Bar-Ilan University
אוניברסיטת בר־אילן

Distributed Algorithms - 83453 - Homework 2

due: 23/12/2024

1) Consider the GHS construction of an MST. What goes wrong if edges do not have unique weights? Pinpoint the argument(s) in the proof that breaks.

2) Consider an **anonymous** ring where party $i \in \{1,2,\ldots,n\}$ gets the input $x_i \in \{0,1\}$

   a. Prove that there is no uniform synchronous algorithm for computing the AND of the input bits: AND$(x_1, \ldots, x_n)$.

      Hint: Suppose such an algorithm exists and consider how it behaves if all $x_i = 1$. Then, consider a much larger ring that has a single party j with input $x_j = 0$.

   b. Design an asynchronous (non-uniform) algorithm that computes the AND function with message complexity $O(n^2)$ in the worst case.

   c. Prove that $\Omega(n^2)$ is a lower bound on the complexity of any asynchronous algorithm for AND over rings.

   d. Show a synchronous algorithm for the AND function over rings, with message complexity of $O(n)$ in the worst case.

3) Assume that $O(\Delta^2)$-coloring can be done in general graphs in t synchronous rounds. Based on this assumption, explain how to obtain $(\Delta + 1)$-coloring in time $O(t + \Delta \log \Delta)$.

   bonus (hard!): can you get $(\Delta + 1)$-coloring in time $O(t + \Delta)$?

4) In the algorithm for 6-coloring a tree in time $O(\log^* n)$, we assumed that the root takes some default index i (say, i=0), and sets its new color to $i \circ \varphi(root)[i]$.

   a. Show that if the root chooses a <u>fixed</u> default color (say, the all-zero string, or some other fixed value) then the algorithm fails.

   b. In light of part (a.) above, explain why the algorithm <u>succeeds</u> if the root begins with color=0000…0 and keeps changing its color to the all-zero string.