9+NeuralNetwork

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1 神经网络

1.1 一、概念

神经网络是时下最热的人工智能话题，而神经网络的历史也由来已久，近年来的算力大爆发使人工智能和神经网络发现了彼此。

神经网络通过神经元进行组织，数据从上一层神经元流向下一层神经元直到输出神经元，损失函数衡量预测和输出之间的差距，再通过反向传播更新各层神经元的参数。

神经网络由如下元素构成：

1. 输入层：数据从输入层进入模型
2. 隐藏层：数据在隐藏层中进行交互和组合
3. 输出层：输出层输出预测结果
4. 激活函数：各个神经元的对上一层的输入进行非线性处理的函数
5. 损失函数：衡量预测结果和实际结果的差距
6. 优化器：即以何种方式更新参数

1.2 二、符号说明

- X：输入数据，$X \in R^{p \times n}$，p 代表变量数，n 代表样本数
- $\hat{Y}$：输出数据，$\hat{Y} \in R^{k \times n}$，n 代表样本数，多分类时 k 代表分类数，二分类和回归时 k 为 1
- Y：实际结果，$Y \in R^{k \times n}$，n 代表样本数，多分类时 k 代表分类数，二分类和回归时 k 为 1
- p_i：第 i 层的神经元数，$p_0 = p$
- W_i：从第 i-1 层向第 i 层传播的矩阵，$W_i \in R^{p_i \times p_{i-1}}$，输入层为第 0 层时，$W_1 \in R^{p_1 \times p}$
- $\alpha(z)$：激活函数，每一层每一个神经元的激活函数都可以不同，此处统一用
- $g(z)$：输出层的激活函数，通常和隐藏层的激活函数不同

- b_i：第 i 层的偏置项，$b_i \in R^{p_{i+1}}$
- Z_i：上一层激活函数的线性组合，$Z_i \in R^{p_i \times n}$
- A_i：线性组合的激活函数值，$A_i \in R^{p_i \times n}$
- * ：逐元素相乘

1.3 三、Feed Forward 前向传播

1.3.1 1. 从输入层到第一个隐藏层

首先是对输入数据的线性组合，由于偏执项是一个向量，对所有 n 个数据来说都相等。虽然此处维度按照线性代数并不能严格成立（因为 $W_1X \in R^{p_1 \times n}$，$b_1 \in R^{p_1 \times 1}$），但是由于 numpy 中的广播（broadcast）机制存在，在编程中以下公式是成立的。如果非要按照数学定义上成立可以对 b_1 乘上一个 $1 \times n$ 的值全为 1 的向量。

$$\begin{aligned} Z_1 &= W_1X + b_1 \quad \in R^{p_1 \times n} \\ \Leftrightarrow Z_1 &= W_1X + b_1 1^{1 \times n} \end{aligned}$$

然后是对第一层的各个神经元进行 ''激活''，对线性组合进行逐元素的函数计算

$$A_1 = \alpha(Z_1) \quad \in R^{p_1 \times n}$$

1.3.2 2. 从第 i-1 层到第 i 层

与输入不同，此时是将上一层的激活函数值进行线性组合：

$$Z_i = W_i A_{i-1} + b_i \quad \in R^{p_i \times n}$$

$$A_i = \alpha(Z_i) \quad \in R^{p_i \times n}$$

1.3.3 3. 从最后一个隐藏层到输出层

假设输入层是第 0 层，第 1——m-1 层是隐藏层，第 m 层是输出层。如果是二分类、回归等情况，则输出层只有一个神经元，若是多分类等情况则有多个神经元，将在后面介绍，暂时假定只有一个输出：

$$Z_m = W_m A_{m-1} + b_m \quad \in R^{k \times n}$$

$$\hat{Y} = A_m = g(Z_m) \quad \in R^{k \times n}$$

1.4 四、激活函数

激活函数有多种多样，本质上都是为了进行非线性组合，还有易于进行求导运算以便更新参数。此处简单介绍几种激活函数

1.4.1 1.sigmoid 函数

Sigmoid 函数已经在 logistic 回归中介绍过：

$$sigmoid(z) = \frac{1}{1+1^{-z}}$$

它是一种较早期的激活函数，现在多用于最后输出层的激活而不用在隐藏层中，这是因为当 x 远离原点时它的梯度会非常接近 0，会造成非常著名的 ''梯度消失'' 的现象。

考虑 sigmoid 函数的导数：

$$\frac{d}{dz}sigmoid(z) = \frac{e^{-z}}{(1+1^{-z})^2}$$

当 z=0 时其梯度最大为 0.25，当神经网络的层数变深时便是指数倍地降低，这便是 ''梯度消失'' 最直观和简洁的解释。

1.4.2 2.Relu（Rectified Linear Unit, 线性整流函数）

Relu 也曾是红极一时的激活函数，因其简洁的函数形式和导数形式（x 大于零导数为 1，其他情况为 0）使计算成本大大降低，但同时这也带来了神经元没有被激活的情况。这是因为当输入小于 0 时，输出和梯度都为 0，导致神经元 ''死亡''。

$$Relu(z) = max(0,z)$$

$$\frac{d}{dz}Relu(z) = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases}$$

1.4.3 3.leaky Relu

leaky Relu 是我最喜欢的激活函数，因为它兼具了 Relu 的优点，且当输入小于零时不会出现神经元死亡的情况，k 通常的设置为 0.1。

$$leakyRelu(z,k) = max(kz,z)$$

$$\frac{d}{dz}leakyRelu(z) = \begin{cases} 1 & z > 0 \\ k & z \leq 0 \end{cases}$$

1.4.4 4.softmax

softmax 是专门用于多分类的输出层的激活函数，有两种等价形式，一种是针对 K 类有 K 个输出的线性相关的形式（即下式），另一个是针对 K 类有 K-1 个输出的线性无关的形式。

$$softmax(z) = \begin{bmatrix} \frac{e^{z_1}}{\sum_{i=1}^{k} e^{z_i}} \\ \frac{e^{z_2}}{\sum_{i=1}^{k} e^{z_i}} \\ \dots \\ \frac{e^{z_j}}{\sum_{i=1}^{k} e^{z_i}} \\ \dots \\ \frac{e^{z_k}}{\sum_{i=1}^{k} e^{z_i}} \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \dots \\ \hat{y}_i \\ \dots \\ \hat{y}_k \end{bmatrix}$$

它的针对单一分量的偏导数形式和 sigmoid 函数极为相似：

$$\begin{aligned} \frac{\partial}{\partial z_i} softmax(z) &= \frac{d}{dz_i}\frac{e^{z_i}}{a+e^{z_i}} = \frac{ae^{z_i}}{(a+e^{z_i})^2} \\ &= \frac{ae^{z_i}+a^2-a^2}{(a+e^{z_i})^2} \\ &= \frac{a(e^{z_i}+a)-a^2}{(a+e^{z_i})^2} \\ &= \frac{a}{a+e^{z_i}} - \left(\frac{a}{a+e^{z_i}}\right)^2 \\ &= \frac{a}{a+e^{z_i}}\left(1-\frac{a}{a+e^{z_i}}\right) \\ &= \left(1-\frac{e^{z_i}}{a+e^{z_i}}\right)\frac{e^{z_i}}{a+e^{z_i}} \end{aligned}$$

则它的梯度为：

$$\triangledown softmax(z) = \begin{bmatrix} \hat{y}_1(1-\hat{y}_1) \\ \hat{y}_2(1-\hat{y}_2) \\ \dots \\ \hat{y}_i(1-\hat{y}_i) \\ \dots \\ \hat{y}_k(1-\hat{y}_k) \end{bmatrix}$$

1.5 五、损失函数

二分类和回归的损失函数不再赘述，和 logistic 回归和多元线性回归类似，这里介绍多分类的损失函数。

多分类的损失函数和二分类相同，也是通过似然函数进行定义：假设随机变量 Y 一共有 K 个取值，第 i 个样本对第 j 个取值的概率估计值为：

$$P(y_i = j) = \hat{y}_{ij} \;\; j = 1, 2, ..., k$$

则对 n 个样本，其似然函数为：

$$likelihood(Y, \hat{Y}) = \prod_{i=1}^{n}\prod_{j=1}^{k}\hat{y}_{ij}^{I(y_i=j)}$$

对其求自然对数，除以样本数进行标准化取负数：

$$loss(Y, \hat{Y}) = -\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k} I(y_i = j) ln(\hat{y}_{ij})$$

这就是最终的损失函数。

1.6 六、Backward propagation 反向传播

反向传播是神经网络更新参数最经典也是最有效、最具有广泛性的算法。

反向传播的基础仍然是梯度下降法。

1.6.1 1. 输出层到第一个隐藏层的反向传播

$$\frac{\partial}{\partial y_i j} loss = \frac{1}{n}\frac{I(y_i = j)}{\hat{y}_{ij}}$$

由于输出是 $\hat{Y} \in R^{k\times n}$ 向量，损失对输出层的梯度和输出保持一致的维度：

$$\frac{\triangledown loss}{\triangledown \hat{Y}} = \frac{1}{n}\begin{bmatrix} \frac{I(y_1=1)}{\hat{y}_{11}} & \dots & \frac{I(y_1=k)}{\hat{y}_{1k}} \\ & \ddots & \vdots \\ \frac{I(y_n=1)}{\hat{y}_{n1}} & & \frac{I(y_n=k)}{\hat{y}_{nk}} \end{bmatrix}^T \in R^{k\times n}$$

对输出层的激活函数有 $\hat{Y} = g(Z_m) = softmax(Z_m) \in R^{k\times n}$，$Z_m \in R^{k\times n}$

$$\frac{\triangledown \hat{Y}}{\triangledown Z_m} = \begin{bmatrix} \hat{y}_{11}(1-\hat{y}_{11}) & \dots & \hat{y}_{1k}(1-\hat{y}_{1k}) \\ & \ddots & \vdots \\ \hat{y}_{n1}(1-\hat{y}_{n1}) & & \hat{y}_{nk}(1-\hat{y}_{nk}) \end{bmatrix}^T \in R^{k\times n}$$

此时还未涉及到参数的更新，而 $Z_m = W_m A_{m-1} + b_m$ 中 $W_m \in R^{p_m=k\times p_{m-1}}$、$b_m \in R^{k\times 1}$、$A_{m-1} \in R^{p_{m-1}\times n}$ 均为参数，其中前两个好理解，而激活函数值也需要更新是因为它是先前输入的函数，需要通过对激活函数更新使梯度传导到更靠前的隐藏层。

$$\frac{\nabla Z_m}{\nabla W_m} = A_{m-1} \in R^{p_{m-1}\times n}$$

$$\frac{\nabla Z_m}{\nabla A_{m-1}} = W_m^T \in R^{p_{m-1}\times k}$$

$$\frac{\nabla Z_m}{\nabla b_m} = 1^{1\times n} \in R^{1\times n}$$

将其和之前的梯度结合起来：

$$\begin{aligned}
\frac{\nabla loss}{\nabla W_m} &= \left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m}\right)\left(\frac{\nabla Z_m}{\nabla W_m}\right)^T \in R^{p_m = k\times p_{m-1}} \\
\frac{\nabla loss}{\nabla A_{m-1}} &= \frac{\nabla Z_m}{\nabla A_{m-1}}\left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m}\right) \in R^{p_{m-1}\times n} \\
\frac{\nabla loss}{\nabla b_m} &= \left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m}\right) 1^{n\times 1} \in R^{k\times 1}
\end{aligned}$$

1.6.2 2. 第 i 层到第 i-1 层的反向传播

从第 i 层到第 i-1 层的反向传播和从输出层到最后一个隐藏层的推导相似：
假设 $\frac{\nabla loss}{\nabla A_i} \in R^{p_i\times n}$ 已知，

$$\begin{aligned}
A_i &= \begin{bmatrix} \alpha(z_{11}) & \dots & \alpha(z_{1p_i}) \\ & \ddots & \vdots \\ \alpha(z_{n1}) & & \alpha(z_{np_i}) \end{bmatrix} \in R^{p_i\times n} \\
\frac{\nabla A_i}{\nabla Z_i} &= \begin{bmatrix} \alpha'(z_{11}) & \dots & \alpha'(z_{1p_i}) \\ & \ddots & \vdots \\ \alpha'(z_{n1}) & & \alpha'(z_{np_i}) \end{bmatrix} \in R^{p_i\times n}
\end{aligned}$$

其余部分和之前的相同

$$\frac{\nabla Z_i}{\nabla W_i} = A_{i-1} \in R^{p_{i-1}\times n}$$

$$\frac{\nabla Z_i}{\nabla A_{i-1}} = W_i^T \in R^{p_{i-1}\times i}$$

$$\frac{\nabla Z_i}{\nabla b_i} = 1^{1\times n} \in R^{1\times n}$$

将其和之前的梯度结合起来：

$$
\begin{aligned}
\frac{\nabla loss}{\nabla W_i} &= \left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_i}\right)\left(\frac{\nabla Z_i}{\nabla W_i}\right)^T \in R^{p_i \times p_{i-1}} \\
\frac{\nabla loss}{\nabla A_{i-1}} &= \frac{\nabla Z_i}{\nabla A_{i-1}}\left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_m}\right) \in R^{p_{i-1} \times n} \\
\frac{\nabla loss}{\nabla b_i} &= \left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_i}\right) 1^{n\times 1} \in R^{p_i \times 1}
\end{aligned}
$$

1.6.3 3. 从第一层到输入层

从第 1 层到输入层的反向传播和从第 i 层到第 i-1 层的推导相似，区别在于输入是固定的数据，而不再是激活函数值，也就不再需要对输入的数据 X 进行更新：

假设 $\frac{\nabla loss}{\nabla A_1} \in R^{p_i \times n}$ 已知，

$$
A_1 = \begin{bmatrix} \alpha(z_{11}) & \dots & \alpha(z_{1p_1}) \\ & \ddots & \vdots \\ \alpha(z_{n1}) & & \alpha(z_{np_1}) \end{bmatrix} \in R^{p_i \times n}
$$
$$
\frac{\nabla A_i}{\nabla Z_i} = \begin{bmatrix} \alpha'(z_{11}) & \dots & \alpha'(z_{1p_1}) \\ & \ddots & \vdots \\ \alpha'(z_{n1}) & & \alpha'(z_{np_1}) \end{bmatrix} \in R^{p_i \times n}
$$

$$
\begin{aligned}
\frac{\nabla Z_1}{\nabla W_1} &= X \in R^{p\times n} \\
\frac{\nabla Z_1}{\nabla b_1} &= 1^{1\times n} \in R^{1\times n}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\nabla loss}{\nabla W_1} &= \left(\frac{\nabla loss}{\nabla A_1} * \frac{\nabla A_1}{\nabla Z_i}\right)\left(\frac{\nabla Z_i}{\nabla W_i}\right)^T \in R^{p_1 \times p} \\
\frac{\nabla loss}{\nabla b_1} &= \left(\frac{\nabla loss}{\nabla A_1} * \frac{\nabla A_1}{\nabla Z_i}\right) 1^{n\times 1} \in R^{p_1 \times 1}
\end{aligned}
$$

1.7 七、优化器

优化器是指优化得到参数的方法，优化器基本都是基于梯度下降方法。如果你在线性回归中不用正规方程求解参数，而是用梯度下降，你会发现随着梯度不断下降，**梯度不断减小**。而这还不是最麻烦的问题，由于线性回归是凸优化，用梯度下降总会收敛到最小值，而神经网络多是非凸问题，梯度下降很可能会困在局部极值**无法收敛**。而且通常神经网络需要很多的数据进行训练，如果每次都像传统的梯度下降那样把所有数据都传入模型，则**计算成本很大**。

这里先介绍 **SGD**（Stochastic Gradient Descnet，随机梯度下降）优化器。

SGD 不再把所有的数据都用来进行梯度下降，而是只用小批量（**mini batch**）数据进行梯度下降，常见的选择是从 2 的 4 次方（16）到 2 的 10 次方之间，选用 2 的整数次方是根据计算机比特的特点决定的，而之前推导中梯度进行标准化时除以样本数，此时需要除以一批量的样本数。

控制梯度下降停止的条件也有所改变，由于神经网络强大的非线性组合能力，训练到收敛会造成过拟合，于是神经网络中用到最多的是早停法，也即小批量进行训练时将全部样本循环数遍（**epoch**）后就立即停下，避免过拟合。

1.8 八、应用

这次采用的是 minist 手写数字数据集，从 kaggle 的入门赛下载下来的训练数据集，有兴趣的可以把自己训练好的型跑一下 kaggle 上的测试数据集提交一下看看分数。（排名就不必了看了…）

In [21]:
```
import pandas as pd
import numpy as np

train_data = pd.read_csv('minist.csv')
train_data.head()
```

Out[21]:
```
   label  pixel0  pixel1  pixel2  pixel3  pixel4  pixel5  pixel6  pixel7  \
0      1       0       0       0       0       0       0       0       0
1      0       0       0       0       0       0       0       0       0
2      1       0       0       0       0       0       0       0       0
3      4       0       0       0       0       0       0       0       0
4      0       0       0       0       0       0       0       0       0

   pixel8  ...  pixel774  pixel775  pixel776  pixel777  pixel778  pixel779  \
0       0  ...         0         0         0         0         0         0
1       0  ...         0         0         0         0         0         0
2       0  ...         0         0         0         0         0         0
3       0  ...         0         0         0         0         0         0
4       0  ...         0         0         0         0         0         0

   pixel780  pixel781  pixel782  pixel783
0         0         0         0         0
1         0         0         0         0
2         0         0         0         0
3         0         0         0         0
4         0         0         0         0

[5 rows x 785 columns]
```

```
In [22]: n = train_data.shape[0]
         np.random.seed(2099)
         index = np.random.permutation(n)
         train_index = index[0: int(0.7*n)]
         test_index = index[int(0.7*n): n]

         test_data = train_data.iloc[test_index]
         test_label = test_data['label']
         del test_data['label']
         test_data = np.array(test_data)
         test_label = np.array(test_label).reshape([n-int(0.7*n), 1])

         train_data = train_data.iloc[train_index]
         train_label = train_data['label']
         del train_data['label']
         train_data = np.array(train_data)
         train_label = np.array(train_label).reshape([int(0.7*n), 1])
```

```
In [23]: def to_category(label, num_classes):
             n = label.shape[0]
             tmp = np.zeros([n, num_classes])
             j = 0
             for i in label:
                 tmp[j, i]=1
                 j += 1
             return tmp
```

```
In [24]: def soft_max(z):
             """
             :param z: input, an p*n matrix
             :return: p*n matrix
             """
             e = np.exp(z)
             total = np.sum(e, axis=0, keepdims=True)
             weight = e / total
             return weight
```

```
In [25]: def accuracy(y, y_hat):
             y = np.argmax(y, axis=0)
             y_hat = np.argmax(y_hat, axis=0)
             return sum(y == y_hat)/len(y)
```

In [26]:
```
def likelihood(y, y_hat):
    """
    :param y: the ture value
    :param y_hat: the predicted value
    :return: minimizing loss is the same as maximizing likelihood function,
            so we spare np.log
    """
    n = y.shape[1]
    return -np.sum(y * np.log(y_hat)) / n
```

In [27]:
```
def leaky_relu(x, k=0.3):
    return (x > 0)*x + k*(x < 0)*x


def d_leaky_relu(x, k=0.3):
    return (x > 0) + k*(x < 0)
```

In [28]:
```
def init_w(b, a):
    w = np.random.randn(a * b)
    w = np.reshape(w, [b, a])
    return w


def init_b(b):
    b = np.zeros([b, 1])
    return b
```

In [29]:
```
def forward(x, parameter, cache):
    """
    :param x:input data p*n matrix
    :param parameter: a dict storing parameters
    :param cache: a dict storing computation result of each layer
    :return: the predicted value
    """
    cache['C1'] = np.dot(parameter['W1'], x) + parameter['b1']
    cache['A1'] = leaky_relu(cache['C1'])
    cache['C2'] = np.dot(parameter['W2'], cache['A1']) + parameter['b2']
    cache['A2'] = leaky_relu(cache['C2'])
    cache['C3'] = np.dot(parameter['W3'], cache['A2']) + parameter['b3']
    cache['A3'] = soft_max(cache['C3'])
    return cache
```

```
In [30]: def back_propagation(x, y, parameter, cache, step):
             """
             X 784*n / Y 10*n
             dW1 W1 800*784, db1 b1 800*1, A1 C1, 800*n
             dW2 W2 400*800, db2 b2 400*1, A2 C2, 400*n
             dW3 W3 10*400, db3 b3 10*1, A3 C3, 10*n
             :param y: true value
             :param parameter: dictionary storing all parameters
             :param cache: dictionary storing all the computation in process
             :param step: learning rate
             :return: updated parameters
             """
             number = y.shape[1]
             cache['dC3'] = cache['A3'] - y  # 10*n
             cache['dW3'] = np.dot(cache['dC3'], cache['A2'].T)/number  # 10*400
             cache['db3'] = np.sum(cache['dC3'], axis=1, keepdims=True)/number  # 10*1
             parameter['W3'] = parameter['W3'] - step*cache['dW3']  # 10*400
             parameter['b3'] = parameter['b3'] - step*cache['db3']  # 10*1

             cache['dC2'] = np.dot(parameter['W3'].T,
                                   cache['dC3'])*d_leaky_relu(cache['C2'])  # 400*n
             cache['dW2'] = np.dot(cache['dC2'], cache['A1'].T)/number  # 400*800
             cache['db2'] = np.sum(cache['dC2'], axis=1, keepdims=True)/number  # 400*1
             parameter['W2'] = parameter['W2'] - step*cache['dW2']  # 400*800
             parameter['b2'] = parameter['b2'] - step*cache['db2']  # 400*1

             cache['dC1'] = np.dot(parameter['W2'].T,
                                   cache['dC2'])*d_leaky_relu(cache['C1'])  # 800*n
             cache['dW1'] = np.dot(cache['dC1'], x.T)/number  # 800*784
             cache['db1'] = np.sum(cache['dC1'], axis=1, keepdims=True)  # 800*1
             parameter['W1'] = parameter['W1'] - step*cache['dW1']  # 800*784
             parameter['b1'] = parameter['b1'] - step*cache['db1']  # 800*1
             return cache, parameter

In [31]: def train(x, y, learning_rate=0.001, batch_size=128, epoch=5):
             """
             :param x: training data
             :param y: training label
             :param learning_rate: the length of a step
             :param batch_size: numbers of samples we train in a round
```

```
        :param epoch: rounds we train through training data
        :return: a trained set of parameters
        """
        parameter = dict()
        nx = x.shape[1]
        parameter['W1'] = init_w(800, 784)/100
        parameter['b1'] = init_b(800)
        parameter['W2'] = init_w(400, 800)/100
        parameter['b2'] = init_b(400)
        parameter['W3'] = init_w(10, 400)/100
        parameter['b3'] = init_b(10)

        index = np.array([], dtype='int')
        for i in range(0, nx, batch_size):
            index = np.append(index, i)
        index = np.append(index, nx)

        cache = dict()
        for i in range(0, epoch):
            for j in range(0, int(nx/batch_size)+1):
                one_batch_x = x[:, index[j]:index[j+1]]
                one_batch_y = y[:, index[j]:index[j+1]]
                cache = forward(one_batch_x, parameter, cache)
                prob = likelihood(one_batch_y, cache['A3'])
                acc = accuracy(one_batch_y, cache['A3'])
                print(str(i)+'--'+str(j)+'--'+str(index[j+1]))
                print('loss: '+str(prob))
                print('accuracy: '+str(acc))
                [cache, parameter] = back_propagation(one_batch_x, one_batch_y,
                                                      parameter, cache, step=learning_rate)
        return cache, parameter
```

```
In [32]: train_label = to_category(train_label, num_classes=10)
         test_label = to_category(test_label, num_classes=10)

         print(train_label.shape)
         print(test_label.shape)
```

```
(29399, 10)
(12601, 10)
```

```
In [33]: cache, parameter = train(x=train_data.T, y=train_label.T, epoch=5)
```

```
0--0--128
likelihood: 2.5144562796833654
accuracy: 0.0703125
0--1--256
likelihood: 2.2170502380328974
accuracy: 0.1875
0--2--384
likelihood: 2.1658027472486903
accuracy: 0.234375
0--3--512
likelihood: 2.0168412770294104
accuracy: 0.296875
0--4--640
likelihood: 1.8946818751163836
accuracy: 0.40625
0--5--768
likelihood: 1.8244820246098055
accuracy: 0.421875
0--6--896
likelihood: 1.6413233158646316
accuracy: 0.515625
0--7--1024
likelihood: 1.6209324889136416
accuracy: 0.5546875
0--8--1152
likelihood: 1.5990883453285492
accuracy: 0.5390625
0--9--1280
likelihood: 1.3856347927896082
accuracy: 0.6484375
0--10--1408
likelihood: 1.303833712288761
accuracy: 0.703125
0--11--1536
likelihood: 1.3310439935553542
accuracy: 0.6484375
0--12--1664
likelihood: 1.1805366474892718
```

```
accuracy: 0.7265625
0--13--1792
likelihood: 1.260705592177902
accuracy: 0.6640625
0--14--1920
likelihood: 1.1617022979770701
accuracy: 0.7109375
0--15--2048
likelihood: 1.1734547644587505
accuracy: 0.6875
0--16--2176
likelihood: 1.0308806471718963
accuracy: 0.734375
0--17--2304
likelihood: 1.0419973584182476
accuracy: 0.71875
0--18--2432
likelihood: 0.9822141820894781
accuracy: 0.78125
0--19--2560
likelihood: 1.048859187543728
accuracy: 0.7421875
0--20--2688
likelihood: 1.0342548244495977
accuracy: 0.6875
0--21--2816
likelihood: 1.018508665939862
accuracy: 0.765625
0--22--2944
likelihood: 1.051002820444534
accuracy: 0.7109375
0--23--3072
likelihood: 0.9282233902726331
accuracy: 0.7109375
0--24--3200
likelihood: 0.8744240635188474
accuracy: 0.7890625
0--25--3328
likelihood: 0.8244569494972285
```

```
accuracy: 0.7890625
0--26--3456
likelihood: 0.7977779804797372
accuracy: 0.78125
0--27--3584
likelihood: 0.7676662447960514
accuracy: 0.8125
0--28--3712
likelihood: 0.7796647625908663
accuracy: 0.8046875
0--29--3840
likelihood: 0.75440817170944
accuracy: 0.7890625
0--30--3968
likelihood: 0.8070503639703792
accuracy: 0.7421875
0--31--4096
likelihood: 0.7655405946902787
accuracy: 0.796875
0--32--4224
likelihood: 0.8616568699672725
accuracy: 0.75
0--33--4352
likelihood: 0.7552023431289551
accuracy: 0.8046875
0--34--4480
likelihood: 0.7433222356628701
accuracy: 0.84375
0--35--4608
likelihood: 0.7532108013910164
accuracy: 0.796875
0--36--4736
likelihood: 0.814190428647501
accuracy: 0.7890625
0--37--4864
likelihood: 0.6220072638473819
accuracy: 0.8125
0--38--4992
likelihood: 0.6470966796465027
```

```
accuracy: 0.84375
0--39--5120
likelihood: 0.7499685064324348
accuracy: 0.78125
0--40--5248
likelihood: 0.7315055664845772
accuracy: 0.8125
0--41--5376
likelihood: 0.5727038702302014
accuracy: 0.875
0--42--5504
likelihood: 0.6847071274302589
accuracy: 0.8203125
0--43--5632
likelihood: 0.7021457761788055
accuracy: 0.78125
0--44--5760
likelihood: 0.615135992116039
accuracy: 0.8359375
0--45--5888
likelihood: 0.7613747692387356
accuracy: 0.8046875
0--46--6016
likelihood: 0.6592947467208227
accuracy: 0.8359375
0--47--6144
likelihood: 0.5879875441592405
accuracy: 0.8515625
0--48--6272
likelihood: 0.6835816107099396
accuracy: 0.84375
0--49--6400
likelihood: 0.6529306702483856
accuracy: 0.828125
0--50--6528
likelihood: 0.6633618750802753
accuracy: 0.8125
0--51--6656
likelihood: 0.5058631434980134
```

```
accuracy: 0.8828125
0--52--6784
likelihood: 0.6311525600106633
accuracy: 0.859375
0--53--6912
likelihood: 0.5339073062733843
accuracy: 0.84375
0--54--7040
likelihood: 0.5454118112789326
accuracy: 0.875
0--55--7168
likelihood: 0.6613946913247551
accuracy: 0.8203125
0--56--7296
likelihood: 0.5332989237884944
accuracy: 0.859375
0--57--7424
likelihood: 0.6223232868020128
accuracy: 0.8046875
0--58--7552
likelihood: 0.604196518083128
accuracy: 0.8515625
0--59--7680
likelihood: 0.6134196717810418
accuracy: 0.7890625
0--60--7808
likelihood: 0.48942496923702294
accuracy: 0.875
0--61--7936
likelihood: 0.6571832732312971
accuracy: 0.8046875
0--62--8064
likelihood: 0.5421928692676916
accuracy: 0.875
0--63--8192
likelihood: 0.5087934491958308
accuracy: 0.8828125
0--64--8320
likelihood: 0.5005148366749299
```

```
accuracy: 0.8671875
0--65--8448
likelihood: 0.4195915562480516
accuracy: 0.90625
0--66--8576
likelihood: 0.46682100140049426
accuracy: 0.8828125
0--67--8704
likelihood: 0.5070714440826076
accuracy: 0.84375
0--68--8832
likelihood: 0.5842310285878669
accuracy: 0.8125
0--69--8960
likelihood: 0.57634229921275
accuracy: 0.796875
0--70--9088
likelihood: 0.4528906176739568
accuracy: 0.8984375
0--71--9216
likelihood: 0.5432561850787223
accuracy: 0.8515625
0--72--9344
likelihood: 0.6901474553130629
accuracy: 0.75
0--73--9472
likelihood: 0.5315023767273102
accuracy: 0.8046875
0--74--9600
likelihood: 0.6398640322165798
accuracy: 0.8125
0--75--9728
likelihood: 0.49572381231156143
accuracy: 0.84375
0--76--9856
likelihood: 0.6074509097457453
accuracy: 0.8046875
0--77--9984
likelihood: 0.5324287706136872
```

```
accuracy: 0.8671875
0--78--10112
likelihood: 0.48572787122398214
accuracy: 0.875
0--79--10240
likelihood: 0.6006242921716833
accuracy: 0.8515625
0--80--10368
likelihood: 0.595173983422323
accuracy: 0.8203125
0--81--10496
likelihood: 0.48336886031493415
accuracy: 0.8671875
0--82--10624
likelihood: 0.5392963149271844
accuracy: 0.828125
0--83--10752
likelihood: 0.4583418701955213
accuracy: 0.9140625
0--84--10880
likelihood: 0.4680546332684437
accuracy: 0.859375
0--85--11008
likelihood: 0.4195068700496332
accuracy: 0.8984375
0--86--11136
likelihood: 0.42270662468915055
accuracy: 0.8671875
0--87--11264
likelihood: 0.4943372348738676
accuracy: 0.859375
0--88--11392
likelihood: 0.5411924886369339
accuracy: 0.8671875
0--89--11520
likelihood: 0.45092554113525873
accuracy: 0.859375
0--90--11648
likelihood: 0.46491213093299805
```

```
accuracy: 0.8828125
0--91--11776
likelihood: 0.4478056670775561
accuracy: 0.890625
0--92--11904
likelihood: 0.5370311291331552
accuracy: 0.8359375
0--93--12032
likelihood: 0.5630288479275365
accuracy: 0.8203125
0--94--12160
likelihood: 0.41718036914741896
accuracy: 0.859375
0--95--12288
likelihood: 0.46524645357264005
accuracy: 0.8515625
0--96--12416
likelihood: 0.45359123350166575
accuracy: 0.90625
0--97--12544
likelihood: 0.5152949188394549
accuracy: 0.859375
0--98--12672
likelihood: 0.3421918275751196
accuracy: 0.8828125
0--99--12800
likelihood: 0.4812717036524752
accuracy: 0.875
0--100--12928
likelihood: 0.43185703545621723
accuracy: 0.8671875
0--101--13056
likelihood: 0.44599554080358805
accuracy: 0.875
0--102--13184
likelihood: 0.3948813414634825
accuracy: 0.8984375
0--103--13312
likelihood: 0.5328456963084235
```

```
accuracy: 0.8359375
0--104--13440
likelihood: 0.5358278780806194
accuracy: 0.8359375
0--105--13568
likelihood: 0.5926010910582731
accuracy: 0.84375
0--106--13696
likelihood: 0.4149401024217997
accuracy: 0.890625
0--107--13824
likelihood: 0.47152444972215724
accuracy: 0.8203125
0--108--13952
likelihood: 0.2783783482785894
accuracy: 0.9296875
0--109--14080
likelihood: 0.4414253984259714
accuracy: 0.890625
0--110--14208
likelihood: 0.534016753920909
accuracy: 0.828125
0--111--14336
likelihood: 0.45327716127931494
accuracy: 0.8984375
0--112--14464
likelihood: 0.4059201480843447
accuracy: 0.90625
0--113--14592
likelihood: 0.4527460669847329
accuracy: 0.8828125
0--114--14720
likelihood: 0.3802117773858477
accuracy: 0.8828125
0--115--14848
likelihood: 0.46591969568443364
accuracy: 0.84375
0--116--14976
likelihood: 0.464085008545169
```

```
accuracy: 0.90625
0--117--15104
likelihood: 0.3973907757585733
accuracy: 0.890625
0--118--15232
likelihood: 0.43732444977749724
accuracy: 0.859375
0--119--15360
likelihood: 0.2861079983495005
accuracy: 0.9453125
0--120--15488
likelihood: 0.3974343484357141
accuracy: 0.8828125
0--121--15616
likelihood: 0.444039390421915
accuracy: 0.859375
0--122--15744
likelihood: 0.5383207455162855
accuracy: 0.8046875
0--123--15872
likelihood: 0.43430907616143255
accuracy: 0.890625
0--124--16000
likelihood: 0.42499198428230667
accuracy: 0.8984375
0--125--16128
likelihood: 0.33490809940522326
accuracy: 0.9140625
0--126--16256
likelihood: 0.39876944211092635
accuracy: 0.90625
0--127--16384
likelihood: 0.3384190521932443
accuracy: 0.890625
0--128--16512
likelihood: 0.4301409301327888
accuracy: 0.8671875
0--129--16640
likelihood: 0.44087146117967435
```

```
accuracy: 0.8984375
0--130--16768
likelihood: 0.41492866103191167
accuracy: 0.8984375
0--131--16896
likelihood: 0.5487121150878685
accuracy: 0.84375
0--132--17024
likelihood: 0.48033160960419174
accuracy: 0.8671875
0--133--17152
likelihood: 0.2827652008103738
accuracy: 0.9375
0--134--17280
likelihood: 0.5280190354445662
accuracy: 0.84375
0--135--17408
likelihood: 0.4013633538688606
accuracy: 0.90625
0--136--17536
likelihood: 0.4341277717816206
accuracy: 0.90625
0--137--17664
likelihood: 0.36986659277996203
accuracy: 0.890625
0--138--17792
likelihood: 0.3639065606743461
accuracy: 0.90625
0--139--17920
likelihood: 0.37277113417056407
accuracy: 0.875
0--140--18048
likelihood: 0.3281691140953412
accuracy: 0.921875
0--141--18176
likelihood: 0.30284969809562756
accuracy: 0.9375
0--142--18304
likelihood: 0.37247722481492523
```

```
accuracy: 0.8984375
0--143--18432
likelihood: 0.4380878591385008
accuracy: 0.8828125
0--144--18560
likelihood: 0.37506361842601255
accuracy: 0.8984375
0--145--18688
likelihood: 0.390889982281352
accuracy: 0.90625
0--146--18816
likelihood: 0.3599942963969399
accuracy: 0.90625
0--147--18944
likelihood: 0.41143262980696377
accuracy: 0.890625
0--148--19072
likelihood: 0.41031210578035787
accuracy: 0.890625
0--149--19200
likelihood: 0.4600985503329954
accuracy: 0.8671875
0--150--19328
likelihood: 0.33945049774323377
accuracy: 0.8984375
0--151--19456
likelihood: 0.3800270390012476
accuracy: 0.8984375
0--152--19584
likelihood: 0.39472370963933356
accuracy: 0.859375
0--153--19712
likelihood: 0.3841021629213581
accuracy: 0.90625
0--154--19840
likelihood: 0.3205243187648902
accuracy: 0.890625
0--155--19968
likelihood: 0.4992907784343231
```

```
accuracy: 0.859375
0--156--20096
likelihood: 0.420497687191258
accuracy: 0.8828125
0--157--20224
likelihood: 0.4747355647815703
accuracy: 0.890625
0--158--20352
likelihood: 0.3297899245674878
accuracy: 0.9140625
0--159--20480
likelihood: 0.5765076515144314
accuracy: 0.8515625
0--160--20608
likelihood: 0.5689105012752618
accuracy: 0.859375
0--161--20736
likelihood: 0.40266185415624056
accuracy: 0.9296875
0--162--20864
likelihood: 0.40794886381505724
accuracy: 0.890625
0--163--20992
likelihood: 0.40792975673851656
accuracy: 0.875
0--164--21120
likelihood: 0.2872802159388109
accuracy: 0.9140625
0--165--21248
likelihood: 0.28309465528930744
accuracy: 0.9140625
0--166--21376
likelihood: 0.4824761618666853
accuracy: 0.8671875
0--167--21504
likelihood: 0.44351047278586536
accuracy: 0.890625
0--168--21632
likelihood: 0.38322419582237044
```

```
accuracy: 0.90625
0--169--21760
likelihood: 0.3769097063798208
accuracy: 0.890625
0--170--21888
likelihood: 0.3920090935928848
accuracy: 0.8515625
0--171--22016
likelihood: 0.3641771654429299
accuracy: 0.875
0--172--22144
likelihood: 0.5023609432674898
accuracy: 0.8359375
0--173--22272
likelihood: 0.37215579370650775
accuracy: 0.8828125
0--174--22400
likelihood: 0.3576019482144597
accuracy: 0.90625
0--175--22528
likelihood: 0.35543354651101966
accuracy: 0.921875
0--176--22656
likelihood: 0.31797409572887086
accuracy: 0.9140625
0--177--22784
likelihood: 0.3133741169958173
accuracy: 0.921875
0--178--22912
likelihood: 0.3121597299498443
accuracy: 0.921875
0--179--23040
likelihood: 0.3400229507354354
accuracy: 0.9296875
0--180--23168
likelihood: 0.32361553850315117
accuracy: 0.9296875
0--181--23296
likelihood: 0.4994068685631748
```

```
accuracy: 0.859375
0--182--23424
likelihood: 0.3665033227083894
accuracy: 0.9140625
0--183--23552
likelihood: 0.38080760151398285
accuracy: 0.8671875
0--184--23680
likelihood: 0.41112087109827866
accuracy: 0.859375
0--185--23808
likelihood: 0.2529433944756985
accuracy: 0.9296875
0--186--23936
likelihood: 0.4355002307324104
accuracy: 0.859375
0--187--24064
likelihood: 0.5443954670852987
accuracy: 0.875
0--188--24192
likelihood: 0.3275130711997787
accuracy: 0.8828125
0--189--24320
likelihood: 0.45066710246642117
accuracy: 0.8828125
0--190--24448
likelihood: 0.362833898554582
accuracy: 0.890625
0--191--24576
likelihood: 0.4840784045475447
accuracy: 0.84375
0--192--24704
likelihood: 0.3273309235119446
accuracy: 0.921875
0--193--24832
likelihood: 0.3690254735701962
accuracy: 0.875
0--194--24960
likelihood: 0.38617889969368624
```

```
accuracy: 0.90625
0--195--25088
likelihood: 0.3240239176367957
accuracy: 0.9140625
0--196--25216
likelihood: 0.399083965780036
accuracy: 0.8828125
0--197--25344
likelihood: 0.4448369377187385
accuracy: 0.8828125
0--198--25472
likelihood: 0.5084765802822988
accuracy: 0.875
0--199--25600
likelihood: 0.3816273765758413
accuracy: 0.875
0--200--25728
likelihood: 0.33549340057174243
accuracy: 0.90625
0--201--25856
likelihood: 0.42596543164542394
accuracy: 0.84375
0--202--25984
likelihood: 0.23221470966040078
accuracy: 0.9453125
0--203--26112
likelihood: 0.3383040850454929
accuracy: 0.9296875
0--204--26240
likelihood: 0.2855465783348593
accuracy: 0.9296875
0--205--26368
likelihood: 0.46788617478901406
accuracy: 0.859375
0--206--26496
likelihood: 0.34802149240598346
accuracy: 0.90625
0--207--26624
likelihood: 0.3078923210986365
```

```
accuracy: 0.9140625
0--208--26752
likelihood: 0.4087322367446533
accuracy: 0.8828125
0--209--26880
likelihood: 0.36737934022717034
accuracy: 0.8984375
0--210--27008
likelihood: 0.3055128566212699
accuracy: 0.9140625
0--211--27136
likelihood: 0.34598678541517935
accuracy: 0.8984375
0--212--27264
likelihood: 0.39143554155932353
accuracy: 0.9140625
0--213--27392
likelihood: 0.4220776143540002
accuracy: 0.875
0--214--27520
likelihood: 0.22968103295881812
accuracy: 0.9453125
0--215--27648
likelihood: 0.4166844996841923
accuracy: 0.8671875
0--216--27776
likelihood: 0.32720253762644524
accuracy: 0.890625
0--217--27904
likelihood: 0.31349499399099046
accuracy: 0.90625
0--218--28032
likelihood: 0.2527009781201379
accuracy: 0.921875
0--219--28160
likelihood: 0.3087098991516306
accuracy: 0.8984375
0--220--28288
likelihood: 0.4390202959658971
```

```
accuracy: 0.875
0--221--28416
likelihood: 0.35484139744046456
accuracy: 0.921875
0--222--28544
likelihood: 0.3450215101025608
accuracy: 0.890625
0--223--28672
likelihood: 0.24524699720009102
accuracy: 0.9296875
0--224--28800
likelihood: 0.46874000176229386
accuracy: 0.859375
0--225--28928
likelihood: 0.35788559457331603
accuracy: 0.890625
0--226--29056
likelihood: 0.36447116394594686
accuracy: 0.8828125
0--227--29184
likelihood: 0.4235807543379243
accuracy: 0.8828125
0--228--29312
likelihood: 0.3624121500369032
accuracy: 0.9140625
0--229--29399
likelihood: 0.3521769928408441
accuracy: 0.896551724137931
1--0--128
likelihood: 0.4784413933361167
accuracy: 0.8671875
1--1--256
likelihood: 0.280998404043543
accuracy: 0.90625
1--2--384
likelihood: 0.5224073266557672
accuracy: 0.859375
1--3--512
likelihood: 0.3235940035634032
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accuracy: 0.890625
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likelihood: 0.4186513298844987
accuracy: 0.8515625
1--5--768
likelihood: 0.3722485682686894
accuracy: 0.8828125
1--6--896
likelihood: 0.3577078536245744
accuracy: 0.90625
1--7--1024
likelihood: 0.3102148202754932
accuracy: 0.90625
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likelihood: 0.2817566739136048
accuracy: 0.90625
1--9--1280
likelihood: 0.37031080057203847
accuracy: 0.8671875
1--10--1408
likelihood: 0.2691861072171289
accuracy: 0.9140625
1--11--1536
likelihood: 0.34609346389358664
accuracy: 0.875
1--12--1664
likelihood: 0.2758241364742746
accuracy: 0.921875
1--13--1792
likelihood: 0.27419080654011907
accuracy: 0.921875
1--14--1920
likelihood: 0.2939112637745687
accuracy: 0.90625
1--15--2048
likelihood: 0.3102518988493187
accuracy: 0.8828125
1--16--2176
likelihood: 0.33204120765980805
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likelihood: 0.33787754391845404
accuracy: 0.921875
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likelihood: 0.38841739289347205
accuracy: 0.8671875
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likelihood: 0.28647193344995925
accuracy: 0.921875
1--21--2816
likelihood: 0.3095969068754494
accuracy: 0.8828125
1--22--2944
likelihood: 0.394890877853302
accuracy: 0.875
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likelihood: 0.3488029876208922
accuracy: 0.8671875
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likelihood: 0.3037214476423902
accuracy: 0.921875
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likelihood: 0.28530917506398634
accuracy: 0.90625
1--26--3456
likelihood: 0.2931464378248877
accuracy: 0.9140625
1--27--3584
likelihood: 0.23507186548665399
accuracy: 0.9296875
1--28--3712
likelihood: 0.24446196276152982
accuracy: 0.96875
1--29--3840
likelihood: 0.300079059540142
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accuracy: 0.90625
1--31--4096
likelihood: 0.37485050138090525
accuracy: 0.8984375
1--32--4224
likelihood: 0.40481754889162824
accuracy: 0.875
1--33--4352
likelihood: 0.3097313623292942
accuracy: 0.921875
1--34--4480
likelihood: 0.35920632785927703
accuracy: 0.8984375
1--35--4608
likelihood: 0.38687265900328316
accuracy: 0.8984375
1--36--4736
likelihood: 0.41814952218396384
accuracy: 0.8515625
1--37--4864
likelihood: 0.24292502593632326
accuracy: 0.921875
1--38--4992
likelihood: 0.3298984284439357
accuracy: 0.90625
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likelihood: 0.35398397172634577
accuracy: 0.8984375
1--40--5248
likelihood: 0.39377776996960023
accuracy: 0.8984375
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likelihood: 0.2149024317936732
accuracy: 0.9453125
1--42--5504
likelihood: 0.30713244667517786
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1--44--5760
likelihood: 0.3134226331172354
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likelihood: 0.5137701880222669
accuracy: 0.8828125
1--46--6016
likelihood: 0.38857735940757243
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1--47--6144
likelihood: 0.2625033273357935
accuracy: 0.921875
1--48--6272
likelihood: 0.31463599821842064
accuracy: 0.8984375
1--49--6400
likelihood: 0.32417084676958396
accuracy: 0.875
1--50--6528
likelihood: 0.42416906936426124
accuracy: 0.859375
1--51--6656
likelihood: 0.20548885540668366
accuracy: 0.9453125
1--52--6784
likelihood: 0.35278242712110924
accuracy: 0.921875
1--53--6912
likelihood: 0.23284593703537668
accuracy: 0.9140625
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1--55--7168
likelihood: 0.41709150237123177
```

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accuracy: 0.9296875
1--57--7424
likelihood: 0.36704233540956954
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accuracy: 0.9375
1--59--7680
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accuracy: 0.90625
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likelihood: 0.21049285400482146
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1--66--8576
likelihood: 0.21814655111015233
accuracy: 0.953125
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likelihood: 0.22486617728784986
accuracy: 0.9375
1--68--8832
likelihood: 0.3442469318671014
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likelihood: 0.3140544232785493
accuracy: 0.890625
1--70--9088
likelihood: 0.25127564374757205
accuracy: 0.9296875
1--71--9216
likelihood: 0.3393408632041585
accuracy: 0.8984375
1--72--9344
likelihood: 0.4864639160924068
accuracy: 0.8359375
1--73--9472
likelihood: 0.34721989284360544
accuracy: 0.8671875
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likelihood: 0.4607340927267417
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1--75--9728
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accuracy: 0.8984375
1--76--9856
likelihood: 0.35825968686244286
accuracy: 0.875
1--77--9984
likelihood: 0.3049351727502574
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1--78--10112
likelihood: 0.3121919562288198
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1--79--10240
likelihood: 0.4295420232389461
accuracy: 0.8984375
1--80--10368
likelihood: 0.3983019736381338
accuracy: 0.8828125
1--81--10496
likelihood: 0.2665409958645021
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likelihood: 0.3532870695502658
accuracy: 0.890625
1--83--10752
likelihood: 0.2779423227207185
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1--84--10880
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1--85--11008
likelihood: 0.21878340370101995
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1--86--11136
likelihood: 0.25071160511379764
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1--87--11264
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accuracy: 0.8671875
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likelihood: 0.3967208036707414
accuracy: 0.890625
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likelihood: 0.2868004728390251
accuracy: 0.9296875
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likelihood: 0.27743950034042697
accuracy: 0.921875
1--92--11904
likelihood: 0.39847535276724294
accuracy: 0.8984375
1--93--12032
likelihood: 0.3709176959356433
accuracy: 0.8984375
1--94--12160
likelihood: 0.27052894195736626
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likelihood: 0.2904986149067824
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likelihood: 0.33194582380738397
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1--97--12544
likelihood: 0.37430277906733755
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1--98--12672
likelihood: 0.1999417758621406
accuracy: 0.9375
1--99--12800
likelihood: 0.37162875343143553
accuracy: 0.9296875
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likelihood: 0.28508558399260997
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likelihood: 0.20992930406145704
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1--103--13312
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1--104--13440
likelihood: 0.36639050533154693
accuracy: 0.8984375
1--105--13568
likelihood: 0.42127941663895585
accuracy: 0.8984375
1--106--13696
likelihood: 0.3056380339349436
accuracy: 0.90625
1--107--13824
likelihood: 0.30391790671947205
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accuracy: 0.8984375
1--110--14208
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likelihood: 0.2736493509320972
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likelihood: 0.3034338647475618
accuracy: 0.859375
1--116--14976
likelihood: 0.3678895103481914
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likelihood: 0.311873122562798
accuracy: 0.921875
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likelihood: 0.1822308884938655
accuracy: 0.9609375
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likelihood: 0.2775342971148901
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accuracy: 0.9296875
1--122--15744
likelihood: 0.3900269043747379
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likelihood: 0.308254262534455
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likelihood: 0.30314833682584785
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likelihood: 0.2897914942258314
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likelihood: 0.3910639523172186
accuracy: 0.8671875
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likelihood: 0.16981852223661753
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likelihood: 0.2727374270007044
accuracy: 0.9140625
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likelihood: 0.2811987983786787
accuracy: 0.9375
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likelihood: 0.26033479606459353
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accuracy: 0.90625
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1--153--19712
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accuracy: 0.921875
1--154--19840
likelihood: 0.21570631567943793
accuracy: 0.9296875
1--155--19968
likelihood: 0.3939596911595955
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1--156--20096
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accuracy: 0.9453125
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likelihood: 0.46845843504096696
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likelihood: 0.3213300303503326
accuracy: 0.921875
1--162--20864
likelihood: 0.3323971150616603
accuracy: 0.921875
1--163--20992
likelihood: 0.309013077594275
accuracy: 0.8984375
1--164--21120
likelihood: 0.20684476378976666
accuracy: 0.9453125
1--165--21248
likelihood: 0.18287534562771526
accuracy: 0.9453125
1--166--21376
likelihood: 0.36682480166017384
accuracy: 0.8984375
1--167--21504
likelihood: 0.3733493286387235
accuracy: 0.8984375
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accuracy: 0.9296875
1--169--21760
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accuracy: 0.9296875
1--170--21888
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accuracy: 0.90625
1--171--22016
likelihood: 0.266503238653585
accuracy: 0.9140625
1--172--22144
likelihood: 0.3711276663502726
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likelihood: 0.21697089647196888
accuracy: 0.921875
1--178--22912
likelihood: 0.23386858464233243
accuracy: 0.921875
1--179--23040
likelihood: 0.27789676113326756
accuracy: 0.9296875
1--180--23168
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accuracy: 0.9296875
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accuracy: 0.890625
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likelihood: 0.1923340955850364
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accuracy: 0.90625
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accuracy: 0.9140625
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accuracy: 0.9453125
1--196--25216
likelihood: 0.35118332739436897
accuracy: 0.90625
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likelihood: 0.36331189238058564
accuracy: 0.8984375
1--198--25472
likelihood: 0.4398789757697274
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likelihood: 0.3395914595424176
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likelihood: 0.18778093420870268
accuracy: 0.9453125
1--203--26112
likelihood: 0.2865569033984523
accuracy: 0.9296875
1--204--26240
likelihood: 0.21680094487446883
accuracy: 0.9296875
1--205--26368
likelihood: 0.40039280850551406
accuracy: 0.8828125
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likelihood: 0.2612819367944928
accuracy: 0.9296875
1--207--26624
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likelihood: 0.3499042615529311
accuracy: 0.890625
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accuracy: 0.9140625
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accuracy: 0.921875
1--211--27136
likelihood: 0.24778017494325777
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1--215--27648
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1--216--27776
likelihood: 0.2690853334690969
accuracy: 0.9375
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likelihood: 0.2573507878034015
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likelihood: 0.2919803677785158
accuracy: 0.9296875
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likelihood: 0.28305228358316903
accuracy: 0.921875
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accuracy: 0.9375
1--224--28800
likelihood: 0.4062359630325349
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likelihood: 0.28241731071444065
accuracy: 0.9140625
1--227--29184
likelihood: 0.38286820305802305
accuracy: 0.8984375
1--228--29312
likelihood: 0.28646853430256464
accuracy: 0.9375
1--229--29399
likelihood: 0.26452557987173797
accuracy: 0.9425287356321839
2--0--128
likelihood: 0.4129748022730617
accuracy: 0.8828125
2--1--256
likelihood: 0.2095900004345317
accuracy: 0.90625
2--2--384
likelihood: 0.44988712294392996
accuracy: 0.8515625
2--3--512
likelihood: 0.24945104584172284
accuracy: 0.921875
2--4--640
likelihood: 0.3541436495270512
accuracy: 0.8828125
2--5--768
likelihood: 0.2939646110547809
accuracy: 0.890625
2--6--896
likelihood: 0.3091939706757324
accuracy: 0.9140625
2--7--1024
likelihood: 0.2571216924935017
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accuracy: 0.9375
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likelihood: 0.20645909197731965
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accuracy: 0.890625
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likelihood: 0.2184377474424078
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likelihood: 0.29064085525491307
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likelihood: 0.22491479480563942
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accuracy: 0.9296875
2--14--1920
likelihood: 0.2423380305127414
accuracy: 0.921875
2--15--2048
likelihood: 0.2489024956256468
accuracy: 0.90625
2--16--2176
likelihood: 0.2825886696565393
accuracy: 0.9453125
2--17--2304
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accuracy: 0.9453125
2--18--2432
likelihood: 0.2926194451140871
accuracy: 0.9375
2--19--2560
likelihood: 0.32324067560597225
accuracy: 0.8671875
2--20--2688
likelihood: 0.21445326906897427
```

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accuracy: 0.921875
2--21--2816
likelihood: 0.24953987833615443
accuracy: 0.8984375
2--22--2944
likelihood: 0.3165146470961461
accuracy: 0.9140625
2--23--3072
likelihood: 0.2774181219834073
accuracy: 0.8828125
2--24--3200
likelihood: 0.24805744825849274
accuracy: 0.9296875
2--25--3328
likelihood: 0.23212184482675405
accuracy: 0.9140625
2--26--3456
likelihood: 0.243634536569421
accuracy: 0.9140625
2--27--3584
likelihood: 0.17555438548800228
accuracy: 0.9375
2--28--3712
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accuracy: 0.96875
2--29--3840
likelihood: 0.24317040363624626
accuracy: 0.9140625
2--30--3968
likelihood: 0.23315855760406998
accuracy: 0.9296875
2--31--4096
likelihood: 0.3276961566327781
accuracy: 0.90625
2--32--4224
likelihood: 0.34578156299598417
accuracy: 0.890625
2--33--4352
likelihood: 0.25333003369876583
```

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accuracy: 0.9453125
2--34--4480
likelihood: 0.30433017965901404
accuracy: 0.9140625
2--35--4608
likelihood: 0.3291496506818022
accuracy: 0.8984375
2--36--4736
likelihood: 0.32612436895112773
accuracy: 0.8984375
2--37--4864
likelihood: 0.19844040141225544
accuracy: 0.921875
2--38--4992
likelihood: 0.30154239358182244
accuracy: 0.9140625
2--39--5120
likelihood: 0.2949081740330273
accuracy: 0.9140625
2--40--5248
likelihood: 0.33970352800268444
accuracy: 0.9140625
2--41--5376
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accuracy: 0.9375
2--42--5504
likelihood: 0.23739734169580798
accuracy: 0.9296875
2--43--5632
likelihood: 0.2087288566117066
accuracy: 0.921875
2--44--5760
likelihood: 0.2664887074780929
accuracy: 0.9296875
2--45--5888
likelihood: 0.48318595663357666
accuracy: 0.8828125
2--46--6016
likelihood: 0.33411854859954526
```

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2--47--6144
likelihood: 0.22126630810781892
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2--48--6272
likelihood: 0.24762541315395326
accuracy: 0.9140625
2--49--6400
likelihood: 0.2618460908693576
accuracy: 0.9140625
2--50--6528
likelihood: 0.38869727542848564
accuracy: 0.8828125
2--51--6656
likelihood: 0.15288891478661581
accuracy: 0.953125
2--52--6784
likelihood: 0.29776325572174006
accuracy: 0.9453125
2--53--6912
likelihood: 0.18113345898406846
accuracy: 0.9453125
2--54--7040
likelihood: 0.22483104816901783
accuracy: 0.9140625
2--55--7168
likelihood: 0.3613008146370009
accuracy: 0.8828125
2--56--7296
likelihood: 0.2109964121373635
accuracy: 0.9453125
2--57--7424
likelihood: 0.3081717897691191
accuracy: 0.921875
2--58--7552
likelihood: 0.32521651014020203
accuracy: 0.9296875
2--59--7680
likelihood: 0.37369855368240246
```

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2--60--7808
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accuracy: 0.8984375
2--62--8064
likelihood: 0.2502321834652378
accuracy: 0.921875
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likelihood: 0.2769833967055245
accuracy: 0.9375
2--64--8320
likelihood: 0.2508444807950252
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2--65--8448
likelihood: 0.17624664860032632
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2--66--8576
likelihood: 0.1695451298105187
accuracy: 0.9609375
2--67--8704
likelihood: 0.16605490863698502
accuracy: 0.96875
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likelihood: 0.28904051210435777
accuracy: 0.9140625
2--69--8960
likelihood: 0.2474821702686613
accuracy: 0.9140625
2--70--9088
likelihood: 0.20861731371885953
accuracy: 0.9375
2--71--9216
likelihood: 0.2825250780616856
accuracy: 0.90625
2--72--9344
likelihood: 0.404378257158427
```

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2--74--9600
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accuracy: 0.9140625
2--75--9728
likelihood: 0.2796643443951746
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2--76--9856
likelihood: 0.28306431578429436
accuracy: 0.9140625
2--77--9984
likelihood: 0.2528199115950023
accuracy: 0.9453125
2--78--10112
likelihood: 0.2703128443962728
accuracy: 0.9140625
2--79--10240
likelihood: 0.373338510285906
accuracy: 0.90625
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likelihood: 0.34076243920984
accuracy: 0.9140625
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accuracy: 0.9296875
2--82--10624
likelihood: 0.30417232967701024
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2--83--10752
likelihood: 0.22517690243272198
accuracy: 0.921875
2--84--10880
likelihood: 0.21302591309926117
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2--85--11008
likelihood: 0.16478433959066557
```

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accuracy: 0.9765625
2--86--11136
likelihood: 0.2121135236552824
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2--87--11264
likelihood: 0.32445617370757185
accuracy: 0.875
2--88--11392
likelihood: 0.3528661070679706
accuracy: 0.90625
2--89--11520
likelihood: 0.17373668364259803
accuracy: 0.96875
2--90--11648
likelihood: 0.22992210437707244
accuracy: 0.953125
2--91--11776
likelihood: 0.22804092422966646
accuracy: 0.9375
2--92--11904
likelihood: 0.3633538410500681
accuracy: 0.890625
2--93--12032
likelihood: 0.31185641448693435
accuracy: 0.890625
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likelihood: 0.22168841598813654
accuracy: 0.9296875
2--95--12288
likelihood: 0.24210022962972927
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2--96--12416
likelihood: 0.2852777315806002
accuracy: 0.953125
2--97--12544
likelihood: 0.3300314858428248
accuracy: 0.8984375
2--98--12672
likelihood: 0.1695380125611306
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likelihood: 0.34444742748367224
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2--100--12928
likelihood: 0.2344509230005629
accuracy: 0.90625
2--101--13056
likelihood: 0.24135903284966692
accuracy: 0.9375
2--102--13184
likelihood: 0.15821932983029224
accuracy: 0.9609375
2--103--13312
likelihood: 0.3149574167856294
accuracy: 0.9140625
2--104--13440
likelihood: 0.3174074558323629
accuracy: 0.8984375
2--105--13568
likelihood: 0.35893248500338715
accuracy: 0.9140625
2--106--13696
likelihood: 0.2725521057482611
accuracy: 0.90625
2--107--13824
likelihood: 0.24901236937188248
accuracy: 0.921875
2--108--13952
likelihood: 0.1491854100742272
accuracy: 0.9453125
2--109--14080
likelihood: 0.2657316662022928
accuracy: 0.90625
2--110--14208
likelihood: 0.3246584645910152
accuracy: 0.890625
2--111--14336
likelihood: 0.3326466515068667
```

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accuracy: 0.921875
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accuracy: 0.9296875
2--113--14592
likelihood: 0.27232160899680324
accuracy: 0.9296875
2--114--14720
likelihood: 0.24184169560747548
accuracy: 0.9140625
2--115--14848
likelihood: 0.25579112019798844
accuracy: 0.9140625
2--116--14976
likelihood: 0.3346408837307868
accuracy: 0.9296875
2--117--15104
likelihood: 0.2865716975225735
accuracy: 0.9296875
2--118--15232
likelihood: 0.2842565280211326
accuracy: 0.921875
2--119--15360
likelihood: 0.15538020811969724
accuracy: 0.96875
2--120--15488
likelihood: 0.23394176019223806
accuracy: 0.9453125
2--121--15616
likelihood: 0.27684753801984474
accuracy: 0.9453125
2--122--15744
likelihood: 0.34217822744201
accuracy: 0.8828125
2--123--15872
likelihood: 0.24768730908900313
accuracy: 0.9375
2--124--16000
likelihood: 0.20952110997057605
```

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accuracy: 0.9453125
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likelihood: 0.16079385670478683
accuracy: 0.9609375
2--126--16256
likelihood: 0.23832031661395198
accuracy: 0.9375
2--127--16384
likelihood: 0.1819166619802945
accuracy: 0.9453125
2--128--16512
likelihood: 0.2632972538770256
accuracy: 0.9140625
2--129--16640
likelihood: 0.2538021127031236
accuracy: 0.921875
2--130--16768
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accuracy: 0.9296875
2--131--16896
likelihood: 0.32429166872073456
accuracy: 0.8984375
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likelihood: 0.38590803523100325
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likelihood: 0.13423707209333413
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2--134--17280
likelihood: 0.3726634636439494
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2--135--17408
likelihood: 0.30706562614343175
accuracy: 0.921875
2--136--17536
likelihood: 0.2637006784032402
accuracy: 0.9296875
2--137--17664
likelihood: 0.23543975786663862
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likelihood: 0.20712106106030054
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2--140--18048
likelihood: 0.20028584516431272
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2--141--18176
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accuracy: 0.9609375
2--142--18304
likelihood: 0.28160381558916414
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2--143--18432
likelihood: 0.3075902570563753
accuracy: 0.9140625
2--144--18560
likelihood: 0.23262041070688994
accuracy: 0.921875
2--145--18688
likelihood: 0.23078906587533574
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2--146--18816
likelihood: 0.2332947331744768
accuracy: 0.9375
2--147--18944
likelihood: 0.29122206283057905
accuracy: 0.9375
2--148--19072
likelihood: 0.3226329447371223
accuracy: 0.9140625
2--149--19200
likelihood: 0.348405764100443
accuracy: 0.90625
2--150--19328
likelihood: 0.18193152412620817
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accuracy: 0.9453125
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likelihood: 0.23706438327201318
accuracy: 0.921875
2--153--19712
likelihood: 0.26355196858347874
accuracy: 0.9296875
2--154--19840
likelihood: 0.17753591538695163
accuracy: 0.9609375
2--155--19968
likelihood: 0.33834344892620816
accuracy: 0.90625
2--156--20096
likelihood: 0.25595326780822314
accuracy: 0.9296875
2--157--20224
likelihood: 0.36013234788819243
accuracy: 0.9140625
2--158--20352
likelihood: 0.19457870534273003
accuracy: 0.953125
2--159--20480
likelihood: 0.4165983639589841
accuracy: 0.890625
2--160--20608
likelihood: 0.44905285553257623
accuracy: 0.8984375
2--161--20736
likelihood: 0.28328074415803906
accuracy: 0.921875
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likelihood: 0.30240956304893885
accuracy: 0.9375
2--163--20992
likelihood: 0.2568513982365574
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likelihood: 0.14699429659741065
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2--166--21376
likelihood: 0.3170605890294504
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2--167--21504
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2--169--21760
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accuracy: 0.9296875
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accuracy: 0.9296875
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2--187--24064
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accuracy: 0.9140625
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likelihood: 0.3367019328005427
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likelihood: 0.26800486477491714
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2--191--24576
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likelihood: 0.2130926247786701
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2--193--24832
likelihood: 0.26249411182650456
accuracy: 0.890625
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likelihood: 0.2911193561797826
accuracy: 0.921875
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likelihood: 0.18439977161928411
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likelihood: 0.334585838882031
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accuracy: 0.9140625
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likelihood: 0.29950389239497976
accuracy: 0.890625
2--202--25984
likelihood: 0.16835276051997342
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accuracy: 0.9609375
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likelihood: 0.18991310727076
accuracy: 0.9296875
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accuracy: 0.90625
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likelihood: 0.1436075425045376
accuracy: 0.9609375
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likelihood: 0.3121079381468088
```

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accuracy: 0.9140625
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likelihood: 0.23118648951350987
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2--222--28544
likelihood: 0.2536757968269545
accuracy: 0.9375
2--223--28672
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likelihood: 0.24341947850466178
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accuracy: 0.890625
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likelihood: 0.2504936925562804
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likelihood: 0.1765381238641272
accuracy: 0.921875
3--2--384
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accuracy: 0.8828125
3--3--512
likelihood: 0.20920549231952765
accuracy: 0.9296875
3--4--640
likelihood: 0.3209794164394881
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likelihood: 0.25340201324637246
accuracy: 0.890625
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likelihood: 0.28704157529416
accuracy: 0.9375
3--7--1024
likelihood: 0.22639839524472719
accuracy: 0.9375
3--8--1152
likelihood: 0.1661334074684365
accuracy: 0.953125
3--9--1280
likelihood: 0.27589608871360144
accuracy: 0.9140625
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likelihood: 0.1892542385135712
accuracy: 0.953125
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likelihood: 0.2658084948304313
```

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accuracy: 0.921875
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likelihood: 0.1910417954679629
accuracy: 0.9296875
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likelihood: 0.17638583339019942
accuracy: 0.9375
3--14--1920
likelihood: 0.21448026487541083
accuracy: 0.9375
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likelihood: 0.22016247088891672
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likelihood: 0.25270473910653196
accuracy: 0.9375
3--17--2304
likelihood: 0.17981911637562045
accuracy: 0.96875
3--18--2432
likelihood: 0.2669384703608221
accuracy: 0.9375
3--19--2560
likelihood: 0.28442579007151547
accuracy: 0.8828125
3--20--2688
likelihood: 0.17813960353638758
accuracy: 0.9140625
3--21--2816
likelihood: 0.21865110870211152
accuracy: 0.921875
3--22--2944
likelihood: 0.27509824083084006
accuracy: 0.921875
3--23--3072
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accuracy: 0.8984375
3--24--3200
likelihood: 0.21767698820698217
```

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accuracy: 0.9296875
3--25--3328
likelihood: 0.20286901854667827
accuracy: 0.921875
3--26--3456
likelihood: 0.21659859824812155
accuracy: 0.9296875
3--27--3584
likelihood: 0.14589050644142126
accuracy: 0.9609375
3--28--3712
likelihood: 0.15476434958466345
accuracy: 0.9765625
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likelihood: 0.20963772021794144
accuracy: 0.921875
3--30--3968
likelihood: 0.19256686096944586
accuracy: 0.9375
3--31--4096
likelihood: 0.2935553955288488
accuracy: 0.9140625
3--32--4224
likelihood: 0.31115805380222594
accuracy: 0.890625
3--33--4352
likelihood: 0.22228210539054372
accuracy: 0.9453125
3--34--4480
likelihood: 0.27013937008115496
accuracy: 0.9296875
3--35--4608
likelihood: 0.29128728078691596
accuracy: 0.9140625
3--36--4736
likelihood: 0.26385767368979207
accuracy: 0.90625
3--37--4864
likelihood: 0.1741716231151212
```

```
accuracy: 0.9375
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3--39--5120
likelihood: 0.2646953684743941
accuracy: 0.9296875
3--40--5248
likelihood: 0.3062922665270055
accuracy: 0.921875
3--41--5376
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accuracy: 0.953125
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accuracy: 0.9453125
3--43--5632
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accuracy: 0.9609375
3--44--5760
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accuracy: 0.9453125
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3--47--6144
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3--49--6400
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accuracy: 0.9296875
3--50--6528
likelihood: 0.3678140432302166
```

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likelihood: 0.12557251065903696
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accuracy: 0.9296875
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likelihood: 0.3240276571713446
accuracy: 0.8828125
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accuracy: 0.953125
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likelihood: 0.2700015345309253
accuracy: 0.9296875
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likelihood: 0.29954638458505284
accuracy: 0.9296875
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likelihood: 0.3496061618709995
accuracy: 0.8984375
3--60--7808
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accuracy: 0.9609375
3--61--7936
likelihood: 0.3829977772486499
accuracy: 0.8984375
3--62--8064
likelihood: 0.21824070092788705
accuracy: 0.9296875
3--63--8192
likelihood: 0.25641544606764977
```

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3--66--8576
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accuracy: 0.9765625
3--67--8704
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likelihood: 0.2548034913682581
accuracy: 0.921875
3--69--8960
likelihood: 0.21249455199073933
accuracy: 0.9296875
3--70--9088
likelihood: 0.1841882532467936
accuracy: 0.9375
3--71--9216
likelihood: 0.2458342334458985
accuracy: 0.9140625
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3--73--9472
likelihood: 0.24346184984273495
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likelihood: 0.3397753006041879
accuracy: 0.921875
3--75--9728
likelihood: 0.24977602972556887
accuracy: 0.9140625
3--76--9856
likelihood: 0.2405953563977826
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3--78--10112
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accuracy: 0.921875
3--79--10240
likelihood: 0.3346375985492658
accuracy: 0.921875
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accuracy: 0.9296875
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likelihood: 0.19321882182395939
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likelihood: 0.13565538699071408
accuracy: 0.984375
3--86--11136
likelihood: 0.1873672214306457
accuracy: 0.9375
3--87--11264
likelihood: 0.3005203685340699
accuracy: 0.8828125
3--88--11392
likelihood: 0.32509252712755105
accuracy: 0.90625
3--89--11520
likelihood: 0.14436656144871424
```

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3--91--11776
likelihood: 0.199206717368618
accuracy: 0.9375
3--92--11904
likelihood: 0.3392661709258577
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3--93--12032
likelihood: 0.2729901199704835
accuracy: 0.90625
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likelihood: 0.19162003928997212
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accuracy: 0.953125
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3--98--12672
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accuracy: 0.96875
3--99--12800
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accuracy: 0.9375
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likelihood: 0.20262561838601317
accuracy: 0.9375
3--101--13056
likelihood: 0.21068553752962627
accuracy: 0.9453125
3--102--13184
likelihood: 0.13012513994354524
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3--104--13440
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accuracy: 0.90625
3--105--13568
likelihood: 0.31554719153324196
accuracy: 0.9140625
3--106--13696
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accuracy: 0.9140625
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likelihood: 0.2922304045253362
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likelihood: 0.20976433386778456
accuracy: 0.921875
3--113--14592
likelihood: 0.24475433993601695
accuracy: 0.9375
3--114--14720
likelihood: 0.222466999229747
accuracy: 0.9140625
3--115--14848
likelihood: 0.23105961423305357
```

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accuracy: 0.921875
3--116--14976
likelihood: 0.3116676254120633
accuracy: 0.921875
3--117--15104
likelihood: 0.26789192257455996
accuracy: 0.9296875
3--118--15232
likelihood: 0.25444517732221655
accuracy: 0.9296875
3--119--15360
likelihood: 0.13993826454596636
accuracy: 0.9609375
3--120--15488
likelihood: 0.20500716513045575
accuracy: 0.9453125
3--121--15616
likelihood: 0.25226860763003534
accuracy: 0.953125
3--122--15744
likelihood: 0.3120215876278157
accuracy: 0.890625
3--123--15872
likelihood: 0.22439185976795564
accuracy: 0.953125
3--124--16000
likelihood: 0.1799474290230641
accuracy: 0.953125
3--125--16128
likelihood: 0.14147879798266644
accuracy: 0.9609375
3--126--16256
likelihood: 0.21350193557383743
accuracy: 0.9453125
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likelihood: 0.15531874735301482
accuracy: 0.9453125
3--128--16512
likelihood: 0.2367926220344379
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accuracy: 0.9375
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likelihood: 0.22197865983411125
accuracy: 0.9375
3--130--16768
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3--131--16896
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3--132--17024
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3--133--17152
likelihood: 0.11338385456927326
accuracy: 0.9765625
3--134--17280
likelihood: 0.3415443389693034
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3--136--17536
likelihood: 0.22854720930886968
accuracy: 0.9453125
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likelihood: 0.20921180277170026
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3--139--17920
likelihood: 0.18388880044315836
accuracy: 0.9375
3--140--18048
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3--141--18176
likelihood: 0.13937151315372437
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likelihood: 0.28571718449796507
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3--145--18688
likelihood: 0.19763393788576372
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3--146--18816
likelihood: 0.2193465554697649
accuracy: 0.9296875
3--147--18944
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likelihood: 0.32431990873370853
accuracy: 0.9140625
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likelihood: 0.16382241998975583
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3--151--19456
likelihood: 0.18563720728314487
accuracy: 0.953125
3--152--19584
likelihood: 0.2065674503717709
accuracy: 0.9296875
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accuracy: 0.9296875
3--154--19840
likelihood: 0.1548725005860651
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likelihood: 0.298470001184726
accuracy: 0.9140625
3--156--20096
likelihood: 0.23064277066708594
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3--157--20224
likelihood: 0.3233230351495567
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3--158--20352
likelihood: 0.17361598648958626
accuracy: 0.953125
3--159--20480
likelihood: 0.38208914295336505
accuracy: 0.8984375
3--160--20608
likelihood: 0.4104934854413441
accuracy: 0.8984375
3--161--20736
likelihood: 0.25651673267690256
accuracy: 0.921875
3--162--20864
likelihood: 0.28110807373276336
accuracy: 0.9296875
3--163--20992
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3--164--21120
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3--165--21248
likelihood: 0.12567062033902904
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3--167--21504
likelihood: 0.31510017108319704
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likelihood: 0.17936424944034754
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likelihood: 0.20269827750146835
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3--172--22144
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accuracy: 0.9296875
3--173--22272
likelihood: 0.21502885393132234
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likelihood: 0.18279046493003864
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accuracy: 0.9296875
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likelihood: 0.17347179831609896
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accuracy: 0.8984375
3--206--26496
likelihood: 0.2169866154607956
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accuracy: 0.953125
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3--213--27392
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likelihood: 0.1314429630482407
accuracy: 0.9609375
3--219--28160
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3--222--28544
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3--223--28672
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accuracy: 0.96875
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accuracy: 0.8828125
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accuracy: 0.9296875
3--226--29056
likelihood: 0.2158970796009546
accuracy: 0.9296875
3--227--29184
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accuracy: 0.90625
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accuracy: 0.9375
4--2--384
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4--6--896
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4--8--1152
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4--9--1280
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4--12--1664
likelihood: 0.16522813941787873
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likelihood: 0.14974199139824937
accuracy: 0.9453125
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likelihood: 0.19369690187848493
accuracy: 0.9375
4--15--2048
likelihood: 0.20220245584743524
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accuracy: 0.8984375
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likelihood: 0.15476600160173345
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4--21--2816
likelihood: 0.19643573223148397
accuracy: 0.9453125
4--22--2944
likelihood: 0.24517442339337242
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4--23--3072
likelihood: 0.21200875264688832
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likelihood: 0.19505017818358522
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accuracy: 0.9296875
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accuracy: 0.9375
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likelihood: 0.12611286227673485
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4--28--3712
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4--38--4992
likelihood: 0.2677910279698471
accuracy: 0.9375
4--39--5120
likelihood: 0.24277599304824807
accuracy: 0.9375
4--40--5248
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accuracy: 0.921875
4--41--5376
likelihood: 0.1465806537936306
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likelihood: 0.21250169780913694
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likelihood: 0.2702537618802683
accuracy: 0.921875
4--47--6144
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4--48--6272
likelihood: 0.17867901965222555
accuracy: 0.9375
4--49--6400
likelihood: 0.19651405392506305
accuracy: 0.9453125
4--50--6528
likelihood: 0.3506149040563447
accuracy: 0.9140625
4--51--6656
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accuracy: 0.9609375
4--52--6784
likelihood: 0.24051398980742802
accuracy: 0.9453125
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accuracy: 0.96875
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likelihood: 0.17101238811748826
```

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accuracy: 0.9453125
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likelihood: 0.29441668060912407
accuracy: 0.90625
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accuracy: 0.9609375
4--57--7424
likelihood: 0.24082398873956268
accuracy: 0.9296875
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likelihood: 0.2778889909123381
accuracy: 0.9375
4--59--7680
likelihood: 0.3298402637834492
accuracy: 0.8984375
4--60--7808
likelihood: 0.1553944237373288
accuracy: 0.9609375
4--61--7936
likelihood: 0.354852550009836
accuracy: 0.90625
4--62--8064
likelihood: 0.19485823470721408
accuracy: 0.9296875
4--63--8192
likelihood: 0.23878496423497722
accuracy: 0.9375
4--64--8320
likelihood: 0.2045332184594949
accuracy: 0.9296875
4--65--8448
likelihood: 0.14264787525536632
accuracy: 0.96875
4--66--8576
likelihood: 0.11739757367912226
accuracy: 0.984375
4--67--8704
likelihood: 0.11724783137212931
```

```
accuracy: 0.984375
4--68--8832
likelihood: 0.22946253865868732
accuracy: 0.9296875
4--69--8960
likelihood: 0.18877786673706287
accuracy: 0.9296875
4--70--9088
likelihood: 0.16647309722415377
accuracy: 0.9375
4--71--9216
likelihood: 0.2168692546995762
accuracy: 0.921875
4--72--9344
likelihood: 0.30045068795348506
accuracy: 0.8984375
4--73--9472
likelihood: 0.21345709275904295
accuracy: 0.9140625
4--74--9600
likelihood: 0.29990337361538294
accuracy: 0.9375
4--75--9728
likelihood: 0.22800479409998742
accuracy: 0.921875
4--76--9856
likelihood: 0.21340339241198036
accuracy: 0.9296875
4--77--9984
likelihood: 0.1946250906128487
accuracy: 0.953125
4--78--10112
likelihood: 0.21743654820372293
accuracy: 0.9375
4--79--10240
likelihood: 0.30261909374368307
accuracy: 0.9296875
4--80--10368
likelihood: 0.27454156512842964
```

```
accuracy: 0.9375
4--81--10496
likelihood: 0.1464114812107803
accuracy: 0.9296875
4--82--10624
likelihood: 0.25514153396439165
accuracy: 0.921875
4--83--10752
likelihood: 0.1709068042614178
accuracy: 0.9453125
4--84--10880
likelihood: 0.1674356471169789
accuracy: 0.96875
4--85--11008
likelihood: 0.11638199466205285
accuracy: 0.984375
4--86--11136
likelihood: 0.16856729568199957
accuracy: 0.9375
4--87--11264
likelihood: 0.27941457242146955
accuracy: 0.8828125
4--88--11392
likelihood: 0.30546422572225723
accuracy: 0.9140625
4--89--11520
likelihood: 0.12453490668140726
accuracy: 0.9765625
4--90--11648
likelihood: 0.17515790705420362
accuracy: 0.9609375
4--91--11776
likelihood: 0.17861904539316115
accuracy: 0.9375
4--92--11904
likelihood: 0.31995014502137664
accuracy: 0.8984375
4--93--12032
likelihood: 0.24364885241293147
```

```
accuracy: 0.90625
4--94--12160
likelihood: 0.17035569269236372
accuracy: 0.9296875
4--95--12288
likelihood: 0.18831394732996193
accuracy: 0.9609375
4--96--12416
likelihood: 0.22949011741845587
accuracy: 0.953125
4--97--12544
likelihood: 0.27616845829241937
accuracy: 0.921875
4--98--12672
likelihood: 0.1469535852373859
accuracy: 0.9765625
4--99--12800
likelihood: 0.30826419021214185
accuracy: 0.9375
4--100--12928
likelihood: 0.18105837965519433
accuracy: 0.9375
4--101--13056
likelihood: 0.18731779436385568
accuracy: 0.953125
4--102--13184
likelihood: 0.11130700355349205
accuracy: 0.984375
4--103--13312
likelihood: 0.24891680748961392
accuracy: 0.921875
4--104--13440
likelihood: 0.25547673533514453
accuracy: 0.90625
4--105--13568
likelihood: 0.2818721579774377
accuracy: 0.921875
4--106--13696
likelihood: 0.22940536771990566
```

```
accuracy: 0.9140625
4--107--13824
likelihood: 0.19020368644408786
accuracy: 0.9375
4--108--13952
likelihood: 0.11716078346317026
accuracy: 0.9453125
4--109--14080
likelihood: 0.21700612276952325
accuracy: 0.9375
4--110--14208
likelihood: 0.26972577169365375
accuracy: 0.921875
4--111--14336
likelihood: 0.2887339675476138
accuracy: 0.921875
4--112--14464
likelihood: 0.18881120132325035
accuracy: 0.9375
4--113--14592
likelihood: 0.22230948023186786
accuracy: 0.9375
4--114--14720
likelihood: 0.2090757623249052
accuracy: 0.921875
4--115--14848
likelihood: 0.21548706857381994
accuracy: 0.9375
4--116--14976
likelihood: 0.29120060812002957
accuracy: 0.921875
4--117--15104
likelihood: 0.2516875210804145
accuracy: 0.9375
4--118--15232
likelihood: 0.22774257295586298
accuracy: 0.9375
4--119--15360
likelihood: 0.12830747834828649
```

```
accuracy: 0.9609375
4--120--15488
likelihood: 0.18224703284315363
accuracy: 0.9453125
4--121--15616
likelihood: 0.23220465376461621
accuracy: 0.9609375
4--122--15744
likelihood: 0.2878933457801678
accuracy: 0.8984375
4--123--15872
likelihood: 0.20799687716837525
accuracy: 0.953125
4--124--16000
likelihood: 0.16005518551703848
accuracy: 0.96875
4--125--16128
likelihood: 0.12793902310279098
accuracy: 0.9609375
4--126--16256
likelihood: 0.19433623125283483
accuracy: 0.9453125
4--127--16384
likelihood: 0.13553740913317208
accuracy: 0.9453125
4--128--16512
likelihood: 0.21666334408277954
accuracy: 0.9453125
4--129--16640
likelihood: 0.19770677363120415
accuracy: 0.9453125
4--130--16768
likelihood: 0.1971772671028427
accuracy: 0.9609375
4--131--16896
likelihood: 0.24574046325169374
accuracy: 0.90625
4--132--17024
likelihood: 0.33301069539701794
```

```
accuracy: 0.890625
4--133--17152
likelihood: 0.09897151645755026
accuracy: 0.9765625
4--134--17280
likelihood: 0.3160848621145217
accuracy: 0.9140625
4--135--17408
likelihood: 0.275097292545167
accuracy: 0.9375
4--136--17536
likelihood: 0.2021883310457841
accuracy: 0.953125
4--137--17664
likelihood: 0.19067923928799974
accuracy: 0.9296875
4--138--17792
likelihood: 0.1382382358255548
accuracy: 0.96875
4--139--17920
likelihood: 0.16702620005478624
accuracy: 0.9375
4--140--18048
likelihood: 0.16477472685298683
accuracy: 0.953125
4--141--18176
likelihood: 0.12279997343387836
accuracy: 0.96875
4--142--18304
likelihood: 0.24827728173044322
accuracy: 0.9453125
4--143--18432
likelihood: 0.26831408180612426
accuracy: 0.921875
4--144--18560
likelihood: 0.18633816460813352
accuracy: 0.9453125
4--145--18688
likelihood: 0.1725950447940985
```

```
accuracy: 0.96875
4--146--18816
likelihood: 0.2093610539330486
accuracy: 0.921875
4--147--18944
likelihood: 0.24438779063885963
accuracy: 0.9296875
4--148--19072
likelihood: 0.2821565693861163
accuracy: 0.9140625
4--149--19200
likelihood: 0.30178085789229325
accuracy: 0.9140625
4--150--19328
likelihood: 0.15150407132577826
accuracy: 0.9453125
4--151--19456
likelihood: 0.16357548720795784
accuracy: 0.96875
4--152--19584
likelihood: 0.1837738349619601
accuracy: 0.9453125
4--153--19712
likelihood: 0.21960075676874743
accuracy: 0.9375
4--154--19840
likelihood: 0.1391354621446731
accuracy: 0.9609375
4--155--19968
likelihood: 0.268029712248375
accuracy: 0.9140625
4--156--20096
likelihood: 0.21165631545517294
accuracy: 0.9296875
4--157--20224
likelihood: 0.2920929382788837
accuracy: 0.9296875
4--158--20352
likelihood: 0.158938231339212
```

```
accuracy: 0.9609375
4--159--20480
likelihood: 0.3540270337134941
accuracy: 0.90625
4--160--20608
likelihood: 0.3765765420653161
accuracy: 0.8984375
4--161--20736
likelihood: 0.2362343120438447
accuracy: 0.9296875
4--162--20864
likelihood: 0.2632897469588855
accuracy: 0.9296875
4--163--20992
likelihood: 0.19239745556935745
accuracy: 0.9453125
4--164--21120
likelihood: 0.15658280690668555
accuracy: 0.953125
4--165--21248
likelihood: 0.11018677510618216
accuracy: 0.96875
4--166--21376
likelihood: 0.25330075341720126
accuracy: 0.9375
4--167--21504
likelihood: 0.2948214211722401
accuracy: 0.9296875
4--168--21632
likelihood: 0.24964040144112387
accuracy: 0.9375
4--169--21760
likelihood: 0.15654102018619617
accuracy: 0.9375
4--170--21888
likelihood: 0.21115331076879118
accuracy: 0.9609375
4--171--22016
likelihood: 0.186894443692652
```

```
accuracy: 0.9453125
4--172--22144
likelihood: 0.224789668422549
accuracy: 0.9609375
4--173--22272
likelihood: 0.1973216498892326
accuracy: 0.9453125
4--174--22400
likelihood: 0.2036453410062981
accuracy: 0.9375
4--175--22528
likelihood: 0.20504239674110777
accuracy: 0.9375
4--176--22656
likelihood: 0.13756878484257082
accuracy: 0.953125
4--177--22784
likelihood: 0.14588562538751845
accuracy: 0.953125
4--178--22912
likelihood: 0.16834311299517724
accuracy: 0.9296875
4--179--23040
likelihood: 0.20978379717515488
accuracy: 0.9375
4--180--23168
likelihood: 0.1581502732882275
accuracy: 0.9609375
4--181--23296
likelihood: 0.30142223475813135
accuracy: 0.9375
4--182--23424
likelihood: 0.20020181201133075
accuracy: 0.9453125
4--183--23552
likelihood: 0.16621796078384202
accuracy: 0.9453125
4--184--23680
likelihood: 0.20047903038117573
```

```
accuracy: 0.9296875
4--185--23808
likelihood: 0.134886633515319
accuracy: 0.9609375
4--186--23936
likelihood: 0.2642442750992303
accuracy: 0.9140625
4--187--24064
likelihood: 0.3634470304287074
accuracy: 0.90625
4--188--24192
likelihood: 0.22454976873230104
accuracy: 0.921875
4--189--24320
likelihood: 0.29377585742803103
accuracy: 0.9375
4--190--24448
likelihood: 0.21915623527578884
accuracy: 0.9453125
4--191--24576
likelihood: 0.28880519091003937
accuracy: 0.9375
4--192--24704
likelihood: 0.1700751531583608
accuracy: 0.9296875
4--193--24832
likelihood: 0.2198196906714845
accuracy: 0.9140625
4--194--24960
likelihood: 0.25078148984770054
accuracy: 0.9296875
4--195--25088
likelihood: 0.14153178801175983
accuracy: 0.9609375
4--196--25216
likelihood: 0.3023061913760594
accuracy: 0.921875
4--197--25344
likelihood: 0.3033680329707075
```

```
accuracy: 0.8984375
4--198--25472
likelihood: 0.36265597128156274
accuracy: 0.90625
4--199--25600
likelihood: 0.19358156279830613
accuracy: 0.9296875
4--200--25728
likelihood: 0.18762840981456683
accuracy: 0.9296875
4--201--25856
likelihood: 0.254522099241759
accuracy: 0.90625
4--202--25984
likelihood: 0.1448301971377703
accuracy: 0.9609375
4--203--26112
likelihood: 0.21639550644067523
accuracy: 0.953125
4--204--26240
likelihood: 0.1602651412142808
accuracy: 0.953125
4--205--26368
likelihood: 0.31387583246728834
accuracy: 0.90625
4--206--26496
likelihood: 0.2074435565648816
accuracy: 0.9453125
4--207--26624
likelihood: 0.13474342046189247
accuracy: 0.96875
4--208--26752
likelihood: 0.2687900375139215
accuracy: 0.9140625
4--209--26880
likelihood: 0.2025481835604518
accuracy: 0.9296875
4--210--27008
likelihood: 0.2139836265641175
```

```
accuracy: 0.9453125
4--211--27136
likelihood: 0.1491441271215847
accuracy: 0.9609375
4--212--27264
likelihood: 0.25480025944582774
accuracy: 0.9375
4--213--27392
likelihood: 0.23771575672541975
accuracy: 0.921875
4--214--27520
likelihood: 0.1163620971705219
accuracy: 0.9609375
4--215--27648
likelihood: 0.2646070282105851
accuracy: 0.921875
4--216--27776
likelihood: 0.21227390437637356
accuracy: 0.9453125
4--217--27904
likelihood: 0.19590521896421653
accuracy: 0.9609375
4--218--28032
likelihood: 0.11751416633383338
accuracy: 0.9609375
4--219--28160
likelihood: 0.18846191980064797
accuracy: 0.9296875
4--220--28288
likelihood: 0.33051941285664554
accuracy: 0.921875
4--221--28416
likelihood: 0.22392232273574922
accuracy: 0.9453125
4--222--28544
likelihood: 0.21932307874218604
accuracy: 0.9296875
4--223--28672
likelihood: 0.1165998121947568
```

```
accuracy: 0.96875
4--224--28800
likelihood: 0.3165181200996261
accuracy: 0.8984375
4--225--28928
likelihood: 0.18950658569351692
accuracy: 0.9453125
4--226--29056
likelihood: 0.1946931597125708
accuracy: 0.9453125
4--227--29184
likelihood: 0.32022406517372276
accuracy: 0.921875
4--228--29312
likelihood: 0.20897876309324015
accuracy: 0.9609375
4--229--29399
likelihood: 0.1699488396563522
accuracy: 0.9655172413793104
```

```
In [34]: hat_label = forward(test_data.T, parameter, cache)
         hat_label.keys()

Out[34]: dict_keys(['C1', 'A1', 'C2', 'A2', 'C3', 'A3', 'dC3', 'dW3', 'db3', 'dC2', 'dW2', 'db2',

In [35]: hat_label = hat_label['A3']
         hat_label.shape

Out[35]: (10, 12601)

In [36]: likelihood(test_label.T, hat_label)

Out[36]: 0.2211812376862138

In [ ]:
```