

02_logistic_regression

December 22, 2019

1 Kaggle Titanic survival - logistic regression model

Can we predict which passengers would survive the sinking of the Titanic?

See:

<https://www.kaggle.com/c/titanic/overview/evaluation>

https://gitlab.com/michaelallen1966/1908_coding_club_kaggle_titanic

The original data comes from:

<https://www.kaggle.com/c/titanic/data>

Though we will download and use a data set that has been pre-processed ready for machine learning.

The data includes:

| Variable | Definition |
|----------|---|
| survival | Survival (0 = No, 1 = Yes) |
| pclass | Ticket class |
| sex | Sex |
| Age | Age in years |
| sibsp | # of siblings / spouses aboard the Titanic |
| parch | # of parents / children aboard the Titanic |
| ticket | Ticket number |
| fare | Passenger fare |
| cabin | Cabin number |
| embarked | Port of Embarkation(C=Cherbourg, Q=Queenstown, S=Southampton) |

1.1 Logistic regression

In this example we will use logistic regression (see https://en.wikipedia.org/wiki/Logistic_regression).

For an introductory video on logistic regression see: <https://www.youtube.com/watch?v=yIYKR4sgzI8>

Logistic regression takes a range of features (which we will normalise/standardise to put on the same scale) and returns a probability that a certain classification (survival in this case) is true.

We will go through the following steps:

- Download and save pre-processed data
- Split data into features (X) and label (y)
- Split data into training and test sets (we will test on data that has not been used to fit the model)
- Standardise data
- Fit a logistic regression model (from sklearn learn)
- Predict survival of the test set, and assess accuracy
- Review model coefficients (weights) to see importance of features
- Show probability of survival for passengers

1.2 Load modules

A standard Anaconda install of Python (<https://www.anaconda.com/distribution/>) contains all the necessary modules.

```
[1]: import numpy as np
import pandas as pd
# Import machine learning methods
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
```

1.3 Load data

The section below downloads pre-processed data, and saves it to a subfolder (from where this code is run). If data has already been downloaded that cell may be skipped.

Code that was used to pre-process the data ready for machine learning may be found at: https://github.com/MichaelAllen1966/1804_python_healthcare/blob/master/titanic/01_preprocessing.ipynb

```
[2]: download_required = True

if download_required:

    # Download processed data:
    address = 'https://raw.githubusercontent.com/MichaelAllen1966/' + \
              '1804_python_healthcare/master/titanic/data/processed_data.csv'

    data = pd.read_csv(address)

    # Create a data subfolder if one does not already exist
    import os
    data_directory = './data/'
    if not os.path.exists(data_directory):
        os.makedirs(data_directory)
```

```
# Save data
data.to_csv(data_directory + 'processed_data.csv')
```

```
[3]: data = pd.read_csv('data/processed_data.csv')
```

1.4 Examine loaded data

The data is in the form of a Pandas DataFrame, so we have column headers providing information of what is contained in each column.

We will use the DataFrame `.head()` method to show the first few rows of the imported DataFrame. By default this shows the first 5 rows. Here we will look at the first 10.

```
[4]: data.head(10)
```

```
[4]:
```

| | Unnamed: 0 | Unnamed: 0.1 | Unnamed: 0.1.1 | Unnamed: 0.1.1.1 | PassengerId | \ |
|---|------------|--------------|----------------|------------------|-------------|---|
| 0 | 0 | 0 | 0 | 0 | 1 | |
| 1 | 1 | 1 | 1 | 1 | 2 | |
| 2 | 2 | 2 | 2 | 2 | 3 | |
| 3 | 3 | 3 | 3 | 3 | 4 | |
| 4 | 4 | 4 | 4 | 4 | 5 | |
| 5 | 5 | 5 | 5 | 5 | 6 | |
| 6 | 6 | 6 | 6 | 6 | 7 | |
| 7 | 7 | 7 | 7 | 7 | 8 | |
| 8 | 8 | 8 | 8 | 8 | 9 | |
| 9 | 9 | 9 | 9 | 9 | 10 | |

| | Survived | Pclass | Age | SibSp | Parch | ... | Embarked_missing | CabinLetter_A | \ |
|---|----------|--------|------|-------|-------|-----|------------------|---------------|---|
| 0 | 0 | 3 | 22.0 | 1 | 0 | ... | 0 | 0 | |
| 1 | 1 | 1 | 38.0 | 1 | 0 | ... | 0 | 0 | |
| 2 | 1 | 3 | 26.0 | 0 | 0 | ... | 0 | 0 | |
| 3 | 1 | 1 | 35.0 | 1 | 0 | ... | 0 | 0 | |
| 4 | 0 | 3 | 35.0 | 0 | 0 | ... | 0 | 0 | |
| 5 | 0 | 3 | 28.0 | 0 | 0 | ... | 0 | 0 | |
| 6 | 0 | 1 | 54.0 | 0 | 0 | ... | 0 | 0 | |
| 7 | 0 | 3 | 2.0 | 3 | 1 | ... | 0 | 0 | |
| 8 | 1 | 3 | 27.0 | 0 | 2 | ... | 0 | 0 | |
| 9 | 1 | 2 | 14.0 | 1 | 0 | ... | 0 | 0 | |

| | CabinLetter_B | CabinLetter_C | CabinLetter_D | CabinLetter_E | CabinLetter_F | \ |
|---|---------------|---------------|---------------|---------------|---------------|---|
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 0 | 0 | 0 | |
| 2 | 0 | 0 | 0 | 0 | 0 | |
| 3 | 0 | 1 | 0 | 0 | 0 | |
| 4 | 0 | 0 | 0 | 0 | 0 | |
| 5 | 0 | 0 | 0 | 0 | 0 | |
| 6 | 0 | 0 | 0 | 1 | 0 | |

| | | | | | |
|---|---|---|---|---|---|
| 7 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 |

| | CabinLetter_G | CabinLetter_T | CabinLetter_missing |
|---|---------------|---------------|---------------------|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 |
| 5 | 0 | 0 | 1 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 1 |
| 8 | 0 | 0 | 1 |
| 9 | 0 | 0 | 1 |

[10 rows x 30 columns]

We can also show a summary of the data with the `.describe()` method

```
[5]: data.describe()
```

```
[5]:      Unnamed: 0  Unnamed: 0.1  Unnamed: 0.1.1  Unnamed: 0.1.1.1  \
count  891.000000    891.000000    891.000000    891.000000
mean   445.000000    445.000000    445.000000    445.000000
std    257.353842    257.353842    257.353842    257.353842
min      0.000000      0.000000      0.000000      0.000000
25%    222.500000    222.500000    222.500000    222.500000
50%    445.000000    445.000000    445.000000    445.000000
75%    667.500000    667.500000    667.500000    667.500000
max    890.000000    890.000000    890.000000    890.000000
```

| | PassengerId | Survived | Pclass | Age | SibSp |
|-------|-------------|------------|------------|------------|------------|
| count | 891.000000 | 891.000000 | 891.000000 | 891.000000 | 891.000000 |
| mean | 446.000000 | 0.383838 | 2.308642 | 29.361582 | 0.523008 |
| std | 257.353842 | 0.486592 | 0.836071 | 13.019697 | 1.102743 |
| min | 1.000000 | 0.000000 | 1.000000 | 0.420000 | 0.000000 |
| 25% | 223.500000 | 0.000000 | 2.000000 | 22.000000 | 0.000000 |
| 50% | 446.000000 | 0.000000 | 3.000000 | 28.000000 | 0.000000 |
| 75% | 668.500000 | 1.000000 | 3.000000 | 35.000000 | 1.000000 |
| max | 891.000000 | 1.000000 | 3.000000 | 80.000000 | 8.000000 |

| | Parch | ... | Embarked_missing | CabinLetter_A | CabinLetter_B |
|-------|------------|-----|------------------|---------------|---------------|
| count | 891.000000 | ... | 891.000000 | 891.000000 | 891.000000 |
| mean | 0.381594 | ... | 0.002245 | 0.016835 | 0.052750 |
| std | 0.806057 | ... | 0.047351 | 0.128725 | 0.223659 |
| min | 0.000000 | ... | 0.000000 | 0.000000 | 0.000000 |

| | | | | | |
|-----|----------|-----|----------|----------|----------|
| 25% | 0.000000 | ... | 0.000000 | 0.000000 | 0.000000 |
| 50% | 0.000000 | ... | 0.000000 | 0.000000 | 0.000000 |
| 75% | 0.000000 | ... | 0.000000 | 0.000000 | 0.000000 |
| max | 6.000000 | ... | 1.000000 | 1.000000 | 1.000000 |

| | CabinLetter_C | CabinLetter_D | CabinLetter_E | CabinLetter_F | \ |
|-------|---------------|---------------|---------------|---------------|---|
| count | 891.000000 | 891.000000 | 891.000000 | 891.000000 | |
| mean | 0.066218 | 0.037037 | 0.035915 | 0.014590 | |
| std | 0.248802 | 0.188959 | 0.186182 | 0.119973 | |
| min | 0.000000 | 0.000000 | 0.000000 | 0.000000 | |
| 25% | 0.000000 | 0.000000 | 0.000000 | 0.000000 | |
| 50% | 0.000000 | 0.000000 | 0.000000 | 0.000000 | |
| 75% | 0.000000 | 0.000000 | 0.000000 | 0.000000 | |
| max | 1.000000 | 1.000000 | 1.000000 | 1.000000 | |

| | CabinLetter_G | CabinLetter_T | CabinLetter_missing |
|-------|---------------|---------------|---------------------|
| count | 891.000000 | 891.000000 | 891.000000 |
| mean | 0.004489 | 0.001122 | 0.771044 |
| std | 0.066890 | 0.033501 | 0.420397 |
| min | 0.000000 | 0.000000 | 0.000000 |
| 25% | 0.000000 | 0.000000 | 1.000000 |
| 50% | 0.000000 | 0.000000 | 1.000000 |
| 75% | 0.000000 | 0.000000 | 1.000000 |
| max | 1.000000 | 1.000000 | 1.000000 |

[8 rows x 25 columns]

The first column is a passenger index number. We will remove this, as this is not part of the original Titanic passenger data.

```
[6]: # Drop Passengerid (axis=1 indicates we are removing a column rather than a row)
      # We drop passenger ID as it is not original data

      data.drop('PassengerId', inplace=True, axis=1)
```

1.5 Looking at a summary of passengers who survived or did not survive

Before running machine learning models, it is always good to have a look at your data. Here we will separate passengers who survived from those who died, and we will have a look at differences in features.

We will use a *mask* to select and filter passengers

```
[7]: mask = data['Survived'] == 1 # Mask for passengers who survive
      survived = data[mask] # filter using mask

      mask = data['Survived'] == 0 # Mask for passengers who died
```

```
died = data[mask] # filter using mask
```

Now let's look at average (mean) values for survived and died

```
[8]: survived.mean()
```

```
[8]: Unnamed: 0          443.368421
      Unnamed: 0.1       443.368421
      Unnamed: 0.1.1     443.368421
      Unnamed: 0.1.1.1   443.368421
      Survived           1.000000
      Pclass             1.950292
      Age               28.291433
      SibSp              0.473684
      Parch              0.464912
      Fare              48.395408
      AgeImputed         0.152047
      EmbarkedImputed    0.005848
      CabinLetterImputed 0.602339
      CabinNumber        18.961988
      CabinNumberImputed 0.611111
      male               0.318713
      Embarked_C         0.271930
      Embarked_Q         0.087719
      Embarked_S         0.634503
      Embarked_missing   0.005848
      CabinLetter_A      0.020468
      CabinLetter_B      0.102339
      CabinLetter_C      0.102339
      CabinLetter_D      0.073099
      CabinLetter_E      0.070175
      CabinLetter_F      0.023392
      CabinLetter_G      0.005848
      CabinLetter_T      0.000000
      CabinLetter_missing 0.602339
      dtype: float64
```

```
[9]: died.mean()
```

```
[9]: Unnamed: 0          446.016393
      Unnamed: 0.1       446.016393
      Unnamed: 0.1.1     446.016393
      Unnamed: 0.1.1.1   446.016393
      Survived           0.000000
      Pclass             2.531876
      Age               30.028233
      SibSp              0.553734
```

| | |
|---------------------|-----------|
| Parch | 0.329690 |
| Fare | 22.117887 |
| AgeImputed | 0.227687 |
| EmbarkedImputed | 0.000000 |
| CabinLetterImputed | 0.876138 |
| CabinNumber | 6.074681 |
| CabinNumberImputed | 0.885246 |
| male | 0.852459 |
| Embarked_C | 0.136612 |
| Embarked_Q | 0.085610 |
| Embarked_S | 0.777778 |
| Embarked_missing | 0.000000 |
| CabinLetter_A | 0.014572 |
| CabinLetter_B | 0.021858 |
| CabinLetter_C | 0.043716 |
| CabinLetter_D | 0.014572 |
| CabinLetter_E | 0.014572 |
| CabinLetter_F | 0.009107 |
| CabinLetter_G | 0.003643 |
| CabinLetter_T | 0.001821 |
| CabinLetter_missing | 0.876138 |

dtype: float64

We can make looking at them side by side more easy by putting these values in a new DataFrame.

```
[10]: summary = pd.DataFrame() # New empty DataFrame
summary['survived'] = survived.mean()
summary['died'] = died.mean()
```

Now let's look at them side by side. See if you can spot what features you think might have influenced survival.

```
[11]: summary
```

```
[11]:
```

| | survived | died |
|--------------------|------------|------------|
| Unnamed: 0 | 443.368421 | 446.016393 |
| Unnamed: 0.1 | 443.368421 | 446.016393 |
| Unnamed: 0.1.1 | 443.368421 | 446.016393 |
| Unnamed: 0.1.1.1 | 443.368421 | 446.016393 |
| Survived | 1.000000 | 0.000000 |
| Pclass | 1.950292 | 2.531876 |
| Age | 28.291433 | 30.028233 |
| SibSp | 0.473684 | 0.553734 |
| Parch | 0.464912 | 0.329690 |
| Fare | 48.395408 | 22.117887 |
| AgeImputed | 0.152047 | 0.227687 |
| EmbarkedImputed | 0.005848 | 0.000000 |
| CabinLetterImputed | 0.602339 | 0.876138 |

| | | |
|---------------------|-----------|----------|
| CabinNumber | 18.961988 | 6.074681 |
| CabinNumberImputed | 0.611111 | 0.885246 |
| male | 0.318713 | 0.852459 |
| Embarked_C | 0.271930 | 0.136612 |
| Embarked_Q | 0.087719 | 0.085610 |
| Embarked_S | 0.634503 | 0.777778 |
| Embarked_missing | 0.005848 | 0.000000 |
| CabinLetter_A | 0.020468 | 0.014572 |
| CabinLetter_B | 0.102339 | 0.021858 |
| CabinLetter_C | 0.102339 | 0.043716 |
| CabinLetter_D | 0.073099 | 0.014572 |
| CabinLetter_E | 0.070175 | 0.014572 |
| CabinLetter_F | 0.023392 | 0.009107 |
| CabinLetter_G | 0.005848 | 0.003643 |
| CabinLetter_T | 0.000000 | 0.001821 |
| CabinLetter_missing | 0.602339 | 0.876138 |

1.6 Divide into X (features) and y (lables)

We will separate out our features (the data we use to make a prediction) from our label (what we are truing to predict). By convention our features are called X (usually upper case to denote multiple features), and the label (survvive or not) y.

```
[12]: X = data.drop('Survived',axis=1) # X = all 'data' except the 'survived' column
      y = data['Survived'] # y = 'survived' column from 'data'
```

1.7 Divide into training and tets sets

When we test a machine learning model we should always test it on data that has not been used to train the model. We will use sklearn's `train_test_split` method to randomly split the data: 75% for training, and 25% for testing.

```
[13]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25)
```

1.8 Standardise data

We want all of out features to be on roughly the same scale. This generally leads to a better model, and also allows us to more easily compare the importance of different features.

One simple method is to scale all features 0-1 (by subtracting the minimum value for each value, and dividing by the new remaining maximum value).

But a more common method used in many machine learning methods is standardisation, where we use the mean and standard deviation of the training set of data to normalise the data. We subtract the mean of the test set values, and divide by the standard deviation of the training data. Note

that the mean and standard deviation of the training data are used to standardise the test set data as well.

Here we will use sklearn's `StandardScaler` method. This method also copes with problems we might otherwise have (such as if one feature has zero standard deviation in the training set).

```
[14]: def standardise_data(X_train, X_test):  
  
    # Initialise a new scaling object for normalising input data  
    sc = StandardScaler()  
  
    # Set up the scaler just on the training set  
    sc.fit(X_train)  
  
    # Apply the scaler to the training and test sets  
    train_std=sc.transform(X_train)  
    test_std=sc.transform(X_test)  
  
    return train_std, test_std
```

```
[15]: X_train_std, X_test_std = standardise_data(X_train, X_test)
```

1.9 Fit logistic regression model

Now we will fit a logistic regression model, using sklearn's `LogisticRegression` method. Our machine learning model fitting is only two lines of code! By using the name `model` for our logistic regression model we will make our model more interchangeable later on.

```
[16]: model = LogisticRegression()  
model.fit(X_train_std,y_train)
```

```
/home/michael/anaconda3/lib/python3.7/site-  
packages/sklearn/linear_model/logistic.py:432: FutureWarning: Default solver  
will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.  
FutureWarning)
```

```
[16]: LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,  
    intercept_scaling=1, l1_ratio=None, max_iter=100,  
    multi_class='warn', n_jobs=None, penalty='l2',  
    random_state=None, solver='warn', tol=0.0001, verbose=0,  
    warm_start=False)
```

1.10 Predict values

Now we can use the trained model to predict survival. We will test the accuracy of both the training and test data sets.

```
[17]: # Predict training and test set labels
y_pred_train = model.predict(X_train_std)
y_pred_test = model.predict(X_test_std)
```

1.11 Calculate accuracy

In this example we will measure accuracy simply as the proportion of passengers where we make the correct prediction. In a later notebook we will look at other measures of accuracy which explore false positives and false negatives in more detail.

```
[18]: accuracy_train = np.mean(y_pred_train == y_train)
accuracy_test = np.mean(y_pred_test == y_test)

print ('Accuracy of predicting training data =', accuracy_train)
print ('Accuracy of predicting test data =', accuracy_test)
```

```
Accuracy of predicting training data = 0.8383233532934131
Accuracy of predicting test data = 0.7219730941704036
```

Not bad - about 80% accuracy. You will probably see that accuracy of predicting the training set is usually higher than the test set. Because we are only testing one random sample, you may occasionally see otherwise. In later note books we will look at the best way to repeat multiple tests, and look at what to do if the accuracy of the training set is significantly higher than the test set (a problem called ‘over-fitting’).

1.12 Examining the model coefficients (weights)

Not all features are equally important. And some may be of little or no use at all, unnecessarily increasing the complexity of the model. In a later notebook we will look at selecting features which add value to the model (or removing features that don’t).

Here we will look at the importance of features – how they affect our estimation of survival. These are known as the model *coefficients* (if you come from a traditional statistics background), or model *weights* (if you come from a machine learning background).

Because we have standardised our input data the magnitude of the weights may be compared as an indicator of their influence in the model. Weights with higher negative numbers mean that that feature correlates with reduced chance of survival. Weights with higher positive numbers mean that that feature correlates with increased chance of survival. Those weights with values closer to zero (either positive or negative) have less influence in the model.

We access the model weights by examining the model `coef_` attribute. The model may predict more than one outcome label, in which case we have weights for each label. Because we are predicting a single label (survive or not), the weights are found in the first element (`[0]`) of the `coef_` attribute.

```
[19]: co_eff = model.coef_[0]
co_eff
```

```
[19]: array([ 0.01988011,  0.01988011,  0.01988011,  0.01988011, -0.64190162,
          -0.48495764, -0.59905658,  0.05392963,  0.16280539, -0.0198423 ,
           0.          ,  0.00743732, -0.00396614, -0.41859326, -1.43512196,
           0.16442981,  0.03172587, -0.16525695,  0.          ,  0.04356281,
          -0.0897397 , -0.13531804,  0.0855177 ,  0.21086001,  0.0267764 ,
          -0.12826602,  0.          ,  0.00743732])
```

So we have an array of model weights.

Not very readable for us mere humans is it?!

We will transfer the weights array to a Pandas DataFrame. The array order is in the same order of the list of features of X, so we will put that those into the DataFrame as well. And we will sort by influence in the model. Because both large negative and positive values are more influential in the model we will take the *absolute* value of the weight (that is remove any negative sign), and then sort by that absolute value. That will give us a more readable table of most influential features in the model.

```
[20]: co_eff_df = pd.DataFrame() # create empty DataFrame
co_eff_df['feature'] = list(X) # Get feature names from X
co_eff_df['co_eff'] = co_eff
co_eff_df['abs_co_eff'] = np.abs(co_eff)
co_eff_df.sort_values(by='abs_co_eff', ascending=False, inplace=True)
```

```
[21]: co_eff_df
```

```
[21]:
```

| | feature | co_eff | abs_co_eff |
|----|--------------------|-----------|------------|
| 14 | male | -1.435122 | 1.435122 |
| 4 | Pclass | -0.641902 | 0.641902 |
| 6 | SibSp | -0.599057 | 0.599057 |
| 5 | Age | -0.484958 | 0.484958 |
| 13 | CabinNumberImputed | -0.418593 | 0.418593 |
| 23 | CabinLetter_E | 0.210860 | 0.210860 |
| 17 | Embarked_S | -0.165257 | 0.165257 |
| 15 | Embarked_C | 0.164430 | 0.164430 |
| 8 | Fare | 0.162805 | 0.162805 |
| 21 | CabinLetter_C | -0.135318 | 0.135318 |
| 25 | CabinLetter_G | -0.128266 | 0.128266 |
| 20 | CabinLetter_B | -0.089740 | 0.089740 |
| 22 | CabinLetter_D | 0.085518 | 0.085518 |
| 7 | Parch | 0.053930 | 0.053930 |
| 19 | CabinLetter_A | 0.043563 | 0.043563 |
| 16 | Embarked_Q | 0.031726 | 0.031726 |
| 24 | CabinLetter_F | 0.026776 | 0.026776 |
| 0 | Unnamed: 0 | 0.019880 | 0.019880 |
| 1 | Unnamed: 0.1 | 0.019880 | 0.019880 |
| 3 | Unnamed: 0.1.1.1 | 0.019880 | 0.019880 |
| 2 | Unnamed: 0.1.1 | 0.019880 | 0.019880 |
| 9 | AgeImputed | -0.019842 | 0.019842 |

| | | | |
|----|---------------------|-----------|----------|
| 11 | CabinLetterImputed | 0.007437 | 0.007437 |
| 27 | CabinLetter_missing | 0.007437 | 0.007437 |
| 12 | CabinNumber | -0.003966 | 0.003966 |
| 18 | Embarked_missing | 0.000000 | 0.000000 |
| 10 | EmbarkedImputed | 0.000000 | 0.000000 |
| 26 | CabinLetter_T | 0.000000 | 0.000000 |

So are three most influential features are:

- male (being male reduces probability of survival)
- Pclass (lower class passengers, who have a higher class number, reduces probability of survival)
- age (being older reduces probability of survival)

1.13 Show predicted probabilities

The predicted probabilities are for the two alternative classes 0 (does not survive) or 1 (survive).

Ordinarily we do not see these probabilities - the `predict` method used above applies a cut-off of 0.5 to classify passengers into survived or not, but we can see the individual probabilities for each passenger.

Later we will use these to adjust sensitivity of our model to detecting survivors or non-survivors.

Each passenger has two values. These are the probability of not surviving (first value) or surviving (second value). Because we only have two possible classes we only need to look at one. Multiple values are important when there are more than one class being predicted.

```
[22]: # Show first ten predicted classes
      classes = model.predict(X_test_std)
      classes[0:10]
```

```
[22]: array([1, 0, 1, 0, 1, 0, 1, 0, 0, 0])
```

```
[23]: # Show first ten predicted probabilities
      # (note how the values relate to the classes predicted above)
      probabilities = model.predict_proba(X_test_std)
      probabilities[0:10]
```

```
[23]: array([[0.01601246, 0.98398754],
             [0.92886988, 0.07113012],
             [0.18822189, 0.81177811],
             [0.86324032, 0.13675968],
             [0.28225111, 0.71774889],
             [0.82048446, 0.17951554],
             [0.00801209, 0.99198791],
             [0.93478968, 0.06521032],
             [0.97193619, 0.02806381],
             [0.91580078, 0.08419922]])
```