Maths assignment-2

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P_1

1) 8 girls can sit around table by 7! ways which in between boys will be sitting

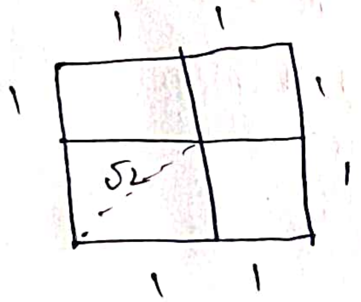
$\Rightarrow 7!$

And boys can sit together in 8! ways

So total is $7! \times 8!$

$= 203212800$

2) Given there are 5 points $\Rightarrow$ 5 pegions and



within a distance of $\sqrt{2}$, there will be in total of 4 Squares $\Rightarrow$ 4 pigeon holes.

$\Rightarrow \left\lfloor \frac{n-1}{m} \right\rfloor + 1$

$= \left\lfloor \frac{5-1}{4} \right\rfloor + 1$

$= 1+1$

$= 2$

$\Rightarrow$ 2 Points are within $\sqrt{2}$ distance.

$|A| =$ divisible by 2 $= \left\lfloor \frac{1000}{2} \right\rfloor = 500$

$|B| =$ divisible by 3 $= \left\lfloor \frac{1000}{3} \right\rfloor = 333$

$|C| =$ divisible by 7 $= \left\lfloor \frac{1000}{7} \right\rfloor = 142$

$|A \cap B| =$ divisible by 2,3 $= \left\lfloor \frac{1000}{6} \right\rfloor = 166$

$|B \cap C| =$ divisible by 3,7 $= \left\lfloor \frac{1000}{21} \right\rfloor = 47$

$|A \cap C| =$ divisible by 2,7 $= \left\lfloor \frac{1000}{14} \right\rfloor = 71$

$|A \cap B \cap C| =$ divisible by 2,3,7 $= \left\lfloor \frac{1000}{42} \right\rfloor = 23$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |C \cap A| + |A \cap B \cap C|$

$= 500 + 333 + 142 - 166 - 71 - 47 + 23$

$|A \cup B \cup C| = 714$

4) $28844m + 15712n = 4$

$$
\begin{array}{l}
15712)\,28844\,(1 \\
\quad 15712 \\
\quad 13132)\,15712\,(1 \\
\qquad 13132 \\
\qquad 2580)\,13132\,(5 \\
\qquad\quad 12900 \\
\qquad\quad 232)\,2580\,(11 \\
\qquad\qquad 2552 \\
\qquad\qquad 28)\,232\,(8 \\
\qquad\qquad\quad 224 \\
\qquad\qquad\quad 8)\,28\,(3 \\
\qquad\qquad\qquad 24 \\
\qquad\qquad\qquad 4)\,8\,(2
\end{array}
$$

$28844 = 15712(1) + 13132$

$15712 = 13132(1) + 2580$

$13132 = 2580(5) + 232$

$2580 = 232(11) + 28$

$232 = 28(8) + 8$

$28 = 8(3) + \boxed{4} \Rightarrow$ GCD

$8 = 4(2) + 0$

From eucledean division

$28 = 8(3) + 4$

$28 - 8(3) = 4$

$28 - \left(232 - 28(8)\right)3 = 4$

$28 - 232(3) + 28(24) = 4$

$28(25) - 232(3) = 4$

$\left(2580 - 232(11)\right)25 - 232(3) = 4$

$2580(25) - 232(275) - 232(3) = 4$

$2580(25) - 232(278) = 4$

$2580(25) - \left(13132 - 2580(5)\right)278 = 4$

$2580(25) - 13132(278) + 2580(1390) = 4$

$2580(1415) - 13132(278) = 4$

$\cancel{13132 =}\ (15712 - 13132)(1415) - 13132(278) = 4$

$15712(1415) - 13132(1415) - 13132(278) = 4$

$15712(1415) - 13132(1693) = 4$

$15712(1415) - (28844 - 15712)1693 = 4$

$15712(3108) - 28844(1693) = 4 \Rightarrow m = -1693,\ n = 3108$

(12345, 54321)

$54321 = 4 \times 12345 + 4941$

$12345 = 2 \times 4941 + 2463$

$4941 = 2 \times 2463 + 15$

$2463 = 164 \times 15 + \boxed{3}$

$15 = 5 \times 3 + 0$

$\Rightarrow GCD(12345, 54321) = 3$

6) LHS

p	q	r	A: $p \to q$	B: $p \to r$	LHS $(A \wedge B)$	$q \wedge r$	$p \to q \wedge r$ (RHS)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

LHS = RHS

$\Rightarrow$ Logical equiralane of $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ holds

1) $p \to q$

Converse – $q \to p$

Inverse – $\sim p \to \sim q$

Contrapositive – $\sim q \to \sim p$

2) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology

LHS

$\Rightarrow [(p \vee q) \wedge ((\sim p \wedge \sim q) \vee r)] \to r$ [distributive]

$\Rightarrow [(p \vee q) \wedge ((\sim p \vee q) \vee r)] \to r$

$\Rightarrow [((p \vee q) \wedge \sim (p \vee q)) \vee ((p \vee q) \wedge r))] \to r$ [Distributive]

$\Rightarrow [F \vee ((p \vee q)) \wedge r)] \to r$ [Negation]

$\Rightarrow [(p \vee q) \wedge r] \to r$ [Identity]

$\Rightarrow \sim [(p \vee q) \wedge r] \vee r$

$\Rightarrow \sim (p \vee q) \vee \sim r \vee r$ [D Morgn's]

$\Rightarrow \sim (p \vee q) \vee T$ [Negation]

$\Rightarrow T$ [Dominant]

$\Rightarrow$ RHS

$\Rightarrow$ Given expression is a tautology.

Base step:

Put $n=1$

LHS $\Rightarrow 1!$ RHS $2^{1-1} = 1$

$\Rightarrow P(n)$ is true for $n=1$

Assumption: $P(k)$ is true

$P(k) = k! \geq 2^{k-1}$

To prove: $P(k+1)$ is true

$P(k+1) = (k+1)!$

$\Rightarrow (k!)(k+1) \geq (k+1) 2^{k-1}$

~~$\Rightarrow (k!)(k+1) \geq 2 \cdot 2^{k-1}$~~

$\Rightarrow (k!)(k+1) \geq (k+1)\, 2^{m-1}$

$\geq 2 \cdot 2^{m-1}$

$\geq 2^{m}$

$\Rightarrow$ The result is true for $k+1$

$\Rightarrow P(n)$ is true.

10)

1) $p \to \sim q$ [Rule P]

2) $\sim s \to p$ [Rule P]

3) $q \vee r$ [Rule P]

4) $\sim r$ [Rule P]

5) q [Rule T, disjunction syllogism, 2,4]

6) $\sim p$ [Rule T, Modus Tollens, 1,5]

7) s [Rule T, Modus Tollens, 2,6]

$\Rightarrow$ Given premises will give conclusion s.