T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 7 ©

J A N U A R Y 1 3 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2017-2018 TMSCA Mathematics Test Seven

1. **Evaluate:** $2+3^0\times 9-18\div(1\times 5)$.

   (A) $25\frac{2}{5}$ **(B)** 8 **(C)** 26 **(D)** $7\frac{2}{5}$ **(E)** $8\frac{3}{5}$

2. **Find the sum of the multiples of 5 that are greater than 0 and less than 142.**

   (A) 2015 **(B)** 2030 **(C)** 2000 **(D)** 700 **(E)** 1420

3. **Simplify** $\frac{(n+3)!}{(n+1)!}\times\frac{1}{n}\div\frac{(n+2)!}{n!}$.

   (A) $\frac{n+3}{n^2+n}$ **(B)** $\frac{n+3}{n+1}$ **(C)** $\frac{n^2+3n}{n+1}$ **(D)** $\frac{n+3}{n^2+2n}$ **(E)** $\frac{n^2+3n}{n+2}$

4. **Points A and B have coordinates** $(3,-5)$ **and** $(-3,19)$ **respectively. Which of the following points is on the perpendicular bisector of** $\overline{AB}$ **?**

   (A) $(4,5)$ **(B)** $(1,3)$ **(C)** $(-4,6)$ **(D)** $(1,11)$ **(E)** $(0,5)$

5. **Events A and B are independent events such that** $P(B)=4P(A)$ **and** $P(A\cup B)=0.84$**. Find** $P(A)$.

   (A) 0.46 **(B)** 0.80 **(C)** 0.24 **(D)** 0.20 **(E)** 0.96

6. **If** $S=\{s,q,u,a,r,e\}$, $H=\{h,e,x,a,g,o,n\}$ **and** $D=\{d,e,c,a,g,o,n\}$ **then** $(S\cap H)\cup D$ **contains how many distinct elements?**

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 9

7. **Three-fifths is the same part of one-fourth as one-third is of_____.**

   (A) $7\frac{1}{5}$ **(B)** $\frac{4}{5}$ **(C)** $1\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{5}{36}$

8. **Tom can paint a fence in 9 hours. His friend Huck can paint the same fence in 6 hours. How long will it take the two boys to paint the fence if they work together? (nearest minute)**

   (A) 3 hrs 36 min **(B)** 3 hrs 45 min **(C)** 3 hrs 30 min **(D)** 3 hrs 52 min **(E)** 3 hrs 38 min

9. **Which of the following multiples of 4 is not the mean of two consecutive prime numbers?**

   (A) 4 **(B)** 40 **(C)** 60 **(D)** 64 **(E)** 108

10. **If** $\frac{x+8}{x-5}+\frac{x-5}{x+8}$ **is written as the mixed number** $A\frac{B}{C}$ **then** $B=?$

    (A) 13 **(B)** 3 **(C)** 26 **(D)** 40 **(E)** 169

11. **If** $\int_2^k \frac{1}{x+2}dx=\ln 2$ **find the value of** k.

    (A) 2 **(B)** 4 **(C)** 6 **(D)** 0 **(E)** 8

12. **If** $2x^2-17x+30=(2x-a)(x+b)$ **then** $a+b=$ **?**

    (A) 11 **(B)** −11 **(C)** −5 **(D)** −1 **(E)** 6

13. The first term of an infinite geometric sequence is 125, while the third term is 45. The sequence has two possible sums. The larger possible sum is:

(A) $78\frac{1}{8}$ **(B)** 245 **(C)** 255 **(D)** $312\frac{1}{2}$ **(E)** 625

14. A box is designed in the shape of a pentagonal prism to ship a decorative lamp. If the perimeter of the pentagonal base is $1\frac{2}{3}$ **yards, what is the length of the longest possible support that will fit in the box parallel to the base? (nearest inch)**

(A) 12 in **(B)** 19 in **(C)** 17 in **(D)** 22 in **(E)** 20 in

15. A fair 6-sided die has sides labelled 1, 1, 1, 2, 2 and 3. What is the expected value of a single roll?

(A) $1\frac{2}{3}$ **(B)** 1 **(C)** $2\frac{1}{6}$ **(D)** $1\frac{5}{6}$ **(E)** $1\frac{1}{6}$

16. Four brothers decided to go on an outing together. Dave didn't have any money at all. Abe, Barry and Carl gave Dave one-fourth, one-third and three-fourths respectively of the money in their possession. All three gave Dave the same amount of money, and Dave wound up with \$45. How much money did the four have all together?

(A) \$90 **(B)** \$170 **(C)** \$125 **(D)** \$120 **(E)** \$145

17. Using the following pattern of numbers, determine the eighth term in the 15th row.

					1						(row 0)
				1		1					(row 1)
			1		2		1				(row 2)
		1		3		3		1			(row 3)
	1		4		6		4		1		(row 4)
1		5		10		10		5		1	(row 5)
					…						…

(A) 5720 **(B)** 4004 **(C)** 910 **(D)** 6435 **(E)** 2048

18. If $\log(7x-2)-\log(3x-1)=2\log(2)$ **then** $x=$ **?**

(A) $\frac{2}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{4}{5}$ **(E)** $\frac{5}{4}$

19. A function $g(x)=x^2+bx+c$**, exists such that** $g(-1)=1$ **and** $g(4)+g(-3)=-2$**. Find** $g(2)$**.**

(A) 13 **(B)** −14 **(C)** −9 **(D)** −23 **(E)** −17

20. Which of the following mathematicians is associated with the "Stepped Reckoner", the first hand-cranked calculator that could perform all four arithmetic operations?

(A) Diophantus **(B)** Newton **(C)** Leibniz **(D)** Mandelbrot **(E)** Archimedes

21. Find k **if** $LCM(18,k)=144$ **and** $GCF(18,k)=6$**.**

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 48

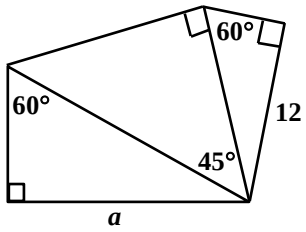
22. Given that $(x+3)$ **is a factor of** $f(x)=6x^3-11x^2+kx+105$**, find the value of** k**.**

(A) −6 **(B)** −156 **(C)** −26 **(D)** −52 **(E)** −14

23. How many distinct 4-letter words can be made from the letters in "PARALLEL"?

(A) 120 **(B)** 286 **(C)** 252 **(D)** 148 **(E)** 142

24. Find the value of *a* on the picture shown.



(A) $8\sqrt{3}$ **(B)** 18 **(C)** 12 **(D)** $12\sqrt{2}$ **(E)** $6\sqrt{6}$

25. Bob's Icy Treats serves 12 flavors of ice cream in 4 types of containers. How many distinct 2-scoop orders are possible?

(A) 264 **(B)** 66 **(C)** 78 **(D)** 312 **(E)** 364

26. Carrie drives the same route to work every weekday. Last week, her average speed on her commute to work was 52.0 mph. On the first 4 days of the week, her speeds were 45.0 mph, 61.0 mph, 48.0 mph and 59.0 mph respectively. What was her speed on Friday? (nearest tenth)

(A) 50.8 mph **(B)** 51.0 mph **(C)** 50.6 mph **(D)** 50.5 mph **(E)** 51.6 mph

27. Given that $s(x)$ is a slant asymptote of $f(x)=\dfrac{2x^3+3x^2-8}{x^2-9}$, calculate $s(2)$.

(A) 17 **(B)** 20 **(C)** –5 **(D)** 13 **(E)** 7

28. If $7x+y=2$, $x-2y=-1$ and $kx+7y=4$, then k =?

(A) –1 **(B)** 2 **(C)** $1\frac{5}{6}$ **(D)** $-1\frac{1}{6}$ **(E)** –7

29. Determine the amplitude of $f(x)=3+5\cos(2x+1)$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** π

30. Find the range of the function $y=3|2-5x|+1$ if the domain is limited to $\{x|x\in\mathbb{R},-1\le x\le 5\}$.

(A) $\{y|y\in\mathbb{R},-1\le y\le 22\}$ **(B)** $\{y|y\in\mathbb{R},1\le y\le 22\}$ **(C)** $\{y|y\in\mathbb{R},0.4\le y\le 70\}$

(D) $\{y|y\in\mathbb{R},1\le y\le 70\}$ **(E)** $\{y|y\in\mathbb{R},0.4\le y\le 22\}$

31. The distance between Los Angeles, CA and New York, NY is approximately 2700 miles. An airplane flying from L.A. to N.Y.C can make the trip in 4 hours flying with the wind. The trip back against the wind takes 4.5 hours. What is the speed of the wind?

(A) 32.5 mph **(B)** 37.5 mph **(C)** 35.0 mph **(D)** 40.0 mph **(E)** 39.5 mph

32. A cube is distorted so that the length is increased by 12%, the height is increased by 15% and the width is decreased by 2%. What is the percent change in the volume of the cube? (nearest 1%)

(A) 31% **(B)** 29% **(C)** 25% **(D)** 26% **(E)** 21%

33. Linus, Marcus, Neill, Olive, Pete and Quinton sit randomly in a row of six chairs. What is the probability that Neill and Pete sit next to each other? (nearest percent)

(A) 25% **(B)** 33% **(C)** 40% **(D)** 20% **(E)** 17%

34. $\left(\sin\theta+\cos\theta\right)^2=$

(A) $2\sin\theta-1$ **(B)** $1+2\sin\theta$ **(C)** $\sin\left(2\theta\right)-1$ **(D)** $1+\sin\left(2\theta\right)$ **(E)** $1-\sin\left(2\theta\right)$

35. Quadrilateral ABCD has vertices $\left(-7,3\right)$, $\left(-4,6\right)$, $\left(2,1\right)$ **and** $\left(4,-5\right)$. **What is the area of ABCD?**

(A) 47.5 **(B)** 53.5 **(C)** 51.5 **(D)** 43.5 **(E)** 41.5

36. Evaluate $\lim\limits_{\theta\to0}\dfrac{\sin\left(2\theta\right)}{\theta}$.

(A) 0 **(B)** $\dfrac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** does not exist

37. Coming into the playoffs, the probability that the Tigers will win any individual game is 0.4. The probability that the Bulldogs will win any individual game is 0.5. Ties are allowed. What is the probability that the Tigers will win a best of three game series?

(A) 0.364 **(B)** 0.304 **(C)** 0.064 **(D)** 0.144 **(E)** 0.164

38. Find the eccentricity of the ellipse, $\dfrac{\left(x-3\right)^2}{16}+\dfrac{y^2}{7}=100$.

(A) $\dfrac{7}{16}$ **(B)** $\dfrac{3}{4}$ **(C)** 1 **(D)** $\dfrac{5}{6}$ **(E)** $\dfrac{13}{15}$

39. $1+\dfrac{1}{3}+\dfrac{1}{6}+\dfrac{1}{10}+\dfrac{1}{15}+\ldots+\dfrac{1}{66}+\dfrac{1}{78}+\dfrac{1}{91}$ = ?

(A) 2 **(B)** $1\dfrac{6}{7}$ **(C)** $1\dfrac{11}{13}$ **(D)** $1\dfrac{13}{15}$ **(E)** $1\dfrac{5}{6}$

40. Let $f''\left(x\right)=3x^2-6x+4$, $f\left(2\right)=-4$ **and** $f\left(-2\right)=8$. **Find** $f\left(1\right)$.

(A) 3 **(B)** 1.25 **(C)** −7.75 **(D)** 2.25 **(E)** −1.25

41. Find $a+b$ **if** $\begin{bmatrix}5&2\\-3&9\end{bmatrix}\bullet\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}7\\57\end{bmatrix}$.

(A) 6 **(B)** −1 **(C)** −4 **(D)** −6 **(E)** 5

42. Find the value of $3A+4B+5C$ **where** A, B **and** C **are integers greater than 0 and**

$$\frac{33}{25}=A+\cfrac{1}{B+\cfrac{1}{C+1}}$$

(A) 11 **(B)** 45 **(C)** 22 **(D)** 50 **(E)** 61

43. If $\left(a+bi\right)^4=41-840i$ **and** $\left(a+bi\right)^5=-1475-4282i$ **then** $a+b=$

(A) 3 **(B)** −1 **(C)** −2 **(D)** 1 **(E)** 0

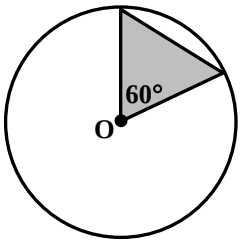
44. If *p* and *q* are the zeros of the function $f(x)=11x^2+9x-13$ then $pq^2+p^2q=$?

(A) $\frac{234}{121}$ **(B)** $-\frac{99}{169}$ **(C)** $\frac{117}{121}$ **(D)** $\frac{81}{143}$ **(E)** $-\frac{198}{169}$

45. Evaluate: $\int_{a-1}^{1-a}(3x+1)dx$

(A) $2-2a$ **(B)** 0 **(C)** $-2a$ **(D)** $2a-2$ **(E)** **does not exist**

46. Maria throws a dart that hits the circle with center O and having a diameter of 8'. What are the odds the dart hits in the shaded area? (nearest whole percent)



(A) 14% **(B)** 17% **(C)** 18% **(D)** 16% **(E)** 19%

47. $(220202_3-2002_3)\times 22_3 =$ _____$_3$

(A) 20210100 **(B)** 20201100 **(C)** 2020110 **(D)** 20222110 **(E)** 2201100

48. A UIl Academics coach has 6 girls and 5 boys competing in calculator. How many distinct 3-person teams can he put together for a meet if he wants to include at least one boy?

(A) 145 **(B)** 75 **(C)** 135 **(D)** 60 **(E)** 165

49. Find the circumference of the circle, $x^2+y^2-6x+4y=156$. (nearest tenth)

(A) 40.8 **(B)** 39.6 **(C)** 79.2 **(D)** 81.7 **(E)** 70.7

50. Let $\|V_1\|=9$ and $\|V_2\|=13$, where the direction angles of V_1 and V_2 are 45° and 60° respectively. Find the direction angle of $\|V_1+V_2\|$. (nearest degree)

(A) 57° **(B)** 49° **(C)** 51° **(D)** 56° **(E)** 54°

51. The fraction $\frac{12}{20}$ in base 5 can be written as which of the following decimals in base 5?

(A) 0.323232… **(B)** 0.222… **(C)** 0.3222… **(D)** 0.3444… **(E)** 0.434343…

52. Given the function $f(x)=3x^2+3$, find the slope of the secant line between the points on $f(x)$ where $x=2$ and $x=7$.

(A) 30 **(B)** 42 **(C)** 27 **(D)** 36 **(E)** 45

53. The sum of the prime factors of 306 is?

(A) 22 **(B)** 20 **(C)** 21 **(D)** 5 **(E)** 56

54. Let $f_0=0,\ f_1=1,\ f_2=1,\ f_3=2,\ f_4=3,\ldots$ be the terms of the Fibonacci sequence. If $f_n=1,346,269$ then *n* is:

(A) 28 **(B)** 29 **(C)** 30 **(D)** 31 **(E)** 32

55. Find the value of the constant term in the expansion of $\left(x-\frac{2}{x^2}\right)^{12}$.

(A) 7920 (B) 4096 (C) 3072 (D) 6008 (E) 4540

56. Consider the point (a,b) in the Cartesian plane. The transformation $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$ results in a rotation of _____about the origin.

(A) 60° counter-clockwise (B) 120° clockwise (C) 30° clockwise

(D) 120° counter-clockwise (E) 60° clockwise

57. How many 5-digit numbers can be created from the set of digits where the digits are not repeated?

(A) 15,120 (B) 30,240 (C) 28,728 (D) 29,484 (E) 27,216

58. The sequence 2.5, a, b, 1 is a harmonic sequence. Find the value of $a+b$

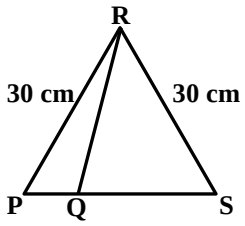
(A) $2\frac{2}{5}$ (B) 5 (C) $2\frac{11}{12}$ (D) $4\frac{1}{6}$ (E) $2\frac{1}{4}$

59. Let $f(x)=ax^6+bx^4+cx^2+x$, where a, b and c are all constants. If $f(6)=-19$ then $f(-6)=$?

(A) -19 (B) -37 (C) -2 (D) -31 (E) -25

60. Given PQ = 9 cm, find $m\angle PRQ$ on the equilateral triangle shown below. (nearest degree)

(A) 13° (B) 12° (C) 17° (D) 16° (E) 19°



2017-2018 TMSCA Mathematics Test Seven Answers

1. D	**21. E**	**41. E**
2. B	**22. D**	**42. D**
3. A	**23. B**	**43. A**
4. C	**24. D**	**44. C**
5. D	**25. D**	**45. A**
6. D	**26. C**	**46. D**
7. E	**27. E**	**47. B**
8. A	**28. A**	**48. A**
9. B	**29. D**	**49. D**
10. E	**30. D**	**50. E**
11. C	**31. B**	**51. C**
12. D	**32. D**	**52. C**
13. D	**33. B**	**53. A**
14. B	**34. D**	**54. D**
15. A	**35. E**	**55. A**
16. C	**36. D**	**56. E**
17. D	**37. A**	**57. E**
18. A	**38. B**	**58. C**
19. D	**39. B**	**59. D**
20. C	**40. C**	**60. C**

2017-2018 TMSCA Mathematics Test Seven Selected Solutions

5. Solve: $0.84 = a + 4a + 4a^2$ **for** $a = 0.15$

7. $\dfrac{\frac{3}{5}}{\frac{1}{4}} = \dfrac{\frac{1}{3}}{x}$ **for** $x = \dfrac{5}{36}$

10. $\left[(x+8)-(x-5)\right]^2 = 169$

11. $\left[\ln(x+2)\right]_2^k = \ln(k+2) - \ln(4) = \ln\left(\dfrac{k+2}{4}\right)$ **for** $\dfrac{k+2}{4} = 2$ **and** $k = 6$

13. $45 = 125r^2$ **for** $r = \pm\dfrac{3}{5}$ **with the greater sum of** $\dfrac{125}{1-\frac{3}{5}} = 312\dfrac{1}{2}$.

14. Each side of the pentagonal base has a length of 12 inches, and the length of the longest diagonal of the pentagon is $\sqrt{12^2+12^2-2(12)(12)\cos 108^\circ} \approx 19$ **in.**

16. Each of the other three brothers gave Dave $15, so originally Abe had $60, Barry had $45 and Carl had $20 for a total of $125.

23. There are 3-L's, 2-A's and one each of P, R and E. All the possible 4-letter arrangements are:

No repeats: ${}_5P_4 = 120$

One double: $2\times {}_4C_2 \times \dfrac{4!}{2!} = 144$

Two doubles: $\dfrac{4!}{2!\times 2!} = 6$

Triple L's: ${}_4C_1 \times \dfrac{4!}{3!} = 16$

For a total of 286.

25. ${}_{12+2-1}C_2 \times 4 = 312$

26. Solve: $52.0 = \dfrac{5}{\frac{1}{45}+\frac{1}{61}+\frac{1}{48}+\frac{1}{59}+\frac{1}{x}}$ **for** $x \approx 50.6$ **mph.**

33. There are two ways that Neill and Pete can sit next to each other in two chairs, then treat that group of two as one block and arrange the other four around them for the numerator, then the 6! is the total of all the possible arrangements $\dfrac{2\times 5!}{6!} = \dfrac{1}{3} \approx 33\%$

36. Use $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1$ **relationship for**

$$\lim_{\theta\to 0}\frac{2}{2}\times\frac{\sin(2\theta)}{\theta} = 2\times\lim_{\theta\to 0}\frac{\sin(2\theta)}{2\theta} = 2\times 1 = 2$$

37. 3 Tiger wins: $(0.4)^3$

2 Tiger wins and tie: $3\times(0.4)^2(0.1)$

2 Tiger wins and 1 Bulldog win: $3(0.4)^2(0.1)$

1 Tiger win and 2 ties: $3(0.4)(0.1)^2$

For a total of 0.364

42. $\dfrac{33}{25} = 1+\dfrac{8}{25} = 1+\dfrac{1}{\frac{25}{8}} = 1+\dfrac{1}{3+\frac{1}{7+1}}$ **for** $3A+4B+5C = 50$

44. $pq^2 + p^2q = pq(q+p) = \dfrac{c}{a}\times -\dfrac{b}{a} = \dfrac{-13}{11}\times\left(-\dfrac{9}{11}\right) = \dfrac{117}{121}$

50. $\arctan\dfrac{9\sin 45^o + 13\sin 60^o}{9\cos 45^o + 13\cos 60^o} \approx 54^o$

55. ${}_{12}C_8(x)^8\left(-\dfrac{2}{x^2}\right)^4 = 7920$

56. Use the point $(a,b) = (1,0)$ **for**

$$\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}$$

or a rotation of 60° clockwise.

58. The reciprocals of the terms of the harmonic sequence will form an arithmetic sequence, so $\dfrac{1}{2.5}, \dfrac{1}{a}, \dfrac{1}{b}, 1$ **are terms of an arithmetic sequence and** $\dfrac{1}{2.5} + 3d = 1$ **and** $d = 0.2$, $\dfrac{1}{a} = 0.6$, $\dfrac{1}{b} = 0.8$ **and** $a+b = 2\dfrac{11}{12}$.