T M S C A   H I G H   S C H O O L
M A T H E M A T I C S
T E S T # 6 (UIL C) ©
D E C E M B E R   1 , 2 0 1 8

Academic Excellence in Mathematics and Science through Competition

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. **Evaluate:** $1 + 2 \times 2 - 2 \div 0! + 1 \times 8$

   (A) 40　　　　(B) 32　　　　(C) 13　　　　(D) 11　　　　(E) undefined

2. Let L = {1, 4, 5, 7, 8, 9}, M = {1, 4, 8, 9, 10, 11}, and W = {1, 6, 7, 9, 11}. The sum of the elements in $(L \cap M) \cup W$ is:

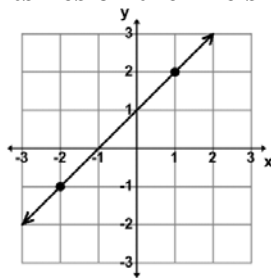   (A) 61　　　　(B) 56　　　　(C) 46　　　　(D) 34　　　　(E) 28

3. If 25% of P is 25% more than Q and Q is 50% less than R, then P is what per cent of R?

   (A) 40%　　　　(B) 50%　　　　(C) 90%　　　　(D) 110%　　　　(E) 250%

4. If the sales tax on $18.75 is $1.26, then the sales tax on $43.50 is? (nearest cent)

   (A) $1.66　　　　(B) $2.52　　　　(C) $2.92　　　　(D) $3.78　　　　(E) $4.18

5. Which of the following points lies on the line shown?

   (A) (11, 10)　　　(B) (8, 9)　　　(C) (6, 6)　　　(D) (−7, −8)　　　(E) (−12, −14)

6. Let $12x^2 - 8x - 15 = (ax + b)(cx + d)$. Find $ad - bc$, where $a < c$.

   (A) 28　　　　(B) 16　　　　(C) −3　　　　(D) −8　　　　(E) −11

7. Which of the following is not an example of a function?

   (A) $y = 3x - 4$　　(B) $x = 3y + 4$　　(C) $y = 3x^2 + 4x$　　(D) $y = \frac{3x - 4}{3x^2 + 4}$　　(E) $x = |3y - 4|$

8. Which one of the following sets of numbers is not closed under any of the four basic operations of addition, subtraction, multiplication, or division?

   (A) Evens　　　(B) Odds　　　(C) Rationals　　　(D) Reals　　　(E) Primes

9. The faces of a regular dodecahedron are _____.

   (A) triangles　　(B) pentagons　　(C) squares　　(D) rectangles　　(E) hexagons

10. If two chords intersect within a circle, then the measure of the angle formed is _____ the sum of the measures of the intercepted arcs.

    (A) equal to　　(B) twice　　(C) one-half　　(D) two-thirds　　(E) greater than

11. **Find x. (drawing is not to scale)**

25 cm   3 cm

5 cm   X

(A) 8 cm   (B) $5\frac{3}{5}$ cm   (C) 15 cm   (D) 13 cm   (E) $4\frac{2}{3}$ cm

12. **Saul Smoke is at the top of a 200-ft lookout tower located on a flat plain. He spots a fire at an angle of depression of 3° from the top of his tower. How far away is the fire? (nearest foot)**

(A) 4,350 ft   (B) 3,816 ft   (C) 3,200 ft   (D) 2,003 ft   (E) 1,048 ft

13. **If $x + 2y = -3$ and $4x - 5y = 6$ then 7x equals ?**

(A) $-1\frac{8}{13}$ (B) $-\frac{9}{13}$ (C) $\frac{3}{13}$ (D) $3\frac{5}{13}$ (E) $4\frac{11}{13}$

14. **Let $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} d & e \\ f & g \end{bmatrix}$. Find g.**

(A) -3 (B) -1.5 (C) 0   (D) 2   (E) 2.5

15. **Let $A + B = 18$ and $A \times B = 34$. Find $B - A$, where $A < B$.**

(A) $2\sqrt{153}$ (B) 16   (C) $2\sqrt{47}$ (D) $3\sqrt{51}$ (E) $\sqrt{47}$

16. **Ginger Gold can pick enough apples to fill a barrel in 30 minutes. Granny Smith takes 42 minutes to do it. How long would it take to fill the barrel if they work together?**

(A) 36 min   (B) 34 min   (C) 24.5 min   (D) 18 min   (E) 17.5 min

17. **Which of the following equations can be obtained from the graph of the parent function $y = \cos(x)$ by applying a vertical stretch of 5 units, a vertical shift of -3 units, and phase shift of 1?   y =**

(A) $3 + \cos(x + 5)$ (B) $5 + 3\cos(x - 1)$ (C) $5\cos(x + 1) - 3$
(D) $1 - 3\sin(x - 5)$ (E) $5\cos(x - 1) - 3$

18. **The expression $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$ is equivalent to:**

(A) 2csc A   (B) $\frac{1}{\sec A}$ (C) 1   (D) $\frac{2 + \cos A}{\sin A}$ (E) csc 2A

19. **Determine the amplitude of $f(x) = 1 - 2\sin[3(x + \frac{\pi}{4})]$.**

(A) -2 (B) $-\frac{1}{4}$ (C) 1   (D) 2   (E) 3

20. **Find the sum of the coefficients of the x^3y^2 term and the xy^4 term in the expansion of $(2x - 3y)^5$.**

(A) 1,170   (B) 1,530   (C) 1,800   (D) 1,960   (E) 2,340

21. When k is divided by 9, the remainder is 7. What is the remainder when 3k is divided by 9?

    (A) 0   (B) 1   (C) 3   (D) 5   (E) 7

22. The number in the unshaded box is found by adding the numbers connected with it from the row above it. (ex. 11 is found using 5 and 6). What is the value of X?
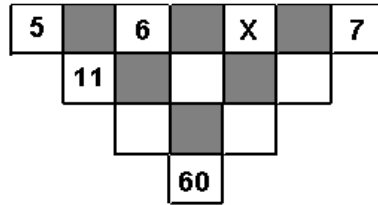
    (A) 8   (B) 9   (C) 10   (D) 11   (E) 12

23. Willie Bankett invested some money in the stock market. His investment increased 4% by the end of the first year, decreased 3% by the end of the second year, increased 5% by the end of the third year and decreased 1% by the end of the fourth year. What was Willie's average rate of return over the four year period? (nearest tenth)

    (A) 4.5%   (B) 3.3%   (C) 2.8%   (D) 1.3%   (E) 1.2%

24. Let $f(x) = 2x^4 - 3x^2 + 4x - 5$. Find $f'(1)$.

    (A) 6   (B) 1   (C) 0   (D) -2 (E) -5

25. Find the slope of the line tangent to $f(x) = 12 - x^2$ at $x = 1$.

    (A) 11   (B) 10   (C) 8   (D) -4 (E) -2

26. Find the area of the shaded region. (square units)
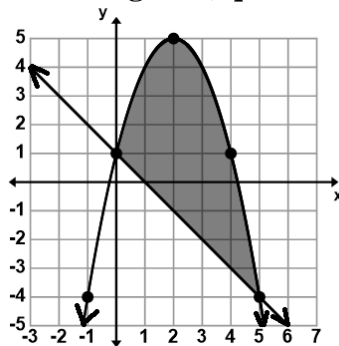

    (A) $22\frac{1}{6}$ (B) $20\frac{5}{6}$ (C) $20\frac{1}{3}$ (D) $19\frac{2}{3}$ (E) $18\frac{1}{2}$

27. Willie Pickett has a math club with 7 boys and 5 girls. If he picks two club members at random what is the probability that both are girls? (nearest whole percent)

    (A) 15%   (B) 17%   (C) 22%   (D) 40%   (E) 42%

28. **Willie Pickett has a math club with 7 boys and 5 girls. How many different science teams consisting of 6 club members can he choose if 4 are boys and 2 are girls?**

   (A) 45   (B) 225   (C) 350   (D) 210   (E) 924

29. **Willie Pickett has a math club with 7 boys and 5 girls. If he picks two club members at random what is the probability that the second one is a girl given that the first one picked was a boy? (nearest whole percent)**

   (A) 27%   (B) 35%   (C) 40%   (D) 45%   (E) 53%

30. **The 'fangs' of the 'vampire' number 1,827 are f and v. Find f + v.**

   (A) 36   (B) 90   (C) 99   (D) 108   (E) 153

31. **Which of the following mathematicians proved the following theorem? The size of the power set of A is strictly larger than the size of A, even when A is an infinite set.**

   (A) George Boole (B) Freda Porter (C) Alicia Stott (D) Alan Turing (E) Georg Cantor

32. $55_{27} + 66_9 = $ _____ $_3$.

   (A) 21102   (B) 11012   (C) 20112   (D) 11212   (E) 22102

33. **How many fluid ounces of water would remain in a two-gallon pail of water if you removed three quarts of water from the pail, then poured a pint of water back in the pail, and drank three cups of water from the pail?**

   (A) 168 fl. oz   (B) 152 fl. oz   (C) 136 fl. oz   (D) 100 fl. oz   (E) 88 fl. oz

34. **Let $(3x - 4)(5x + 2) = ax^2 + bx + c$. Find $a + b - c$.**

   (A) -7 (B) 6   (C) 9   (D) 20   (E) 21

35. **Find the area of $\triangle ABC$.**
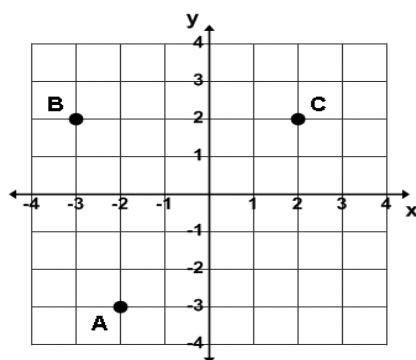
   (A) 12.5 units^2 (B) 15 units^2 (C) 16.5 units^2 (D) 14 units^2 (E) 11.5 units^2

36. If $a_1 = -1$, $a_2 = 0$, $a_3 = 1$ and $a_n = (a_{n-2} - a_{n-1})(a_{n-3})$, where $n \geq 4$, then a_6 equals:

   (A) -2 (B) -1 (C) 0   (D) 1   (E) 2

37. The distance from Sumware to Enyware is 105 km at a bearing of 80°. The distance from Enyware to Knoware is 75 km at a bearing of 205°. Find, to the nearest degree, the bearing from Knoware to Sumware.

    (A) 89 °   (B) 120 °   (C) 269 °   (D) 305 °   (E) 325 °

38. If $f(x) = x^2 - 3x + 2$, where $x \leq 1.5$ then $f^{-1}(6) = ?$

    (A) 4   (B) 1.5   (C) $\frac{1}{20}$ (D) -1 (E) $-\frac{1}{4}$

39. Jack went to the river to get a pail of water then took it to Jill. What is the minimum total distance Jack can travel to the river then to Jill to deliver the water? (nearest hundreth)
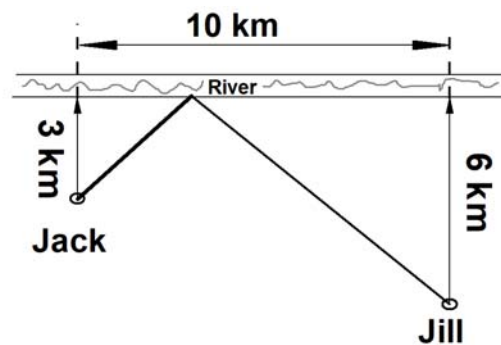
    (A) 9.42 km   (B) 18.83 km   (C) 4.44 km   (D) 13.45 km   (E) 20.18 km

40. How many distinct 3-letter code words can be made from the letters in the words "DA GURU"?

    (A) 120   (B) 72   (C) 60   (D) 36   (E) 20

41. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of theFibonacci sequence. How many digits are in f_{25}.

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

42. Les Change went to the *Big Dawg Hut* and ordered three super pup dogs at $1.39 each, two orders of curly fries at 79¢ each, a 99¢ medium drink and a $1.25 large drink. He gave the cashier a ten dollar bill. How much change should he get back?

    (A) $5.58   (B) $3.40   (C) $4.40   (D) $2.63   (E) $2.01

43. R.U. Kannfuzed divided $80.00 among Peter, Paul, and Mary. Paul received twice as much as Peter, and Mary received $5.00 less than Paul. How much did Mary receive?

    (A) $34.00   (B) $24.00   (C) $39.00   (D) $29.00   (E) $17.00

44. ABCD is an isosceles trapezoid with legs $\overline{AD}$ and $\overline{BC}$ and height $\overline{DE}$. Find CD if AD = 10'', DE = 6'' and the area of ABCD is 246 sq. inches.

   (A) 49''   (B) 41''   (C) 36.5''   (D) 33''   (E) 30.75''

45. The harmonic mean of the roots of $x^5 - 11x^4 + 45x^3 - 85x^2 + 74x - 24 = 0$ is ? (nearest tenth)

   (A) 2.3   (B) 1.6   (C) 1.8   (D) 3.6   (E) 1.2

46. Let $(-\sqrt{3} + i)^{-6} = (a + bi)$. Find $a + b$.

   (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) $\frac{1}{3}$ (D) $\frac{1}{27}$ (E) $\frac{1}{32}$

47. $(412_8 + 511_8 \times 6_8) \div 7_8$ has a remainder of _____.

   (A) 0   (B) 1   (C) 3   (D) 4   (E) 6

48. Let $f(x) = \begin{cases} 2 - x & \text{if } x \le 0 \\ x + 3 & \text{if } x > 0 \end{cases}$. Which of the following is/are false?

   1. $\lim\limits_{x \to 0^+} f(x)$ exists   2. $\lim\limits_{x \to 0^-} f(x)$ exists   3. $f(x)$ is continuos

   (A) none of these   (B) all of them   (C) 1 only   (D) 2 only   (E) 3 only

49. P-Q-R is the combination needed to open the safe with the combination dial shown below. How many distinct combinations exist if P is a prime number, Q is a Fibonacci number greater than zero, and R is a factor of 40?
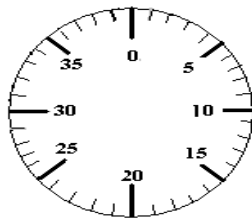
   (A) 28   (B) 192   (C) 672   (D) 704   (E) 768

50. If the following pattern continues, find the sum of the first and last numbers in row 30.

$$1 + 2 = 3 \qquad \text{row 1}$$
$$4 + 5 + 6 = 7 + 8 \qquad \text{row 2}$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15 \qquad \text{row 3}$$

   (A) 1,741   (B) 1,860   (C) 1,924   (D) 1,985   (E) 2,050

51. The digits 1, 2, 3, 4, and 9 are each used once to form the smallest possible five-digit even number. What is the digit in the tens place?

   (A) 9   (B) 4   (C) 3   (D) 2   (E) 1

52. How many integers, n, satisfy the inequality $-\frac{3}{4} < \frac{2n}{5} < \frac{1}{6}$

(A) 1   (B) 2   (C) 3   (D) 5   (E) 7

53. Point P(— 2, 3) lies on the x-y plane. P is translated vertically 4 units down to point Q. Then Q is translated horizontally 5 units to the right to point R. Then R is reflected across the x-axis to point S with coordinates (x, y). Find $x + y$

(A) — 6   (B) — 4   (C) 1   (D) 2   (E) 4

54. If $\sqrt[3]{x\sqrt{x\sqrt[4]{x}}} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 26   (B) 24   (C) 13   (D) 12   (E) 11

55. Find the sum of the x-values in $\left\{x \mid \sin(x)\cos(x) + \cos(x) = 0, x \in [0, 2\pi]\right\}$. (nearest hundredth)

(A) 6.28   (B) 7.20   (C) 7.85   (D) 9.42   (E) 11.00

56. *Hero of Alexander* determined that his heronian mean of 5 and 10 was greater than the harmonic mean of 5 and 10. How much bigger was his mean? (hundredths place)

(A) 0.14   (B) 0.29   (C) 0.40   (D) 0.69   (E) 0.98

57. Find the volume of the solid generated by revolving the region bounded by $y = x^2 + 2$ and $y = x + 4$ about the x-axis. (nearest whole cubic unit)

(A) 180 units^3 (B) 161 units^3 (C) 118 units^3 (D) 102 units^3 (E) 59 units^3

58. Betty Gesses knows that her probability of getting exactly 1 problem correct on a 2 problem true-false test is 50%. Betty only takes true-false tests with an even number of problems on it. What is the least number of problems on such a test so that the probability of getting exactly half the questions correct is less than 25%?

(A) 6   (B) 8   (C) 10   (D) 16   (E) 20

59. The nine congruent circles in the figure have 10 cm diameters and each is tangent to its neighbors. What fraction best approximates the area of the square that is not shaded?

(A) $\frac{3}{14}$ (B) $\frac{7}{22}$ (C) $\frac{4}{31}$ (D) $\frac{3}{7}$ (E) $\frac{7}{41}$

60. What is the smallest composite number generated by $p^2 - p - 1$, where p is prime?

(A) 115   (B) 155   (C) 123   (D) 135   (E) 217

1. D	21. C	41. C
2. C	22. C	42. E
3. E	23. E	43. D
4. C	24. A	44. D
5. B	25. E	45. B
6. A	26. B	46. A
7. E	27. A	47. A
8. E	28. C	48. E
9. B	29. D	49. C
10. C	30. D	50. B
11. C	31. E	51. A
12. B	32. A	52. B
13. A	33. B	53. E
14. B	34. C	54. C
15. C	35. A	55. A
16. E	36. D	56. D
17. E	37. D	57. D
18. A	38. D	58. C
19. D	39. D	59. A
20. B	40. B	60. B