

The earthquake source is represented by the point force \mathbf{f} , which may be written in terms of a moment tensor \mathbf{M} as (omitting the source time function)

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) \quad (1)$$

The point force \mathbf{f} in 3D:

$$\begin{cases} f_x = -[M_{xx}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{xy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{xz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \\ f_y = -[M_{xy}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{yz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \\ f_z = -[M_{xz}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yz}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{zz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \end{cases} \quad (2)$$

Multiplying equations (2) by the time-independent test functions, we can obtain

$$\begin{cases} F_x = -\int [M_{xx}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{xy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{xz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_x(x, y, z)d\Omega \\ F_y = -\int [M_{xy}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{yz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_y(x, y, z)d\Omega \\ F_z = -\int [M_{xz}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yz}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{zz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_z(x, y, z)d\Omega \end{cases} \quad (3)$$

where φ_x , φ_y and φ_z are the test functions, respectively.

Applying the identity of the Dirac delta function

$$\begin{cases} F_x(x_s, y_s, z_s) = \left(M_{xx} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial x} + M_{xy} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial y} + M_{xz} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial z} \right) \\ F_y(x_s, y_s, z_s) = \left(M_{xy} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial x} + M_{yy} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial y} + M_{yz} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial z} \right) \\ F_z(x_s, y_s, z_s) = \left(M_{xz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial x} + M_{yz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial y} + M_{zz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial z} \right) \end{cases} \quad (4)$$

If we can find the dimensionless parameters $(\xi_s, \eta_s, \gamma_s)$ satisfying

$$\begin{cases} x_s = \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) x_{pqr} \\ y_s = \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) y_{pqr} \\ z_s = \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) z_{pqr} \end{cases} \quad (5)$$

or,

$$\left\{ \begin{aligned} \xi_s &= \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \xi_{pqr} \\ \eta_s &= \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \eta_{pqr} , \\ \gamma_s &= \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \gamma_{pqr} \end{aligned} \right. \quad (6)$$

Applying the chain rule, (4) can be written

$$\left\{ \begin{aligned} F_x(x_s, y_s, z_s) &= M_{xx} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial x} \right) \\ &\quad M_{xy} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial y} \right) \\ &\quad M_{xz} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial z} \right) \\ F_y(x_s, y_s, z_s) &= M_{xy} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial x} \right) \\ &\quad M_{yy} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial y} \right) \\ &\quad M_{yz} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial z} \right) \\ F_z(x_s, y_s, z_s) &= M_{xz} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial x} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial x} \right) \\ &\quad M_{yz} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial y} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial y} \right) \\ &\quad M_{zz} \left(\frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \xi} \frac{\partial \xi_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \eta} \frac{\partial \eta_s}{\partial z} + \frac{\partial \varphi_x(\xi_s, \eta_s, \gamma_s)}{\partial \gamma} \frac{\partial \gamma_s}{\partial z} \right) , \end{aligned} \right. \quad (7)$$

where,

$$\left\{ \begin{aligned} \varphi_x(\xi_s, \eta_s, \gamma_s) &= \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \varphi_{xklm} \\ \varphi_y(\xi_s, \eta_s, \gamma_s) &= \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \varphi_{yklm} , \\ \varphi_z(\xi_s, \eta_s, \gamma_s) &= \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \varphi_{zklm} \end{aligned} \right. \quad (8)$$

Substituting (6) and (8) into (7), we can obtain

$$\begin{aligned}
F_z(x_s, y_s, z_s) = & M_{xz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial x} \\
& + M_{xz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial x} \\
& + M_{xz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial x} \\
& + M_{yz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial y} \\
& + M_{yz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial y} \\
& + M_{yz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial y} \\
& + M_{zz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial z} \\
& + M_{zz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial z} \\
& + M_{zz} \sum_{k=1}^N \sum_{l=1}^N \sum_{m=1}^N \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial z} .
\end{aligned} \tag{9c}$$

Therefore, the source force at each GLL node can be written:

$$\begin{aligned}
F_x(k, l, m) = & M_{xx} \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial x} \\
& + M_{xx} \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial x} \\
& + M_{xx} \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial x} \\
& + M_{xy} \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial y} \\
& + M_{xy} \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial y} \\
& + M_{xy} \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial y} \\
& + M_{xz} \ell'_k(\xi_s) \ell_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \xi_{pqr}}{\partial z} \\
& + M_{xz} \ell_k(\xi_s) \ell'_l(\eta_s) \ell_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \eta_{pqr}}{\partial z} \\
& + M_{xz} \ell_k(\xi_s) \ell_l(\eta_s) \ell'_m(\gamma_s) \sum_{p=1}^N \sum_{q=1}^N \sum_{r=1}^N \ell_p(\xi_s) \ell_q(\eta_s) \ell_r(\gamma_s) \frac{\partial \gamma_{pqr}}{\partial z} ,
\end{aligned} \tag{10a}$$

