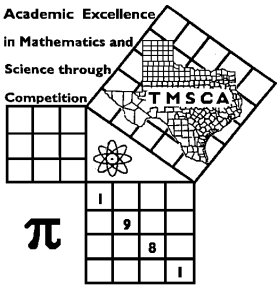


TMSCA HIGH SCHOOL MATHEMATICS TEST # 13 (UIL D) ©

MARCH 3, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. **Evaluate the following to the nearest tenth:** $\sqrt[3]{8} + 4! \div (2)^5 - 7 \times (1)^{-6}$

   (A) -6.1875 **(B)** -4.25 **(C)** $-3.58333...$ **(D)** -2.6 **(E)** $-1.9333...$

2. $-\frac{3}{8}$ P Q R $\frac{3}{4}$ S

   <----------|----------|----------|----------|----------|----------|---------->

   The distances between the hash marks (|) are equal. Find P + Q + R − S.

   (A) $-\frac{15}{32}$ **(B)** $-\frac{9}{32}$ **(C)** $\frac{9}{32}$ **(D)** $1\frac{1}{8}$ **(E)** $1\frac{19}{32}$

3. **Les Dough has a bag of coins consisting of pennies, nickels, and dimes. The ratio of pennies to dimes is 3 to 2 and the ratio of dimes to nickels is 4 to 1. The total amount of money in the bag is $8.16. How many coins are in the bag?**

   (A) 64 **(B)** 80 **(C)** 112 **(D)** 160 **(E)** 176

4. **Find k if GCF(64, k) = 16 and LCM(64, k) = 192.**

   (A) 128 **(B)** 80 **(C)** 48 **(D)** 32 **(E)** 4

5. **Simplify:** $\left(\frac{8x^2 - 22x + 5}{12x^2 - 7x + 1}\right)\left(\frac{27x^3 - 3x}{5 + 13x - 6x^2}\right)$

   (A) -1 **(B)** $\frac{6x^2 - 15x}{-2x + 5}$ **(C)** $-3x$ **(D)** $\frac{3x}{2x - 5}$ **(E)** $\frac{-1}{5 - 2x}$

6. **Let $12x^2 + ax - 15 = (6x - b)(2x + c)$, where a, b, and c are positive integers less than 10. Find $a(b + c)$.**

   (A) 120 **(B)** 64 **(C)** 43 **(D)** 28 **(E)** 16

7. **The equation of a line through point P (− 1, 3) and perpendicular to the line $4x + 5y = 2$ is $ax + by = c$. Find $a + b + c$, where $a > 0$ and gcf(a, b, c) = 1.**

   (A) -26 **(B)** -21 **(C)** -16 **(D)** -8 **(E)** 2

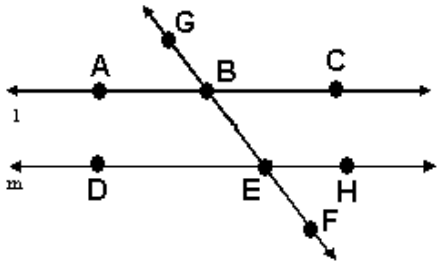
8. **Which of the following mathematicians was one of the first to use computer graphics to create and display fractal geometric images?**

   (A) John Napier **(B)** Alan Turing **(C)** Aryabhata
   (D) Christian Goldbach **(E)** Benoit Mandelbrot

9. **M.T. Tank needs to fill his water tank in 2 hours. He has one pump that can fill the tank in 6 hours by itself. He has a second pump that can fill it in 8 hours by itself. If he uses these two pumps together plus a third pump, he can fill the tank in 2 hours. How long would it take to fill the tank using the third pump by itself?**

   (A) 7 hrs **(B)** 4.8 hrs **(C)** 4.666... hrs **(D)** 2.8 hrs **(E)** 2.4 hrs

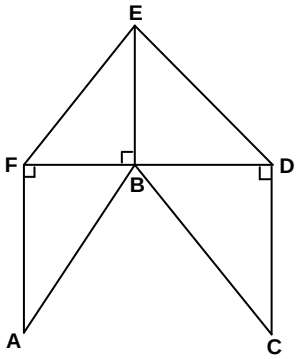
10. The three lines in the figure are coplanar with $m \parallel \ell$.
Which of the following are true statements?



1. ∠ABG & ∠CBE are vertical angles 2. m∠BED = m∠ABG
3. m∠FEH + m∠ABE = 180° 4. ∠DEF & ∠CBG are alternate exterior angles

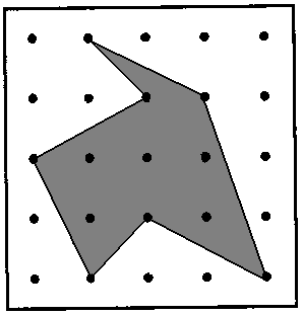
(A) 1 & 4 (B) 1, 2, & 3 (C) 2 & 3 (D) 2 & 3 (E) 1, 2, 3, & 4

11. Given: m∠BEF = 45°, m∠BDE = 30°, m∠BCD = 30°, m∠BAF = 30°,and AB = 6 inches.
Find the perimeter of hexagon ABCDEF. (nearest tenth).



(A) 40.8 " (B) 38.4 " (C) 38.0 " (D) 37.8 " (E) 37.4 "

12. A rubber band was stretched on the geoboard to form this septagon.
What is the area of the septagon?



(A) 7.5 sq. units (B) 7 sq. units (C) 6.5 sq. units (D) 6 sq. units (E) 5.5 sq. units

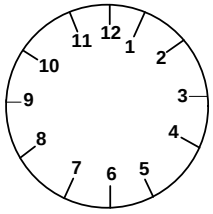
13. If y varies inversely as x and x = 10 when y = 6, find x when y = 40.

(A) 24 (B) 20 (C) 7.5 (D) 1.5 (E) 0.666...

14. Approximate to the nearest tenth: $(\log_2 4)(\log_4 6)(\log_6 8)(\log_8 10)$

(A) 3.3 (B) 2.4 (C) 1.6 (D) 1.1 (E) 0.4

15. Willie Ternrite can't remember the combination to the padlock shown. He knows that the first number is a composite number, the second number is a pentagonal number, and the third is a factor of 12. How many combinations can he try to open the lock?



(A) 216 (B) 90 (C) 108 (D) 144 (E) 126

16. Let $\frac{Ax-B}{5x-3}+\frac{3x-4}{x-5}=\frac{22x^2-66x+22}{5x^2-28x+15}$, where A and B are constants, Find $A+B$.

(A) 2 (B) 5 (C) 7 (D) 9 (E) 14

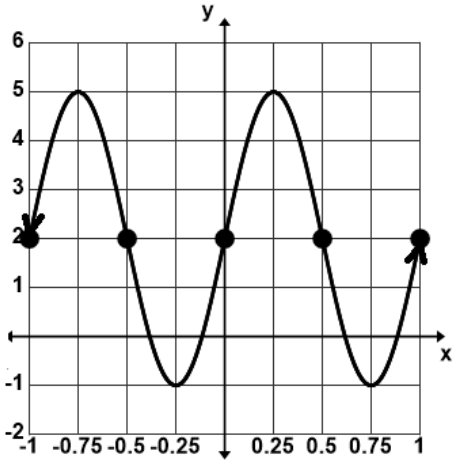
17. Ten tennis balls are numbered from 11 to 20. The balls are placed in a box and mixed up. A single ball is randomly drawn out. What are the odds that the ball drawn out has a prime number on it?

(A) $33\frac{1}{3}\%$ (B) 50% (C) 60% (D) $66\frac{2}{3}\%$ (E) 40%

18. Determine the frequency of $y = 1 - 3\cos(6x+10)$. (nearest tenth)

(A) 0.1 (B) 0.6 (C) 1.0 (D) 2.0 (E) 2.1

19. The equation $y = A\sin(Bx+C)+D$ will produce this graph. Find $A+B+C+D$, where $-1 \le x \le 1$. (nearest tenth)



(A) 11.3 (B) 9.1 (C) 6.3 (D) 3.1 (E) 2.9

20. $(\tan\theta)(\sin\theta)+\cos\theta$ is equivalent to:

(A) $\csc\theta$ (B) $\frac{1}{\cot\theta}$ (C) $\frac{1}{\sec\theta}$ (D) 1 (E) $\sec\theta$

21. A guy wire fastened to the ground on one end is fastened to a radio station tower on the other end. The base of the tower is 100 feet from the end of the guy wire on the ground. The angle of inclination of the wire on the ground to the tower is 50°. At what height does the guy wire connect to the tower? (nearest ft)

(A) 77 ft (B) 84 ft (C) 119 ft (D) 143 ft (E) 173 ft

22. The difference between the 5^{th} tetrahedral number and the 5^{th} triangular number is:

(A) 0 (B) 5 (C) 10 (D) 20 (E) 50

23. The coefficient of the 2nd term of the expansion of $(2x + 3)^6$ is:

(A) 576 (B) -1296 (C) 720 (D) -60 (E) 96

24. Find $m - n$ if $\begin{bmatrix} -2 & 1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

(A) -8 (B) -4 (C) -0.8 (D) 1.8 (E) 5

25. An infinite geometric sequence has a common ratio of $\frac{3}{5}$ and a sum of 25. What is the first term of the sequence?

(A) -10 (B) -5 (C) 0.4 (D) 15 (E) 10

26. Dr. When Zenn's science team consists of 6 students from his AP science class. There are 7 boys and 5 girls in his AP class. How many different science teams consisting of 4 boys and 2 girls could he make up?

(A) 350 (B) 924 (C) 72 (D) 860 (E) 45

27. Let $f(x) = 2x^2 - 3x - 1$ and $g(x) = 2x + 3$. Find $g(f'(-1))$

(A) 2 (B) 1 (C) -1 (D) -9 (E) -11

28. The function $f(x) = ax^3 + bx^2 + 1$ has an inflection point at $(-1, 2)$. Find $a + b$.

(A) $-\frac{1}{2}$ (B) -1 (C) $\frac{1}{2}$ (D) $1\frac{1}{2}$ (E) 2

29. Which of the following polar equations has a graph of a circle with a radius of 1?

(A) $r = 2$ (B) $\theta = 1$ (C) $r = 2\cos\theta$ (D) $r\cos 2\theta = 1$ (E) $r = \sin\theta$

30. How many non-negative proper fractions in lowest terms have a denominator of 72?

(A) 36 (B) 30 (C) 24 (D) 18 (E) 12

31. Which of the following points is NOT in the solution set for $y < |3x - 2| + 1$?

(A) (— 3, 8) **(B) (— 1, 5)** **(C) (0, 0)** **(D) (2, 2)** **(E) (2, 6)**

32. If $x + y = 2$ and $xy = 4$ then $x^3 + y^3 = ?$

(A) 72 **(B) 32** **(C) 8** **(D) — 16** **(E) — 24**

33. If the line $y = mx + b$ contains the point $(3, -4)$ and has a slope of $\frac{2}{5}$ then the y-intercept is:

(A) (0, 5.6) **(B) (0, 2.8)** **(C) (0, — 0.4)** **(D) (0, — 5.2)** **(E) (0, — 3.2)**

34. If $a_1 = -1$, $a_2 = 2$ and $a_n = [(a_{n-1}) + (a_{n-2})] \times (a_{n-1})$ for $n \geq 3$, then a_5 equals:

(A) — 1 **(B) 2** **(C) 8** **(D) 40** **(E) 80**

35. The polar equation $r = \frac{6}{2\cos\theta - 3\sin\theta}$ written in rectangular form is:

(A) $2x - 3y = 6$ **(B) $3x + 2y = 6$** **(C) $x + y = -1$** **(D) $3x - 2y = 1$** **(E) $x - y = 6$**

36. Let $f(x) = 2x^2 - 3x + 4$. Find k if $f(k - 1) - f(k) = 5$.

(A) 2 **(B) $1\frac{1}{4}$** **(C) 0** **(D) $-\frac{1}{4}$** **(E) — 1**

37. The y-intercept of the line that is tangent to $y = x^3 - 2x + 5$ at $x = 2$ is (x, y). Find $x + y$.

(A) 1.1 **(B) 9** **(C) 10** **(D) — 11** **(E) — 29**

38. Find the area (in square units) of the region bounded by $x = 3 - y^2$ and $x = -1$.

(A) 16 **(B) $12\frac{2}{3}$** **(C) $11\frac{1}{3}$** **(D) $10\frac{2}{3}$** **(E) 10**

39. What is the instantaneous rate of change of $f(x) = \frac{x}{x-4}$ at $x = 2$?

(A) — 2 **(B) — 1** **(C) 1** **(D) 2** **(E) 4**

40. Phil Badd was absent the day of the geometry quiz. He took the quiz the next day. His quiz grade was 85, and it raised the class average from 74 to 75. How many students, including Phil, took the quiz?

(A) 15 **(B) 14** **(C) 12** **(D) 11** **(E) 10**

41. Phil Witwater has a right cylindrical water tank that is 12 feet high and has an inside diameter of 5 feet. The tank is 75% full. How many gallons of water is in Phil's tank? (neatest gallon)

(A) 1,763 gal **(B) 1,728 gal** **(C) 1,058 gal** **(D) 1,610 gal** **(E) 1,322 gal**

42. The distance from Wickett, Tx to Waskom, Tx by way of I-20 is 561 miles. Willie Makett leaves Wickett on I-20 at 8:00 a.m. driving toward Waskom at 70 mph. Betty Wont leaves Waskom on I-20 at 7:00 a.m. driving toward Wickett at 60 mph. How far will Willie have driven when they meet? (nearest mile)

(A) 270 miles (B) 275 miles (C) 281 miles (D) 286 miles (E) 291 miles

43. Roland Bones rolls a pair of dice. What is the probability that the sum of top faces he rolls is 2, 3, or 12?

(A) $\frac{1}{8}$ (B) $\frac{1}{12}$ (C) $\frac{1}{9}$ (D) $\frac{1}{11}$ (E) $\frac{1}{4}$

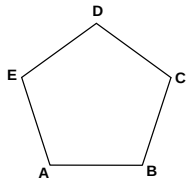
44. Saul Wood cut a six-foot dowel rod into four pieces. The lengths of the pieces are 24 inches, 20 inches, 18 inches, and 10 inches. How many different acute triangles can he make using three pieces at a time?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

45. Les Square added 12 cm to two opposite sides of a square, and subtracted 8 cm from the other two opposite sides. Find the area of the new rectangle if the perimeter of the original square was 96 cm. (nearest cm^2)

(A) 288 cm^2 (B) 576 cm^2 (C) 104 cm^2 (D) 584 cm^2 (E) 157 cm^2

46. Given the regular pentagon shown with DE = 1.6", find AC + AD + BE + BD + CE. (nearest tenth)



(A) 8.0" (B) 12.9" (C) 16.0" (D) 6.5" (E) 4.9"

47. The depth of the water at a dock at Saul Tee's marina varies sinusoidally with time. The depth is 3 ft at low tide at 2:00 a.m. and 21 feet at high tide, which occurs every 5 hours. Find the depth of the water at the dock at 3:00 p.m. (nearest inch).

(A) 3' 1" (B) 9' 3" (C) 18' 9" (D) 19' 3" (E) 20' 7"

48. A right triangle, △ABC, with leg lengths 12" and 22" and the right angle at vertex B is congruent to right triangle, △BDE, with the right angle at vertex D. Point C lies on segment BD and points A and E are on the same side of segment BD. Find the distance between points A and E. (nearest fourth of an inch).

(A) 25" (B) $24\frac{1}{4}$" (C) 22" (D) $20\frac{3}{4}$" (E) $15\frac{1}{2}$"

49. Lotta Dough invested some money in the stock market. Her investment increased 4.5% by the end of the first year, increased 3% by the end of the second year, and decreased 1.5% by the end of the third year. What was Lotta's average rate of return over the three year period? (nearest tenth)

(A) 2.0% **(B) 2.4%** **(C) 2.5%** **(D) 2.6%** **(E) 3.0%**

50. Let $f(x) = (2x - 1)^2$. Let the tangent to f(x) at (x, y) exist where the tangent is perpendicular to $x = 4 - 2y$. Find $x + y$.

(A) 1 **(B) $\frac{29}{64}$** **(C) $\frac{3}{16}$** **(D) $\frac{37}{64}$** **(E) $\frac{1}{2}$**

51. The probability that statement P is true is 2/5, and the probability that statement Q is true is 1/4. Determine the probability that $P \rightarrow Q$ is false.

(A) $\frac{1}{10}$ **(B) $\frac{3}{20}$** **(C) $\frac{7}{20}$** **(D) $\frac{9}{20}$** **(E) $\frac{3}{10}$**

52. Let function f = {(2, 3), (3, 5), (5, 7)} and function g = {(3, 3), (5, 2), (7, 9)}. Which of the following is a member of the function g(f)?

(A) (2, 3) **(B) (2, 5)** **(C) (2, 7)** **(D) (3, 5)** **(E) (5, 7)**

53. A *square-free semiprime* is a composite number that is the product of two different primes. How many *square-free semiprimes* exist such that the two different prime factors are single digits?

(A) 8 **(B) 6** **(C) 4** **(D) 2** **(E) 0**

54. Let $f(x) = \sqrt{8 - \sqrt{5x - 3}}$. The domain of f(x) is $\{x \mid p \leq x \leq q\}$. Find $\frac{P+Q}{2}$.

(A) 8 **(B) 7** **(C) 6.7** **(D) 6.4** **(E) 3**

55. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the sum of the numbers in the first 10 rows of the 10th column.

1				1				2				3				5		
			2				3				5				8			
		3				5				8				13				21
	5				8				13				21				34	
8				13				21				34				55		
			21				34				55				89			
		34				55				89				144				233

(A) 246 **(B) 241** **(C) 212** **(D) 157** **(E) 117**

56. Let $(230_b) \times 4_b = k_b$, where k is a 4-digit number. Find 123_b if $k_b = 476$ in base 10.

(A) 66 **(B) 51** **(C) 38** **(D) 102** **(E) 83**

57. Let x = 3,27A,5B1, where A and B are positive digits. Find the largest sum, A + B, if x divided by 9 has a remainder is 3.

(A) 3 **(B) 6** **(C) 9** **(D) 12** **(E) 15**

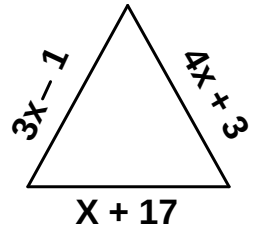
58. If $y < x$ and $y < 0$, which of the following is never greater than any of the others?

(A) $x + y$ **(B) $x - y$** **(C) $x + |y|$** **(D) $x - |y|$** **(E) $-|x + y|$**

59. The fraction $\frac{12}{20}$ base 7 can be written as which of the following decimals in base 7?

(A) $0.363636..._7$ **(B) $0.4333..._7$** **(C) $0.4222..._7$** **(D) $0.1555..._7$** **(E) 0.6_7**

60. Find the greatest possible perimeter of this triangle given that it is isosceles but not equilateral.



(A) 51 **(B) 56** **(C) 91** **(D) 93** **(E) 100**

2017-18 TMSCA HS Math Test #13
Answer Key

1. B	**21. C**	**41. E**
2. A	**22. D**	**42. A**
3. E	**23. A**	**43. C**
4. C	**24. C**	**44. C**
5. C	**25. E**	**45. B**
6. B	**26. A**	**46. B**
7. C	**27. E**	**47. D**
8. E	**28. E**	**48. B**
9. B	**29. C**	**49. A**
10. E	**30. C**	**50. A**
11. A	**31. E**	**51. E**
12. A	**32. D**	**52. A**
13. D	**33. D**	**53. B**
14. A	**34. E**	**54. B**
15. C	**35. A**	**55. D**
16. D	**36. C**	**56. A**
17. D	**37. D**	**57. D**
18. C	**38. D**	**58. E**
19. A	**39. B**	**59. B**
20. E	**40. D**	**60. C**