TMSCA HIGH SCHOOL MATHEMATICS TEST #7 ©

JANUARY 5, 2019

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Seven

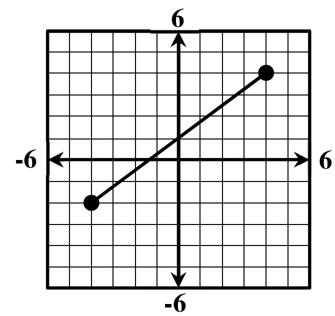
1. Evaluate $5.4 \div \left(\frac{5}{2}\right)^{-1} - 3! + 9.8$.

(A) 17.3 (B) 5.96 (C) –0.7 (D) –12.04 (E) 19.3

2. Caroline had a rope that was 15 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:12 with 10 inches of string left over. How long was the longest piece?

(A) 9' 8" (B) 9' 11" (C) 6' 3" (D) 10' 1" (E) 10'

3. What is the x – intercept of the perpendicular bisector of the line segment shown?



(A) $\left(0,\frac{3}{4}\right)$ (B) $\left(-\frac{4}{3},0\right)$ (C) $\left(\frac{3}{4},0\right)$ (D) $(0,1)$ (E) $(1,0)$

4. If $1 = 3x + y$, $-x + 2y = 16$ and $ax + 4y = 0$, then $a = ?$

(A) 2 (B) 7 (C) 16 (D) 14 (E) –7

5. Kim currently has an average of 87 in her math class. The only remaining grade is her final exam. What minimum grade does she need to earn on the final to earn at least a 90 in the class if the final exam is 30% of her grade and her teacher does not round.

(A) 97 (B) 93 (C) 95 (D) 99 (E) 98

6. A 12-sided die is numbered with the numbers 1 to 12. If the die is rolled, what are the odds that the outcome will be a perfect number?

(A) 1:12 (B) 1:5 (C) 1:6 (D) 1:11 (E) 1:3

7. Events A and B are independent events such that $P(B) = 4P(A)$ and $P(A \cup B) = 0.66$. Find $P(A)$.

(A) 0.15 (B) 0.20 (C) 0.48 (D) 0.16 (E) 0.12

8. Simplify: $\frac{n!(n+3)!}{(n-2)!} \div \frac{(n+1)!(n+2)!}{(n-1)!}$.

(A) $\frac{n^2-2n-3}{n-1}$ (B) $\frac{n^2-n-2}{n-1}$ (C) $\frac{n^2-n-2}{n+1}$

(D) $\frac{n^2+2n-3}{n-1}$ (E) $\frac{n^2+2n-3}{n+1}$

9. If x is 20% less than y and y is 75% greater than z, then x is what part of z?

(A) 90% (B) 70% (C) 15% (D) 35% (E) 140%

10. A box contains 10 black marbles, 8 red marbles and 12 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 black marbles?

(A) 6:197 (B) 1:32 (C) 2:73 (D) 2:75 (E) 6:203

11. $\angle A$ and $\angle B$ are complementary, and $m\angle B$ is 2° more than seven times $m\angle A$. Find the measure of the supplementary angle to $\angle A$.

(A) 101° (B) 169° (C) 91° (D) 112° (E) 153°

12. 10,540 inches per second is the same as ______________ miles per hour. (nearest mile per hour)

(A) 1288 (B) 429 (C) 120 (D) 599 (E) 359

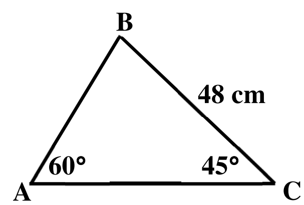
13. If $\dfrac{Ax+B}{x+2}+\dfrac{x-5}{2x-1}=\dfrac{11x^2-4x-12}{2x^2+3x-2}$, where A and B are constants, then $A+B=$?

(A) 12 (B) 7 (C) 5 (D) 10 (E) 15

14. The four brothers Lester, Morris, Nigel and Porter wanted to go on a road trip, but Lester had no money. Morris, Nigel and Porter each gave Lester one-tenth, one-eighth and one-sixth of his money respectively. If each gave Lester the same amount, what fraction of the money did Lester possess after the exchange?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{3}{8}$ (E) $\dfrac{1}{3}$

15. The area of triangle ABC is ________cm^2. (nearest square centimeter)



(A) 1241 (B) 621 (C) 1817 (D) 909 (E) 1038

16. Let $f(x)=3x^3+3x^2-18x+2$. Find the largest value of x for which $f'(x)=6$.

(A) 2 (B) -2 (C) $\dfrac{4}{3}$ (D) $\dfrac{3}{4}$ (E) $\dfrac{1}{2}$

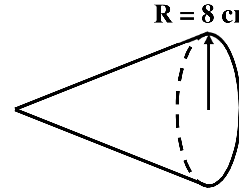
17. $\left(2+\sqrt{-27}\right)\left(7-\sqrt{-75}\right)=$

(A) $14+45i$ (B) $14-45i$ (C) $59-11\sqrt{3}\,i$ (D) $59+11\sqrt{3}i$ (E) undefined

18. If p and q are the zeros of the function $f(x)=6x^2+13x-5$ then $pq^2+p^2q=$

(A) $\dfrac{65}{72}$ (B) $\dfrac{65}{36}$ (C) $-\dfrac{65}{72}$ (D) $\dfrac{65}{144}$ (E) $-\dfrac{65}{72}$

19. The lateral surface area of the cone shown is 136π cm^2. The volume of the cone is ______cm^3.



(A) 160π (B) 363π (C) 121π (D) 180π (E) 320π

20. Let $f(x)=x^2+1$ and $g(x)=x^4$. Calculate $g\left(f'(2)\right)$.

(A) 64 (B) 16 (C) 256 (D) 25 (E) 577

21. Carrie drives the same route to work every weekday. Last week, her average speed on her commute to work was 52.0 mph. On the first 4 days of the week, her speeds were 48.0 mph, 61.0 mph, 48.0 mph and 59.0 mph respectively. What was her speed on Friday? (nearest tenth)

(A) 44.0 mph (B) 51.0 mph (C) 50.6 mph (D) 47.3 mph (E) 51.6 mph

22. Using the following pattern of numbers, determine the fifth term in the 15th row.

					1						(row 0)
				1		1					(row 1)
			1		2		1				(row 2)
		1		3		3		1			(row 3)
	1		4		6		4		1		(row 4)
1		5		10		10		5		1	(row 5)
					…						…

(A) 3003 (B) 455 (C) 364 (D) 1001 (E) 1365

23. An operation "Ω" is defined by $a\Omega b = b^2 - ab$. What is the value of $(-1\Omega 2)\Omega(3\Omega 2)$?

(A) 24 (B) 16 (C) –8 (D) 48 (E) –24

24. Determine the range of $f(x) = 5 + 6\sin[\pi x + 2\pi]$.

(A) $[-6,6]$ (B) $[-11,1]$ (C) $[-11,11]$ (D) $[-1,11]$ (E) $[1,11]$

25. The Real value solution set for $\left|\frac{1}{2} + 3x\right| - 7 \geq 1$ is:

(A) $\left(-\infty, -\frac{5}{2}\right] \cup \left[\frac{17}{6}, \infty\right)$ (B) $\left[-\frac{17}{6}, \frac{5}{2}\right]$ (C) $\left[-\frac{5}{2}, \frac{17}{6}\right]$

(D) $\left(-\infty, -\frac{17}{6}\right] \cup \left[-\frac{5}{2}, \infty\right)$ (E) $\left(-\infty, -\frac{17}{6}\right] \cup \left[\frac{5}{2}, \infty\right)$

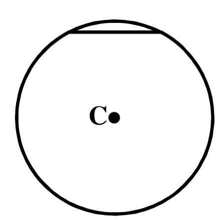
26. Which of the following functions is neither even nor odd? $f(x) =$ _______.

(A) $\sin(x^2)$ (B) $x^2 - 6$ (C) $\sqrt[3]{x} - x$ (D) $\sqrt[3]{x-7}$ (E) $\cos(2x)$

27. Frederick wants to construct a 5-letter password using letters from his first name. If Frederick is willing allow unlimited repeats of all letters, how many possible passwords could he make?

(A) 16,807 (B) 59,049 (C) 6,561 (D) 2,401 (E) 32,768

28. The circle C shown has a diameter of 8.2 feet, and the chord shown is 4 feet from the center of the circle. Find the length of the chord. (not drawn to scale)



(A) 7.2 ft (B) 1.8 ft (C) 3.6 ft (D) 0.9 ft (E) 2.7 ft

29. If $\frac{A}{x+5} + \frac{B}{2x+3} = \frac{5x+4}{2x^2+13x+15}$, then $A + B = ?$

(A) 2 (B) 7 (C) –2 (D) 4 (E) –5

30. What is the probability that a proper factor of 234 is a multiple of 13?

(A) $\frac{3}{7}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{2}{5}$ (E) $\frac{2}{7}$

31. Calculate $5^3 + 6^3 + 7^3 + ... + 22^3 + 23^3 = ?$

(A) 76,176 (B) 53,361 (C) 76,076 (D) 53,261 (E) 18,821,096

32. A meal at Ron's Burgers consists of a burger, fries and a milkshake and contains 59 g of fat. The burger has 13 more grams of fat than a milkshake. The fat content of fries and a milkshake together exceeds the fat content of a burger by 3 grams. Find the fat content of the milkshake.

(A) 13 g (B) 15 g (C) 16 g (D) 14 g (E) 26 g

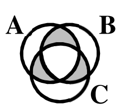
33. Let $A = \begin{bmatrix} 7 & -9 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. Find the sum of the elements in AB.

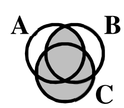
(A) −27 (B) −126 (C) 126 (D) −30 (E) 27

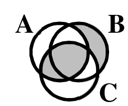
34. If $a - b = 7$ and $ab = -4$, then $a^3 - b^3 = ?$

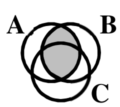
(A) 592 (B) 448 (C) 315 (D) 427 (E) 259

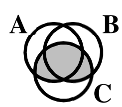
35. The shaded region in which of the following Venn diagrams represents the set $(A \cup C)' \cup (A \cap C)$?

(A) 

(B) 

(C) 

(D) 

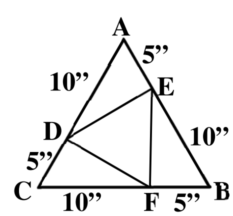
(E) 

36. Coach Euclid's UIL team consists of 9 boys and 6 girls. How many different 3-member teams consisting of at least one girl could he make up?

(A) 216 (B) 71 (C) 351 (D) 371 (E) 238

37. The area of triangle DEF is _______ in^2 (nearest tenth)

(A) 32.5 (B) 25.0 (C) 10.8 (D) 28.8 (E) 21.7



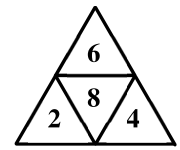
38. Evaluate $\lim_{\theta \to 0} \frac{18\sin(3\theta)}{\theta}$.

(A) 54 (B) 6 (C) 9 (D) 18 (E) does not exist

39. Simplify: $\left(a^3 \div b^5\right)^{-4} \div b^{-3} \times a^6$.

(A) $\dfrac{b^{23}}{a^{18}}$ (B) $\dfrac{1}{a^6 b^{17}}$ (C) $\dfrac{b^{23}}{a^6}$ (D) $\dfrac{b^{17}}{a^6}$ (E) $\dfrac{b^{17}}{a^{18}}$

40. Morgan folds the net shown into a fair tetrahedral die. She rolls her tetrahedral die and adds the three visible sides. What is the expected value for the sum?



(A) 14 (B) 16 (C) 18 (D) 17 (E) 15

41. The intersection of the altitudes of a triangle of a triangle is called the_________.

(A) Incenter (B) Orthocenter (C) Center (D) Centroid (E) Circumcenter

42. How many distinct solutions exist for $2\cos^2 x = 2\sin x$, where $-\pi \le x \le \pi$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

43. Consider the Fibonacci-type sequence: -1, a, 1, 3, b, 7, c, … . Find the value of $a+b+c$.

(A) 13 (B) 6765 (C) 3610 (D) 17 (E) 5473

44. Which of the following functions expresses the perimeter, *P*, of a regular hexagon in terms of the length of the apothem, *a*?

(A) $P = 4a\sqrt{3}$ (B) $P = 6a$ (C) $P = 3a\sqrt{3}$ (D) $P = 6a\sqrt{3}$ (E) $P = 2a\sqrt{3}$

45. Which of the following statements is a false statement for $f(x) = \begin{cases} 3x^2 - 7 & \text{if } x < 3 \\ 20 & \text{if } x = 3 \\ 6x + 2 & \text{if } x > 3 \end{cases}$.

(A) $f(3)$ exists (B) f is continuous at 3 (C) $\lim_{x \to 3} f(x)$ exists

(D) $f'(3)$ exists (E) none of these

46. Evaluate: $\int_{a-1}^{1-a} (2x-1)\,dx$

(A) $2-2a$ (B) 0 (C) $-2a$ (D) $2a-2$ (E) does not exist

47. If the length of one edge of a regular icosahedron is 2 cm, the total surface area of the icosahedron is _______cm²?

(A) $16\sqrt{3}$ (B) $20\sqrt{3}$ (C) 35 (D) 32 (E) $18\sqrt{3}$

48. Calculate the probability that when nine books are arranged on a shelf at random, that three particular books are together.

(A) $\dfrac{1}{720}$ (B) $\dfrac{1}{60480}$ (C) $\dfrac{1}{84}$ (D) $\dfrac{1}{72}$ (E) $\dfrac{1}{12}$

49. The repeating decimal $0.4111\ldots$ in base 5 can be written as which of the following fractions in base 5 in simplified terms?

(A) $\frac{31}{40_5}$ (B) $\frac{31}{44_5}$ (C) $\frac{32}{40_5}$ (D) $\frac{32}{44_5}$ (E) $\frac{23}{40_5}$

50. Find the slope of the normal line to the graph of $2x^2+3y^2=11$ at the point $(-2,1)$.

(A) $-\frac{19}{3}$ (B) $-\frac{4}{3}$ (C) $-\frac{3}{4}$ (D) $\frac{3}{4}$ (E) $\frac{4}{3}$

51. A lightbulb company produces bulbs that are faulty on average 3.7% of the time. If 10 bulbs are packaged together, what is the probability that at least one of the bulbs is faulty? (nearest tenth)

(A) 99.0% (B) 31.4% (C) 36.0% (D) 64.0% (E) 68.6%

52. The sequence of numbers $252, 66, 8, 6, 12, 2, -24, \ldots$ can be modelled with a polynomial function. Find the value of the 10th term of the sequence.

(A) 552 (B) –4 (C) –800 (D) 162 (E) $\frac{23}{40_5}$

53. What is the constant term in the binomial expansion of $\left(2x-\frac{1}{x^3}\right)^8$?

(A) 1792 (B) 1120 (C) 64 (D) 512 (E) 1536

54. Solve: $\log_2\left(5x^2-x-2\right)=2+2\log_2 x$

(A) 4 (B) 2 (C) –2 (D) 3 (E) –1

55. If $\frac{x+9}{x-9}+\frac{x-9}{x+9}=2+\frac{B}{(x-9)(x+9)}$ where $B\in\mathbb{Z}^+$ then $B=$?

(A) 27 (B) 324 (C) 162 (D) 81 (E) 54

56. $\frac{1-\cos(2\theta)}{\sin(2\theta)}=$

(A) $\tan(2\theta)$ (B) $\tan(2\theta)$ (C) $\cot(2\theta)$ (D) $1+\tan\theta$ (E) $\tan\theta$

57. The lengths of the sides of triangle PQR are the roots of $f(x)=2x^3-31x^2+159x-270$. The perimeter of triangle PQR is 15.5. Find the area of triangle PQR. (nearest tenth)

(A) 10.1 (B) 11.0 (C) 22.0 (D) 16.5 (E) 5.0

58. Which of the following are the side lengths of a scalene obtuse triangle?

(A) 5, 9, 9 (B) 8, 10, 12 (C) 5, 24, 25 (D) 5, 12, 13 (E) 9, 9, 14

59. Consider the point (a,b) in the Cartesian plane. The transformation $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$ results in a rotation of _____about the origin.

(A) 60° counter-clockwise (B) 120° clockwise (C) 30° clockwise

(D) 120° counter-clockwise (E) 60° clockwise

60. What is the digit in the ten-thousandth place in the sum: $1+3.4+\frac{3.4^2}{2}+\frac{3.4^3}{6}+\frac{3.4^4}{24}+\frac{3.4^5}{120}+\ldots$.

(A) 0 (B) 4 (C) 5 (D) 1 (E) 8

2018-2019 TMSCA Mathematics Test Seven Answers

1. A	21. D	41. B
2. E	22. E	42. C
3. C	23. B	43. D
4. D	24. D	44. A
5. A	25. E	45. D
6. D	26. D	46. D
7. A	27. A	47. B
8. E	28. B	48. E
9. E	29. A	49. C
10. A	30. B	50. C
11. B	31. C	51. B
12. D	32. B	52. D
13. B	33. D	53. A
14. A	34. E	54. B
15. D	35. C	55. B
16. C	36. D	56. E
17. D	37. A	57. B
18. B	38. A	58. C
19. E	39. C	59. E
20. C	40. E	60. D

2018-2019 TMSCA Mathematics Test Seven Solutions

7. Solve $P(A)+4P(A)-P(A)\times 4P(A)=0.66$ for $P(A)=0.15$

10. $P(BBB)=\frac{10}{30}\times\frac{9}{29}\times\frac{8}{28}=\frac{6}{203}$ with odds 6:(203-6) = 6:197

12. Lester has 3 portions while his brothers are left with 9, 7 and 5 portions, so Lester has 3 out of 24 portions or $\frac{1}{8}$ of the money.

15. Drawing the altitude divides the triangle into 2 special right triangles with $b=24\sqrt{2}+8\sqrt{6}$ and $h=24\sqrt{2}$ for $A=\frac{1}{2}bh\approx 909$.

21. The distance of the trip does not matter as long as it is the same every day, so let it just be 1 mile, then the total time for the week should be equal to the sum of the daily times: $\frac{5}{52.0}=\frac{1}{48.0}+\frac{1}{61.0}+\frac{1}{48.0}+\frac{1}{59.0}+\frac{1}{x}$ for $x\approx 47.3$ miles per hour.

22. The binomial expansion of $(1+1)^n$ gives the terms of each row of Pascal's triangle, so for the 5th term of the 15th row use the binomial theorem: ${}_{15}C_{11}=1365$.

27. There are 7 distinct letters in Frederick's name. If he allows unlimited repeating, then the number of passwords will be $7^5=16{,}807$.

31. Use $\sum_{k=1}^{n}k^3=\left(\frac{n(n+1)}{2}\right)^2$ for

$$\left(\frac{23(24)}{2}\right)^2-1-8-27-64=76{,}076$$

32. Solve the system of equations:

$b+f+m=59$

$b-13=m$ for m = 15

$f+m=b+3$

36. Calculate the number of ways in which each of the three compositions of teams can be formed, the add the results.

$$\left({}_6C_3\right)+\left({}_6C_2\times{}_9C_1\right)+\left({}_6C_1\times{}_9C_2\right)=371$$

37. The sides of the triangle DEF are the long legs of special right tringles with lengths $5\sqrt{3}$. Then $\frac{\left(5\sqrt{3}\right)^2\sqrt{3}}{4}\approx 32.5$

38. Use $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ for $\lim_{\theta\to 0}\frac{3(18)\sin 3\theta}{3\theta}=54(1)=54$

40. There are 4 sums each with a ¼ probability of occurring, so the expected value for the roll is:

$$\frac{1}{4}(18+16+14+12)=15$$

45. The function is defined when $x=3$. All three pieces would have a value of 20 when $x=3$, so the function is continuous and the limit exists as $x\to 3$. The slopes of the functions to the left and right are not the same, so the slope does not exist at $x=3$.

47. The icosahedron has 20 faces, each of which is an equilateral triangle with a side length of 2, so the total surface area is $20\left(\frac{2^2\sqrt{3}}{4}\right)=20\sqrt{3}$

48. The number of arrangements with the 3 together is $(7!)(3!)=30240$ while the total number of possible arrangements is 9!. The probability that the books will be shelved together is $\frac{30240}{9!}=\frac{1}{12}$.

53. $\frac{A}{B}+\frac{B}{A}=2+\frac{(A-B)^2}{AB}$ for $18^2=324$

57. The semi-perimeter is 7.75, $A=\sqrt{s(s-a)(s-b)(s-c)}$ which in this case will be $\sqrt{7.75\times\frac{f(7.75)}{2}}\approx 11.0$, the 2 comes from the coefficient of the highest degree term.

60. This is the MacClaurin series expansion of $f(x)=e^x$ at $x=3.4$, so evaluate $e^{3.4}\approx 29.964\underline{1}$ with a "1" in the ten-thousandth place.