

Algorithmics

Correction Midterm Exam #1

(Version profs)

UNDERGRADUATE 1st YEAR S1# – EPITA

Solution 1 (Abstract types: Vector (errors and extension) – 6 points)

1. There are two types of problems:

- a completeness problem (lack of axioms) with a missing axiom applying `isinit` to `vect`.
- a consistency problem (ambiguity between axioms) with `i≠j` on one of the axioms applying `isinit` to `modify`.

The corrected declaration of the type `vector` should be:

TYPES

vector

USES

integer, element, boolean

OPERATIONS

`vect` : integer \times integer \rightarrow vector
`modify` : vector \times integer \times element \rightarrow vector
`nth` : vector \times integer \rightarrow element
`isinit` : vector \times integer \rightarrow boolean
`lowerlimit` : vector \rightarrow integer
`upperlimit` : vector \rightarrow integer

PRECONDITIONS

`nth(v,i)` if-and-only-if `lowerlimit(v) ≤ i ≤ upperlimit(v)` & `isinit(v,i)=vrai`

AXIOMS

`lowerlimit(v) ≤ i ≤ upperlimit(v)` ~~\wedge~~ `nth(modify(v,i,e),i) = e`
`lowerlimit(v) ≤ i ≤ upperlimit(v)` & `lowerlimit(v) ≤ j ≤ upperlimit(v)` & `i≠j`
 \Rightarrow `nth(modify(v,i,e),j) = nth(v,j)`
`isinit(vect(i,j),k)=Faux`
`lowerlimit(v) ≤ i ≤ upperlimit(v)` ~~\wedge~~ `isinit(modify(v,i,e),i)=vrai`
`lowerlimit(v) ≤ i ≤ upperlimit(v)` & `lowerlimit(v) ≤ j ≤ upperlimit(v)` & `i≠j`
 \Rightarrow `isinit(modify(v,i,e),j)=isinit(v, j)`
`lowerlimit(vect(i,j))=i`
`lowerlimit(v) ≤ i ≤ upperlimit(v)` ~~\wedge~~ `lowerlimit(modify(v,i,e))=lowerlimit(v)`
`upperlimit(vect(i,j))=j`
`lowerlimit(v) ≤ i ≤ upperlimit(v)` ~~\wedge~~ `upperlimit(modify(v,i,e))=upperlimit(v)`

WITH

vector v
 integer i, j, k
 element e

2. Extension of the type *vector*

- (a) There is no precondition. It is an internal operation defined on the bounds of the vector. Its limits will be precised if necessary.
- (b) The axioms are the following:

AXIOMS

```

lowerlimit(v) ≤ i ≤ upperlimit(v) ⇒ isinit(reinitialize(v,i),i)=faux
lowerlimit(v) ≤ i ≤ upperlimit(v) & lowerlimit(v) ≤ j ≤ upperlimit(v) & i≠j
    ⇒ isinit(reinitialize(v,i),j) = isinit(v,j)
lowerlimit(v) ≤ i ≤ upperlimit(v) & lowerlimit(v) ≤ j ≤ upperlimit(v) & i≠j
    ⇒ nth(reinitialize(v,i),j) = nth(v,j)
lowerlimit(reinitialize(v,i))=lowerlimit(v)
upperlimit(reinitialize(v,i))=upperlimit(v)

```

WITH

```

vector      v
integer     i, j

```

Solution 2 (Insertion Sort – 7 points)**1. Specifications:**

The function `insert x l comp` adds the element `x` at its place in the list `l` sorted according to the comparison function `comp`.

```

# let rec insert x list comp =
  match list with
  | [] -> x::[]
  | e::l when comp x e -> x::e::l
  | e::l -> e :: insert x l comp ;;
val insert : 'a -> 'a list -> ('a -> 'a -> bool) -> 'a list = <fun>

```

2. Specifications:

The function `insertion_sort comp list` sorts the list `list` in order according to the function `comp`.

```

# let rec insertion_sort comp = function
  [] -> []
  | e::l -> insert e (insertion_sort comp l) comp ;;
val insertion_sort : ('a -> 'a -> bool) -> 'a list -> 'a list = <fun>

```

Tail-recursive version:

```

# let insertion_sort_term comp l =
  let rec sort accu = function
    [] -> accu
    | e::l -> sort (insert e accu comp) l
  in
  sort [] l ;;
val insertion_sort_term : ('a -> 'a -> bool) -> 'a list -> 'a list = <fun>

```

Solution 3 (Association – 5 points)**Specifications:**

The function `assoc k list` returns the value corresponding to the key `k` in `list`: a list of couples (*key*, *value*) (with *key* > 0) sorted in increasing order with respect to keys. It raises an exception if `k` is not a valid key or if it does not correspond to any couple.

```

# let rec assoc k list =
  if k <= 0 then
    invalid_arg "k not a natural"
  else
    let rec search = function
      [] -> failwith "not found"
      | (key, value)::l -> if key = k then
        value
      else
        if k < key then
          failwith "not found"
        else
          search l
    in
    search list ;;

val assoc : int -> (int * 'a) list -> 'a = <fun>

```

Solution 4 (Mystery – 2 points)

1. Specifications:

Give the results of the successive evaluations of the following phrases.

```

# let mystery = function
  [] -> failwith "..."
  | e::f::l -> (let rec aux_mystery m1 m2 = function
    [] -> m2
    | e::l -> if e < m1 then aux_mystery e m1 l
              else if e < m2 then aux_mystery m1 e l
              else aux_mystery m1 m2 l
    in if e < f then aux_mystery e f l else aux_mystery f e l);;
val mystery : ('a list) -> 'a = <fun>

mystery [1;3;4;2];;
- : int = 2

mystery [3.5;8.2;9.5;4.0];;
- : float = 4.0

mystery ['a'];;
Une erreur (Exception : match_failure)

```

2. Specifications:

What is the return value of `mystery` ?

The function `mystery` returns the second smallest element of the list if it exists