PAPAI MONDAL

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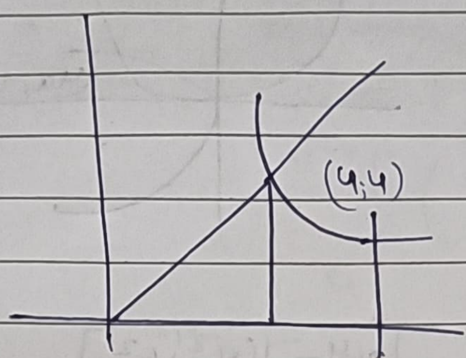
Date

DELTA Pg No.

Maths Assignment 1

17)



$xy = 16$

$D = (4,4)$

When $x = 8$

$xy = 16$

$8y = 16$

$y = 2$

$P(8,2)$

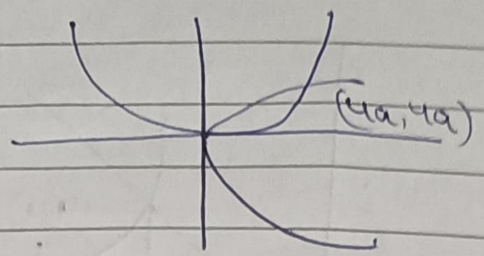
$$\iint x^2\,dx\,dy = \int_{x=0}^{4}\int_{y=0}^{x} x^2\,dy\,dx + \int_{4}^{8}\int_{y=0}^{16/x} x^2\,dy\,dx$$

$$= \int_{0}^{4} x^2 [y]_0^x + \int_{4}^{8} x^2 [y]_0^{16/x}\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^4 + 16\left[\frac{x^2}{2}\right]_4^8 = \underline{\underline{448}}$$

Q7 $x^2 = 4ay \quad y^2 = 4ax$

$= x^4 = 16a^2(4ax)$



$\Rightarrow x^4 = 64a^3 x$

$x^4 - 64a^3 x = 0 \qquad [x^4 = 16a^2 y^2]$

$= (0,0), (4a, 4a)$

$$I = \int_0^{4a} \int_{y^2/4a}^{2\sqrt{ay}} dx\,dy$$

$$= \int_0^{4a} \left[2\sqrt{ay} - \frac{y^2}{4a}\right] dy$$

$$I = 2a^{1/2}\left[\frac{y^{3/2}}{3/2}\right]_0^{4a} - \left[\frac{y^3}{12a}\right]_0^{4a}$$

$$= \frac{2x^2}{3} a^{1/2}[4a]^{3/2} - \left[\frac{4a}{12a}\right]^3$$

$$= \frac{32a^2 - 16a^2}{3} = \frac{16a^2}{3}$$

3) $\int_0^{\pi/2} \left[\frac{x^3}{3}\right]_{a(1-\cos\theta)}^{a} d\theta = \int_0^{\pi/2} \left(\frac{a^3}{3} - a^3\left(\frac{1-\cos\theta}{3}\right)^3\right) d\theta$

$= \frac{a^3}{3} \int_0^{\pi/2} \left(1 - \left(\frac{1-\cos\theta}{3}\right)^3\right) d\theta$

$= \frac{a^3}{3} \int_0^{\pi/2} \left[3\cos\theta - 3\cos^2\theta + \cos^3\theta\right) d\theta$

$= \frac{a^3}{3} \int_0^{\pi/2} \left[3\cos\theta - \frac{3}{2} - \frac{3\cos 2\theta}{2} + \frac{\cos 3x}{4} + \frac{3\cos x}{4}\right] d\theta$

$= \frac{a^3}{3} \int_0^{\pi/2} \left[12\cos\theta - 6 - 6\cos 2\theta + \cos 3x + 3\cos x\right] d\theta$

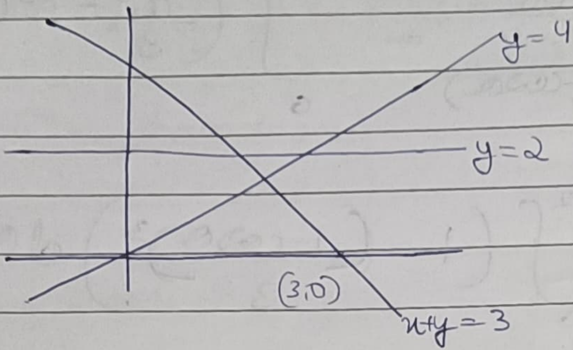
$= \frac{a^3}{12} \int_0^{\pi/2} \left[15\cos\theta - 6\cos 2\theta + \cos 3\theta - 6\right) d\theta$

$= \frac{a^3}{12} \left[15 + 0 - \frac{1}{3} - 6\frac{\pi}{2}\right]$

$= \frac{a^3}{\cancel{12}} \times \cancel{2} \left(\frac{44 - 9\pi}{6}\right)$

$= \frac{a^3}{36} \left(44 - 9\pi\right)$

4)



$$I = \int_{y=0}^{2}\int_{x=y/4}^{x=3-y} (x^2+y^2)\,dx\,dy$$

$$I = \int_0^2 \left[\frac{x^3}{3}\right]_{y/4}^{3-y} dy + y^2\,[x]_{y/4}^{3-y}\,dy$$

$$= \int_0^2 \left[\left(\frac{(3-y)^3}{3}\right) - \left(\frac{y}{4}\right)^3 \cdot \frac{1}{4}\right] dy - \int_0^2 y^2\left(3-y-\frac{y}{4}\right) dy$$

$$= \int_0^2 \left[\frac{27-27y+3y^2-y^3}{3} - \frac{y^3}{192}\right] dy + \int_0^2 \left(3y^2 - y^3 - \frac{y^3}{4}\right) dy$$

$$= \int_0^2 9\,dy - 9y\,dy + 3y^2\,dy - \frac{y^3}{3}\,dy - \frac{y^3}{192} + \int_0^2 3y^2\,dy - y^3\,dy - \frac{y^3}{4}\,dy$$

$$= \cancel{18} - \cancel{18} + 8 - \frac{16}{12} - \frac{16}{768} + 8 - \frac{16}{4} - \frac{16}{16}$$

$$= 8 - \frac{4}{3} - \frac{1}{48} + 8 - 4 - 1 =$$

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$I = 11 - \frac{4}{3} - \frac{1}{48}$

$I = \frac{528 - 64 - 1}{48}$

$= \frac{463}{48}$

5) Changing in order of integration

$I_1, x = 0 \longrightarrow x = y$

$y = 0 \longrightarrow y = \frac{a}{\sqrt{2}}$

$I_2, = x = 0 \longrightarrow x = \sqrt{a^2 - y^2}$

$y = \frac{a}{\sqrt{2}} \longrightarrow y = a$

$$I = \int_0^{a/\sqrt{2}} \int_0^y y^2 \, dx \, dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-y^2}} y^2 \, dx \, dy$$

$$= \int_0^{a/\sqrt{2}} y^3 \, dy + \int_{a/\sqrt{2}}^{a} y^2 \sqrt{a^2 - y^2} \, dy$$

$$= \int_0^{a/\sqrt{2}} y^3 \, dy + \int_{a/3}^{a} y^2 \sqrt{a^2 - y^2} \, dy$$

$$= \left[\frac{y^4}{4} \right]_0^{a/\sqrt{2}} + I_3$$

$$I_3 = \int_{a/\sqrt{2}}^{4} y^2 \sqrt{a^2 - y^2}\, dy$$

$$y = a \sin\theta$$

$$y^2 - a^2 \sin^2\theta$$

$$I = \int_{\pi/4}^{+\pi/2} a^4 \sin^2\theta \cos^2\theta \, d\theta$$

$$I_3 = \int_{\pi/4}^{\pi/2} \frac{a^4}{4} \sin^2 2\theta \, d\theta$$

$$\frac{a^4}{8} \int_{\pi/4}^{\pi/2} \frac{a^2}{4} \left(\frac{1 - \cos 4\theta}{2} \right) d\theta$$

$$= \frac{a^4}{8} \int_{\pi/4}^{\pi/2} (1 - \cos 4\theta)\, d\theta$$

$$= \frac{a^4}{8} \left[[x]_{\pi/2}^{\pi/4} - \left[\frac{\sin 4\theta}{4} \right]_{\pi/4}^{\pi/2} \right]$$

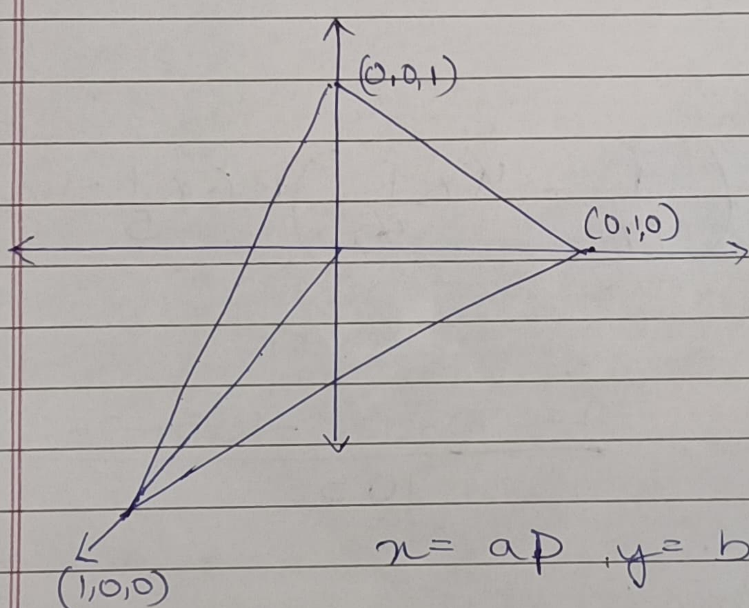
$$= \frac{a^4}{8} \left[\pi/4 \right] - 0$$

$$= a^4 \frac{\pi}{32}$$

$$\rightarrow I = \frac{a^4}{16} + \frac{a^4 \pi}{32}$$

$$= \left(\frac{2+\pi}{32}\right) a^4$$

6)



$x = ap,\ y = bq,\ z = cr$

$dx\,dy\,dz = abc \cdot dp \cdot dq$

Volume

$$V = \int_0^1 \int_0^{1-p} \int^{1-p-q} abc\, a^2 p^2\, bq\, cr\, dp\, dq\, dr$$

$$= \frac{1}{2} a^3 b^2 c^2 \int_0^1 \int_0^{1-p} p^2 q (1-p-q)^2\, dp\, dq$$

$$= \frac{1}{2} a^3 b^3 c^3 \int_0^1 \int_0^{1-p} p^2 q \left[(1-p)^2 - 1(1-p)q + q^2 \right] dp\, dq$$

$$= \frac{a^3 b^2 c^2}{2} \int_0^1 p^2 \frac{(1-p)^4}{12} \, dp$$

$$= \cancel{\frac{a^3 b^2 c^2}{2} \int_0^1 p^2 \frac{(1-p)}{12}}$$

$$= \frac{a^3 b^2 c^2}{24} \int_0^1 (p^6 - 4p^5 + 6p^4 - 4p^3 + p^2)\, dp$$

$$= \frac{a^3 b^2 c^2}{24} \left(\frac{1}{7} - \cancel{4}^{2} \times \frac{1}{\cancel{6}_{3}} + 6 \times \frac{1}{5} - 4 \times \frac{1}{4} + \frac{1}{3} \right)$$

$$= \frac{a^3 b^2 c^2}{24} \times \frac{15 - 70 + 126 - 105 + 35}{105}$$

$$= \frac{a^3 b^2 c^2}{24} \times \frac{1}{105}$$

$$= \frac{a^3 b^2 c^2}{2520}$$