1 Particle Swarm Optimization Algorithm:

Particle Swarm Optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution. It solves a problem by having a population of candidate solutions (i.e particles) and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity. Each particle's movement is influenced by its local best known position, but is also guided toward the global best known positions in the search-space.

Our task is to prove that the search space is convex in nature. Below is the description on how the search space is defined with certain constraints.

Search space condition:

$$\{x \in \mathbb{R}^n | constraints\}$$

Above condition refers to determining a multi dimensional real valued search space with a set of constraints.

The search space is defined as follows,

$$\{(A_i, B_i, C_i, P_i)\}_t$$

where, $1 \leq i \leq M$, $1 \leq t \leq T$, $P \in [-P_{max}, P_{max}] \subseteq \mathbb{R}$ and A,B,C are the 3 phases

The parameters in the search space(A_i, B_i, C_i, P_i) are the phase and power of i^{th} EV's, M is the maximum number of EV's and T is the endding time.

Hence the dimension of the search space becomes :

$$\mathbb{R}^{MT}$$

The following are the constraints based on which the boundaries for the search space is defined,

1. Phase Constraints:
   Only one phase is chosen out of the 3 phases A,B and C For the tuple (A_i, B_i, C_i, P_i) the constraint is as follows,

   $$A_i + B_i + C_i = 1$$

   where $(A_i, B_i, C_i) \in \{0, 1\}$

2. Grid constraints:
   The grid constraints are defined in such a way that the power consumption should not exceed 4.6KW

   $$I(t) = f\left(P_A(t), P_B(t), P_C(t)\right)$$

where,
$\forall t \in [1, T] : I(t) \leq 4.6$ KW

$P_A(t) = \sum_{i=1}^{M} A_i \cdot P_i(t)$

$P_B(t) = \sum_{i=1}^{M} B_i \cdot P_i(t)$

$P_C(t) = \sum_{i=1}^{M} C_i \cdot P_i(t)$

$f(P_A(t), P_B(t), P_C(t)) : max\,\{|P_A(t) - P_B(t)|, |P_B(t) - P_C(t)|, |P_C(t) - P_A(t)|\} \leq 4.6$

This can be further linearized as follows,

$|P_A(t) - P_B(t)| \leq 4.6 \implies -4.6 \leq P_A(t) - P_B(t) \leq 4.6$

$|P_B(t) - P_C(t)| \leq 4.6 \implies -4.6 \leq P_B(t) - P_C(t) \leq 4.6$

$|P_C(t) - P_A(t)| \leq 4.6 \implies -4.6 \leq P_C(t) - P_A(t) \leq 4.6$

3. SOC constraints:

   General constraint:

   The constraint is that the energy consumed and/or produced (E_i^t) by an EV must be greater than 0 and must be less than the maximum battery capacity (E_{max}).

$$\forall t_x \in [T_S, T_D] : \qquad E_i^t = \sum_{t=T_S}^{t_x} t \cdot P_i(t) \cdot \epsilon$$

   where, $E_i^t \geq 0$,$E_i^t \leq E_{max}$ and $\epsilon = 0.95$ is the efficiency value which provides the losses when charging and discharging.

   User constraint:

   The constraint is set so that the energy consumed and/or produced by an EV must be equal to (E_{req}).

$$E_{int} + \sum_{t=T_S}^{T_D} t \cdot P_i(t) \cdot \epsilon = E_{req}$$

   With an assumption that the initial value of the energy level of an EV battery (E_{int}) is provided by user.

4. Power consumed at t $\notin [T_S, T_D]$:

   This constraint defines that the power drawn is set to 0 during the time when the EV is not plugged in the charging station.

$$\forall t \in [1, T_S] \cup [T_D, T] : \qquad P_i(t) = 0$$

The total cost is calculated as follows,

The time duration is discretized with a fixed time slot duration and the time slot is enumerated with $1 \leq t \leq T$

$C = \sum_{i=1}^{M} C_i$

$C_i = \sum_{t=T_S}^{T_D} C_i(t)$ where $C_i(t) = E_i^t \cdot c(t)$ and $E_i^t = t \cdot P_i(t) \cdot \epsilon$

where, c(t) is the cost from market in ct/kwh, E_i^t is the energy of i^{th} EV at time t in kwh and $C_i(t)$ is the cost of i^{th} EV at time t in ct (euro cent).

Finally the particle swarm optimization algorithm is used to minimize the total cost.

i.e $min \rightarrow C$