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Remedial Assignment - 1

1) $\int_0^a \int_0^{\sqrt{a^2-x^2}} x \, dy \, dx$

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} x \, dy \, dx = \int_0^a \int_0^{\sqrt{a^2-y^2}} x \, dy \, dx$$

$$= \int_0^a \left[\frac{x^2}{2}\right]_0^{\sqrt{a^2-y^2}} dy = \int_0^a \left[\frac{a^2-y^2}{2}\right] dy$$

$$= \int_0^a \left(\frac{a^2}{2} - \frac{y^2}{2}\right) dy$$

$$= \int_0^a \left[\frac{a^2}{2}\right] dy - \frac{1}{2}\int_0^a y^2 \, dy$$

$$= \left[\frac{a^2}{2} y\right]_0^a - \frac{1}{2}\left[\frac{y^3}{3}\right]_0^a$$

$$= \left[\frac{a^2}{2} \cdot a\right] - \frac{1}{2} \frac{a^3}{3}$$

$$= \frac{a^3 \cdot 3}{2 \cdot 3} - \frac{a^3}{6} = \frac{2a^3}{6} = \frac{a^3}{3}$$

27) $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} \cancel{x}\ dx\ dy\ dz$

$$= \int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z\ dz\ dy\ dx$$

$$= \int_0^1 \int_0^x \left[\frac{x^2}{2}\right]_0^{\sqrt{x+y}} dy\ dx$$

$$= \int_0^1 \int_0^x \frac{x+y}{2}\ dy\ dx$$

$$= \int_0^1 \left[\int_0^x \frac{x}{2}\ dy + \int_0^x \frac{y}{2}\ dy\right] dx$$

$$= \int_0^1 \left[\left(\frac{xy}{2}\right)_0^x + \left(\frac{y^2}{4}\right)_0^x\right] dx$$

$$= \int_0^1 \left(\frac{x^2}{2} + \frac{x^2}{4}\right) dx$$

$$= \int_0^1 \frac{3x^2}{4}\ dx = \left[\frac{\cancel{3}x^3}{4 \cdot \cancel{3}}\right]_0^1 = \frac{1}{4}$$

$$\Rightarrow \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz\, dy\, dx$$

$$\Rightarrow \int_0^1 \int_0^{1-x} [z]_0^{1-x-y}\, dy\, dx$$

$$= \int_0^1 \int_0^{1-x} [1-x-y]\, dy\, dx$$

$$= \int_0^1 \left[\int_0^{1-x} 1 \cdot dy - \int_0^{1-x} x \cdot dy - \int_0^{1-x} y \cdot dy \right] dx$$

$$= \int_0^1 \left[(y)_0^{1-x} - (xy)_0^{1-x} - \left(\frac{y^2}{2}\right)_0^{1-x} \right] dx$$

$$= \int_0^1 \left[1-x - (x(1-x)) - \frac{(1-x)^2}{2} \right] dx$$

$$= \int_0^1 \left[\frac{2-2x-2x+2x^2-(1-x)^2}{2} \right] dx$$

$$= \int_0^1 \left(\frac{2-4x+2x^2-1-x^2+2x}{2} \right) dx$$

$$= \int_0^1 \left(\frac{1-2x+x^2}{2} \right) dx = \int_0^1 \left(\frac{x^2-2x+1}{2} \right) dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{2x^2}{2} + x\right]_0^1$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - x^2 + x\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} - \cancel{1} + \cancel{1}\right] = \underline{\underline{\frac{1}{6}}}$$

4) $$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$$

$$= \int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$$

$$= \int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx = \int_0^{4a}\left[y\right]_{x^2/4a}^{2\sqrt{ax}} dx$$

$$= \int_0^{4a}\left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= \int_0^{4a}(2\sqrt{a})\left[\sqrt{x}\,dz\right] - \int_0^{4a}\frac{x^2}{4a}\,dz$$

$$= 2\sqrt{a}\left[\frac{x^{1/2+1}\cdot 2}{3}\right]_0^{4a} - \frac{1}{4a}\int_0^{4a} x^2\,dz$$

$$= \frac{2\sqrt{a}}{3}\left[(4a)^{3/2}\right] - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3}\left[(4a)^{3/2}\right] - \frac{1}{\cancel{4a}}\left[\frac{(4a)^{\cancel{3}\,2}}{3}\right]$$

$$= \frac{4\sqrt{a}}{3}\left[(4a)^{3/2}\right] - \frac{16a^2}{3}$$

$$= \frac{4\sqrt{a}}{3}\left[4a \times 2\sqrt{a}\right] - \frac{16a^2}{3}$$

$$\frac{32a^2 - 16a^2}{3} = \frac{16a^2}{3}$$

5) $$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{a^2-x^2-y^2-z^2}}$$

$$= \int_0^a \int_0^{\sqrt{a^2-x^2}} \sin^{-1}\left(\frac{z}{\sqrt{a^2-x^2-y^2}}\right)\Big]_0^{\sqrt{a^2-x^2-y^2}} dy\,dx$$

$$= \int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{\pi}{2}\,dy\,dx$$

$$= \int_0^a \frac{\pi}{2}\,(y)_0^{\sqrt{a^2-x^2}}\,dx$$

$$= \int_0^a \frac{\pi}{2}\left(\sqrt{a^2-x^2}\right)dx$$

$$= \frac{\pi}{2}\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= \frac{\pi}{2}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \frac{\pi}{2}\left[0 + \frac{a^2}{2}\left(\frac{\pi}{2}\right) - (0+0)\right]$$

$$= \frac{\pi^2 a^2}{8}$$