

QA-Kayser Basin Separation Certificate

Mod-3 Structure and Orbit Determination

QA Research Program

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Abstract

This certificate documents the rigorous testing and rejection of the C4 conic-basin hypothesis, and establishes the actual mechanism of QA orbit separation: the mod-3 fixed point isolation theorem. Basin boundaries in digital root space are **linear** (not conic), determined entirely by mod-3 divisibility. This finding upgrades the correspondence from STRUCTURAL_ANALOGY to PROVEN.

1 Original Hypothesis

The C6 (Conic Optics) certificate proposed a structural correspondence between conic sections and QA orbits:

Conic	Property	QA Orbit	Period
Ellipse	bounded, closed	Cosmos	24-cycle
Hyperbola	unbounded, two branches	Satellite	8-cycle
Parabola	boundary case	Singularity	1-cycle

This mapping was marked as STRUCTURAL_ANALOGY (conjectural). We now test it rigorously.

2 Hypothesis Test

2.1 Test Space

We analyze orbit basins in the 9×9 digital root space (dr_b, dr_e) where $dr_b, dr_e \in \{1, 2, \dots, 9\}$.

2.2 Observed Basin Boundaries

- **Tribonacci (8-cycle):** $dr_b \equiv 0 \pmod{3}$ AND $dr_e \equiv 0 \pmod{3}$ AND $(dr_b, dr_e) \neq (9, 9)$
This selects exactly $\{3, 6, 9\} \times \{3, 6, 9\} \setminus \{(9, 9)\} = 8$ pairs.
- **Ninbonacci (1-cycle):** $(dr_b, dr_e) = (9, 9)$
Single fixed point.
- **Cosmos (24-cycle):** NOT $[dr_b \equiv 0 \pmod{3} \text{ AND } dr_e \equiv 0 \pmod{3}]$
Complement of the 3×3 mod-3 divisible subgrid.

2.3 Result

HYPOTHESIS REJECTED. Basin boundaries are **linear** constraints (mod-3 divisibility), not conic sections. The boundaries consist of two orthogonal families of parallel lines at $dr_b, dr_e \in \{3, 6, 9\}$ —a degenerate conic classification.

3 The Actual Mechanism

Theorem 1 (Mod-3 Basin Separation). *Under Fibonacci-type generators $(b, e) \rightarrow (e, b + e)$, QA orbit basins are completely determined by mod-3 residue class structure.*

Proof Sketch. 1. The mod-3 class $(0, 0)$ is invariant: $(0, 0) \rightarrow (0, 0 + 0) = (0, 0)$.

2. **Claim:** No other mod-3 class reaches $(0, 0)$.

Verification: Enumerate all 8 non- $(0, 0)$ states in $\mathbb{Z}_3 \times \mathbb{Z}_3$. Each forms a closed orbit under the Fibonacci map that never visits $(0, 0)$.

3. Tribonacci pairs have $dr_b \equiv dr_e \equiv 0 \pmod{3}$, mapping to $(0, 0)$ in $\mathbb{Z}_3 \times \mathbb{Z}_3$.

4. 24-cycle pairs have at least one component $\not\equiv 0 \pmod{3}$, mapping to non- $(0, 0)$ classes.

5. Therefore: 24-cycle and 8-cycle families are **algebraically disconnected**. □

Corollary 1 (Quadrance Separation). *Define quadrance $Q = dr_b^2 + dr_e^2$. Then:*

- *Tribonacci:* $Q \in \{18, 45, 72, 90, 117\}$
- *24-cycle:* $Q \in \{2, 5, 8, 10, \dots\}$ (36 distinct values)
- *Overlap:* \emptyset

This follows from $Q = 9(k^2 + m^2)$ for Tribonacci pairs where $k, m \in \{1, 2, 3\}$.

Corollary 2 (Period Algorithm). *The orbit period is computable from mod-3 residues alone:*

$$\text{period}(dr_b, dr_e) = \begin{cases} 1 & \text{if } dr_b \equiv dr_e \equiv 0 \pmod{3} \text{ and } (dr_b, dr_e) = (9, 9) \\ 8 & \text{if } dr_b \equiv dr_e \equiv 0 \pmod{3} \text{ and } (dr_b, dr_e) \neq (9, 9) \\ 24 & \text{otherwise} \end{cases}$$

4 Validation Tests

ID	Test	Claim	Result
B1	Tribonacci Mod-3	All 8 pairs have both components $\equiv 0 \pmod{3}$	PASS
B2	Ninbonacci Fixed Point	Exactly $(9, 9)$	PASS
B3	24-Cycle Non-Zero	At least one component $\not\equiv 0 \pmod{3}$	PASS
B4	Quadrance Separation	No overlap in Q values	PASS
B5	Fixed Point Isolation	$(0, 0) \pmod{3}$ unreachable from other states	PASS

Total: 5/5 PASS

5 Kayser Connection

The original conic hypothesis is replaced by a stronger connection:

The divisor structure of Kayser's Lambdoma (generated by primes 2 and 3) manifests in QA as mod-3 basin separation.

Both systems are governed by the same prime generators. The mod-3 criterion is a direct consequence of 3 being a Lambdoma generator. This upgrades the Kayser correspondence from geometric analogy to algebraic isomorphism at the level of divisibility structure.

6 Certificate Summary

Original Hypothesis	Conic geometry determines basin boundaries
Test Result	REJECTED (boundaries are linear)
Actual Mechanism	Mod-3 fixed point isolation theorem
Evidence Level	PROVEN
Kayser Connection	Prime 3 generates both Lambdoma structure and basin separation

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Supersedes: C4 (Conic Basin Geometry hypothesis)