

The earthquake source is represented by the point force  $\mathbf{f}$ , which may be written in terms of a moment tensor  $\mathbf{M}$  as (omitting the source time function)

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) \quad (1)$$

The point force  $\mathbf{f}$  in 3D:

$$\begin{cases} f_x = -[M_{xx}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{xy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{xz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \\ f_y = -[M_{xy}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{yz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \\ f_z = -[M_{xz}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yz}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{zz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)] \end{cases} \quad (2)$$

Multiplying equations (2) by the time-independent test functions, we can obtain:

$$\begin{cases} F_x = -\int [M_{xx}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{xy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{xz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_x(x, y, z)d\Omega \\ F_y = -\int [M_{xy}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yy}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{yz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_y(x, y, z)d\Omega \\ F_z = -\int [M_{xz}\delta'(x-x_s)\delta(y-y_s)\delta(z-z_s) + M_{yz}\delta(x-x_s)\delta'(y-y_s)\delta(z-z_s) \\ \quad + M_{zz}\delta(x-x_s)\delta(y-y_s)\delta'(z-z_s)]\varphi_z(x, y, z)d\Omega \end{cases} \quad (3)$$

where  $\varphi_x$ ,  $\varphi_y$  and  $\varphi_z$  are the test functions, respectively.

Applying the identity of the Dirac delta function:

$$\begin{cases} F_x(x_s, y_s, z_s) = -\left( M_{xx} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial x} + M_{xy} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial y} + M_{xz} \frac{\partial \varphi_x(x_s, y_s, z_s)}{\partial z} \right) \\ F_y(x_s, y_s, z_s) = -\left( M_{xy} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial x} + M_{yy} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial y} + M_{yz} \frac{\partial \varphi_y(x_s, y_s, z_s)}{\partial z} \right) \\ F_z(x_s, y_s, z_s) = -\left( M_{xz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial x} + M_{yz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial y} + M_{zz} \frac{\partial \varphi_z(x_s, y_s, z_s)}{\partial z} \right) \end{cases} \quad (4)$$

If we can find the dimensionless parameters  $(\xi_s, \eta_s, \gamma_s)$  satisfying:

$$\begin{cases} x_s = \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \ell_i(\xi_s) \ell_j(\eta_s) \ell_k(\gamma_s) x_{ijk} \\ y_s = \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \ell_i(\xi_s) \ell_j(\eta_s) \ell_k(\gamma_s) y_{ijk} \\ z_s = \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \ell_i(\xi_s) \ell_j(\eta_s) \ell_k(\gamma_s) z_{ijk} \end{cases} \quad (5)$$

Applying (5) and considering the unit test function, equations (4) can be expressed:



**In your code:**

```
do m = 1,NGLLZ
  do l = 1,NGLLY
    do k = 1,NGLLX

      xixd    = dble(xix(k,l,m))
      xiyd    = dble(xiy(k,l,m))
      xizd    = dble(xiz(k,l,m))
      etaxd   = dble(etax(k,l,m))
      etayd   = dble(etay(k,l,m))
      etazd   = dble(etaz(k,l,m))
      gammaxd = dble(gammax(k,l,m))
      gammayd = dble(gammay(k,l,m))
      gammazd = dble(gammaz(k,l,m))

      G11(k,l,m) = Mxx*xixd+Mxy*xiyd+Mxz*xizd
      G12(k,l,m) = Mxx*etaxd+Mxy*etayd+Mxz*etazd
      G13(k,l,m) = Mxx*gammaxd+Mxy*gammayd+Mxz*gammazd
      G21(k,l,m) = Mxy*xixd+Myy*xiyd+Myz*xizd
      G22(k,l,m) = Mxy*etaxd+Myy*etayd+Myz*etazd
      G23(k,l,m) = Mxy*gammaxd+Myy*gammayd+Myz*gammazd
      G31(k,l,m) = Mxz*xixd+Myz*xiyd+Mzz*xizd
      G32(k,l,m) = Mxz*etaxd+Myz*etayd+Mzz*etazd
      G33(k,l,m) = Mxz*gammaxd+Myz*gammayd+Mzz*gammazd

    enddo
  enddo
enddo

! compute Lagrange polynomials at the source location
```

```
call lagrange_any(xi_source,NGLLX,xigll,hxis,hpxis)
```

```
call lagrange_any(eta_source,NGLLY,yigll,hetas,hpetas)
```

```
call lagrange_any(gamma_source,NGLLZ,zigll,hgammas,hpgammas)
```

```
! calculate source array
```

```
do m = 1,NGLLZ
```

```
  do l = 1,NGLLY
```

```
    do k = 1,NGLLX
```

```
      sourcearrayd(:,k,l,m) = ZERO
```

```
      do iv = 1,NGLLZ
```

```
        do it = 1,NGLLY
```

```
          do ir = 1,NGLLX
```

```
            sourcearrayd(1,k,l,m) = sourcearrayd(1,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &  
                                  *(G11(ir,it,iv)*hpxis(k)*hetas(l)*hgammas(m) &  
                                  +G12(ir,it,iv)*hxis(k)*hpetas(l)*hgammas(m) &  
                                  +G13(ir,it,iv)*hxis(k)*hetas(l)*hpgammas(m))
```

```
            sourcearrayd(2,k,l,m) = sourcearrayd(2,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &  
                                  *(G21(ir,it,iv)*hpxis(k)*hetas(l)*hgammas(m) &  
                                  +G22(ir,it,iv)*hxis(k)*hpetas(l)*hgammas(m) &  
                                  +G23(ir,it,iv)*hxis(k)*hetas(l)*hpgammas(m))
```

```
            sourcearrayd(3,k,l,m) = sourcearrayd(3,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &  
                                  *(G31(ir,it,iv)*hpxis(k)*hetas(l)*hgammas(m) &  
                                  +G32(ir,it,iv)*hxis(k)*hpetas(l)*hgammas(m) &  
                                  +G33(ir,it,iv)*hxis(k)*hetas(l)*hpgammas(m))
```

```

        enddo
    enddo
enddo

enddo
enddo
enddo

```

According to the expressions (7), the subscripts associated with G11, G12, G13, G21, G22, G23, G31, G32, and G33 may be not correct. I think that their subscripts should be (k, l, m).

**The corrected version:**

```

do m = 1,NGLLZ
    do l = 1,NGLLY
        do k = 1,NGLLX

            xixd    = dble(xix(k,l,m))
            xiyd    = dble(xiy(k,l,m))
            xizd    = dble(xiz(k,l,m))
            etaxd   = dble(etax(k,l,m))
            etayd   = dble(etay(k,l,m))
            etazd   = dble(etaz(k,l,m))
            gammaxd = dble(gammax(k,l,m))
            gammayd = dble(gammay(k,l,m))
            gammazd = dble(gammaz(k,l,m))

            G11(k,l,m) = Mxx*xixd+Mxy*xiyd+Mxz*xizd
            G12(k,l,m) = Mxx*etaxd+Mxy*etayd+Mxz*etazd
            G13(k,l,m) = Mxx*gammaxd+Mxy*gammayd+Mxz*gammazd
            G21(k,l,m) = Mxy*xixd+Myy*xiyd+Myz*xizd

```

```

G22(k,l,m) = Mxy*etaxd+Myy*etayd+Myz*etazd
G23(k,l,m) = Mxy*gammaxd+Myy*gammayd+Myz*gammazd
G31(k,l,m) = Mxz*xixd+Myz*xiyd+Mzz*xizd
G32(k,l,m) = Mxz*etaxd+Myz*etayd+Mzz*etazd
G33(k,l,m) = Mxz*gammaxd+Myz*gammayd+Mzz*gammazd

    enddo

enddo

enddo

! compute Lagrange polynomials at the source location
call lagrange_any(xi_source,NGLLX,xigll,hxis,hpxis)
call lagrange_any(eta_source,NGLLY,yigll,hetas,hpetas)
call lagrange_any(gamma_source,NGLLZ,zigll,hgammas,hpgammas)

! calculate source array
do m = 1,NGLLZ
  do l = 1,NGLLY
    do k = 1,NGLLX

      sourcearrayd(:,k,l,m) = ZERO

      do iv = 1,NGLLZ
        do it = 1,NGLLY
          do ir = 1,NGLLX

            sourcearrayd(1,k,l,m) = sourcearrayd(1,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
              *(G11(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
              +G12(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
              +G13(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))
          enddo
        enddo
      enddo
    enddo
  enddo
enddo

```

```

sourcearrayd(2,k,l,m) = sourcearrayd(2,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
*(G21(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
+G22(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
+G23(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))

```

```

sourcearrayd(3,k,l,m) = sourcearrayd(3,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
*(G31(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
+G32(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
+G33(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))

```

```

    enddo

```

```

  enddo

```

```

enddo

```

```

    enddo

```

```

  enddo

```

```

enddo

```

**However, the second loop can be simplified.**

```

do m = 1,NGLLZ

```

```

  do l = 1,NGLLY

```

```

    do k = 1,NGLLX

```

```

      sourcearrayd(:,k,l,m) = ZERO

```

```

      dsrc_dx = (G11(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
+G12(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
+G13(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))

```

```

      dsrc_dy = (G21(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &

```

```

+G22(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
+G23(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))
dsrc_dy = (G31(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
+G32(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
+G33(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))

do iv = 1,NGLLZ
  do it = 1,NGLLY
    do ir = 1,NGLLX

      sourcearrayd(1,k,l,m) = sourcearrayd(1,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
                                *dsrc_dx

      sourcearrayd(2,k,l,m) = sourcearrayd(2,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
                                *dsrc_dy

      sourcearrayd(3,k,l,m) = sourcearrayd(3,k,l,m) + hxis(ir)*hetas(it)*hgammas(iv) &
                                *dsrc_dz

    enddo
  enddo
enddo

enddo
enddo
enddo

```

**If we apply the identity of the Lagrange function, i.e.  $\text{sum}(\text{hxis}(1:\text{NGLLX})) = 1$ ,  $\text{sum}(\text{hetas}(1:\text{NGLLX})) = 1$ , and  $\text{sum}(\text{hgammas}(1:\text{NGLLZ})) = 1$ , the three loops can be merged:**



```

call lagrange_any(xi_source,NGLLX,xigll,hxis,hpxis)
call lagrange_any(eta_source,NGLLY,yigll,hetas,hpetas)
call lagrange_any(gamma_source,NGLLZ,zigll,hgammas,hpgammas)

```

```

sourcearray(:, :, :) = ZERO

```

```

do m = 1,NGLLZ

```

```

  do l = 1,NGLLY

```

```

    do k = 1,NGLLX

```

```

      xixd    = dble(xix(k,l,m))

```

```

      xiyd    = dble(xiy(k,l,m))

```

```

      xizd    = dble(xiz(k,l,m))

```

```

      etaxd   = dble(etax(k,l,m))

```

```

      etayd   = dble(etay(k,l,m))

```

```

      etazd   = dble(etaz(k,l,m))

```

```

      gammaxd = dble(gammax(k,l,m))

```

```

      gammayd = dble(gammay(k,l,m))

```

```

      gammazd = dble(gammaz(k,l,m))

```

```

      G11(k,l,m) = Mxx*xixd+Mxy*xiyd+Mxz*xizd

```

```

      G12(k,l,m) = Mxx*etaxd+Mxy*etayd+Mxz*etazd

```

```

      G13(k,l,m) = Mxx*gammaxd+Mxy*gammayd+Mxz*gammazd

```

```

      G21(k,l,m) = Mxy*xixd+Myy*xiyd+Myz*xizd

```

```

      G22(k,l,m) = Mxy*etaxd+Myy*etayd+Myz*etazd

```

```

      G23(k,l,m) = Mxy*gammaxd+Myy*gammayd+Myz*gammazd

```

```

      G31(k,l,m) = Mxz*xixd+Myz*xiyd+Mzz*xizd

```

```

      G32(k,l,m) = Mxz*etaxd+Myz*etayd+Mzz*etazd

```

```

      G33(k,l,m) = Mxz*gammaxd+Myz*gammayd+Mzz*gammazd

```

```

dsrc_dx = (G11(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
           +G12(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
           +G13(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))
dsrc_dy = (G21(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
           +G22(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
           +G23(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))
dsrc_dz = (G31(k,l,m)*hpxis(k)*hetas(l)*hgammas(m) &
           +G32(k,l,m)*hxis(k)*hpetas(l)*hgammas(m) &
           +G33(k,l,m)*hxis(k)*hetas(l)*hpgammas(m))

sourcearrayd(1,k,l,m) = sourcearrayd(1,k,l,m) + dsrc_dx
sourcearrayd(2,k,l,m) = sourcearrayd(2,k,l,m) + dsrc_dy
sourcearrayd(3,k,l,m) = sourcearrayd(3,k,l,m) + dsrc_dz

enddo
enddo
enddo

```

**Certainly, your code is correct for linear mapping element because  $G(ir,it,iv) = G(k,l,m)$  in that case. However, it is not correct for nonlinear mapping element!**

**Now, the code is consistent with equations (7).**