Key to Final Exam S1
Computer Architecture

Duration: 1 hr. 30 min.

Last name: .... First name: .... Group: ....

Write answers only on the worksheet.
Do not show any calculation unless you are explicitly asked.
Do not use red ink.

Exercise 1 (2 points)

Convert the following numbers from the source form into the destination form. Do not write down the result in a fraction or a power form (e.g. write down 0.25 and not ¼ or 2^{-2}).

| Number to Convert	Source Form	Destination Form	Result
101011101.0101	Binary	Decimal	**349.3125**
1E2.5	Hexadecimal	Decimal	**482.3125**
750	Decimal	Base 5	**11000**
1707.66	Hexadecimal	Base 8	**13407.314**

Exercise 2 (5 points)

Perform the following 8-bit binary operations (the two operands and the result are 8 bits wide). Then, convert the result into unsigned and signed decimal values. If an overflow occurs, write down 'ERROR' instead of the decimal value.

Operation	Binary Result	Decimal Value	
		Unsigned	**Signed**
10110111 + 00101101	11100100	**228**	**-28**
01011010 - 10001101	11001101	**ERROR**	**ERROR**
01110111 + 11111000	01101111	**ERROR**	**111**
10000001 - 10000000	00000001	**1**	**1**
11010111 - 01111100	01011011	**91**	**ERROR**

Exercise 3 (3 points)

We want to simplify the following circuit diagram:
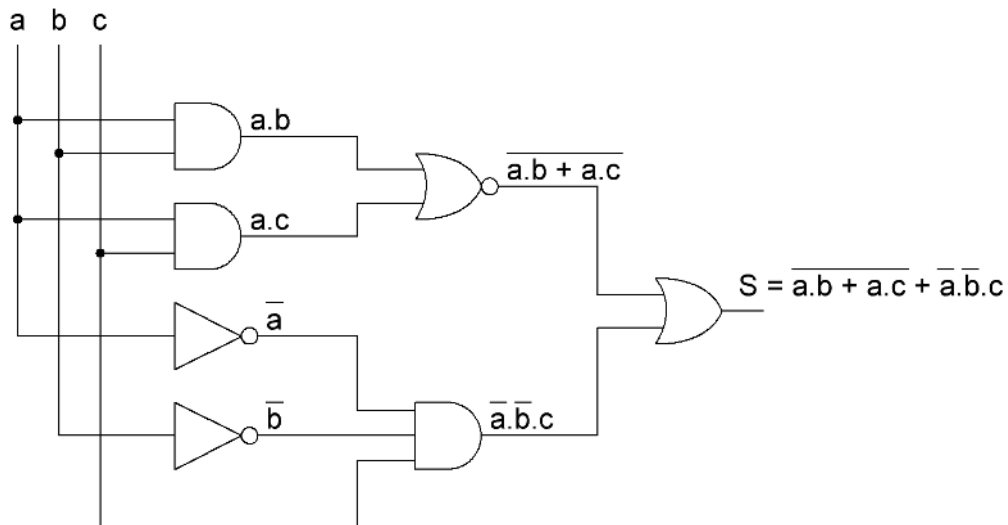
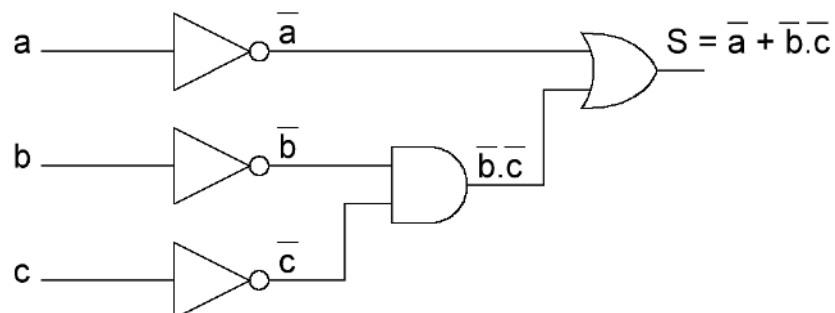1. Without any simplifications, give the S output in terms of *a*, *b* and *c*.

$$S = \overline{a.b + a.c} + \overline{a}.\overline{b}.c$$

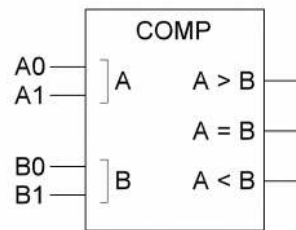2. Give the most simplified expression of S.

$$S = \overline{a} + \overline{b}.\overline{c}$$

3. From the most simplified expression, draw a new circuit diagram by using three NOT gates, one two-input AND gate and one two-input OR gate.

Exercise 4 (4 points)

We want to design the following comparator:

The *A* and *B* inputs are 2-bit unsigned integers (*A0* and *B0* are the LSBs):

- If $A > B$, the '*A > B*' output is set to 1 and the other outputs are set to 0.
- If $A = B$, the '*A = B*' output is set to 1 and the other outputs are set to 0.
- If $A < B$, the '*A < B*' output is set to 1 and the other outputs are set to 0.

1. Complete the following truth table:

A1	A0	B1	B0	A > B	A = B	A < B
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

2. Without using Karnaugh maps, give the most simplified expression of the '*A = B*' output. **Use the EXCLUSIVE-OR operator to simplify the expression.**

$$\text{'A = B'} = \overline{A0 \oplus B0} \cdot \overline{A1 \oplus B1}$$

3. Complete the Karnaugh maps below (circles included) and give the most simplified expressions of the $'A > B'$ and $'A < B'$ outputs. **No points will be given to an expression if its Karnaugh map is wrong.**

B1 B0

A > B	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	1
10	1	1	0	0

A1 A0

B1 B0

A < B	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

A1 A0

$$'A > B' = A1.\overline{B1} + A0.\overline{B0}.\overline{B1} + A0.A1.\overline{B0}$$

$$'A < B' = \overline{A1}.B1 + \overline{A0}.B0.B1 + \overline{A0}.\overline{A1}.B0$$

Exercise 5 (6 points)

Let us consider the truth tables below. *A*, *B*, *C* and *D* are the inputs. *U*, *V*, *W*, *X*, *Y* and *Z* are the outputs.

A	B	C	U	V
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	0	1

A	B	C	D	Y	Z
0	0	0	0	0	1
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	Φ	0
0	1	0	0	0	0
0	1	0	1	0	Φ
0	1	1	0	0	0
0	1	1	1	Φ	Φ
1	0	0	0	Φ	1
1	0	0	1	Φ	0
1	0	1	0	1	1
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	Φ
1	1	1	0	1	0
1	1	1	1	1	Φ

A	B	C	W	X
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	0

1. Write down the minterm canonical form of *U*.

$$U = \overline{A}.\overline{B}.C + \overline{A}.B.C$$

2. Write down the maxterm canonical form of *V*.

$$V = (\overline{A} + B + \overline{C}).(\overline{A} + \overline{B} + C)$$

3. Complete the Karnaugh maps below (circles included) and give the most simplified expression for each output. **No points will be given to an expression if its Karnaugh map is wrong. For the time being, do not simplify by using the EXCLUSIVE-OR operator.**

BC

W	00	01	11	10
0	0	1	0	1
1	0	1	0	1

A

$$W = \overline{B}.C + B.\overline{C}$$

C

X	0	1
00	0	1
01	1	1
11	0	0
10	1	0

AB

$$X = \overline{A}.B + \overline{A}.C + A.\overline{B}.\overline{C}$$

CD

Y	00	01	11	10
00	0	0	Φ	0
01	0	0	Φ	0
11	1	1	1	1
10	Φ	Φ	1	1

AB

$$Y = A$$

CD

Z	00	01	11	10
00	1	0	0	1
01	0	Φ	Φ	0
11	0	Φ	Φ	0
10	1	0	0	1

AB

$$Z = \overline{B}.\overline{D}$$

4. See if some of the *W*, *X*, *Y* and *Z* outputs can be simplified by using the EXCLUSIVE-OR operator. If so, simplify them and write down the new expressions (do not show any calculation).

$$W = \overline{B}.C + B.\overline{C}$$
$$\mathbf{W = B \oplus C}$$

$$X = \overline{A}.B + \overline{A}.C + A.\overline{B}.\overline{C}$$
$$X = \overline{A}.(B + C) + A.\overline{(B + C)}$$
$$\mathbf{X = A \oplus (B + C)}$$

Feel free to use the blank space below if you need to: