TMSCA HIGH SCHOOL MATHEMATICS TEST #7 © JANUARY 18, 2020

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2019-2020 TMSCA High School Math Test 7

1. **Evaluate:** $6+12\times 4-(7-3)!-(3\times 5)$

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 15

2. **Andrew cut a 24-foot-long string into four pieces that have the ratio 3:4:5:6. Find the combined length of the shortest piece and the longest piece.**

   (A) 11 ft 4 in **(B)** 11 ft 8 in **(C)** 12 ft **(D)** 12 ft 4 in **(E)** 12 ft 8 in

3. **Line L_1 contains the points $(4, 2)$ and $(-3, 12)$. Line L_2 contains the point $(0, 2)$ and is parallel to L_1. Which of the following points lies on L_2?**

   (A) $(-14, 24)$ **(B)** $(-7, 10)$ **(C)** $(7, -8)$ **(D)** $(14, -16)$ **(E)** $(21, -30)$

4. **Ruocheng bought a 6.25 section ranch in the Texas Panhandle near Pampa. A section is one square mile. How many labors of land did he purchase? (nearest tenth of a labor)**

   (A) 22.6 **(B)** 23.8 **(C)** 25.0 **(D)** 26.2 **(E)** 27.4

5. **Ethan has exactly \$10.00 in nickels, dimes and quarters. He has a total of 72 coins. He has 4 more quarters than nickels. How many quarters does he have?**

   (A) 20 **(B)** 22 **(C)** 24 **(D)** 26 **(E)** 28

6. **Dulles is hosting a fundraiser for their UIL Math team. They are bringing in R. Srinivasa Varadhan to speak on probability theory and to elaborate on creating a unified theory of large deviation. Expenses include a round trip plane ticket costing \$455, two nights in a motel costing \$165 each night, 6 meals costing \$12.50 each, and a speaking fee of \$1500. If tickets cost \$15 each, what is the minimum number of tickets they need to sell to raise \$5000 for the math team?**

   (A) 488 **(B)** 491 **(C)** 494 **(D)** 497 **(E)** 500

7. **Find the range of the function** $y = 2\cdot 3^x + 5$.

   (A) $(-\infty, \infty)$ **(B)** $(0, \infty)$ **(C)** $(3, \infty)$ **(D)** $(5, \infty)$ **(E)** $(7, \infty)$

8. **Given:** $\begin{aligned} 3x+6y&=11 \\ 5x-2y&=-8 \end{aligned}$ **If the solution to the system is (a, b), then $a+b=$ _____.**

   (A) $\frac{5}{4}$ **(B)** $\frac{47}{36}$ **(C)** $\frac{49}{36}$ **(D)** $\frac{17}{12}$ **(E)** $\frac{53}{36}$

9. $\left(4\sqrt{x}+3\right)\left(5-2\sqrt{x}\right)=$ _____

   (A) $8x+14\sqrt{x}+15$ **(B)** $-8x+14\sqrt{x}+15$ **(C)** $-8x-14\sqrt{x}+15$
   (D) $-8x+14\sqrt{x}-15$ **(E)** $8x+14\sqrt{x}-15$

10. Consider right triangle ABC where $\angle C = 90°$, $BC = 4.44$ and $\angle B = 60°$. Find the perimeter of the triangle. (nearest tenth)

(A) 20.1 (B) 20.4 (C) 20.7 (D) 21.0 (E) 21.3

11. The 26-foot-tall red oak tree in Larry's yard cast a shadow that is 47 inches long. If the length of Larry's shadow is 10.58 inches how tall is Larry? (nearest quarter inch)

(A) 5 ft $9\frac{1}{4}$ in (B) 5 ft $9\frac{3}{4}$ in (C) 5 ft $10\frac{1}{4}$ in (D) 5 ft $10\frac{3}{4}$ in (E) 5 ft $11\frac{1}{4}$ in

12. Point B lies on $\overleftrightarrow{AC}$ between A and C and point E lies on $\overleftrightarrow{DF}$ between D and F. $\overleftrightarrow{AC}$ is parallel to $\overleftrightarrow{DF}$. If $m\angle CBE = 42°$, then $m\angle FEB =$ _____.

(A) 42° (B) 48° (C) 90° (D) 132° (E) 138°

13. The center of a circle is point O and points A and C lie on the circle. The diameter of the circle is 52. The distance from O to $\overline{AC}$ is 10. Find the length of $\overline{AC}$. (nearest whole number)

(A) 40 (B) 42 (C) 44 (D) 46 (E) 48

14. Consider regular hexagon ABCDEF with perimeter = 72. AE = _____. (nearest tenth)

(A) 20.8 (B) 21.6 (C) 22.4 (D) 23.2 (E) 24.0

15. Consider triangle ABC with vertices $(3, 5)$, $(-2, -6)$ and $(2b, b)$. If $b > 0$ and the area of the triangle is 30, then b = _____.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

16. Triangle ABC is similar to triangle DEF. AB = 9, BC = 8, AC = 7, and DE = 6. EF + DF = _____. (nearest tenth)

(A) 9.0 (B) $9.\overline{3}$ (C) $9.\overline{6}$ (D) 10.0 (E) $10.\overline{3}$

17. A right circular cone has a diameter of 10 and a slant height of 13. Find the volume of the cone.

(A) 314.2 (B) 316.4 (C) 318.6 (D) 320.8 (E) 323.

18. Consider triangle ABC with point D on $\overline{AC}$. $AB = 20$, $BC = 24$, and $AC = 33$. $\overrightarrow{BD}$ bisects $\angle B$ and $CD > AD$. DC = _____. (nearest tenth)

(A) 17.8 (B) 18.0 (C) 18.2 (D) 18.4 (E) 18.6

19. The graph of $x^2 + y^2 - 10x + 4y - 7 = 0$ is a circle with radius r and center (h, k). $h + k - r =$ _____.

(A) –3 (B) –1 (C) 1 (D) 3 (E) 5

20. Let $f(x) = ax^2 + bx + 4$ where a and b are integers. If $f(3) = 13$ and $f(-3) = 49$, then $f(5) =$ _____.

(A) 40 **(B)** 43 **(C)** 46 **(D)** 49 **(E)** 52

21. Let $f(x) = \dfrac{x^2 + 6x - 8}{x + 2}$ and let s(x) be the slant asymptote of f(x). $s(6) - f(6) =$ _____. (nearest tenth)

(A) 1.2 **(B)** 1.4 **(C)** 1.6 **(D)** 1.8 **(E)** 2.0

22. If -4 is a root of $x^4 + 4x^3 - 7x^2 + cx + 24 = 0$, then c = _____.

(A) -22 **(B)** -20 **(C)** -18 **(D)** -16 **(E)** -14

23. Consider the equation $\ln(x^2 + x) = \ln(20)$. The sum of the solutions to this equation is _____.

(A) -1 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

24. If $A = \begin{bmatrix} 6 & -9 \\ 4 & -5 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $a + b + c + d =$ _____.

(A) 0 **(B)** 0.5 **(C)** 1 **(D)** 1.5 **(E)** 2

25. Consider the sequence $\frac{1}{4}, \frac{3}{8}, \frac{7}{16}, \frac{15}{32}, \frac{31}{64}, \frac{a}{b}$. $b - a =$ _____.

(A) 56 **(B)** 59 **(C)** 62 **(D)** 65 **(E)** 68

26. Find the acute angle between the lines with equations $2x - y = -6$ and $4x + y = -5$.

(A) 37.5° **(B)** 40.6° **(C)** 43.7° **(D)** 46.8° **(E)** 49.9°

27. Find the eccentricity of the ellipse with equation $4x^2 + 9y^2 - 24x + 36y + 36 = 0$. (nearest thousandth)

(A) 0.599 **(B)** 0.667 **(C)** 0.725 **(D)** 0.745 **(E)** 0.800

28. $(2x + 3)^2(x - 5) = ax^3 + bx^2 + cx + d$. $a - b - c - d =$ _____.

(A) 4 **(B)** 30 **(C)** 56 **(D)** 82 **(E)** 108

29. Find the sum of the series. $11 + 14 + 17 + 20 + 23 + \ldots + 80$

(A) 1076 **(B)** 1080 **(C)** 1084 **(D)** 1088 **(E)** 1092

30. Simplify: $\dfrac{\left(2x^2-13x-7\right)}{\left(6x^2-x-2\right)} \div \dfrac{\left(3x^2-19x-14\right)}{\left(3x^2+10x-8\right)}$

(A) $\dfrac{x+4}{3x+2}$ **(B)** $\dfrac{x+4}{3x-2}$ **(C)** $\dfrac{x-4}{3x+2}$ **(D)** $\dfrac{x-4}{3x-2}$ **(E) 1**

31. $\begin{bmatrix} -3 & 4 \\ -7 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 6 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $a+b+c+d =$ _____

(A) –128 **(B) –126** **(C) –124** **(D) –122** **(E) –120**

32. Given a table of values for f(x), find the absolute maximum of f(x). (nearest thousandth)

x	-2	-1	0	1	2	3
f(x)	-54	-11	-4	-3	-2	-19

(A) –1.917 **(B) –1.796** **(C) –1.675** **(D) –1.554** **(E) –1.433**

33. Consider a parabola with a horizontal axis of symmetry, a vertex at $(-1, 3)$ and a directrix with equation $x = -4$. The focus is (a, b). $a + b =$ _____.

(A) 2 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

34. Naveen left the Fort Bend Walmart at 12:00 PM and cycled at a velocity (28 mph, 40° East of North) until 3:15 PM. Eric left the Fort Bend Walmart at 12:45 and cycled at a velocity (32 mph, 10° South of West) until 3:15 PM. How far apart were they at 3:15 PM? (nearest mile)

(A) 158 mi **(B) 161 mi** **(C) 164 mi** **(D) 167 mi** **(E) 170 mi**

35. $\dfrac{x-2}{x^2+4x+3} = \dfrac{A}{x+1} + \dfrac{B}{x+3}$ $A + B =$ _____.

(A) 0 **(B) 0.5** **(C) 1** **(D) 1.5** **(E) 2**

36. Convert the polar equation $r = 6\cos\theta$ to a rectangular equation.

(A) $x^2+y^2-6x-6y=0$ **(B)** $x^2+y^2+6y=0$ **(C)** $x^2+y^2-6y=0$
(D) $x^2+y^2+6x=0$ **(E)** $x^2+y^2-6x=0$

37. Find the area of the region bounded by the graphs of $y = .5x^2 - 6$ and $y = -.25x^2 + 6$.

(A) 60 **(B) 62** **(C) 64** **(D) 66** **(E) 68**

38. Given: $f''(x) = 6,\ f(-1) = 14,\ f(1) = 4.\ \ f'(2) =$ _____.

(A) 1 **(B) 3** **(C) 5** **(D) 7** **(E) 9**

39. Find the sum of the local maximum and the local minimum of $f(x) = 2x^3 - 4x^2 + x + 2$. (nearest hundredth)

(A) 2.85 **(B) 2.96** **(C) 3.07** **(D) 3.18** **(E) 3.29**

40. If $f(x) = 3x^4 + 6x^2$, then the y-intercept of the line tangent to f(x) when $x = -1$ is _____.

(A) -15 **(B) -13** **(C) -11** **(D) -9** **(E) -7**

41. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = -.5(x-4)^2 + 9$ and $y = .25x + 3$ about the line $x = -4$. (nearest whole number)

(A) 988 **(B) 1000** **(C) 1012** **(D) 1024** **(E) 1036**

42. $\lim_{x \to 2}\left(\frac{1}{(x-2)^2}\right) =$

(A) -2 **(B) -1** **(C) 1** **(D) 2** **(E) ∞**

43. A spherical balloon is inflated at a rate of 900 cubic centimeters of air per minute. How fast is the radius of the balloon increasing at the instant the radius is 15 centimeters? (nearest thousandth)

(A) 0.196 cm/min **(B) 0.318 cm/min** **(C) 0.440 cm/min** **(D) 0.562 cm/min** **(E) 0.684 cm/min**

44. Logan decided to evaluate $\int \frac{e^{\frac{1}{x}}}{x^2} dx$ using the method of u-substitution. What should he choose for u?

(A) e^x **(B) x^2** **(C) $2x$** **(D) $\frac{1}{x}$** **(E) $\frac{1}{x^2}$**

45. Find the sum of the series. $1 + \ln(6) + \frac{(\ln(6))^2}{2} + \frac{(\ln(6))^3}{6} + \frac{(\ln(6))^4}{24} + \ldots$

(A) ln(6) **(B) $5.8\overline{6}$** **(C) 6** **(D) $6.1\overline{4}$** **(E) e^6**

46. Jackie's bowling scores from Tuesday night were 144, 136, 162, 144, 154, and 130. Find the sum of the mode, median, mean and range of the scores.

(A) 461 **(B) 463** **(C) 465** **(D) 467** **(E) 469**

47. Anthony's mean score for a round of frisbee golf is 82 with a standard deviation of 12. What is the probability that he will score no lower than 70 and no higher than 94 when he plays a round of frisbee golf? (nearest hundredth)

(A) 0.68 (B) 0.70 (C) 0.72 (D) 0.74 (E) 0.76

48. Keith is going to do a linear regression to predict the weight of fish from their lengths. To do this, he should place the _____ of the lengths in list 1 and the weights in list 2.

(A) cube root (B) square root (C) log (D) square (E) cube

49. At Whitney High School, 45% of the seniors take AP Biology, 40% take AP Chemistry, and 25% take both courses. A senior is selected at random. What is the probability that this student takes at least one of these courses? (nearest hundredth)

(A) 0.56 (B) 0.60 (C) 0.63 (D) 0.67 (E) 0.70

50. Suppose each child of Larry and Kathy has a 36% chance of having red hair. They decide to have five kids. What is the probability that at least 3 of their kids will have red hair? (nearest hundredth)

(A) 0.19 (B) 0.21 (C) 0.23 (D) 0.25 (E) 0.27

51. A poll of eligible voters in Austin found that 56% of those polled were going to vote for John Smith for president. However, the poll had not contacted any of the local college students who live on campus and this likely produced an unreliable result. This is an example of _____.

(A) undercoverage (B) voluntary response (C) sampling frame error (D) nonresponse (E) cluster sampling

52. Darren analyzed some data and he generated a modified box plot with a five-number summary. The five number summary includes the quartiles, the maximum and minimum values, and the _____.

(A) median (B) mean (C) mode (D) range (E) IQR

53. Assume that the mean IQ for the students who compete in UIL mathematics is 128 with a standard deviation of 8. Colby has an IQ of 156. Find the standardized value (z-score) of his IQ.

(A) 2.50 (B) 2.75 (C) 3.00 (D) 3.25 (E) 3.50

54. In statistics, a data point in a set of results that is much smaller or much larger than most of the other values is called a/an _____.

(A) aberration (B) quirk (C) outlier (D) irregularity (E) deviation

55. Consider the geometric sequence $24, a, b, c, 9.8304$ $a+b+c$ could equal _____.

(A) –46.848 (B) –16.128 (C) –8.764 (D) 29.162 (E) 36.454

56. $555_6 + 666_7 + 777_8 =$ _____$_{14}$.

(A) 530 **(B) 542** **(C) 553** **(D) 564** **(E) 575**

57. Given the Fibonacci characteristic sequence 3, 7, 10, 17, 27, a, b, c, d, 301 Find the sum of the numbers in this sequence.

(A) 773 **(B) 777** **(C) 781** **(D) 785** **(E) 789**

58. This mathematician is known as the "father of geometry" and he wrote a book called <u>The Elements</u>.

(A) Euclid **(B) Archimedes** **(C) Diophantus** **(D) Ptolemy** **(E) Descartes**

59. Which of the following numbers is a "happy prime" number?

(A) 5 **(B) 11** **(C) 17** **(D) 19** **(E) 29**

60. $3^0 + 3^1 + 3^2 + 3^3 + ... + 3^9 =$

(A) 29500 **(B) 29506** **(C) 29512** **(D) 29518** **(E) 29524**

2019 – 2020 TMSCA High School Mathematics Test 7
Answer Key

1. E	**21. E**	**41. E**
2. C	**22. A**	**42. E**
3. C	**23. A**	**43. B**
4. A	**24. C**	**44. D**
5. D	**25. D**	**45. C**
6. B	**26. B**	**46. C**
7. D	**27. D**	**47. A**
8. E	**28. E**	**48. E**
9. B	**29. E**	**49. B**
10. D	**30. A**	**50. D**
11. C	**31. A**	**51. A**
12. E	**32. C**	**52. A**
13. E	**33. D**	**53. E**
14. A	**34. B**	**54. C**
15. C	**35. C**	**55. B**
16. D	**36. E**	**56. D**
17. A	**37. C**	**57. C**
18. B	**38. D**	**58. A**
19. A	**39. B**	**59. D**
20. D	**40. A**	**60. E**

19-20 TMSCA HSMA Test 7 Selected Solutions

$18x = 24(12)$

2. $x = 16$

$9x = 144 = 12 \text{ ft}$

$m = -\frac{10}{7}$

3. $y - 2 = -\frac{10}{7}x$

$(7, -8)$

4. $\frac{(6.25)(640)}{177.1} = 22.6$

$n + d + q = 72$

5. $5n + 10d + 25q = 1000$

$q = n + 4$

$q = 26$

8. $\frac{455 + 2(165) + 6(12.5) + 1500 + 5000}{15} = 491$

10. $4.44 + 8.88 + (4.44)\sqrt{3} = 21.0$

$\frac{(26)(12)}{47} = \frac{x}{10.58}$

11. $x = 70.233$

$5 \text{ ft } 10\frac{1}{4} \text{ in}$

12. $180 - 42 = 138°$

13. $x^2 + (10)^2 = (26)^2$

$2x = 48$

14. $c^2 = 12^2 + 12^2 - 2(12)(12)\cos(120°)$

$c = 20.8$

15. $30 = (.5)\begin{vmatrix} 3 & 5 & 1 \\ -2 & -6 & 1 \\ 2b & b & 1 \end{vmatrix}$

$b = 4$

16. $\frac{9}{6} = \frac{8}{x} = \frac{7}{w}$

$x + w = 10$

17. $5^2 + h^2 = 13^2$

$h = 12$

$V = \frac{1}{3}\pi(5)^2(12)$

$V = 314.2$

$\frac{20}{x} = \frac{24}{33 - x}$

18. $x = 15$

$33 - x = 18$

$x^2 - 10x + 25 + y^2 + 4y + 4 = 7 + 25 + 4$

19. $(x - 5)^2 + (y + 2)^2 = 36$

$5 - 2 - 6 = -3$

$9a + 3b + 4 = 13$

20. $9a - 3b + 4 = 49$

$a = 3, \ b = -6$

$3(5)^2 - 6(5) + 4 = 49$

21. $s(x) = x + 4$

$s(6) - f(6) = 2$

$x^2 + x = 20$

23. $x = -5, 4$

$-5 + 4 = -1$

24. $A^{-1} = \begin{bmatrix} -\frac{5}{6} & \frac{3}{2} \\ -\frac{2}{3} & 1 \end{bmatrix}$

$\text{sum} = 1$

25. $a = 63, \ b = 128$

$128 - 63 = 65$

$m_1 = 2, \ m_2 = -4$

26. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\theta = 40.6°$

$4\left(x^2 - 6x + 9\right) + 9\left(y^2 + 4y + 4\right) = -36 + 36 + 36$

27. $\frac{(x - 3)^2}{9} + \frac{(y + 2)^2}{4} = 1$

$9 = 4 + c^2, \ c = \sqrt{5}$

$e = \frac{c}{a} = \frac{\sqrt{5}}{3} = .745$

28. $4 + 8 + 51 + 45 = 108$

29. $\frac{24}{2}(11 + 80) = 1092$

31. $\begin{bmatrix} -26 & -30 \\ -24 & -48 \end{bmatrix}$

-128

32. $f(x) = -x^4 + 3x^3 - 2x^2 + x - 4$

$\max = -1.675$

34. $c^2 = 80^2 + 91^2 - 2(80)(91)\cos(140°)$

$c = 161$

35. $-\frac{3}{2} + \frac{5}{2} = 1$

36. $r^2 = 6r\cos\theta$

$x^2 + y^2 = 6x$

$x^2 + y^2 - 6x = 0$

37. $\int_{-4}^{4} (y_2 - y_1)\,dx = 64$

38. $f' = 6x + c$

$f = 3x^2 + cx + d$

$f(-1) = 14 = 3 - c + d$

$f(1) = 4 = 3 + c + d$

$c = -5,\ f' = 6x - 5$

$f'(2) = 7$

39. $2.0670885 - 0.89587 = 2.96$

41. $V = 2\pi \int_{.5778556}^{6.92214} (x+4)(y1 - y2)\,dx$

$V = 1036$

43. $V = \frac{4}{3}\pi R^3$

$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$

$900 = 4\pi(15)^2 \frac{dR}{dt}$

$\frac{dR}{dt} = 0.318$

45. $e^{\ln(6)} = 6$

46. $144 + 144 + 145 + 32 = 465$

47. one SD each side of mean

Area = 0.68

49. $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$.45 + .40 - .25 = .60$

50. binomialCdf = .25

53. $z = \frac{156 - 128}{8} = 3.5$

55. $24 \cdot r^4 = 9.8304$

$r = -.8, .8$

$-19.2 + 15.36 + (-12.288) = -16.128$

56. $215 + 342 + 511 = 1068$

$1068_{10} = 564_{14}$

57. $71 \times 11 = 781$