Electronic Commerce Models HW2

1)

$$F_A(v) = \frac{v^2}{25}$$

$$F_B(v) = \frac{1 - \exp(-v)}{1 - \exp(-5)}$$

a) computer the v.v function

$$\phi_i = v_i - \left(\frac{1 - G_i(v_i)}{g_i(v_i)}\right)$$
$$g_i = G_i{}'$$

$$\phi_A(v) = v - \left[\frac{\left(1 - \frac{v^2}{25}\right)}{\frac{2v}{25}}\right] = v - \frac{25 - v^2}{2v} = \frac{3v}{2} - \frac{25}{2v}$$

$$\phi_B = v - \left(\frac{1 - \left(\frac{1 - \exp(-v)}{1 - \exp(-5)}\right)}{\frac{\exp(-v)}{1 - \exp(-5)}}\right)$$
$$t = 1 - \exp(-5)$$
$$\phi_B = v - \left(\frac{1 - \left(\frac{1 - \exp(-v)}{t}\right)}{\frac{\exp(-v)}{t}}\right) = v - \frac{\exp(-v) - \exp(-5)}{\exp(-v)}$$
$$\phi_B = v - 1 + \frac{\exp(-5)}{\exp(-v)} = v - 1 + \exp(-5)\exp(v)$$

b) Verify that the virtual value function is monotone non-decreasing

$$\phi_A(v) = \frac{3v}{2} - \frac{25}{2v}$$
$$\phi_A(v)' = \frac{3}{2} + \frac{25}{2x^2} \geq 0 \rightarrow non\ decresing$$

$$\phi_B = v - 1 + \frac{\exp(-5)}{\exp(-v)}$$

$$\phi_B' = 1 + \exp(x-5) \geq 0 \rightarrow non\ decresing$$

c)

$$\phi_A(v) = 0 = \frac{3v}{2} - \frac{25}{2v}$$

$$v = \pm\sqrt{\frac{25}{3}} = \pm 5\sqrt{3}\frac{1}{3}$$

Because $v \geq 0$

$$v = 5 \cdot \left(\frac{1}{\sqrt{3}}\right)$$

$$\phi_B = 0 = v - 1 + \exp\,(x-5)$$

$$v = 1 - W\left(\frac{1}{e^4}\right) \rightarrow W \text{ Lambert function}$$

2)

a) One item single player for a where the reserve price is

$$v_0 = 5 \cdot \left(\frac{1}{\sqrt{3}}\right)$$

$$E[Revenue] = \int_{v_0}^{5} v \cdot \left[\frac{3v}{2} - \frac{25}{2v}\right] = \left[\frac{1}{2} \cdot x \cdot (x^2 - 25)\right]_{v_0}^{5} \approx 24.056$$

b) One item single player for b where the reserve price is

$$v_0 = 1 - W\left(\frac{1}{e^4}\right) \approx 0.982 \rightarrow W \text{ Lambert function}$$

$$E[Revenue] = \int_{v_0}^{5} v \cdot [v - 1 + \exp(x-5)] = -\frac{v_0^2}{2} + v_0 - e^{v_0 - 5} + \frac{17}{2}$$

$$E[Revenue] = -\frac{0.982^2}{2} + 0.982 - e^{0.982-5} + \frac{17}{2} \approx 8.98$$

c)

1.4.2)

To prove that is strategy proof we need to prove

Monotonicity – if you have higher value you still get the item.

threshold payment – each player have a R/k payment he will have to pay.