HMMT November 2024

November 09, 2024

Team Round

1. **[20]** The integers from 1 to 9 are arranged in a 3×3 grid. The rows and columns of the grid correspond to 6 three-digit numbers, reading rows from left to right, and columns from top to bottom. Compute the least possible value of the largest of the 6 numbers.

2. **[20]** Compute the sum of all positive integers x such that $(x-17)\sqrt{x-1}+(x-1)\sqrt{x+15}$ is an integer.

3. **[30]** Rectangle R with area 20 and diagonal of length 7 is translated 2 units in some direction to form a new rectangle R'. The vertices of R and R' that are not contained in the other rectangle form a convex hexagon. Compute the maximum possible area of this hexagon.

4. **[35]** Albert writes down all of the multiples of 9 between 9 and 999, inclusive. Compute the sum of the digits he wrote.

5. **[40]** Let $ABCD$ be a convex quadrilateral with area 202, $AB = 4$, and $\angle A = \angle B = 90^\circ$ such that there is exactly one point E on line CD satisfying $\angle AEB = 90^\circ$. Compute the perimeter of $ABCD$.

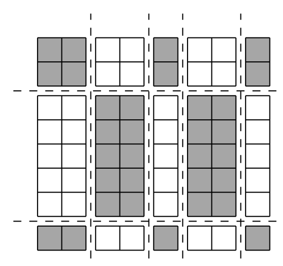
6. **[45]** There are 5 people who start with 1, 2, 3, 4, and 5 cookies, respectively. Every minute, two different people are chosen uniformly at random. If they have a and b cookies and $a \neq b$, the person with more cookies eats $|a-b|$ of their own cookies. If $a = b$, the minute still passes with nothing happening.

   Compute the expected number of minutes until all 5 people have an equal number of cookies.

7. **[50]** A *weird checkerboard* is a coloring of an 8×8 grid constructed by making some (possibly none or all) of the following 14 cuts:

   - the 7 vertical cuts along a gridline through the entire height of the board,
   - and the 7 horizontal cuts along a gridline through the entire width of the board.

   The divided rectangles are then colored black and white such that the bottom left corner of the grid is black, and no two rectangles adjacent by an edge share a color. Compute the number of weird checkerboards that have an equal amount of area colored black and white.



8. **[50]** Compute the unique real number $x < 3$ such that

$$\sqrt{(3-x)(4-x)} + \sqrt{(4-x)(6-x)} + \sqrt{(6-x)(3-x)} = x.$$

9. **[55]** Let P be a point inside isosceles trapezoid $ABCD$ with $AB \parallel CD$ such that

$$\angle PAD = \angle PDA = 90^\circ - \angle BPC.$$

   If $PA = 14$, $AB = 18$, and $CD = 28$, compute the area of $ABCD$.

10. **[55]** For each positive integer n, let $f(n)$ be either the unique integer $r \in \{0, 1, \ldots, n-1\}$ such that n divides $15r - 1$, or 0 if such r does not exist. Compute

$$f(16) + f(17) + f(18) + \cdots + f(300).$$