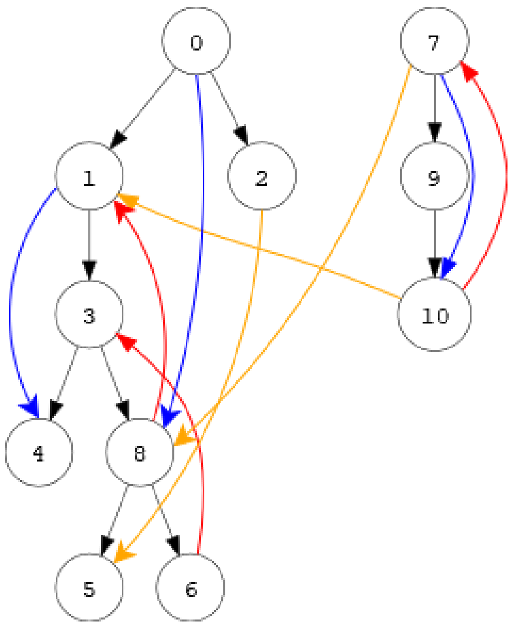
# Algorithmics
# Correction Final Exam #3 (P3)

Undergraduate $2^{nd}$ year - S3 – Epita

*January 5, 2021* - 9 : 30

**_Solution 1_ (In the depth of the spanning forest – _3 points_)**

1. *Spanning forest and extra-edges for the depth-first search of the graph in figure 1:*




⟶ arc avant (forward edge)

⟶ arc retour/arrière (back edge)

⟶ arc croisé (cross edge)

Figure 1: DFS: Spanning forest

2. *Meeting orders in prefix **pref** and suffix **suff**:*

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **pref** | 1 | 2 | 14 | 3 | 4 | 7 | 9 | 17 | 6 | 18 | 19 |
| **suff** | 16 | 13 | 15 | 12 | 5 | 8 | 10 | 22 | 11 | 21 | 20 |

### *Solution 2* (Union-Find – *4 points*)

1. Number of vertices of each connected component:

   $C_1$ : 4 $C_2$ : 6 $C_3$ : 4

2. Edges to add: **two** among 5 – 8 8 – 12 5 – 12 for example...

3. Among the following chains, those which can not exist in $G$:

   ☐ 3 ⇝ 7 ☒ **11 ⇝ 6** ☒ **0 ⇝ 13** ☐ 4 ⇝ 9

4. Vector $p$ after adding the edge 7–4:

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | 5 | 8 | 5 | 8 | **12** | -4 | 5 | **8** | **-10** | 12 | 8 | 12 | 8 | 8 |

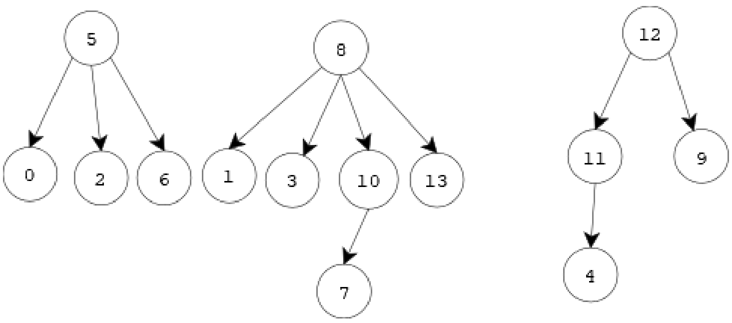



Figure 2: before

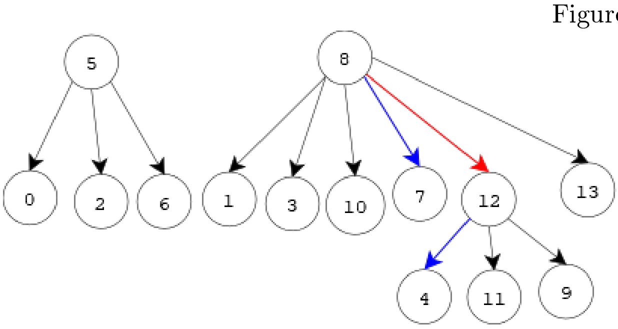



Figure 3: after edge 7 – 4 added

### *Solution 3* (Distance from start – *5 points*)

**Specifications:** `dist_range`($G$, $src$, $dmin$, $dmax$) returns the list of vertices that are at a distance between $dmin$ and $dmax$ from the vertex $src$ in the graph $G$ (with $0 < dmin \leq dmax$).

```
def dist_range(G, src, dmin, dmax):
    dist = [None] * G.order
    q = queue.Queue()
    q.enqueue(src)
    dist[src] = 0
    L = []
    while not q.isempty():
        x = q.dequeue()
        if dist[x] >= dmin:
            L.append(x)
        if dist[x] < dmax:
            for y in G.adjlists[x]:
                if dist[y] == None:
                    dist[y] = dist[x] + 1
                    q.enqueue(y)
    return L
```

***Solution 4*** **(Get cycle –** ***5 points*****)**

**Specifications:**

the function `get_cycle`$(G)$ returns a cycle of the undirected graph $G$, an empty list if $G$ is acyclic.

**Version 1:** using parent vector

```
def __get_cycle(G, x, parent):      # DFS on G from x, interrupted at first back edge found
    for y in G.adjlists[x]:         # parent: vertices marked with their parent
        if parent[y] == None:       # return first back edge found (x, y) or None
            get = __get_cycle(G, y, parent)
            if get != None:
                return get
        else:
            if y != parent[x]:
                return (x, y)
    return None

def get_cycle(G):
    parent = [None] * G.order
    s = 0
    get = None
    while s < G.order and get == None:
        if parent[s] == None:
            parent[s] = -1
            get = __get_cycle(G, s, parent)
        s += 1
    L = []
    if get != None:
        (x, y) = get
        L = [x]
        while x != y:
            x = parent[x]
            L.append(x)
        L.append(L[0])
    return L
```

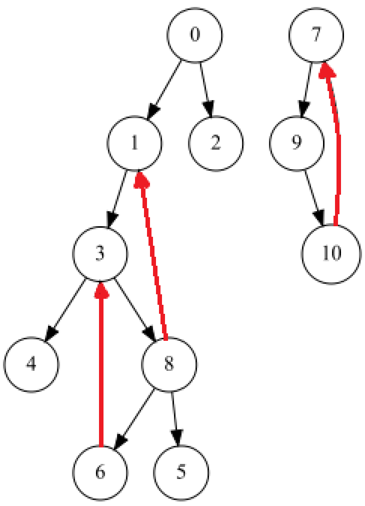
**Version 2:** the cycle is built by the recursive function in going up
Many ways to do it. The difficulty: no longer add vertices when the cycle is complete.

```python
def __get_cycle2(G, x, M, p):
    """
        DFS on G from x
        M: mark vector (boolean)
        p: x's parent
        return (cycle, done):
        - cycle = the vertices of the first cycle found, [] if no cycle
        - done: boolean: is the cycle completed?
    """
    M[x] = True
    for y in G.adjlists[x]:
        if not M[y]:
            (cycle, done) = __get_cycle2(G, y, M, x)
            if cycle:
                if done:
                    return (cycle, True)
                if cycle[0] != y:
                    cycle.append(y)
                return (cycle, cycle[0] == y)
        else:
            if y != p:
                return ([y], False)
    return ([], False)

    def get_cycle_2(G):
        M = [False] * G.order
        for s in range(G.order):
            if not M[s]:
                cycle, done = __get_cycle2(G, s, M, -1)
                if cycle:
                    return cycle + [cycle[0]]
        return []
```

***Solution 5*** **(What is this? –** ***3 points*****)**

1. The built graph ($NG$):




2. For each vertex $s$, during the traversal:

   (a) What does `D[`$s$`]` represent?

   `D[`$s$`]` is `None` if the vertex $s$ has not been met. Otherwise it is the depth of $s$ in the spanning forest of the DFS.

   (b) What does `P[`$s$`]` represent?

   `P[`$s$`]` indicates whether $s$ was encountered in suffix.