HMMT February 2025

February 15, 2025

Geometry Round

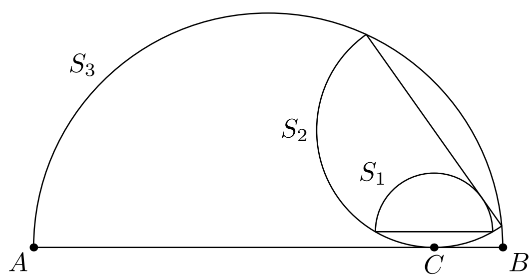
1. Equilateral triangles $\triangle ABC$ and $\triangle DEF$ are drawn such that points B, E, F, and C lie on a line in this order, and point D lies inside triangle $\triangle ABC$. If $BE = 14$, $EF = 15$, and $FC = 16$, compute AD.

2. In a two-dimensional cave with a parallel floor and ceiling, two stalactites of lengths 16 and 36 hang perpendicularly from the ceiling, while two stalagmites of heights 25 and 49 grow perpendicularly from the ground. If the tips of these four structures form the vertices of a square in some order, compute the height of the cave.

3. Point P lies inside square $ABCD$ such that the areas of $\triangle PAB$, $\triangle PBC$, $\triangle PCD$, and $\triangle PDA$ are 1, 2, 3, and 4, in some order. Compute $PA \cdot PB \cdot PC \cdot PD$.

4. A semicircle is inscribed in another semicircle if the smaller semicircle's diameter is a chord of the larger semicircle, and the smaller semicircle's arc is tangent to the diameter of the larger semicircle.

   Semicircle S_1 is inscribed in a semicircle S_2, which is inscribed in another semicircle S_3. The radii of S_1 and S_3 are 1 and 10, respectively, and the diameters of S_1 and S_3 are parallel. The endpoints of the diameter of S_3 are A and B, and S_2's arc is tangent to AB at C. Compute $AC \cdot CB$.



5. Let $\triangle ABC$ be an equilateral triangle with side length 6. Let P be a point inside triangle $\triangle ABC$ such that $\angle BPC = 120°$. The circle with diameter $\overline{AP}$ meets the circumcircle of $\triangle ABC$ again at $X \neq A$. Given that $AX = 5$, compute XP.

6. Trapezoid $ABCD$, with $AB \parallel CD$, has side lengths $AB = 11$, $BC = 8$, $CD = 19$, and $DA = 4$. Compute the area of the convex quadrilateral whose vertices are the circumcenters of $\triangle ABC$, $\triangle BCD$, $\triangle CDA$, and $\triangle DAB$.

7. Point P is inside triangle $\triangle ABC$ such that $\angle ABP = \angle ACP$. Given that $AB = 6$, $AC = 8$, $BC = 7$, and $\frac{BP}{PC} = \frac{1}{2}$, compute $\frac{[BPC]}{[ABC]}$.

   (Here, $[XYZ]$ denotes the area of $\triangle XYZ$).

8. Let $ABCD$ be an isosceles trapezoid such that $CD > AB = 4$. Let E be a point on line CD such that $DE = 2$ and D lies between E and C. Let M be the midpoint of $\overline{AE}$. Given that points A, B, C, D, and M lie on a circle with radius 5, compute MD.

9. Let $ABCD$ be a rectangle with $BC = 24$. Point X lies inside the rectangle such that $\angle AXB = 90°$. Given that triangles $\triangle AXD$ and $\triangle BXC$ are both acute and have circumradii 13 and 15, respectively, compute AB.

10. A plane $\mathcal{P}$ intersects a rectangular prism at a hexagon which has side lengths 45, 66, 63, 55, 54, and 77, in that order. Compute the distance from the center of the rectangular prism to $\mathcal{P}$.