

09+NeuralNetwork

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1 神经网络

1.1 一、概念

神经网络是时下最热的人工智能话题，而神经网络的历史也由来已久，近年来的算力大爆发使人工智能和神经网络发现了彼此。

神经网络通过神经元进行组织，数据从上一层神经元流向下一层神经元直到输出神经元，损失函数衡量预测和输出之间的差距，再通过反向传播更新各层神经元的参数。

神经网络由如下元素构成：

1. 输入层：数据从输入层进入模型
2. 隐藏层：数据在隐藏层中进行交互和组合
3. 输出层：输出层输出预测结果
4. 激活函数：各个神经元的对上一层的输入进行非线性处理的函数
5. 损失函数：衡量预测结果和实际结果的差距
6. 优化器：即以何种方式更新参数

1.2 二、符号说明

- X : 输入数据, $X \in R^{p \times n}$, p 代表变量数, n 代表样本数
- \hat{Y} : 输出数据, $\hat{Y} \in R^{k \times n}$, n 代表样本数, 多分类时 k 代表分类数, 二分类和回归时 k 为 1
- Y : 实际结果, $Y \in R^{k \times n}$, n 代表样本数, 多分类时 k 代表分类数, 二分类和回归时 k 为 1
- p_i : 第 i 层的神经元数, $p_0 = p$
- W_i : 从第 $i-1$ 层向第 i 层传播的矩阵, $W_i \in R^{p_i \times p_{i-1}}$, 输入层为第 0 层时, $W_1 \in R^{p_1 \times p}$
- $\alpha(z)$: 激活函数, 每一层每一个神经元的激活函数都可以不同, 此处统一用
- $g(z)$: 输出层的激活函数, 通常和隐藏层的激活函数不同

- b_i : 第 i 层的偏置项, $b_i \in R^{p_{i+1}}$
- Z_i : 上一层激活函数的线性组合, $Z_i \in R^{p_i \times n}$
- A_i : 线性组合的激活函数值, $A_i \in R^{p_i \times n}$
- $*$: 逐元素相乘

1.3 三、Feed Forward 前向传播

1.3.1 1. 从输入层到第一个隐藏层

首先是对输入数据的线性组合, 由于偏执项是一个向量, 对所有 n 个数据来说都相等。虽然此处维度按照线性代数并不能严格成立 (因为 $W_1 X \in R^{p_1 \times n}$, $b_1 \in R^{p_1 \times 1}$), 但是由于 `numpy` 中的广播 (broadcast) 机制存在, 在编程中以下公式是成立的。如果非要按照数学定义上成立可以对 b_1 乘上一个 $1 \times n$ 的值全为 1 的向量。

$$\begin{aligned} Z_1 &= W_1 X + b_1 \in R^{p_1 \times n} \\ \Leftrightarrow Z_1 &= W_1 X + b_1 1^{1 \times n} \end{aligned}$$

然后是对第一层的各个神经元进行“激活”, 对线性组合进行逐元素的函数计算

$$A_1 = \alpha(Z_1) \in R^{p_1 \times n}$$

1.3.2 2. 从第 $i-1$ 层到第 i 层

与输入不同, 此时是将上一层的激活函数值进行线性组合:

$$Z_i = W_i A_{i-1} + b_i \in R^{p_i \times n}$$

$$A_i = \alpha(Z_i) \in R^{p_i \times n}$$

1.3.3 3. 从最后一个隐藏层到输出层

假设输入层是第 0 层, 第 1—— $m-1$ 层是隐藏层, 第 m 层是输出层。如果是二分类、回归等情况, 则输出层只有一个神经元, 若是多分类等情况则有多个神经元, 将在后面介绍, 暂时假定只有一个输出:

$$Z_m = W_m A_{m-1} + b_m \in R^{k \times n}$$

$$\hat{Y} = A_m = g(Z_m) \in R^{k \times n}$$

1.4 四、激活函数

激活函数有多种多样，本质上都是为了进行非线性组合，还有易于进行求导运算以便更新参数。此处简单介绍几种激活函数

1.4.1 1.sigmoid 函数

Sigmoid 函数已经在 logistic 回归中介绍过：

$$\text{sigmoid}(z) = \frac{1}{1 + 1^{-z}}$$

它是一种较早期的激活函数，现在多用于最后输出层的激活而不用在隐藏层中，这是因为当 x 远离原点时它的梯度会非常接近 0，会造成非常著名的“梯度消失”的现象。

考虑 sigmoid 函数的导数：

$$\frac{d}{dz} \text{sigmoid}(z) = \frac{e^{-z}}{(1 + 1^{-z})^2}$$

当 $z=0$ 时其梯度最大为 0.25，当神经网络的层数变深时便是指数倍地降低，这便是“梯度消失”最直观和简洁的解释。

1.4.2 2.Relu (Rectified Linear Unit, 线性整流函数)

Relu 也曾是红极一时的激活函数，因其简洁的函数形式和导数形式 (x 大于零导数为 1，其他情况为 0) 使计算成本大大降低，但同时这也带来了神经元没有被激活的情况。这是因为当输入小于 0 时，输出和梯度都为 0，导致神经元“死亡”。

$$\text{Relu}(z) = \max(0, z)$$

$$\frac{d}{dz} \text{Relu}(z) = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases}$$

1.4.3 3.leaky Relu

leaky Relu 是我最喜欢的激活函数，因为它兼具了 Relu 的优点，且当输入小于零时不会出现神经元死亡的情况， k 通常的设置为 0.1。

$$\text{leakyRelu}(z, k) = \max(kz, z)$$

$$\frac{d}{dz} \text{leakyRelu}(z) = \begin{cases} 1 & z > 0 \\ k & z \leq 0 \end{cases}$$

1.4.4 4.softmax

softmax 是专门用于多分类的输出层的激活函数，有两种等价形式，一种是针对 K 类有 K 个输出的线性相关的形式（即下式），另一个是针对 K 类有 K-1 个输出的线性无关的形式。

$$\text{softmax}(z) = \begin{bmatrix} \frac{e^{z_1}}{\sum_{i=1}^k e^{z_i}} \\ \frac{e^{z_2}}{\sum_{i=1}^k e^{z_i}} \\ \dots \\ \frac{e^{z_j}}{\sum_{i=1}^k e^{z_i}} \\ \dots \\ \frac{e^{z_k}}{\sum_{i=1}^k e^{z_i}} \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \dots \\ \hat{y}_i \\ \dots \\ \hat{y}_k \end{bmatrix}$$

它的针对单一分量的偏导数形式和 sigmoid 函数极为相似：

$$\begin{aligned} \frac{\partial}{\partial z_i} \text{softmax}(z) &= \frac{d}{dz_i} \frac{e^{z_i}}{a + e^{z_i}} = \frac{ae^{z_i}}{(a + e^{z_i})^2} \\ &= \frac{ae^{z_i} + a^2 - a^2}{(a + e^{z_i})^2} \\ &= \frac{a(e^{z_i} + a) - a^2}{(a + e^{z_i})^2} \\ &= \frac{a}{a + e^{z_i}} - \left(\frac{a}{a + e^{z_i}} \right)^2 \\ &= \frac{a}{a + e^{z_i}} \left(1 - \frac{a}{a + e^{z_i}} \right) \\ &= \left(1 - \frac{e^{z_i}}{a + e^{z_i}} \right) \frac{e^{z_i}}{a + e^{z_i}} \end{aligned}$$

则它的梯度为：

$$\nabla \text{softmax}(z) = \begin{bmatrix} \hat{y}_1(1 - \hat{y}_1) \\ \hat{y}_2(1 - \hat{y}_2) \\ \dots \\ \hat{y}_i(1 - \hat{y}_i) \\ \dots \\ \hat{y}_k(1 - \hat{y}_k) \end{bmatrix}$$

1.5 五、损失函数

二分类和回归的损失函数不再赘述，和 logistic 回归和多元线性回归类似，这里介绍多分类的损失函数。

多分类的损失函数和二分类相同，也是通过似然函数进行定义：假设随机变量 Y 一共有 K 个取值，第 i 个样本对第 j 个取值的概率估计值为：

$$P(y_i = j) = \hat{y}_{ij} \quad j = 1, 2, \dots, k$$

则对 n 个样本，其似然函数为：

$$likelihood(Y, \hat{Y}) = \prod_{i=1}^n \prod_{j=1}^k \hat{y}_{ij}^{I(y_i=j)}$$

对其求自然对数，除以样本数进行标准化取负数：

$$loss(Y, \hat{Y}) = -\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^k I(y_i = j) \ln(\hat{y}_{ij})$$

这就是最终的损失函数。

1.6 六、Backward propagation 反向传播

反向传播是神经网络更新参数最经典也是最有效、最具有广泛性的算法。

反向传播的基础仍然是梯度下降法。

1.6.1 1. 输出层到第一个隐藏层的反向传播

$$\frac{\partial}{\partial y_{ij}} loss = \frac{1}{n} \frac{I(y_i = j)}{\hat{y}_{ij}}$$

由于输出是 $\hat{Y} \in R^{k \times n}$ 向量，损失对输出层的梯度和输出保持一致的维度：

$$\frac{\nabla loss}{\nabla \hat{Y}} = \frac{1}{n} \begin{bmatrix} \frac{I(y_1=1)}{\hat{y}_{11}} & \cdots & \frac{I(y_1=k)}{\hat{y}_{1k}} \\ & \ddots & \vdots \\ \frac{I(y_n=1)}{\hat{y}_{n1}} & & \frac{I(y_n=k)}{\hat{y}_{nk}} \end{bmatrix}^T \in R^{k \times n}$$

对输出层的激活函数有 $\hat{Y} = g(Z_m) = softmax(Z_m) \in R^{k \times n}$, $Z_m \in R^{k \times n}$

$$\frac{\nabla \hat{Y}}{\nabla Z_m} = \begin{bmatrix} \hat{y}_{11}(1 - \hat{y}_{11}) & \cdots & \hat{y}_{1k}(1 - \hat{y}_{1k}) \\ & \ddots & \vdots \\ \hat{y}_{n1}(1 - \hat{y}_{n1}) & & \hat{y}_{nk}(1 - \hat{y}_{nk}) \end{bmatrix}^T \in R^{k \times n}$$

此时还未涉及到参数的更新，而 $Z_m = W_m A_{m-1} + b_m$ 中 $W_m \in R^{p_m=k \times p_{m-1}}$ 、 $b_m \in R^{k \times 1}$ 、 $A_{m-1} \in R^{p_{m-1} \times n}$ 均为参数，其中前两个好理解，而激活函数值也需要更新是因为它是先前输入的函数，需要通过激活函数更新使梯度传导到更靠前的隐藏层。

$$\frac{\nabla Z_m}{\nabla W_m} = A_{m-1} \in R^{p_{m-1} \times n}$$

$$\frac{\nabla Z_m}{\nabla A_{m-1}} = W_m^T \in R^{p_{m-1} \times k}$$

$$\frac{\nabla Z_m}{\nabla b_m} = 1^{1 \times n} \in R^{1 \times n}$$

将其和之前的梯度结合起来：

$$\begin{aligned} \frac{\nabla loss}{\nabla W_m} &= \left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m} \right) \left(\frac{\nabla Z_m}{\nabla W_m} \right)^T \in R^{p_m = k \times p_{m-1}} \\ \frac{\nabla loss}{\nabla A_{m-1}} &= \frac{\nabla Z_m}{\nabla A_{m-1}} \left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m} \right) \in R^{p_{m-1} \times n} \\ \frac{\nabla loss}{\nabla b_m} &= \left(\frac{\nabla loss}{\nabla \hat{Y}} * \frac{\nabla \hat{Y}}{\nabla Z_m} \right) 1^{n \times 1} \in R^{k \times 1} \end{aligned}$$

1.6.2 2. 第 i 层到第 i-1 层的反向传播

从第 i 层到第 i-1 层的反向传播和从输出层到最后一个隐藏层的推导相似：

假设 $\frac{\nabla loss}{\nabla A_i} \in R^{p_i \times n}$ 已知，

$$\begin{aligned} A_i &= \begin{bmatrix} \alpha(z_{11}) & \dots & \alpha(z_{1p_i}) \\ & \ddots & \vdots \\ \alpha(z_{n1}) & & \alpha(z_{np_i}) \end{bmatrix} \in R^{p_i \times n} \\ \frac{\nabla A_i}{\nabla Z_i} &= \begin{bmatrix} \alpha'(z_{11}) & \dots & \alpha'(z_{1p_i}) \\ & \ddots & \vdots \\ \alpha'(z_{n1}) & & \alpha'(z_{np_i}) \end{bmatrix} \in R^{p_i \times n} \end{aligned}$$

其余部分和之前的相同

$$\frac{\nabla Z_i}{\nabla W_i} = A_{i-1} \in R^{p_{i-1} \times n}$$

$$\frac{\nabla Z_i}{\nabla A_{i-1}} = W_i^T \in R^{p_{i-1} \times i}$$

$$\frac{\nabla Z_i}{\nabla b_i} = 1^{1 \times n} \in R^{1 \times n}$$

将其和之前的梯度结合起来：

$$\begin{aligned}\frac{\nabla loss}{\nabla W_i} &= \left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_i} \right) \left(\frac{\nabla Z_i}{\nabla W_i} \right)^T \in R^{p_i \times p_{i-1}} \\ \frac{\nabla loss}{\nabla A_{i-1}} &= \frac{\nabla Z_i}{\nabla A_{i-1}} \left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_m} \right) \in R^{p_{i-1} \times n} \\ \frac{\nabla loss}{\nabla b_i} &= \left(\frac{\nabla loss}{\nabla A_i} * \frac{\nabla A_i}{\nabla Z_i} \right) 1^{n \times 1} \in R^{p_i \times 1}\end{aligned}$$

1.6.3 3. 从第一层到输入层

从第 1 层到输入层的反向传播和从第 i 层到第 i-1 层的推导相似，区别在于输入是固定的数据，而不再是激活函数值，也就不再需要对输入的数据 X 进行更新：

假设 $\frac{\nabla loss}{\nabla A_1} \in R^{p_1 \times n}$ 已知，

$$\begin{aligned}A_1 &= \begin{bmatrix} \alpha(z_{11}) & \dots & \alpha(z_{1p_1}) \\ & \ddots & \vdots \\ \alpha(z_{n1}) & & \alpha(z_{np_1}) \end{bmatrix} \in R^{p_1 \times n} \\ \frac{\nabla A_i}{\nabla Z_i} &= \begin{bmatrix} \alpha'(z_{11}) & \dots & \alpha'(z_{1p_1}) \\ & \ddots & \vdots \\ \alpha'(z_{n1}) & & \alpha'(z_{np_1}) \end{bmatrix} \in R^{p_i \times n} \\ \frac{\nabla Z_1}{\nabla W_1} &= X \in R^{p \times n} \\ \frac{\nabla Z_1}{\nabla b_1} &= 1^{1 \times n} \in R^{1 \times n}\end{aligned}$$

$$\begin{aligned}\frac{\nabla loss}{\nabla W_1} &= \left(\frac{\nabla loss}{\nabla A_1} * \frac{\nabla A_1}{\nabla Z_i} \right) \left(\frac{\nabla Z_i}{\nabla W_i} \right)^T \in R^{p_1 \times p} \\ \frac{\nabla loss}{\nabla b_1} &= \left(\frac{\nabla loss}{\nabla A_1} * \frac{\nabla A_1}{\nabla Z_i} \right) 1^{n \times 1} \in R^{p_1 \times 1}\end{aligned}$$

1.7 七、优化器

优化器是指优化得到参数的方法，优化器基本都是基于梯度下降方法。如果你在线性回归中不用正规方程求解参数，而是用梯度下降，你会发现随着梯度不断下降，**梯度不断减小**。而这还不是最麻烦的问题，由于线性回归是凸优化，用梯度下降总会收敛到最小值，而神经网络多是非凸问题，梯度下降很可能会困在局部极值**无法收敛**。而且通常神经网络需要很多的数据进行训练，如果每次都像传统的梯度下降那样把所有数据都传入模型，则**计算成本很大**。

这里先介绍 **SGD**（Stochastic Gradient Descent，随机梯度下降）优化器。

SGD 不再把所有的数据都用来进行梯度下降，而是只用小批量（**mini batch**）数据进行梯度下降，常见的选择是从 2 的 4 次方（16）到 2 的 10 次方之间，选用 2 的整数次方是根据计算机比特的特点决定的，而之前推导中梯度进行标准化时除以样本数，此时需要除以一批量的样本数。

控制梯度下降停止的条件也有所改变，由于神经网络强大的非线性组合能力，训练到收敛会造成过拟合，于是神经网络中用到最多的是早停法，也即小批量进行训练时将全部样本循环数遍（**epoch**）后就立即停下，避免过拟合。

1.8 八、应用

这次采用的是 **minist** 手写数字数据集，从 **kaggle** 的入门赛下载下来的训练数据集，有兴趣的可以自己训练好的型跑一下 **kaggle** 上的测试数据集提交一下看看分数。（排名就不必看了...）

```
In [21]: import pandas as pd
import numpy as np
```

```
train_data = pd.read_csv('data_set/minist.csv')
train_data.head()
```

```
Out[21]:
```

	label	pixel0	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	\
0	1	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	
2	1	0	0	0	0	0	0	0	0	
3	4	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	

	pixel8	...	pixel774	pixel775	pixel776	pixel777	pixel778	pixel779	\
0	0	...	0	0	0	0	0	0	
1	0	...	0	0	0	0	0	0	
2	0	...	0	0	0	0	0	0	
3	0	...	0	0	0	0	0	0	
4	0	...	0	0	0	0	0	0	

	pixel780	pixel781	pixel782	pixel783
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

```
[5 rows x 785 columns]
```



```

In [22]: n = train_data.shape[0]
         np.random.seed(2099)
         index = np.random.permutation(n)
         train_index = index[0: int(0.7*n)]
         test_index = index[int(0.7*n): n]

         test_data = train_data.iloc[test_index]
         test_label = test_data['label']
         del test_data['label']
         test_data = np.array(test_data)
         test_label = np.array(test_label).reshape([n-int(0.7*n), 1])

         train_data = train_data.iloc[train_index]
         train_label = train_data['label']
         del train_data['label']
         train_data = np.array(train_data)
         train_label = np.array(train_label).reshape([int(0.7*n), 1])

```

```

In [23]: def to_category(label, num_classes):
         n = label.shape[0]
         tmp = np.zeros([n, num_classes])
         j = 0
         for i in label:
             tmp[j, i]=1
             j += 1
         return tmp

```

```

In [24]: def soft_max(z):
         """
         :param z: input, an p*n matrix
         :return: p*n matrix
         """
         e = np.exp(z)
         total = np.sum(e, axis=0, keepdims=True)
         weight = e / total
         return weight

```

```

In [25]: def accuracy(y, y_hat):
         y = np.argmax(y, axis=0)
         y_hat = np.argmax(y_hat, axis=0)
         return sum(y == y_hat)/len(y)

```

```

In [26]: def likelihood(y, y_hat):
        """
        :param y: the ture value
        :param y_hat: the predicted value
        :return: minimizing loss is the same as maximizing likelihood function,
                 so we spare np.log
        """
        n = y.shape[1]
        return -np.sum(y * np.log(y_hat)) / n

In [27]: def leaky_relu(x, k=0.3):
        return (x > 0)*x + k*(x < 0)*x

        def d_leaky_relu(x, k=0.3):
            return (x > 0) + k*(x < 0)

In [28]: def init_w(b, a):
        w = np.random.randn(a * b)
        w = np.reshape(w, [b, a])
        return w

        def init_b(b):
            b = np.zeros([b, 1])
            return b

In [29]: def forward(x, parameter, cache):
        """
        :param x: input data p*n matrix
        :param parameter: a dict storing parameters
        :param cache: a dict storing computation result of each layer
        :return: the predicted value
        """
        cache['C1'] = np.dot(parameter['W1'], x) + parameter['b1']
        cache['A1'] = leaky_relu(cache['C1'])
        cache['C2'] = np.dot(parameter['W2'], cache['A1']) + parameter['b2']
        cache['A2'] = leaky_relu(cache['C2'])
        cache['C3'] = np.dot(parameter['W3'], cache['A2']) + parameter['b3']
        cache['A3'] = soft_max(cache['C3'])
        return cache

```

```

In [30]: def back_propagation(x, y, parameter, cache, step):
    """
    X 784*n / Y 10*n
    dW1 W1 800*784, db1 b1 800*1, A1 C1, 800*n
    dW2 W2 400*800, db2 b2 400*1, A2 C2, 400*n
    dW3 W3 10*400, db3 b3 10*1, A3 C3, 10*n
    :param y: true value
    :param parameter: dictionary storing all parameters
    :param cache: dictionary storing all the computation in process
    :param step: learning rate
    :return: updated parameters
    """
    number = y.shape[1]
    cache['dC3'] = cache['A3'] - y # 10*n
    cache['dW3'] = np.dot(cache['dC3'], cache['A2'].T)/number # 10*400
    cache['db3'] = np.sum(cache['dC3'], axis=1, keepdims=True)/number # 10*1
    parameter['W3'] = parameter['W3'] - step*cache['dW3'] # 10*400
    parameter['b3'] = parameter['b3'] - step*cache['db3'] # 10*1

    cache['dC2'] = np.dot(parameter['W3'].T,
                           cache['dC3'])*d_leaky_relu(cache['C2']) # 400*n
    cache['dW2'] = np.dot(cache['dC2'], cache['A1'].T)/number # 400*800
    cache['db2'] = np.sum(cache['dC2'], axis=1, keepdims=True)/number # 400*1
    parameter['W2'] = parameter['W2'] - step*cache['dW2'] # 400*800
    parameter['b2'] = parameter['b2'] - step*cache['db2'] # 400*1

    cache['dC1'] = np.dot(parameter['W2'].T,
                           cache['dC2'])*d_leaky_relu(cache['C1']) # 800*n
    cache['dW1'] = np.dot(cache['dC1'], x.T)/number # 800*784
    cache['db1'] = np.sum(cache['dC1'], axis=1, keepdims=True) # 800*1
    parameter['W1'] = parameter['W1'] - step*cache['dW1'] # 800*784
    parameter['b1'] = parameter['b1'] - step*cache['db1'] # 800*1
    return cache, parameter

In [31]: def train(x, y, learning_rate=0.001, batch_size=128, epoch=5):
    """
    :param x: training data
    :param y: training label
    :param learning_rate: the length of a step
    :param batch_size: numbers of samples we train in a round

```

```

:param epoch: rounds we train through training data
:return: a trained set of parameters
"""
parameter = dict()
nx = x.shape[1]
parameter['W1'] = init_w(800, 784)/100
parameter['b1'] = init_b(800)
parameter['W2'] = init_w(400, 800)/100
parameter['b2'] = init_b(400)
parameter['W3'] = init_w(10, 400)/100
parameter['b3'] = init_b(10)

index = np.array([], dtype='int')
for i in range(0, nx, batch_size):
    index = np.append(index, i)
index = np.append(index, nx)

cache = dict()
for i in range(0, epoch):
    for j in range(0, int(nx/batch_size)+1):
        one_batch_x = x[:, index[j]:index[j+1]]
        one_batch_y = y[:, index[j]:index[j+1]]
        cache = forward(one_batch_x, parameter, cache)
        prob = likelihood(one_batch_y, cache['A3'])
        acc = accuracy(one_batch_y, cache['A3'])
        print(str(i)+'--'+str(j)+'--'+str(index[j+1]))
        print('loss: '+str(prob))
        print('accuracy: '+str(acc))
        [cache, parameter] = back_propagation(one_batch_x, one_batch_y,
                                              parameter, cache, step=learning_rate)

    return cache, parameter

```

```

In [32]: train_label = to_category(train_label, num_classes=10)
        test_label = to_category(test_label, num_classes=10)

        print(train_label.shape)
        print(test_label.shape)

```

```
(29399, 10)
```

```
(12601, 10)
```

```
In [33]: cache, parameter = train(x=train_data.T, y=train_label.T, epoch=5)
```

```
0--0--128
likelihood: 2.5144562796833654
accuracy: 0.0703125
0--1--256
likelihood: 2.2170502380328974
accuracy: 0.1875
0--2--384
likelihood: 2.1658027472486903
accuracy: 0.234375
0--3--512
likelihood: 2.0168412770294104
accuracy: 0.296875
0--4--640
likelihood: 1.8946818751163836
accuracy: 0.40625
0--5--768
likelihood: 1.8244820246098055
accuracy: 0.421875
0--6--896
likelihood: 1.6413233158646316
accuracy: 0.515625
0--7--1024
likelihood: 1.6209324889136416
accuracy: 0.5546875
0--8--1152
likelihood: 1.5990883453285492
accuracy: 0.5390625
0--9--1280
likelihood: 1.3856347927896082
accuracy: 0.6484375
0--10--1408
likelihood: 1.303833712288761
accuracy: 0.703125
0--11--1536
likelihood: 1.3310439935553542
accuracy: 0.6484375
0--12--1664
likelihood: 1.1805366474892718
```

accuracy: 0.7265625
0--13--1792
likelihood: 1.260705592177902
accuracy: 0.6640625
0--14--1920
likelihood: 1.1617022979770701
accuracy: 0.7109375
0--15--2048
likelihood: 1.1734547644587505
accuracy: 0.6875
0--16--2176
likelihood: 1.0308806471718963
accuracy: 0.734375
0--17--2304
likelihood: 1.0419973584182476
accuracy: 0.71875
0--18--2432
likelihood: 0.9822141820894781
accuracy: 0.78125
0--19--2560
likelihood: 1.048859187543728
accuracy: 0.7421875
0--20--2688
likelihood: 1.0342548244495977
accuracy: 0.6875
0--21--2816
likelihood: 1.018508665939862
accuracy: 0.765625
0--22--2944
likelihood: 1.051002820444534
accuracy: 0.7109375
0--23--3072
likelihood: 0.9282233902726331
accuracy: 0.7109375
0--24--3200
likelihood: 0.8744240635188474
accuracy: 0.7890625
0--25--3328
likelihood: 0.8244569494972285

accuracy: 0.7890625
0--26--3456
likelihood: 0.7977779804797372
accuracy: 0.78125
0--27--3584
likelihood: 0.7676662447960514
accuracy: 0.8125
0--28--3712
likelihood: 0.7796647625908663
accuracy: 0.8046875
0--29--3840
likelihood: 0.75440817170944
accuracy: 0.7890625
0--30--3968
likelihood: 0.8070503639703792
accuracy: 0.7421875
0--31--4096
likelihood: 0.7655405946902787
accuracy: 0.796875
0--32--4224
likelihood: 0.8616568699672725
accuracy: 0.75
0--33--4352
likelihood: 0.7552023431289551
accuracy: 0.8046875
0--34--4480
likelihood: 0.7433222356628701
accuracy: 0.84375
0--35--4608
likelihood: 0.7532108013910164
accuracy: 0.796875
0--36--4736
likelihood: 0.814190428647501
accuracy: 0.7890625
0--37--4864
likelihood: 0.6220072638473819
accuracy: 0.8125
0--38--4992
likelihood: 0.6470966796465027

accuracy: 0.84375
0--39--5120
likelihood: 0.7499685064324348
accuracy: 0.78125
0--40--5248
likelihood: 0.7315055664845772
accuracy: 0.8125
0--41--5376
likelihood: 0.5727038702302014
accuracy: 0.875
0--42--5504
likelihood: 0.6847071274302589
accuracy: 0.8203125
0--43--5632
likelihood: 0.7021457761788055
accuracy: 0.78125
0--44--5760
likelihood: 0.615135992116039
accuracy: 0.8359375
0--45--5888
likelihood: 0.7613747692387356
accuracy: 0.8046875
0--46--6016
likelihood: 0.6592947467208227
accuracy: 0.8359375
0--47--6144
likelihood: 0.5879875441592405
accuracy: 0.8515625
0--48--6272
likelihood: 0.6835816107099396
accuracy: 0.84375
0--49--6400
likelihood: 0.6529306702483856
accuracy: 0.828125
0--50--6528
likelihood: 0.6633618750802753
accuracy: 0.8125
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likelihood: 0.5058631434980134

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likelihood: 0.6311525600106633
accuracy: 0.859375
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likelihood: 0.5339073062733843
accuracy: 0.84375
0--54--7040
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accuracy: 0.875
0--55--7168
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accuracy: 0.8203125
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accuracy: 0.859375
0--57--7424
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accuracy: 0.8046875
0--58--7552
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accuracy: 0.8515625
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likelihood: 0.6134196717810418
accuracy: 0.7890625
0--60--7808
likelihood: 0.48942496923702294
accuracy: 0.875
0--61--7936
likelihood: 0.6571832732312971
accuracy: 0.8046875
0--62--8064
likelihood: 0.5421928692676916
accuracy: 0.875
0--63--8192
likelihood: 0.5087934491958308
accuracy: 0.8828125
0--64--8320
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accuracy: 0.8671875
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likelihood: 0.57634229921275
accuracy: 0.796875
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likelihood: 0.5432561850787223
accuracy: 0.8515625
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likelihood: 0.5315023767273102
accuracy: 0.8046875
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accuracy: 0.8515625
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accuracy: 0.8203125
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accuracy: 0.8671875
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likelihood: 0.22800479409998742
accuracy: 0.921875
4--76--9856
likelihood: 0.21340339241198036
accuracy: 0.9296875
4--77--9984
likelihood: 0.1946250906128487
accuracy: 0.953125
4--78--10112
likelihood: 0.21743654820372293
accuracy: 0.9375
4--79--10240
likelihood: 0.30261909374368307
accuracy: 0.9296875
4--80--10368
likelihood: 0.27454156512842964

accuracy: 0.9375
4--81--10496
likelihood: 0.1464114812107803
accuracy: 0.9296875
4--82--10624
likelihood: 0.25514153396439165
accuracy: 0.921875
4--83--10752
likelihood: 0.1709068042614178
accuracy: 0.9453125
4--84--10880
likelihood: 0.1674356471169789
accuracy: 0.96875
4--85--11008
likelihood: 0.11638199466205285
accuracy: 0.984375
4--86--11136
likelihood: 0.16856729568199957
accuracy: 0.9375
4--87--11264
likelihood: 0.27941457242146955
accuracy: 0.8828125
4--88--11392
likelihood: 0.30546422572225723
accuracy: 0.9140625
4--89--11520
likelihood: 0.12453490668140726
accuracy: 0.9765625
4--90--11648
likelihood: 0.17515790705420362
accuracy: 0.9609375
4--91--11776
likelihood: 0.17861904539316115
accuracy: 0.9375
4--92--11904
likelihood: 0.31995014502137664
accuracy: 0.8984375
4--93--12032
likelihood: 0.24364885241293147

accuracy: 0.90625
4--94--12160
likelihood: 0.17035569269236372
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4--95--12288
likelihood: 0.18831394732996193
accuracy: 0.9609375
4--96--12416
likelihood: 0.22949011741845587
accuracy: 0.953125
4--97--12544
likelihood: 0.27616845829241937
accuracy: 0.921875
4--98--12672
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4--99--12800
likelihood: 0.30826419021214185
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4--100--12928
likelihood: 0.18105837965519433
accuracy: 0.9375
4--101--13056
likelihood: 0.18731779436385568
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4--102--13184
likelihood: 0.11130700355349205
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4--103--13312
likelihood: 0.24891680748961392
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4--104--13440
likelihood: 0.25547673533514453
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4--105--13568
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4--106--13696
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4--107--13824
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4--108--13952
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4--109--14080
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4--110--14208
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4--111--14336
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accuracy: 0.921875
4--112--14464
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4--113--14592
likelihood: 0.22230948023186786
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4--114--14720
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4--115--14848
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accuracy: 0.9375
4--116--14976
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accuracy: 0.921875
4--117--15104
likelihood: 0.2516875210804145
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4--118--15232
likelihood: 0.22774257295586298
accuracy: 0.9375
4--119--15360
likelihood: 0.12830747834828649

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4--120--15488
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4--121--15616
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4--122--15744
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4--123--15872
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4--124--16000
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4--125--16128
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4--126--16256
likelihood: 0.19433623125283483
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4--127--16384
likelihood: 0.13553740913317208
accuracy: 0.9453125
4--128--16512
likelihood: 0.21666334408277954
accuracy: 0.9453125
4--129--16640
likelihood: 0.19770677363120415
accuracy: 0.9453125
4--130--16768
likelihood: 0.1971772671028427
accuracy: 0.9609375
4--131--16896
likelihood: 0.24574046325169374
accuracy: 0.90625
4--132--17024
likelihood: 0.33301069539701794

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4--133--17152
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4--134--17280
likelihood: 0.3160848621145217
accuracy: 0.9140625
4--135--17408
likelihood: 0.275097292545167
accuracy: 0.9375
4--136--17536
likelihood: 0.2021883310457841
accuracy: 0.953125
4--137--17664
likelihood: 0.19067923928799974
accuracy: 0.9296875
4--138--17792
likelihood: 0.1382382358255548
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4--139--17920
likelihood: 0.16702620005478624
accuracy: 0.9375
4--140--18048
likelihood: 0.16477472685298683
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4--141--18176
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4--142--18304
likelihood: 0.24827728173044322
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4--143--18432
likelihood: 0.26831408180612426
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4--144--18560
likelihood: 0.18633816460813352
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4--145--18688
likelihood: 0.1725950447940985

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4--146--18816
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4--149--19200
likelihood: 0.30178085789229325
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4--151--19456
likelihood: 0.16357548720795784
accuracy: 0.96875
4--152--19584
likelihood: 0.1837738349619601
accuracy: 0.9453125
4--153--19712
likelihood: 0.21960075676874743
accuracy: 0.9375
4--154--19840
likelihood: 0.1391354621446731
accuracy: 0.9609375
4--155--19968
likelihood: 0.268029712248375
accuracy: 0.9140625
4--156--20096
likelihood: 0.21165631545517294
accuracy: 0.9296875
4--157--20224
likelihood: 0.2920929382788837
accuracy: 0.9296875
4--158--20352
likelihood: 0.158938231339212

accuracy: 0.9609375
4--159--20480
likelihood: 0.3540270337134941
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4--160--20608
likelihood: 0.3765765420653161
accuracy: 0.8984375
4--161--20736
likelihood: 0.2362343120438447
accuracy: 0.9296875
4--162--20864
likelihood: 0.2632897469588855
accuracy: 0.9296875
4--163--20992
likelihood: 0.19239745556935745
accuracy: 0.9453125
4--164--21120
likelihood: 0.15658280690668555
accuracy: 0.953125
4--165--21248
likelihood: 0.11018677510618216
accuracy: 0.96875
4--166--21376
likelihood: 0.25330075341720126
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4--167--21504
likelihood: 0.2948214211722401
accuracy: 0.9296875
4--168--21632
likelihood: 0.24964040144112387
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4--169--21760
likelihood: 0.15654102018619617
accuracy: 0.9375
4--170--21888
likelihood: 0.21115331076879118
accuracy: 0.9609375
4--171--22016
likelihood: 0.186894443692652

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4--172--22144
likelihood: 0.224789668422549
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4--173--22272
likelihood: 0.1973216498892326
accuracy: 0.9453125
4--174--22400
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accuracy: 0.9375
4--175--22528
likelihood: 0.20504239674110777
accuracy: 0.9375
4--176--22656
likelihood: 0.13756878484257082
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4--177--22784
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4--178--22912
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4--179--23040
likelihood: 0.20978379717515488
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4--183--23552
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4--184--23680
likelihood: 0.20047903038117573

accuracy: 0.9296875
4--185--23808
likelihood: 0.134886633515319
accuracy: 0.9609375
4--186--23936
likelihood: 0.2642442750992303
accuracy: 0.9140625
4--187--24064
likelihood: 0.3634470304287074
accuracy: 0.90625
4--188--24192
likelihood: 0.22454976873230104
accuracy: 0.921875
4--189--24320
likelihood: 0.29377585742803103
accuracy: 0.9375
4--190--24448
likelihood: 0.21915623527578884
accuracy: 0.9453125
4--191--24576
likelihood: 0.28880519091003937
accuracy: 0.9375
4--192--24704
likelihood: 0.1700751531583608
accuracy: 0.9296875
4--193--24832
likelihood: 0.2198196906714845
accuracy: 0.9140625
4--194--24960
likelihood: 0.25078148984770054
accuracy: 0.9296875
4--195--25088
likelihood: 0.14153178801175983
accuracy: 0.9609375
4--196--25216
likelihood: 0.3023061913760594
accuracy: 0.921875
4--197--25344
likelihood: 0.3033680329707075

accuracy: 0.8984375
4--198--25472
likelihood: 0.36265597128156274
accuracy: 0.90625
4--199--25600
likelihood: 0.19358156279830613
accuracy: 0.9296875
4--200--25728
likelihood: 0.18762840981456683
accuracy: 0.9296875
4--201--25856
likelihood: 0.254522099241759
accuracy: 0.90625
4--202--25984
likelihood: 0.1448301971377703
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4--203--26112
likelihood: 0.21639550644067523
accuracy: 0.953125
4--204--26240
likelihood: 0.1602651412142808
accuracy: 0.953125
4--205--26368
likelihood: 0.31387583246728834
accuracy: 0.90625
4--206--26496
likelihood: 0.2074435565648816
accuracy: 0.9453125
4--207--26624
likelihood: 0.13474342046189247
accuracy: 0.96875
4--208--26752
likelihood: 0.2687900375139215
accuracy: 0.9140625
4--209--26880
likelihood: 0.2025481835604518
accuracy: 0.9296875
4--210--27008
likelihood: 0.2139836265641175

accuracy: 0.9453125
4--211--27136
likelihood: 0.1491441271215847
accuracy: 0.9609375
4--212--27264
likelihood: 0.25480025944582774
accuracy: 0.9375
4--213--27392
likelihood: 0.23771575672541975
accuracy: 0.921875
4--214--27520
likelihood: 0.1163620971705219
accuracy: 0.9609375
4--215--27648
likelihood: 0.2646070282105851
accuracy: 0.921875
4--216--27776
likelihood: 0.21227390437637356
accuracy: 0.9453125
4--217--27904
likelihood: 0.19590521896421653
accuracy: 0.9609375
4--218--28032
likelihood: 0.11751416633383338
accuracy: 0.9609375
4--219--28160
likelihood: 0.18846191980064797
accuracy: 0.9296875
4--220--28288
likelihood: 0.33051941285664554
accuracy: 0.921875
4--221--28416
likelihood: 0.22392232273574922
accuracy: 0.9453125
4--222--28544
likelihood: 0.21932307874218604
accuracy: 0.9296875
4--223--28672
likelihood: 0.1165998121947568

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accuracy: 0.96875
4--224--28800
likelihood: 0.3165181200996261
accuracy: 0.8984375
4--225--28928
likelihood: 0.18950658569351692
accuracy: 0.9453125
4--226--29056
likelihood: 0.1946931597125708
accuracy: 0.9453125
4--227--29184
likelihood: 0.32022406517372276
accuracy: 0.921875
4--228--29312
likelihood: 0.20897876309324015
accuracy: 0.9609375
4--229--29399
likelihood: 0.1699488396563522
accuracy: 0.9655172413793104
```

```
In [34]: hat_label = forward(test_data.T, parameter, cache)
         hat_label.keys()
```

```
Out[34]: dict_keys(['C1', 'A1', 'C2', 'A2', 'C3', 'A3', 'dC3', 'dW3', 'db3', 'dC2', 'dW2', 'db2',
```

```
In [35]: hat_label = hat_label['A3']
         hat_label.shape
```

```
Out[35]: (10, 12601)
```

```
In [36]: likelihood(test_label.T, hat_label)
```

```
Out[36]: 0.2211812376862138
```

```
In [ ]:
```