שאלה 1:

סעיף ג)

Step 1)

**Verify that each component of k(x) integrates to 0.25**

The probability density function k(x) is defined as:

We observe that each piece of k(x) corresponds to half of the respective density functions f(x) and g(x):

* The **first part** is half of the function , mirrored over the y-axis.
* The **second part** is half of the function , mirrored over the y-axis.
* The **third** and **fourth parts** are directly half of the two respective parts of f(x).

Let us now verify that each section integrates to 0.25

Since the total area under a PDF must be 1, and the previous three regions summed to 0.75, this section must integrate to 0.25 as well (which aligns with f(x)’s normalization).

Step 2: Constructing the Sampling Algorithm for k(x)

To sample from k(x), observe that it consists of mirrored and scaled versions of f(x) and g(x). Specifically:

* The **left half** of k(x) (i.e., for x<0) is a reflection and scaling of g(x).
* The **right half** (i.e., for x>0) is a scaling of f(x).

Thus, we can use the following **sampling algorithm**:

#### **Algorithm to Sample from** k(x)**:**

1. Flip a fair coin which we can do by sample from f(x) value and if x is bentween 0 and 1 we set B=0, else B=1. We already showed before that both section have 0.5 to occur.
2. **If** B=0:  
   Sample v∼g(x), then return k(-v).
   * + **This covers from** minus infinity to 0
3. **Else** B=1:  
   Sample v∼f(x), return k(v).
   * + This covers from zero to infinity

#### **Remarks:**

* The mirroring ensures that the density is correctly reproduced on x<0.
* The scaling by 0.5​ is already incorporated into the construction of k(x), so sampling from f(x) or g(x) and randomly selecting left/right gives the full distribution.