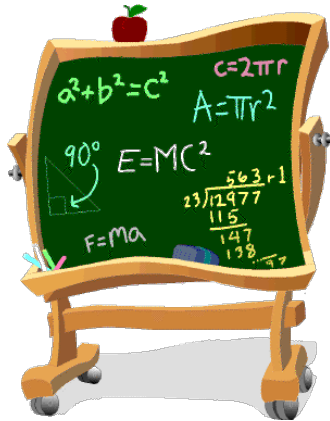# Mathematics

Invitational A • 2024

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Luke, Paige, Ryan and Elliot went to the Salado Pizza Palace for supper. They ordered two large pizzas which cost $15.95 each and two pitchers of coke which cost $5.95 each. They agreed to share the cost equally. If the tax rate is 8.125% and they added a 25% after tax tip, how much did Luke pay?

   (A) $14.68   (B) $14.80   (C) $14.92   (D) $15.04   (E) $15.16

2. The Calhoun math team did a fundraiser in October. Stacy's favorite band, Santana, agreed to come to Port Lavaca and perform. A student ticket cost $12.50 and an adult ticket cost $21.50. If they sold a total of 498 tickets and they grossed $9,213 from ticket sales, how many adult tickets were sold?

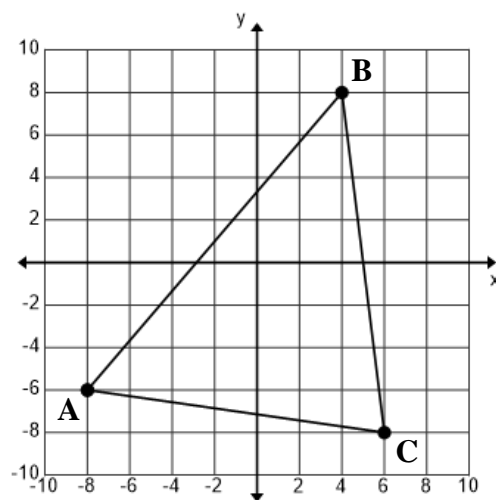   (A) 326   (B) 328   (C) 330   (D) 332   (E) 334

3-6. Consider △ABC shown on the right.

3. The point $P(4, b)$ lies on the perpendicular
   bisector of $\overline{BC}$. b = _____ .

   (A) -0.125
   (B) 0
   (C) 0.125
   (D) 0.25
   (E) 0.375

4. Find the perimeter of △ABC.
   (nearest tenth)

   (A) 48.1
   (B) 48.4
   (C) 48.7
   (D) 49.0
   (E) 49.3

Problems 3, 4, 5, 6

5. Find the area of △ABC.

   (A) 102   (B) 104   (C) 106   (D) 108   (E) 110

6. Point D, not shown, lies on $\overline{AB}$. If $\overline{CD}$ is the median from point C to side $\overline{AB}$, then CD = _____ .
   (nearest tenth)

   (A) 11.4   (B) 11.7   (C) 12.0   (D) 12.3   (E) 12.6

7. The roots of $ax^3 + bx^2 + cx + d = 0$ are $-2, 1$ and 3. If a = 2, then c = _____ .

   (A) -10 (B) -8 (C) -6 (D) -4 (E) -2

8. The number of fleas in a house is directly related to the number of dogs and inversely related to the square of the number of ferrets. Landon's house has 240 fleas, 6 dogs and 2 ferrets. If Schafer's house has 4 ferrets and 8 dogs, how many fleas are in his house?

   (A) 72   (B) 74   (C) 76   (D) 78   (E) 80

UIL Mathematics

9. When $2x^3 + 6x^2 + 4x + 8$ is divided by $x + 2$, the remainder is _____.

(A) 0   (B) 2   (C) 4   (D) 6   (E) 8

10. Matthew is taking Linear Algebra this year at Southwestern University during 7th period. Each of the 4 test grades count once and the final exam grade counts twice when averaging his grade for the course. His test grades are 86, 92, 91 and 87. He needs to have an average of at least 90.0 to earn an A for the course. What is the minimum score he needs to make on the final exam to earn an A?

(A) 90   (B) 91   (C) 92   (D) 93   (E) 94

11. Points A, B, C and D lie on a circle such that $\overleftrightarrow{BD}$ intersects $\overrightarrow{AC}$ at point E. If BE = 4, AE = 6 and DE = 9, then CE = _____. (nearest tenth)

(A) 5.8   (B) 6.0   (C) 6.2   (D) 6.4   (E) 6.6

12. Points F, G, H and I lie on a circle such that $\overleftrightarrow{HF}$ intersects $\overrightarrow{IG}$ at point J.

Given: minor arcs GF and IH with mGF = 88° and mIH = 108°. Find m∠HJG.

(A) 80°   (B) 82°   (C) 84°   (D) 86°   (E) 88°

13-14. ΔABC is similar to ΔDEF. Point G lies on $\overline{AC}$ and $\overrightarrow{BG}$ bisects ∠ABC. Given: AB = 22, BC = 20, AC = 14 and EF = 15.

13. Find the perimeter of ΔDEF. (nearest tenth)

(A) 42.0   (B) 42.5   (C) 43.0   (D) 43.5   (E) 44.0

14. CG = _____. (nearest tenth)

(A) 6.7   (B) 6.9   (C) 7.1   (D) 7.3   (E) 7.5

15. Consider rhombus PQRS with PR = 12.274 and QS = 25.166. The area of the rhombus is _____. (nearest whole number)

(A) 150   (B) 152   (C) 154   (D) 156   (E) 158

16-17. A large cylindrical container has a diameter of 4 feet and a height of 8 feet.

16. Find the total area of the cylinder. (nearest whole number)

(A) 117 ft^2 (B) 120 ft^2 (C) 123 ft^2 (D) 126 ft^2 (E) 129 ft^2

17. How many gallons of water are required to completely fill the container? (nearest whole number)

(A) 740   (B) 743   (C) 746   (D) 749   (E) 752

18-19. Consider a circle with center O and diameter $\overline{BD}$. Chord $\overline{AC}$ is perpendicular to $\overline{BD}$. $\overline{AC}$ intersects $\overline{BD}$ at point E. Given: AC = 80 and BE = 32.

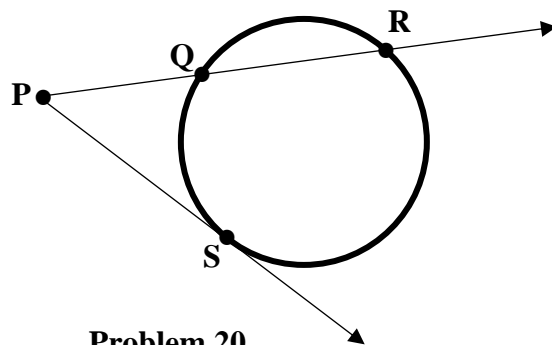**18.** Find the area of $\triangle OEC$. (nearest whole number)

   (A) 180   (B) 182   (C) 184   (D) 186   (E) 188

19. Find the length of minor arc CD. (nearest tenth)

   (A) 72.6   (B) 72.9   (C) 73.2   (D) 73.5   (E) 73.8

20. $\overrightarrow{PS}$ is tangent to the circle at point S.
   PQ = 16 and QR = 22.
   PS = _____. (nearest tenth)

   (A) 24.1
   (B) 24.4
   (C) 24.7
   (D) 25.0
   (E) 25.3

Problem 20

21. Leonardo has a collection of 24 marbles that are identical in size, but vary in color. Twenty of them are green, 2 are red and 2 are blue. How many different ways can he arrange them in a row?

   (A) 7,695   (B) 15,939   (C) 31,878   (D) 63,756   (E) 127,512

22. Consider the sequence 7, 11, 15, 19, 23, 27,… Find the sum of the first 19 terms.

   (A) 813   (B) 814   (C) 815   (D) 816   (E) 817

23. Consider the sequence 4, 6, 9, 13.5, 20.25, 30.375,… Find the sum of the first 11 terms. (nearest tenth)

   (A) 681.8   (B) 682.9   (C) 684.0   (D) 685.1   (E) 682.2

24-25. The measure of an interior angle of a regular polygon is 135°. The length of each side is 12.

24. Find the area of the polygon. (nearest whole number)

   (A) 686   (B) 689   (C) 692   (D) 695   (E) 698

25. Find the area of the circle that is inscribed in the polygon. (nearest whole number)

   (A) 656   (B) 659   (C) 662   (D) 665   (E) 668

26. $f(x) = \dfrac{2x+7}{3x-2}$ and $g(x) = \dfrac{4x-1}{5x+2}$. $f^{-1}(2) - g^{-1}(2) = $ _____.

(A) $3\dfrac{5}{12}$ (B) $3\dfrac{1}{2}$ (C) $3\dfrac{7}{12}$ (D) $3\dfrac{2}{3}$ (E) $3\dfrac{3}{4}$

27. Consider the function $g(x) = \sqrt{\dfrac{5x+4}{3x}}$. Find the range of $g(x)$.

(A) $(-\infty, \infty)$ (B) $(-0.8, \infty)$ (C) $\left(\sqrt{\dfrac{5}{3}}, \infty\right)$ (D) $[0, \infty)$ (E) $(0, \infty)$

28-29. Consider the polynomial function $f(x) = 3x^5 + x^4 - 5x^3 + dx^2 - 8x + 4$.

28. If $f(-1) = 20$, then $d = $ _____.

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

29. The polynomial function $f(x)$ has _____ real zeros.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

30. On June 1st of 2018, Ian's grandpa placed $25,000 into an account for Ian that earns interest at a rate of 7.25% compounded monthly. Ian plans to withdraw all of the money in the account on June 1st of 2024 and use it toward the purchase of a new RAV4 Hybrid from Georgetown Toyota. If the total cost including tax, title and license is $38,773.56, how much money will Ian have to come up with?

(A) $100 (B) $200 (C) $300 (D) $400 (E) $500

31. Wyatt is playing 5 card poker with his friends on the Georgetown High School track team. They are using a standard deck of 52 cards. What is the probability that he will be dealt a hand (5 cards) with at least one queen? (nearest thousandth)

(A) 0.329 (B) 0.335 (C) 0.341 (D) 0.347 (E) 0.353

32. Find the period of the graph of $y = 3 + 2\cos\left(\dfrac{\pi x}{3} - \dfrac{\pi}{6}\right)$.

(A) 2 (B) 3 (C) 6 (D) π (E) 2π

33. Six chairs are arranged in a row. Six students from Calhoun are to sit in the chairs. If Seth insists on sitting next to Doreen, how many distinct seating arrangements are possible?

(A) 36 (B) 120 (C) 240 (D) 480 (E) 720

34. Ariana leaves Georgetown and flies on a bearing of 210° for 120 miles. She lands at an airport and picks up her mom and then she flies on a bearing of 300° for 180 miles and lands at a different airport near her lake home. After 4 days, Ariana flies directly back to Georgetown. If her Bell 407 helicopter travels at an average speed of 112 mph, how much time will it take her to fly directly from her lake house to Georgetown? (nearest minute)

  (A) 1 hr 44 min   (B) 1 hr 47 min   (C) 1 hr 50 min   (D) 1 hr 53 min   (E) 1 hr 56 min

35. Consider a hard drive disk with a diameter of 3.5 inches. If it is operating at 7200 rpm, what is the linear velocity of a point on the outer edge of the disk? (nearest whole number)

  (A) 69 mph   (B) 71 mph   (C) 73 mph   (D) 75 mph   (E) 77 mph

36. The equation of an ellipse that is centered at $(4,2)$, with vertices $(0,2)$ and $(8,2)$, and with eccentricity $= \dfrac{\sqrt{7}}{4}$ is $ax^2 + by^2 + cx + dy + e = 0$. e = _____.

  (A) 16   (B) 32   (C) 64   (D) 80   (E) 96

37. The y-intercept of the graph of the polar equation $r = \dfrac{-6}{2\sin\theta - \cos\theta}$ is $(0, b)$. b = _____.

  (A) -6 (B) -5 (C) -4 (D) -3 (E) -2

38. Find the angle between the vectors $u = 3i + 5j$ and $v = 4i + 7j$. (nearest tenth)

  (A) 1.2°   (B) 2.3°   (C) 3.4°   (D) 4.5°   (E) 5.6°

39. Consider the curve represented by the parametric equations $x = 2\sec(\theta)$ and $y = 4\tan(\theta)$. Find the distance between the foci. (nearest tenth)

  (A) 8.7   (B) 8.9   (C) 9.1   (D) 9.3   (E) 9.5

40. Find the distance between the point $P(1, 2, 3)$ and the plane $2x - 3y + 5z = 4$. (nearest tenth)

  (A) 1.1   (B) 1.3   (C) 1.5   (D) 1.7   (E) 1.9

41. Given: $f(0.5) = 0.5$, $f(6.5) = 0.5$, and $f'(x) = \dfrac{2\pi\cos^2\left(\dfrac{\pi x}{6}\right) - \pi}{3}$. Find the sum of the values of x in the open interval $(0.5, 6.5)$ that satisfy the Mean Value Theorem for the function f on the closed interval $[0.5, 6.5]$? (rad) (nearest tenth)

  (A) 5.6   (B) 5.8   (C) 6.0   (D) 6.2   (E) 6.4

42. Consider the function $h(x) = .5e^x$. At what value of x is the slope of the line tangent to the graph of $y = h(x)$ equal to 2.2? (nearest hundredth)

   (A) 1.46   (B) 1.48   (C) 1.58   (D) 1.60   (E) 1.62

43-44. Consider the graphs of $y = f(x)$ and $y = h(x)$.

43. Find the area bounded by the graphs of $y = f(x)$ and $y = h(x)$. (nearest tenth)

   (A) 14.2   (B) 14.4   (C) 14.6
   (D) 14.8   (E) 15.0

44. Find the volume generated by revolving the region bounded by the graphs of $y = f(x)$ and $y = h(x)$ about the line $x = -4$. (nearest tenth)

   (A) 602.2   (B) 603.3   (C) 604.4   (D) 605.5   (E) 606.6

Problems 43, 44

45-46. Consider the graph of $y = f(x)$.

45. The function f(x) is continuous for $-5 \le x \le 5$. The graph of y = f(x) consists of five line segments as shown. The average value of f(x) on the interval $-5 \le x \le 5$ is _____.
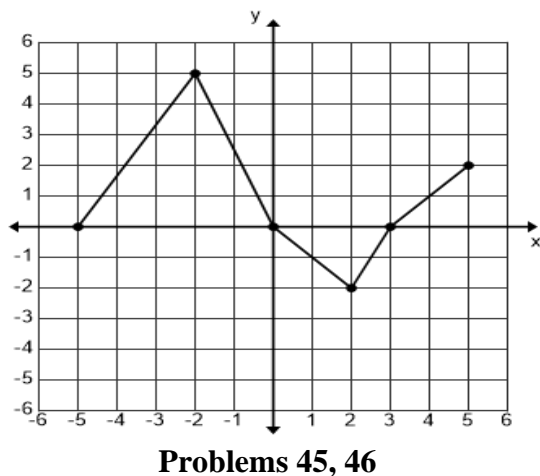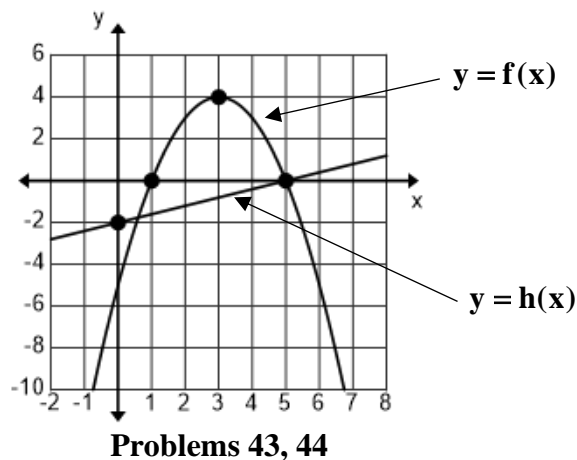
   (A) $\dfrac{11}{10}$ (B) $\dfrac{23}{20}$ (C) $\dfrac{6}{5}$

   (D) $\dfrac{5}{4}$ (E) $\dfrac{13}{10}$

46. Find the value of $\displaystyle\int_{-5}^{5} f'(x)dx$. (nearest tenth)

Problems 45, 46

   (A) 1.7   (B) 2.0   (C) 2.3   (D) 2.6   (E) 2.9

47. Let F(x) be an antiderivative of $\dfrac{x^2 + 2\cos(x)}{x^3 - 7}$. If $F(4) = 3$, then $F(12) = $ _____.
   (nearest hundredth)

   (A) 3.92   (B) 4.03   (C) 4.14   (D) 4.25   (E) 4.36

48. Given: $\int\limits_{2}^{-6} f(x)dx = -7$ and $\int\limits_{2}^{8} f(x)dx = 9$. $\int\limits_{-6}^{8} f(x)dx =$ _____ .

    (A) -2 (B) 2   (C) 10   (D) 16   (E) 22

49. Given: $3x^2 - 2xy + y^2 = 6$. At the point $P(1, b)$, $b < 0$, $\dfrac{dy}{dx} =$ _____ . (nearest tenth)

    (A) 1.6   (B) 1.8   (C) 2.0   (D) 2.2   (E) 2.4

50-52. The position of a particle is given by the parametric equations $x(t) = e^{-3t}$ and $y(t) = \ln(2t^2 + 5)$
     for $0 \le t \le 10$.

50. Find the velocity vector when $t = 8$. (nearest hundredth)

    (A) $\langle 3.19, 0.36 \rangle$ (B) $\langle 3.22, 0.33 \rangle$ (C) $\langle 3.25, 0.30 \rangle$ (D) $\langle 3.28, 0.27 \rangle$ (E) $\langle 3.31, 0.24 \rangle$

51. Find the speed of the particle at $t = 8$. (nearest tenth)

    (A) 2.9   (B) 3.1   (C) 3.3   (D) 3.5   (E) 3.7

52. Find the total distance traveled by the particle from $t = 2$ to $t = 4$. (nearest hundredth)

    (A) 1.51   (B) 1.62   (C) 1.73   (D) 1.84   (E) 1.95

Test #	1	2	3	4	5	6	7	8	9
Score	274	310	328	337	355	319	337	391	346

Dhilan took nine old number sense tests this week to prepare for Saturday's UIL meet. His scores are listed in the table above. Use the table above for problems 53 and 54.

53. Find the sum of the mean, median and mode of the scores.

    (A) 1005   (B) 1006   (C) 1007   (D) 1008   (E) 109

54. How many of the scores are classified as outliers?

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

55. The Dublin Bottling Company produces bottles of Crème Soda and stores them in a huge warehouse until they are shipped. The mean amount of soda in each bottle is 2.00 liters with a standard deviation of 0.02 liters. The Brownwood Safeway placed a large order and when the shipment arrived, the store manager randomly selected 12 bottles and measured the amount of soda in each bottle. If three or more bottles have less than 1.96 liters, the shipment will be rejected. Find the probability that this order was rejected. Assume the amounts of soda in the bottles are independent of each other. (nearest ten-thousandth)

    (A) 0.0022   (B) 0.0033   (C) 0.0044   (D) 0.0055   (E) 0.0066

Miles per Week	44	50	54	59	66	69
5-K time (minutes)	18.2	17.5	17.3	16.9	16.2	15.9

Aryan keeps a record of his weekly mileage and his 5-K times at cross country meets. Use the table above for problems 56 and 57.

56. Aryan plotted the data and observed a strong, negative, linear relationship between his weekly mileage and his 5-K times. Statistical software generated a LSRL. Find the value of the residual for the data point (50 miles per week, 17.5 minutes). (nearest thousandth)

   (A) −0.127   (B) −0.123   (C) −0.119   (D) −0.115   (E) −0.111

57. Find the time predicted by the LSRL if Aryan increases his mileage to 80 miles per week. (nearest hundredth of a minute)

   (A) 14.95   (B) 14.98   (C) 15.01   (D) 15.04   (E) 15.07

58. A survey asked a random sample of 500 California high school students to name their favorite college mascot. From the sample, 39% selected Willie the Wave from Pepperdine University. Construct a 90% confidence interval for the proportion of all California high school students who would select Willie the Wave as their favorite college mascot. (nearest ten-thousandth)

   (A) {.3511, .4229} (B) {.3521, .4239} (C) {.3531, .4249} (D) {.3541, .4259} (E) {.3551, .4269}

Score	1	2	3	4	5
Probability	.02	.06	.14	.33	.45

A large number of students from the Idaho Falls STEM Academy take the AP Electricity and Magnetism exam each spring. Based on results over the last ten years, the counselor created the table above. Define X to be the AP score on the AP Electricity and Magnetism exam for a randomly selected student. Use the table above for problems 59 and 60.

59. Compute the mean of the random variable X. (nearest hundredth)

   (A) 4.07   (B) 4.09   (C) 4.11   (D) 4.13   (E) 4.15

60. Compute the standard deviation of the random variable X. (nearest thousandth)

   (A) 0.985   (B) 0.989   (C) 0.993   (D) 0.997   (E) 1.001

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2024
Answer Key

1. B	21. D	41. C
2. D	22. E	42. B
3. A	23. C	43. A
4. C	24. D	44. E
5. E	25. B	45. B
6. C	26. C	46. B
7. A	27. D	47. C
8. E	28. C	48. D
9. E	29. C	49. C
10. C	30. B	50. E
11. B	31. C	51. C
12. B	32. C	52. D
13. A	33. C	53. C
14. A	34. E	54. A
15. C	35. D	55. A
16. D	36. C	56. B
17. E	37. D	57. A
18. A	38. A	58. D
19. D	39. B	59. D
20. C	40. A	60. D

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2024
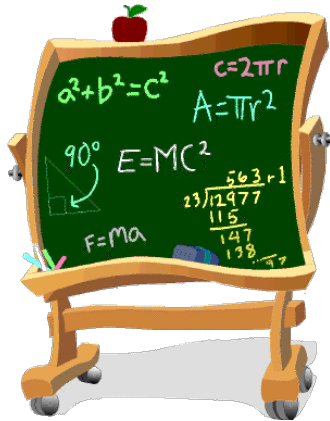
DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Denise drove to Lubbock and flew to Boise to spend some time with her cousins. A round-trip ticket cost $815. She rented a car at the Boise Airport for $37.95 per day plus $0.25 per mile. She stayed for 10 days and drove a total of 225 miles while she was in Boise. Find the total cost of her trip.

   (A) $1247.75   (B) $1250.75   (C) $1253.75   (D) $1256.75   (E) $1259.75

2. Julian is taking Calculus III this semester from Mrs. Edens. Sudan High School has a standard grading rubric. Forty percent of your semester grade is your homework average, 40% is your test average and 20% is your score on the semester exam. Julian has a 91 homework average. Julian's test scores are 93, 86, 84 and 91. An overall average of 90.00 or higher earns an A for the semester. What is the minimum score Julian can make on the final to earn an A?

   (A) 85   (B) 87   (C) 89   (D) 91   (E) 93

3-7. Consider $\triangle ABC$ shown on the right.

3. Point D is the midpoint of $\overline{BC}$. The y-intercept of $\overrightarrow{AD}$ is the point $(0, b)$. b = _____.

   (A) $-\dfrac{14}{13}$ (B) -1 (C) $-\dfrac{12}{13}$

   (D) $-\dfrac{11}{13}$ (E) $-\dfrac{10}{13}$
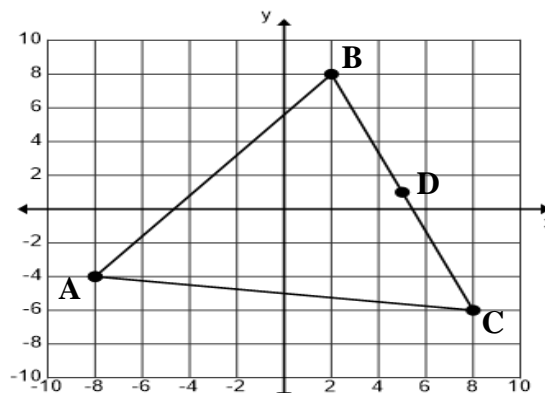
Problems 3, 4, 5, 6, 7

4. Find the perimeter of $\triangle ADC$. (nearest tenth)

   (A) 37.1   (B) 37.3   (C) 37.5   (D) 37.7   (E) 37.9

5. Find the area of $\triangle ABD$. (nearest whole number)

   (A) 45   (B) 47   (C) 49   (D) 51   (E) 53

6. Find the measure of $\angle ABC$. (nearest tenth)

   (A) 63.0°   (B) 63.3°   (C) 63.6°   (D) 63.9°   (E) 64.2°

7. Find the length of the longest median of $\triangle ADC$. (nearest tenth)

   (A) 14.4   (B) 14.6   (C) 14.8   (D) 15.0   (E) 15.2

8. Riley went for a bike ride on Saturday. First, Riley left Sudan and headed north on Hwy 303 for 12 miles. Next, Riley headed east on Hwy 70 for 6 miles. Then, Riley headed south on Hwy 1055 for 16 miles. Finally, Riley headed straight back to Sudan on Hwy 84. If Riley averaged 25 mph, how long did the bike ride take? (nearest minute)

   (A) 1 hr 33 min   (B) 1 hr 35 min   (C) 1 hr 37 min   (D) 1 hr 39 min   (E) 1 hr 41 min

UIL Mathematics

9. Donald can mow Oscar's property by himself in 7.5 hours. On Saturday, Donald started mowing at 8:00. At 10:00, Jaysean came to help Donald and together they finished mowing the property at 12:00. How long would it take Jaysean to mow Oscar's property by himself? (nearest minute)

   (A) 238 min   (B) 241 min   (C) 251 min   (D) 254 min   (E) 257 min

10. The equation of state of a hypothetical ideal gas is given by $PV = nRT$, where P is the pressure in pascals, V is the volume in cubic meters, n is the number of moles, R is a constant equal to 8.31446, and T is the temperature in kelvins of the gas. Laynee has 2.24 moles of a gas at 333 K. If it has a volume of 0.042 m³, what is the pressure of the gas. (nearest whole number)
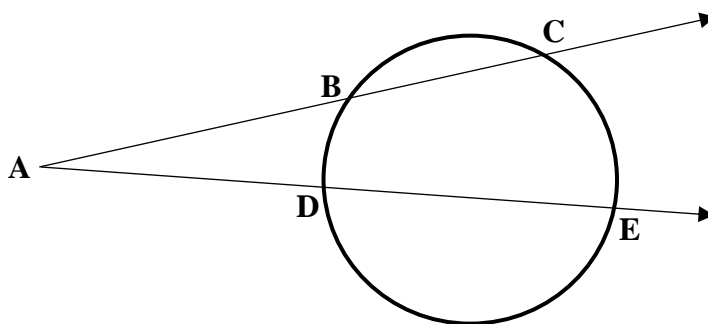
   (A) 147,659 Pa   (B) 147,662 Pa   (C) 147,665 Pa   (D) 147,668 Pa   (E) 147,671 Pa

11-12. Consider the circle shown with minor arcs **CE** and **BD** .

   Given: $mCE = 86°$, $mBD = 52°$, AB = 14,
   BC = 12, and AD = 13.

11. $m\angle CAE =$ _____. (nearest tenth)

   (A) 17.0°
   (B) 17.2°
   (C) 17.4°
   (D) 17.6°
   (E) 17.8°
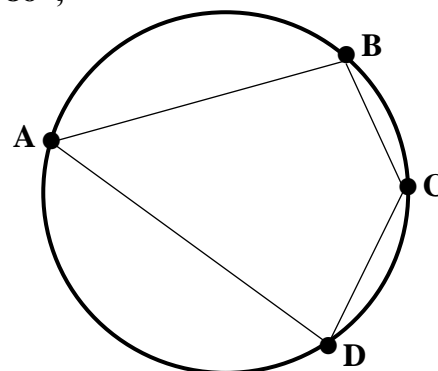
Problems 11, 12

12. DE = _____. (nearest tenth)

   (A) 14.6   (B) 14.8   (C) 15.0   (D) 15.2   (E) 15.4

13-14. Consider the circle shown with $m\angle DAB = 62°$, $m\angle ADC = 86°$,
   and minor arc **BC** with $mBC = 56°$.

13. $m\angle BCD =$ _____.

   (A) 116°
   (B) 117°
   (C) 118°
   (D) 119°
   (E) 120°

Problems 13, 14

14. The measure of minor arc **AD** is _____. (nearest whole number)

   (A) 114°   (B) 116°   (C) 118°   (D) 120°   (E) 122°

15. A cone has a volume of 2654.65 and a diameter of 26. Find the total surface area of the cone. (nearest whole number)

   (A) 1330   (B) 1333   (C) 1336   (D) 1339   (E)   1342

UIL Mathematics

16-17. Given: $\triangle ABC$ with $m\angle ABC = 90°$. Point D lies on $\overline{AC}$ such that $m\angle ADB = 90°$.
AD = 10 and CD = 18.

16. Find the perimeter of $\triangle ABC$. (nearest tenth)

(A) 65.6   (B) 66.0   (C) 66.4   (D) 66.8   (E) 67.2

17. Find the area of $\triangle BDC$. (nearest whole number)

(A) 119   (B) 121   (C) 123   (D) 125   (E) 127

18-19. Given: $\triangle DEF$ with DE = 16 and EF = 24. Point G lies on $\overline{DF}$ such that FG = 16 and $\overrightarrow{EG}$
bisects $\angle DEF$.

18. DG = _____ .

(A) $\dfrac{28}{3}$ (B) $\dfrac{29}{3}$ (C) 10   (D) $\dfrac{31}{3}$ (E) $\dfrac{32}{3}$

19. $m\angle GEF$ = _____ . (nearest tenth)

(A) 40.5°   (B) 40.8°   (C) 41.1°   (D) 41.4°   (E) 41.7°

20. The sum of the measures of the interior angles of an icosagon (20 sides) is _____ .

(A) 3240°   (B) 3312°   (C) 3384°   (D) 3456°   (E) 3600°

21-22. Given: The length of a side of a regular icosagon is 8. The icosagon is inscribed in a circle.

21. The area of the icosagon is _____ . (nearest tenth)

(A) 2012.0   (B) 2014.1   (C) 2016.2   (D) 2018.3   (E) 2020.4

22. The area of the circle is _____ . (nearest tenth)

(A) 2048.7   (B) 2050.8   (C) 2052.9   (D) 2054.0   (E) 2056.1

**23. A hawk is perched at the edge of the roof of the Three Rivers State bank. The hawk spots a mouse
at an angle of depression of 37° on the ground below. The mouse is located 100 feet from the base of
the bank. How tall is the Three Rivers State bank? (nearest inch)**

(A) 74 ft 2 in   (B) 74 ft 3 in   (C) 75 ft 4 in   (D) 76 ft 5 in   (E) 77 ft 6 in

24. Given: $f^{-1}(x) = \dfrac{7x+b}{-2x+3}$ and $f^{-1}(1) = 12$. Evaluate $f(-3)$.

(A) -15 (B) -14 (C) -13 (D) -12 (E) -11

UIL Mathematics

25. At the 2023 graduation ceremony for Rankin High School, 6 chairs were placed in the front row for the honor graduates. If Madison insisted on sitting next to Jaxon, how many different seating arrangements for the front row were possible?

    (A) 80   (B) 240   (C) 400   (D) 560   (E) 720

26. Consider the circle $x^2 + y^2 + ax + by + c = 0$. The center of the circle is the point $(-4, 6)$ and the diameter is 18. $a + b + c =$ _____.

    (A) −35   (B) −33   (C) −31   (D) −29   (E) −27

27. Find the number that is $\dfrac{3}{4}$ of the way from $-2\dfrac{1}{3}$ to $5\dfrac{5}{6}$.

    (A) $\dfrac{87}{24}$ (B) $\dfrac{11}{3}$ (C) $\dfrac{89}{24}$ (D) $\dfrac{15}{4}$ (E) $\dfrac{91}{24}$

28. The equation of the line that is equidistant from the points $(-5, 9)$ and $(7, -6)$ is $8x + by + c = 0$. $b + c =$ _____.

    (A) -7 (B) -5 (C) -3 (D) -1 (E) 1

29. Find the number of ways 10 charms can be arranged on a charm bracelet.

    (A) 40,320   (B) 181,440   (C) 362,880   (D) 1,814,400   (E) 3,628,800

30. Consider the parabola with a vertex at $(3, 4)$ and a focus at $(5, 4)$. If the point $(11, b)$ lies on the graph of the parabola and $b > 0$, then $b =$ _____.

    (A) 10   (B) 11   (C) 12   (D) 13   (E) 14

31-32. The vertices of an ellipse are $(2, -4)$ and $(10, -4)$. The length of the minor axis is 6.

31. The eccentricity of the ellipse is _____. (nearest hundredth)

    (A) 0.64   (B) 0.66   (C) 0.68   (D) 0.70   (E) 0.72

32. Find the distance from the center of the ellipse to the line $y = .75x + 6$. (nearest tenth)

    (A) 10.7   (B) 11.0   (C) 11.3   (D) 11.6   (E) 11.9

33. My clock shows that it is exactly 3:00. How long will I have to wait until both hands (hour hand and minute hand) of my clock point in the same direction for the second time? (nearest second)

    (A) 81 min 46 sec (B) 81 min 49 sec (C) 81 min 52 sec (D) 81 min 55 sec (E) 81 min 58 sec

34. **A population of Goliath Birdeater spiders is increasing exponentially in Gaines County. Anthony introduced a population of n Birdeaters at t = 0. At t = 60 days, the population reached 72 Birdeaters. At t = 150 days, the population reached 374 Birdeaters. Find n.**

    (A) 18   (B) 20   (C) 22   (D) 24   (E) 26

35. **A ball is dropped from a height of 200 feet. On each bounce, the ball rebounds three-fifths of the distance it fell. How far does the ball rebound on the 12th bounce? (nearest inch)**

    (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

36-37. The equation of a conic is $2x^2 + 6xy - y^2 + 4x + 6y + 12 = 0$.

36. **Classify the conic.**

    (A) circle   (B) parabola   (C) ellipse   (D) hyperbola   (E) line

37. **The angle of rotation is _____. (nearest tenth)**

    (A) 30.5°   (B) 30.8°   (C) 31.1°   (D) 31.4°   (E) 31.7°

38. **Three times the complement of angle A exceeds the supplement of angle A by 18°. Find the measure of angle A.**

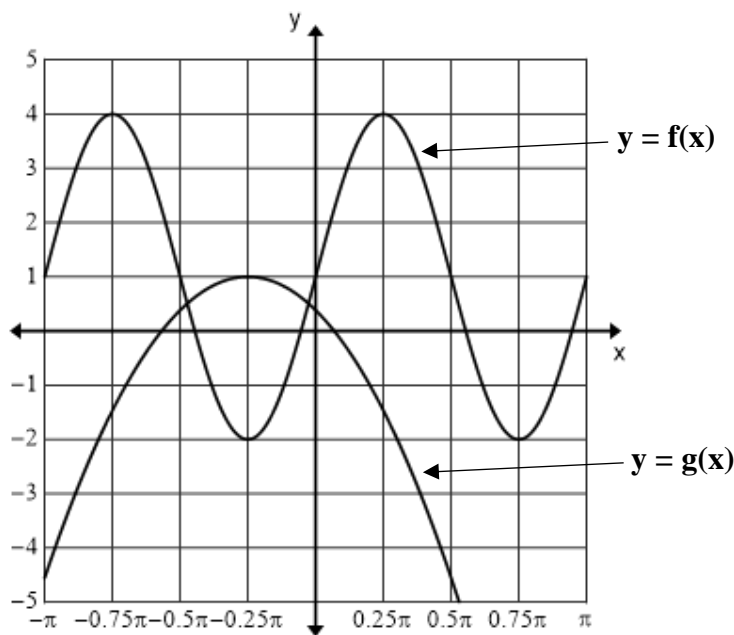   (A) 36°   (B) 38°   (C) 40°   (D) 42°   (E) 44°

39-43. Consider the graph shown on the right. The equation of $y = f(x)$ is of the form $f(x) = c + a\sin(bx)$.

39. $a + b + c =$ _____.

(A) 2   (B) 3
(C) 4   (D) 5
(E) 6

40. The graph of $g(x) = -\left(x + \dfrac{\pi}{4}\right)^2 + 1$

intersects the graph of f(x) at points E and F. EF = _____.
(nearest hundredth)

(A) 1.35   (B) 1.37
(C) 1.39   (D) 1.41
(E) 1.43

(Graph on right showing y = f(x) and y = g(x), with x-axis labeled at $-\pi$, -0.75π, -0.5π, -0.25π, 0.25π, 0.5π, 0.75π, π)

Problems 39, 40, 41, 42, 43

41. The x-intercept of the line tangent to the graph of $y = g(x)$ when $x = \dfrac{\pi}{6}$ is the point

(j, 0). j = _____. (nearest hundredth)

(A) 0.21   (B) 0.23   (C) 0.25   (D) 0.27   (E) 0.29

42. Find the area bounded by the graphs of $y = f(x)$ and $y = g(x)$. (nearest tenth)

(A) 2.5   (B) 2.7   (C) 2.9   (D) 3.1   (E) 3.3

43. Find the volume generated when the region bounded by the graphs of $y = f(x)$ and $y = g(x)$ is revolved about the line $x = -\pi$. (nearest whole number)

(A) 36   (B) 37   (C) 38   (D) 39   (E) 40

44. The derivative of the function f(x) is given by $f'(x) = -\dfrac{\pi}{x} \cdot \sin(0.5x - \pi)$. If f(x) is continuous for all positive real numbers, then the maximum value of $f(x)$ for $2\pi \le x \le 4\pi$ is _____.

(A) $f(2\pi)$ (B) $f(2.5\pi)$ (C) $f(3\pi)$ (D) $f(3.5\pi)$ (E) $f(4\pi)$

45. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x^2 + y$ with the initial condition $f(0) = 3$. Find the approximation for f(−1) obtained using Euler's method with two equal steps starting at x = 0. (nearest thousandth)

(A) 0.625   (B) 0.636   (C) 0.647   (D) 0.658   (E) 0.669

UIL Mathematics

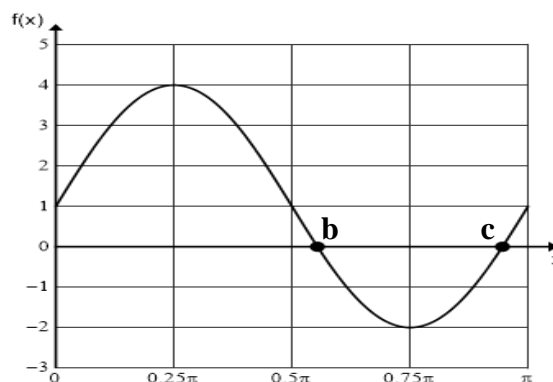46. Given: $\dfrac{dy}{dx} = 2x(3-y)$ and $y = -1$ when $x = 0$. Find the value of y when $x = 2$. (nearest tenth)

 (A) 2.1 (B) 2.3 (C) 2.5 (D) 2.7 (E) 2.9

47. Consider the graph of f(x) shown on the right.
 Arrange in order from least to greatest.

 I. $\displaystyle\int_0^b f(x)dx$ II. $\displaystyle\int_0^c f(x)dx$

 III. $\displaystyle\int_0^c |f(x)|dx$ IV. $\displaystyle\int_b^c f(x)dx$

Problem 47

 (A) I, II, III, IV (B) I, IV, III, II
 (C) IV, II, I ,III (D) IV, II, III, I
 (E) III, IV, I, II

48-49. Consider the polar curve $r = 3 + 3\sin(\theta)$.

48. Find the area of the region in the fourth quadrant bounded by the graph of this polar curve and the x-axis. (nearest tenth)

 (A) 1.4 (B) 1.6 (C) 1.8 (D) 2.0 (E) 2.2

49. The y-intercept of the line tangent to the graph of this polar curve at $\theta = \dfrac{7\pi}{4}$ is the point $(0, b)$.

 $b =$ _____. (nearest hundredth)

 (A) -0.45 (B) -0.42 (C) -0.39 (D) -0.36 (E) -0.33

50. The radius of a sphere is increasing at a rate of 9.2 inches per minute. What is the volume of the sphere at the instant that the volume is increasing at 462 cubic inches per minute? (nearest tenth)

 (A) 33.1 in^3 (B) 33.3 in^3 (C) 33.5 in^3 (D) 33.7 in^3 (E) 33.9 in^3

51. Find the radius of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(x+3)^n}{n \cdot 4^{n+1}}$.

 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

52. Ron just purchased a certified used car. The salesman told him that in the coming year, the probability of a transmission issue is 0.03, the probability of an alternator issue is 0.02, and the probability of a braking issue is 0.06. If these issues occur independently of each other, what is the probability that the car will not have one of these issues next year? (nearest thousandth)

 (A) 0.861 (B) 0.872 (C) 0.883 (D) 0.894 (E) 0.905

| Tire A | 44,000 | 48,500 | 52,200 | 39,000 | 47,800 | 50,600 | 49,700 |
| Tire B | 51,000 | 50,200 | 47,900 | 55,600 | 53,000 | 50,200 | 49,800 |

Ron claims that tires made by company B will outlast tires made by company A. Seven tires from each company were tested independently and the miles were recorded. Use the table above for problems 53-55.

53. Find the sum of the mean, median and mode of the Tire B results. (nearest whole number)

(A) 151,488   (B) 151,492   (C) 151,496   (D) 151,500   (E) 151,554

54. Find the IQR of the Tire A results.

(A) 6,400   (B) 6,500   (C) 6,600   (D) 6,700   (E) 6,800

55. Assuming that all conditions for inference are met, an appropriate test was conducted to see if company B tires do last longer than company A tires based on the data. The p-value of the test was _____. (nearest thousandth)

(A) 0.033   (B) 0.044   (C) 0.055   (D) 0.066   (E) 0.077

56-57. Assume that the distribution of the heights of high school senior boys in Allen High School is approximately normal with a mean of 70 inches and a standard deviation of 5.5 inches.

56. Mark is 5 ft 6 in tall. What percentile does that place Mark at?

(A) 17th   (B) 19th   (C) 21st   (D) 23rd   (E) 25th

57. What is the IQR for the heights of senior boys in Allen High School? (nearest hundredth)

(A) 7.30   (B) 7.33   (C) 7.36   (D) 7.39   (E) 7.42

| Pies | Apple | Cherry | Chocolate | Lemon | Blueberry |
| # of students | 26 | 28 | 24 | 12 | 10 |

Caroline believes that the students at Argyle High School like all five kinds of pies that are sold in the cafeteria equally. She randomly selected 100 students and asked them to identify their favorite type of pie. The results are in the table above. Use the table above for problems 58 and 59.

58. Caroline assumed that all conditions for inference were met and she conducted an appropriate test. The p-value of the test was _____. (nearest thousandth)

(A) 0.007   (B) 0.010   (C) 0.013   (D) 0.016   (E) 0.019

59. The blueberry cell contributed _____ to the chi-square statistic. (nearest hundredth)

(A) 4.22   (B) 4.44   (C) 4.66   (D) 4.88   (E) 5.00

60. Sarah rolls a fair die 10 times. What is the probability that she will get a 5 at least 3 times? (nearest thousandth)

(A) 0.217   (B) 0.221   (C) 0.225   (D) 0.229   (E) 0.232

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2024
Answer Key

1. B	21. E	41. C
2. D	22. D	42. B
3. C	23. C	43. E
4. D	24. B	44. A
5. E	25. B	45. A
6. A	26. B	46. E
7. B	27. E	47. C
8. D	28. C	48. B
9. E	29. B	49. D
10. C	30. C	50. C
11. A	31. B	51. D
12. C	32. D	52. D
13. C	33. B	53. D
14. D	34. D	54. C
15. E	35. B	55. B
16. E	36. D	56. D
17. B	37. E	57. E
18. E	38. A	58. A
19. A	39. E	59. E
20. A	40. D	60. C

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2024
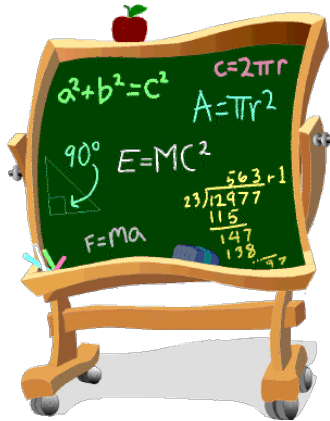
DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Tommy took Telena to the Trail Dust for her birthday. He ordered the 8-oz Center Cut Filet for $23.99 and iced tea for $2.15. She ordered the 16-oz Ribeye for $26.99 and iced tea for $2.15. They shared a slice of chocolate cake which cost $6.99. The tax rate was 8.125%. Tommy paid with four $20 bills and he told the waitress to keep the change as a tip. How much was her tip?

    (A) $12.34   (B) $12.45   (C) $12.56   (D) $12.67   (E) $12.78

2. Coach Hood persuaded the Canadian Education Foundation to purchase some new calculators for the High School and the Middle School UIL teams. A Swiss DM 32 calculator cost $249.00 and an HP Prime calculator cost $144.99. The foundation agreed to purchase a total of 40 calculators at a total cost of $8087.82. How many Swiss DM 32 calculators did they purchase?

    (A) 20   (B) 21   (C) 22   (D) 23   (E) 24

3. A round trip ticket on American Airlines from Dallas to Idaho Falls cost $840. Avis will rent a RAV4 for $57 per day. Arlene estimates that food and other expenses will cost $55 per day. She has budgeted $2500 and she wants to stay as long as possible. According to her projected costs, what is the maximum length of her stay in Idaho if she does not exceed her budget?

    (A) 10 days   (B) 12 days   (C) 14 days   (D) 16 days   (E) 18 days

4. Last summer, Anthony took a 4-day trip from Seminole to Astoria, Oregon. The total distance was 1785 miles and the total drive time was 27 hr 37 min. On day one, he traveled 525 miles at an average speed of 63 mph. On day two, he traveled 475 miles at an average speed of 67 mph. On day three, he traveled 425 miles at an average speed of 71 mph. What was his average speed on day four? (nearest whole number)

    (A) 58 mph   (B) 60 mph   (C) 62 mph   (D) 64 mph   (E) 66 mph

5. Melania left her home in Afton and cycled due north for 30 minutes at an average speed of 26 mph. Then she turned east and cycled for 45 minutes at an average speed of 24 mph. Next, she turned south and cycled for 75 minutes at an average speed of 22 mph. She stopped for lunch and calculated that she was _____ miles from home. (nearest tenth)

    (A) 23.1   (B) 23.4   (C) 23.7   (D) 24.0   (E) 24.3

6. Diezel can milk 8 cows in 50 minutes. Zeven can milk 12 cows in 70 minutes. Monday morning, Diezel began milking cows at The Afton Dairy at 5:00 AM. Zeven arrived at 6:45 AM and they worked together until all 97 cows at the dairy had been milked. What time was it when they finished? (nearest minute)

    (A) 10:35 AM   (B) 10:41 AM   (C) 10:47 AM   (D) 10:53 AM   (E) 10:59 AM

7. Consider the function $f(x) = \dfrac{7 - 3x}{9 - 4x}$. If $g(x)$ is the inverse function of $f(x)$, then $g(1) =$ _____.

(nearest tenth)

    (A) 1.4   (B) 1.6   (C) 1.8   (D) 2.0   (E) 2.2

8. Walt made a 176 on Test A, a 182 on Text B, a 184 on Test C, a 186 on Test D and a 198 on Test E. What score will he need to make on Test F to have an overall average of 190?

   (A) 212   (B) 214   (C) 216   (D) 218   (E) 220

9. Dylan has a small farm west of Brock where he raises frogs, hogs and dogs. The number of frogs is 7 more than 5 times the number of hogs. The number of hogs is 3 times the number of dogs. If there are a total of 121 creatures to take care of on the farm, how many frogs does Dylan have?

   (A) 91   (B) 93   (C) 95   (D) 97   (E) 99

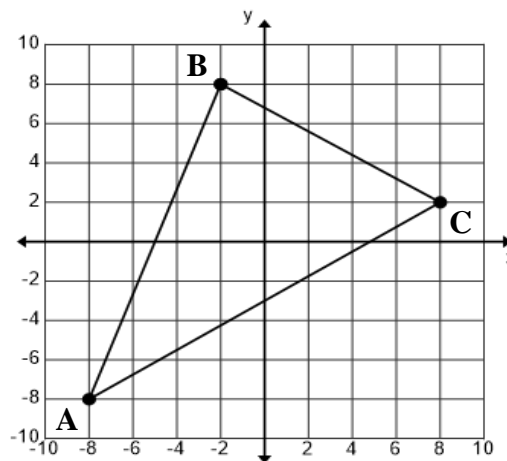**10.** The area of △ABC is _____. (nearest tenth)

   (A) 97.2   (B) 97.4   (C) 97.6
   (D) 97.8   (E) 98.0

11. The graph of $y = f(x)$ is the perpendicular bisector of $\overline{AB}$. $f(-21) =$ _____. (nearest tenth)

   (A) 5.8   (B) 6.0   (C) 6.2
   (D) 6.4   (E) 6.6

12. The length of the longest median of △ABC is _____. (nearest tenth)

   (A) 17.0   (B) 17.2   (C) 17.4
   (D) 17.6   (E) 17.8

Problems 10, 11, 12

13-14. Consider △DEF with DE = 12, EF = 9, and $m\angle DEF = 90°$. Point G lies on $\overline{DF}$ such that $m\angle EGF = 90°$.

13. DG = _____. (nearest tenth)

   (A) 7.2   (B) 7.8   (C) 8.4   (D) 9.0   (E) 9.6

14. The area of △EGF = _____. (nearest hundredth)

   (A) 19.22   (B) 19.33   (C) 19.44   (D) 19.55   (E) 19.66

15. Consider △HIJ with HI = 16, IJ = 25, and $m\angle HIJ = 56°$. Point K lies on $\overline{HJ}$ such that ray $\overrightarrow{IK}$ bisects $\angle HIJ$. If HK = 8.1265, then KJ = _____. (nearest tenth)

   (A) 12.1   (B) 12.3   (C) 12.5   (D) 12.7   (E) 12.9

16. If the area of regular hexagon ABCDEF is 62.3798, then AE = _____. (nearest tenth)

   (A) 7.7   (B) 7.9   (C) 8.1   (D) 8.3   (E) 8.5

UIL Mathematics

17. The total area of a pyramid with a square base is 464.49. If the diagonal of the base is 16.55, what is the volume of the pyramid? (nearest whole number)

   (A) 577   (B) 580   (C) 583   (D) 586   (E) 589

18. Consider an arithmetic sequence in which 5 times the third term equals the 13th term. If the 8th term is 60, what is the sum of the first 18 terms?

   (A) 1280   (B) 1284   (C) 1288   (D) 1292   (E) 1296

19. The frequency of a vibrating string, fixed at both ends, varies directly with the square root of the tension the string is under and inversely with the effective length of the string. A string that was initially under a tension of 60 N, with an effective length of 42 cm, produced sound with a frequency of 512 Hz. What will the frequency be if the tension is increased to 120 N and the effective length is reduced to 36 cm? (nearest whole number)

   (A) 833 Hz   (B) 836 Hz   (C) 839 Hz   (D) 842 Hz   (E) 845 Hz

20. A ball is dropped from a height of 18 feet. On each bounce, it rebounds two-thirds of the distance it fell. How far does the ball fall on its 10th fall? (nearest tenth)

   (A) 4.0 in   (B) 4.4 in   (C) 4.8 in   (D) 5.2 in   (E) 5.6 in

21. Two different samples of radioactive materials are decaying. Sample A has an initial mass of 3.22 g and has a half-life of 96 hours. Sample B has an initial mass of 6.86 g and has a half-life of 64 hours. Find the time required for the remaining amount in sample A to equal the remaining amount in sample B. (nearest tenth)

   (A) 207.3 hr   (B) 208.4 hr   (C) 209.5 hr   (D) 210.6 hr   (E) 211.7 hr

22. Jacob left port at 1:00 PM and sailed due east at 18 mph. Benjamin left port at 2:00 PM and sailed at 30° south of west at 15 mph. How far apart are they at 6:00 PM? (nearest whole number)

   (A) 143 mi   (B) 145 mi   (C) 147 mi   (D) 149 mi   (E) 151 mi

23-24. The swimming pool at Teresa's house has a pipe that can fill the pool in 72 hours. The drain can empty the pool in 90 hours. The pool is shaped like a regular pentagon with each side 24 feet long and it has a constant depth of 4 feet. The pool was empty at t = 0.

23. The fill pipe was turned on at t = 0. The drain was accidentally opened 12 hours later. The pool was finally full at t = _____ hours. (nearest whole number)

   (A) 296   (B) 300   (C) 304   (D) 308   (E) 312

24. When the pool is completely full, how many gallons of water does it hold? (nearest whole number)

   (A) 29,653   (B) 29,657   (C) 29,661   (D) 29,665   (E) 29,669

25. $\dfrac{36(\cos 150° + i\sin 150°)}{9(\cos 30° + i\sin 30°)} = $ _____ .

   (A) $-2 - 2\sqrt{3}\,i$ (B) $-2\sqrt{3} + 2i$ (C) $-2 + 2\sqrt{3}\,i$ (D) $-2\sqrt{3} - 2i$ (E) $2 + 2\sqrt{3}\,i$

26. Grandpa told Michael that he would pay for his first year of graduate school at A&M. The current estimated cost for one year is $29,266. Michael anticipates entering graduate school in 4 years. How much will Grandpa need to place in an account that earns 6% annual interest compounded monthly to pay for Michael's first year if the estimated cost does not increase? (nearest dollar)

   (A) $23,002   (B) $23,013   (C) $23,024   (D) $23,035   (E) $23,046

27. Find the acute angle formed by the two intersecting lines shown on the right. (nearest tenth)

   (A) 61.3°   (B) 61.6°   (C) 61.9°
   (D) 62.2°   (E) 62.5°

28. The two lines are the asymptotes of a hyperbola. The equation of the hyperbola is of the form $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$. Given: a and b are integers with $a < b < 9$. One of the foci is the point $(h, k+c)$. $a + b + c = $ _____ . (nearest tenth)

   (A) 13.0   (B) 13.2   (C) 13.4
   (D) 13.6   (E) 13.8
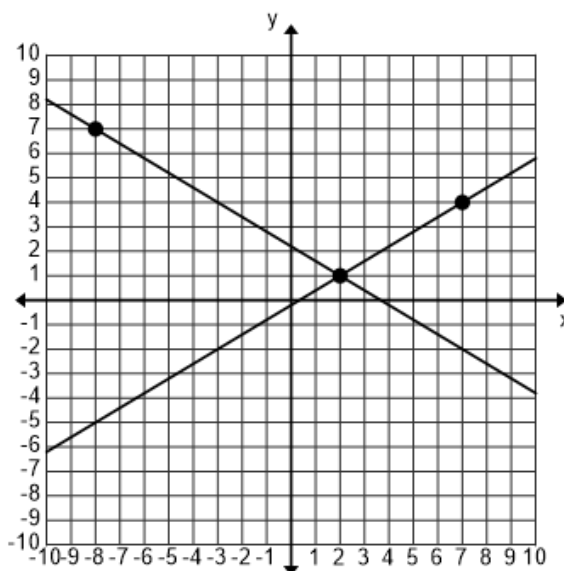
Problems 27, 28

29. If the area of the circle $x^2 + y^2 - 12x + 4y + f = 0$ is 49π, then f = _____ .

   (A) -15 (B) -12 (C) -9 (D) -6 (E) -3

30. Three of the zeros of $f(x) = x^4 + bx^3 + cx^2 + dx + h$ are $-4, 3,$ and $1 + \sqrt{7}$. If b, c, d, and h are integers, then $f(4) = $ _____ .

   (A) 16   (B) 18   (C) 20   (D) 22   (E) 24

31. Russell shoots free throws every day after practice. On Tuesday, after 15 minutes, he had made only 60% of his free throws. At this point, Becci came into the gym and Russell began to focus. He got on a hot streak and made 40 free throws in a row. If this increased his free throw percentage for the day to 70%, how many free throws did he attempt on Tuesday?

   (A) 154   (B) 156   (C) 158   (D) 160   (E) 162

32. If $f(x) = 2x^2 - 3$ and $h(x) = x^2 \div 25$, then $\left(h \circ f\right)(-3) = $ _____ .

   (A) 3   (B) 6   (C) 9   (D) 12   (E) 15

UIL Mathematics

33. The angle of elevation from a mouse on the ground to a hawk on the edge of the roof of the Canadian State Bank was $36°$. The mouse cautiously moved toward the bank and 80 seconds later, the angle of elevation had changed to $48°$. If the height of the bank building is 126 feet, at what rate did the mouse move toward the bank? (nearest tenth)

    (A) 8.8 in/s   (B) 9.0 in/s   (C) 9.4 in/s   (D) 9.6 in/s   (E) 9.8 in/s

34. A parabola is concave down with the point $(-3, -2)$ as the vertex. The point $(3, -11)$ lies on the parabola. If the point $(-6, b)$ also lies on the parabola, then b = _____. (nearest hundredth)

    (A) −5.00   (B) −4.75   (C) −4.50   (D) −4.25   (E) −4.00

35-36. Consider $\triangle ABC$ with vertices $A(-4, 3, 5)$, $B(2, -5, 3)$, and $C(7, -6, 8)$.

35. $m\angle BAC =$ _____. (nearest tenth)

    (A) 26.8°   (B) 27.0°   (C) 27.2°   (D) 27.4°   (E) 27.6°

36. Find the area of $\triangle ABC$. (nearest tenth)

    (A) 33.6   (B) 33.9   (C) 34.2   (D) 34.5   (E) 34.8

37. The point in rectangular coordinates, $(a, 5)$, is on the polar graph $r^2 \sin(2\theta) = 24$. a = _____. (nearest tenth)

    (A) 2.0   (B) 2.1   (C) 2.2   (D) 2.3   (E) 2.4

38. Consider the unit circle with an angle θ in quadrant III such that $\cos\theta = -0.5$ and with an angle α also in quadrant III such that $\sin\alpha = -0.5$. $\cos(\theta - \alpha) =$ _____.

    (A) $-\dfrac{\sqrt{3}}{2}$ (B) $-\dfrac{1}{2}$ (C) 0   (D) $\dfrac{1}{2}$ (E) $\dfrac{\sqrt{3}}{2}$

39-40. Consider the conic $5x^2 + 4xy + 3y^2 - 32 = 0$.

39. The graph of the conic is _____.

    (A) an ellipse   (B) a parabola   (C) a hyperbola   (D) a line   (E) 2 parallel lines

40. The angle of rotation of the graph of the conic is _____. (nearest tenth) (nearest tenth)

    (A) 30.9°   (B) 31.1°   (C) 31.3°   (D) 31.5°   (E) 31.7°

41. The slope of the line tangent to the graph of
$y = f(x)$ at $x = -2$ is _____.
(nearest tenth)

(A) 2.0   (B) 2.1   (C) 2.2

(D) 2.3   (E) 2.4

42. Find the area bounded by the graphs of
$y = f(x)$ and $y = g(x)$. (nearest tenth)

(A) 28.0   (B) 28.2   (C) 28.4
(D) 28.6   (E) 28.8

43. Find the volume of the solid generated when
the region bounded by the graphs of
$y = f(x)$ and $y = g(x)$ is revolved about the
line $y = -8$. (nearest whole number)

(A) 1744   (B) 1748   (C) 1752   (D) 1756   (E) 1760

Problems 41, 42, 43, 44

y = f(x)

y = g(x)

44. What is the arc length of $y = f(x)$ between $x = -3$ and $x = 1$? (nearest hundredth)

(A) 12.73   (B) 12.76   (C) 12.79   (D) 12.82   (E) 12.85

45. Given: $\dfrac{dy}{dx} = 4 - e^{-x} - y$ and $y(0) = 1$. Use Euler's method with a step size of h = 0.1 to approximate
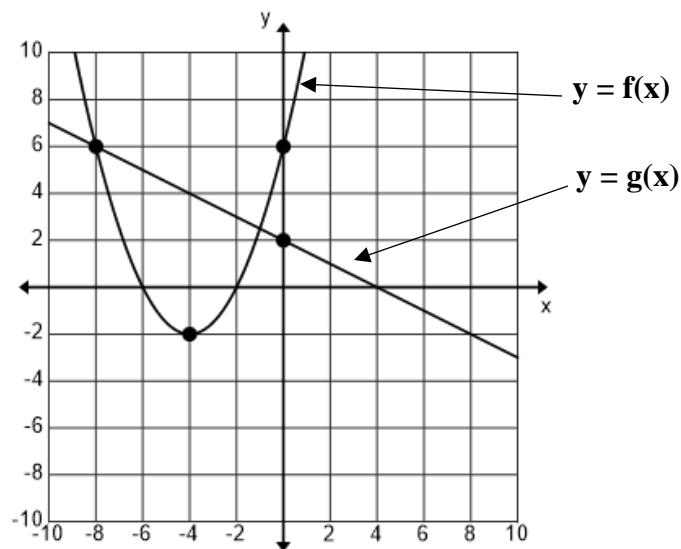the value of $y(0.2)$. (nearest thousandth)

(A) 1.386   (B) 1.388   (C) 1.390   (D) 1.392   (E) 1.394

46. Given: $F(x) = \displaystyle\int_{0}^{3x} \cos(2t)dt$. $F'(2) =$ _____. (nearest hundredth)

(A) 2.53   (B) 2.55   (C) 2.57   (D) 2.59   (E) 2.61

47. Find the average value of $f(x) = 2\sin(3x) - 0.1e^{0.2x}$ over the interval $[0, 12]$. (nearest thousandth)

(A) −0.366   (B) −0.355   (C) −0.344   (D) −0.333   (E) −0.322

UIL Mathematics

48-49. Suppose that the quail population in Hemphill County was essentially zero in 2019. Suppose also that the Panhandle Conservation Club released 100 quail into Hemphill County on March 1, 2020. On March 1, 2023, the population reached 180 quail. Professors from TAMU in Canyon estimate that Hemphill County can sustain no more than 500 quail.

48. Find a logistic differential equation that models the rate of change of the quail population. The greatest rate of increase of the quail population according to the model is _____ quail per year. (nearest whole number)

    (A) 28   **(B) 30**   **(C) 32**   **(D) 34**   **(E) 36**

49. Find a general solution to this logistic differential equation that models the quail population at any time t. This model predicts that the quail population on March 1, 2040 will be _____ quail. (nearest whole number)

    (A) 483   **(B) 485**   **(C) 487**   **(D) 489**   **(E) 491**

50. Find the interval of convergence of $\sum_{n=0}^{\infty} \dfrac{(-1)^n (x+2)^n}{3^n}$.

    (A) $(-2, 2)$ **(B) $[-5, 1]$**   **(C) $(-1, 5)$**   **(D) $(-5, 1)$**   **(E) $(-\infty, \infty)$**

51-52. The continuous function f shown on the right is defined for $-10 \le x \le 10$. Let h be the function defined by $h(x) = \displaystyle\int_{-10}^{x} f(t)\,dt$.

51. Find $h(10) = $ _____. (nearest whole number)

    (A) 89   **(B) 91**   **(C) 93**
    (D) 95   **(E) 97**

Problems 51, 52

52. The graph of h is concave down over the interval (a, b). $a + b = $ _____.

    (A) -14   **(B) -12**   **(C) -10**   **(D) -8**   **(E) -6**

53. The OEA reported in January that the mean SAT score for the 2024 seniors at Mac High was 980 with a standard deviation of 120. In March, the OEA said an error had been found in the scoring and the corrected scores could be found by adding 20 points to your original score and then multiplying by 1.1. If Pistol's corrected score was 1280, what percentile does that put her in?

    (A) 91st   **(B) 93rd**   **(C) 95th**   **(D) 97th**   **(E) 99th**
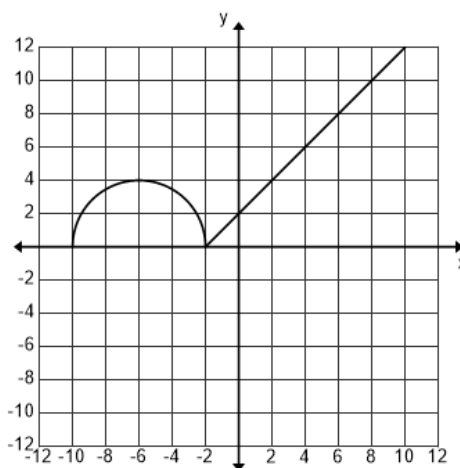
54-55. Assume that the mean height of the trees in the California Redwoods National Park is 380 feet with a standard deviation of 20 feet.

54. What proportion of the trees are over 400 feet tall? (nearest hundredth)

(A) 0.16   (B) 0.18   (C) 0.20   (D) 0.22   (E) 0.24

55. If a group of 12 trees is randomly selected, what is the probability that at least 4 of the trees will be taller than 400 feet tall? (nearest hundredth)

(A) 0.11   (B) 0.13   (C) 0.15   (D) 0.17   (E) 0.19

56. Are students at A&M more fit than the students at UT? A random sample of 180 students at A&M found that 122 of them exercise regularly. A random sample of 200 students at UT found that 114 of them exercise regularly. Calculate a 96% confidence interval for the difference between the proportions of students at the two universities who exercise regularly.

(A) {.0046, .2074}   (B) {.0052, .2080}   (C) {.0058, .2086}   (D) {.0064, .2092}   (E) {.0070, .2098}

57. Suppose 62% of the adult men over 40 are overweight. Of these, 36% are on an exercise program. Of the adult men over 40 who are not overweight, 44% are on an exercise program. Given that an adult man over 40 is on an exercise program, what is the probability that he is overweight? (nearest hundredth)

(A) 0.51   (B) 0.54   (C) 0.57   (D) 0.60   (E) 0.63

58. Randy and Tommy are retired buddies who play golf together 3 days a week. The distribution of Randy's drives off the tee is approximately normal with a mean of 260 yd and a standard deviation of 8 yd. The distribution of Tommy's drives off the tee is also normal, with a mean of 280 yd and a standard deviation of 12 yd. What is the probability that Randy will drive the ball off the tee farther than Tommy on a randomly selected hole on the golf course? (nearest hundredth)

(A) 0.08   (B) 0.11   (C) 0.14   (D) 0.17   (E) 0.20

# of tests	3	6	9	12	15	18
Meet Score	112	133	152	176	193	215

59-60. Mr. Newberry has a student who is very talented in Number Sense, but has never practiced very much. Mr. Newberry convinced him to increase the number of practice tests he takes each week by 3 after each meet. The results of the first 6 meets are in the table above. Mr. Newberry plotted the data in the table and calculated a LSRL for the data.

59. Find the value of the residual for the week the student takes 12 practice tests. (nearest tenth)

(A) 1.4   (B) 1.6   (C) 1.8   (D) 2.0   (E) 2.2

60. Mr. Newberry convinces the student to take 30 practice tests the week of the state meet. Use the LSRL to predict the student's score at the state meet. (nearest whole number)

(A) 291   (B) 293   (C) 295   (D) 297   (E) 299

DO NOT DISTRIBUTE BEFORE OR DURING THE CONTEST

University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2024
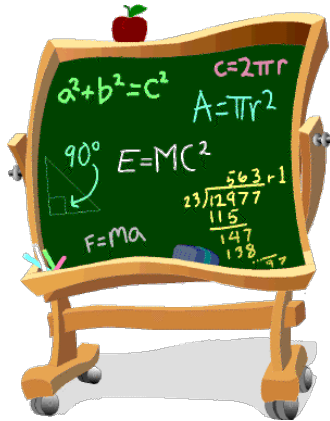### Answer Key

1. D	21. C	41. A
2. C	22. B	42. D
3. C	23. B	43. E
4. A	24. A	44. B
5. A	25. C	45. C
6. C	26. D	46. A
7. D	27. C	47. B
8. B	28. E	48. D
9. D	29. C	49. E
10. E	30. A	50. D
11. B	31. D	51. E
12. A	32. C	52. D
13. E	33. B	53. A
14. C	34. D	54. A
15. D	35. B	55. A
16. E	36. A	56. D
17. B	37. E	57. C
18. E	38. E	58. A
19. E	39. A	59. E
20. E	40. E	60. D

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2024

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. The rental fee from AAA Car Rentals includes a charge for each day the car is rented as well as a charge for each mile the car is driven. Tom rented a car for four days, drove 177 miles, and his bill was $177.56. Diane rented a car for six days, drove 208 miles, and her bill was $250.24. Find the charge for each mile driven.

   (A) $0.24   (B) $0.26   (C) $0.28   (D) $0.30   (E) $0.32

2-5. Given points $A(-4,2)$, $B(6,8)$, $C(2,-4)$, $D(a,b)$ and $E(5,c)$.

2. If $\overleftrightarrow{AB} \perp \overrightarrow{CD}$ and $a = -10$, then $b =$ _____ .

   (A) 13   (B) 14   (C) 15   (D) 16   (E) 17

3. Find the perimeter of $\triangle ABC$. (nearest tenth)

   (A) 32.8   (B) 33.0   (C) 33.2   (D) 33.4   (E) 33.6

4. If $\overrightarrow{AB}$ is parallel to $\overrightarrow{CE}$, then $c =$ _____. (nearest tenth)

   (A) −2.2   (B) −2.0   (C) −1.8   (D) −1.6   (E) −1.4

5. If F is the midpoint of $\overline{AB}$ and G is the midpoint of $\overline{BC}$, then $FG =$ _____. (nearest tenth)

   (A) 3.6   (B) 3.8   (C) 4.0   (D) 4.2   (E) 4.4

6. Anthony left Seminole on his bike and headed west at 24 mph. After 90 minutes, he turned and headed north at 25 mph for two hours. Then he turned and headed east at 30 mph for 40 minutes. Then he stopped for lunch and calculated that he was _____ miles from Seminole. (nearest tenth)

   (A) 51.7   (B) 51.9   (C) 52.1   (D) 52.3   (E) 52.5

7. The frequency of a vibrating string fixed at both ends varies directly as the square root of the tension the string is under and inversely as the length of the string. When the tension is 80 N and the length is 75 cm, the frequency is 512 Hz. What is the frequency if the tension is increased to 96 N and the length is decreased to 60 cm? (nearest whole number)

   (A) 689 Hz   (B) 692 Hz   (C) 695 Hz   (D) 698 Hz   (E) 701 Hz

8. The UIL Math Booster Club held a fundraiser to support the Salado math team. They flew in the Quebe Sisters to perform for the community. Adult tickets cost $35 and student tickets cost $20. If they sold 1110 tickets and raised $34,020, how many adult tickets were sold?

   (A) 786   (B) 788   (C) 790   (D) 792   (E) 794

9. Grades for Mr. Cabaniss's calculus class are based on six tests. Grades on tests 1 through 5 count once and the grade on test 6 counts twice. Jason's grades on the first 5 tests are 89, 93, 91, 84 and 85. What is the lowest score he can make on test 6 and still have an average of 90.0 or higher?

   (A) 91   (B) 92   (C) 93   (D) 94   (E) 95

10. **Austin is 218 miles from Argyle. At 9:00 AM, a van leaves Argyle heading toward Austin traveling at 55 mph. At 9:45 AM, a second van leaves Argyle traveling at 75 mph heading toward Austin. How far are the vans from Austin when the second van catches the first van? (nearest whole number)**

    (A) 63 mi   (B) 65 mi   (C) 67 mi   (D) 69 mi   (E) 71 mi

11-12. **Dennis's pool is rectangular in shape with a length of 24 feet, a width of 18 feet, and a constant depth of 4 feet. He uses two pipes to fill the pool. Each pipe can fill the pool by itself in 48 hours. The drain can empty the pool in 60 hours. The first pipe is turned on at noon on Monday. At 6:00 PM on Monday, the second pipe is turned on. At 10:00 PM on Monday, the drain is accidentally opened.**

11. **How many gallons of water does the pool contain when full? (nearest whole number)**

    (A) 12,926   (B) 12,930   (C) 12,934   (D) 12,938   (E) 12,942

12. **What is the earliest time on Wednesday that the pool will be full?**

    (A) 2:00 AM   (B) 2:10 AM   (C) 2:20 AM   (D) 2:30 AM   (E) 2:40 AM

13-14. **Consider a circle with points A, B, C and D on the circle. Point O is the center of the circle. $\overline{AC}$ intersects $\overline{BD}$ at point O. AB = 8 and AC = 16.**

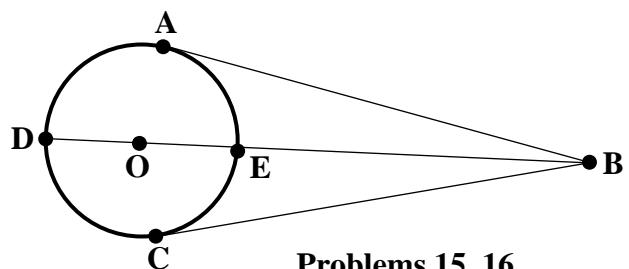13. **Find the area of ∆BOA. (nearest tenth)**

    (A) 26.9   (B) 27.1   (C) 27.3   (D) 27.5   (E) 27.7

14. **Find the arc length of minor arc AD. (nearest tenth)**

    (A) 16.2   (B) 16.4   (C) 16.6   (D) 16.8   (E) 17.0

15-16. **Point O is the center of the circle. DO = 12 and DB = 60. $\overline{BA}$ is tangent at point A and $\overline{BC}$ is tangent at point C.**

15. **Find the area of ∆DCB. (nearest whole number)**

    (A) 337   (B) 340   (C) 343
    (D) 346   (E) 349

Problems 15, 16

16. **Find the area of the region inside ∆AOB, but outside sector AOE. (nearest whole number)**

   (A) 181   (B) 184   (C) 187   (D) 190   (E) 193

17. **The measures of the angles of a triangle are in the extended ratio 4:5:6. Find the measure of the smallest angle.**

    (A) 40°   (B) 44°   (C) 48°   (D) 52°   (E) 56°

UIL Mathematics

18. Consider a right circular cylinder with a volume of 2714 cm³. If the diameter of each base is 12 cm, then the total surface area of the cylinder is _____ cm². (nearest whole number)

    (A) 1131   (B) 1134   (C) 1137   (D) 1140   (E) 1143

19. Consider a regular hexagon with an inscribed circle and a circumscribed circle. If the area of the inscribed circle is 763 cm², then the area of the circumscribed circle is _____ cm². (nearest whole number)

    (A) 1008   (B) 1011   (C) 1014   (D) 1017   (E) 1020

20. If 7 is a zero of $3x^3 - kx^2 - 99x + 105 = 0$, then k = _____.

    (A) 6   (B) 9   (C) 12   (D) 15   (E) 18

21. A boat leaves port at 1:00 PM and travels on a bearing of 236° at 24 knots. At 2:00 PM, a second boat leaves port and travels on a bearing of 24° at 20 knots. How far apart will the boats be at 6:00 PM? (1 knot = 1.15 mph) (nearest whole number)

    (A) 215 mi   (B) 218 mi   (C) 221 mi   (D) 224 mi   (E) 227 mi

22. Joe placed $110,000 into an account at a bank in Brock that earns 5.06% annual interest compounded quarterly. On the same day, Joe placed $110,000 into an account at a bank in Weatherford that earns 4.94% annual interest compounded monthly. At the end of 3 years, how much more money does he have in his account in Brock than he has in his account in Weatherford? (nearest dollar)

    (A) $377   (B) $379   (C) $381   (D) $383   (E) $385

23-24. The time on my grandpa's old clock is exactly 6:09 PM.

23. The obtuse angle between the hour hand and the minute hand is _____°. (nearest tenth)

    (A) 129.4   (B) 130.5   (C) 131.6   (D) 132.7   (E) 133.8

24. How many minutes until the hour hand and the minute hand align for the second time? (nearest hundredth)

    (A) 86.45 min   (B) 87.36 min   (C) 88.27 min   (D) 89.18 min   (E) 90.09 min

25. If $f(x) = \dfrac{4x-5}{6x+7}$ and $h(x) = \dfrac{2-8x}{5+4x}$, then $\left(h^{-1} \circ f^{-1}\right)(1) =$ _____.

    (A) -3 (B) -2 (C) -1 (D) 1   (E) 2

26. The elk population in Caribou County has been doubling every 6 years. If there were 112 elk in 2004, in what year should the number of elk reach 7168 if the population continues to double every 6 years?

    (A) 2038   (B) 2040   (C) 2042   (D) 2044   (E) 2046

UIL Mathematics

27. Find the sum of the first ten terms of the sequence $24, 18, 13\frac{1}{2}, 10\frac{1}{8}, 7\frac{19}{32}, ...$ (nearest tenth)

   (A) 90.0   (B) 90.2   (C) 90.4   (D) 90.6   (E) 90.8

28. A hungry hawk that is perched on top of a 78-ft-tall pole spots a mouse on the ground. The angle of depression from the hawk to the mouse is 20°. The mouse begins moving directly toward the pole at 1.00 ft/s. Exactly 80 seconds later, the mouse stops. What is the angle of depression from the hawk to the mouse at this point? (nearest tenth)

   (A) 30.1°   (B) 30.4°   (C) 30.7°   (D) 31.0°   (E) 31.3°

29. Given: The equation of an ellipse is $\frac{(x-3)^2}{a^2} + \frac{(y+2)^2}{b^2} = 1$, $a+b = 20$, a and b are integers. If the area of the ellipse is 311, then the eccentricity of the ellipse is _____. (nearest thousandth)

   (A) 0.575   (B) 0.586   (C) 0.597   (D) 0.608   (E) 0.619

30. The graph of $x^2 - 6xy + 25y^2 + 4x - 12 = 0$ is a/an _____.

   (A) ellipse   (B) hyperbola   (C) parabola   (D) circle   (E) line

31. Assume that the life expectancy for men living in American Falls is 77.5 years and for women it is 83.6 years. If the total average is 80.9 years, what is the number of women divided by the number of men? (nearest hundredth)

   (A) 1.20   (B) 1.23   (C) 1.26   (D) 1.29   (E) 1.32

32. Consider the function $f(x) = 6 - 4\csc 2\left(x - \frac{\pi}{4}\right)$. Which of the following are true?

   I. The range of $f(x)$ is $(-\infty, 2] \cup [10, \infty)$.

   II. The domain of f(x) is all $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$, where n is an integer.

   III. The period of f(x) is 2π.
   IV. The graph of f(x) is symmetric about the y-axis.

   (A) II, III only   (B) I, IV only   (C) I, II, IV only
   (D) I, II, III only   (E) I, II, III, IV

33. Assume that the temperature on February 16th at Steve's cabin near Anchorage varies sinusoidally with a low of 15° at 6:00 AM and a high of 27° at 6:00 PM. The temperature is equal to or above 18° on February 16th for _____ hours. (nearest tenth)

   (A) 15.4   (B) 15.6   (C) 15.8   (D) 16.0   (E) 16.2

UIL Mathematics

34. Consider the graph of the ellipse represented by the parametric equations $x = 4\cos\theta - 6$ and $y = 2\sin\theta + 4$. The distance between the foci is _____. (nearest tenth)

    (A) 6.9   (B) 7.1   (C) 7.3   (D) 7.5   (E) 7.7

35. The graph of the polar equation $r = 6 - 6\sin\theta$ is a _____.

    (A) circle   (B) lemniscate   (C) cardioid
    (D) rose curve with 6 petals (E) rose curve with 12 petals

36. Consider the vectors $\mathbf{u} = \langle 4, n, -3 \rangle$ and $\mathbf{v} = \langle -2, n+1, -5 \rangle$. If the angle between the vectors is 44.735°, then n = _____. (n > 0) (nearest whole number)

    (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

37. Montana Institute of Technology admitted 240 freshmen last fall. Of those, 86 took E&M, 90 took DE, 108 took Cal III, 18 took E&M and DE but not Cal III, 16 took DE and Cal III but not E&M, 12 took all 3, and 34 did not take any of these courses. How many took E&M, but not DE or Cal III?

    (A) 32   (B) 34   (C) 36   (D) 38   (E) 40

38. Carter invented a new card game consisting of 64 cards. There are 15 red cards, 15 green cards, 15 blue cards, 15 yellow cards, and 4 wild cards. Each player is randomly dealt a 6-card hand. What is the probability that a hand will contain exactly 2 wild cards? (nearest thousandth)

    (A) 0.036   (B) 0.039   (C) 0.042   (D) 0.045   (E) 0.048

39-41. Consider the parabola shown on the right and the function $f(x) = x - 3$.

39. The equation of the directrix is x = _____.

    (A) 1   (B) 2   (C) 3
    (D) 3.5   (E) 3.75

40. If the point $(a, 11)$ lies on the graph of the parabola, then a = _____.

    (A) 28   (B) 30   (C) 32
    (D) 34   (E) 36

Problems 39, 40, 41

41. The area bounded by the graph of the parabola and the graph of $y = f(x)$ is _____. (nearest tenth)

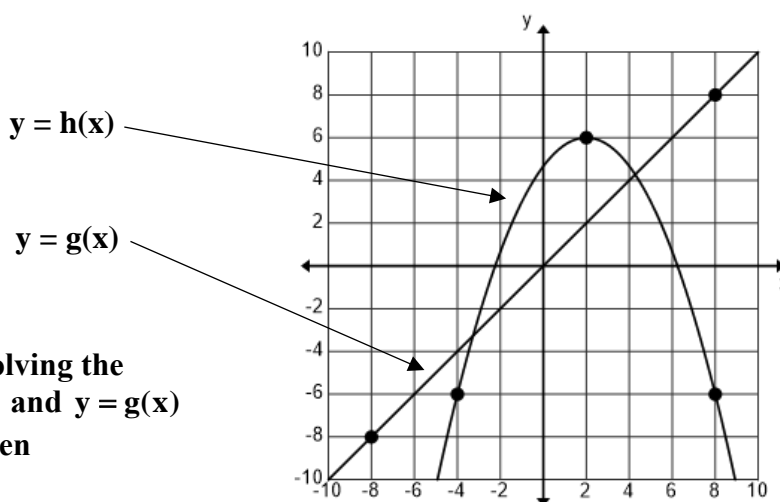   (A) 7.3   (B) 7.5   (C) 7.7   (D) 7.9   (E) 8.1

UIL Mathematics
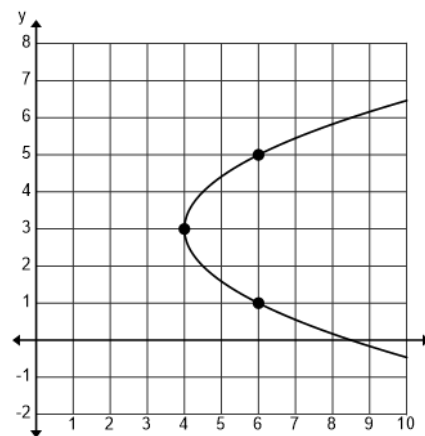
42. The line tangent to $y = h(x)$ has a slope of –1.6 when x = _____. (nearest tenth)

(A) 4.4
(B) 4.6
(C) 4.8
(D) 5.0
(E) 5.2

$y = h(x)$

$y = g(x)$

43. The volume of the solid generated by revolving the region bounded by the graphs of $y = h(x)$ and $y = g(x)$ about the line y = b is 961.39. If $b < 0$, then b = _____. (nearest whole number)

(A) – 8   (B) – 7   (C) – 6
(D) – 5   (E) – 4

Problems 42, 43

44. Rancher Rob is designing a jogging track on his property. It will consist of a rectangle with a semicircle on each end. The perimeter of the track will be 600 meters. If Rob wants to maximize the rectangular area, what is this maximum value? (nearest whole number)

(A) 14,308 m²   (B) 14,312 m²   (C) 14,316 m²   (D) 14,320 m²   (E) 14,324 m²

45. Let $f(x) = \dfrac{3}{8}x^3 + 2x - 1$. Find the value of $\left(f^{-1}\right)'(6)$. (nearest thousandth)

(A) 0.154   (B) 0.165   (C) 0.176   (D) 0.187   (E) 0.198

46-47. In April, researchers from Texas A&M found that the Lake Marvin trout population was zero. They believe the lake can support no more than 5000 trout. On May 1ˢᵗ, 1000 trout were released into the lake. They believe that the rate of increase of the population of trout after t weeks can be modeled by the logistic differential equation $\dfrac{dP}{dt} = 0.00003P(5000 - P)$.

46. How long after May 1ˢᵗ will the trout population be growing the fastest? (nearest tenth)

(A) 8.8 weeks   (B) 9.0 weeks   (C) 9.2 weeks   (D) 9.4 weeks   (E) 9.6 weeks

47. The model derived from solving the differential equation predicts the trout population will reach _____ trout 25 weeks after the lake was stocked. (nearest whole number)

(A) 4549   (B) 4556   (C) 4563   (D) 4570   (E) 4577

UIL Mathematics

x	2	6	11	14	18
f(x)	3	7	9	10	11

48. Use the values in the table above to approximate the value of $\int_{2}^{18} f(x)dx$ using the trapezoidal

   approximation method with four subintervals. (nearest tenth)

   (A) 129.4   (B) 130.5   (C) 131.6   (D) 132.7   (E) 133.8

49. Consider the region M in the first quadrant bounded by the graph of $y_1 = -x + 8$, $y_2 = x \cdot \sin(.25x)$,
   and the y-axis. The vertical line $x = c$ divides M into two regions of equal area. $c =$ _____.
   (radians) (nearest hundredth)

   (A) 1.32   (B) 1.34   (C) 1.36   (D) 1.38   (E) 1.40

50-51. Consider the graph of a rose curve given by $r = 4\sin(3\theta)$.

50. Find the area of one leaf of the rose curve. (nearest tenth)

   (A) 3.6   (B) 3.8   (C) 4.0   (D) 4.2   (E) 4.4

51. Find the perimeter of one leaf of the rose curve. (nearest tenth)

   (A) 8.5   (B) 8.7   (C) 8.9   (D) 9.1   (E) 9.3

52. Consider the curve $5y^2 - 4xy + 2x^3 - 3y = 14$. Find the equation of the line tangent to the curve at the
   point $(2, c)$, $c > 1$. The y-intercept of the tangent line is the point $(0, b)$. $b =$ _____. (nearest tenth)

   (A) 4.4   (B) 4.7   (C) 5.0   (D) 5.3   (E) 5.6

53. Suppose 25% of the students at Latexo plan to attend Harvard. Of those, 40% plan to major in
   mathematics. Of the 75% who do not plan to attend Harvard, 20% plan to major in
   mathematics. Given that a randomly selected student plans to major in math, what is the probability
   that this student plans to attend Harvard? (nearest hundredth)

   (A) 0.30   (B) 0.35   (C) 0.40   (D) 0.45   (E) 0.50

54. According to the Idaho Insurance Agency, the state accident rate for 18-year-old males was 0.125
   accidents for every driver during 2020. A random sample of 120 of the 18-year-old males in Boise
   found that 18 of them had been in an accident in 2020. If the accident rate for males in Boise is the
   same as the accident rate statewide, what is the probability of getting a sample whose accident rate is
   0.150 or greater? (nearest hundredth)

   (A) 0.20   (B) 0.23   (C) 0.26   (D) 0.29   (E) 0.32

Week	1	2	3	4	5	6
# Pushups	12	15	19	25	27	30

55-57. Mr. Cantu decided to start an exercise program in January. On Fridays, he finishes his workout by seeing how many pushups he can do. The results from the first 6 weeks are shown in the table above. Mr. Cantu analyzed the data by calculating a LSRL.

55. The absolute value of the residual for week 4 is _____ . (nearest hundredth)

   (A) 1.34   (B) 1.45   (C) 1.56   (D) 1.67   (E) 1.78

56. Based on the LSRL, how many push ups does Mr. Canto expect to do at the end of his workout on week 9?

   (A) 38   (B) 40   (C) 42   (D) 44   (E) 46

57. Calculate the standard deviation of the residuals to find the approximate size of a typical prediction error. (nearest hundredth)

   (A) 0.97   (B) 1.01   (C) 1.05   (D) 1.09   (E) 1.13

58. The mean score on the first exam in Professor Stat's class was 82. The scores on the first exam were approximately normal. If Justin's score of 89 placed him at the 90th percentile, what was the standard deviation of the scores on the first exam? (nearest hundredth)

   (A) 5.15   (B) 5.46   (C) 5.77   (D) 6.08   (E) 6.39

	N	Mean	Standard Deviation
BMI Male	88	27.7	3.8
BMI Female	96	26.3	3.6

59. The table above shows the BMI (Body Mass Index) of two independent samples of 70-year-old people in Idaho. The first sample consisted of 88 randomly selected males and the second sample consisted of 96 randomly selected females. It is thought that the BMI of 70-year-old males in Idaho is significantly higher than the BMI of 70-year-old females in Idaho. Professor Stat used an appropriate test to analyze the data. The P-value of the test is _____. (nearest thousandth)

   (A) 0.006   (B) 0.009   (C) 0.012   (D) 0.015   (E) 0.018

60. It is known that 28% of the seniors at Pokie High enroll in ISU after graduating from high school. If a group of 18 seniors is randomly selected, what is the probability that at least 6 of them will enroll at Idaho State? (nearest hundredth)

   (A) 0.37   (B) 0.39   (C) 0.41   (D) 0.43   (E) 0.45

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Region ● 2024
Answer Key

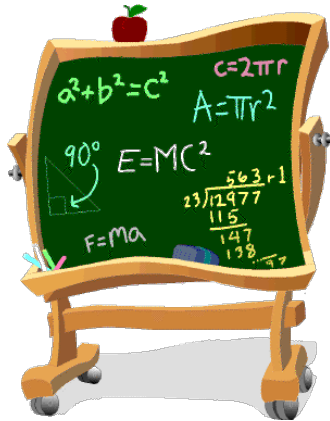|		
1. C	21. C	41. B
2. D	22. A	42. A
3. A	23. B	43. E
4. A	24. D	44. E
5. D	25. B	45. A
6. E	26. B	46. C
7. E	27. D	47. D
8. B	28. A	48. B
9. D	29. A	49. B
10. A	30. A	50. D
11. A	31. C	51. C
12. C	32. C	52. E
13. E	33. D	53. C
14. D	34. A	54. A
15. E	35. C	55. E
16. B	36. C	56. C
17. C	37. C	57. C
18. A	38. B	58. B
19. D	39. D	59. A
20. B	40. E	60. B

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2024

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Consider the formula for a thin lens, $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}$, where f is the focal length of the lens, d_o is the

   distance from the lens to the object, and d_i is the distance from the lens to the image. If the focal
   length of the lens is 24 cm and the distance from the lens to the object is 36 cm, find the distance from
   the lens to the image.

   (A) 24 cm   (B) 36 cm   (C) 48 cm   (D) 60 cm   (E) 72 cm

2. During the tax-free weekend, Penelope, Sydney and Alexa went to Academy Sports and purchased
   some cross country attire. Penelope purchased 4 shirts, 3 shorts and some shoes. The shoes cost $82
   and she spent a total of $196. Alexa purchased 6 shirts, 4 shorts and some shoes. The shoes cost $98
   and she spent a total of $260. All shirts were the same price and all shorts were the same price. If
   Sydney purchased 2 shirts and 3 shorts, how much did she spend?

   (A) $80   (B) $82   (C) $84   (D) $86   (E) $88

3. Five times Rose's age is 2 more than Carrie's age. In 10 years, Arlene will be twice as old as Carrie.
   Two years ago, Arlene was 14 times as old as Rose. How old is Arlene?

   (A) 84   (B) 86   (C) 88   (D) 90   (E) 92

4-6. Consider the points $A(-6, 2)$, $B(8, 4)$, $C(2, -6)$ and $D(-10, -4)$.

4. Find the distance from point A to the midpoint of $\overline{BC}$. (nearest tenth)

   (A) 11.4   (B) 11.6   (C) 11.8   (D) 12.0   (E) 12.2

5. Given: $\overleftrightarrow{AC}$ is parallel to $\overleftrightarrow{DE}$. If the coordinates of point E are $(a, 2)$, then a = _____.

   (A) −18   (B) −17   (C) −16   (D) −15   (E) −14

6. Given: $\overleftrightarrow{FG}$ is the perpendicular bisector of $\overleftrightarrow{AB}$. If the coordinates of point F are $(3, b)$,
   then b = _____.

   (A) −14   (B) −13   (C) −12   (D) −11   (E) −10

7. Caleb took a three-day trip from Sanger to Aberdeen, a distance of 1383 miles. On day one, he drove
   577 miles at an average speed of 62 mph. On day two, he drove 464 miles at an average speed of
   72 mph. If the total driving time on the trip was 20 hr 48 min, what was his average speed on day
   three? (nearest tenth)

   (A) 66.5 mph   (B) 66.8 mph   (C) 67.1 mph   (D) 67.4 mph   (E) 67.7 mph

8. Ronita flew from Portland to Dallas to visit her favorite sister. The round-trip ticket cost $882. She
   rented a car for $48 per day plus $0.32 per mile. She drove a total of 448 miles during the 12 days she
   was there. She also spent $366 on Texas Rangers shirts and hats. How much did the trip cost?

   (A) $1967.36   (B) $1968.36   (C) $1969.36   (D) $1970.36   (E) $1971.36

9. Consider an arithmetic sequence in which the fourth term is 37 and the eleventh term is 93. Find the sum of the first 16 terms.

    (A) 1162   (B) 1164   (C) 1166   (D) 1168   (E) 1170

10. Consider the circle on the right. If AB = 14, BC = 18, and AE = 16, then DE = _____.

    (A) 9
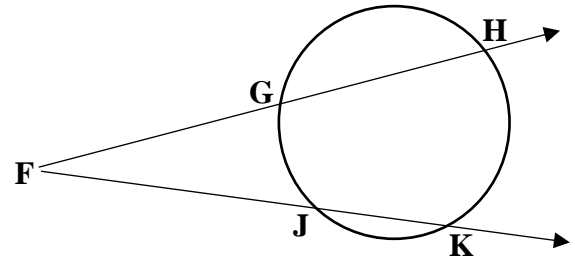   (B) 10
    (C) 11
    (D) 12
    (E) 13
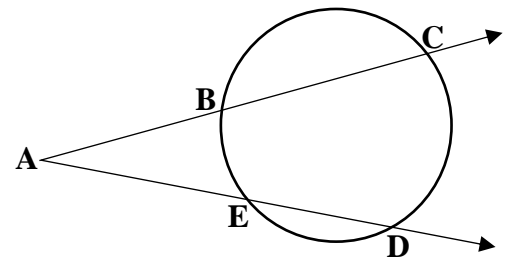
Problem 10

11. Consider equilateral triangle PQR with a circumscribed circle. If the area of the circle is 339, then the area of triangle PQR is _____. (nearest whole number)

    (A) 134   (B) 136   (C) 138   (D) 140   (E) 142

12. Consider the circle on the right. If the measure of minor arc $HK = 128°$ and the measure of $\angle GFJ = 33°$, then the measure of minor arc GJ = _____.

    (A) 56°
    (B) 58°
    (C) 60°
    (D) 62°
    (E) 64°

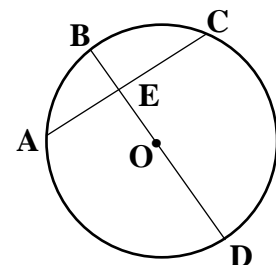**Problem 12**

13. The total area of a cylinder with a radius of 14 cm is 3343 cm². The volume of the cylinder is _____ cm³. (nearest whole number)

    (A) 14,764   (B) 14,768   (C) 14,772   (D) 14,776   (E) 14,780

14. Consider the circle on the right with center O. Chord $\overline{AC}$ intersects diameter $\overline{BD}$ at point E. $\overline{AC} \perp \overline{BD}$, BD = 18, and AC = 14. BE = _____. (nearest tenth)

    (A) 3.3   (B) 3.5   (C) 3.7
    (D) 3.9   (E) 4.1

Problem 14

15. Given: $\triangle ABC$ is inscribed in a circle with $m\angle C = 90°$, AC = 7, and the perimeter of the triangle is 56. The area of the circle = _____. (nearest whole number)

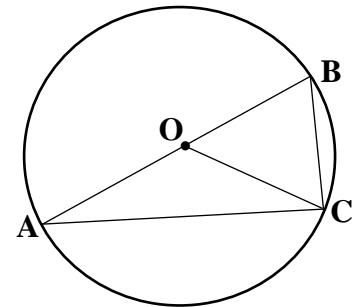   (A) 491   (B) 494   (C) 497   (D) 500   (E) 503

16. Consider △ABC with m∠ABC = 90°. Point D lies on $\overline{AC}$ such that m∠ADB = 90°.
    If AD = 6 and CD = 13, then the perimeter of △ABC = _____. (nearest tenth)

    (A) 44.5   (B) 44.8   (C) 45.1   (D) 45.4   (E) 45.7

17-18. The circle shown on the right has an area of 707.
       The measure of ∠BAC is 30°.
       Point O is the center of the circle.

17. Find the area of △AOC. (nearest tenth)

    (A) 97.1   (B) 97.4   (C) 97.7
    (D) 98.0   (E) 98.3

    Problems 17, 18

18. Find the area of the region bounded by chord $\overline{BC}$ and

    minor arc $\overline{BC}$. (nearest tenth)

    (A) 19.8   (B) 20.1   (C) 20.4   (D) 20.7   (E) 21.0

19-21. Given: △ABC is similar to △DEF, AB = 36, BC = 39, AC = 42, and DF = 28.

19. Point G is the midpoint of $\overline{DF}$. EG = _____. (nearest tenth)

    (A) 20.3   (B) 20.5   (C) 20.7   (D) 20.9   (E) 21.1

20. Point H lies on $\overline{AC}$ and ray $\overrightarrow{BH}$ bisects ∠ABC. AH = _____. (nearest hundredth)

    (A) 20.13   (B) 20.16   (C) 20.19   (D) 20.22   (E) 20.25

21. The area of △BHC = _____. (nearest whole number)

    (A) 326   (B) 329   (C) 332   (D) 335   (E) 338

22. Rachel accepted a job with a salary of $95,000 the first year. During the next 19 years, she was given
    a 6% raise each year. Find the total compensation she received over the 20-year period.
    (nearest dollar)

    (A) $3,494,628   (B) $3,494,631   (C) $3,494,634   (D) $3,494,637   (E) $3,494,640

23. Given: $\sin(u) = -\dfrac{24}{25}$ and $\cos(v) = -\dfrac{3}{5}$. Both u and v are in quadrant III. Evaluate $\sec(u - v)$.

    (A) $\dfrac{121}{117}$ (B) $\dfrac{41}{39}$ (C) $\dfrac{125}{117}$ (D) $\dfrac{127}{117}$ (E) $\dfrac{44}{39}$

24. Consider the sequence 2, 5, 9, 14, 20, 27, 35, …. The sum of the first 24 terms is _____.

    (A) 2896   (B) 2900   (C) 2904   (D) 2908   (E) 2912

UIL Mathematics

25. **Audrey invested $100,000 for 4 years. If the interest was compounded monthly rather than quarterly, she would have made $345.93 more. What was the annual interest rate? (nearest hundredth)**

    (A) 4.22%   (B) 5.33%   (C) 6.44%   (D) 7.55%   (E) 8.66%

26. **Assume July temperatures vary sinusoidally in Denali National Park with a low of 48° at 4:00 AM and a high of 68° at 4:00 PM. The number N of brown bears that are visible from Keith's campsite is given by $N(t) = (T - 46°)$, $48° \le T \le 68°$, where $N(t) =$ the number of brown bears visible at time t and T is the temperature. How many brown bears are visible from Keith's campsite at 12:00 PM?**

    (A) 16   (B) 17   (C) 18   (D) 19   (E) 20

27. **The circle $(x-6)^2 + (y-12)^2 = 20$ is tangent to the circle $x^2 + y^2 = 80$. The common internal tangent is a line with x-intercept $(a, 0)$ and y-intercept $(0, b)$. $a + b =$ _____ . (nearest whole number)**

    (A) 26   (B) 28   (C) 30   (D) 32   (E) 34

28. **Justin obtained a sample of radioactive plutonium 234 at 5:00 AM on Wednesday. Only 1.510 g remained at 5:00 AM on Thursday and only 0.501 g remained at 7:00 PM on Thursday. Find the amount of plutonium Justin originally obtained. (nearest thousandth)**

    (A) 10.008 g   (B) 10.082 g   (C) 10.156 g   (D) 10.230 g   (E) 10.304 g

29-30. **Consider the graph of a parabola with vertex $V(2, -6)$. Points $P(0, -4)$ and $Q(0, -8)$ both lie on the graph of the parabola.**

29. **The equation of the directrix of the graph of the parabola is $x =$ _____ .**

    (A) $\dfrac{17}{8}$ (B) $\dfrac{9}{4}$ (C) $\dfrac{5}{2}$ (D) 3   (E) 4

30. **Point $T(a, 0)$ lies on the graph of the parabola and point $F(e, f)$ is the focus of the graph of the parabola. $FT =$ _____ . (nearest tenth)**

    (A) 18.5   (B) 18.7   (C) 18.9   (D) 19.1   (E) 19.3

31. **Computer World in Big Timber, Montana currently has 20 computers in stock. Fifteen have 16 GB RAM and five have 8 GB RAM. If Rancher Rob randomly selects four computers to purchase, what is the probability that at least two of the computers have 16 GB RAM? (nearest thousandth)**

    (A) 0.940   (B) 0.947   (C) 0.954   (D) 0.961   (E) 0.968

32. **Consider $\overleftrightarrow{AB}$ such that every point on $\overleftrightarrow{AB}$ is the same distance from point $P(-6, 4)$ as the distance from point $Q(8, -2)$. If point $R(13, c)$ lies on $\overleftrightarrow{AB}$, then $c =$ _____ . (nearest tenth)**

    (A) 29.0   (B) 30.1   (C) 31.2   (D) 32.3   (E) 33.4

33. Ship A leaves port at 1:00 PM and travels at an average speed of 18 mph on a bearing of 144°. Ship B leaves port at 3:00 PM and travels at an average speed of 24 mph on a bearing of 284°. At what time will the ships be 155 miles apart? (nearest minute)

    (A) 6:01 PM   (B) 6:04 PM   (C) 6:07 PM   (D) 6:10 PM   (E) 6:13 PM

34. Consider an ellipse such that for any point $P(e, f)$ that lies on the ellipse, the distance from P to the point $(2, 4)$ plus the distance from P to the point $(14, 4)$ equals 40. If the equation of the ellipse is $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, then b = _____. (nearest tenth)

    (A) 18.3   (B) 18.5   (C) 18.7   (D) 18.9   (E) 19.1

35. The graph of $4x^2 + 5xy + 2y^2 - 16 = 0$ is an ellipse in which the axes have been rotated _____°. (nearest whole number)

    (A) 28   (B) 30   (C) 32   (D) 34   (E) 36

36. The graph of the parametric equations $x = 4\sec\theta + 3$ and $y = 3\tan\theta - 2$ is a hyperbola. The asymptote with positive slope has an x-intercept of $(e, 0)$. e = _____. (nearest tenth)

    (A) 5.3   (B) 5.5   (C) 5.7   (D) 5.9   (E) 6.1

37. The graph of the polar equation $r = \dfrac{4}{3 + 2\sin\theta}$ is an ellipse centered at the point $P(a, b)$. a + b = _____. (nearest tenth)

    (A) −1.8   (B) −1.7   (C) −1.6   (D) −1.5   (E) −1.4

38. Find the distance from the point $Q(2, 3, 5)$ to the plane $x - 2y + 3z = 6$. (nearest tenth)

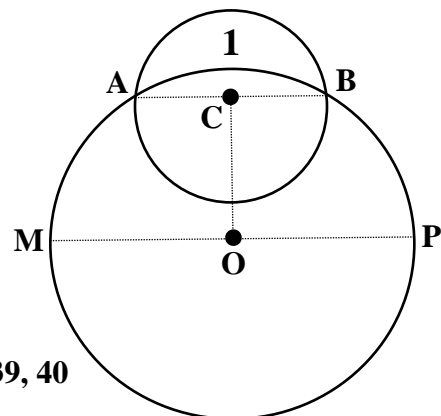   (A) 1.1   (B) 1.3   (C) 1.5   (D) 1.7   (E) 1.9

39-40. The radius of the large circle with center O is 10 and the radius of the small circle with center C is 6. The crescent-shaped region (1) is called a lune. $\overline{CO} \perp \overline{AB}$ and $\overline{CO} \perp \overline{MP}$.

39. The perimeter of the lune is _____. (nearest tenth)

    (A) 31.1   (B) 31.4   (C) 31.7
    (D) 32.0   (E) 32.3

40. The area of the lune is _____. (nearest tenth)

    (A) 39.4   (B) 39.6   (C) 39.8
    (D) 40.0   (E) 40.2

Problems 39, 40

UIL Mathematics

41. Find the area of the region bounded by the
graphs of $y = h(x)$ and $y = f(x)$.
(nearest tenth)

(A) 30.6   (B) 30.9   (C) 31.2
(D) 31.5   (E) 31.8

42. Find the volume of the solid formed when
the region bounded by the graphs of
$y = h(x)$ and $y = f(x)$ is revolved about the
line $y = -10$. (nearest tenth)

(A) 2233.1   (B) 2244.2   (C) 2255.3
(D) 2266.4   (E) 2277.5

Problems 41, 42, 43, 44

43. Find the volume of the solid whose base is the region bounded by the graphs of $y = h(x)$ and $y = f(x)$
and whose cross sections perpendicular to the x-axis are semicircles. (nearest tenth)

(A) 63.5   (B) 63.8   (C) 64.1   (D) 64.4   (E) 64.7

44. If the arc length of the graph of $y = h(x)$ on the interval $[a, 7.4]$ is 24.51, and $a < 0$, then $a = $ _____.
(nearest tenth)

(A) −2.2   (B) −2.0   (C) −1.8   (D) −1.6   (E) −1.4

45-46. A particle is moving along the x-axis so that at any time t, in seconds, the acceleration of the
particle is given by $a(t) = 1 + 8\cos(t)$, $t \geq 0$, where $a(t)$ is the acceleration in cm/s². At t = 0, the
particle's position is at x = 3 cm and the particle's velocity is 2 cm/s to the right. Consider the path
of the particle from t = 0 to t = 6 seconds. (radians)

45. The position of the particle at t = 4 seconds is at x = _____ cm. (nearest tenth)

(A) 32.2   (B) 32.5   (C) 32.8   (D) 33.1   (E) 33.4

46. The maximum speed of the particle when it is traveling to the left is _____ cm/s.
(nearest hundredth)

(A) 1.35   (B) 1.46   (C) 1.57   (D) 1.68   (E) 1.79

47. The number of fire ants in Mr. Garcia's backyard is given by a differentiable function f, where f(t)
is the number of fire ants present and t is measured in weeks. The number of fire ants is increasing
according to the equation $\dfrac{df}{dt} = kf$, where k is a constant. At t = 0, the number of fire ants is 450 and
is increasing at the rate of 150 fire ants per week. Find the expected number of fire ants
at t = 6 weeks. (nearest whole number)

(A) 3303   (B) 3314   (C) 3325   (D) 3336   (E) 3347

UIL Mathematics
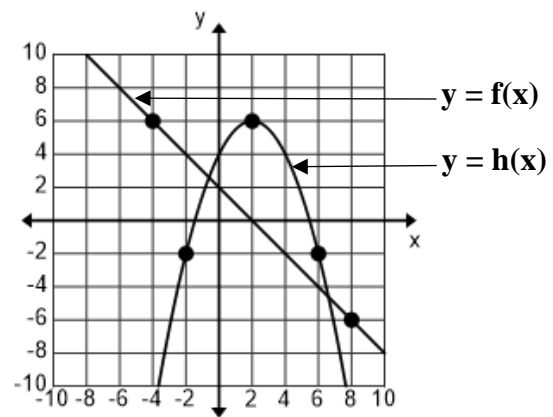
t (min)	0	13	22	34	48
r(t) (in/min)	0.098	0.074	0.061	0.048	0.035

48-49. Rancher Rob has an elk ranch near Driggs. He stores rainwater in a large cylindrical tank which has a radius of 4 feet and a height of 3 feet. The top of the tank has been removed so that rain can fill the tank and the elk can drink from the tank. The depth of the water in the tank was 2 feet when a storm blew in and it began raining. The rate at which the depth of the water in the tank is increasing is shown for various values of t in the table above.

48. Use the table above to estimate the increase in the depth of the water in the tank from $t = 0$ to $t = 48$ minutes. Use a right Riemann sum, RRAM, with four subintervals. (nearest hundredth)

(A) 2.55 in   (B) 2.58 in   (C) 2.61 in   (D) 2.64 in   (E) 2.67 in

49. Rob developed a mathematical model for the rate at which the depth of the water in the tank is increasing. His model is the function r where $r(t) = 0.098(0.979)^t$, $0 \leq t \leq 48$, and where $r(t)$ is measured in inches per minute and t is measured in minutes. Find the amount of water in the tank at $t = 48$ minutes using Rob's model. (nearest gallon)

(A) 835 gal   (B) 838 gal   (C) 841 gal   (D) 844 gal   (E) 847 gal

50. Given: $y^2 + 5x^2y^3 + x^4 = 37$. Evaluate $\dfrac{dy}{dx}$ when x = 2. (nearest hundredth)

(A) −0.96   (B) −0.93   (C) −0.90   (D) −0.87   (E) −0.84

51. Let f be a function with third derivative $(2x+8)^{\frac{2}{3}}$. The coefficient of x^4 in the Taylor series for f about x = 0 is _____.

(A) $\dfrac{1}{72}$ (B) $\dfrac{1}{36}$ (C) $\dfrac{1}{24}$ (D) $\dfrac{1}{12}$ (E) $\dfrac{1}{6}$

52. Find the area of the region lying between the inner and outer loops of the polar graph of $r = 1 - 2\cos(\theta)$. (nearest hundredth)

(A) 8.31   (B) 8.34   (C) 8.37   (D) 8.40   (E) 8.43

53. The distribution of the amount of water in a 20 oz bottle of Olney Natural Springs Water is approximately normal with a mean of 20 oz and a standard deviation of 0.45 oz. Approximately what proportion of bottles have less than 19 oz? (nearest thousandth)

(A) 0.013   (B) 0.016   (C) 0.019   (D) 0.022   (E) 0.025

54. Assume that the length of a fully grown Lesser Bandicoot Rat has a roughly normal distribution with a mean of 36 cm and a standard deviation of 1.5 cm. Find the interquartile range of this distribution. (nearest tenth)

(A) 1.8 cm   (B) 2.0 cm   (C) 2.2 cm   (D) 2.4 cm   (E) 2.6 cm

55. A survey planned to determine how much personal debt people in the 25 to 34 age group have due to home mortgages, car loans and credit cards. Of the following, which is the minimum number of people in this age group that researchers should plan to survey to be within $1,000 of the true mean with 90% confidence? A previous study found that the standard deviation of the personal debt of people in this age group was $15,500.

   (A) 449   (B) 550   (C) 651   (D) 752   (E) 853

Student	1	2	3	4	5	6	7	8	9
Test 1	82	88	77	81	90	74	83	85	82
Test 2	84	91	78	85	93	76	84	88	86

56-57. Professor Satterfield randomly selected 9 of his statistics students to participate in a small study. He wanted to see if offering a group study session with a T.A. the night before major tests would improve their scores. Assume test 1 and test 2 were of equal difficulty. Students were not offered a study session before test 1, but they were required to attend a study session before test 2. Results are shown in the table above. When evaluating the results of the study, the null hypothesis was H_0 : The study session had no effect on test scores. The alternative hypothesis was

   H_a : The study session improved test scores. The significance level was $\alpha = 0.10$.

56. Using an appropriate test, he should reject H_0 if the test statistic is greater than _____.
   (nearest thousandth)

   (A) 1.286   (B) 1.397   (C) 1.508   (D) 1.619   (E) 1.730

57. The P-value obtained from using the appropriate test was _____. (nearest hundred-thousandth)

   (A) 0.00007   (B) 0.00010   (C) 0.00013   (D) 0.00016   (E) 0.00019

58. A Lottery ticket cost $10. In the Lottery, six numbers are randomly chosen without repetition from the numbers 1 to 40. If you select all 6 numbers, you win $10,000,000. If you only select 5 of the 6, you win $100,000. If you only select 4 of the 6, you win $100. Find the expected value of a lottery ticket.

   (A) −$2.02   (B) −$1.98   (C) −$1.94   (D) −$1.90   (E) −$1.86

59-60. Professor Stat randomly selected 50 students at ISU for a study. He collected information about their parents. When he analyzed the data, he noticed that a strong positive linear relationship exists between a student's final grade (FG) in English 101 and the college grade average (CA) of the student's mother. The results of computing a LSRL from the data were:
   FG mean = 88, FG standard deviation = 4, CA mean = 92, CA standard deviation = 3, $r^2 = 0.81$.

59. Find the predicted final grade of a student whose mother had a CA of 98.
   (nearest whole number)

   (A) 92   (B) 93   (C) 94   (D) 95   (E) 96

60. His analysis predicts that for each increase of one point in a mother's college grade average, there is a corresponding increase of _____ points in a student's final grade. (nearest tenth)

   (A) 1.2   (B) 1.4   (C) 1.6   (D) 1.8   (E) 2.0

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2024
Answer Key

1. E	21. E	41. C
2. C	22. B	42. E
3. B	23. C	43. B
4. A	24. B	44. C
5. C	25. E	45. A
6. D	26. B	46. A
7. E	27. C	47. C
8. A	28. A	48. B
9. D	29. C	49. D
10. D	30. A	50. E
11. D	31. E	51. B
12. D	32. A	52. B
13. E	33. B	53. A
14. A	34. E	54. B
15. A	35. D	55. C
16. D	36. C	56. B
17. B	37. C	57. A
18. C	38. B	58. E
19. C	39. C	59. D
20. B	40. E	60. A