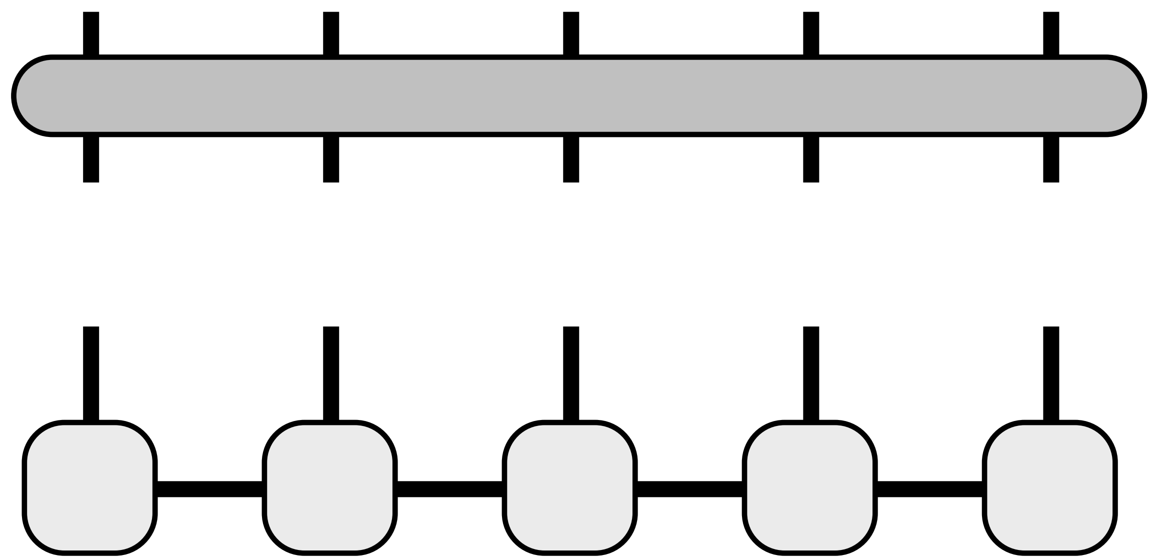
06 MPO

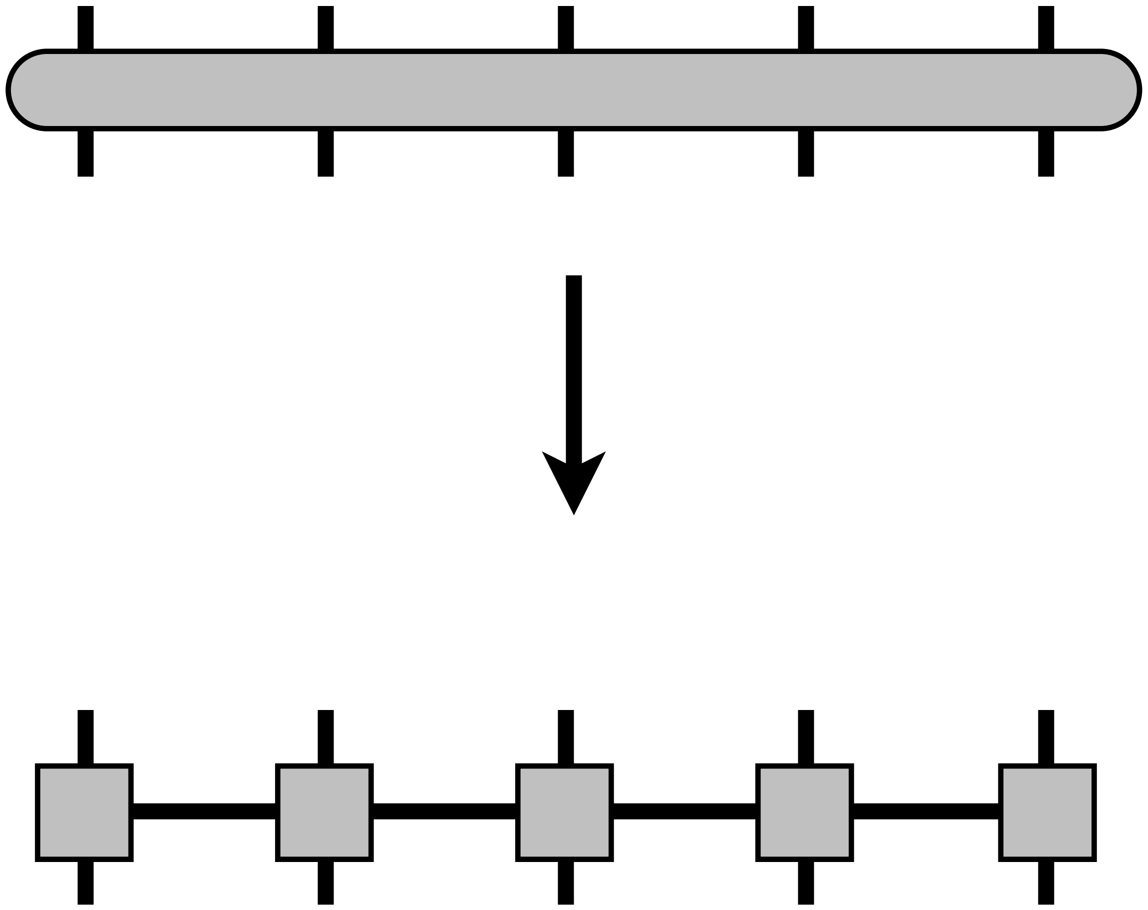
We have seen a Hamiltonian looks like this:



$$\hat{H}|\Psi\rangle$$

Does a 1d Hamiltonian have a local form/factorization like an MPS?

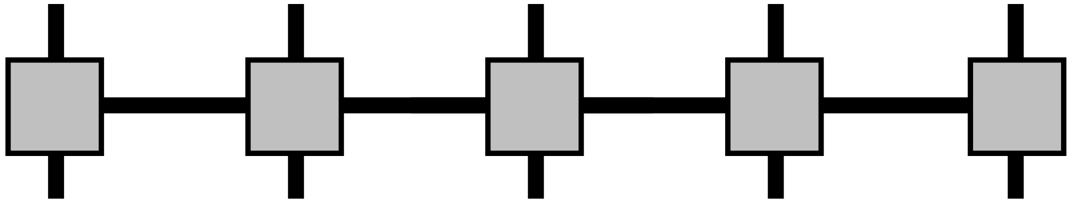
Want something like



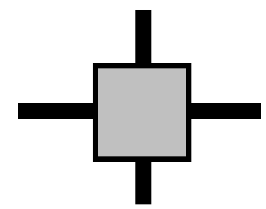
Operator (H) as product of “matrices”

matrix product operator

Focus on just one tensor

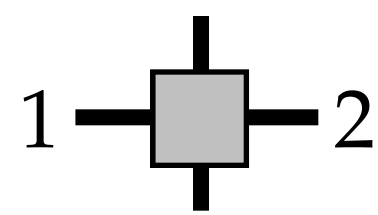


Focus on just one tensor



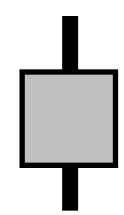
Focus on just one tensor

Specific values for horizontal bonds
gives site operator



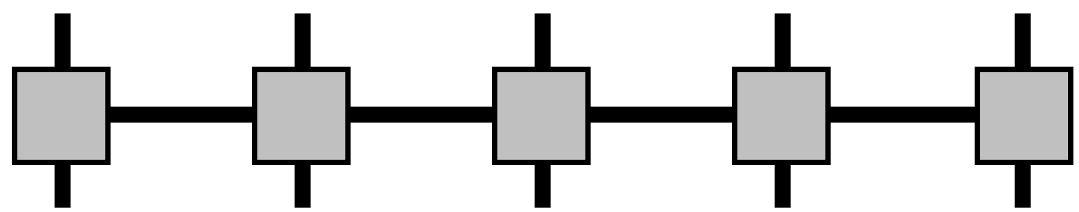
Focus on just one tensor

Specific values for horizontal bonds
gives site operator



Focus on just one tensor

Specific values for horizontal bonds
gives site operator



Each tensor a matrix of site operators!

$\longrightarrow$ Each tensor a matrix of site operators!

Hamiltonians can be written

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

→ Each tensor a matrix of site operators!

Multiply out

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\longrightarrow$ Each tensor a matrix of site operators!

Multiply out

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} \\ \hat{\sigma}^z \end{bmatrix}$$

→ Each tensor a matrix of site operators!

Multiply out

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} \\ \hat{\sigma}^z \end{bmatrix}$$

$$\hat{\sigma}_1^z \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{\sigma}_2^z$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & \\ \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

This Hamiltonian is

$$H = \sum_i \hat{\sigma}_i^z$$

More complicated example

$$\begin{bmatrix}0\\0\\1\end{bmatrix}^{\mathsf{T}} \quad 3 \begin{bmatrix}\hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I}\end{bmatrix} \quad \begin{bmatrix}\hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I}\end{bmatrix} 1 \quad \begin{bmatrix}1\\0\\0\end{bmatrix}$$

More complicated example

$$
\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^{\mathsf{T}}
\quad
\begin{array}{cc}
\begin{matrix} \\ \\ 3 \end{matrix} &
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
\end{array}
\quad
\begin{array}{cc}
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix} &
\begin{matrix} 1 \\ \\ \\ \end{matrix}
\end{array}
\quad
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$\hat{\sigma}^z$

More complicated example

$$\begin{bmatrix}0\\0\\1\end{bmatrix}^{T}
\begin{array}{cc}
 & \begin{array}{ccc} \phantom{-h\hat{\sigma}^x} & 2 & \phantom{\hat{I}} \end{array} \\
\begin{array}{c} \\ \\ 3 \end{array} & \begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
\end{array}
\begin{array}{ccc}
 & \begin{array}{ccc} 1 & \phantom{\hat{\sigma}^z} & \phantom{\hat{I}} \end{array} & \\
\begin{array}{c} \\ 2 \\ \\ \end{array} & \begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix} & \begin{array}{c} 1 \\ \\ \\ \end{array}
\end{array}
\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$\hat{\sigma}^z$ $\hat{\sigma}^z$

More complicated example

$$\begin{bmatrix}0\\0\\1\end{bmatrix}^{\mathsf{T}} \quad 3 \begin{bmatrix}\hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I}\end{bmatrix} \quad \begin{bmatrix}\hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I}\end{bmatrix} 1 \quad \begin{bmatrix}1\\0\\0\end{bmatrix}$$

More complicated example

$$
\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^{\mathsf{T}}
\quad
\begin{array}{c}
\\
\\
3
\end{array}
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
\quad
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
\begin{array}{c}
1 \\
\\
\\
\end{array}
\quad
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$-h\hat{\sigma}^x$$

More complicated example

$$
\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^{T}
\quad
\begin{array}{cc}
 & \begin{array}{ccc} 1 & \phantom{\hat{\sigma}^z} & \phantom{\hat{I}} \end{array} \\
\begin{array}{c} \\ \\ 3 \end{array} &
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
\end{array}
\quad
\begin{array}{ccc}
 & \begin{array}{ccc} 1 & \phantom{\hat{\sigma}^z} & \phantom{\hat{I}} \end{array} & \\
\begin{array}{c} 1 \\ \\ \\ \end{array} &
\begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix}
& \begin{array}{c} 1 \\ \\ \\ \end{array}
\end{array}
\quad
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$-h\hat{\sigma}^x \qquad\qquad \hat{I}$$

$$\bullet \qquad\qquad \bullet$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^{\mathsf{T}} \quad \begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix} \begin{bmatrix} \hat{I} & & \\ \hat{\sigma}^z & 0 & \\ -h\hat{\sigma}^x & \hat{\sigma}^z & \hat{I} \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Hamiltonian is

$$\hat{H} = \sum_j \hat{\sigma}_j^z \sigma_{j+1}^z - h\hat{\sigma}_j^x$$