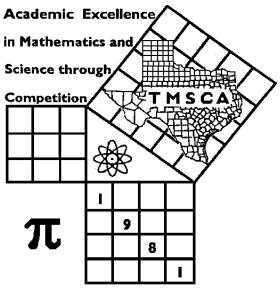


TMSCA HIGH SCHOOL MATHEMATICS TEST # 11 © FEBRUARY 17, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2017-2018 TMSCA Mathematics Test Eleven

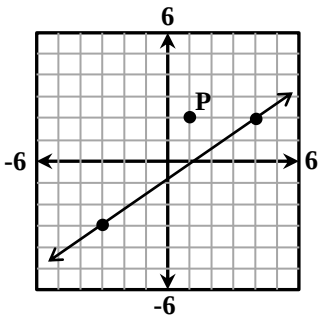
1. **Evaluate** $2.8 \div \left(\frac{5}{4}\right)^{-1} - 5! + 2.3$.

   (A) -117.56 **(B)** -115.46 **(C)** -120.06 **(D)** -114.2 **(E)** -118.8

2. **Caroline had a rope that was 60 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:12 with 40 inches of string left over. How long was the longest piece?**

   (A) 6 ft. 8 in. **(B)** 10 ft. **(C)** 40 ft. **(D)** 13 ft. 4 in. **(E)** 38 ft. 9 in.

3. **The *x*-intercept of the line through point P that is perpendicular to the line shown is** (x, y). **Find** $x + y$.



   (A) $-\frac{3}{7}$ **(B)** $\frac{5}{6}$ **(C)** 3

   (D) $\frac{17}{5}$ **(E)** $\frac{17}{7}$

4. **If** $-7 = 3x - y$, $x - 4y = -17$ **and** $x + ay = 0$, **then** $a = ?$

   (A) 0.25 **(B)** -2.5 **(C)** 1.5 **(D)** 0.4 **(E)** 5

5. **A survey of 75 homes in a particular neighborhood reveals that 38 of the homes have cable but not a phone line. Sixteen of the houses have cable and phone lines. Nine have phone lines but not cable. How many houses have neither cable nor phone lines?**

   (A) 17 **(B)** 5 **(C)** 21 **(D)** 12 **(E)** 15

6. **Simplify:** $\frac{(n+5)!}{(n-1)!} \div \frac{1}{n} \div \frac{(n+3)!}{n!}$

   (A) $n^4 + 9n^3 + 20n^2$ **(B)** $n^2 + 9n + 20$ **(C)** $n^3 + 5n$

   (D) $n^2 + 8n + 15$ **(E)** $\frac{n^2 + 9n + 20}{n^2}$

7. **If** $\theta = 5\lambda$ **and** $\theta = \alpha + \varphi$, **then** $5\lambda = \alpha + \varphi$. **This is an example of the _____ property.**

   (A) Distributive (B) Transitive (C) Commutative (D) Associative (E) Reflexive

8. **1075 inches per second is the same as _____ miles per hour. (nearest mile per hour)**

   (A) 1 **(B)** 5 **(C)** 12 **(D)** 61 **(E)** 305

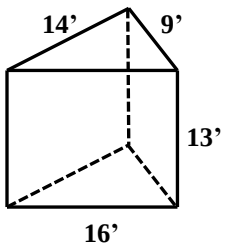
9. **If *p* and *q* are the zeros of the function** $f(x) = 17x^2 + 3x - 32$ **then** $pq^2 + p^2q =$

   (A) $-\frac{96}{289}$ **(B)** $\frac{48}{289}$ **(C)** $\frac{225}{289}$ **(D)** $-\frac{48}{289}$ **(E)** $\frac{96}{289}$

10. $\angle A$ **and** $\angle B$ **are supplementary. If** $m\angle A = (x^2 - 3x)^\circ$ **and** $m\angle B = (9x - 7)^\circ$, **find the measure of the larger angle.**

    (A) 62° **(B)** 88° **(C)** 92° **(D)** 28° **(E)** 98°

11. The lateral surface area of the right triangular prism shown is _____ft^2.



(A) 633 (B) 819 (C) 515 (D) 647 (E) 507

12. The four sisters Lesley, Michelle, Nora and Patty wanted to go on a road trip, but Lesley had no money. Michelle, Nora and Patty each gave Lesley one-fifth, one-sixth and one-seventh of her money respectively. If each gave Lesley the same amount, what fraction of the money did Lesley possess after the exchange?

(A) $\frac{1}{6}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{67}{210}$ **(E)** $\frac{1}{5}$

13. Lily has 11 non-fiction books to shelve. She wants to keep her 4 math books together and her 3 grammar books together, but otherwise any order is fine. In how many different ways can Lily organize her shelf?

(A) 17,280 (B) 8,640 (C) 103,680 (D) 3,456 (E) 34,560

14. Let $x^2 + x$ **and** $g(x) = x^5$**. Calculate** $f(g(-2))$**.**

(A) $-1,023$ **(B) 1025 (C) 32 (D) 992 (E)** -1025

15. Find the exact value of *x* satisfying $\left(3^{2x+1}\right)\left(4^x\right) = 6^{x+2}$**.**

(A) $\ln(2)$ **(B)** $\ln(6)$ **(C)** $\frac{\ln(12)}{\ln(6)}$ **(D)** $\frac{\log(3)}{\log(4)}$ **(E)** $\frac{\log(6)}{\ln(2)}$

16. An operation "Ω" is defined by $a \Omega b = b^a - ab$**. What is the value of** $(3\Omega - 1) + (3\Omega 2)$**?**

(A) 4 (B) 6 (C) 9 (D) 3 (E) 7

17. There are two values of *k* for which $\begin{vmatrix} k+1 & 5 \\ 3 & 2k \end{vmatrix} = 25$**. The sum of those two values is**

(A) 1 (B) 17 (C) 3 (D) –2 (E) –1

18. Given the sequence 4, 9, 18, 31, 48, …, *k*, 193, … find *k*.

(A) 123 (B) 121 (C) 189 (D) 156 (E) 152

19. Point $P(-5,2)$ **lies on the x-y plane. Point P is reflected over the line** $y = -x$ **to point Q. Point Q is translated +4 units horizontally to point R. Point R is rotated 270° clockwise around the origin to point S. The coordinates of point S are** (x, y)**. Find** $x + y$**.**

(A) –1 (B) 11 (C) 1 (D) 4 (E) –3

20. The intersection of the three perpendicular bisectors of the sides in a triangle is called the_____.

(A) Centroid (B) Incenter (C) Orthocenter (D) Center (E) Circumcenter

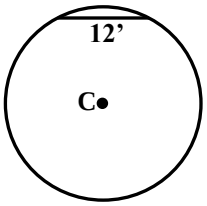
21. Which of the following functions is neither even nor odd? $f(x) =$ _____.

(A) $y = \sin(2x)$ **(B)** $y = |x| + 7$ **(C)** $y = |x - 2|$ **(D)** $y = 3x^5 - 2x$ **(E)** $y = 3\cos x$

22. How many distinct 4-letter arrangements can be formed using the letters in "GEOMETRY"?

(A) 1,680 **(B)** 1,020 **(C)** 840 **(D)** 3,720 **(E)** 5,160

23. The circle C shown has a diameter of 20 feet. How far is the chord shown from the center of the circle?



(A) 16 ft. **(B)** 12 ft. **(C)** 9 ft. **(D)** 8 ft. **(E)** $4\sqrt{2}$ ft.

24. If $\frac{A}{x+3} + \frac{B}{3x-4} = \frac{17x-14}{3x^2+5x-12}$ **, then** $A + B =$ **?**

(A) –6 **(B)** 7 **(C)** –2 **(D)** 10 **(E)** –5

25. Let $f(x) = 9x^3 - 7x^2 - 52$ **. Find** $f'(-2)$ **.**

(A) 132 **(B)** –152 **(C)** 172 **(D)** 156 **(E)** 136

26. Let the "1" at the top of Pascal's triangle be row 0. Determine the fifth number in row 23.

(A) 33,649 **(B)** 10,626 **(C)** 42,504 **(D)** 100,947 **(E)** 8,855

27. Which of the following is an equation of the tangent line of $f(x) = 2x^2 - x + \frac{32}{x}$ **when** $x = 2$ **?**

(A) $y = x + 20$ **(B)** $y = -x + 24$ **(C)** $y = -x + 26$ **(D)** $y = x + 24$ **(E)** $y = -x + 20$

28. Which of the following is the reference angle for 3105°?

(A) 60° **(B)** 0° **(C)** 30° **(D)** 45° **(E)** None of these

29. If $2 + 3i$ **is one of the zeros the polynomial** $f(x) = 7x^3 - 31x^2 + 103x - 39$**, then another of its zeros is:**

(A) $\frac{2}{3}$ **(B)** $\frac{5}{11}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{6}{13}$ **(E)** $\frac{2}{5}$

30. Let $A = \begin{bmatrix} 7 & -9 \\ -2 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 5 & 3 \\ 2 & -7 \end{bmatrix}$**. Find the sum of the elements in** BA**.**

(A) 55 **(B)** 151 **(C)** –30 **(D)** 90 **(E)** –53

31. Given that $x - y = -15$ **and** $xy = 28$**, find** $x^3 - y^3$**.**

(A) –4,635 **(B)** –4,215 **(C)** –3,795 **(D)** –2,955 **(E)** –2,115

32. Which of the following equations in rectangular form can be written as $r - 10\sin\theta = 0$ **in polar form?**

(A) $x^2 + y^2 = 25$ **(B)** $x^2 + y^2 = 5$ **(C)** $x^2 + y^2 - 10y = 0$

(D) $x^2 - 10x + y^2 = 0$ **(E)** $x^2 + y^2 - 10y + 50 = 0$

33. $2222_5 + 2222_6 + 2222_7 = \_\_\_\_\_{}_8$.

(A) 1,170 **(B)** 3,226 **(C)** 1,630 **(D)** 3,136 **(E)** 2,360

34. Find the sum of the digits in the ones and tens places of 7^{107} **.**

(A) 1 **(B)** 7 **(C)** 13 **(D)** 9 **(E)** 3

35. The formulas that relate the coefficients of a polynomial to the sums and product of its roots are named after which of the following mathematicians?

(A) Franciscus Vieta **(B)** Sophie Germain **(C)** Marin Mersenne

(D) Zeno of Alea **(E)** Freda Porter

36. If $\left[(5+3i)(2-3i)\right] \div (2-i) = a+bi$ **, then** $a+b =$ **?**

(A) 9.2 **(B)** 9.4 **(C)** 0.2 **(D)** 0.4 **(E)** 9.6

37. If $g(x) \le f(x) \le h(x)$ **for all** x**,** k **in** $[a,b]$**, where** $x \neq k$ **, and** $\lim_{x\to k} g(x) = L$ **and** $\lim_{x\to k} h(x) = L$ **then** $\lim_{x\to k} f(x) = L$ **. This theorem is known as:**

(A) Rolle's Theorem **(B)** Sandwich Theorem **(C)** Fundamental Theorem of Calculus

(D) Intermediate Value Theorem **(E)** Fundamental Theorem of Algebra

38. What is the angle between the minute and hour hands on a circular clock at 11:17 pm?

(A) 120° **(B)** 122.5° **(C)** 140.5° **(D)** 123.5° **(E)** 127.5°

39. Simplify: $\left(a^4 \div b^5\right)^{-3} \div b^3 \times a^9$ **.**

(A) $\frac{1}{a^3 b^{18}}$ **(B)** $\frac{a^{21}}{b^{18}}$ **(C)** $\frac{b^{12}}{a^3}$ **(D)** $\frac{a^{10}}{b^5}$ **(E)** $a^{10}b^5$

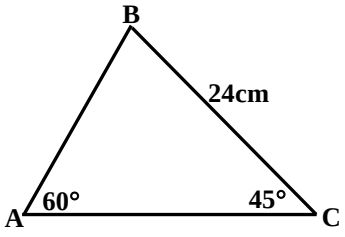
40. Let $f(x) = \begin{cases} 5x-7, & \text{if } x \le 4 \\ kx+2, & \text{if } x > 4 \end{cases}$ **. Find the value of** k **that makes** $f(x)$ **continuous on** $(-\infty, \infty)$**.**

(A) 2.75 **(B)** 2 **(C)** 3 **(D)** 2.25 **(E)** 3.25

41. Find the perimeter of triangle ABC. (nearest centimeter)

(A) 70 cm. **(B)** 62 cm. **(C)** 57 cm.

(D) 59 cm. **(E)** 64 cm.



42. How many distinct solutions exist for $2\sin^2 x = 2\cos x$, where $0 < x < 3\pi$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

43. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ be the terms of the Fibonacci sequence. Find $f_{20} - f_{17}$.

(A) 4,181 (B) 8,362 (C) 5,778 (D) 6,765 (E) 5,168

44. Which of the following functions expresses the perimeter, *P*, of an equilateral triangle in terms of the length of the apothem, *a*?

(A) $P = 6\sqrt{3}\,a$ (B) $P = 3\sqrt{3}\,a$ (C) $P = 3\sqrt{3}\,a^2$ (D) $P = 2\sqrt{3}\,a$ (E) $P = 2\sqrt{3}\,a^2$

45. The repeating decimal 0.113113113…in base 8 can be written as which of the following fractions in base 8 in simplified form?

(A) $\frac{31}{250_8}$ (B) $\frac{113}{777_8}$ (C) $\frac{16}{111_8}$ (D) $\frac{11}{111_8}$ (E) $\frac{75}{511_8}$

46. A polyhedron has 30 faces and 32 vertices, how many edges does it have?

(A) 92 (B) 52 (C) 58 (D) 62 (E) 60

47. A contestant on a game show rolls a single, fair, standard die. The player loses $100 if an odd number is rolled. If he rolls an even prime, he gets a $400 payout. If he rolls a perfect number, he gets $1000 payout. Otherwise, nothing happens. What are his expected winnings?

(A) $133.33 (B) $250 (C) $350 (D) $183.33 (E) $275

48. Given $y = \ln(5x - 9)$, find the value of *x* for which $\frac{dy}{dx} = \frac{dx}{dy}$.

(A) 0.8 (B) 1.6 (C) 2.5 (D) 2.8 (E) 2

49. The Real value solution set for $13 < |9 + 5x| - 8$ is?

(A) $\{x | -6 < x < 2.4\}$ (B) $\{x | -2.4 < x < 6\}$ (C) $\{x | \{x < -2.4\} \cup \{x > 6\}\}$

(D) $\{x | -6 < x < -2.4\}$ (E) $\{x | \{x < -6\} \cup \{x > 2.4\}\}$

50. If $\frac{x-8}{x+8} + \frac{x+8}{x-8} = 2 + \frac{A}{x^2 - 64}$ where $A \in \mathbb{Z}^+$ then $A =$?

(A) 256 (B) 64 (C) 16 (D) 32 (E) 0

51. A lightbulb company produces bulbs that are faulty on average 3.7% of the time. If 6 bulbs are packaged together, what is the probability that at least one of the bulbs is faulty? (nearest tenth)

(A) 22.2% (B) 19.7% (C) 18.7% (D) 20.2% (E) 19.9%

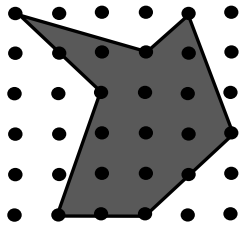
52. $9^3 + 10^3 + 11^3 + \ldots + 18^3 + 19^3 =$

(A) 34,804 (B) 27,945 (C) 29,241 (D) 27,216 (E) 34,075

53. The inequality $x+3<\dfrac{16}{x-3}$ is true for what percentage of the real number values on the interval $[-4,6]$?

(A) 40% (B) 60% (C) 80% (D) 70% (E) 20%

54. Find the area of the shaded region on the illustration shown if the dots on the grid are 5 mm. apart both horizontally and vertically.



(A) 57 mm^2 (B) 375 mm^2 (C) 325 mm^2 (D) 350 mm^2 (E) 65 mm^2

55. An investor invested \$1200 for 5 years. The chart below shows the percentage growth each year. What was the average yearly percentage growth for the 5-year period? (nearest hundredth percent)

Year	1	2	3	4	5
Growth %	3.7%	1.85%	-2.31%	−6.25%	0.75%

(A) −0.51% (B) −0.45% (C) −2.24% (D) −2.54% (E) −1.97%

56. What is the constant term in the expansion of $\left(x^2-\dfrac{2}{x}\right)^9$?

(A) 64 (B) 8,064 (C) 512 (D) 5,376 (E) 6,720

57. Simplify to the nearest ten-thousandth place: $1+2.6+\dfrac{2.6^2}{2!}+\dfrac{2.6^3}{3!}+\dfrac{2.6^4}{4!}+\ldots.$

(A) 12.8035 (B) 13.4637 (C) 13.3919 (D) 13.2326 (E) 11.8134

58. The lengths of the sides of ΔPQR are the roots of $f(x)=x^3-11x^2+36x-30$. Find the area of ΔPQR. (nearest tenth)

(A) 6.7 (B) 3.4 (C) 9.0 (D) 7.2 (E) 3.0

59. $\begin{vmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{vmatrix} =$

(A) $\sin(2\theta)$ (B) $\sec^2\theta$ (C) $\cos(2\theta)$ (D) $\csc^2\theta$ (E) 1

60. The line $\overline{AB}$ passes through the point $(8,1)$ and bounds a right triangle in the first quadrant along with the lines $x=0$ and $y=0$. What is the smallest possible area for such a triangle?

(A) 10 (B) 20 (C) 4 (D) 16 (E) 24

2017 – 2018 TMSCA Mathematics Test Eleven Answers

1. D	**21. C**	**41. A**
2. C	**22. B**	**42. D**
3. E	**23. D**	**43. E**
4. A	**24. B**	**44. A**
5. D	**25. E**	**45. B**
6. A	**26. E**	**46. E**
7. B	**27. B**	**47. D**
8. D	**28. D**	**48. D**
9. E	**29. C**	**49. E**
10. C	**30. C**	**50. A**
11. E	**31. A**	**51. D**
12. A	**32. C**	**52. A**
13. C	**33. D**	**53. E**
14. D	**34. B**	**54. C**
15. C	**35. A**	**55. A**
16. A	**36. E**	**56. D**
17. E	**37. B**	**57. B**
18. D	**38. D**	**58. E**
19. E	**39. C**	**59. D**
20. E	**40. A**	**60. D**

2017-2018 TMSCA Mathematics Test Eleven Select Solutions

9. This is the sum of the roots times the product of the roots for $\left(-\frac{3}{17}\right)\left(\frac{-32}{17}\right)=\frac{96}{289}$

12. Let Lesley have \$3, then the other sisters have \$4 ,\$5 and \$6 left after giving money to Lesley, so Lesley has $\frac{3}{18}=\frac{1}{6}$ **of the money.**

13. Treat the math and grammar books each as one item with 4 other for 6! arrangements, then there are 4! arrangements of the math books and 3! arrangements of grammar books. The total number of arrangements will $(6!)(4!)(3!)=103{,}680$

14. $(2x+1)\ln 3+x\ln 4=(x+2)\ln 6$ **for**

$x(2\ln 3+\ln 4-\ln 6)=2\ln 6-\ln 3$ **and** $x=\frac{\ln 12}{\ln 6}$

22. There are 7 distinct letters with 2-E's. The two possibilities are:
No repeats: ${}_7P_4=840$
Repeat E's: ${}_6C_2\times\frac{4!}{2!}=180$
For a total of 1020.

23. The shortest distance from the center to the chord will form a right angle and bisect the 12'. Finish out the right triangle with a radius of 10' then $10^2-6^2=d^2$ **and** $d=8$ **'**

26. The numbers in each row are the coefficients of a binomial expansion. The 5th term in row 23 will be ${}_{23}C_4=8855$

31. $x^3-y^3=(-15)\left[(-15)^2+3(28)\right]=-4635$

32. The graph of this polar equation is a circle centered at $(0,5)$ **with a radius of 5 for** $x^2+(y-5)^2=25$ **and** $x^2+y^2-10y=0$

34. The last 2 digits form a repeating pattern 7, 49, 43, 01, 7, 49, 43, 01…. 107 divided by 4 has a remainder of 3 for the last two digits 43 and a sum of 7.

40. Solve $20-7=4k+2$ **for** $k=2.75$

46. Use Euler's formula F+V-2=E for 60 edges.

47. $\frac{1}{2}(-100)+\frac{1}{6}(400)+\frac{1}{6}(1000)$ **for \$183.33**

48. Take the derivative and solve either $\frac{5}{5x-9}=1$ **or** $\frac{5}{5x-9}=-1$ **with only one solution in the domain of the original function** $x=2.8$

51. Calculate 1-no faulty or $1-(1-0.037)^6\approx 20.2\%$

54. $A=\frac{2I+P}{2}-1=13$ **units on the grid with each square unit on the grid equal to 25 mm² for a total of 325 mm²**

55. $(1+0.037)(1+.0185)(1-0.0231)(1-0.0625)(1+0.0075)$ **for 0.975 then take the 5th root for 0.998 and -0.051% growth.**

60. Let the *y*-intercept be $(0,b)$ **and the *x*-intercept of** $(a,0)$ **then** $A=\frac{1}{2}ab$ **and** $\frac{b-1}{0-8}=\frac{0-1}{a-8}$ **for** $b=\frac{a}{a-8}$ **and** $A=\frac{1}{2}a\left(\frac{a}{a-8}\right)$ **a minimum area of 16.**