Bar-Ilan University
אוניברסיטת בר־אילן

Distributed Computing - 83453 - Homework 1

Due: 2/12/2024

1) Assume we are given a network G and a Tree T with a root. The root begins with k items $\{I_1, \ldots, I_k\}$ of size $O(\log n)$ each, that it needs to deliver to the parties. Each item is designated to a single party, $I_i = (\text{party}_i, \text{value}_i)$ (each party might get zero or more items). Show a synchronous CONGEST algorithm that delivers all items in time $O(k + \text{depth}(T))$

2) A 2-approximation of a function $f : G \to \mathbb{R}$, is an algorithm that computes a value $g \in \mathbb{R}$ such that for any graph G we get that $\frac{1}{2} f(G) \leq g \leq 2f(G)$.

   Show a synchronous CONGEST algorithm for computing a 2-approximation of the diameter D of the network G in time $O(D)$.
   (You can assume the existence of a designated node r that begins the computation and needs to give the output)

3) Suppose G is a ***directed*** graph, so that the directed edge $(v \to u)$ only allows the node v to send messages to u (but not in the other direction).

   Reminder: a strongly connected directed graph G is such any vertex is reachable from any other vertex by a directed path in G.

   a. Prove that if G is not strongly connected, then broadcast is impossible in general.
   b. Prove an existential lower bound of $\Omega(n)$ rounds for broadcast on a directed G.
   c. Is $\Omega(n)$ also a global lower bound for broadcast on directed G? Prove or give a counterexample.

4) Here is the pseudocode for α-Synchronizer, which executes the synchronous algorithm A over an asynchronous network

```
init: i = 1
At node v, repeat:
  "Execute round i of A":
     Send round i messages to all neighbours
     Receive round i message from each neighbour
     and reply with an ACK for any such received message
  wait until all v's sent messages of round i are ACK'd
  send SAFE to all neighbors
  wait until v receives SAFE messages from all neighbors
  set i = i+1
```

a. Prove that the above algorithm executes any synchronous algorithm A, over a LOCAL asynchronous network.
   Specifically, assume A takes L rounds and show that when some node sets i = L+1, it output the correct output of A.
   Argue that all nodes reach i= L+1.
b. Prove that at any given time, the difference between the value of i for any two neighboring nodes is at most +-1.
c. Analyze the message complexity of the above Synchronizer α on A. Assume again that A takes L rounds.