CS471_HW4

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1 MDP

1.1 (a)

According to the question, we could get that V(G) = 0. And we know that when we are at C state, we would always choose to jump. Otherwise, we would always choose to go right.
Therefore, we could get that:
$V^{\pi}(F) = 1 \times (10 + 1 \times V(G)) = 10$
$V^{\pi}(E) = 1 \times (1 + 1 \times V(F)) = 11$
$V^{\pi}(D) = 1 \times (1 + 1 \times V(E)) = 12$
$V^{\pi}(C) = 0.5 \times (1 + 1 \times V(D)) + 0.5 \times (4 + 1 \times V(E)) = 14$

1.2 (b)

Yes, the policy is optimal. Because for all states (except C), if we go left, the value of the policy would always be smaller than going right since the reward of going left is a negative number. As for C, if we do not choose jump, the reward of going left is also negative. This guarantees the final value of policy would be smaller than jump.

1.3 (c)

For the initial iteration, the value of all states are 0. Therefore, we could get the following table:

A	B	C	D	E	F	G
0	0	0	0	0	0	0

After the first iteration, we could get the following table:

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	1	2.5	1	1	10	0

For all the states that are having V as 1:
$1 + 1 \times 0 = 1$
For state C:

$0.5 \times (1 + 1 \times 0) + 0.5 \times (4 + 1 \times 0) = 2.5$
For state F:
$10 + 1 \times 0 = 10$
After the second iteration, we could get the following table:

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	1	2.5	1	1	10	0
2	3.5	3.5	2	11	10	0

In this iteration, we could get:
$Q(B, right) = 1 + 1 \times 2.5 = 3.5$
$Q(B, left) = -1.5 + 1 \times 1 = -0.5$
$V_2(B) = max\{Q(B, right), Q(B, left)\} = 3.5$

2 Naive Bayes

2.1 (a)

According to the dataset, we could get that: P(Y = true) = $\frac{3}{8}$ = 0.375
P(Y = false) = 1 - 0.375 = 0.625

2.2 (b)

$P(X_2 = true|Y = true) = 1$
$P(X_2 = false|Y = true) = 0$
$P(X_2 = true|Y = false) = \frac{2}{5} = 0.4$
$P(X_2 = false|Y = false) = \frac{3}{5} = 0.6$
Therefore, we could get a table:

X_2	$P(X_2\|Y = true)$	$P(X_2\|Y = false)$
1	1	$\frac{2}{5}$
0	0	$\frac{3}{5}$

After we apply Laplace Rule, we could get a new table:

X_2	$P(X_2\|Y = true)$	$P(X_2\|Y = false)$
1	$\frac{3+3}{3+6} = \frac{2}{3}$	$\frac{2+3}{5+6} = \frac{5}{11}$
0	$\frac{0+3}{3+6} = \frac{1}{3}$	$\frac{3+3}{5+6} = \frac{6}{11}$

2.3 (c)

From the given data, we could get the following tables:

X_1	$P(X_1\|Y = true)$	$P(X_1\|Y = false)$
1	$\frac{2}{3}$	$\frac{3}{5}$
0	$\frac{1}{3}$	$\frac{2}{5}$

X_2	$P(X_2\|Y = true)$	$P(X_2\|Y = false)$
1	1	$\frac{2}{5}$
0	0	$\frac{3}{5}$

X_3	$P(X_3\|Y = true)$	$P(X_3\|Y = false)$
1	0	1
0	1	0

After Laplace Smoothing, we could get the modified tables:

X_1	$P(X_1\|Y = true)$	$P(X_1\|Y = false)$
1	$\frac{2+3}{3+6} = \frac{5}{9}$	$\frac{3}{5} = \frac{6}{11}$
0	$\frac{1+3}{3+6} = \frac{4}{9}$	$\frac{2}{5} = \frac{5}{11}$

X_2	$P(X_2\|Y=true)$	$P(X_2\|Y=false)$
1	$\frac{3+3}{3+6}=\frac{2}{3}$	$\frac{2+3}{5+6}=\frac{5}{11}$
0	$\frac{0+3}{3+6}=\frac{1}{3}$	$\frac{3+3}{5+6}=\frac{6}{11}$

X_3	$P(X_3\|Y=true)$	$P(X_3\|Y=false)$
1	$\frac{3}{9}=\frac{1}{3}$	$\frac{8}{11}$
0	$\frac{6}{9}=\frac{2}{3}$	$\frac{3}{11}$

According to Naive Bayes, we could get the following:

$P(X_1=0,X_2=1,X_3=1)$
$=P(X_1=0,X_2=1,X_3=1|Y=1)P(Y=1)+P(X_1=0,X_2=1,X_3=1|Y=0)P(Y=0)$
$=P(X_1=0|Y=1)P(X_2=1|Y=1)P(X_3=1|Y=1)P(Y=1)+P(X_1=0|Y=0)P(X_2=1|Y=0)P(X_3=1|Y=0)P(Y=0)$
$=\frac{4}{9}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{3}{8}+\frac{5}{11}\cdot\frac{5}{11}\cdot\frac{8}{11}\cdot\frac{5}{8}$
$=\frac{4706}{355937}$

Give new example, we could get the calculations:

$P(Y=1|X_1=0,X_2=1,X_3=1)$
$=\frac{P(X_1=0,X_2=1,X_3=1|Y=1)P(Y=1)}{P(X_1=0,X_2=1,X_3=1)}$
$=\frac{P(X_1=0|Y=1)P(X_2=1|Y=1)P(X_3=1|Y=1)P(Y=1)}{P(X_1=0,X_2=1,X_3=1)}$
$=\frac{\frac{4}{9}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{3}{8}}{\frac{4706}{35937}}$
$=\frac{1331}{4706}$

$P(Y=0|X_1=0,X_2=1,X_3=1)$
$=\frac{P(X_1=0,X_2=1,X_3=1|Y=0)P(Y=0)}{P(X_1=0,X_2=1,X_3=1)}$
$=\frac{P(X_1=0|Y=0)P(X_2=1|Y=0)P(X_3=1|Y=0)P(Y=0)}{P(X_1=0,X_2=1,X_3=1)}$
$=\frac{\frac{5}{11}\cdot\frac{5}{11}\cdot\frac{8}{11}\cdot\frac{5}{8}}{\frac{4706}{35937}}$
$=\frac{3375}{4706}$

Therefore, the prediction would be not biased.

3 Approximate Inference

3.1 (a)

With simple sampling, we could get: $P(B = +b) = \frac{3}{5} = 0.6$

3.2 (b)

With rejection sampling, we could get: $P(B = +b|A = -a, C = +c, D = +d) = \frac{1}{2}$

3.3 (c)

According to the give tables and diagram, we could get:
Weight of P(-a, -b, +c, +d) should be $P(-a)P(+c|-b)P(+d|-b) = \frac{2}{3}\frac{1}{4}\frac{1}{4} = \frac{1}{24}$
The weight of P(-a, +b, +c, +d) should be $P(-a)P(+c|+b)P(+d|+b) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{9}$
Therefore, we could get the final answer:
$P(B = +b|A = -a, C = +c, D = +d)$
$= \frac{\frac{1}{9}}{\frac{1}{9}+\frac{1}{24}+\frac{1}{24}}$
$= \frac{4}{7}$