T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 6 (U I L C) ©

D E C E M B E R 7 , 2 0 1 9

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. **Evaluate: $1 - 2^0 \times 7 + 20 \div 2 + 0!$**

   (A) 4 (B) 5 (C) 8 (D) 10 (E) 12

2. **Let P = {2, 3, 5, 7, 11}, O = {1, 3, 5, 7, 9}, and T = {1, 3, 6, 10, 15}. The number of the elements in $O \cap (P \cup T)$ is:**

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. **A B C D E**
   <----------|----------|----------|----------|----------|---------->
   The distances between the hash marks (|) are equal. Find the length of segment AB if A = 0.5 and E = 1.5

   (A) 0.25 (B) 0.5 (C) 0.75 (D) 1 (E) 1.25

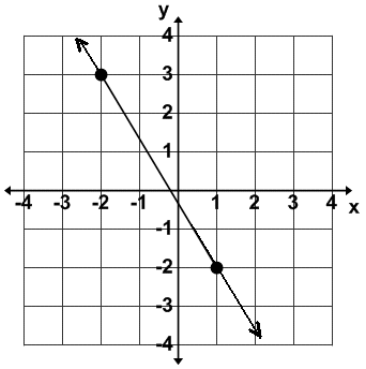
4. **A diagram used in the area of sets and probability was developed by which of the following mathematicians?**

   (A) Mary Rudin (B) John Napier (C) Sophie Germain (D) Alan Turing (E) John Venn

5. **Liquid measurements include teaspoons, tablespoons, gills, pints, quarts, gallons, and barrels. Saul T. Water poured 1 gallon 1 quart 1 pint of liquid out of his 5 gallon bucket of liquid. How much liquid was left in the bucket?**

   (A) 4 gal 2 qt 1 pt (B) 4 gal 1 qt 1 pt (C) 3 gal 2 qt 1 pt (D) 3 gal 1 qt 1 pt (E) 3 gal 1 pt

6. **Find the equation of the line shown below.**



   (A) $5x + 3y = -1$ (B) $3x - 5y = 1$ (C) $5x + 3y = 11$ (D) $3x - 5y = 11$ (E) $5x + 3y = 1$

7. **Let $(2x + P)(4x - 3) = Qx^2 - 2x + R$. Find $P + Q - R$.**

   (A) 3 (B) 6 (C) 9 (D) 10 (E) 12

8. **$\triangle ABC$ exists such that AB = 4", AC = 7", BC = 5", and point M lies on segment AB where AM = BM. Find CM. (nearest tenth)**

   (A) 2.9" (B) 6.3" (C) 5.7" (D) 3.8" (E) 5.1"

9. The area of a regular polygon is _____ the product of the apothem and the perimeter.

(A) equal to (B) twice (C) one-half (D) two-thirds (E) greater than

10. The table shows the distribution of people who entered a contest. If the contest winner is chosen at random what is the probability that the winner will be either female under 40 or male 40 or older?

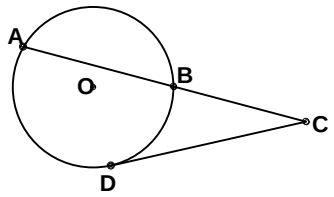
	Under 40	40 plus	Total
Male	12	2	14
Female	8	3	11
Total	20	5	25

(A) 80% (B) 20% (C) 10 % (D) 40% (E) 60%

11. Point P(— 4, — 3) lies on the x-y plane. P is translated vertically 6 units up to point Q. Then Q is rotated 90° clockwise about the origin to point R. Then R is translated horizontally 2 units to the left to point S. Then S is reflected across the y-axis to point T(x, y). Find $x + y$.

(A) — 7 (B) — 4 (C) — 1 (D) 1 (E) 3

12. If AB = 7" and BC = 5" then CD = ? (nearest tenth)



(A) 7.7" (B) 8.4" (C) 5.9" (D) 7.3" (E) 6.0"

13. The sum of the product of the roots taken three at a time of $x^5 - 11x^4 + 45x^3 - 85x^2 + 74x - 24 = 0$ is ?

(A) — 85 (B) — 74 (C) 45 (D) 74 (E) 85

14. If $a_1 = -2$, $a_2 = 1$, $a_3 = 3$ and $a_n = (a_{n-2})^{(a_{n-3})} - (a_{n-1})$, where $n \geq 4$, then a_5 equals:

(A) 6 (B) 5 (C) — 2 (D) — 3 (E) — 13

15. △ABE is a right triangle. Point C lies on BE and point D lies on AE forming a right angle, ∠CDE. Find AE if m∠ACB = 40°, m∠CAD = 20°, CE = 3". (nearest tenth)

(A) 5.6" (B) 4.9" (C) 6.0" (D) 5.8" (E) 5.3"

16. Hi Kerr is walking up a hill with a slope of 15° at a constant rate of 2 mph. How much altitude does Hi gain in 3 hours? (nearest foot)

(A) 2,733 ft (B) 8,199 ft (C) 7,920 ft (D) 8,489 ft (E) 30,601 ft

17. How many real number solutions exist such that $5\sin(x - 4) = 3\cos(x + 2)$, where $x \in [-1, 6]$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

18. $(214_7 + 115_7 \times 6_7) \div 5_7$ has a remainder of _____.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. Which of the following have a remainder of 1?

I. $4^{10} \div 11$ II. $7^{13} \div 13$ III. $9^6 \div 7$ IV. $5^7 \div 9$

(A) I & III (B) I, II, & III (C) II & III (D) II only (E) I only

20. Which of the following recursive rules would produce the sequence 5, 11, 17, 23, ..., 47, ... ?

(A) $a_n = 6(a_{n-1})$ (B) $a_n = (a_{n-1}) + 3$ (C) $a_n = 2(a_{n-1}) + 1$
(D) $a_n = (a_{n-1}) + 6$ (E) $a_n = 3(a_{n-1}) + 1$

21. Chip Picker has a box containing four chips numbered 1, 2, 3, 4. He wants to randomly select two chips without replacement creating ordered pairs. How many elements are in the sample space of this event?

(A) 4 (B) 12 (C) 16 (D) 20 (E) 24

22. Willie When randomly selects two distinct numbers from the set of digits. What is the probability both numbers are prime numbers?

(A) 40% (B) $7\frac{1}{7}\%$ (C) $13\frac{1}{3}\%$ (D) $17\frac{1}{7}\%$ (E) 20%

23. Which of the following are considered to be "emirp primes" ?

I. 59 II. 73 III. 89 IV. 113

(A) I & III (B) I, II & IV (C) III only (D) IV only (E) II & IV

24. $1AC2_{16} - 3B4_{16} =$ _____ $_{16}$.

(A) 1714 (B) 170E (C) 1708 (D) 160E (E) 161E

25. If the triangular pattern continues as shown below then D + G equals ?.

					1						(row 1)
				1	1	1					(row 2)
			1	2	3	2	1				(row 3)
		1	3	6	7	6	3	1			(row 4)
	1	4	10	16	19	16	10	4	1		(row 5)
1	A	B	C	D	E	F	G	H	I	1	(row 6)

(A) 75 (B) 90 (C) 52 (D) 60 (E) 57

26. The *Deep Sleep* store sells its mattresses for 15% off of the retail price. If someone pays cash, they take 10% off of the discounted price. If someone pays with a credit card, they add 3% of the discounted price. How much would a buyer save by paying cash for a mattress that retails for $650 instead of using a credit card? (nearest cent)

(A) $38.68 (B) $45.50 (C) $56.92 (D) $71.83 (E) $84.50

27. *Chik N A Box* charges 40¢ for a chicken breast, 25¢ for a chicken leg or wing, and 50¢ for the box. Les Peeces has $5.00 to pay for a box of chicken containing 2 legs, 2 wings, and some chicken breasts. What is the greatest number of chicken breasts could be in the box?

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

28. Tu Yung is k years old. Her sister, Soh, is k^2 years old. In 8 years, Soh will be twice as old as Tu will be. How old will Tu be in 5 years?

(A) 21 (B) 16 (C) 10 (D) 9 (E) 7

29. Point B is 3 units due south and 2 units due west of point A. Point C is 2 units due south and 4 units due east of point B. How far is point C to point A? (nearest tenth of a unit)

(A) 5.4 (B) 4.9 (C) 3.6 (D) 3.3 (E) 2.2

30. Rose Thorn's rectangular garden is 12 feet wide and 25 feet long. She wants to add 3 feet to the width and shorten the length by 3 feet. What is the difference in the area of her original garden and her new garden?

(A) 0 ft^2 (B) 9 ft^2 (C) 18 ft^2 (D) 20 ft^2 (E) 30 ft^2

31. The distance from Hear to Thare is 80 km on a bearing of 240°. The distance from Hear to Yondare is 60 km on a bearing of 120°. What is the distance from Thare to Yondare? (nearest km)

(A) 130 km (B) 72 km (C) 98 km (D) 144 km (E) 122 km

32. If a 60 question test was scored on a 0 to 60 scale and you scored 48, what would your score be on a 0 to 100 scale?

(A) 88 (B) 28.8 (C) 66.666... (D) 86.666... (E) 80

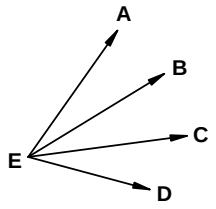
33. Simplify: $\left(\frac{x^2-3x-10}{x^2+2x-35}\right) \div \left(\frac{x^2+9x+14}{x^2+4x-21}\right)$

(A) $\frac{x-5}{x+7}$ **(B)** $\frac{x+5}{x+2}$ **(C)** $\frac{x-3}{x+2}$ **(D)** $\frac{x-3}{x+7}$ **(E)** $\frac{x+2}{x-5}$

34. Find k if $(\sin(x) - 2\cos(x))^2 + 3\sin^2(x) + 2\sin(2x) + k = 0$.

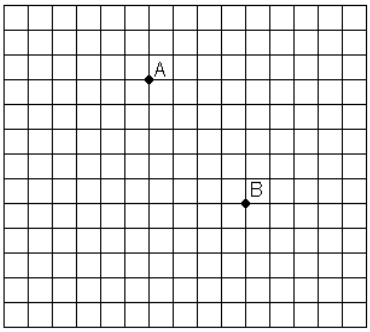
(A) −4 (B) −2 (C) 0 (D) 1 (E) 5

35. Angle AED is an acute angle. How many other acute angles are shown?



(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

36. In the graph below, no axes or origin is shown. If point A's coordinates are (— 4, 2), and B's coordinates are (p, q), then p + q = ?



(A) — 3 **(B) — 2** **(C) — 1** **(D) 6** **(E) 7**

37. If $\frac{x-3}{x-2} + \frac{x+2}{x+3} = \frac{ax^2+bx+c}{x^2+x-6}$, then $a+b+c$ equals:

(A) — 15 **(B) — 13** **(C) — 11** **(D) 5** **(E) 13**

38. If $\frac{d}{dx}\left(\frac{3x-1}{2x+5}\right) = \frac{a(bx+c)-b(ax-d)}{(bx+c)^2}$, then a + b + c + d = ?

(A) 1 **(B) 5** **(C) 7** **(D) 9** **(E) 11**

39. Roland Dye tosses a fair die twice. The results are recorded in order to form a two-digit number. What are the odds that the number is a perfect square?

(A) $11\frac{1}{9}\%$ **(B) $12\frac{1}{2}\%$** **(C) 25%** **(D) $33\frac{1}{3}\%$** **(E) 50%**

40. If you start at (— 1, 0) on a unit circle and travel — 75 radians, where will you come to a stop on the unit circle?

(A) y-axis **(B) QI** **(C) QII** **(D) QIII** **(E) QIV**

41. An infinite geometric sequence has a common ratio of $-\frac{1}{2}$ and a sum of 20. What is the first term of the sequence?

(A) 40 **(B) 30** **(C) 10** **(D) — 10** **(E) — 40**

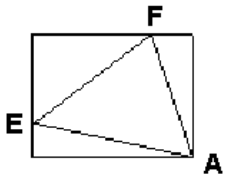
42. How many arrangements of five letters are possible using three letters from the word SQUARE and two letters from the word FOOT?

(A) 7,200 (B) 120 (C) 8,400 (D) 70 (E) 720

43. Which of the following functions is a decreasing function?

(A) $f(x) = x^3, x < 0$ (B) $f(x) = x^2, x > 0$ (C) $f(x) = |x|, x > 0$
(D) $f(x) = -2x + 1$ (E) $f(x) = \log x, x > 0$

44. Equilateral $\triangle AEF$ is inscribed in the 6" square as shown. Find EF. (nearest hundredth)



(A) 6.19" (B) 6.21" (C) 6.25" (D) 6.30" (E) 6.32"

45. Log (x) has a characteristic of — 2 and a mantissa of 0.534. Find x. (nearest ten-thousandth)

(A) 0.0342 (B) 0.0340 (C) 0.0292 (D) 0.0147 (E) 0.0029

46. Given: AB||CD, AB < CD, AC = 25 cm, BD = 18 cm, CD = 37 cm, $m\angle ABD = 105°$, points E and F lie on segment CD forming right triangles, $\triangle$ CEA and $\triangle$ DFB. Find AB. (nearest tenth)

(A) 12.7 cm (B) 13.3 cm (C) 14.4 cm (D) 15.0 cm (E) 16.4 cm

47. The sum of 49 consecutive integers is $(7)^5$. What is the median of these 49 integers?

(A) 129.641... (B) 343 (C) 2,401 (D) 8,403.5 (E) not enough information

48. Find the sum of the solutions to the equation $3(3^{2x}) - 28(3^x) + 3^2 = 0$.

(A) -2 (B) -1 (C) 1 (D) 2 (E) 3

49. The sum of all of the x and y values of the positive integer ordered pairs (x, y) that satisfy $x^2 + 7 = xy + 3x$ is?

(A) 6 (B) 8 (C) 12 (D) 13 (E) 18

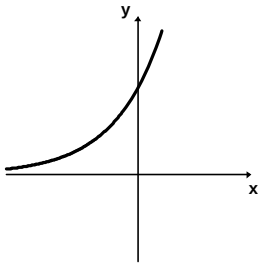
50. The largest prime divisor of $87! + 88!$ is _____.

(A) 79 (B) 83 (C) 89 (D) 93 (E) cannot be determined

51. The intersection of a right circular cone and a plane that is parallel to the edge of the cone is a:

(A) circle (B) ellipse (C) hyperbola (D) intersecting lines (E) parabola

52. Which of the following functions could be represented by this graph?



(A) $y = ax^n, n > 1$ **(B)** $y = \log_a(x)$ **(C)** $y = ax$ **(D)** $y = ab^x, b > 1$ **(E)** $y = \frac{P(x)}{Q(x)}$

53. R.U. Shur is taking a unusual math test. The test has fifteen 8-point questions and fifteen 6-point questions. He can only get credit for twenty questions and the number of 6-point questions must be more than half of the number of 8-point questions. What could his maximum score be?

(A) 160 **(B) 150** **(C) 148** **(D) 146** **(E) 120**

54. Lotta Dough puts $250.00 into her retirement account at the end of each quarter of the year. The account pays 4% interest compounded quarterly. How much will be in Lotta's account at the end of the first year? (nearest cent)

(A) $767.73 **(B) $1,015.10** **(C) $1,035.45** **(D) $1,172.58** **(E) $1,275.25**

55. The set of numbers, {2, 5, 3, 7, 2, 6, 3}, are the number of times different Millersview-ites missed the monthly luncheon this past year. Find the *variance* of this set of data? (nearest hundredth)

(A) 1.85 **(B) 2.15** **(C) 3.00** **(D) 3.43** **(E) 4.00**

56. Let $f(x) = \begin{cases} -x & \text{if } x < 1 \\ x - 1 & \text{if } x > 1 \end{cases}$ **. At what value assigned to $f(1)$ will make f continuous at x = 1?**

(A) -1 **(B) 0** **(C) 1** **(D) 2** **(E) no value exist**

57. Dee Joker is dealt a hand of 5 cards from a standard deck of 52 cards. What is the probability that all 5 cards will be the same suit given that the first 3 cards are of the same suit? (nearest tenth)

(A) 0.5% **(B) 1.0%** **(C) 3.8%** **(D) 9.4%** **(E) 15.3%**

58. Let $k = 9 \times n$, **where** $n \in$ **{natural numbers} and all of the digits of k are even. The sum of the digits of the smallest possible k is?**

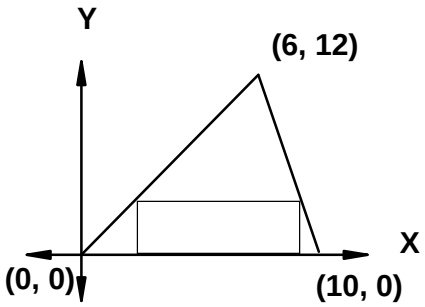
(A) 7 **(B) 9** **(C) 16** **(D) 18** **(E) 24**

59. Given that the set of natural numbers continue in the triangular pattern shown below, find the first number in row 10.

				1					(row 1)
			2	3	4				(row 2)
		5	6	7	8	9			(row 3)
	10	11	12	13	14	15	16		(row 4)
17	18	19	20	21	22	23	24	25	(row 5)
				...					(...)

(A) 97 **(B) 90** **(C) 85** **(D) 82** **(E) 81**

60. A rectangle is inscribed in a triangle as shown below such that it has the maximum possible area. What is the length of the rectangle? (drawing not to scale)



(A) 11 units **(B) 3 units** **(C) 8 units** **(D) 5 units** **(E) 6 units**

2019-20 TMSCA HS Math Test #6
Answer Key

1. B	**21. B**	**41. B**
2. D	**22. C**	**42. C**
3. A	**23. E**	**43. D**
4. E	**24. B**	**44. B**
5. C	**25. A**	**45. A**
6. A	**26. D**	**46. C**
7. E	**27. B**	**47. B**
8. C	**28. D**	**48. C**
9. C	**29. A**	**49. E**
10. D	**30. E**	**50. C**
11. E	**31. E**	**51. E**
12. A	**32. E**	**52. D**
13. E	**33. D**	**53. D**
14. B	**34. A**	**54. B**
15. A	**35. C**	**55. D**
16. B	**36. A**	**56. E**
17. C	**37. C**	**57. C**
18. A	**38. E**	**58. D**
19. A	**39. B**	**59. D**
20. D	**40. C**	**60. D**