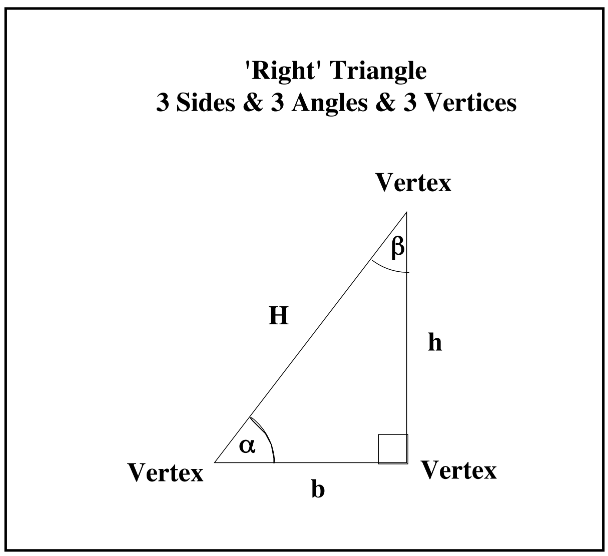
The 'Right' Triangle is a member of a family of shapes called triangles.

The Greek prefix word 'tri' means three. Every triangle has three sides forming three angles.

Each of the three corners formed by each pair of sides is called a 'vertex'. The plural form is 'vertices'. Any triangle has three 'vertices'.

In the 'Right' triangle one of these angles is a 90° or a 'Right' angle. In the special case of the divided Square the other two sides each form an angle equal to 45°. These angles were formed by dividing a pair of 90° angles into two halves.

The upper 'Right' triangle contains the other halves of the divided pair of 90° angles, the two other sides of the square and the other 90° angle opposite the lower 90° angle.



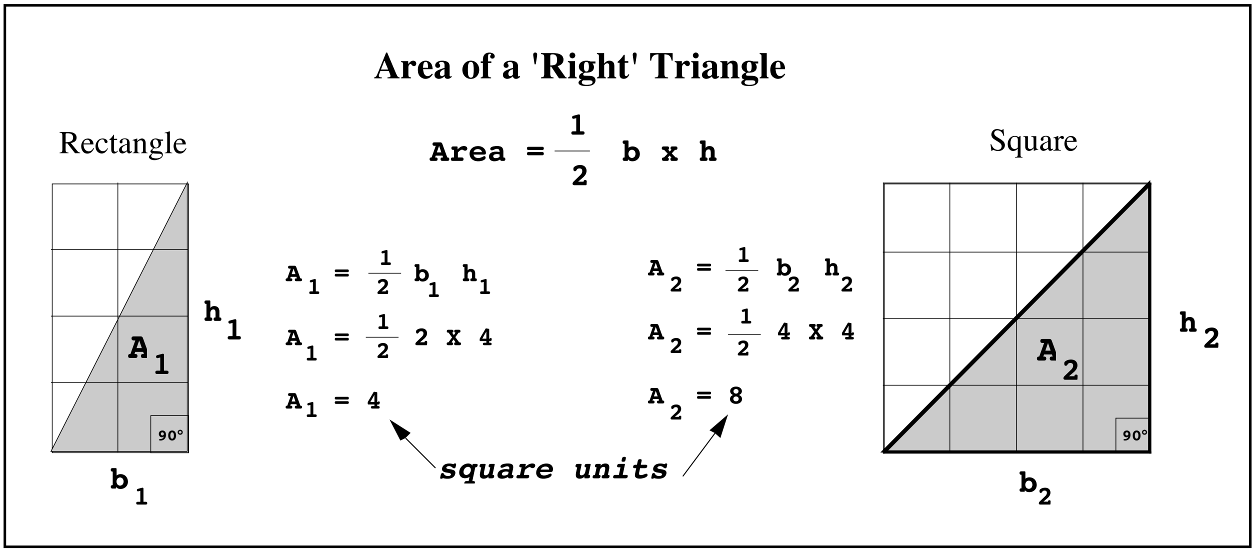
One side of the 'Right' triangle is called the 'base' and will be represented by that name or the letter 'b' made bold - **b**. The side at a 90° angle to the base is called the 'height' and will be represented by that name or the letter 'h' made bold - **h**.

The line forming the height **h** of the 'Right' triangle is perpendicular to the line forming the base **b**; the two sides being at a 90° angle one to the other.

The third side is called by the Greek word 'hypotenuse' and will be represented by that name or by the capital letter 'H' made bold - **H**. More will be said about this side later.

The 'Right' triangles of the divided Square are a special case of the 'Right' triangle family - each contains two 45° angles and one 90° angle. The length of the base **b** and the height **h** are exactly equal to one another as these lengths were the former sides **s** of the Square.

In all triangles the sum of the three angles totals 180°. We label two of the angles alpha **α** and beta **β.** The third angle is the 90° angle and is represented by a small square at the the intersection of the base **b** and height **h** lines.



Area of The 'Right' Triangle

$$\text{Area } A = \frac{1}{2} \times b \times h$$

The area **A** of any 'Right' triangle is equal to $1/2$ the base **b** times the height **h**.

Whenever two quantities are to be multiplied together the letters which represent them will be place next to one another with a space or letter 'x' between them. Either the space or the letter 'x' means that the two quantities should be multiplied.

Calculating Area Using the Right Triangle

In the 'right' triangle the contained area is found by multiplying one half of the length of the base (**b**) by the height (**h**). The base and the height are the two line lengths forming the 90° angle of the 'right' triangle.

Of A Rectangle

On the left hand side of our illustration we have the shape of a Rectangle. A rectangle is a shape made of Squares. The rectangle has been divided in two by drawing a line between opposite corners just as was our square. Two right triangles have been formed.

In the rectangle on the left we have a base length of two units and a height of four units. Area of the 'Right' triangle equals $1/2$ times the base (2 units) times the height (4 units). **$1/2$ times 2 times 4 equals 4 *square* units of area.**

The rectangle formed by the same measure of base and height has an area equal to 2 units times 4 units equaling 8 *square* units. We can clearly see in our drawing that the 'right' triangle has one half of the *square units of area* contained by the rectangle.

Of a Square

We repeat the example using the Square on the right hand side of our illustration. The Square has been divided into two 'Right' triangles by drawing a line between opposite corners. The Square has sides **s** with a length of four units.

The shaded area delineating the 'Right' triangle has a base of four units and a height of four units. We again use our formula for area of a 'right' triangle: area **A** equals **$^1/_2$ times 4 units of length times 4 units of length**. When two or more numbers have been multiplied together the result is called a product. Our 'right' triangle has a product equal to 8 *square units* of area.

The Square in this example has sides **s** with a length of 4 units. To find the area we multiply the length of the base side times the length of the height sides. 4 units times 4 units equals 16 *square units* of area. Our triangle had 8 square units of area or one half the area of the square.

Proportion & Area

The Square contains two of our 'right' triangles and each of those 'right' triangles has $^1/_2$ of the area of that Square. The same could be said of each 'measuring' square (these are the small squares contained by the Rectangle and Square). There are two 'right' triangles in each 'measuring' square.
Each 'right' triangle of the 'measuring' square has a base length of $^1/_2$ inch and a height of $^1/_2$ inch. Each 'right' triangle contained in the 'measuring' square has an area of **$^1/_2$ times $^1/_2$ inch times $^1/_2$ inch**; the product equals $^1/_8$ square inch. There are two right triangles in each 'measuring' square making an area for the measuring square of 2 times $^1/_8$ square inches; the product equals $^1/_4$ square inch.

Notice that the area of the measuring square can also be calculated as $^1/_2$ inch squared. $^1/_2$ inch times $^1/_2$ inch equals $^1/_4$ square inch. The sum of the area of the two right triangles *in* the 'measuring' square is exactly equal to the area *of* the 'measuring' square.

If we multiply the true area of the 'measuring' square times the number of measuring squares in the large square we find that the area of the large square is $^1/_4$ square inch times 16 which equals 4 square inches. The area of the large right triangle was calculated to be 8 square units of area. $^1/_4$ square inch times 8 square units gives an area of 2 square inches. The 2 square inch area of the right triangle is $^1/_2$ the size of the 4 square inch area of the larger Square.

This exercise has demonstrated the property of proportion of length and area between, and within, the two shapes of the 'right' triangle and the Square.

> *When one shape is a fixed part of another the proportion of area is maintained between the two shapes independent of their absolute size or the standard of measure used to describe them.*

Fractions

Mathematically we have also seen that 1 divided by 2 is equal to 8 divided by 16. We notice that 8 is equal to 1 multiplied by 8 and 16 is equal to 2 multiplied 8. It is a general rule that if the numerator (top part) of fraction and the denominator (bottom part) of a fraction are multiplied by the same number then the fractional relationship remains unchanged. Fractional relationships are a measure between two things or between part of an object and the sum of all of the parts of that object.

A second mathematical observation can be made: A Square can be divided into two right triangles; each right triangle representing 1/2 of the square. These two 'right' triangles, when added together form the original square in all of its measure of length, area and shape.

Any quantity when divided into parts becomes whole, or unity, or one when all of the parts are added together in exactly the way that they were divided.

Any number can be divided into one. The result is a fractional part of the whole one. Multiplying the fractional part by the original number gives back the whole or one. The whole or one is equal to the sum of all of its parts.

Any number (or quantity) when divided by another number (or quantity) returns to the original number (or quantity) when multiplied by the dividing number (or quantity).

Quantity of Measure

A quantity contains both number and units of measure. For example 7 days, 4 miles, 6 feet or 4 square inches. Only like quantities can be added or subtracted. The numbers are added or subtracted, the units of measure remain unchanged. For example, 7 days plus 4 days equal 11 days. 4 square inches minus 3 square inches equal 1 square inch.

Both like and unlike quantities can be multiplied or divided. The numbers are multiplied or divided. The units are changed if the multiplying or dividing quantity has units of measure. The units are not changed when multiplying or dividing by a number having no units of measure: Two inches times two inches equal four *square* inches. Two feet times 12 inches (per foot) equals 24 inches. 8 times 2 inches equals 16 inches. 16 square inches divided by 16 square inches equal 1 (with no units of measure).

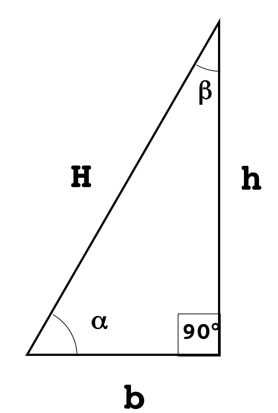
Pythagoras and the 'Right" Triangle

We are now going to study the 'Right' Triangle in much more detail. By so doing we will establish important relationships between the angle formed between the base and the hypotenuse sides of the triangle and the length of the three sides forming the 'Right' triangle.

The 'Right' Triangle and its measure were extensively studied by Greek mathematicians. One name in particular is remembered - that of Pythagoras who lived in the years around 580 BC to 500 BC. Pythagoras and his fellow students studied geometric shapes and numbers as a means of gaining insights into the deep mysteries of life.

Pythagorean Theorem

$$H^2 = b^2 + h^2$$



Pythagoras is credited with having recognized, that given a right triangle, the length squared of the long side opposite of the 90° angle is equal to the sum of the squares of each of the other two sides. This relationship between the sides of the Right Triangle is called the Pythagorean Theorem.

Comment

It is important to note that the relationships existing in the 'Right' triangle are ever present; they are not the discovery of any man. Pythagoras and his fellow students (several of which were women) are honored because they clearly described this relationship and demonstrated the value of such a relationship to others.

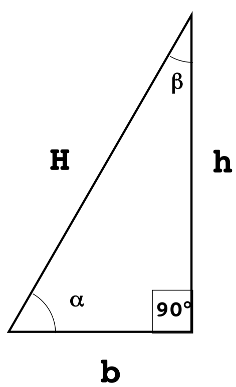
"Certainly many examples of this theorem were known and used by the Babylonians a good thousand years earlier. At least by 600 BC, before Pythagoras was born, the theorem was stated with details of construction in an ancient Hindu Handbook for temple builders, the *Sulbasutram*, by Baudhayana" (Quote from The Prentice-Hall Encyclopedia of Mathematics P. 461).

Other peoples of other traditions have their great discoverers. How many men and women of known and lost cultures over many millenniums (1,000's of years) may have discovered this and other mathematical relationships? Original discovery is always available to the sincere and inquiring mind. Nature does not keep from one what it has openly shown to another nor does it hide its patterns but repeats them endlessly until the mind of man can finally grasp them. This is the purpose of our paper - to illuminate what has always been.

Calculating the Length and Angles of the 'Right' Triangle

The side opposite the two sides forming the 90° angle is called the hypotenuse after the Greek words '*hypo*' meaning under and '*teinein*' meaning 'to stretch' conveying to us the concept of 'stretched between' or 'under tension'.

Angles and Side Lengths of a 'Right' Triangle



$$H^2 = b^2 + h^2$$ **Pythagorean Equatio**

$$H = \sqrt{b^2 + h^2}$$ **length of side H, the hypotenuse**

$$h = \sqrt{H^2 - b^2}$$ **length of side h, the height**

$$b = \sqrt{H^2 - h^2}$$ **length of side b, the base**

The hypotenuse is designated by the capital letter **H**. The length of the hypotenuse (**H**) is calculated by multiplying the length of the base (**b**) times it's self (squaring), multiplying the length of the height (**h**) by it's self (squaring) adding the two squares together and then extracting the square root of this combined sum.

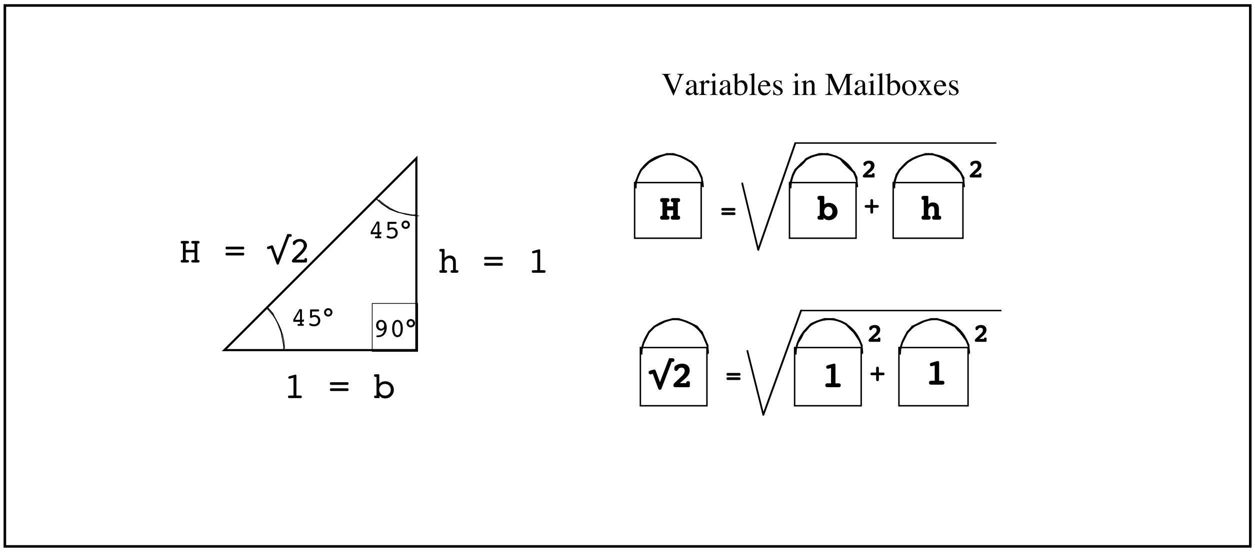
If any two lengths are known then the third can be calculated using the formula `H squared equals b squared plus h squared`. The process of producing or rearranging a known equation to find any one quantity when the others are known is called Algebra. We will be using algebra, geometry and relationships of the Right Triangle to find the value of π.

In the box above I have rearranged the Pythagorean Equation to allow any one of the sides of the 'Right' Triangle to be calculated if the other two are known.

The description below contains important concepts that are used through out this paper.

Think of the letters in our formula as being special mailboxes. We can place any number in each of the two mailboxes that are on the same side of the equal sign (=).

Once having placed the known numbers in the mail boxes we then perform the mathematical operations called for. In this case we square the number in mailbox **b**, square the number in mailbox **h** and add the two numbers obtained.



Having the sum of the squares we are now asked to extract the square root. This means that there exists some number - that when multiplied by its self - will equal the number under the square root sign (√).

Let us use the real numbers found for the 45° 'Right' triangle. Both the base and height had a length of one unit. The angle alpha α has a measure of 45° (degrees of angle).

Since the length of the base is 1 unit then the number in mailbox **b is 1**. The height of the triangle is 1 unit so the number in mailbox **h is 1**. Squaring 1, the value in mailbox **b,** equals 1 *square* . Squaring 1, the value in mailbox **h,** equals 1 *square* .

One square plus one square equals 2 *squares*. **H** equals square root √ of 2 squares. When we extract the square root of 2 squares we find, $\sqrt{2}$ has a decimal value of 1.41421 (plus additional digits) and the √ *of the unit square is equal to the unit length.*

If you have a calculator that has a square root (√) function perform this process by entering 2 and pressing the √ key. The number displayed will be the $\sqrt{2}$ and this is the value for the length of side **H**. Multiply $\sqrt{2}$ times the $\sqrt{2}$. The result will be 2.

To confirm the relationship between 'extracting the square root' and 'squaring' - multiply the number obtained for **H** times its self - square it. The new number will equal the number before we extracted the square root.

Solving Algebraic Equations by Rearranging

Now we have the clue to allow us to reorganized mathematical formulae to find any value that we desire.

Think of the equal sign as being a balancing point. Whatever numbers and mathematical operations are located on one side of the equal sign - when calculated - must exactly equal the calculated number for all the number values and operations on the other side.

We can think of the number obtained for H as always having been in mailbox H, however, we could not open its door until we performed the required mathematical operation. This thought is very real because the 'right' triangle with specific side lengths existed *before* we started to solve the mathematical equation. In fact all of the elements of the triangle were in place as soon as we thought of the shape.

If length **H** and length **b** are known, then sitting in mail box **h** is the number that makes the formula `H equals square root of the sum of b squared plus h squared` balanced. Performing any operation on one side requires that the other side have a similar operation performed.

Square Root and Squaring

Below is a table that reinforces the concept of Square root and Squaring as being inverse operations. We list the original number, extract the square root, square it thereby returning the original number, square that number and again extract the square root returning the original number.

Number	$\sqrt{N}$	$(\sqrt{N})^2$	N^2	$\sqrt{(N^2)}$
1	1.00000	1	1	1
2	1.41421	2	4	2
3	1.73205	3	9	3
4	2.0000	4	16	4
5	2.23607	5	25	5
6	2.44945	6	36	6
7	2.64575	7	49	7
8	2.828438	8	64	8
9	3.00000	9	81	9
10	3.16227	10	100	10

By doing an inverse process on a number we can find the original number.

Extracting the square root is the inverse of squaring, subtraction is the inverse of addition, division is the inverse of multiplication.

The Trigonometric Functions

To find the relationship of the sides of a 45° 'Right' triangle we divided the Square in half. It was very easy to find the lengths of the base and height of the triangle formed since they were equal to the sides of the Square. The length of the Hypotenuse was easily calculated using the Pythagorean Theorem.

In general it is much more difficult to find the relationships of the three sides of the 'Right' Triangle and time consuming drawings are neither sufficiently accurate nor quickly made. For these reasons and because of the many uses of triangular measurement the trigonometric functions were developed.

There is an exact formula which is used to calculate the three trigonometric functions called Sine, Cosine and Tangent. I will not go into these formulas as they involve a series of mathematical calculations having a length determined by the number of decimal digits required. We will note that for most angles the trigonometric functions produce a series of digits not having a repeating pattern.

We will show that the trigonometric functions are simply numbers representing the ratios of the length of each of the three pairs of sides of the 'Right' triangle.

Sine, Cosine and Tangent Relationships

Sine

Taking the sine of angle **α,** represented as sin **α**, produces a number equal to **h** divided by **H**.
We use the sine function to find relationships between the height, hypotenuse and angle **α**.

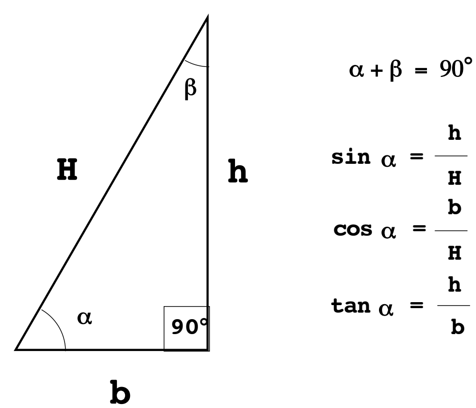
Cosine

Taking the cosine of angle **α**, represented as cos **α**, produces a number equal to **b** divided by **H**. We use the cosine function to find relationships between the base, hypotenuse and angle **α**.

Tangent

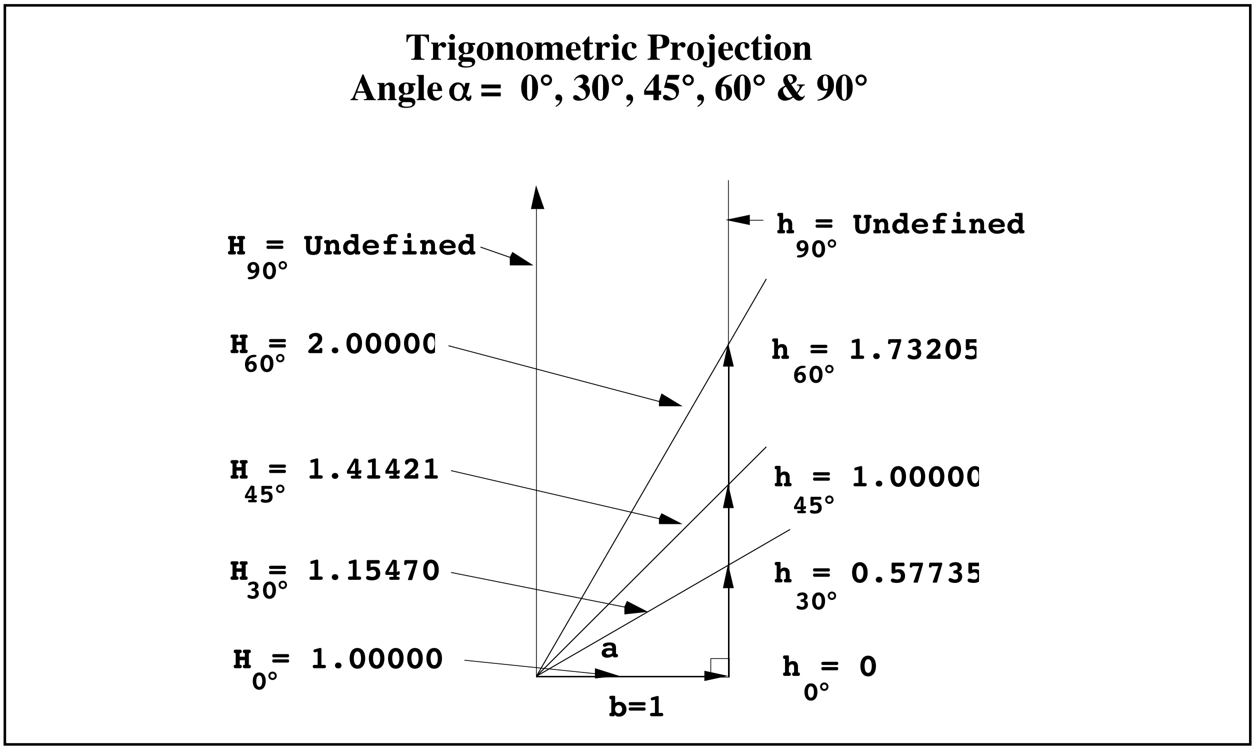
Taking the tangent of angle **α,** represented as tan **α**, produces a number equal to **h** divided by **b**. We use the tangent function to find relationships between the base, height and angle **α**.

Angles and Side Lengths of a 'Right' Triangle



The Projected 'Right' Triangle

There is another way to look at the sides of the 'Right' triangle and that is to see them as being the projection of the line forming the hypotenuse, onto line **h**. See the illustration. Angle **α** formed between the base and the hypotenuse determines the point at which line **H** will strike perpendicular line **h**.



Hypotenuse line **H** must have a length stretching between it's point of connection to line **b** and it's intersection with line **h**. Line **h** will have a length from it's connecting point on base **b** to the point of interception with **H**. All of the previously introduced formulae will be satisfied when we measure or calculate the lengths of lines **h, b** and **H**.

The illustration uses a base length of 1. The selected angles are 0°, 30°, 45°, 60° and 90°. The values for height **h** and hypotenuse **H** length are found by precise measurement or using the trigonometric functions tangent (tan) and cosine (cos).

The trigonometric relationship, tangent of angle α equals the length of side **h** divided by the length of side **b.** This formula allows us to easily find the length of side **h**.

Since our base has a length of 1 then the length of side **h** is simply the numerical value of the tangent of angle α. Using our calculator having the tangent function we simply enter the selected five angles and record the displayed values. If you do a scale drawing similar to the one above you will also measure these same values for **h**.

To find the length of the hypotenuse **H** we note that the cosine function relates side **b** to the hypotenuse: cosine angle α equals side **b** divided by side **H**. Since **b** equals 1 then the cosine of angle α equals 1/**H**. To get **H** we divide both sides of the equation by 1/**H** and obtain **H** = 1/cos α.

To get the length of side **H** we use our calculator to find the cosine of the selected angle and then divide the number obtained into 1.

Comparing Trigonometric and Pythagorean Equalities

If we square each of the values found for each of the sides using the trigonometric functions we should find that the Pythagorean relationships hold true. That is: The hypotenuse H squared must equal the sum of the squares of the other two sides. We will check using the values calculated above. Note that the base always has a length of 1 and that 1 squared equals 1 square.

$$H^2 = b^2 + h^2$$
$$1^2 = 1^2 + 0^2$$
$$1.00000 = 1.00000$$

0° Angle

H has a length of 1, the base has a length of 1, and **h** has a length of 0; there being no difference between **H** and **b**.

The obvious solution 1 = 1 results confirming the relationships.

30° Angle

H has a length of 1.15470. **h** has a length of 0.57735. Squaring both gives 1.33333 and 0.33333 respectively. Adding 1 to **h** squared gives the equality 1.33333 = 1.33333. The relationship is again confirmed.

$$H^2 = b^2 + h^2$$
$$1.15470^2 = 1^2 + 0.57735^2$$
$$1.33333 = 1.333333$$

45° Angle

H has a length of √2 dividing this number out equals 1.41421. **h** has a length of 1.

$$H^2 = b^2 + h^2$$
$$1.41421^2 = 1^2 + 1.00000^2$$
$$2.00000 = 2.00000$$

The square root of 2 squared equals 2 and 1 squared plus 1 squared also equals 2. The equality is confirmed.

60° Angle

H has a length of 2 . **h** has a length of √3 dividing this number out equals 1.73205.

2 squared equals 4. 1.73205 squared equals 3. 3 plus 1 equals 4. The equality is confirmed.

$$H^2 = b^2 + h^2$$
$$2.00000^2 = 1^2 + 1.73205^2$$
$$4.00000 = 4.00000$$

90° Angle

H has no intersection with **h**. Therefore no 'Right' triangle exists that has two 90° angles.

Summary

In this section we have completely explored the measurement of area using the 'Right' triangle. Introduced were concepts of form, fractions, the unit length and unit area, squaring, the inverse function of square root and trigonometric relationships.

We are now ready to combined the measurement of the Regular Polygons with the shape of those figures.

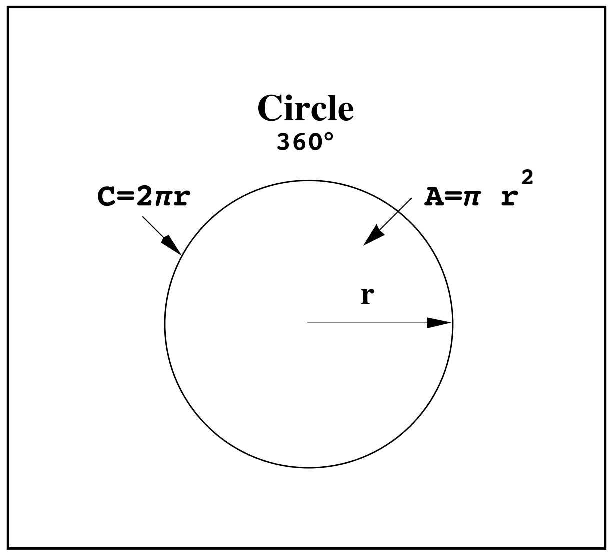
The table below is included for reference.

Angle	b	h	H	tan α	cos α
0°	1.00000	0	1.00000	0	1.00000
30°	1.00000	$1/\sqrt{3}$	$2/\sqrt{3}$	0.57735	0.86602
45°	1.00000	1	$\sqrt{2}$	1.00000	0.70711
60°	1.00000	$\sqrt{3}$	2	1.73205	0.50000
90°	1.00000	---	---	---	0

Measuring the Polygon

The circle and Right Triangle can be used to describe and measure all of the parameters of any Regular polygon. We do this be drawing a circle, marking the vertices of the regular shape on the circumference of the circle, constructing the polygon, and then drawing lines from the center of the circle to each of the regular vertices and midpoints of each face. The inscribed circle and it's radius are then constructed. The completed drawing has divided the regular shape into twice as many Right Triangles as there are vertices. Using formulas previously derived we can determine the size and number of triangles forming any measure of the polygon of interest.

The Circle



The circle has no vertices nor sides. The circumference is one continuous curved line assigned a measure of 360° of arc. The measure of this line is described by the ratio of the diameter (**d**) or twice the radius (**r**) to the circumference.

The formula for circumference **C** equals 2 times Pi times **r** is the measure of length of the circumference line. As we learned earlier the area (**A**) can be expressed as being equal to 1/2 of the circumference times the radius (**r**) of the inscribed circle.

The circle has but one radius making the formula for area equal to:

Area (**A**) equals Pi times radius **(r)**^{2}

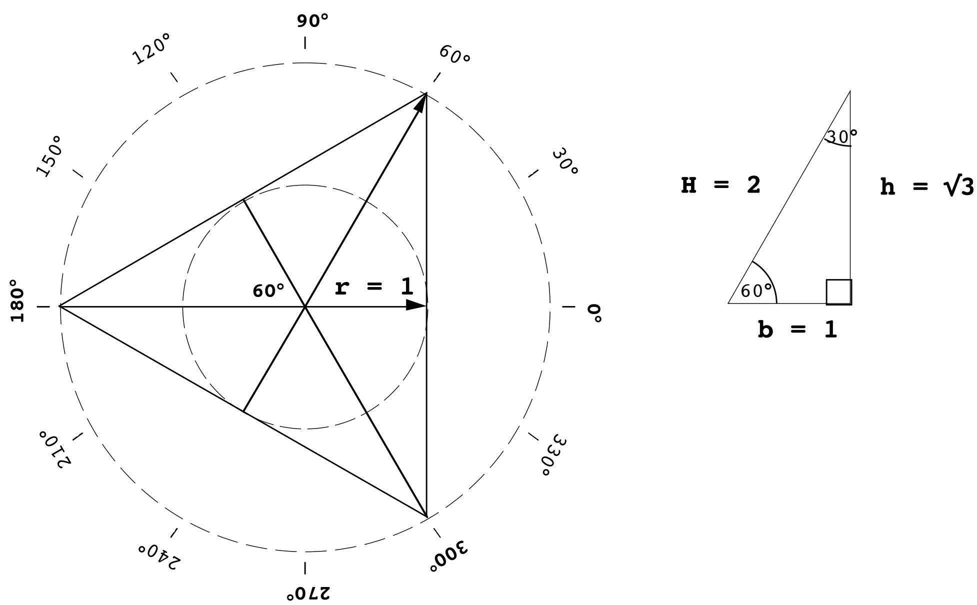
Measuring The Equilateral Triangle

Make a circle and mark the location of the three vertices on the circumference of the circle. Each vertex location will be spaced 120° form it's neighbor. We will choose a starting angle of 60° and proceed around the circle marking each angular point. Connect each pair of adjacent points using as straight line. This forms the sides of the Equilateral Triangle. Refer to the the illustration on the the next page.

Draw lines from each vertex to the center of the circumscribed circle. The length of each line is equal to the radius of the circumscribed circle. Bisect (divide by two) each angle formed by the lines drawn from the center of the polygon to each pair of vertices. Do this by locating the 0°, 120° and 240° angles on the circumscribed circle. Draw three new lines from the center of the circle towards each of the newly located points. Terminate each line on the midpoint of each polygon face. The length of this line is equal to the radius of the inscribed circle. Draw the inscribed circle. Each of the three new lines divides each side in half and is perpendicular to each side. Notice <u>six</u> right triangles with an internal angle (α) of 60° has been formed.

The six, 60° right triangles internal to the Equilateral triangle allows us to calculate the perimeter length, length of a side and total area of the polygon. The radius of the inscribed circle we will assign a value of 1 unit of length and carry this value as the variable **r**.

Equilateral Triangle Divided into Six 60° 'Right' Triangles



The value of this approach is the development of one simple pattern of formulae applicable to all polygons *and to the circle*.

For a 60° triangle with a base length of 1 **r** the hypotenuse **H** has a length of 2 **r**. Using the Pythagorean Theorem, height **h** equals $\sqrt{(4-1)}$ **r** = $\sqrt{3}$ **r.** We could also have used the formula: tan 60° = 1.73205 to find the height ($\sqrt{3}$ when reduced to decimal form equals 1.7320, the two quantities are equivalent).

Height (h) = √3 r

Each side of the Equilateral Triangle (see illustration at top of page) is made from two 'Right' Triangles having a height equal to the $\sqrt{3}$. This gives each side a length of $2\sqrt{3}$**.**

Side (s) = 2√3 r

The perimeter is composed of three sides.
We multiply the length of one side by 3: **Perimeter (P) = 6√3 r**

The area of each 'right' triangle is equal to $1/2$ the base (1 **r**) times the height ($\sqrt{3}$ **r**) . The area becomes: $A = 1/2\ \sqrt{3}\ \mathbf{r}^2$. Since there are six such triangles the total area is found by multiplying the area of one triangle 6 times.

Area = $1/2$ P r

Notice that the area could be expressed as one half Perimeter, $1/2\ 6\sqrt{3}$ **r** = $3\sqrt{3}$ **r,** times **r** giving the same result of $3\sqrt{3}\ \mathbf{r}^2$.

The diameter of the Equilateral Triangle is equal to the maximum straight line length that can be contained. This length is the radius of the circumscribed circle with is equa1 to 2 **r** plus the radius of the inscribed circle which is 1 **r**: Diameter = 2 **r** + 1 **r**

Diameter = 3 r

This is the length of the line perpendicular to the midpoint of a face and terminating on the opposite vertex. All diameter lines pass through the center of the polygon and inscribed circle.

Constructing the Square

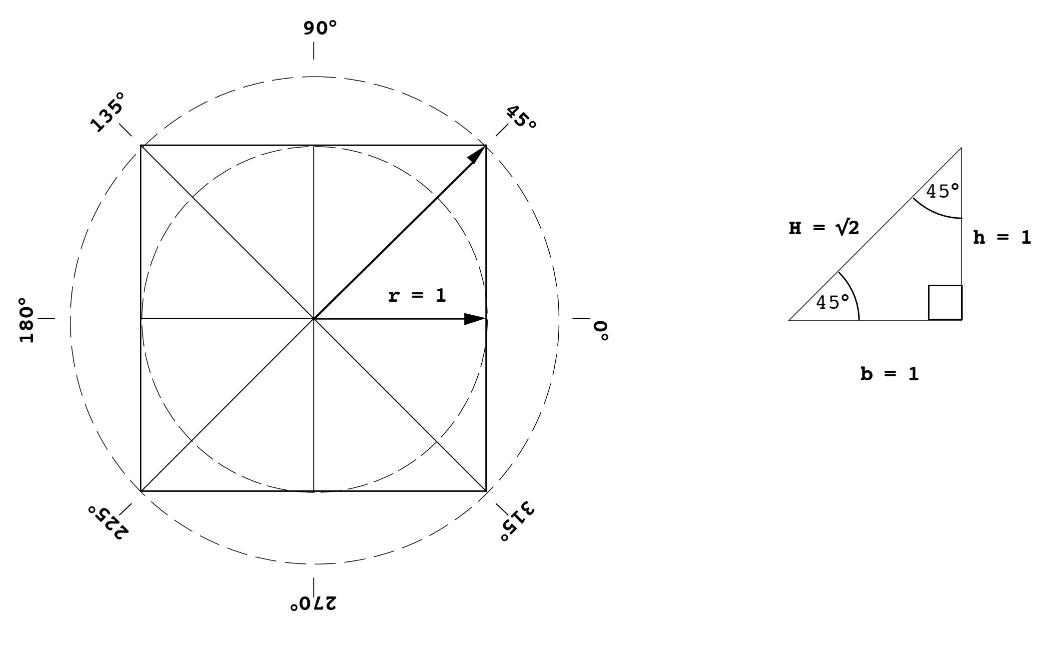
Construct the Square using the same procedure as used previously. The angular spacing between the vertices is 90°. Choose a starting angle of 45° and proceed around the circle marking each angular point. Connect each pair of adjacent points using a straight line. This forms the sides of the Square.

Refer to Illustration - Square.

Draw lines from each vertex to the center of the circumscribed circle. The length of these lines is equal to the radius of the circumscribed circle. Bisect (divide by two) each angle formed by the lines drawn from the center to vertex. Do this by locating the 0°, 90°, 180° and 270° angles on the circumscribed circle. Draw four new lines from the center of the circle towards each of the newly located points. Terminate each line on the midpoint of each of the four sides. The length of these lines are equal to the radius of the inscribed circle. Draw the inscribed circle. Each of the four new lines divide each side in half and are perpendicular to each side. Notice that eight right triangles with an internal angle (α) of 45° have been formed.

The eight, 45° right triangles internal to the the Square allows us to calculate the perimeter length, length of a side, total area and relationship of these parameters to that of the circle.

Square Divided into Eight 45° 'Right' Triangles



The 45° 'right' triangle has a base length of 1 **r** and the height is also equal to 1 **r**. The tangent of 45° equals 1. The hypotenuse **H** becomes $\sqrt{2}$ **r**.

Height (h) = 1 r

Each side then has a length equal to the height of two triangles. **Side (s) = 2 r**

The perimeter has four sides so we multiply the length of one side by 4:

Perimeter (P) = 8 r

The area of each 'right' triangle is $1/2$ the base (1 **r**) times the height (1 **r**) which equals $1/2$ times $\mathbf{r}^2$. Since there are eight such triangles the **area equals 8 times $1/2$ $\mathbf{r}^2$**.

Area = 4 $\mathbf{r}^2$

Notice that the area could be expressed as one half Perimeter, $1/2$ 8 **r** = 4 **r,** times **r** giving the same result of 4 $\mathbf{r}^2$.

Area = $1/2$ P r

The diameter of the Square is equal to the maximum straight line length that can be contained. The square has two such lengths. The distance between the sides is equal to the diameter of the inscribed circle 2 **r**. The distance between two opposed vertices is equal to the diameter of the circumscribed circle, $2\sqrt{2}$ **r**.

Diameter (min) = 2 r

Diameter (max) = $2\sqrt{2}$ r

Any polygon with an even number of sides has a minimum diameter equal to the diameter of the inscribed circle and a maximum diameter equal to the diameter of the circumscribed circle. The measure of the circumscribed diameter compared to the radius of the inscribed circle is unique for each polygon.

The Polygon Formulas

There is one set of general formulae that maybe used to calculate the area and perimeter of all polygons and the circle.

The height **h** of each right triangle forms one half of the length of each side. The angle α of each right triangle becomes 360° divided by twice the number of sides (**n**) of the polygon. Each side of the polygon has a length of **s=2h**. The perimeter has a total length equal to the length of each side (**s=2 h**) times the total number of sides (**n**). Therefore the perimeter **P** equals 2 times **n** times **h**. The area of the total polygon is equal to the area of 1 triangle times the total number of triangles - this number being **2 n**. The total area (**A**) becomes **A=(1/2 b h) times (2 n)** or total area **A = n b h**.

We have defined the radius of the inscribed circle **r** to equal 1 and have made this the base **b** of each right triangle. The interior angle α and the base **b** can be used to calculated the height **h**. The calculation of height **h** from α and base **b** we recognized as being the trigonometric relationship **tan a = h/b**. Since **b = 1 r** then **h** becomes equal to the tangent of angle α: **h = tan**

α.

We now have the final relationship of height **h** being equal to the tangent of the interior angle **α.** The angle alpha is simply 360° divided by two times the number of sides (**n**) of the polygon. We have reduced the description of all polygons and the circle to the two variables: **n** the number of sides and **r** the radius of the inscribed circle.

The radius of the circumscribed circle is equal to the hypotenuse **H.** The length of the hypotenuse we have already shown to be equal to the cosine of angle alpha divided into 1. The radius of the circumscribed circle is exactly the same be equal to `1/(cos 360°/2n) r.`

One Set of Polygon Formulae

Inscribed circle radius (**r**)	**`r = r`**
Interior angle	α **`= 360°/ 2n`**
Circumscribed circle radius	r_c **`= 1/(cos 360°/2n) r`**
Height **h** of 1 triangle **`= (tan`** α**`) r;`**	**`h = (tan 360°/2n) r`**
Length of 1 side (**s**) **`= 2 h`**	**`s = (2 tan 360°/n) r`**
Perimeter (**P**) **`= n s`**	**`P = (2n tan 360°/2n) r`**
Area (**A**) = 1/2 the perimeter (**P**) times **r**	
`Area (A)`	**`A = (n tan 360°/2n)`** r^2

The General Formula Set For The Circle And All Regular Polygons

For any regular polygon the form factor is determined by the height and number of internal triangles present. This form factor is equal to the tangent of the internal angle α where α **`is equal to 360° divided by twice the number of sides`**. All other parameters become a product of this number, the number of sides, and the radius of the inscribed circle.

A table has been prepared on the following page containing calculated values for diameter, radius of circumscribed circle, side length, total perimeter and area. Included are the first five basic shapes and the general formula for any regular polygon. Only two parameters, number of sides **n** and radius of inscribed circle **r,** are required to calculate any parameter of any regular polygon.

Formulae and Relationships between n sided Regular Polygons - Including the Circle

Shape	n	Form Factor	Perimeter	Side	Area	Diameter
Equi. Triangle	3	tan 60° = √3	6√3 r	2√3 r	3√3 r2	3 r
Square	4	tan 45° = 1	8 r	2 r	4 r2	2√2 r
Pentagon	5	tan 36° = 0.7265+	7.2654 r	1.4530 r	3.6327 r2	2.2360 r
Hexagon	6	tan 30° = 1/√3	12/√3 r	2/√3 r	6/√3 r2	4/√3 r
Polygon	n	tan 360°/2n	(2n tan 360°/2n) r	(2 tan 360°/2n) r	(n tan 360°/2n) r2	--
Circle	Curved line	tan 360°= 0	2π r	---	π r2	2 r

Using the data in the table above let us evaluate the perimeter for each of the shapes in decimal form. We do this by writing down the decimal value of the perimeter (see table below).

Notice that the perimeter of the Equilateral Triangle has the largest number and the circle has the least. A polygon of any number of sides will have a perimeter value greater than that of the unit circle (see analysis Chapter 7 as there is a point at which this statement is not true for a circle described by ∏).

Form Factor And Perimeter About The Unit Circle

Shape	n	Form Factor	Perimeter
Equi. Triangle	3	tan 60° = √3	10.3923+
Square	4	tan 45° = 1	8.0000
Pentagon	5	tan 36° = 0.7265+	7.2654+
Hexagon	6	tan 30° = 1/√3	6.9282+
Polygon	n	tan 360°/2n	(2n tan 360°/2n) r
Circle	Curved line	tan 360°= 0	6.2831+

A Many Sided Polygon

We are going to calculate the perimeters and areas of two more polygons to illustrate how the approximated value of Pi was obtained. When we do these calculations we are going to carry ten display digits because the exact and true value of Pi that we are seeking is only slightly different in decimal value from that obtained using the polygon approximation method. I also want to show that given a very large number of sides on a polygon, the perimeter seems to approach one final numeric value. *Although, the digits to the very most right of the decimal point do continue to change as more sides are added to the polygon.*

The formula for the Perimeter of a polygon is 2 **n** tan 360°/2**n** times the radius of the inscribed circle. **n** we know to be the number of sides and tangent 360°/2n the height of an individual 'Right' triangle.

Let the number of sides **n** equal some large number such as 6,144 and 1,000,000.

A Polygon with 6,144 sides

Calculate the perimeter of a polygon with 6,144 sides and an inscribed circle with a radius of 1:

r = 1.000,000,000

The angle α is equal to 360° divided by 2. Dividing we find angle α equals 0.029,296,875°. The tangent of this angle equals 0.000,511,326,97.

The Perimeter is made up of 12,288 'Right' triangles each having a height of 0.000,511,326,97. Upon multiplying the tan of α (0.000,511,326,97) 12,288 times we obtain a perimeter of:

P = 6.283,185,855 r

By referring to the chart on the preceding page we notice that the perimeter value appears to be very similar to that of our unit circle (6.2831+).

Area Of A Polygon Having 6,144 Sides

The formula for the area of this shape is simply ½ Perimeter times inscribed radius **r.** To find the area we divide the number obtained for the perimeter by two and multiply by **r:**

Area = 3.141,592,928 r^2

The radius of the circumscribed circle is (1/cos360°/2n) **r** = 1/cos 0.029,296,875°

= 1/0.999,999,869 **r**

Circumscribed radius = 1.000,000,131 r.

Notice that the radius of the inscribed circle and circumscribed circle are very nearly of the same size. The value for the perimeter of the inscribed and circumscribed circles will also be very nearly equal since circumference equals Pi times radius. We will use the 6,144 sided polygon calculations in a relationship developed in the next chapter.

n = 1,000,000

Increase the number of sides of the polygon to one million (**n** = 1,000,000) and perform the same calculations. In this series of calculations and analysis we will show exactly now the value of π is approximated by the polygon and bring up possible shortcomings in using this approach.

The 1 million sided polygon is divided into 2,000,000 right triangles each having an interior angle of 360°/2,000,000. The tangent of that small angle becomes 0.000,003,141,592,654 which is the height of each interior triangle having a base length of 1 **r**.

The sum of all of the heights is obtained by multiplying the individual height two million times (there being two 'right' triangles per side). This product is the perimeter of the 1,000,000 sided polygon. The value obtained is: **P = 6.283,185,308 r**. Dividing this number by **2 r** we obtain **3.141,592,654**.

This number, the first 10 digits being given, has been called Pi and represented by the Greek letter π because a polygon formed by a very large number of sides *appears* to be identical to the circle, and the definition of Pi is such that the *circumference* of a circle divided by twice the radius of the circle equals Pi.

If we calculate the radius of the circumscribed circle using the formula of 1/cos 360°/2n we get an answer of 1.000,000,000 on a calculator that has a resolution of 10 digits. It would seem that a vanishingly small difference exists between the true circle, the inscribed and circumscribed circles and the perimeter of the 1,000,000 sided polygon.

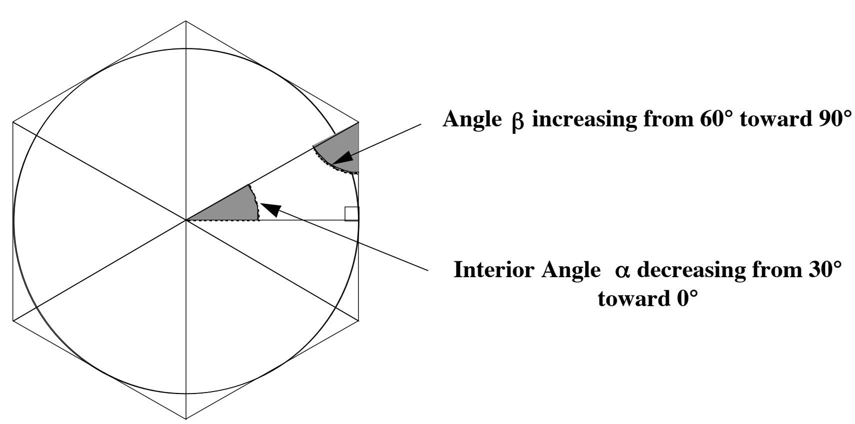
The difference is very small between the inscribed and circumscribed circles but has the perimeter of the polygon *become* a circle? Remember that we divided the polygon into 2,000,000 right triangles to find the height of one of these triangles so that it would match the circumference very closely. To get the total perimeter we *multiplied* this very small number 2,000,000 times. The small difference between the straight line height of the triangle and the actual curved line arc was also multiplied by this same factor of 2,000,000 times. As was shown in the drawing of the arc connecting two adjacent points A and B in Chapter 2 page 1 (The Circular Shape), the curved line length of an arc is always longer than any number of straight line segments added together.

This number, 3.141,592,654, is the ratio of the perimeter of a very many sided *straight line* polygon to the diameter of that polygon. Represented as such it is a perfectly accurate description of the polygon. The number, however, is not Pi for it is <u>not</u> the circumference of a *curved line* circle divided by the circles radius. The circumference of a circle is equal to the *straight line* length containing all of the area of the circle but located *outside* of that circle as was shown by the wheel track example.

No 'right' Triangle Has Two 90° Angles

In the previous section of this paper we have shown that the sum of any number of straight lines connecting two adjacent points will always be less than the length of the continuous arc con-

Hexagon with Inscribed Circle



necting the same two points.

Now we add the additional truth that no 'Right' triangle can exist having two 90° angles.

In the drawing below we have circumscribed a hexagon having six sides around a circle. Notice the angles formed by a 'Right' triangle drawn with its base on the radius line and it's vertex at the vertex oof the circumscribed hexagon. The interior angle α is equal to 30°. The opposite angle β is equal to 60° (90° minus 30° equals 60°). A 90° angle is formed between the base line and the height line.

Now imagine that the polygon circumscribed about our circle is the 1,000,000 sided polygon that we have just been evaluating. The corresponding 'Right' triangle interior angle will be 0.000,180 degrees. The opposite angle will be 89.999,820 degrees. The tangent of angle β is 318,309.8862. To perfectly match the circle angle β must measure exactly 90° and this it has not done by measure of angle and there being a finite tangent value.

As has was shown in Chapter 4 no 'Right' triangle exists that has two 90° angles since the hypotenuse line will never intercept the height line. And, since we have shown that any real polygon can be divided into and measured by the 'Right' triangle, it follows that *no polygon exists that can have a perimeter exactly equal to the circumference of a curved line circle*.

The one and only correct way to find Pi is to solve 'the Quadrature of the Circle' problem - where the area of a circle inscribed within a square of unit area (each side, the diameter and the area of the square all equal 1) is known to be exactly equal to Pi/ 4.

Perimeter and Area of a Polygon 'Normalized' to Those of the Unit Circle

All regular shapes of straight lines have their area equal to half their perimeter multiplied by the least radius which the shape contains - which is always the radius of an inscribed circle. The area of the circle is similarly calculated since the inscribed circle is simply the radius of the original circle - for they are one and the same. All regular geometric shapes (circle, triangle, square and polygon) have been shown to have one formula for area. The one formula being: Area equals $^1/_2$ Perimeter times the radius of the inscribed circle **r**.

The radius of the inscribed circle is the common element when determining the area for all polygons and circles.

The number of sides of a polygon produces a specific form factor which allows calculation of a ratio of perimeter to the circumference of an inscribed circle and also area to the area of that inscribed circle. This relationship is fixed at all times and is independent of size.

The diameter of any polygon of a given number of sides and the diameter of its inscribed circle have another specific ratio. This relationship is fixed at all times and is independent of size.

The Table on the following page was calculated by dividing the parameter of Perimeter (**P**), Area (**A**) and diameter (**d**) of each primary polygon by the corresponding parameters for the inscribed circle having a radius of 1. We see that by so doing the relationship of each shape to the circle is described by one numerical ratio. For the Equilateral Triangle this ratio is **(3 √3) / Pi** units of perimeter per unit of circumference, and also, **(3 √3) / Pi** units of area per unit of area of the inscribed circle.

For the Square the factor becomes **4 / Pi**. The Square has **4/Pi** units of area compared to that of the inscribed circle. The inscribed circle has the fractional amount of **Pi/4** units of area of the circumscribed Square.

The table at the top of the next page summarizes all of our calculations and shows the fixed ratio of area and perimeter of the polygon to its inscribed circle.

Perimeter, Area, and Diameter Normalized To The Unit Circle

(Divide each parameter by the corresponding parameter for an inscribed circle with a radius of one)

Shape	n	Perimeter	Area	Diameter
Equi. Triangle	3	$(3\sqrt{3})/\pi$	$(3\sqrt{3})/\pi$	3/2
Square	4	$4/\pi$	$4/\pi$	1
Pentagon	5	$(1.8163)/\pi$	$(1.8163)/\pi$	1.1180
Hexagon	6	$(6/\sqrt{3})/\pi$	$(6/\sqrt{3})/\pi$	1
Polygon	n	$(n \tan 360°/2n)/\pi$	$(n \tan 360°/2n)/\pi$	$(1 + 1/\cos 360°/2n)/2$ *or* 1
Circle	Curved line	1	1	1

Summary

In this chapter we have calculated all of the measures of several polygons and given the general set of formulae allowing any polygon to be drawn and its exact shape and area evaluated by division into 'Right' triangles. We have shown the basic and fixed nature of the inscribed circle and the form factor of any polygon.

We have demonstrated how π is obtained by the polygon approximation and we have shown three short comings in this approximation. The three short comings are:

1) the sum of any number of straight line segments does not total the length of one complete circular arc.

2) The circumference line is a line which must enclose all of the area of the circle (the circumscribed polygon does not do this *for an equivalent area circle* as will be demonstrated in the next chapter).

3) As the polygon is completely measured by its contained 'Right' triangles, and no 'Right' triangle exists having two 90° angles, which is a requirement to exactly match the circle, then, no circumscribed polygon exists that can exactly match the circle.

We move to the next chapter and finally solve for the value of Pi using the areas and proper relationships of the Circle, Equilateral Triangle and Square.

Quadrature of the Circle Solved

This section will show that a Square (or any regular polygon) can be made to have an area exactly equal to that of a given circle using a four step procedure. After having demonstrated the validity of this procedure and the short comings of the multi-sided polygon approximation for Pi, we will use the unit Square, Equilateral Triangle and Circle to reveal the one, and only one true value of Pi.

The Four Step Procedure Making Area of any Circle Equal to The Area of Any Polygon

Because the circle is common to all polygons and the area of any polygon can be made equal to the area of any circle by equality or by scale (a ratio between two like things such as shape, area, perimeter, radius or diameter) then the problem of the Quadrature Of The Circle can be solved and so can the exact and perfect value of Pi can be determined.

We give here a four step procedure for making the area of any regular shape equal to the area of any given circle.

Four Step Procedure for Making Area of a Circle and a Polygon Equal

Step 1, Begin with any circle, let us call it Circle E having a radius **r**. Calculate the area of the circle using the standard formula: $\text{Area} = \pi \mathbf{r}^{2}$.

Step 2, Make the perimeter of any regular shape F equal to the circumference of Circle E. Calculate the area of F from its perimeter value.

Step 3, Make the area of another circle U equal to the area of regular shape F. Calculate the radius of circle U.

Step 4, Make circle U the inscribed circle for the shape chosen in step 2. The area of the final regular shape V will be equal to the area of the original circle E.

Pi is used as a constant in each of the four steps. We convert a perimeter and an area from a circle to a polygon and then back to a circle again. The value chosen for π cancels out when the final values of area are calculated. Because of this any value of Pi may be used giving a full equality of area between circle E and polygon V.

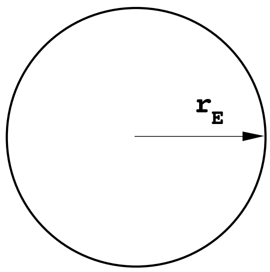
The radii of the circles will only be exact if an exact value of π is available. We will use the commonly accepted value of Pi until we fully prove the existence of a superior value. The common value we will be using is: π = 3.141,592,654. This value also allows comparative relationships to be displayed between the intermediate shapes; all as commonly accepted.

Illustration - Four Step Procedure

Refer to the illustration on page 6.2 of the Four Step Procedure as we do the step by step calculations and explanation on page 6.3.

Four Step Procedure
Making
Area of Square V Equal To Area of Circle E

Step 1 E



Begin with Circle E

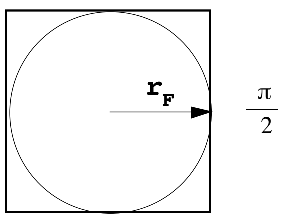
$C_E = 2\pi$

$r_E = 1.000,000,000$

$\text{Area E} = \pi r^2$

$\text{Area E} = 3.141,592,654$

Step 2 F



Make Perimeter of F Equal to Circumference of E

$P_F = C_E$

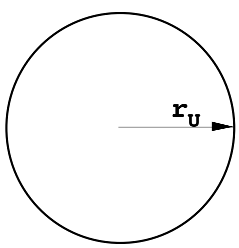
$r_F = (2\pi)/8$

$r_F = 0.785,398,164$

$\text{Area F} = 4\ r^2$

$\text{Area F} = 2.467,401,101$

Step 3 U



Make Area of U equal Area of F

$\text{Area U} = \text{Area F}$

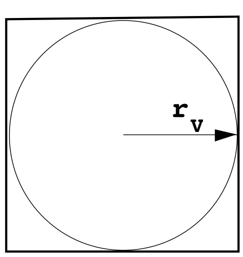
$\pi\ r_U^2 = \text{Area F}$

$r_U{}^2 = 2.467,401,101\ /\pi$

$r_U{}^2 = 0.785,398,164$

$r_U = 0.886,226,926$

Step 4 V



Make U the inscribed circle for V. Calculate area of V.

$r_V = r_U$

$r_V = 0.886,226,926$

$\text{Area V} = 4\ r_V{}^2$

$\text{Area V} = 4\ 0.785,398,164$

$\textbf{Area V = 3.141,592,654}$

Step 1:

The perimeter of circle E was chosen to be 2 π giving a radius of exactly one.
radius **r = 1.000,000,000**

The **area of E** equals π $\mathbf{r^2}$ = 3.141,592,654 times 1 (square) **= 3.141,592,654**

Step 2:

The chosen shape is the Square. The selected Square F has been given a perimeter of 2π. The radius of the inscribed circle of any square is equal to the perimeter divided by 8.

The **radius of inscribed circle F** is **2π/8 = 0.785,398,164**.

The area of F becomes 4 times $\mathbf{r^2}$
= 4 times$(0.785,398,164)^2$
= 4 times 0.616,850,275.

Area F = 2.467,401,101

Step 3:

The circle U is given an area equal to the area of F.
Area U = 2.467,401,101.

The radius of U equals √ of quantity area of U divided by 2 π

r = √(2.467,401,101/(2 times 3.141,592,654))
r = √0.785,398,164

Radius of U = 0.886,226,926

Step 4:

Circle U is made the inscribed circle of Square V.

The area of V equals 4 times the quantity radius of the inscribed circle squared - which is the radius of circle U.

Area V = 4 times $(0.886,226,926)^2$
= 4 times (0.785,398,164)

Area of V = 3.141,592,654

Comparing the area of Square V in step 4 to the area calculated for Circle E in step 1 we find that they are equal to each other.

Area of Square V = 3.141,592,654 = Area of Circle E

The four Step process can produce a polygon having the exact area of any given circle and, by reversing the steps, any circle can be made to have an area exactly equal to that of any given polygon.

Observations and Deductions

Compare the radii of the three shapes E, V and F.

The original circle E has a radius of 1.000,000,000. Square V - the square that has all of the area of the circle E - has an inscribed circle radius of 0.886,226,926. Square F has all of the perimeter of circle E and an inscribed radius of 0.785,398,164.

The radius of circle E having all of the area of square V is larger than the radius of the inscribed circle of V. The radius of the circle inscribed in V is larger than the radius of the inscribed circle of square F which contains all of the perimeter of Circle E. From these relationships *we can see that regardless of how many sides polygon* V *may have the radius of the circle* (circle E)*which has all of the area of that polygon* will *be larger than that of the inscribed circle of that polygon and the circumference of this circle* (circle E)*will be smaller than the perimeter of that polygon* .

Compare The Area And Perimeter Of A 6,144 Sided Polygon To The Area And Perimeter Of An Inscribed Circle

Another way to look at the picture is to consider the inscribed circle of polygon V as having a radius of 1 and *this* circle is the original circle. We will use the 6,144 sided polygon calculated earlier and reverse the order of the four steps to show these relationships.

Step 1

Begin with inscribed circle V having a radius of exactly **1**. The general formula for the perimeter and area of a 6,144 sided polygon were solved yielding a value for perimeter equal to 6.283,185,855 **r** . The area is equal to $^1/_2$ Perimeter **P** times the radius of the inscribed circle. Since the radius of the inscribed circle is exactly 1 and 1^2 is equal to 1 then area is equal to **P/2**.

Area of V = 3.141,592,927

Step 2

We calculate the area of circle U which has the same radius as that of circle V; this being **1**. The area of circle U is equal to $\pi \mathbf{r}^2$ where $\mathbf{r}$ =1. We use the accepted value for π, 3.141,592,654, and multiply by 1^2 which equals 1. We calculate:

Area of Circle U = 3.141,592,654

Step 3

Set the area of Polygon F equal to the area of Circle U. Solve for the perimeter of F by calculating the radius of the inscribed circle first.

We are given the area of F as being that of circle U and our general formula gives the area as being 3.141,592,927 times the radius of the inscribed circle squared

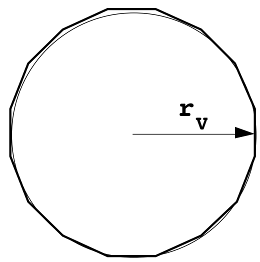
Set the area equal to the formula for area. Divide the area of U, 3.141,592,654, by the area constant for the 6,144 sided polygon: 3.141,592,927. Take the square root of the division to obtains the radius of the inscribed circle of figure F:

radius of F = 0.999,999,956

Area of Circle E Equals Area of 6,144 Sided Polygon V

V

Step 1



$\mathbf{r_V = 1.000,000,000}$

$P_{6,144} = 6.283,185,855\ r$

$\text{Perimeter V} = 6.283,185,855$

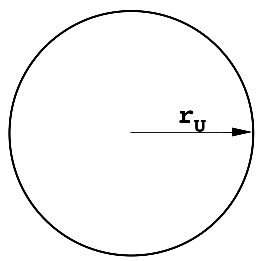
$\text{Area V} = 1/2\ P_V\ r_V$

$A_{6,144} = 3.141,592,927\ r^2$

$\mathbf{Area\ V = 3.141,592,927}$

U

Step 2



$\mathbf{r_U = r_V}$

$r_U = 1.000,000,000$

$r_U{}^2 = 1.000,000,000$

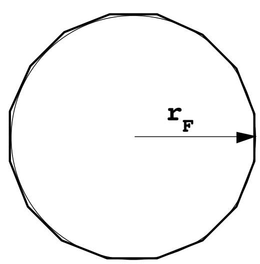
$\text{Area U} = \pi\ r_U^2$

$\text{Area U} = 3.141,592,654 \cdot 1^2$

$\text{Area U} = 3.141,592,654$

F

Step 3



$\mathbf{Area\ F = Area\ U = 3.141,592,654}$

$\text{Area F} = 3.141,592,927\ r^2$

To find r: Divide area of circle by area constant for polygon and extract the square root of the divided quantity

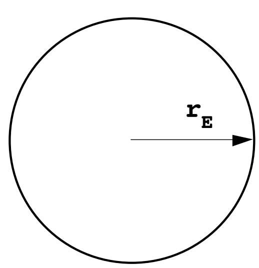
$r_F = 0.999,999,956$

$P_F = 6.283,185,855\ r_F$

$\mathbf{P_F = 6.283,185,581}$

E

Step 4



$\mathbf{C_E = P_F = 6.283,185,581}$

$r_E = P/2\ \pi$

$= \dfrac{6.283,185,581}{(2)\ 3.141,592,654}$

$r_E = 1.000,000,044$

$\text{Area E} = \pi\ r^2$

$\mathbf{Area\ E = 3.141,592,930}$

The perimeter of F is equal to 6.283,185,855 times the value of radius of the inscribed circle 0.999,999,956 equaling 6.283,185,581.

Perimeter of F = 6.283,185,581

Step 4

The circumference of circle E is equal to the perimeter of F. The radius of Circle E is equal to Circumference divided by 2 π. 6.283,185,581 divided by 2 times 3.141,592,654

radius of E = 1.000,000,044.

The area becomes: π times r^2 = 3.141,592,930.

(The value for the area of circle E is slightly larger than that of Polygon V. This is due to rounding error in my calculator as we progressed through the several steps of calculation.)

Compare The Size Of The Radii Of Circle V And Circle E

Circle E has all of the area of polygon V and a radius of 1.000,000,044; slightly larger than the radius of the inscribed circle of V. The circumference of Circle E is smaller than that of the perimeter of polygon V by the ratio of 6.283,185,<u>581</u> to 6.283,185,<u>855</u>.

Circle E has the simultaneous properties of having *all* of the area of polygon V and a *larger* diameter than that of the circle that is inscribed to V and yet a perimeter that is *less* than that of V.

We may say: *The circle inscribed to V has not the area of Circle E nor the perimeter of Circle E (it's radius being less) so it can not be an accurate representation of either the area or circumference of the true equivalent Circle E.*

From the above analysis we deduce that the true and correct circumference of a circle will be larger than that obtained by the many sided polygon method and since Pi is the ratio of the circumference of a circle to its radius then the correct value of Pi will also be larger than the value approximated by the many sided polygon method.

The Equilateral Triangle Is Opposite To the Circle

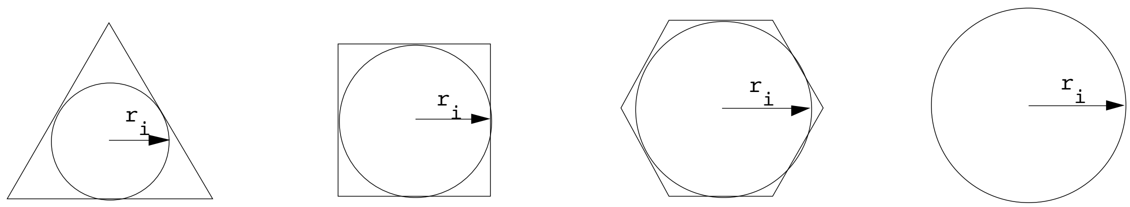
Before beginning the final solution to the true value of Pi one additional fact must be introduced.

We already know that the circle is unique in that it has the *maximum* area and the minimum circumference (perimeter) as compared to any regular polygon. In contrast the Equilateral Triangle has the *minimum* area for a given perimeter of any shape, the fewest number of straight lines of any possible polygon and the maximum perimeter for a given area. The Equilateral Triangle is opposite to the circle in method of construction and in all corresponding parameters of measure. This opposite nature to the circle is particularly obvious when viewing the circle as having an uncountable number of sides residing within the continuously curved line of the circle's circumference.

The four figures on the next page each have a perimeter of 2 π units (we are still using the approximated value of 3.141,592,654). Notice the increase in area as the number of sides increase and the increase in radius of the inscribed circle. The sides change from three, to four, to six, to that of a continuously curved line.

Increase in Area and Radius of Inscribed Circle as the Number (n) of Sides are Increased

n = 3	n = 4	n = 6	n = Curved Line
P = 2π	P = 2π	P = 2π	P = 2π
Area = 1.89941	Area = 2.46740	Area = 2.84911	Area = 3.14159
r = 0.60460	r = 0.78540	r = 0.90690	r = 1.00000



We clearly see that the Equilateral Triangle is least like the circle and marks the extreme boundary of all possible polygons in it's measure of all properties as compared to the circle. Because the Equilateral Triangle and the Circle mark the extremes of difference in shape - and the Square lies between them - we are now ready to make our final calculations.

The True Value Of Pi Calculated By Exact Proportion And Area Of Geometric Shape

We are now ready to calculate the true and exact value of Pi. Having recognized the Equilateral Triangle as being the shape least like the circle and the Square as being the shape by which area is measured we will now construct a set of corresponding relationships that will allow us to find the diameter of a circle which has an exact integer ratio to both the diameter and area of an Equilateral triangle and Square.

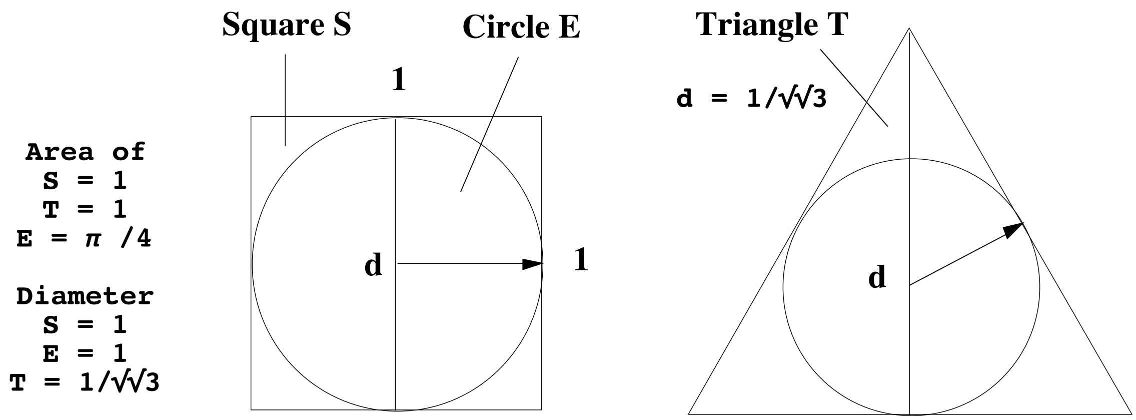
Let us construct a Square (S) having an area of 1. Each side has a length of 1 and the diameter of the inscribed circle also has a diameter of 1. The diameter of the Square is exactly congruent to the diameter of the inscribed Circle. This same relationship will exist for *any* circle (E) that has a diameter of 1. See illustration on the following page.

We have constructed a Square (**S**) having sides of exactly 1 unit of length. This square has a perimeter of 4, a radius of $^1/_2$, and an the area of 1. It's minimum diameter is 1.

Circle **E** is an independent circle that exactly matches the inscribed circle of the square **S**. The diameter of **E** is 1; the same as the minimum diameter of square **S**. A diameter of 1 gives **E** a radius of $^1/_2$. The area of the circle **E** becomes Pi times r^2 **.** Pi times $(^1/_2)^2$ equals Pi times $^1/_4$.

The area **(A)** of Circle E is equal to **Pi times $^1/_4$: `Area = Pi/4`**.

Corresponding Unit Area Square and Equilateral Triangle



Next to our square we construct an Equilateral Triangle **T** with *an area equal to one*. The formula for area we derived earlier and found it to be: area **A** equals $3\sqrt{3}\mathbf{r}^2$ (**r** is the radius of the inscribed circle).

Let us solve for the area in terms of the diameter of the Equilateral Triangle. The diameter was found to equal 3 **r**. When we substitute **d** for 3 **r** the area formula $3\sqrt{3}\,\mathbf{r}^2$ becomes: $\text{area} = \sqrt{3}\,\mathbf{d}^2$. Substituting into the equation our desired area of 1, then $1 = \sqrt{3}\,\mathbf{d}^2$. Dividing $\sqrt{3}$ into 1 and taking the square root of both **d** and the square root of 3 gives: **Diameter** $\mathbf{d = 1/\sqrt{\sqrt{3}}}$.

The diameter **d** of an Equilateral Triangle **T** having an area of 1 is equal to 1 divided by the square root of 3 twice extracted.

We now observe that the area of both the Square and Equilateral Triangle are 1. The diameter of the Square **S** is equal to the diameter of Circle **E.** The diameter **d** of Equilateral Triangle **T** is equal to $\mathbf{1/\sqrt{\sqrt{3}}}$**.** The diameter of Circle **E** has the same correspondence to the diameter of Square **S** as the diameter of Triangle **T** has to $1/\sqrt{\sqrt{3}}$. *We can now solve for the integer value of the diameter of Circle E that is in direct correspondence to the diameter of Triangle T which is equal to* $1/\sqrt{\sqrt{3}}$.

We seek a whole number that when the square root is twice extracted equals 3. Such a number is obtained by squaring 3 twice: $\mathbf{n = (3^2)^2 = 9^2 = 81}$

We extract the square root of 81 twice and find that: $\sqrt{\sqrt{81}} = \sqrt{9} = 3.$ *The* $\sqrt{\sqrt{81}}$ *has the same relationship to 3 as 3 has to* $\sqrt{\sqrt{3}}$ *. In full correspondence: 1/81 is to 1 as 1 is to* $1/\sqrt{\sqrt{3}}$.

Verify 1/81 Is To 1 As 1 Is To 1/√√3

To verify the above correspondence take 1/81 and extract the square root twice.The result is $^1/_3$. This number divided into 1 gives the numerical value 3. The relationship between 1/81 square root twice extracted and *1* is 3.

Take 1 is to 3, the relationship derived in the preceding paragraph, and extract the square root of 3 twice. The result is 1.31607. Divide this number into 1. The result is 0.75984. The number 0.75984 is exactly equal to 1/√√3. The correspondence between √√1/81 to 1 is exactly equal to the correspondence of 1 is to 1/√√3.

We have found the whole number, 81, that corresponds to the diameter of the equilateral triangle with an area of 1. We will use this number as both the numerator and denominator of the diameter of Circle **E**:

Finding Pi By Ratio Of Area Between The Square And Inscribed Circle

We begin with the important ratio: the diameter of circle E has 81 parts each having a size of 1 / 81. 81 parts having a size of 1/81 each provide us with an integer ratio for the diameter of Circle **E** to Square **S**. This ratio, `81/81`, keeps all relationships of area and diameter valid simultaneously. The diameter of **E** remains 1 there being as many parts of diameter as the diameter was divided into. In other words we are multiplying the diameter **E** by `n/n` and for any value of **n**, **`n/n = 1.`**

The diameter of Square **S** is equal to the diameter of Circle **E** therefore it also can be described as having 81 parts each of a size of 1 / 81. The length of 1 side of the Square also becomes equal to 81/81. The area of the square is equal to the length of a side **s** squared and the length of a side is equal to 81 of our 1 / 81 parts. Area of the square **S** equals 81 times 81 = 6,561 of our 1 / 81 parts squared. **`Area of the square equals 6,561 parts of a size of 1/ 6,561 square each`**. *It remains a square of 6,561 / 6,561 = 1.*

We know that the inscribed circle **E** contains an area equal to Pi / 4 of the total area of the Square **S**. We know that the square contains 6,561 parts of a size of 1 / 6,561 each and that the number of parts contained by the circle must be a whole number of 1 / 6,561 parts (because the formula for area is exact and diameter of **E** was pre-divided to produce a whole number of parts based on the exact correspondence to diameter **d** of the Equilateral Triangle).

We also know that the correct value of Pi is slightly larger than the value obtained by the many sided polygon approximation. To find the exact value of Pi we substitute both of the known values for the area of Square **S** and the closest approximated value of Pi (this being π) into the formula: Area **E** = Pi/4 of the Area of **S.** We substitute into our formulas, multiply and then take the whole number above and closest to the numerator value obtained:

The area of Circle E = `π/4 times` the area of the Square

= `π/4 times 6,561` divided by 6,561 parts of the Square

= `(3.141,592,654 /4)` times 6,561 divided by 6,561 *squares*

= `5,152.997,35` divided by 6,561 *squares*

At this point we take the nearest and slightly larger whole number above 5,152.99735 which is obviously 5,153 **:**

The area of Circle E `= 5,153` whole *squares* of `6,561` *squares* of the Square

The true value of Pi `= 4 times 5,153 divided by 6,561`

Therefore **`Pi = 20,612 / 6,561`**

Remembering that Pi is equal to the circumference of a circle divided by twice its radius or 1 diameter, then:

The true value of Pi = 20,612 parts of Circumference to 6,561 parts of diameter

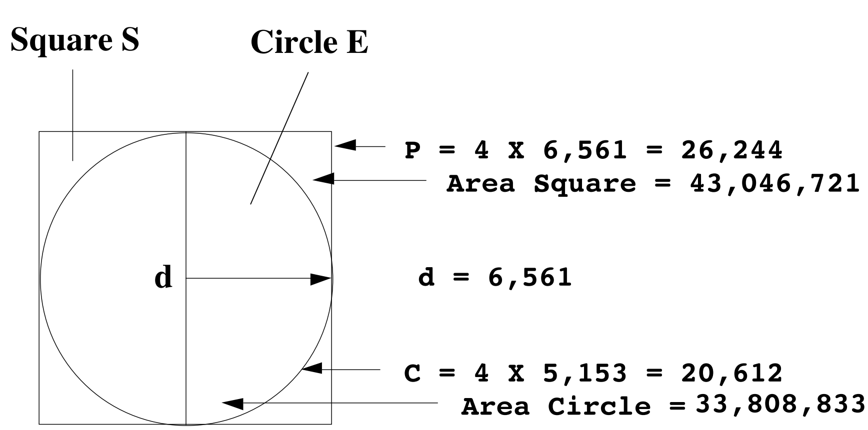
Summary

Using the relationships existing between the unit Square, Circle and Equilateral Triangle we have established a value for Pi that is the ratio of two whole numbers. The formulation of the denominator was established by building on the number 3. To obtain the number 1/6,561we squared 1/3 twice to obtain 1/81. This value was used as the diameter of both circle and square. 1/81 was twice squared to obtain 1/6,561. This number represented the number of squares in the unit Square.

Multiplying $\pi/4$ by the unit ratio 6,561/6,561 gave the fractional number 5152.997351divided by 6,561. Taking the whole number 5,153 as the numerator we established the fraction 5153/6561 as being equal to Pi/4. Multiplying by 4 gives a value for Pi of 20,612 divided by 6,561.

Applying these numbers to our unit Circle and Square produces a very finely divided pair of shapes. The diameter of the circle is now divided into 6,561 parts with a circumference of 20,612 parts. To obtain an area for the circle we multiply $^1/_2$ circumference times radius. This calculation produces an area equal to 33,808,833 square units. In a similar fashion we calculate the perimeter and area of the Square. The results are a perimeter of 26,244 units and an area of 43,046,721 square units.

Square and Inscribed Circle Measurement



Pi Values Compared

In this chapter we will do an in-depth comparison of the two values of Pi; the larger value 20,612/6,561 just computed and the value produced by polygon approximation. It is necessary to compare the two values completely as we have made a jump from straight line geometry to that involving curved lines.

Straight line geometry and mathematics are exact and accurate when applied to the triangle, tangent line and the polygon. The difference between the two approaches does produce a conflict when the two methods are compared.

We will explore the straight line method in more detail so as to find the point of divergence between Mr. Parker's system and the trigonometric/polygon approach.

I am also going to use the capitalized greek letter '∏' for Pi having the value of 20,612/6,561 and non capitalized letter 'π' to represent the smaller value that is derived using straight line approximation.

The Difference Between ∏ And The Polygon Approximation

The first 10 digits of the decimal value of ∏	= 3.141,594,269
The many sided polygon approximation of π	= 3.141,592,654
The difference between the two values becomes	= 0.000,001,615
Dividing the difference by ∏ we obtain:	= 0.000,000,514.

Multiply this difference by the American 1 Billion (1,000,000,000) to obtain the difference in terms of parts per Billion:

∏ is 514 parts per Billion larger
than that obtained by a many sided polygon approximation method

Using ∏ in the Four Step Procedure

In good science or mathematics a new finding is tested against what is already believed to be known. In this case we have the derivations for the polygon about the unit radius Circle and the four step process. The larger value of ∏ when combined with a 6,144 sided polygon and the four step procedure produced a result indicating that a polygon of 6,144 sides must have an inscribed radius of 1.000,000,213 (see page 7.2) to equal the area of a circle with a radius of 1. However, the inscribed circle of the 6,144 sided polygon is a circle that was originally defined as having a radius of 1 **r**.

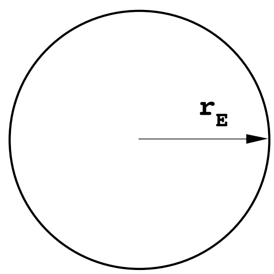
If a polygon having 1,000,000 sides was subjected to the 4 Step procedure the inscribed radius would be 1.000,000,257. The ratio of the radii is equal to the square root of the quantity 2 times Pi divided by the perimeter of the polygon. This is a formula applicable to the ratio of the radius of the inscribed circle of any polygon to the radius of a circle having an equivalent area.

The four step procedure analysis performed in Chapter 6 on page 6.6 predicted that the diameter of the polygon must be made larger for the many sided polygon to contain the same area as that of the perfect circle, however, one circle with a radius of 1 can not be larger or smaller than another circle with a radius of 1.

Four Step Procedure
Making
Area of 6,144 Sided Polygon V Equal To Area of Circle E

$\Pi = \dfrac{20,612}{6,561}$

Step 1 **E**



Begin with Circle E

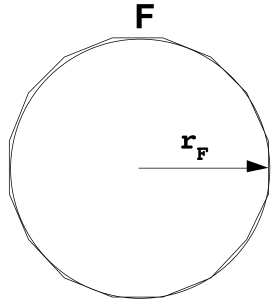
$r_E = 1.000,000,000$

$\mathbf{C_E = 2\ \Pi\ r = 6.283,188,538}$

$\text{Area E} = \Pi\ r^2$

$\mathbf{Area\ E = 3.141,594,269}$

Step 2 **F**



Make Perimeter of F Equal to Circumference of E

$\mathbf{P_F = C_E}$

$\mathbf{P_{6,144} = 6.283,185,855}\ r_F$

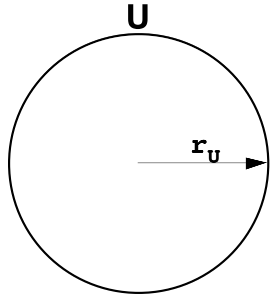
$P_E = 6.283,185,855\ r_F = 6.283,188,538$

$r_F = 1.000,000,427$

$\text{Area F} = 3.141,592,928\ r^2$

$\text{Area F} = 3.141,595,611$

Step 3 **U**



Make Area of U equal Area of F

$\mathbf{Area\ U = Area\ F}$

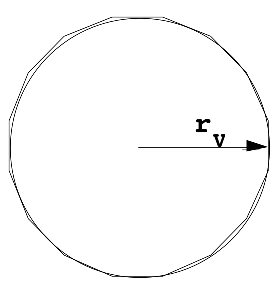
$\Pi\ r_U^2 = \text{Area F} = 3.141,595,611$

$r_U{}^2 = 3.141,595,611 / \Pi$

$r_U{}^2 = 1.000,000,427$

$r_U = 1.000,000,213$

Step 4 **V**



Make U the inscribed circle for V. Calculate area of V.

$\mathbf{r_V = r_U}$

$r_V = 1.000,000,213$

$\text{Area V} = 3.141592928\ r_V^2$

$\mathbf{Area\ V = 3.141,594,266}$

Finding The Difference

Where does the difference come from? Mr. Parker might say that it is due to the difference between curved and straight lines, the circumference of the circle lying outside of the circle or that all real shapes have lines of an identifiable width that width must be accounted for in area calculations.

I wondered if the trigonometric tables are possibly inaccurate for very small angles of measure? Or does the whole circle contain 1 additional unit of measure of some inherent center circle that is not recognized by the polygon 'divide and multiply' method?

Standard mathematics would say that our 'squaring of the circle' has produced an erroneous result. Who is correct? Both systems appear to be based on sound principles. Are the mathematics of straight and curved lines slightly different? Was the jump from the fractional number 5,152.99735 square units to 5,153 whole square units a leap of faith that can not be justified by straight line mathematics?

These are the questions that came to my mind when beginning work on this chapter. This was the most difficult of the chapters to write. The effort to answer the questions listed above did produce a mathematical conflict that that I could not explain but we do find the key formula and assumptions that are behind all of the straight line mathematics used to describe geometry. The final vindication of ∏ had to come in Chapter 8 using real world and 'heavenly' observations. These bodies exist in the natural form of relationships and are not produced by straight line approximations.

Difference In Area Using ∏ and Common π

We will begin to answer our questions be eliminating possibilities. First we will show the size of the difference in area between the two approaches. Second, we will develop a method to find the parameters of the line, circle and polygon without the use of trigonometric functions. Third we will compare our findings.

We use our newly discovered circle having a diameter of 6,561 units. The circumference of such a circle is 20,612 units using our larger value for Pi. The area becomes 1/2 of the circumference times 1/2 of the diameter. This is simply our original 5,153 times 6,561 giving an area of 33,808,833 square units.

Using common π: circumference equals diameter 6,561 times 3.141,592,654 or 20,611.98940 units. The area becomes π times $6{,}561^2$ or 33,808,815.62 units. We summarize below:

Pi Name	Value	Diameter	Circumference	Area
∏	3.141,594,269	6,561	20,612.000,000	33,808,833.00
Common	3.141,592,654	6,561	20,611.989,400	33,808,815.62
Difference	0.000,001,615	None	0.010,600	17.38

As can be seen the differences are very small between the two values.

To determine where differences exist between common π and what I have called ∏ the approach that is most easy to see and is mathematically exact is that of an X,Y graph. We begin by drawing the shapes on the graph. Specific points of interest are located by developing and solving the mathematical equations which represent the circle and lines of interest.

From these equations we can directly determine trigonometric values. These values are calculated from the direct measures of points on a circle. We will use a circle with a diameter of 6,561 units to insure that all measures are consistent with the relationships that we have previously developed.

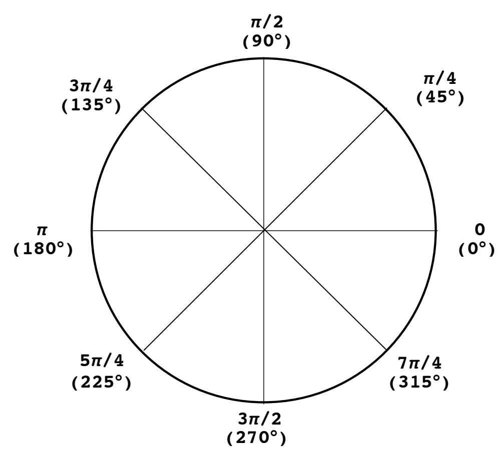
Investigating The Definition of Tangent and The Radian

First however we will examine the the tangent function of trigonometry because we use the tangent of 360°/2**n** to determine the height of the triangles used in our polygon calculations. In this sub section we will expand upon the idea of the tangent trigonometric function and a line tangent to a circle. If this amount of mathematical detail is too great please feel free to skip to the following section or the following chapter.

The trigonometric functions have a set of specific mathematical definitions that is rather complicated to explain fully. They are derived using calculus and expressed as a series of mathematical formulas called a "Taylor Series' of terms. These series are defined for each trigonometric function of sine, cosine and tangent. They are an unending series of terms and have a reminder expression. For any given precision the number of calculations performed (mathematical terms evaluated) must be such as to leave a remainder less than that of the least significant digit desired.

The mathematical series must use an angular measure called the radian rather than angles of degrees which we have been using. We will begin by introducing the radian.

The Circle Measured In Radians



The Radian

When calculating the trigonometric expression for an angle the angle is expressed in a new unit of measure. We already know that the distance around the circumference of the circle is equal to 2 Pi times the radius of the circle. An angle is formed by locating a point on the circumference of the circle that is 1 radius of length traveled long the circumference line of the circle. A line drawn from the center of the circle through the beginning and ending points of this distance traveled around the circumference of the circle is called an angle of 1 radian.

There are 2 Pi possible radians in a circle and there are also 360°. One radian has a measure in degrees equal to 360°/2Pi. This value is 57.2957+°. Each 45° point on the circle is equal to 2Pi/8 or Pi/4 radians. At the 45° angle the tangent of that angle is exactly 1. Correspondingly the tangent of Pi/4 radians is also equal to 1. It is from this relationship that small π is actually determined.

$$\text{arctan (angle in radians X)} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots\cdots$$

$$\text{Remainder} \leq \frac{|x|^{2n+3}}{2n+3}$$

As can be seen from the two formulae at the right calculating an angle or tangent of that angle is a very complicated task. The value **x** is the measure of the angle in radians. **x** is placed in the equation and then the process of evaluating each term is performed. **x** is raised to an odd number power and then divided by an expression called a 'factorial'. Any number followed by an explanation mark, **n!**, means " multiply all of the whole number digits from the number 1 to the number **n**". **3!** is equal to 1 times 2 times 3 which is 6. **5!** would equal the product of all the integer digits from 1 to 5. **5!** equals 60.

The remainder expression tells us how many terms must we evaluate to meet our required level of precision. If we wanted to find π in the form 'the arctangent of π/4 is equal to 1', then the value of **x** is 1. If we want an answer accurate to the 10th decimal place the remainder value must be less than 0.000,000,001. Substituting 1 for the value of **x** and 0.000,000,001 for the remainder and solving for n gives a value for **n** of nearly 500 million terms. This means nearly 500 million additions and subtractions need to be performed each to a precision of better than 10 digits with allowances for rounding and truncation errors. Fortunately the number 1 raised to any power remains 1 so this complication is not encountered.

A computer was used to calculate π to more than 2,000,000 digits by Kazunori Miyoshi and

$$\pi = 32 \arctan \left(\frac{1}{10}\right) - 4 \arctan \left(\frac{1}{239}\right) - 16 \arctan \left(\frac{1}{515}\right)$$

Kazuhiko Nakayama of Japan in 1981 using the trigonometric identity below. Using this identity we could solve for the first 10 digits of π by evaluating just 6 terms of arctan(1/10) using the arctangent formula given on page 7.4.

Numerically we could solve the above equations but doing so would be an exercise in mathematical process but would not confirm the trigonometric relationships to primary geometric shapes.

Back to the Circle and the Line

Another method is needed that is mathematically exact, does not require use of the radian angular measure and can have the 10 digit precision needed. Such a solution exits using the X,Y graph, basic mathematical formulae for the circle, straight line and the bisection of an angle process.

Once having established these fundamental relationships we may use the bisection process to divide an angle as many times as desired and so create an unlimited series of polygons.

The X,Y Graph

An X,Y graph is a way of drawing a picture and being able to identify every point on that picture with mathematical precision. To make an X,Y graph the paper is divided by equally spaced horizontal and vertical lines. One set of perpendicular lines are chosen to be the reference lines. The horizontal line is called the X axis; the vertical line the Y axis. The point where they cross is called the origin. The origin is given a value of 0,0 and becomes the reference point for all other lines.

The number pair 0,0 is called a pair of X,Y coordinates. The first number is the number of units from the X axis and second number the number of units from the Y axis. All of the other lines are given numbers indicating their distance from the X or Y axis that they are parallel to.

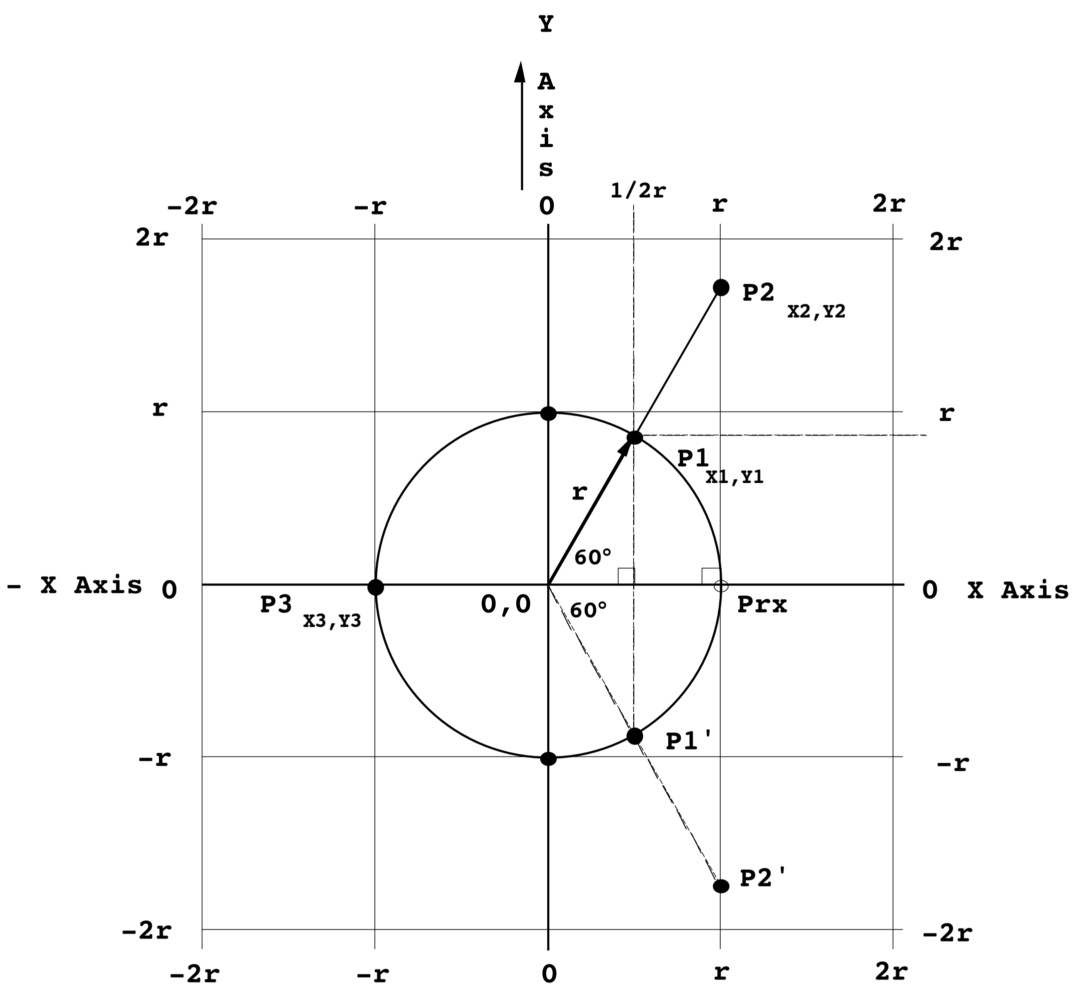
Any line or circle that we chose to draw or plot on the X,Y graph will also have a mathematical formula relating the X,Y coordinates of points located on the lines. If we have a line and an X coordinate on that line then there will be a specific Y coordinate to pair with that X coordinate to describe that point on the line.

On this page is plotted a circle with a radius of r units and each of the major divisions of the graph are also labeled as being r units in size. The center of the circle is located at the origin of the graph. The coordinate values at the origin are X equals 0 and Y equals 0. A 60° line has been drawn from the center of the circle through point P1 and terminating at point P2.

Each point on the circumference of the circle, on the 60° line or at any point on the graph has a set of coordinate values described by it's distance from the 0,0 location. By drawing dotted lines from a point of interest to the X and Y axis a set of X,Y coordinate values are displayed.

Several points have been located on the graph using small circles. Four of the points are located on the circumference line of the circle. They are located at the points where each of the axes

X,Y Graph , Circle, and 60° Line



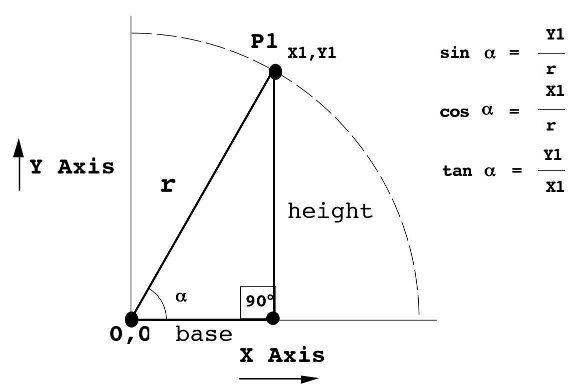
crosses the circumference line. Each point has two numbers associated with it; the first number is the distance from the origin along the X axis and the second number is the distance from the origin along the Y axis. The numbers are named X and Y and have the subscript corresponding to the point that they are associated with. As an example Prx has the coordinate pair $Prx_{X1} = 1$ **r** and $Prx_{Y1} = 0$. As we proceed around the circumference line in a counter clockwise direction the point at the top of the circle has coordinates X=0,Y=1 **r**, Point P3 has coordinates X=-1**r**,Y= 0, the point as the bottom of the circle has the coordinates X=0 and Y=-**r**.

Formula For The Circle

$$X^2 + Y^2 = r^2$$

$$Y = \pm \sqrt{r^2 - X^2}$$

Trigonometric Functions As A Point On A Circle With X,Y Coordinates



The Mathematical Formulae For Lines In Terms Of X,Y Coordinate Pairs

Any straight or curved line may be described by a formula which contains X and Y as variables. If either an X or Y value is known or selected then the value of the other variable must form a coordinate pair of a point exactly lying on the line described by the formula. If two or more lines intersect then one set of X and Y coordinate values at the point of intersection must satisfy the equations for both lines.

Formula For The Circle

The formula for a circle is shown at the right for a circle having its center at the 0,0 coordinate point and a radius of **r** units. X can have a range of values from negative **r** to plus **r**. Y can have the same range -**r** to +**r**. When X equals **r** the value of Y will be zero. When X equals zero then Y must equal **r**.These are the 4 coordinate points discussed previously.

A circle has 4 quadrant symmetry. This means that for any X value there will be two Y values - one positive and the other negative. Also for any Y value there will be two X values one positive and the other negative. Any intermediate values of X or Y will pair with it's mates along the line forming the circle. In all cases the combined distance from the center of the circle for each coordinate pair will be 1 radius **r**.

Points P1 and P1' form the two points having the same X coordinate. Their Y coordinates have the same magnitude but opposite signs (+ &-). The angles between each point and point Prx on the X axis will also have the same magnitude and opposite sign. P1 has a measure of +60° and P1' -60°.

Y Values for Point P1 and P1'

$$Y = \pm \sqrt{r^2 - X^2}$$

$$X = 1,640.25$$

$$Y = \pm \sqrt{3,280.5^2 - 1,640.25^2}$$

$$Y = \pm \sqrt{10,761,680.25 - 2,690,420.063}$$

$$Y = \pm \sqrt{8,071,260.187}$$

$$Y = \pm 2,840.996,337$$

The Trigonometric Functions Are Formed By The Point On The Circle

Notice that the line drawn from the center of the circle that passes through point P1 and terminates on point P2 forms a 60° angle with the X axis line. Point P1 is located at the intersection of the line forming the 60° angle and the circumference line of the circle. If we draw a line perpendicular to the X axis and terminating on P1 a triangle is formed between the center of the circle, the X axis and point P1 as a vertex. This triangle is a 'Right' triangle which we have previously studied.

$$\text{Cos } 60° = \frac{X_1}{r} = \frac{1,640.250,000}{3,280.500,000} = 0.500,000,000$$

$$\text{Sin } 60° = \frac{Y_1}{r} = \frac{2,840.996,337}{3,280.500,000} = 0.866,025,404$$

$$\text{Tan } 60° = \frac{Y_1}{X_1} = \frac{2,840.996,337}{1,640.250,000} = 1.732,050,808$$

The radius of the circle forms the hypotenuse H of the triangle. The X coordinate value of the point on the circle gives the length of the base line which lies on the X axis. The Y coordinate value gives the length of the height line which is parallel to the Y axis. All three line lengths forming the right triangle are completely described by the X,Y coordinates of a point on the circle.

As we remember the trigonometric function 'sine' is defined as being the ratio of the height of the triangle divided by the Hypotenuse. Similarly the 'cosine' is the length of the base divided by the hypotenuse. The 'tangent' is equal to the height divided by the base.

The radius of our circle is **r** making the hypotenuse **r**. The X coordinate value gives the measure of the base length. The X coordinate value, when divided by the radius value, becomes the cosine of a 60° angle.

The Y coordinate value divided by the radius becomes the sine of a 60° angle.

The Y coordinate value divided by the X coordinate value becomes the tangent of the 60° angle.

Formula For A Line

$$Y = m\ X + Y_{0,0}$$

$$\text{Slope } m = \frac{Y_1 - Y_0}{X_1 - X_0}$$

Let the radius of the circle have a value of 3,280.5 units; this number being 1/2 of a circle with a diameter of 6,561 units. For a 60° angle the X coordinates value of P1 is 1/2 of the radius value. P1x coordinate becomes 1,640.25 units.

We substitute these values into our formula for the circle to find the Y coordinated value for point P1. This calculation is described in the box at the right.

Next we substitute the values for coordinates at point P1 and use our radius value of 3,280.5 into the trigonometric formulas labeled on the previous page.

Solving For a 60° Line

$$\text{Slope } m = \frac{2{,}840.996{,}337}{1{,}640.250{,}000}$$

$$m = 1.732{,}050{,}808$$

$$Y_{0,0} = 0$$

Formula For 60° Line

$$Y = 1.732{,}050{,}808\ X$$

We find that the values produce exactly the same numbers for the trigonometric identities of a 60° angle as do the geometric, Pythagorean or Taylor Series approaches.

Use your 'scientific' calculator and the trigonometric function keys to verify the numbers given.

The Formula Of A Line

The formula for the 60° line must meet the requirements for each X,Y pair of coordinates for any point on the line. The line begins at the center of the circle and the origin of the graph. Thus the first point is 0,0. P1 has coordinates of X = 1,640.25 and Y = 2,840.996,337. Point P2 can be seen to have an X coordinate value of 2 **r** or 3.280.5 units. We have not yet calculated the Y coordinate value.

Formula For Distance

$$d = \sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2}$$

The general formula for a straight line is Y = m X + Yo,o. Where Y the value of the coordinate that pairs with X when the formula is evaluated. **m** is called the slope of the line, and Yo,o is the coordinate value of Y when X has a value zero.

The Slope Of A 60° Line

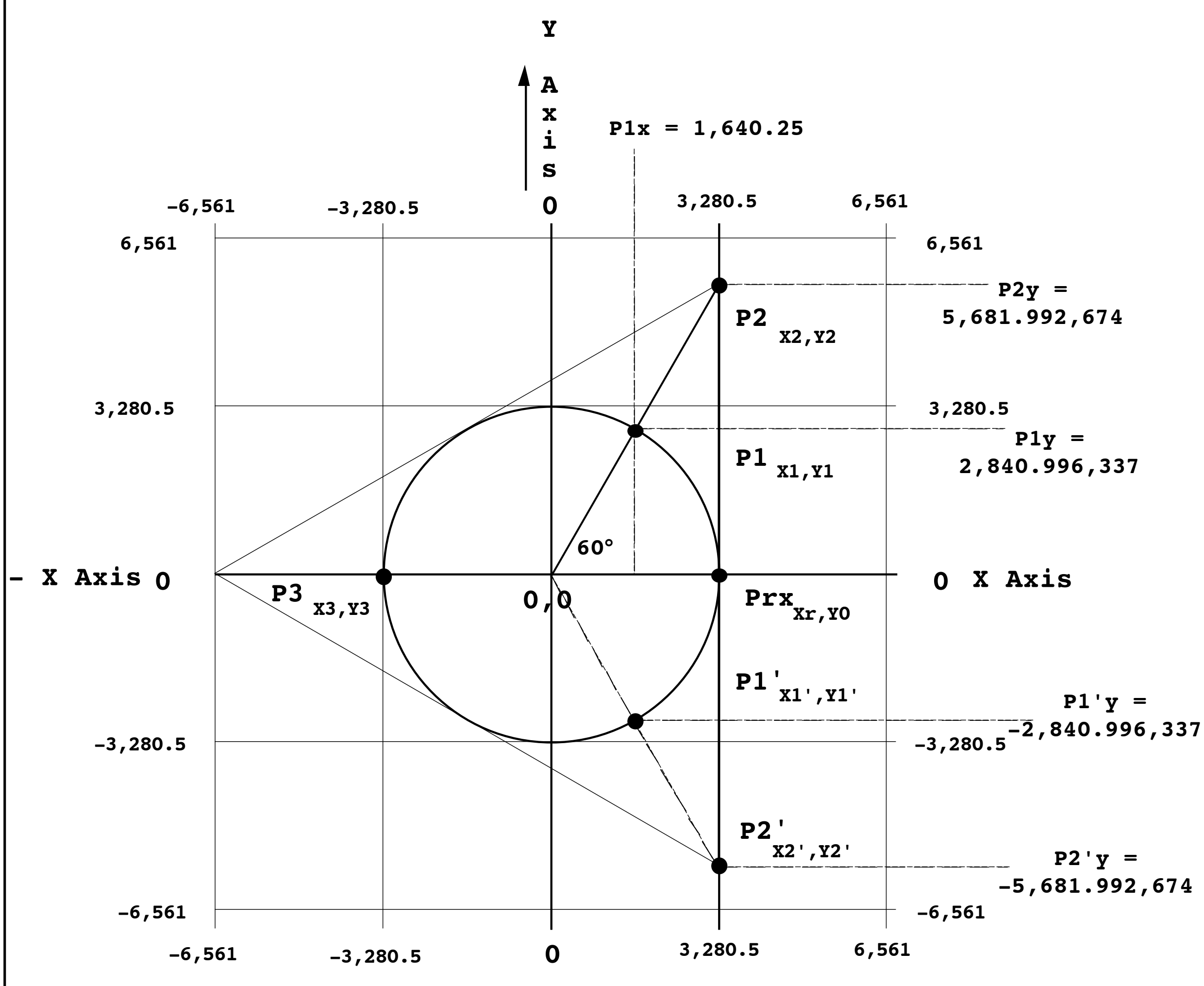
The slope of a line is calculated by selecting two points on the line. The difference in the two Y coordinate values divided by the difference in the two X coordinate values is equal to the slope of the line.

To calculate the slope of the 60° line we chose two points on the line; one being at the center of the circle having coordinates of 0,0 and the second being P1 having coordinates of X = 1,640.25 and Y = 2,840.996,337. Subtracting 0,0 from P1's coordinates we find that P1 has gained 2,840.996,337 units of vertical height over a horizontal distance of 1,640.25 units. Dividing the two quantities we obtain 1.732,050,808.

The formula for any line may be described as: Y coordinate value equals the slope **m** times the X coordinate value plus the value of Y at the location 0,0. This formula is in the box above. For the 60° line drawn in our example the slope is 1.732,050,808 and the value of Y coordinate at location 0,0 is zero.

The find the Y coordinate value of point P2 we multiply the slope **m** times the X coordinate value of P2: 1.732,050,808 times 3,280.5 equals 5,681.992,674. If we divide the Y coordinate value of P2 by the X coordinate value of P2 we obtain the same value for the slope of the line and the tangent of a 60°angle.

X,Y Graph of Circle, Equilateral Triangle and Bisecting Line



Distance Between Two Points On The X,Y Graph

The distance **d** between any two points on the X,Y graph or between two points on the line is equal to the square root of the sum of the difference in the X coordinate values squared plus the difference in the Y coordinate values squared.

The similarity of this formula to that of the circle is obvious. The radius becomes the distance **d** and the X0,Y0 becomes the coordinate pair at the center of the circle. This same distance **d** is also recognizable as the length of the hypotenuse **H** of the 'right' triangle. Coordinate values X1 and Y1 becomes the measure of the base and height of the right triangle.

The loci of the points on the circumference line of the circle are derived by solving a straight line equation for the 'right' triangle. This is a critical formula that unites all straight line mathematics of the polygon, trigonometric formulae and the standard circle. *This equation must be the source of similarity and incompatibility with curved line geometry consisting of continuous curved lines.* This formula correctly locates each point on the circumference of the circle as being a fixed distance from the center point of the circle but this formulas does not make any distinction nor allowance for the *continuity* of the continuous curved line.

Formulas for the Circumscribed and Inscribed Polygon Defined By A Line And Point On A Circle

All of the preceding effort has been done to allow us to accurately determine the length of a side of polygon that is circumscribed or inscribed about a circle without dependency upon trigonometric functions. In the drawing below we will bring these points together and introduce the bisection process whereby other polygons can be created from an existing polygon.

Refer to the drawing below. Point P2 and P2' form one side of the circumscribed Equilateral Triangle. Point Prx and the center of the circle form the inscribed radius line r that we have used as the base measurement of the two 'right' triangles that form one side of a circumscribed polygon. The line that forms the hypotenuse of the right triangle has been found to have a slope **m** that has also been shown to be equal to the tangent of the interior angle a of the right triangle. This same line also bisects the angle formed between the sides of the Equilateral Triangle at vertex point P2.

Mid Cord Coordinates

$$X = X_{rx} + \frac{X_1 - X_{rx}}{2} \quad , \quad Y = Y_{rx} + \frac{Y_1 - Y_{rx}}{2}$$

$$X = r + \frac{X_1 - r}{2} \quad , \quad Y = + \frac{Y_1}{2}$$

Formulae For The Circumscribed Polygon

Our graphical technique with formulae for the circle and line has allowed us to find the tangent value of the interior angle a which we also know to be equal to the tangent of 360° divided by the twice the number of sides (2**n**) in terms of the slope **m** of that line. Calculating the X and Y coordinates of point P1 on the circle and then dividing the Y value by the X value gives both the slope of the line **m** and the tangent of 360°/2**n**.

Multiplying the slope of the line **m** times the radius **r** of the circle gives the Y coordinate value of the vertex point P2. The length of one side of circumscribed polygon is the distance between points P2 and P2' where P2 and P2' have an X coordinate equal to the radius of the circle. Since point P2' has the same X coordinate as point P2 and has the same Y coordinate but of opposite sign then the length of 1 side of the circumscribed polygon is equal to 2 times the Y coordinate value of point P2.

Coordinate Points On The Circle Of A Bisecting Line, Given The Coordinate Points Of The Initial Line Crossing The Circle

New tangent = New slope = m = $\left(\dfrac{\frac{Y_1}{2}}{r + \frac{X_1 - r}{2}}\right)$

$$X_{\frac{P_1}{2}} = r\sqrt{\frac{1}{1+m^2}} \qquad Y_{\frac{P_1}{2}} = r\,m\sqrt{\frac{1}{1+m^2}}$$

Length of 1 side **s** of the circumscribed polygon equals **$s = 2\ m\ r$**

The perimeter **P** of the circumscribed circle is equal to **n** times the length of one side.

$$\mathbf{P = 2\ n\ m\ r}$$

The area **A** is equal to **n** times the area of the triangle formed by the center of the circle, point P2 and point Prx. Since the Y coordinate value is equal to the slope of the line **m** times the distance to point Prx then the area for the two triangles becomes equal to **n** times the slope **m** times $\mathbf{r^2}$.

$$\mathbf{Area = n\ m\ r^2}$$

Formulae For The Inscribed Circle

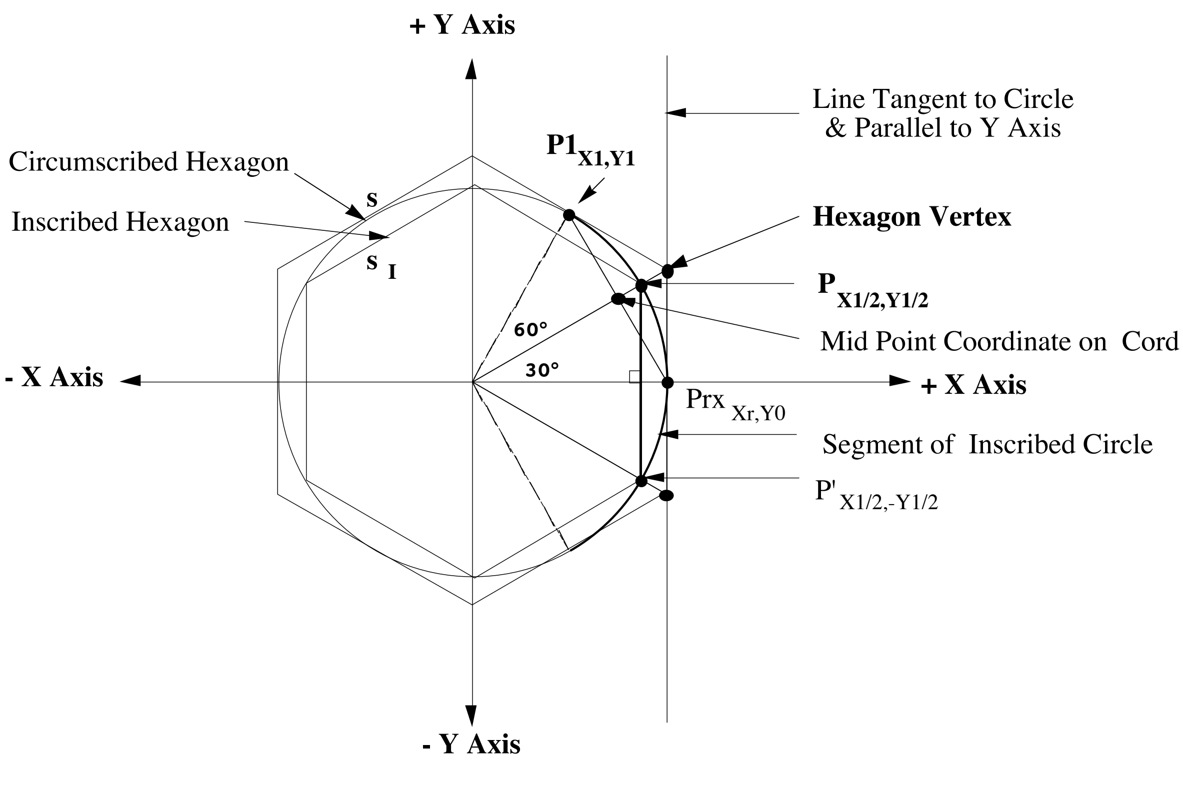
Looking at our graph on the previous page we see that if lines connect points P1, P1' and P3 then the <u>inscribed</u> polygon is formed. The length of one side $\mathbf{s_i}$ is the distance between points P1 and P1'. This value becomes equal to 2 times the Y coordinate value of point P1.

$$\mathbf{s_i = 2\ Y_{P1}}$$

The perimeter $\mathbf{P_I}$ of the inscribed polygon becomes n times the Y coordinate value of point P1.

$$\mathbf{P_I = 2\ n\ Y_{P1}}$$

Formation of Hexagons and Circle Segment Length Using Bisection



The area $\mathbf{A_I}$ becomes **n** times Y coordinate of P1 times X coordinate of P1.

$$\mathbf{A_I = n\, Y_{P1}\, X_{P1}}$$

Arc Segment Located Between Points On The Circle

The arc of the circle located between points P1 and P1' becomes the circumference of the circle divided by the number of sides **n** of the polygon.

For the standard value of Pi (π) **Arc = (2 π r)/n**

For the larger value of Pi ($\prod$) **Arc = (41,224 divided by 6,561) divided by n**

Forming Other Polygons By Multiple Bisection Of One Side Of The Circumscribed Equilateral Triangle

We now wish to bisect the 60° angle formed by the 60° line and the X axis line. When we bisected an angle on a circle using a drawing compass we placed the metal point of the drawing compass on the two points on the circle where the sides forming the angle crossed the circle. In this case the two points are P1, the point where the 60° line crosses the circle and point Prx where the circle crosses the X axis.

We will draw a line between the two points P1 and Prx. A line drawn between two points on a circle is called a cord. The bisecting line will pass through the mid point of the cord connecting point P1 and Prx.

The distance between P1 and the midpoint will exactly equal the distance between the mid point and Prx. Substituting into our formula for distance **d** the coordinate values of the midpoint becomes equal to the Prx coordinates plus 1/2 the difference between the coordinate values of each of the two points. P1 has the coordinates X1 = 1,640.25, Y1 = 2,840.996,337. Prx has the coordinates Xrx = 3,280.5, Yrx = 0.0.

Substituting into the equation for mid point coordinates and calculating X and Y we find that: X = 2,460.375, Y = 1,420.498,169. The line drawn from the center of the circle through the midpoint coordinates is the line bisecting the 60° angle.

As the midpoint and the bisecting line share the point just calculated then we may use this coordinate point to determine the slope of the line. The slope is equal to Y divided by X and this is also equal to the tangent of the angle formed by the bisecting line and the the X axis. Performing the calculation we find: 1,420.498,169 divided by 2,460.375 equals 0.577,350,269.

To check our formula we use a calculator to take the arctangent of 0.577,350,269. We find it to be an angle of 30°which is indeed one half of 60°.

To find the point on the circle where the new bisecting line crosses we solve for the X,Y coordinate pair that simultaneously satisfies the 30° line equation, Y = 0.577,350,269 X, and the equation for the circle, $Y= \sqrt{(r^2-X^2)}$. We set the two equations equal to one another, substitute the known values, and solve for the value of the X coordinate where by the one common Y value is located.

We calculate $X = 3280.5/\sqrt{(1+0.577{,}350{,}269^2)}$ = 2,840.996,338. Y equals 0.577,350,269 times X

Circumscribed Polygons Using Coordinate Points Located On The Circumference Of A Circle Having An Inscribed Diameter Of 6,561 Units

Number Of Sides	**3**	**2,529**	**6,144**	**393,216**
Angle 360°/n	120°	0.142,348,754°	0.058,593,750°	0.000,915,527,34+
Slope m	1.732,050,807	0.001,242,227,864+	0.000,511,326,973+	0.000,007,989,483+
Coordinate on Circle				
X coordinate	1,640.250,000	3,280.497,469	3,280.499,571	3,280.499,999,895
Y coordinate	2,840.996,337	4.075,125,366	1.677,407,918	0.026,209,499+
Coordinate at Vertex				
X coordinate	3,280.500,000	3,280.500,000	3,280.500,000	3,280.500,000
Y coordinate	5,681.99,2674	4.075,128,510	1.677,408,138	0.026,209,499+
Trigonometric Functions				
Cos (360°/2n) = X/r	0.500,000,000	0.999,992,284	0.999,998,693	0.999,999,999,968
Sin (360/2n) = Y/r	0.866,025,404	0.001,242,226,906	0.000,511,326,907	0.000,007,989,483+
Tan (360°/2n) = Y/X	1.732,050,808	0.001,242,227,864	0.000,511,326,974	0.000,007,989,483+
Side Length				
Inscribed	5,681.992,674	8.150,250,731	3.354,815,837	0.052,418,999,735+
Arc (π)	6,870.663,133	8.150,252,827	3.354,815,983	0.052,418,999,736+
Arc (∏)	6,870.666,667	8.150,257,019	3.354,817,708	0.052,419,026,692+
Circumscribed	11,363.985,348	8.150,257,020	3.354,816,275	0.052,418,999,737+
Perimeter				
Inscribed	17,045.978,023	20,611.984,099	20,611.988,502	20,611.989,400
Arc (π)	20,611.989,400	20,611.989,400	20,611.989,400	20,611.989,400
Arc (∏)	20,612.000,000	20,612.000,000	20,612.000,000	20,612.000,000
Circumscribed	34,091.956,045	20,612.000,003	20,611.991,197	20,611.989,401
Area				
Inscribed	13,979,832.73	33,808,780.83	33,808,809.72	33,808,815.61
Arc (π)	33,808,815.61	33,808,815.61	33,808,815.61	33,808,815.61
Arc (∏)	33,808,833.00	33,808,833.00	33,808,833.00	33,808,833.00
Circumscribed	55,919,330.90	33,808,833.00	33,808,818.56	33,808,815.61

giving 1,640.250,000. These are the new coordinates of the point on the circle where the 30° line crosses.

Combined Formulas To Obtain Bisecting Coordinate Points On The Circle

The box at the bottom of page 7.12 gives the general formula set for the point on the circle where the bisecting line crosses the circumference line of the circle referred to an initial line crossing the circle at point P1 having coordinates Y1 and X1. The coordinates of the new point on the circle are described as Y $P_{1/2}$ and X $P_{1/2}$ to show that they are related to the point P1 by bisection.

We are able to continue the process of bisection as many times as desired. We draw a new cord from the point located at Y $P_{1/2}$ and X $P_{1/2}$ and point Prx and again calculating the mid point and the second bisecting line on the circle. The bisected angle has 1/2 the angular measure of the preceding angle. The coordinates of consecutive points $P_{1/2}$ accurately allow calculation of the three primary trigonometric functions and all of the measures of the inscribed, circumscribed and arc segments corresponding to the polygons and circles of interest.

We have developed a way to bisect an angle accurately and calculate the measure of sides belonging to the inscribed, circumscribed and circular shapes using algebraic functions only. The trigonometric functions are available from this method of calculation but are not used to make the calculation. The process is easily chained to allow an unlimited number of bisections of an original angle with known slope. The graph on page 7.13 shows the results of one bisection of the Equilateral Triangle to form the six sided hexagon.

Bisecting The Equilateral Triangle To Produce The 6,144 Sided Polygon

The 6,144 sided polygon can be produced by bisecting the 120° angle of an Equilateral Triangle eleven times. Each step of bisection produces twice as many angles each with one half of the angular measure.

As demonstration: 2 multiplied by its self 11 times equal 2,048. 2,048 times 3 sides equals 6,144 sides. Each of the three original 120° angles is divided by 2,048. This gives 6,144 angles each of a size of 120° divided by 2,048 or 0.058,593,750°. For our calculations we bisect one additional time to produce two 'right' triangles per side. The angle of interest has a value of 0.0292,968,750°.

Using the bisection coordinate formulae, a personal computer and available spreadsheet program we are able to calculate all of the points, lines and areas of interest with greater than the 10 digit precision desired.

Results From Spreadsheet Calculations

Two series of bisections were computed; one beginning with the three sided Equilateral Triangle and the other beginning with the four sided Square. Each series was bisected 19 times. The number of sides based on the Equilateral Triangle ranged from 3 to 786,432 and for the Square from 4 to 1,048,576 sides. The complete results of this undertaking are reproduced in Appendix A.2.

When the number of sides reached 393,216 there was no perceptible difference between the inscribed and circumscribed polygon perimeter or area with a display having 10 decimal digits of resolution. Both polygons matched all measures of the inscribed circle segment based on common π. .

A third series of calculations were performing using the standard trigonometric tables to examine the circumscribed polygon with 2,529 and 2,530 sides. The reason for these additional calculations will be explained shortly.

To eliminate the vast array of numbers produced only a summary of the results for polygons having 3, 2529, 6144 and 396,216 sides are given in the table on the next page.

Looking At The Table Of Calculations On The Next Page

The X,Y coordinates are those corresponding to the point on the circle where the line drawn from the center of the circle crosses the circumference line of the circle. The terminus of this line is at the vertex of the side of the circumscribed polygon which is closest to and above the X axis. A mirror image point (P1') and line joining the center of the circle to the vertex point immediately below the X axis center line delimits the side of the polygons and the arc segment under consideration.

The slope **m** of the line is equal to the Y coordinate on the circle divided by the X coordinate at the point of crossing. The vertex of circumscribed polygon is found by multiplying the slope of the line times the radius of the circle.

The results showed a smooth convergence of inscribed and circumscribed polygons to the arc based on common π. A polygon with 393,216 or more sides, either inscribed, circumscribed or taken as an arc (using common π) and calculated to 10 decimal digits of accuracy, had a perimeter of 20,611.989,400 units and an area of 33,808,815.61 units.

Looking At The Rows Containing The Larger Value Of Pi (Π)

When the number of sides reached 2,529 the perimeter and area of the circumscribed polygon equalled that of the circle as mathematically determined using Π . As the number of sides was increased above 2,529 the area and perimeter of the circumscribed polygon dropped below those measures of the reference circle (based upon Π). Such a happening could only occur if the perimeter of the polygon became less than the circumference of the circle. The numbers show the circumscribed polygon to have both a smaller perimeter and area. Is this a reasonable happening?

To compare area we locate the vertex point on the circumscribed polygon having 2,529 sides and the coordinate values for the point on the circle corresponding to the line connecting the center of the circle to the vertex of the 2,529 sided polygon. The two sets of values are given at the top of the following page.

n = 2,529

Point at vertex	**Xv**	**=**	**3,280.500,000,000**	**and**	**Yv**	**=**	**4.075,128,510**
Point on the circle	**Xc**	**=**	**3,280.497,468,884**	**and**	**Yc**	**=**	**4.075,125,366**
Subtracting	**X**	**= +**	**0.002,531,116**	**and**	**Y**	**=**	**+0.000,003,144**

The difference between both the X and Y values are positive numbers indicating that the point at the vertex of the polygon is farther from the center of the circle than is the point on the circle. Therefore the the vertex of the circumscribed polygon lies outside of the circle.

Another measure is to calculate the radius of the circumscribed circle passing through the vertex points. This distance **d** is equal to the square root of the sum of the squares of the vertex coordinates (the standard formula for distance) and is also equal to our previously found formula for the radius of the circumscribed circle: $\mathbf{r_c = [1/cos(360°/2n)]\ r}$. Performing the calculations we find that the circumscribed radius is equal to 3,280.502,532 units. This is exactly the number that would be obtained by using the formula based on the trigonometric functions and multiplying by the radius of 3,280.5.

This 'sameness' of the above quantities shows that had we performed our analysis on the unit Circle rather than a circle having a diameter of 6,561 units the results would produce exactly the same ratios between all the quantities calculated. Therefore we have not found any special qualities by using a circle with a diameter of 6,561 units.

What is implied is that the length of one side of the 2,529 sided polygon has exactly the same length as the arc segment lying around the circumference of the circle as calculated using Π. This would be a logical conclusion supported by the non equivalency of the continuous arc and the sum of many straight line segments.

Three Gravitating Bodies

Our solar system, or part of it, is composed of many material bodies moving in orbits about the sun. The shapes of all planets and large moons are nearly spherical. Their orbits are also very nearly circular in shape.

The movement of the planetary bodies is the mechanism that has been used for millennium to define our units of time; whether it be the passing minute, day or year. Great cycles such as the 25,800 year cycle of polar precession, the rotation of the galaxies about the Universal Center or even the creation and dissolution of the Universe are all accompanied by the constant movement of matter.
In all of these cycles the circle is to be found as the foundation shape of an orbit, sphere or spiral. We now apply our knowledge of the circle, area and circumference to the measure of time, cyclical periods and size of our three most significant bodies - the moon, earth and sun.

Time and Space

More than 125 years ago - 30 years before Mr. Einstein advanced his famous theorem of Relativity - Mr. Parker gave us these words:

> *First*, Time (or perhaps I should be better understood by saying the standard by which time is measured) is nothing else but the relations existing between light and motion. Therefore time is *altogether relative*, and the motion of the revolving bodies by which time is measured, being measured by time, is *also relative*.
>
> *Second*, Time and space are, for all purposes of calculation with respect to motion, *one and the same thing*, because the measure of time is the circumference of a circle, and its length or duration is the revolution of a circle. Therefore the circumference or area of *one* circle may be reduced to time, and the length of a day or a year may be considered and treated as circumference or area.

Time and Space Combined via Light

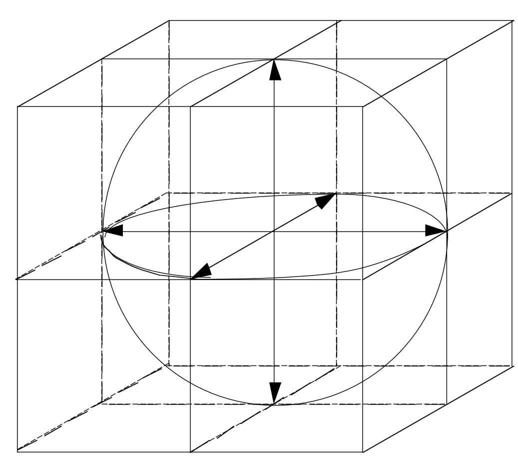
The connecting link between time and space is the velocity and nature of light. The grand distances, volumes and spheres of astronomical bodies can all be referred to the light ray.

Light rays appear to propagate through the medium of uniform space in a straight line and at a fixed rate of speed. The velocity of propagation has been measured by certain scientific techniques to be constant between two or more bodies moving in the same frame of reference.

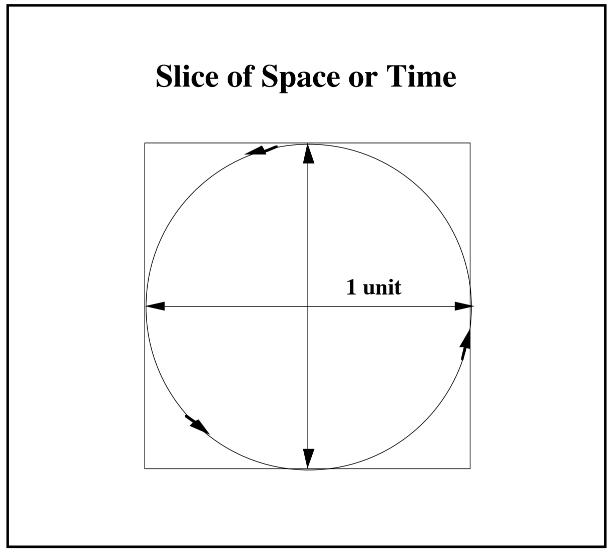
Light is the perfect measuring rod for both distance and time. In 1 unit of time light will travel 1 unit of distance. If we chose any point in space then the light rays will radiate radially in all directions. The result will form a sphere having a radius of 1 unit. The sphere would be the measure of 1 unit of time and of space.

See the figure to the right

Light Sphere and Cube



In 1 unit of time the six arrows would form the maximum dimension in the six possible straight line directions North, South, East, West, Up and Down. Each ray would touch the center face of a six sided cube circumscribed about the sphere. If we were to take a slice through the cube at the points where the sphere touches cube the resulting geometric pattern would be that of the circle inscribed to the square. This slice would represent 1 unit of space-time related to the geometry of the circle contained by the square and 1 Period of Circular Orbit.



Mathematically the several possible measures of the circle and sphere are the radius, diameter, circumference, area, surface area of a sphere and the volume of a sphere. Each unit of measure consists of Pi multiplied by a numeric factor such as 1, 2, 4, or the fraction 4/3 and the radius or diameter of the circle multiplied times itself 1, 2 or 3 times. Thus all relationships between any of these factors will always contain Pi.

To be specific we know that:
The circumference **C** of the circle is related to the radius by **C = 2 times ∏ times r**.
The area **A** of the circle equals **A = ∏ times** r^2.
The surface area **S** of the 3 dimensional sphere is equal to **S = 4 times ∏ times** r^2.
The volume **V** of the sphere is equal to **V = 4/3 times ∏ times** r^3.

Notice that in each unit of measure the variation is by the product of some number multiplied by ∏ and the radius **r** raised to the first, second or third power. It is these combined properties that allow us to use geometric concepts to describe periods of motion in gravitating bodies, i.e. bodies of matter whose motions are describable by gravitational effects and the circular path.

An additional important concept that may be added is the fact that light is influenced by gravitational fields and gravitational fields are associated with mass. If matter is present then light is bent into arcs as it passes over the body. Space-light-time and gravity are all interrelated phenomena.

Electric and magnetic forces are also related to the motion of the primary forces. The path followed by particles subject to electro magnetic forces in motion is that of a spiral. Actual planetary bodies are sources of gravitational and electromagnetic fields. At any moment in time the spiral motion actually generated by these moving bodies appears to be a portion of a circular arc.

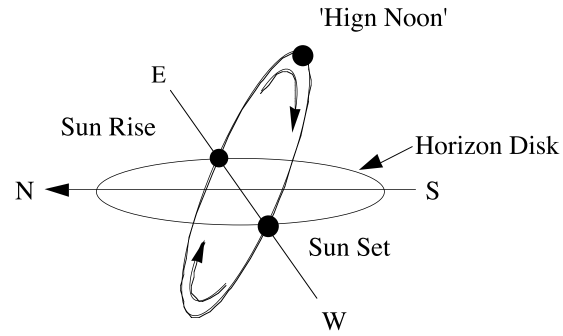
The Solar Day And Measurement Of Time

To an observer standing on the Earth the Sun appears to transit the sky daily. It rises on the Eastern horizon, moves in a steady arc to its highest elevation above the horizon and then declines toward the Western horizon where it passes out of sight having 'set in the West'.

Local 'high Noon' occurs when the sun is at it's highest point above the horizon . The definition of one Solar day is the length of time elapsed between two consecutive local high noons.

One solar day is defined to be exactly 24 hours in duration. This means that as the Earth rotates through one complete cycle of daily time (for as we know the movement of earth and not the sun is the primary reason for the relative motion between the two bodies) the sun appears to move through 360° of arc. Thus a link between time and the circular arc is made.

Apparent Sun Movement



For units of time less than 1 hour in duration (and angles of arc having less than 1 degree of measure) the system of division into smaller units is accomplished by dividing the preceding unit of time (or arc angle) by 60. The table at the right gives the system of divisions.

Divisions of Time

1 Solar Day	**=**	**24 Hours**
1 hour	**=**	**60 minutes (')**
1 minute	**=**	**60 seconds ('')**
1 second	**=**	**60 thirds (''')**
1 third	**=**	**60 forths ('''')**

To produce the high resolution desired for our measurement of time we can express the length of a Solar day as having 5,184,000 thirds and zero fourths. We have calculated these numbers by multiplying the magnitude of each succeeding subdivision by the preceding one beginning with 60 thirds to one second. Performing the calculations based on thirds (''') we have divided one solar day into over 5 million parts.

If we wished to have an even finer division of time (and arc travel of the sun) we would divide

$$\textbf{1 Solar Day} = \frac{\textbf{24 Hours}}{\textbf{1 Solar Day}} \times \frac{\textbf{60 minutes}}{\textbf{1 Hour}} \times \frac{\textbf{60 seconds}}{\textbf{1 minute}} \times \frac{\textbf{60 thirds}}{\textbf{1 second}} = \textbf{5,184,000}'''$$

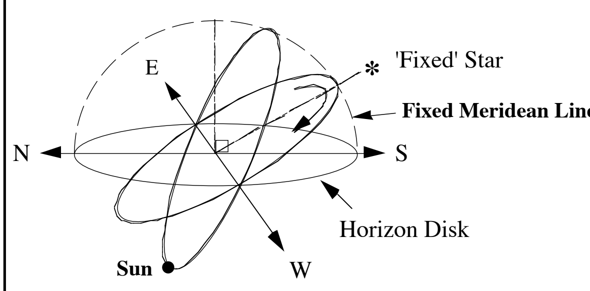
by another factor of 60 to give fourths (''''). In the calculations above we did not perform this finer division so the number of fourths considered are zero.

One Minute Of Arc Is 15 Times Greater Than 1 Minute Of Time

We can measure the passage of time in either hours, minutes and the following subdivisions or by apparent movement of selected bodies such as the sun or stars as they cross a fixed point above the horizon. To do so we might select the units of minutes, seconds or thirds of an arc and begin counting consecutively beginning at some chosen time of day or night. Any selected planetary body would appear to move across the sky at the rate of 15 *thirds* (''') of an arc of degree for every one *third* (''') division of time.

The 15 to 1 ratio comes from there being 360° of arc per day verses 24 hours of time per day. 360° divided by 24 Hrs equals 15° per hour. When we divide each quantity by 60 each fraction is kept constant but now there are 15 1/60th parts of a degree of angle which is called 1 minute of arc per 1 minute of time. The rate of sun travel across the sky is 15 minutes of arc for every 1 minute of time and correspondingly 15''' of arc per 1''' of time.

Sidereal Time



Astronomers have long noted that over the cycle of years and even centuries some of the stars show very little change in their positions relative to each other and to the solar system as seen from earth. These stars are called fixed stars.

The movement of the earth, moon, sun and nearer stars can be compared to those that are fixed. When such a comparison is made the rate of movement of a local body is now compared to those points in space that appear to be stable.

Accurate clocks and accurate observations of the fixed stars proved that the length of a solar day (time elapsed between consecutive noons) is found to vary on a daily bases. The solar day can be as long as 24 hours 30 seconds or as short as 23 hours 59 minutes 39 seconds. The variation is due to the earths orbit around the sun not being exactly circular. We will say more about this topic later.

An additional factor when measuring time by angle between the earth and sun is the fact that the earth is spinning in the same direction as it orbits around the sun. This causes the earth to rotate more than 360° between two consecutive Noons.

To deal with these several problems in the variation of the length of a day another measure of day length has been defined. This day is called the Mean or average Sidereal Day. The length of this day begins and ends when a 'fixed' star is in exactly the same position as viewed by an observer on earth.

1 Mean Sidereal Day

23 Hours 56 minutes 04 seconds 06thirds

5,169,846'''

The sidereal day is an average of the length of many days spanning an integer number of solar years. The length of 1 Mean (average) Sidereal day has been determined to have a duration of **23 hrs 56' 04'' 06'''.** That amount of time equates to 5,169,846 thirds (''')**.**

The Utility Of The Sidereal Day

The length of a day appears to be much more consistent when referred to a 'fixed' star than when compared to the relative position between the Earth and Sun. This is due to the comparative stability of the rotational rate (or spin) of the earth about its axis and the 'fixed' appearance of very distant stars.

The three variables in the sidereal system of time measurement are the gradual change in the speed of rotation of the earth, the actual values of the rate of earth rotation over the interval of time used to calculate the Mean Sidereal day, and the very small but finite movement of the 'fixed' stars with time. Using the Sidereal day as a unit of time measure gives a stability of time measure at least 600,000 times greater than that which would occur by the measurement of any one solar noon cycle on any arbitrarily chosen day of the year.

The second advantage of Sidereal time is the improved simplicity of comparing rotation rates using the universal position of the 'fixed' stars as reference. All bodies have a reference to the greater heavens and locally to the sun but do not have such a central relationship to the orbit of the earth about the sun.

The Circular Day

We now come back to the perfect circle having a circumference of 20,612 units for a diameter of 6,561 units. We remember that the number 20,612 was derived by multiplying 4 times the area of 5,153 square units contained by 1 circle inscribed in 1 Square which had a diameter of 1 divided into 81 parts each of a size of 1/81. The number 6,561 is also the number of square units contained by the Square. This same number, 6561, can also be said to be the diameter of a circle having a circumference of 20,612.

All of these numbers are interrelated and may be viewed as a diameter, an area, circumference, or if taken over 1 circle of circumference, as a measure of time.

The Number 5,153 As A Measure Of Time

We notice that the solar day has a length of 5,184,000 thirds ('''). Let us multiply 5153 by 1000 and treat the quantity as being 5,153,000 thirds(''') of a solar day. This number represents 1 circular day of elapsed time to complete 1 orbit about the circumference of 1 circle perfect circle.

The duration of one Circular day as a fundamental unit of time measure is represented by the number 5,153,000 thirds ('''). Converting this number to the day divided by the customary subdivisions hours, minutes, seconds etc. is performed in the following manner:

First calculate the number of thirds (''') contained in 1 hour of time. Then divide that number into the number of thirds (''') in 1 day. The whole number is noted, subtracted and each decimal re-

Converting 1 Circular Day to Thirds (''')

$$\text{1 Circular Day} = 5{,}153{,}000''' \quad \text{x} \; \frac{1 \text{ hr}}{216{,}000'''} = 23 \text{ hr} + 0.856{,}481{,}48 \text{ hours}$$

$$0.856{,}481{,}48 \text{ hours} \quad \text{x} \; \frac{60 \text{ minutes}}{1 \text{ Hour}} = 51 \text{ minutes} + .388{,}889 \text{ minutes}$$

$$0.388{,}889 \text{ minutes} \quad \text{x} \; \frac{60 \text{ seconds}}{1 \text{ minute}} = 23 \text{ seconds} + 0.333{,}280 \text{ second}$$

$$0.333{,}280 \text{ seconds} \quad \text{x} \; \frac{60 \text{ thirds}}{1 \text{ seconds}} = 20 \text{ thirds} + 0 \text{ forths}$$

$$\text{1 Day Contains} = \frac{60 \text{ minutes}}{1 \text{ Hour}} \text{ x } \frac{60 \text{ seconds}}{1 \text{ minute}} \text{ x } \frac{60 \text{ thirds}}{1 \text{ second}} = 216{,}000'''$$

mainder is then multiplied by 60 to obtain the next number of subdivided units. The process is ccontinued until the final number of thirds are displayed. We can use this process to convert between decimal fractions and subdivided fractions using a base 60 denominator.

Performing the above calculations we find that 1 hour contains 216,000''' and 1 circular day then

1 Circular Day = 23 Hours 51minutes 23 seconds 20 thirds = 5,153,000''' + 0''''

can be said to have the measure of 23 hours 51' 23" 20''' and zero fourths (''''). This calculation is also the prototype for converting between systems of 1/60ths and decimal units. The process is described in the box on the preceding page.

The Significance Of The Three Types Of Day Measurements

We summarize the length of the various days:

1 Solar day =	5,184,000'''	24 h 0' 0" 0"
1 Sidereal day =	5,169,845'''	23 h 56' 4" 5'''
Circular day =	5,153,000'''	23 h 51' 23" 20'''

Each change in Day length measure is due to the rotation of the Earth relative to something else. For the Solar day the duration is between succeeding 'high' noons where the sun is the reference. For the Sidereal day it is between consecutive passages of one of the 'fixed' stars of deep space across a selected celestial meridian (an imaginary vertical line perpendicular to the horizon disk and referenced to the pattern of fixed stars).

The Circular day would be the length of time for one complete rotation of the earth in absolute space-time relative to one 360 degree circle. This is the shortest of the three reference systems used to describe the length of 1 day. In all real systems every available reference is in motion and that motion must be added to the motion of the object under consideration. Only in the realm of mind, number fields and geometry can an object be considered to be in temporary isolation. All real bodies are parts of larger (and smaller) systems and consideration must be given to these surrounding relationships.

The Difference In The Units Of Time Measure

The difference between the day measures is calculated by the process of mathematical subtraction. A brief table has been prepared for convenient reference:

1 Solar day is longer than 1 Sidereal day by	3' 55" 54'''	units of time.
1 Sidereal day is longer than 1 Circular day by	4' 40" 46'''	units of time.
1 Solar day is longer than 1 Circular day by	8' 36" 39'''	units of time.

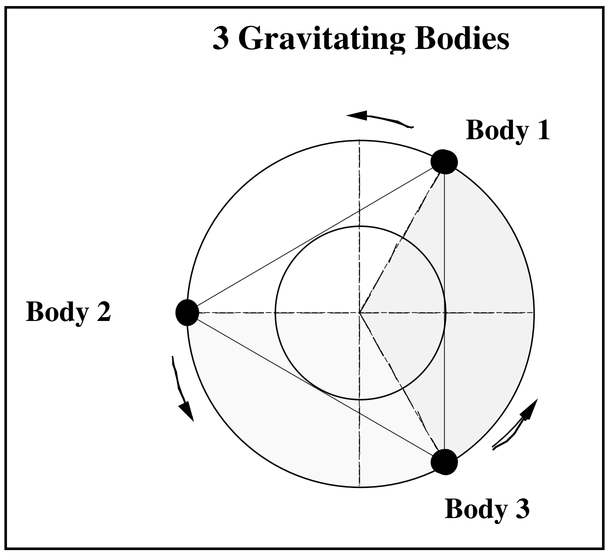
The difference in the Circular to Sidereal day
minus the difference in the Sidereal to Solar day is 0' 44" 52''' units of time.

Because the difference between the absolute Circular day and the Sidereal day is greater than the difference between the Sidereal and Solar day then we know that the Earth/Sun system is in motion relative to absolute space-time.

Three Gavitating Bodies

The Moon, Earth and Sun are known to form a system of three gravitating bodies whereby the moon revolves about the Earth and the Earth/moon revolve about the sun. Direct and extensive astronomical observation has allowed the periods of revolution for these bodies to be determined.

We will use the basic constructions of geometry and the numerical values of the perfect circle as the basic relational reference system to describe planetary bodies. After making our calculations in the pure realm of geometric relationships we will then map those findings into the 'real world' space-time system that they are a part of.



The Geometry Of The Gravitating Bodies

The drawing to the right gives the beginning arrangement for completely equal bodies.

Conceptually we will begin by viewing the trinary system as being made of three exactly identical bodies, equally spaced and revolving about their common center. Each gravitating body sees the several individual forces acting upon it as but one force which guides its path through space. Each body contributes forces and movements of 1 share to the total system. The total system has a value of 3 parts with each body having 1/3 of the total.

About the common center of rotation is drawn 1 perfect circle. Circumscribed about the 1 circle is 1 perfect Equilateral Triangle and circumscribed about that triangle is a second perfect circle having exactly twice the measure of the inner circle (by the law and nature of geometric relationship). What ever measure of radius, diameter or circumference exists in the inner circle, the outer circle will have exactly 2 times that value.

The area of the outer circle becomes exactly 4 times greater than the inner circle area as it has twice the measure of radius and twice the measure of circumference of the inner circle. Area being equal to 1/2 the circumference times the radius gives the product of 2 times 2 equaling 4 times the area of the inner circle.

The rotating bodies, being equal in all things, follow each other in 'lock step' around the circumference of the larger circle. In 1/3 cycle of time each body will move from its beginning position to the position of its neighbor, i.e. after 1/3 cycle of time body 1 occupies the position of body 2, body 2 occupies the position of body 3 and body 3 occupies the position of body 1. The area swept by the movement of each body during the 1/3 cycle of time is equal to 1/3 of the 4 units of area of the larger circle and similarly 1/3 of the area of the inner circle (having 1 unit of area).

As compared to the inner circle, - being a measure of time by circumference or of swept area - the outer body has traversed a measure of distance, time or area 4 times greater than that of the inner circle. Therefore the measure of time, position or area of 1 body gravitating about 1 circle of reference is as 4 times 1/3 or exactly 4 to 3.

The ratio 4 to 3 will hold true for any system of three bodies in a gravitating relationship in reference rotation of one to another. This condition holds true even if the bodies are of disparate magnitudes. It also hold true when the swept area is that of a body following an elliptical path about the larger body and thereby completing 1 circle of orbit by that means.

The Magnitudes Of The Sun, Earth , Moon System

The great differences between three gravitating bodies such as the moon, earth and sun is in the magnitude of their individual contribution to the performance of the whole and to their individual requirement for balance.

The order of their relative magnitudes by size and dependency is such that the Moon has the least measure and the Sun the greatest. Therefore in order for the moon to contribute it's 1/3 share to the complete balance of the system then it must do so via the mechanism of having the greatest number of rotational orbits. The Earth will follow with the next greatest number of rotational orbits.The Sun will perform 1 orbit about the common center and have the least rotational velocity about the common center.

The system is much like that of a clock mechanism with the moon being the smallest wheel with the greatest spin. The Sun represents the hour hand of the clock with the slowest rate of revolution relative to the three body system under consideration. The sun is not fixed in space however. It is believed to be traveling in a great circle about the center of the galaxy - a journey of more than 1,000,000 light years in circumference requiring an estimated 225,000,000 years to complete.

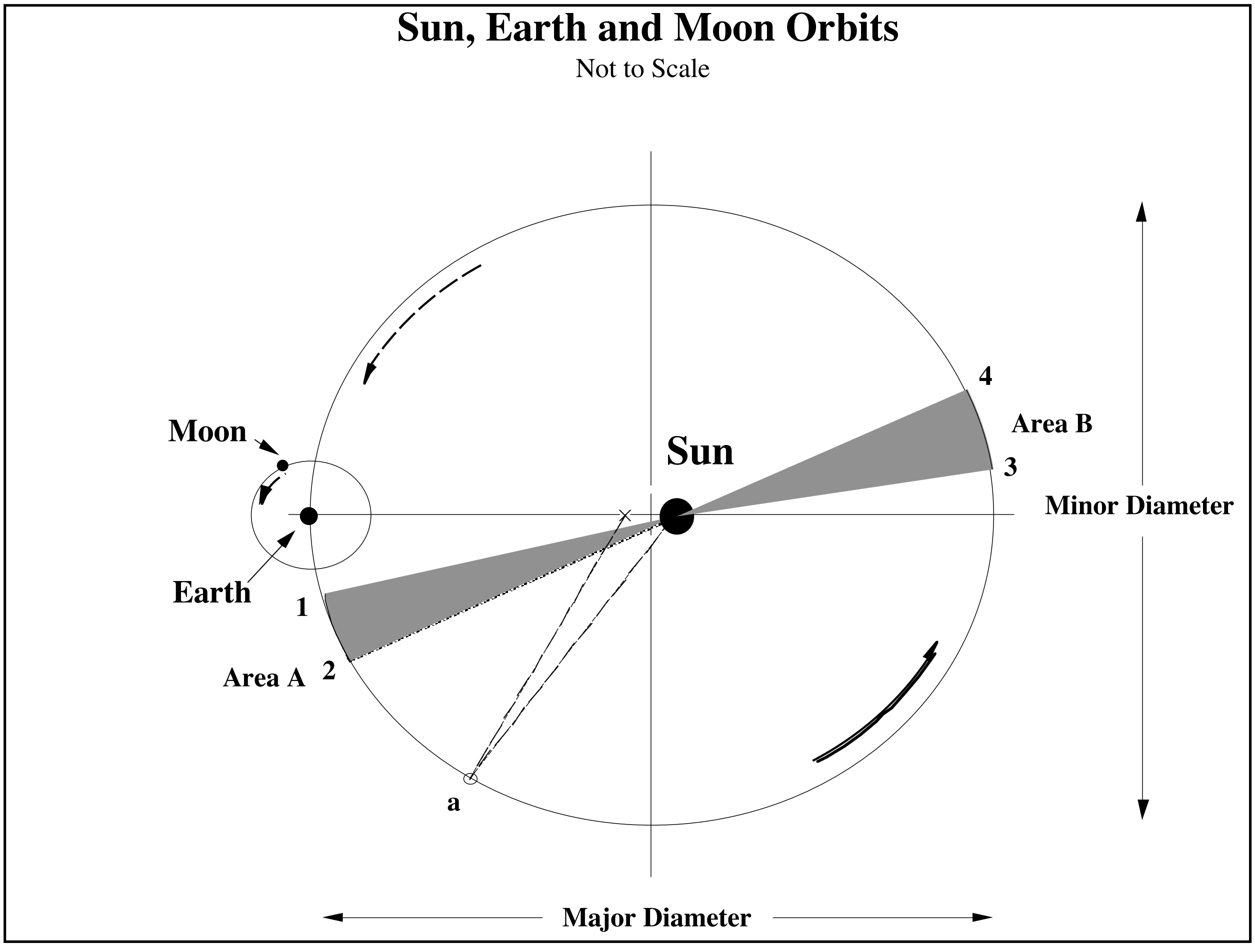
The Moon, Earth and Sun System

The drawing on page 8.9 indicates the comparative magnitude of the three bodies of interest. The scale is not fixed as the sun has a diameter more than 100,000 times that of the earth and the orbit of the earth about the sun has a diameter about 387 times larger than the diameter of the moons orbit about the earth.The drawing does show that by order of magnitude the moon appears to revolve about the Earth and the Moon/Earth together revolve about the Sun.

Elliptical Orbits

Up to this time we have spoken of the perfect circle and have thoroughly analyzed it's characteristics. There is another shape that is related to that of the circle but has several differing properties. This shape is called the ellipse. Real gravitating bodies have been found to have orbital paths following that shape.

In the diagram on page 8.9 you will notice that a centerline has been drawn and the Sun is offset slightly to the right of center. The orbital path of Earth and Moon can be seen to be somewhat 'squashed' with the vertical dimension being less than the horizontal dimension. The center of the sun and the small 'X' to the left of the vertical centerline are called focal points. The path fol-



lowed by the orbiting bodies is such that the sum of the distance from the two points to the center of the orbiting body remains constant at all times. The length of the two dotted lines from the centers of the two focal points to the small circle **a** vary as the body orbits, but the sum of their length remains constant. The resulting orbital path is caused to have a major and minor diameter whose comparative size is dependent upon the distance of the two focal points from the center line of orbital path.

Nearly all planetary orbits and the spherical planets and moons are themselves slightly out of round. The question is, "does this situation invalidate our approach to solving for the measures of gravitating bodies using the circle as the basis of relationships?" The answer is an emphatic "no", and in fact supports the approach that we will employ.

Johannes Kepler's Three Laws

In the year 1609 Johannes Kepler proved that the orbit of the planet Mars was that of an ellipse rather than that of a perfect circle. For 1300 years philosophers and astronomers had thought that the planets must have perfectly circular shapes and orbits because they were 'heavenly' bodies and therefore perfect. As we will show the truth of the perfect circle does still exists and remains a source of accurate measure but the outward path is that as observed by the astronomer.

The effect observed with a non circular orbit is a speeding up of the orbiting body when it is closer to the central body and a slowing down when it is farther away. Kepler observed that if a line were drawn from the center of the central body to the orbiting body - such as is shown by the line drawn from the center of the Sun to point 1 in our drawing - then the area swept by the line in 1 unit of time would be constant.

Thus the shaded area produced by the orbiting body moving between points 1 and 2 in 1 unit of time must have the same number of square units of area measure as the shaded area formed by the movement between points 3 and 4 in that same measure of time. To make the areas equal the orbiting body moved more slowly between points 1 and 2 and faster between points 3 and 4.

Eccentricity

$$e = \frac{d_{minor}}{d_{major}} - 1$$

Elliptical Eccentricity

By measuring the major and minor axis of the ellipse a number can be determined that describes the variation of orbital path from that of a perfect circle. This measure is called the eccentricity of the orbit and is designated by the italicized letter '***e***'.

In our drawing the major diameter lies on the horizontal axis and the minor diameter on the vertical or Y axis. Dividing the minor diameter by the larger diameter and subtracting 1 from the quotient gives a resulting number having a range of 0 for a perfect circle and 1 for a completely closed ellipse.

The orbital paths drawn on page 8.9 have an eccentricity of 0.1. The moon's orbit about the earth has an eccentricity of 0.0549. The orbit of the earth/moon system about the sun has been determined by astronomical measurement to have an eccentricity of 0.0167. From these numbers we deduce that both orbital paths would appear more circular than as shown by the drawing.

Kepler's Third Law Applied To The Solar System

In 1619 Kepler further refined his theories to include a third principle of orbiting bodies. His third principle states that for orbiting planetary bodies (the law is applicable to other orbital processes) the ratio of the square of the time that they take to complete one elliptical orbit $\mathbf{P^2}$ divided by the cube of 1/2 the major diameter $\mathbf{a^3}$ is a constant.

With a perfectly circular orbit the rates of movement would be constant at all points. In either the case of the elliptical or circular orbits the rate of movement measured over 1 complete circle of travel will produce 1 measure of time and 1 measure of average velocity. This average speed must be proportional to the area enclosed by the orbital path. The orbital path can be thought of as being the circumference measure of a circle with an appropriate diameter. Thus the elliptical orbit by measure of area can be described by the measure of 1 circle and the measure of 1 orbital period **P** as that of 1 circular cycle of 1 360° measure of arc.

Listed below are the eccentricity (***e***), periods of rotation (**P**) and average distance (**a**) from the sun for four of the planets in our solar system. The time for the earth to revolve once around the sun is taken as being 1 unit of time. The average distance of the earth from the sun is taken as 1 unit of distance. The rotational periods and the average distance from the center of the sun of the other 3 planets have been converted to these units of measure by division.

Kepler's third law states that the square of the period ($\mathbf{P^2}$) of one orbit divided by the cube ($\mathbf{a^3}$) of the average distance of the orbiting body from the sun will be constant for all bodies in orbit around the common center. Upon dividing $\mathbf{P^2}$ by $\mathbf{a^3}$ the quotient will be found to be 1 in all cases. The ratio is independent of the eccentricity.

The theme of our analytic approach would lead to the similar result that the time for completion of 1 cycle of revolution for one body in a gravitating system will have an exact relationship to another body in the same gravitating system.

	Earth	**Mars**	**Jupiter**	**Saturn**
e	0.0167	0.0934	0.0484	0.0556
P^2	$(1)^2$	$(1.88081)^2$	$(11.86179)^2$	$(29.4566)^2$
a^3	$(1)^3$	$(1.52369)^3$	$(5.2028)^3$	$(9.53884)^3$

In calculations to follow we will show that, by considering only the parameters of the perfect circle and the measured duration of 1 Sidereal day, orbital times can be determined to very high degrees of accuracy. Time measurement is always very precise because it deals with the counting of circular cycles which are the *central* feature of all material manifestation.

Measurement - The Earth As The Standard

When a unit of measure is sought we would like to find one in nature if possible. The Earth itself provides a measure of time by its spin on it's axis and a unit of distance by measure of it's diameter. These measures can be considered as being 1 point of reference as defined by nature. The earth's parameters of measure have been fashioned under the laws of the original first cause and therefore must be in harmonious relationship to the Sun and Moon. They must also represent the fundamental property of proportion and number contained by 1 perfect circle.

The solar day, lunar month and solar year are directly observable cycles as seen by simple observation and accurate time measurement. The Sidereal day, month and year require observation and calculation to produce the observed and averaged numerical values given.

The circular day, circular lunar month and circular year are calculated by fixed ratios using the fundamental properties of the circle and geometry. They are taken from the field of numbers and geometry which form the primary standards of measurement that relativistic space-time expresses.
Solar time is the 'holistic or 'collective' time in which all of the bodies appear to move as we experience them relative to the light-space-time matrix existing at an Earth based viewpoint.

The Lunar Month And Solar Year As Given In Standard Astronomical Tables

One lunar month is the time elapsed between consecutive new moons. It has been measured to have a length of 29.5306 days. The length of 1 Sidereal month is referenced to the fixed stars. It has a length of is 27.32166 days.

One Mean Solar year is the time taken by the earth to orbit the Sun and circumscribe one complete seasonal cycle from Spring Equinox to the following Spring Equinox. The average elapsed time has been determined to be 365 d 5h 48' 46''.

The Moon's Period Of Rotation About The Earth As Determined by Circular Time

The relationship of the circumference of one perfect circle to one diameter is as 20,612 parts of circumference to 6,561 parts of diameter. The measure of time (circular time as differentiated from Sidereal or Solar time) for 1 orbit of one lesser gravitating body rotating about a more primary body (in the same 3 body gravitating system) is the ratio of 4 to 3.

Given the Moon as being the least gravitating body in the Moon/Earth/Sun system and the measure of 1 primary circle as being 20,612 parts of circumference to 6,561 parts of diameter then the time for the moon to orbit the earth in true circular time is 4/3 times 20,612. Performing the calculation produces the number 27,482.666,667 circular units. This number is the measure of one circular moon orbit about the earth as compared to one primary circle having a diameter of 6,561. Multiplying the number 27,482.666,667 circular units by 4/3 gives the number 36,643.555,556. This is the number of circular units in 1 orbit of the Earth/Moon about the sun referred to the diameter of the Earth/Moon orbit.

Mapping Circular Units Of The Lunar Month And Mean Year Into Sidereal And Solar Time

We wish to convert the number 27,482.666,667 circular units for the Sidereal Lunar month and the number 36,643.555,556 circular units for the Mean Year into time as expressed by the mean day.

The earth and sun are represented by the circle and its fundamental relationship of circumference to area. Their respective magnitudes are but some multiple of that relationship. We found that the ratio which mapped into our system of time occurred when 5,153 *thousand* thirds (''') were viewed as being the circular equivalent of 1 solar day. The number 27,482.666,667 circular units is referenced to the same circular time and same primary circle as is the earth/sun system. Dividing 27,482.666,667 by *1000* converts the number into the correct number of 27.482,666,667 circular days per Sidereal lunar month.

Multiplying the lunar cycle of 27.482 days by the quantity 4/3 times 10 gives the correct number of 366.435,555,556 circular days per mean year. This is the same quantity that would have been obtained by dividing the yearly orbit of 36,643.555,556 circular units by 100. The factor of 4/3 times 10 (rather than 4/3 times 1) came into the picture because of the Earth/moon orbit about the sun was now referenced to the larger diameter moon orbit about the earth. In other words the Earth/moon orbit was referenced to the larger <u>diameter</u> of the moon orbit yet still remained a scaled 4/3 multiple of the perfect circle circumference.

To complete the process of time conversion we have previously calculated that 5,153,000 circular thirds (''') of time constitute 1 circular day and we have calculated 1 solar day to contain 5,184,000 thirds (''') . Multiplying 27.482,666,667 circular days times 5,153,000''' equals 141,618,181.3333''' circular thirds per day. Dividing by 5,184,000''' gives a final value of 27.318,322,016 mean solar days for one complete mean lunar orbit. Performing the same calculation on the circular mean year gives 364.244,293,552 mean days for the mean year.

1 Mean Lunar Sidereal Month
27d 7h 43' 3'' 47''' 20'''

To find the length in Sidereal days (as would be observed by an astronomer) we convert this value to days, hours and subdivisions down to fourths and then add the difference between 1 circular earth day and one observed Sidereal earth day.

27.318,322,016 days is equivalent to 27d 7h 38' 23" 1''' 20'''' circular solar days. The difference between 1 circular day and 1 sidereal day is 4' 40" 46'''. Adding we obtain a final value of **27d 7h 43' 3'' 47''' 20''''** . This is the calculated time for one complete Sidereal lunation (one complete orbit of the moon around the earth as observed by the return of earth and moon to the same angular relationship to one of the fixed stars).

The Mean Year

1 Mean Year

365d 5h 48' 50'' 53''' 6''''

The method to calculate the mean year from the circular year of 364.244293552 mean days is some what more involved as additional correction factors must be added. To get the number of days which would be observed to pass by astronomical observation we must add 1 Sidereal day and the relative motion of the combined Earth/Moon/Sun system to the number calculated. 1 Sidereal day is added to the above number because the Earth in making 1 annual orbit around the sun will experience 1 day/night cycle. This appears as 1 added day to the number of days counted by the spin rotation of the earth about it's axis during 1 annual orbit.

To perform the calculations we convert the calculated circular year number to standard divisions of time. This conversion produces the number 364d 5h 51' 46" 57''' 46''''. We now add 1 Sidereal day of 23h 56' 4" 6''' bring the total time to 365d 5h 47' 51" 3''' 46''''.

To the number 365d 5h 47' 51" 3''' 46''' we add the final factor of 4/3 times 0' 44" 52''' to correct for the difference in circular to sidereal and sidereal to solar days. 4/3 times 0' 44" 50''' 52'''' equals 59" 49" 20''''. Adding the final factor we find 1 Mean year to have a length of exactly **365d 5hr 48' 50'' 53''' 6''''**.

The Solar Year Calculated

To obtain the value of 1 mean solar year we take our original *circular* year, add 1 circular day, add the difference between 1 Sidereal day and 1 solar day and add the correction factor for the combined precession of the Moon/Earth/Sun system:

1 circular year plus	364d	5h	51'	46"	57"	46''''
1 circular day plus		23h	51'	23"	20'''	0''''
1 Solar day minus 1 Sidereal day plus			3'	55"	54'''	0''''
1 Correction due to system movement				59"	49'''	20''''
1 Solar year equals	**365d**	**5h**	**48'**	**6''**	**1'''**	**6''''**

The Synodic Lunar Month

This is the time that would be measured between two new moons. We must consider the rotation rates of the both the moon and motion of the earth about the sun. The required alignment is such that the moon lies directly between the earth and sun at the start of the measurement period and ends when the moon again lies exactly centered on a line drawn from the center of the sun to the center of the earth. This calculation is the most complex to make and I will give Mr. Parkers value without proof: 29d 12h 44' 2" 50''' 31''''.

Comparison Of The Calculated Lunar Cycle and Mean Year To The Observed Data

A published value for 1 Mean Sidereal lunation period based upon averaged astronomical observation was given as 27d 7h 43' 11" 36'''. The value calculated using 1 perfect circle circumference in a three body gravitating field was 27d 7h 43' 3" 47'''. Difference between the two values is a mere 7" 49''' (the observed value being larger by this amount). Dividing the observed value by the calculated value gives the ratio of 1 to 1.000,003,311. This is a very high level of correlation.

A published value for 1 Mean Year based upon averaged astronomical observation was given as 365d 5h 48' 46". The value calculated using the 3 gravitating body system is 365d 5h 48' 50" 51'''. The difference is 4" 51''', the calculated value being larger by this amount.

Dividing the calculated value by the observed value gives the ratio of 1 to 1.000,000,135.

This is the very close ratio between the circumference derived for ∏ and the actual circular path of the earth orbit about the sun in real time-space that Mr. Parker (and I) feel that this concurrence verifies the validity of ∏.

The Earth As A Standard Of Space Measurement

The diameter of the earth has been measured by various means and has been found to have differing dimensions at different locations. The average diameter measured about the equator (the location of a circumference line being farthest from the polar spin axis and halfway between them) is listed as being 7,926.41 miles. The average diameter measured along the spin axis and between the poles is 7,899.83 miles. The average between the volumes represented by two spheres having those diameters and reconverted to diameter is 7,913.14 miles.

We will use the value of 7,913.14 miles as being the diameter of one circular point in space.

Calculation Of The Moon's Diameter

We again use the special relationships found through Mr. Parker's methodology.

Each of the measures for the Lunar and Earth orbital times were referenced to the circumference of 1 circle having a circumference of 20,612 parts per 6,561 parts of diameter. Having utilized the numerator to find time periods (**P**) then we can also use the denominator to find diameter. We shall say that the diameter of the moon squared (**dm^2**) divided by the diameter of the earth squared (**de^2**) must be equal to the circular time period for 1 orbit of the moon (**Pm**) about the earth divided by the circular time period for 1 orbit of the earth (**Pe**) about the sun.

The orbital period of the moon (**Pm**) was found to be equal to 20,612 times 4/3 divided by 1000. The orbital period of the earth about the sun (**Pe**) was found to be equal to 20,612 times 4/3 times 4/3 divided by 100. The above equation simplifies to: the moon's diameter equals the earth's diameter times the square root of 3/40. Taking the square root of 3/40 and multiplying by the earth's approximated diameter of 7913.14 miles we obtain a diameter for the moon (**dm**) equal to 2,167.10 miles.

A published measure of the moon's diameter is 2,160 miles. Again an extraordinary correlation of only 6 miles difference between values based upon the numbers of ∏.

The Earth's Distance From The Sun

We found the circular time for 1 orbit of the earth about the sun to be equal to 20,612 parts of circumference times 4/3 times 4/3 again and divided by 100. The distance from the center of the earth to the center of the sun can be found by multiplying the original perfect circle diameter of 6,561 by 4/3 times 4/3 and multiplied by the diameter of the earth. We do this because the earth represents 1 diameter of measure that has been related to the perfect circle and is a product of this solar system.

Performing the multiplication we find 6,561 times 4/3 times 4/3 times equals 11,664. Multiplying 11,664 times 7913.14 miles gives a product of 92,298,865 miles. This is the number of miles from the center of the earth to the center of the sun if the earth's orbit were perfectly circular.

A published number for the average distance between the earth and sun is 92,955,825 miles. Dividing the calculated number by the published average number gives a quotient of 0.992,932+.

Kepler's Third Law Using ∏ Relationships

$$\frac{P^2}{a^3} = \frac{\{[(\frac{4}{3} \times \frac{4}{3} \times \frac{20{,}612}{100}) \times (\frac{5{,}153}{5{,}184})+1]\}^2 \ [\text{Mean Sidereal Solar Day}]^2}{[\frac{4}{3} \times \frac{4}{3} \times 6{,}561 \times 7{,}913.14]^3 \ [\text{miles}]^3}$$

This difference is by far the greatest of any of the measures yet considered but again offers an extraordinary concurrence. As more astronomical information becomes available and if we accurately account for the elliptical orbit of the earth and the gravitational effects on space-time I would expect Mr. Parker's distance between the earth and sun will be found to be the truer of the two values.

Kepler's Third Law As Expressed By ∏

Kepler's Third law can be expressed by the relationships that we have developed in this chapter. The quantities that represent the Earth's time period $\mathbf{P}_E$ to orbit the Sun and the distance between Earth and Sun form an interesting pattern when substituted into Kepler's Third Law formula. The box contains the formula with Earth/Sun parameters. All of the numbers are those of the circle (20,612 and 5,153), time divisions of 1 solar day (5,184) and the relationship 4 to 3. Only the diameter of the Earth expressed in miles appears as an empirical number.

Notice that the units are those of days^2 per cubic mile (miles^3). The days ^{2} units may be thought of a constant rate of time change (derivative of time) or constant area of 'event space' per volume of space. The two drawings at the beginning of this chapter 'Light Sphere and Cube' and 'Slice of Space-Time' are geometric illustrations of the interrelated nature of space-time-matter.

The other planets in our solar system, when their time periods to orbit the Sun **P** and their semi major diameter from the Sun **a** are normalized to the Earth year and Earth distance form the sun again form a constant relationship to the Earth orbital parameters. This implies that the other

Summary

In this section we have shown how the Moon/Earth/Sun gravitating system has been very accurately described using the three numbers 20612, 5153 and 6561 and the simple relationship of 4 parts to 3 parts. These numbers and the relationships represented have formed an accurate and solid link between physical reality and geometric perfection.

The calculations performed have also confirmed the primary significance of those numbers as being true representatives of circumference, time, diameter and area. No straight line method of calculating Pi and expressing that value as an endless decimal series would have produced such a grand and valuable set of correlations.

The subject of light as a Unit of Time and Space measure was introduced but not fully explored. The interested reader may wish to refer to Walter Russll's *The Universal One Volume One* or *the Secret of Light* (also by Walter Russell). These books provide and indepth description of Light as being the central link between Spirit and Matter.

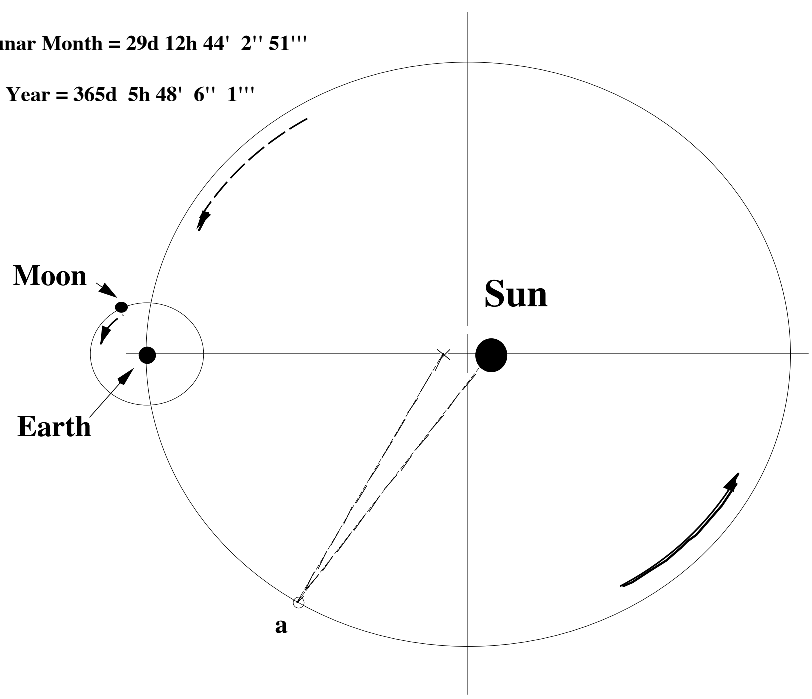
Sun, Earth and Moon Orbits

Not to Scale

1 Mean Sidereal Lunar Month = 27d 43' 3'' 47'''

1 Synodic Lunar Month = 29d 12h 44' 2'' 51'''

1 Solar Year = 365d 5h 48' 6'' 1'''



Moon Diameter = 2,167.10 miles

Earth Diameter = 7,913.14 (By Measuement)

Ideal Average Distance Center of Earth to Center of Sun = 92,298,865 miles

Sacred Geometry with Summary

The Quadrature of the Circle and Π

The Quadrature of the Circle problem has been solved using fundamental relationships of area and geometry between three fundamental shapes; these are the primary Circle, secondary Equilateral Triangle and the tertiary Square. Important concepts were introduced in the first five chapters. Chapter 6 brought all of the points together and produced the final ratio of ***20,612 parts of circumference divided by 6,561 parts of diameter*** as being the complete description of Pi.

The Circle is a Shape Contained in all Regular Polygons

The key to solving the Quadrature problem has been to discover that the circle is the most fundamental of all shapes. Any secondary perfect shape can be described in terms of points and lines drawn on or about the circle. Lines drawn from the center of the polygon to each vertex and the midpoint of each side are the radii of the circumscribed and inscribed circles and also form twice the number of right triangles as there are sides **n** forming the polygon.

Form Factor and Proportion

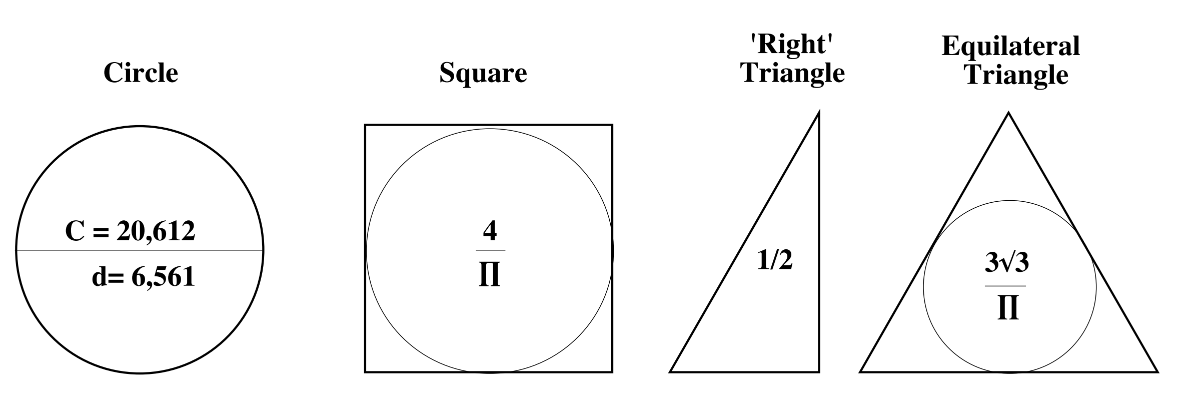
Each polygon has one number that relates it's form to that of the inscribed circle. From this one number the ratio of perimeter of the shape to the circumference of the inscribed circle or area of the shape to area of the inscribed circle can be determined.

The form factor between the circumference and diameter of a circle is the ratio Pi Π and best described as being 20,612 parts of circumference divided by 6,561parts of diameter. For the Equilateral Triangle this number is **(3√3)/Pi**. For the Square the number is **4/Pi**. For any regular polygon the number is **(tan 360°/2n) divided by Pi** where **n** is the number of sides of the shape.

When calculating the area of the Right Triangle and the Square we found that the perimeter or area of each shape was related by the form factor of $^1/_2$.

It is this fact, that both perimeter and area have only one numeric ratio to the inscribed circle, that makes our Four Step method for equating area between any circle and any polygon exact.

Four Basic Shapes



The Difference Between π And ∏

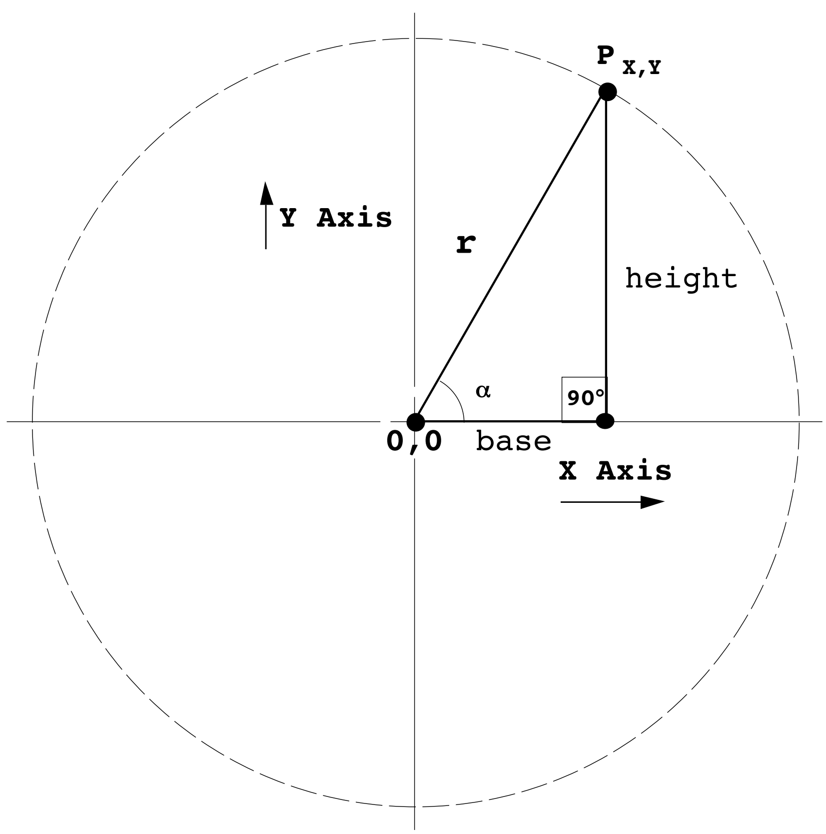
The small but fundamental difference between the straight line polygon method of calculation and the method by ratio of areas was fully explored in Chapter 7. The essential source of similarity and difference between π and true ∏ was found to be the Pythagorean relationship for a 'right' triangle which is also the formula for a circle when described by Cartesian X,Y coordinates.

The Pythagorean relationship is strictly the relationship between straight lines and points on those straight lines. It does not embrace the fundamental difference of a continuously curving line.

The drawing at the bottom of this page shows a circle constructed of Cartesian (axes at right angles) coordinate points. The line is dotted to show that it is made of a series of discrete points and straight line segments. These are located by performing the Pythagorean calculation to keep each point $\mathbf{P}_{X,Y}$ a fixed distance from the center of the circle. The calculated coordinate values for all points of interest are then plotted on the X,Y coordinate graph. the resultant shape is circular.

A Circle Described By The Pythagorean Function And A Point Specified By X,Y Coordinates

$$r^2 = H^2 = X1^2 + Y1^2$$



The dotted circumference line is not that of a continuous curved line. Every point on the curve is located independent of its neighbor; each calculation is based upon a desecrate choice of X or Y coordinate value and has no inherent continuity to it's neighbor by placement nor sequence of calculation.

In real circular systems the path or shape of an object is a continuous and smoothly flowing movement of evolution from its present state into its future or adjacent state in accordance with all of the local relationships that exist within it and act upon it. Thus when we tried to calculate Pi using polygon mathematics we were ignoring the inherent nature of the circle as being a primary *unit* of relationship. Calculating the relationships of diameter and area of a *whole* Circle to a *whole* Equilateral Triangle and to a *whole* Square preserved the inherent *one unit* relationship expressed by each of these figures. *These relationships can not be dissected to reveal any greater truth than that which it expressed by their entirety*. This statement is true for man, nature and the Universe. They will all be best understood when their gestalt wholeness is comprehended first and then, secondarily, that understanding can be applied to the individual forms of expression to predict what *must* be the nature of their interrelationships.

Gravitating System

The 'real life' 3 body gravitating system described in Chapter 8 demonstrated the exact correlation between circumference, time and diameter using the relationships embodied by the three numbers 20612, 5153 and 6561. The planetary orbits and spherical shapes are portions of true continuous curved lines and are not the product of straight line approximations. The final findings of Chapter 8 have fully confirmed the fundamental and true nature of Pi described as being 20,612 parts of circumference divided by 6,561 parts of radius.

In the 3 body gravitating system of Chapter 8 we saw that the number 20,612 as a *unit whole* stood for the representation of circumference and also as 1 period **P** of revolution. Similarly the number 6,561 as a *unit whole* stood for diameter and was used to extrapolate the diameter of the earth to the necessary radius of diameter of the earth orbit about the Sun.

The Elliptical Shape

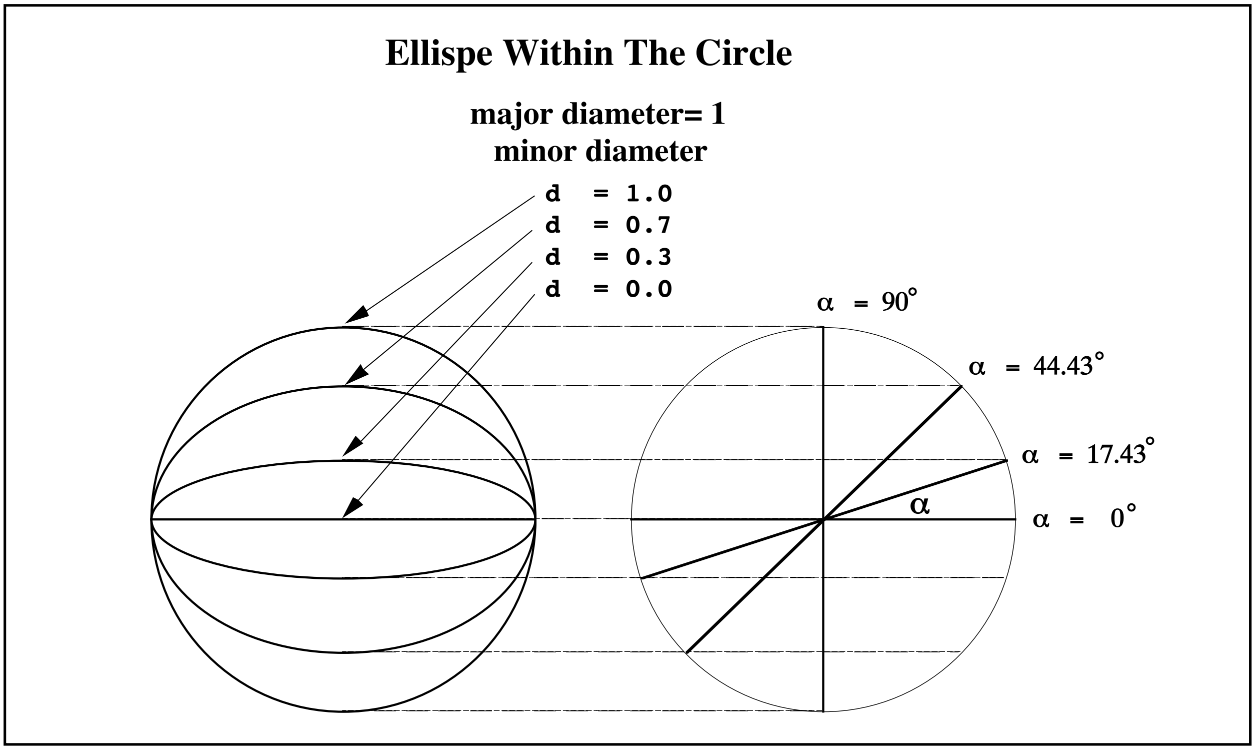
In astronomy the relationships discovered by Kepler have been mainstays for about 380 years. We saw that the Ellipse was the shape most associated with the actual manifestation of planetary orbits. We also applied the properties of the circle to these orbital shapes with out difficulty. I would like to show in a simple graphical way why we could do this.

In the box at the top of page 9.4 are drawn two circles. The left hand circle contains four ellipses all having a major diameter of 1 and a minor diameter ranging from 1 to 0. The right hand circle contains diameter lines at an angel ranging form 90° to 0°. The dotted lines direct the viewers eye showing that the shape of an ellipse can be thought of as a circle drawn at an angle to the viewer.
An ellipse with a major and minor diameter of 1 we recognize as being a circle having a diameter of 1. An ellipse having a minor diameter of 0.7 appears to be a circle drawn at an angle of 44.43°to the viewer. A minor diameter of 0.3 appears to be a circle drawn at an angle of 17.43°. An ellipse or circle drawn at an angle of 0° *appears to be a straight diameter line*. This series of relationships also shows that the diameter *line* of a circle is inherent to the circle. It may be considered a line *or* a point of view.

Ellipse to Circle Correspondence

$$\sin\ \alpha = \frac{\text{diameter}_{\text{minor}}}{\text{diameter}_{\text{major}}}$$

The ratio of the minor diameter of an ellipse divided by the the major diameter is exactly equal to the sine of the angle α at which a circle having a diameter equal to the major diameter is



drawn.

In astronomical calculations the radius line **a** is defined to be exactly equal to $^1/_2$ of the major diameter of the elliptical orbital path of a body. We also know that the diameter line is also equal to twice the length of the radius line **r**. The center of the circle divides the diameter line exactly into two equal lengths.

Kepler's three Laws describe 1) **a** as a measure of distance, 2) swept area as being constant over time and 3) an orbital path as being described by the ellipse. All are factors inherently contained within the circle and remain constants for the circle. The space-time ratios do not need to be corrected for the eccentricity ***e*** of elliptical orbits as was seen in our description of the circle and as was found to be true using Kepler's Third Law (chart page 8.11).

Only the area contained by the elliptical orbit need be converted to the area of a perfect circle for actual measurement of distance. This last factor accounts for most of the difference between the measured distance from the earth to sun and the distance calculated by diameter ratio using Mr. Parker's approach.

20,612

The number 20,612 - equal to 4 times 5,153 - has already been described as the primary representation of circumference, circular measure and orbital period. It is also very nearly the length in British inches of 1,000 Memphis cubits of Ancient Egyptian measurement. An extensive exploration of this topic is contained in the two books, *The Source of Measures* and and *The Secret of the Great Pyramid* referenced in the Bibliography.

6,561

The number 6,561 is the primary representative of diameter and linear measure. It is also equal to three to the eighth power (3^8) giving the numerical series of 1, 3, 9, 27, 81, 243, 729, 2187 and 6561.

5,153

The number 5,153 is equal to 1/4 of the circumference of the circle and the area of an inscribed circle having exactly 5,153 square parts of 6,561 square parts of the Square. 5,153 is also the base number of circular time expressed as 5,153,000''' (thirds) of a mean solar day.

4/3

The fundamental ratio in a 3 body gravitating system that relates 1 superior circle to 1 primary or contained circle.

Sympathetic Vibratory Physics (SVP)

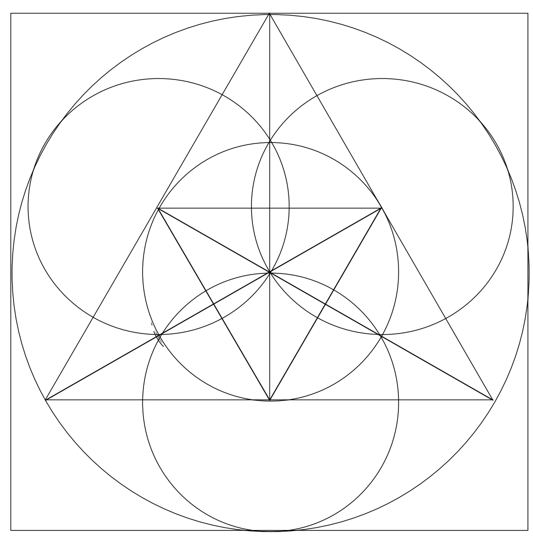
The ratio of 4 to 3 derived in Chapter 8 for three gravitating bodies is of utmost significance. Its applicability to the Moon, Earth, Sun system was shown. We have also seen how dynamic Space-Time-Light-Matter can be represented by fixed geometry figures. Is there another way to look at these relationships that inherently includes all of the above concepts? The answer is "Yes".

The one term that simultaneously captures all of the above concepts is *Vibration*. If we have vibration we must have periodicity, object, medium, sense of time, space and relationship which is locally significant and influential at a distance. A body of scientific thought developed by Mr. John Keely in the mid to late 1800's, described and promoted by Mr. Dale Pond of present days and based on vibration is called *Sympathetic Vibratory Physics* (SVP).

This view describes the Universe as built up of Triads of circular vibration. These Triads of Vibration contain within themselves the 4 to 3 ratio of outer sphere diameter to inner sphere diameter. The interval ratio of the seven musical notes, the 2^n octave and the 3^n cubic power are fundamental relationships between Triads. Levels of Triads form the building blocks of physical matter.

Through sympathetic outreach one group or level influences the actions of all other groups having identical or properly ratioed vibratory rates. It is this outreach and exchange between bodies that sets up the clock work balance that was so very well described by Mr. Parker's astronomical calculations. The same principles can be applied to atomic bodies and living bodies.

The 4/3 Ratio andTriad of Vibrating Spheres



Much more could be said about this topic however the scope of this paper was not intended to cover the extensive topic of SVP. Interested readers should write to Delta Spectrum Research and Mr. Dale Pond for a catalog of available material.

Other Numeric Ratios Associated With Pi

The ratio 355 divided by 113 has been used to approximate Pi. These two numbers are also referenced in the *Source of Measures* and *the Secret Doctrine*. Numerically, upon division, that ratio produces a number which is 1.000,000,085 times larger than the value of π determined using straight line mathematics.

Circular Ratios

$$\frac{355 + \frac{1}{6{,}561}}{113} = \frac{20{,}612}{6{,}561} = \frac{355}{112 + \frac{20{,}611}{20{,}612}}$$

If we take the number 355 and add 1/ 6,561 then divide that quantity by 113 the quotient is found to be exactly equal the ratio of 20,612 divided by 6,561. Similarly if we divide the number 355 by the quantity 112 plus quantity 20,611 divided by 20,612 we again obtain the exact equivalency to the numeric ratio 20,612 divide by 6,561 (see box below). Notice also that 20,611 is 1 unit less than 20,612 and that the quantity 1/6,561 in the upper numerator divided by 1/20,612 in the lower denominator is equal to 20,612/6,561.

The esoteric bases of this identity is described more fully in *The Secret Doctrine* Vol II PP. 38-40 and throughout the book, *The Source of Measures*. The fundamental concept demonstrated by these relationships is that the One Original First Cause is always contained within any manifestation and 1 or One Original First Cause may always be added to make the lessor whole (holy). Such principles may be and must be within the atom, mineral, plant, animal, man, solar system or any of the Universes past, present or future.

The Equilateral Triangle and the Octave

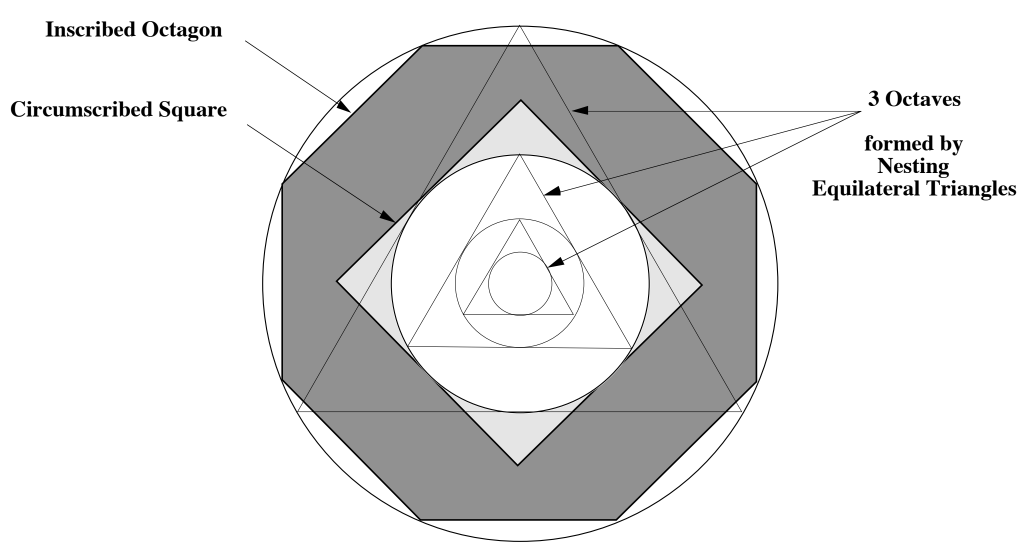
The Equilateral Triangle displays a very interesting relationship between the radii of its interior and exterior circles. The radii of the inscribed and circumscribed circles have a ratio of exactly 1 to 2. Each of the three vertex points of the circumscribed Equilateral Triangle are located on the circumference of the circumscribed circle and each of the three sides of the Equilateral Triangle are equally divided by, and touch, the inner circle (see illustration this page).

If a series of Equilateral Triangles are drawn such that the circumscribed circle of one becomes the inscribed circle of the next then a chain of shapes having a 1 to 2 ratio (octaves) are generated. Each Triangle (and circle) will have exactly twice the diameter, twice the perimeter and four time the area of the preceding triangle. In metaphysical understanding the circle is often associated with the original source and when the original source becomes manifested a trinity is revealed.

The octave is a measurement relationship that is of great importance. Vibration modes in one octave are often repeated in other octaves or are found to form sympathetic relationships to vibrations having exactly a 2^n vibration rate difference. The trinity of the Equilateral Triangle spans the full range of one octave. No other geometric figure has this breadth of range.

Using the Equilateral Triangle as the reference shape and the circumscribed and inscribed circles as reference circles then any other shapes will have either their vertices on the circumscribed circle of the Equilateral Triangle or their sides touching the inscribed circle of the Equilateral Triangle.

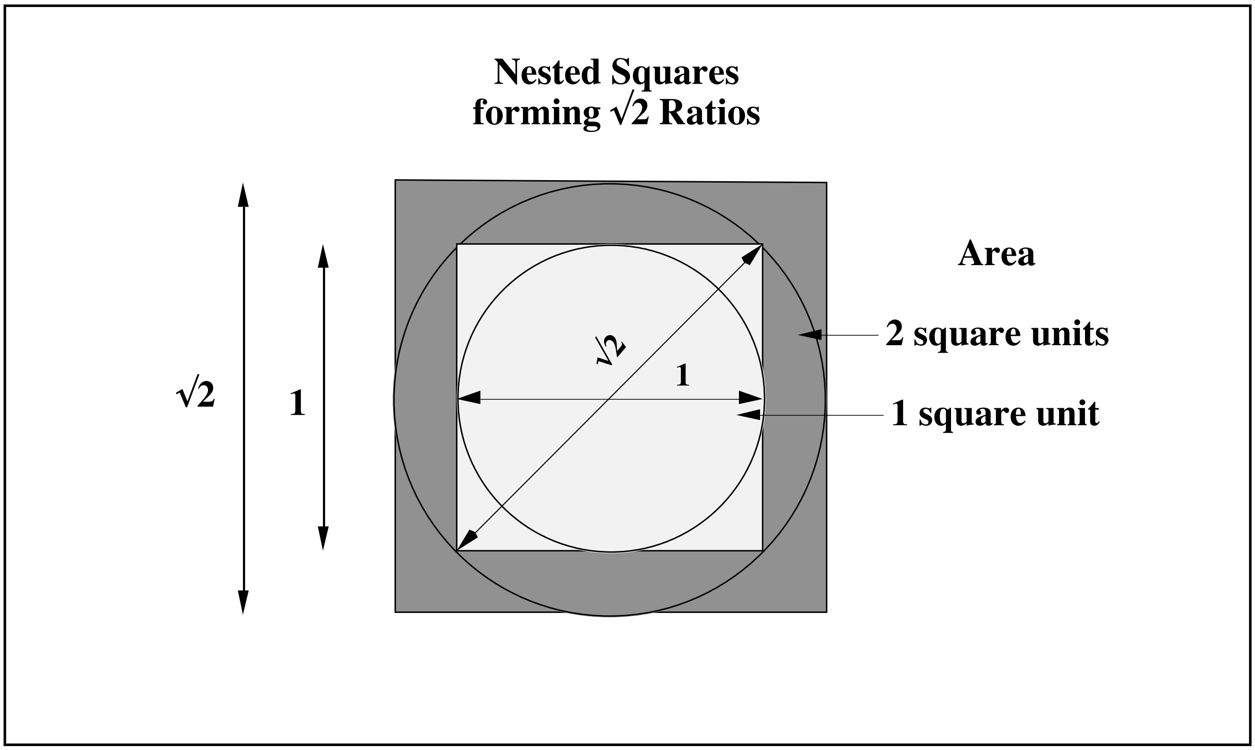
Octagon and Square Nested within One Octave of The Trinity of the Equilateral Triangle



As more sides are added to a polygon it becomes ever more circular but no matter how many sides are added neither approaching shape can become the circle. A new circle is required to initiate a new octave thereby allowing a new trinity to establish the boundaries for additional sizes of perfect shapes.

The Square and the √2

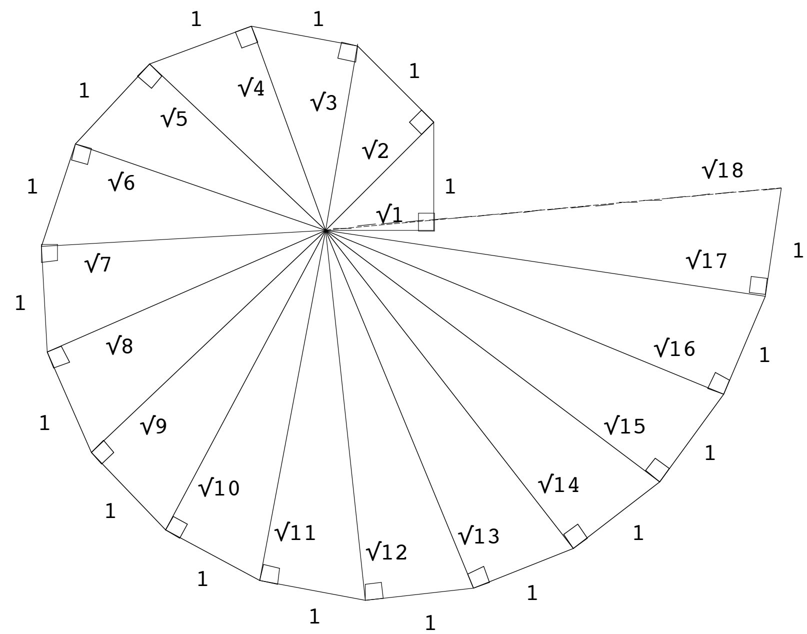
If a series of Squares and Circles are drawn such that the circumscribed circle of one becomes



the inscribed circle of the next - the length of sides, diameters and perimeters of the squares (and circles) will increase in a ratio of √2. The areas will double each time as $(\sqrt{2})^2 = 2$. The drawing below shows the relationships.

The study of many of the shapes contained within this paper can be illuminating to the student. Shapes that cause an increased intuitive understanding of the object or of a larger field of relationships are called mandalas.

Generative Spiral of Roots



The Generative Spiral & The Golden Mean

So far we have talked of shapes that are complete when once drawn. There is another group of shapes that have a natural pattern which, once begun, grow as the pattern is repeated. Such a shape and pattern is shown at the top of the next page. The spiral form is an application of the Pythagorean 'right' triangle relationship whereby 1 unit of length is added at a 'right' angle to the hypotenuse of the previous triangle.

The shape began with two sides of a Square having sides of a length of 1 and at right angles to each other. Think of the side that is the base as also having a length of √1. The hypotenuse of the triangle formed by the sides of two lines perpendicular to each other is the Pythagorean relationship.

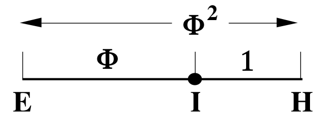
We remember that the length of the hypotenuse will be equal to the Square root (√) of the sum of the lengths squared (Pythagorean Theorem) and that the square of a Square root (√) is simply the original number under the Square root sign.

To begin our spiral we find the hypotenuse: H = $\sqrt{(\sqrt{1}^2 + 1^2)}$ which equal $\sqrt{2}$. If we now use the $\sqrt{2}$ as one side of a new triangle and draw a line perpendicular to that side having a length of 1 the hypotenuse of the new triangle will equal Square Root of $(\sqrt{2})^2$ plus 1^2 equaling $\sqrt{(2+1)}$ which becomes $\sqrt{3}$. Each successive hypotenuse of the new triangle will have the value of Square root ($\sqrt{}$ of **n** plus 1 where **n** was the number previously under the Square root ($\sqrt{}$) sign.

We may repeat the process as many times as desired to generate the square root of any number. Thus the generation of square root lengths by geometric means is quite easily accomplished. The 'mysterious' transcendental numbers (numbers that have an unending string of decimal digits) are graphically displayed and used to consecutively build the next triangle.

A beautiful spiral is formed by the process. Such a shape can be seen in the shells of certain sea creatures particularly the chambered shell of the Nautilus. The spiral also shows how nature builds upon her self to create the next phase of being.

Golden Mean Phi (ϕ)



$$\frac{EI}{IH} = \frac{EH}{EI}$$

$$\frac{\phi}{1} = \frac{\phi + 1}{\phi}$$

$$\phi^2 = \phi + 1$$

$$\phi = \frac{1 \pm \sqrt{5}}{2} = 1.618{,}033+$$

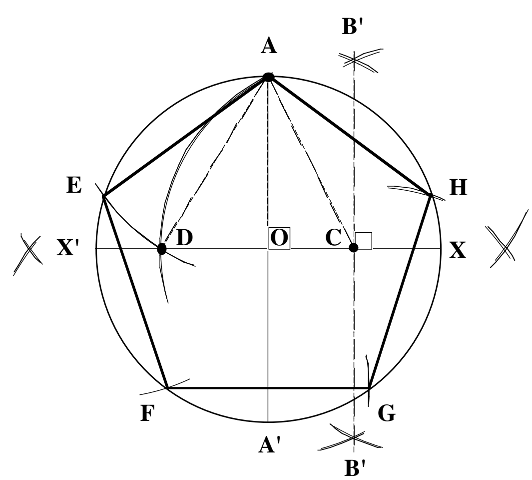
Fibonacci Series

Growth is accomplished by building on what has gone before. In our geometric triangle drawing growth was accomplished by adding 1 unit of length at 'right' angles to 1 preceding unit of length.
In mathematics we find growth of a numerical series as the progressive sum of numbers beginning with 1+1=2, 1+2=3, 2+3=5, 3+5=8 etcetera. The next number in the series is generated by adding the two previous numbers together. The mathematical series has been called the Fibonacci Series in honor of Leonardo of Pisa (1170AD-1250 AD).

The Golden Mean

If we take two adjacent numbers in the additive mathematical (Fibonacci) series and divide the larger by the smaller and interesting pattern develops. For example the numbers 8 divided by 5 give a ratio of 1.6. The next number in the series is 5+8 =13. 13 divided by 8 gives the number 1.625 If we continue the series we will find that the ratio of any two numbers in the series will oscillate about a single number called the Golden Mean.

Constructed Pentagon



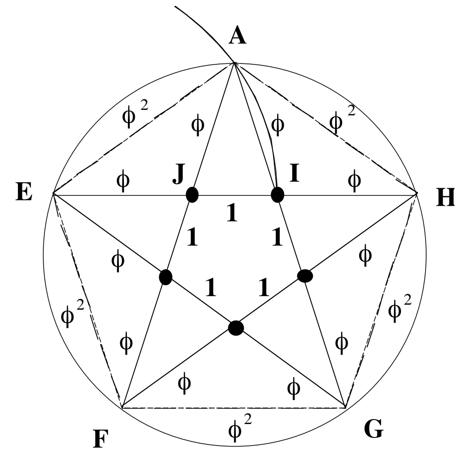
Solving For The Golden Mean Phi (Φ)

The Golden Mean ratio is found by drawing a line (EH) and locating a point 'I' on that line such that the line segments EH divided by EI equals line segment EI divided by IH.

When we perform the calculations we find that the Golden Mean has an exact value of 1 plus the $\sqrt{5}$ quantity divided by 2 [$(1+\sqrt{5})/2$]. The first seven digits of the Golden Mean ratio expressed in decimal form are 1.618,033+.

Notice in the derivation box at the right that $\phi^2 = \phi + 1$. Phi (ϕ) has the additional property that $\phi - 1 = 1/\phi$.

Pentagram



Pentagram Line Lengths

$$\frac{EJ}{IJ} = \frac{EI}{EJ} = \frac{EH}{EI} = \frac{EH}{EA} = \phi$$

The Golden Mean And The Root Spiral

Mathematically the length of each side, the angle of the hypotenuse line and the area of each triangular section of the generative root spiral can be evaluated using the relationships of the 'right' triangle and the slope of a line (**m**) formulas. In addition, as the number of 'right' angle segments becomes large a smooth spiral curve will result.

If we place a straight edge so that it cross the center of the spiral (drawing page 9.8) and measure the distance from the center point to each of the opposite edges then the ratio between the two measured lengths will approach that of the Golden Mean. In our drawing the two lines $\sqrt{7}$ and $\sqrt{18}$ lie nearly in line. Dividing $\sqrt{18}$ by $\sqrt{7}$ produces the quotient number 1.603,567. If we measure the drawing we find the length of the $\sqrt{7}$ line to be about 1.43 inches and the length of the $\sqrt{18}$ line to be about 2.30 inches. Dividing we find the ratio to be 1.61. Using geometric construction we have created the generative spiral and introduced the Golden Mean of progressive growth.

The Pentagon And Phi

When we constructed the Pentagon in Chapter 3 page 3.10 (drawing reproduced at right) we created the length AC by the bisection of line OX. Notice that line AC is the hypotenuse of a right triangle having a base of 1/2 radius line OX and the height of 1 radius AO.

Using the Pythagorean Theorem we find that the Length of line AC is equal to $\sqrt{5}$ divided by 2. Length of line DO becomes equal to Length AC minus 1/2. Substituting we find that length DO equals $\sqrt{5}$ divide by 2 minus 1/2. This quantity is equal to ϕ minus 1. Line length DX is equal to DO plus 1 making DX equal to ϕ.

Line AD is the hypotenuse of a 'Right' triangle with a base length of $1/\phi$ (DO) and height 1. The side AE has this same measure as line AD so line AE - the side of the pentagon - is built upon a triangle having sides of $1/\phi$ and 1.

The Pentagram and Phi

Examples of Phi are also found in the geometric figure of the Pentagram. The Pentagram is a five legged 'star' formed by joining the vertices of the Pentagon with straight lines.

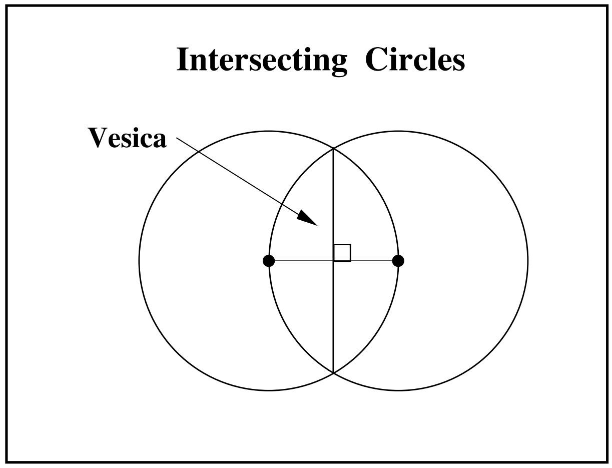
The drawing to the right shows the Pentagon as a dotted figure and the lines forming the Pentagram as solid. When the vertices of the Pentagon are connected with straight lines a smaller inverted Pentagon is formed at the center of the figure.

If we let the length of the sides of the inner pentagon equal 1 then each line extending from the

Pi and Phi Honored Ratios

The ratio of Pi (∏) and the Golden Mean Phi (f) are two ratios of primary significance. These ratios were greatly honored in Ancient days and were used in the design or layout of many of buildings and works of art. Many examples of Phi are described in books devoted to the subject. Examples of Phi in nature and art are included in several of the books listed in the Bibliography at the end of this publication.

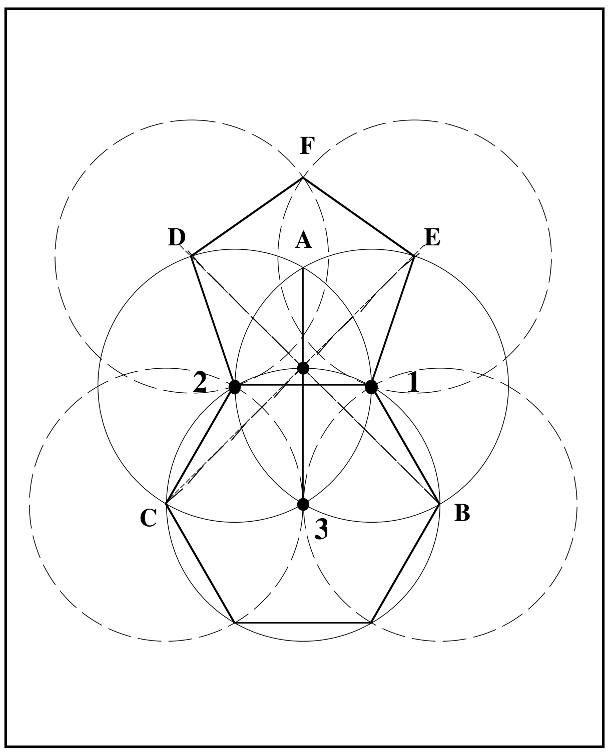
Interlaced Circles (Vescia)



To the right is a drawing showing the intersection of circles with their circumference lines coincident with the center of the other circle. Notice that a vertical perpendicular line having a length of $\sqrt{3}$ **r** and a horizontal line having a length of 1 **r** are formed between points of intersection. The crossed lines also bisect one another.

The next drawing shows the direct generation of the Vescia, Pentagon and Hexagon via the intersection of Seven Circles; three primary and four secondary. Only one initial setting of the drawing compass span is required in this construction; all sides have a length of 1 radius (this drawing is to be found in the book City Of Revelations by John Michell p. 78).

Begin construction by drawing primary circles 1 and 2 having centers lying on the circumference line of one another. The center of primary circle 3 is located at the point of intersection of circumference lines 1 and 2.



Solid line A3 and the circumference line of Circle 3 locates the intersection point needed to draw lines BD and CE. Dashed line BD begins at point B, passes through the intersection point of A3 and circumference of circle 3 and is extended so that the line crosses the circumference line of circle 2. Point D is located at this intersection. Dashed line CE is constructed in the same manner as BD; beginning at point C passing through the common intersection to locate point E on circle 1.

Four dashed line secondary circles are drawn with their centers at points D, E, C and B. The intersections of these circles locate the additional points needed to draw the sides of the Pentagon (point F) and lower half of the Hexagon inscribed to circle 3.

The sides of the polygons are drawn by connecting the several intersection points in sequence. The length of all sides will have a measure of 1 radius.

The Lost Canon (Laws of Proportion) Of Ancient Egypt

In a document called the Lost Canon of Ancient Egypt was said to lay the keys or laws of all of the Sacred Proportions needed for complete harmonious creation. Creations made using these proportions and the complete understanding of Vibration were said allow men/women to be able to cleave rock, levitate monoliths, build pyramids and endow their creations with spiritual energy.

Such claims are hard to justify from the point of present day knowledge but many examples of extraordinary engineering feats in ancient days are available. Certain walls in the cities of Cuzco and Machu Picchu in Peru and the Great Pyramid on the plane of Giza (Al-Jizah) in Egypt involve shaping and placement of hugh stones with a precision that could not be matched today.

The work of Mr. John Keely, an inventor living in Philadelphia in the later 1800's, demonstrated disintegration of water, rock and levitation of metallic and non-metallic bodies. Several demonstrations were given before observers from the United States Navy.

The Theosophical Society was founded in New York City in November of 1875 after an informative dissertation on the 'Lost Canon of Proportions' by Mr. George Felt in the apartments of H.P. Blavatsky on September 7th of that year. The contributions of H.P. Blavatsky, the sources close to her and the Theosophical Society offer a view of the Universe that is magnificent, inspiring and provides information pertinent to all that we have discussed in this paper.

It is for these several reasons that I believe that we can find and implement a improved system of creative science giving life enhancing concepts based upon alignment with the 'Holistic' Original First Cause, Sympathetic Vibratory Physics and Proportion.

Closing

We have explored several topics surrounding the common value of π and the larger value of $\prod$. The larger value was derived from the relationship of area and measure of fundamental geometric shapes. The numbers forming $\prod$ have been shown in the 3 Gravitating Body section (Chapter 8) to have exact and meaningful relationships to both time as expressed by planetary revolution and space as measured by diameter of the planet earth. The concept of time-space contained by form.

The differences between the two values π and $\prod$ are small numerically but mathematically, astronomically and metaphysically very significant. The differences from the stand point of confirmation by nature and usefulness stand dramatically in the favor of $\prod$ (I hope that additional measurements and feedback from those reading this paper will produce additional validation confirming of the ratio 20,612 divided by 6,561 and showing its usefulness in a variety of relationships).

In this chapter we have summarized Mr. Parker's work and expanded upon the realm of numerical, geometrical and Spiritual possibilities. This extended view of numbers and geometry is the essential premise of my work and that of many others. The resulting synthesis of physics and Spiritual metaphysics, without the dogma nor closed mindedness of either, I define as being 'High Science'. This viewpoint I believe to be growing in the minds of many. Mr. Parker himself often referred to the creation and management of the Heavenly Bodies as being works of The Divine Hand of the Creator. Hopefully my additional thoughts expressed in this paper are in

High Precision Circumference Measurement System

I would like to describe a high precision method for obtaining an experimental value for ∏. This design is an extension of the experiment that we set up using the plastic 'CD' disk and our desk top.

This approach uses a very high precision linear translation stage to roll the disk. A laser interferometer to measure the linear distance traveled by the linear translation stage. A magnifying eye piece with cross hairs to detect one complete revolution of the disk. And a very accurately made disk of some material probably metal. Such a disk might be the platter of a large diameter hard disk.

Refer to the drawing page A 1.2.

The disk has a fine line scribed on its 'rolling' edge. A special set of bearing are placed upon the stationary sides of the linear translation table which support the axle of the disk and produce a slight but constant downward pressure on the axle such that the disk will not slip nor be 'pinched' as the translation stage moves beneath it.

Using a constant and slow rate of movement the disk is caused to roll beneath the eye piece focused upon the rim of the disk. When the scribed mark is aligned with the cross hairs on the disk the display readout on the controller is reset to zero. The translation stage must have enough travel to allow one complete revolution of the disk after the position counter has been reset.

After one complete revolution of the disk the scribe mark will again be visible under the eye piece. When the mark is in exactly the same position as when the display was reset this will be the precise circumference of the disk. Using a laser interferometer a resolution of better than 0.000,004 inch can be obtained.

Carefully remove the disk from the axle and lay it on the linear stage. The process of measurement is repeated using the edge of the disk passing under the eye piece as the reference marks. The position of the disk must be adjusted so that the center of the disk passed directly under the eye piece there by assuring that the diameter has been measured.

Divide the measured circumference by the measured diameter. The result should give an excellent approximation of ∏.

Pi Calculations

This Appendix details the spread sheet program developed to calculate the X,Y loci of the inscribed polygon, circumscribed polygon and circles. Results from this spreadsheet were used to compare the two values of Pi in Chapter 7. The thrust of the program is to calculate various parameters of measure to better than 10 digits of accuracy and not use trigonometric 'look up' tables.

The task was admirably accomplished using the formulae for the circle, the line and the process of bisection. A spreadsheet program called 'Wingz' performed very well on an Apple Macintosh Plus computer. This program has 15 digit internal precision.

All quantities are calculated to 15 digits of precision. The resultant calculation may be displayed with any number of digits desired (up to the obvious limit of 15). The accuracy of the number calculated will be dependent upon the accuracy of the original numbers and the effects of truncation and 'round off' errors in the chain of calculations used to reach the displayed result. I sought to minimize the number of calculations in any chain and also included check points at places in the program where the ratio or total of certain intermediate quantities must be some known number. I am very sure that the goal of 10 digits of accuracy in the combined chain and individual calculations was entirely met.

The Cells Of A Spreadsheet Program

This spreadsheet program (as does Lotus 1-2-3 and VP Planner) uses the concept of 'cells' . In a cell a quantity or formula maybe stored. We build our program out of cells having the correct quantities or formulae within them. These cells may be copied, linked or used as sources for quantities used in later calculations. By constructing the pattern of cells correctly an initial set of calculations may be used as the source for a second set of cells that systematically modifies the first set of calculations into the next set desired. If the second set of cells are copied and linked to their neighbors all of the quantities desired can be quickly calculated with little effort.

The Program

To begin the process vertical column B is used to label the quantities calculated. Column C develops the first set of calculations for the Equilateral Triangle. Column D develops the first set of formulae for the Square. Alternate columns beginning with column E are binary divisions of the Equilateral Triangle. Beginning with column F and every second column are binary divisions of the Square. Columns W and X are columns devoted to polygons having 2,529 and 2,530 sides respectively.

The three pages immediately following page A2.3 display the actual formulae in columns B through I. The remaining 10 pages (columns A through AQ) show the results of the calculations for 40 polygons. The least number of sides calculated are 3 and the greatest number 1,048,576.

The following text describes the contents of the first five columns and some of the thinking that produced the qualities within the individual cells.

Column B

Cell B2 contains the value 360°.

Cell B3 contains the number of sides **n** that the polygon will have. This number is a variable and will begin with the number 3.

In cell B4 we label the quantity 360° divided by **n.** This will be the contents of each cell 4. This calculation gives the angle between the vertices of the polygon.

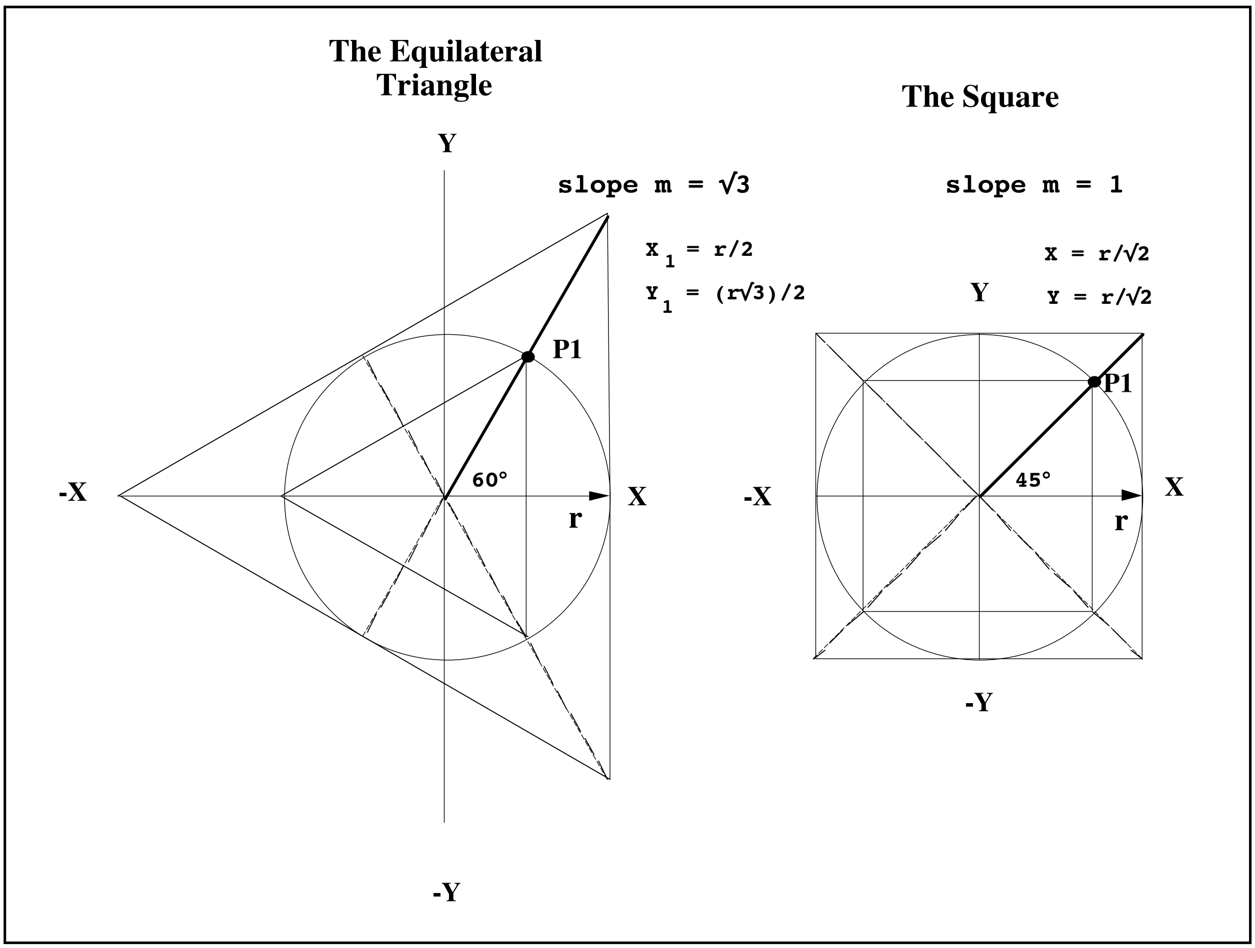
In cell B5 we place the radius **r** of the inscribed circle. This number was selected to be 1/2 of 6561 or 3280.5 units. This number is also the base length of the 'right' triangles used to calculate the height, side length and area of the polygon. From the study of the Equilateral and 60° 'right' triangle we know that the *ratio* of the height divided by the base is equal to the $\sqrt{3}$. The $\sqrt{3}$ is the slope **m** of the line bisecting one 120° angle forming a corner of the Equilateral Triangle. It is also equal to the tangent of the interior angle α where α equals 360°/2**n** and **n** is the number of sides of the circumscribed polygon. The numerical value for the $\sqrt{3}$ is calculated by the program and placed in cell B6. The drawing below contains this information.

In cell B7 we label the future content of all row sevens as containing the value of the X coordinate of the point on the circle intersected by the line with slope **m** contained in cell 6.

In cell B8 we label the future content of all row eights as containing the value of the Y coordinate of the point on the circle intersected by the line with slope **m** contained in cell 6.

In cell B9 we label the future content of all row nines as containing the value of X coordinate of the first polygon vertex above the X axis. This number will always be the same as the radius of the circle and the content of cell B5.

In cell B10 we label the future content of all row tens as containing the value of Y coordinate of



the ffirst polygon vertex above the X axis. This number will always equal the product of the radius of the circle and the slope **m** of the bisecting line. The content of cell B5 multiplied by the content of cell 6 in each column will be the content of cell 10.

Cell B11 is a label indicating the content of all row elevens as being a check sum of cell 7 squared plus cell 8 squared divided by cell 5 squared. The result should always equal exactly 1 because of the Pythagorean Identity for the 'right' triangle.

Cell B12 labels the content of all row twelves as being the Cos of (360°/2**n**) found by dividing the content of cell 7 by content of cell B5. Cos α = X/r.

Cell B13 labels the content of all row thirteens as being the Sin of (360°/2**n**) found by dividing the content of cell 8 by content of cell B5. Sin α = Y/r.

Cell B14 labels the content of all row fourteens as being the Tan of (360°/2**n**) found by dividing the content of cell 8 by content of cell 7. Cos α = Y/X.

Cell B16 labels the content of all row sixteens as being the length $\mathbf{s}_I$ of one side of the inscribed polygon found by multiplying the content of cell 8 by 2.

Cell B17 labels the content of all row seventeens as being twice the length of the cord from the point on the circle to point Prx on the X axis. The quantity is equal to 2 times quantity $\sqrt{[(\text{cell }7)^2 + (\text{cell }8)^2]}$.

Cell B18 labels the content of all row eighteens as being the length of the arc contained between two adjacent vertices of the polygon based on π. The number is found by multiplying π times 360° and dividing by the number of sides of the polygon - this being the content of cell 3.

Cell B19 labels the content of all row nineteens as being the length of the arc contained between two adjacent vertices of the polygon based on $\prod$. The number is found by multiplying $\prod$ times 360° and dividing by the number of sides of the polygon - this being the content of cell 3.

Cell B20 labels the content of all row twenties as being the length $\mathbf{s}_c$ of one side of the circumscribed polygon found by multiplying the twice the radius **r** times the slope **m** of the line. This being 2 times the content of cell B5 times content of cell 6.

Cells 22 through 25 are labeled as being the the perimeter or circumference of the cell quantities calculated in cells 17,19-20. **n** was multiplied by the content of each of those cells in-turn giving the content of each of the cells 22 through 25 as labeled.

Cell B27 labels the content of all row twenty-sevens as being the area of the inscribed polygon. This quantity is equal to the product of the X and Y coordinates of the point on the circle - contents of cell 7 times cell 8 - multiplied by the number of sides **n**.

Cell B28 labels the content of all row twenty-eights as being the area of the circle based on π. This quantity is always the product of π times the radius of the inscribed circle squared ($\mathbf{r}^2$). The formula is written as π times 1/2 the diameter squared $[(d/2)]^2$.

Cell B29 labels the content of all row twenty-nines as being the area of the circle based on $\prod$. This number is equal 1/2 the circumference times 1/2 the diameter. For the radius chosen the area becomes the product of 5,153 times 6,561.

Column C

Column C implements the first calculations for the Equilateral Triangle where the slope is known to be $\sqrt{3}$. The X coordinate for the point on the circle is one half the radius value, because as we have previously calculated, the base length is equal to 1/2 of the hypotenuse length and the hypotenuse is equal to the radius of our inscribed circle.

Column D

Column D implements the first calculations for the Square. The slope is 1 as the base and height each have the same length. The first X coordinate point on the circle is equal to the radius of the inscribed circle divided by the $\sqrt{2}$. We solve for the Y coordinate value by substitution of the X coordinate value into the formula for the circle.

Column E

Column E takes results from column C, bisects the angle using the coordinate points on the circle using the formulae derived in Chapter 7 and is formulated to allow the column to be copied to every other column.

Column F Through Column AP

Column F takes results from column E, bisects the angle using the coordinate points on the circle using the formulae derived in Chapter 7 and is formulated to allow the column to be copied to every other column.

Coordinate Points On The Circle Of A Bisecting Line,
Given The Coordinate Points Of The Initial Line Crossing The Circle

$$\text{New tangent} = \text{New slope} = m = \left(\frac{\frac{Y_1}{2}}{r+\frac{X_1-r}{2}}\right)$$

$$X P_{\frac{1}{2}} = r\sqrt{\frac{1}{1+m^2}} \qquad Y P_{\frac{1}{2}} = r\, m\sqrt{\frac{1}{1+m^2}}$$

Column W & X

Column W & X are for the two values of **n:** 2,529 and 2,530 respectively. The slope **m** is determined by taking the trigonometric tangent of the angle in radians of the quotient of 360° divided by 2**n**: tan α = radian (360°/2**n**).This calculation series was performed to find the exact point where the area of the circumscribed polygon equaled the area of the circle as calculated using Π. By this time I had found no difference between the trigonometric functions as calculated by the slope of a line crossing a circle and the functions based on the Taylor's Series and using an angle specified in radians.

	A	B	C	D
1				
2		360		
3		3	3	4
4		Angle 360°/n	120	90
5		3280.5		
6		slope m	1.7320508076	1
7		X coordinate on circle	1640.25	2319.6637957
8		Y Coordinate on circle	2840.9963371	2319.6637957
9		X coordinate at vertex above X axis	3280.5	3280.5
10		Y Coordinate at vertex above X axis	5681.9926742	3280.5
11		Check X^2 +Y^2 = 3280.5 ^2	1	1
12		Cos (360°/2n) = X/r	0.5	0.70710678119
13		Sin (360°/2n) = Y/r	0.86602540378	0.70710678119
14		Tan (360°/2n) = Y/X	1.7320508076	1
15				
16		Inscribed side = 2 Y	5681.9926742	4639.3275914
17		2*(Cord Px,y to Po,o)	6561	5021.5719995
18		Segment of Circle = 6561*π * a/360°	6870.6631334	5152.9973501
19		Segment of Circle = 20,612 * a/360°	6870.6666667	5153
20		Circumscribed side = 2 r (slope)	11363.985348	6561
21				
22		Inscribed perimeter = n Si	17045.978023	18557.310365
23		Circumference = 6561*π*n*a/360°	20611.9894	20611.9894
24		Circumference = 20,612*n*a/360°	20612	20612
25		Circumscribed Perimeter = n Sc	34091.956045	26244
26				
27		Inscribed area = n*X*Y	13979832.73	21523360.5
28		Area (Common π) = π*((6561/2)^2)	33808815.61	33808815.61
29		Area (∏) =5,153*6561	33808833	33808833
30		Circumscribed Area = n*slope*(6561/2)^2	55919330.9	43046721

	E	F	G	H	I
1					
2					
3	6	8	12	16	24
4	60	45	30	22.5	15
5					
6	0.57735026919	0.41421356237	0.26794919243	0.19891236738	0.13165249759
7	2840.9963371	3030.7868064	3168.7196731	3217.4661124	3252.4348677
8	1640.25	1255.3929999	849.05587746	639.99380137	428.19117358
9	3280.5	3280.5	3280.5	3280.5	3280.5
10	1893.9975581	1358.8275914	879.00732577	652.53202119	431.88601834
11	1	1	1	1	1
12	0.86602540378	0.92387953251	0.96592582629	0.9807852804	0.99144486137
13	0.5	0.38268343237	0.2588190451	0.19509032202	0.13052619222
14	0.57735026919	0.41421356237	0.26794919243	0.19891236738	0.13165249759
15					
16	3280.5	2510.7859997	1698.1117549	1279.9876027	856.38234716
17	3396.2235098	2559.9752055	1712.7646943	1286.1809154	858.21986176
18	3435.3315667	2576.498675	1717.6657834	1288.2493375	858.83289168
19	3435.3333333	2576.5	1717.6666667	1288.25	858.83333333
20	3787.9951162	2717.6551827	1758.0146515	1305.0640424	863.77203667
21					
22	19683	20086.287998	20377.341059	20479.801644	20553.176332
23	20611.9894	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612	20612
25	22727.970697	21741.241462	21096.175818	20881.024678	20730.52888
26					
27	27959665.45	30438628.33	32285040.75	32946533.89	33423933.67
28	33808815.61	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833	33808833
30	37279553.94	35661071.31	34603002.39	34250100.73	34003250

	J	K	L	M	N
1					
2					
3	32	48	64	96	128
4	11.25	7.5	5.625	3.75	2.8125
5					
6	0.09849140336	0.06554346282	0.04912684977	0.03273661041	0.02454862211
7	3264.7034958	3273.4761977	3276.5484941	3278.7435792	3279.5119747
8	321.54522885	214.55496544	160.96650563	107.3349512	80.507500169
9	3280.5	3280.5	3280.5	3280.5	3280.5
10	323.10104871	215.01532977	161.16063067	107.39245046	80.531754828
11	1	1	1	1	1
12	0.99518472667	0.99785892324	0.9987954562	0.99946458748	0.9996988187
13	0.09801714033	0.06540312923	0.04906767433	0.03271908282	0.02454122852
14	0.09849140336	0.06554346282	0.04912684977	0.03273661041	0.02454862211
15					
16	643.0904577	429.10993088	321.93301126	214.66990239	161.01500034
17	643.86602252	429.33980479	322.03000068	214.6986424	161.02712539
18	644.12466876	429.41644584	322.06233438	214.70822292	161.03116719
19	644.125	429.41666667	322.0625	214.70833333	161.03125
20	646.20209743	430.03065953	322.32126134	214.78490092	161.06350966
21					
22	20578.894646	20597.276682	20603.712721	20608.31063	20609.920043
23	20611.9894	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612	20612
25	20678.467118	20641.471657	20628.560726	20619.350488	20616.129236
26					
27	33591994.65	33712347.48	33754531.94	33784683.08	33795239.79
28	33808815.61	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833	33808833
30	33917855.69	33857173.89	33835996.73	33820889.64	33815605.98

	O	P	Q	R	S
1					
2					
3	192	256	384	512	768
4	1.875	1.40625	0.9375	0.703125	0.46875
5					
6	0.01636392214	0.01227246238	0.0081814134	0.00613600016	0.00409063825
7	3280.0608654	3280.2529844	3280.3902145	3280.4382455	3280.4725535
8	53.674660601	40.256781346	26.838228471	20.128769592	13.419226508
9	3280.5	3280.5	3280.5	3280.5	3280.5
10	53.681846565	40.259812836	26.839126671	20.129148517	13.419338782
11	1	1	1	1	1
12	0.99986613791	0.99992470184	0.99996653392	0.99998117528	0.99999163344
13	0.01636173163	0.01227153829	0.0081811396	0.00613588465	0.00409060403
14	0.01636392214	0.01227246238	0.0081814134	0.00613600016	0.00409063825
15					
16	107.3493212	80.513562693	53.676456941	40.257539183	26.838453016
17	107.35291388	80.515078366	53.676906032	40.257728644	26.838509153
18	107.35411146	80.515583595	53.67705573	40.257791797	26.838527865
19	107.35416667	80.515625	53.677083333	40.2578125	26.838541667
20	107.36369313	80.519625672	53.678253342	40.258297034	26.838677563
21					
22	20611.069671	20611.472049	20611.759466	20611.860062	20611.931916
23	20611.9894	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612	20612
25	20613.829081	20613.024172	20612.449283	20612.248081	20612.104369
26					
27	33802781.51	33805421.35	33807307.03	33807967.03	33808438.46
28	33808815.61	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833	33808833
30	33811833.15	33810512.9	33809569.94	33809239.92	33809004.19

	T	U	V	W	X
1					
2					
3	1024	1536	2048	2529	2530
4	0.3515625	0.234375	0.17578125	0.14234875445	0.14229249012
5					
6	0.0030679712	0.00204531057	0.00153398199	0.00124222786	0.00124173686
7	3280.4845613	3280.4931384	3280.4961403	3280.4974689	3280.4974709
8	10.064432161	6.7096272882	5.0322220011	4.0751253656	4.0735146449
9	3280.5	3280.5	3280.5	3280.5	3280.5
10	10.064479526	6.7096413223	5.0322279218	4.0751285098	4.0735177854
11	1	1	1	1	1
12	0.99999529381	0.99999790836	0.99999882345	0.99999922844	0.99999922905
13	0.00306795676	0.00204530629	0.00153398019	0.00124222691	0.00124173591
14	0.0030679712	0.00204531057	0.00153398199	0.00124222786	0.00124173686
15					
16	20.128864322	13.419254576	10.064444002	8.1502507311	8.1470292898
17	20.128888004	13.419261593	10.064446963	8.1502523032	8.1470308601
18	20.128895899	13.419263932	10.064447949	8.1502528273	8.1470313835
19	20.12890625	13.419270833	10.064453125	8.1502570186	8.1470355731
20	20.128959053	13.419282645	10.064455844	8.1502570196	8.1470355708
21					
22	20611.957066	20611.975029	20611.981317	20611.984099	20611.984103
23	20611.9894	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612	20612
25	20612.05407	20612.018142	20612.005568	20612.000003	20611.999994
26					
27	33808603.47	33808721.33	33808762.58	33808780.83	33808780.86
28	33808815.61	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833	33808833
30	33808921.69	33808862.76	33808842.13	33808833	33808832.99

	Y	Z	AA	AB	AC
1					
2					
3	3072	4096	6144	8192	12288
4	0.1171875	0.087890625	0.05859375	0.0439453125	0.029296875
5					
6	0.00102265422	0.00076699054	0.00051132697	0.00038349522	0.00025566347
7	3280.4982846	3280.4990351	3280.4995711	3280.4997588	3280.4998928
8	3.3548153983	2.5161117406	1.6774079185	1.2580559628	0.83870398664
9	3280.5	3280.5	3280.5	3280.5	3280.5
10	3.3548171526	2.5161124807	1.6774081377	1.2580560553	0.83870401405
11	1	1	1	1	1
12	0.99999947709	0.99999970586	0.99999986927	0.99999992647	0.99999996732
13	0.00102265368	0.00076699032	0.00051132691	0.00038349519	0.00025566346
14	0.00102265422	0.00076699054	0.00051132697	0.00038349522	0.00025566347
15					
16	6.7096307967	5.0322234813	3.3548158369	2.5161119257	1.6774079733
17	6.7096316738	5.0322238513	3.3548159466	2.5161119719	1.677407987
18	6.7096319662	5.0322239747	3.3548159831	2.5161119873	1.6774079916
19	6.7096354167	5.0322265625	3.3548177083	2.5161132812	1.6774088542
20	6.7096343052	5.0322249614	3.3548162755	2.5161121107	1.6774080281
21					
22	20611.985807	20611.987379	20611.988502	20611.988895	20611.989176
23	20611.9894	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612	20612
25	20611.996586	20611.993442	20611.991197	20611.990411	20611.989849
26					
27	33808792.04	33808802.35	33808809.72	33808812.3	33808814.14
28	33808815.61	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833	33808833
30	33808827.4	33808822.24	33808818.56	33808817.27	33808816.35

	AD	AE	AF	AG
1				
2				
3	16384	24576	32768	49152
4	0.02197265625	0.0146484375	0.01098632813	0.00732421875
5				
6	0.0001917476	0.00012783173	0.0000958738	0.00006391587
7	3280.4999397	3280.4999732	3280.4999849	3280.4999933
8	0.62902799298	0.41935199675	0.31451399793	0.2096759988
9	3280.5	3280.5	3280.5	3280.5
10	0.62902800454	0.41935200017	0.31451399938	0.20967599923
11	1	1	1	1
12	0.99999998162	0.99999999183	0.9999999954	0.99999999796
13	0.0001917476	0.00012783173	0.0000958738	0.00006391587
14	0.0001917476	0.00012783173	0.0000958738	0.00006391587
15				
16	1.258055986	0.83870399349	0.62902799587	0.4193519976
17	1.2580559917	0.83870399521	0.62902799659	0.41935199782
18	1.2580559937	0.83870399578	0.62902799683	0.41935199789
19	1.2580566406	0.83870442708	0.62902832031	0.41935221354
20	1.2580560091	0.83870400035	0.62902799876	0.41935199846
21				
22	20611.989274	20611.989344	20611.989369	20611.989386
23	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612
25	20611.989653	20611.989512	20611.989463	20611.989428
26				
27	33808814.78	33808815.25	33808815.41	33808815.52
28	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833
30	33808816.03	33808815.8	33808815.72	33808815.66

	AH	AI	AJ	AK
1				
2				
3	65536	98304	131072	196608
4	0.00549316406	0.00366210938	0.00274658203	0.00183105469
5				
6	0.0000479369	0.00003195793	0.00002396845	0.00001597897
7	3280.4999962	3280.4999983	3280.4999991	3280.4999996
8	0.15725699915	0.10483799945	0.0786284996	0.05241899973
9	3280.5	3280.5	3280.5	3280.5
10	0.15725699933	0.10483799951	0.07862849962	0.05241899974
11	1	1	1	1
12	0.99999999885	0.99999999949	0.99999999971	0.99999999987
13	0.0000479369	0.00003195793	0.00002396845	0.00001597897
14	0.0000479369	0.00003195793	0.00002396845	0.00001597897
15				
16	0.3145139983	0.20967599891	0.15725699919	0.10483799947
17	0.31451399839	0.20967599893	0.1572569992	0.10483799947
18	0.31451399842	0.20967599894	0.15725699921	0.10483799947
19	0.31451416016	0.20967610677	0.15725708008	0.10483805339
20	0.31451399866	0.20967599902	0.15725699924	0.10483799948
21				
22	20611.989392	20611.989397	20611.989398	20611.989399
23	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612
25	20611.989416	20611.989407	20611.989404	20611.989402
26				
27	33808815.56	33808815.59	33808815.6	33808815.61
28	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833
30	33808815.64	33808815.63	33808815.62	33808815.62

	A	B	C	D
1				
2				
3	262144	393216	524288	786432
4	0.00137329102	0.00091552734	0.00068664551	0.00045776367
5				
6	0.00001198422	0.00000798948	0.00000599211	0.00000399474
7	3280.4999998	3280.4999999	3280.4999999	3280.5
8	0.0393142498	0.02620949987	0.0196571249	0.01310474993
9	3280.5	3280.5	3280.5	3280.5
10	0.0393142498	0.02620949987	0.0196571249	0.01310474993
11	1	1	1	1
12	0.99999999993	0.99999999997	0.99999999998	0.99999999999
13	0.00001198422	0.00000798948	0.00000599211	0.00000399474
14	0.00001198422	0.00000798948	0.00000599211	0.00000399474
15				
16	0.0786284996	0.05241899974	0.0393142498	0.02620949987
17	0.0786284996	0.05241899974	0.0393142498	0.02620949987
18	0.0786284996	0.05241899974	0.0393142498	0.02620949987
19	0.07862854004	0.05241902669	0.03931427002	0.02620951335
20	0.07862849961	0.05241899974	0.0393142498	0.02620949987
21				
22	20611.9894	20611.9894	20611.9894	20611.9894
23	20611.9894	20611.9894	20611.9894	20611.9894
24	20612	20612	20612	20612
25	20611.989401	20611.989401	20611.9894	20611.9894
26				
27	33808815.61	33808815.61	33808815.61	33808815.61
28	33808815.61	33808815.61	33808815.61	33808815.61
29	33808833	33808833	33808833	33808833
30	33808815.62	33808815.61	33808815.61	33808815.61

	E	F
1		
2		360
3	1048576	Number of Sides
4	0.00034332275	Angle 360°/n
5		3280.5
6	0.00000299606	slope m
7	3280.5	X coordinate
8	0.00982856245	Y Coordinate
9	3280.5	X coordinate
10	0.00982856245	Y Coordinate
11	1	Check X^2 +Y^2 = 3280.5 ^2
12	1	Cos (360°/2n) = X/r
13	0.00000299606	Sin (360°/2n) = Y/r
14	0.00000299606	Tan (360°/2n) = Y/X
15		
16	0.0196571249	Inscribed side = 2 Y
17	0.0196571249	2*(Cord Px,y to P0,0)
18	0.0196571249	Segment of Circle = 6561*π * a/360°
19	0.01965713501	Segment of Circle = 20,612 * a/360°
20	0.0196571249	Circumscribed side = 2 r (slope)
21		
22	20611.9894	Inscribed perimeter = n Si
23	20611.9894	Circumference = 6561*π*n*a/360°
24	20612	Circumference = 20,612*n*a/360°
25	20611.9894	Circumscribed Perimeter = n Sc
26		
27	33808815.61	Inscribed area = n*X*Y
28	33808815.61	Area (Common π) = π*((6561/2)^2)
29	33808833	Area (∏) =5,153*6561
30	33808815.61	Circumscribed Area = n*slope*(6561/2)^2

Bibliography

Blavatsky, H.P.; *The Secret Doctrine*; The Theosophical Publishing House ;Adyar, India; 1979

Encyclopaedia Britannica; *The New Encyclopaedia Britannica*; Encyclopaedia Britannica; 1974

Lawlor, Robert; *Sacred Geometry, Philosophy and Practice*; Thames & Hudson; London England; 1982

Michell, John; *City of Revelation*; Ballantine; New York, New York; 1972

Moore, Patrick; *The New Atlas Of The Universe*; Crown Publishing Company; New York, New York; 1984

Murphet, Howard; *When Daylight Comes - A Biography of Helena Petrovna Blavatsky*; The Theosophical Publishing House; Wheaton, Ill

Parker, John A.; *Quadrature Of The Circle CONTAINING DEMONSTRATIONS OF The Errors Of Geometers In Finding The Approximations In Use; WITH APPENDIX, AND Practical Questions On The Quadrature, APPLIED TO THE ASTRONOMICAL CIRCLES TO WHICH ARE ADDED Lectures On Polar Magnetism And Non-existence Of Projectile Forces In Nature*; John Wiley & Son; New York, New York; 1874 (Photocopy)

Pond, Dale; *A Short Course In Vibration Physics & Philosophy for the non-physicist developed as a Certification Course In Sympathetic Vibratory Physics, Level I - Basic Theory*; Delta Spectrum Research, Peyton, Colorado; 1991

Skinner, Ralston J; *The Source Of Measures Original* (Copyright 1875); Reprint of 1894 Edition; Wizards Bookshelf; San Diego, Ca.; 1982

Russell, Walter; *The Universal One Volume One*; University Of Science And Philosophy; Swannanoa, Waynesboro, Virginia; 1927 Second Edition.

The Secret of Light; Carnegie Hall; New York, New York; 1947

Thomas, George B. Jr; *Calculus And Analytic Geometry*; Addison-Wesley, Inc.; Reading Massachusetts; 1961

Tompkins, Peter; *Secrets Of The Great Pyramid*; Harper & Row; New York, New York; 1971

West, Beverly H. Editor, Et Al.; *The Prentice-Hall Encyclopaedia Of Mathematics*; Prentice-Hall, Inc.; Englewood Cliffs, New Jersey; 1982

List of Headings and Partial Index

List of Headings and Partial Index

List of Headings and Partial Index

List of Headings and Partial Index