

### A Proof of Theorem 3

*Proof.* The proof is presented here in two cases, first for the user  $v_i$  whose edges' privacy levels are not all 2:

$$\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + \text{Lap}(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \mathbb{E}[w_i^{(1)} + w_i^{(2)}] \\
&= \mathbb{E}[\frac{t_i^{(1)} - (1 - p_1)s_i^{(1)}}{2p_1 - 1} + \frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1}] \\
&= \frac{1}{2p_1 - 1}\mathbb{E}[t_i^{(1)} - (1 - p_1)s_i^{(1)}] + \frac{1}{2p_2 - 1}\mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}]
\end{aligned} \tag{8}$$

Let  $s_i^{*(l)} (l \in \{1, 2\})$  be the count of the 2-star counts of user  $v_i$  excluding the triangle part, i.e.,  $s_i^{*(l)} = |(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, a_{j,k} = 0|$ .  $T_i$  denotes the real triangle counts in the local graph for user  $v_i$ .  $t_i^{(l)}$  is the noisy triangle counts, i.e.,  $t_i = |(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, (v_j, v_k) \in G'|$ .  $s_i^{(l)}$  is the true 2-star counts for user  $v_i$ , then we have  $s_i^{(l)} = s_i^{*(l)} + T_i^{(l)}$ . By the properties of RR:

$$\mathbb{E}[t_i^{(l)}] = T_i^{(l)}p + s_i^{*(l)}(1 - p) \tag{9}$$

so,

$$\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \frac{1}{2p_1 - 1}\mathbb{E}[t_i^{(1)} - (1 - p_1)s_i^{(1)}] + \frac{1}{2p_2 - 1}\mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}] \\
&= \frac{1}{2p_1 - 1}\mathbb{E}[T_i^{(1)}p_1 + s_i^{*(1)}(1 - p_1) - (1 - p_1)s_i^{(1)}] \\
&\quad + \frac{1}{2p_2 - 1}\mathbb{E}[T_i^{(2)}p_2 + s_i^{*(2)}(1 - p_2) - (1 - p_2)s_i^{(2)}] \\
&= \frac{(2p_1 - 1)T_i^{(1)}}{2p_1 - 1} + \frac{(2p_2 - 1)T_i^{(2)}}{2p_2 - 1} \\
&= T_i^{(1)} + T_i^{(2)} \\
&= T_i.
\end{aligned} \tag{10}$$

For the user  $v_i$  whose edges all have privacy level 2:

$$\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \mathbb{E}[w_i^{(2)}] \\
&= \frac{1}{2p_2 - 1} \mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}] \\
&= \frac{1}{2p_2 - 1} \mathbb{E}[T_i^{(2)} p_2 + s_i^{*(2)}(1 - p_2) - (1 - p_2)s^{(2)}] \\
&= \frac{(2p_2 - 1)T_i^{(2)}}{2p_2 - 1} \\
&= T_i^{(2)} \\
&= T_i.
\end{aligned} \tag{11}$$

## B Proof of Theorem 4

For a user  $v_i$  whose relationship edges all have a privacy level of 2:

*Proof.*

$$\begin{aligned}
\text{Var}[\hat{w}_i] &= \text{Var}[w_i^{(2)} + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \text{Var}[\frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1} + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \frac{1}{(2p_2 - 1)^2} \text{Var}[t_i^{(2)}] + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_2 q_2 (T_i^{(2)} + s_i^{*(2)})}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_2 q_2 s_i^{(2)}}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&\leq \frac{p_2 q_2}{(2p_2 - 1)^2} \frac{K_i(K_i - 1)}{2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2.
\end{aligned} \tag{12}$$

For a user  $v_i$  whose relationship edges' privacy levels are not all 2:

*Proof.*

$$\begin{aligned}
Var[\hat{w}_i] &= Var[w_i^{(1)} + w_i^{(2)} + Lap(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= Var[\frac{t_i^{(1)} - (1 - p_1)s_i^{(1)}}{2p_1 - 1} + \frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1} + Lap(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \frac{1}{(2p_1 - 1)^2} Var[t_i^{(1)}] + \frac{1}{(2p_2 - 1)^2} Var[t_i^{(2)}] + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 (T_i^{(1)} + s_i^{*(1)})}{(2p_1 - 1)^2} + \frac{p_2 q_2 (T_i^{(2)} + s_i^{*(2)})}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 s_i^{(1)}}{(2p_1 - 1)^2} + \frac{p_2 q_2 s_i^{(2)}}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&\leq \frac{p_1 q_1 (s_i^{(1)} + s_i^{(2)})}{(2p_1 - 1)^2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&\leq \frac{p_1 q_1}{(2p_1 - 1)^2} \frac{K_i(K_i - 1)}{2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2.
\end{aligned} \tag{13}$$