



# TMSCA HIGH SCHOOL MATHEMATICS

TEST # 5 ©

NOVEMBER 18 , 2017

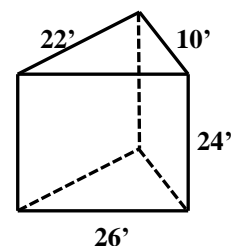
## GENERAL DIRECTIONS

- About this test:
  - You will be given 40 minutes to take this test.
  - There are 60 problems on this test.
- All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
- If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
- You may write anywhere on the test itself. You must write only answers on the answer sheet.
- You may use additional scratch paper provided by the contest director.
- All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
- All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

2017-2018 TMSCA Mathematics Test Five

1. Evaluate:  $(6! + 4^2) \div 23 \times 2^3 - 9$ .  
 (A) 183                      (B) 38                      (C) -5                      (D) 717                      (E) 247
2. Lisa had a large box of cookies. She gave  $\frac{1}{2}$  of them to a class then 25% of what was left to a friend. Later, she kept  $\frac{1}{3}$  of the remaining and gave the rest to her brother Lyle. Lyle gave away  $\frac{5}{6}$  of his cookies and was left with 18. How many cookies did Lynn start out with?  
 (A) 448                      (B) 360                      (C) 720                      (D) 432                      (E) 480
3. Three billion, eighty-six is subtracted from sixteen billion, four hundred sixty-one thousand, eight hundred seventy-two. What is the sum of the digits in the difference?  
 (A) 47                      (B) 36                      (C) 26                      (D) 63                      (E) 39
4. Points A and B have coordinates  $(3, -5)$  and  $(-3, 19)$  respectively. What is the  $x$ -coordinate of the  $x$ -intercept of the perpendicular bisector of the segment  $\overline{AB}$  ?  
 (A) -28                      (B) -3                      (C) 1.75                      (D) -4                      (E) -11
5. A survey of 22 homes in a particular neighborhood reveals that 5 of the homes have cats, but not dogs. Eight houses have cats and dogs. Four houses have dogs, but not cats. How many houses do not have a cat or a dog?  
 (A) 17                      (B) 5                      (C) 10                      (D) 7                      (E) 15
6. A 24-sided die is numbered with the numbers 1 to 24. If the die is rolled, what are the odds that the outcome will be an abundant number?  
 (A) 1:6                      (B) 1:5                      (C) 2:5                      (D) 1:3                      (E) 5:19
7.  $\angle P$  and  $\angle Q$  are complementary.  $\angle Q$  and  $\angle R$  are supplementary. If  $m\angle R = 122^\circ$ , then  $m\angle P =$  ?  
 (A)  $24^\circ$                       (B)  $42^\circ$                       (C)  $18^\circ$                       (D)  $32^\circ$                       (E)  $48^\circ$
8. A right cylinder tank is 9 ft. high and has an interior diameter of 12 ft. The amount of water in the tank is 75% of the maximum capacity. How many gallons of water are in the tank? (nearest gallon)  
 (A) 476 gal                      (B) 5710 gal                      (C) 4110 gal                      (D) 30,457 gal                      (E) 7614 gal
9. If  $\frac{A}{2-x} + \frac{B}{x+5} = \frac{3x+36}{10-3x-x^2}$ , then  $A+B =$  ?  
 (A) 12                      (B) 3                      (C) -3                      (D) 9                      (E) -6
10. The lateral surface area of the right triangular prism shown is \_\_\_\_\_  $\text{ft}^2$ .  
 (A) 1344                      (B) 1564                      (C) 2640                      (D) 136                      (E) 1392



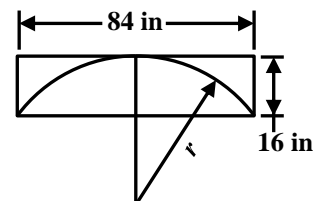
11.  $8x^2 - 22x - 21 = (4x + 3)(ax + b)$  then  $a + b = ?$

- (A) 2 (B) -4 (C) -5 (D) 9 (E) -7

12. Simplify:  $\frac{n!(n+4)!}{(n-2)!} \div \frac{[(n+2)!]^2}{(n-1)!}$ .

- (A)  $\frac{n^3 + 6n^2 + 5n - 12}{n^2 + 3n + 2}$  (B)  $\frac{n^2 + 7n + 12}{n + 2}$  (C)  $\frac{n^3 + 8n^2 + 19n + 12}{n^2 + 3n + 2}$   
 (D)  $\frac{n^2 + 7n + 12}{n^2 + 3n + 2}$  (E)  $\frac{n^3 + 6n^2 + 8n - 12}{n^2 + 3n + 2}$

13. On the diagram, the arc shown is part of a circle with radius  $r$ . What is the value of  $r$ ? (nearest inch)



- (A) 72 in (B) 69 in (C) 59 in (D) 63 in (E) 45 in

14. A jar contains \$8.00 in nickels, dimes and quarters. There are 62 coins in the jar and six more nickels than dimes. How many quarters are in the jar?

- (A) 18 (B) 20 (C) 24 (D) 22 (E) 26

15. If  $p$  and  $q$  are the zeros of the function  $f(x) = 14x^2 + 11x - 15$  then  $pq^2 + p^2q =$

- (A)  $-\frac{165}{196}$  (B)  $\frac{165}{196}$  (C)  $-\frac{165}{392}$  (D)  $\frac{165}{392}$  (E)  $-\frac{11}{14}$

16. Find the exact value of  $x$  satisfying  $(3^x)(4^{2x+1}) = 6^{x+2}$ .

- (A)  $\frac{\ln(3)}{\ln(2)}$  (B)  $\frac{\log(3)}{\log(2)}$  (C)  $\frac{\ln(9)}{\ln(8)}$  (D)  $\frac{\log(3)}{\log(4)}$  (E)  $\frac{\log(6)}{\ln(2)}$

17. Simplify:  $(b^3) \frac{(b^5)^{-2}(\sqrt{b})}{b^{-6}}$

- (A)  $(\sqrt{b})^{-5}$  (B)  $(\sqrt{b})^{-19}$  (C)  $(\sqrt{b})^{-1}$  (D)  $(\sqrt{b})^{-3}$  (E)  $(\sqrt{b})^{-15}$

18. What is the constant term in the expansion of  $\left(2x^3 - \frac{3}{x}\right)^8$ ?

- (A) 2,916 (B) 204,120 (C) 9,072 (D) 81,648 (E) 18,144

19. The intersection of the three perpendicular bisectors of the sides in a triangle is called the\_\_\_\_\_.

- (A) Centroid (B) Incenter (C) Orthocenter (D) Center (E) Circumcenter

20. How many distinct words can be formed using all of the letters in "STEPHENVILLE"?

- (A) 39,916,800 (B) 79,336,000 (C) 3,628,800 (D) 1,814,400 (E) 3,326,400

21. Determine the frequency of  $f(x) = 7 + 3\cos\left[2\left(x - \frac{\pi}{2}\right)\right]$ .
- (A)  $\frac{2}{\pi}$  (B)  $\pi$  (C)  $\frac{\pi}{2}$  (D)  $\frac{1}{\pi}$  (E)  $2\pi$
22. Find the total area of the two regions enclosed by the curves  $y = x^3 - 3x^2 - 9x + 28$  and  $y = x + 4$ .
- (A) 127.50 (B) 101.75 (C) 85.75 (D) 92.75 (E) 106.75
23.  $111_6 + 1111_5 + 11111_4 = \text{_____}_7$ .
- (A) 1410 (B) 501 (C) 6001 (D) 12333 (E) 1401
24. A square has a perimeter of 18 cm. A right triangle with one leg length of 18 cm has the same perimeter as the square. What is the area of the triangle? (nearest square centimeter)
- (A) 216 cm<sup>2</sup> (B) 360 cm<sup>2</sup> (C) 135 cm<sup>2</sup> (D) 180 cm<sup>2</sup> (E) 270 cm<sup>2</sup>
25. Let  $f(x) = 12x^3 - 5x^2 + 8x - 135$ . Find  $f'(-2)$ .
- (A) 132 (B) 162 (C) 172 (D) 156 (E) 116
26. Consider the point  $(a, b)$  in the Cartesian plane. The transformation  $\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$  results in a rotation of \_\_\_\_\_ about the origin.
- (A) 60° counter-clockwise (B) 120° clockwise (C) 30° clockwise  
(D) 120° counter-clockwise (E) 60° clockwise
27. Use the Fibonacci characteristic sequence,  $-3, p, q, 11, r, s, \dots$  to find  $pq - rs$ .
- (A) -379 (B) -313 (C) 418 (D) -390 (E) -362
28. Let  $f(x) = ax^5 + bx^3 + cx - 22$ , where  $a, b$  and  $c$  are all constants. If  $f(6) = -19$  then  $f(-6) = ?$
- (A) -19 (B) 41 (C) -2 (D) -25 (E) -35
29. How many 3-digit numbers exist such that the sum of their digits is 14?
- (A) 82 (B) 75 (C) 70 (D) 88 (E) 72
30. Let  $A = \begin{bmatrix} 7 & -9 \\ -2 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 5 & 3 \\ 2 & -7 \end{bmatrix}$ . Find the sum of the elements in  $AB$ .
- (A) 55 (B) 151 (C) -30 (D) 90 (E) -53
31. The sum of the prime factors of 288 is?
- (A) 6 (B) 9 (C) 5 (D) 14 (E) 15
32. Given that  $\int_{-6}^9 f(x) dx = 45$ , evaluate  $\int_{-6}^9 [-2f(x) + 7] dx$ .
- (A) -135 (B) -83 (C) -63 (D) 15 (E) 21

33. Let  $f(x)$  be a function that is continuous on  $[a, b]$ . If  $y_1$  is between  $f(a)$  and  $f(b)$ , then  $y_1 = f(c)$  for some  $c$  on  $[a, b]$ . This theorem is known as:

- (A) Max-Min Theorem (B) Sandwich Theorem (C) Intermediate Value Theorem  
(D) Fundamental Theorem of Algebra (E) Fundamental Theorem of Calculus

34. Which of the following mathematicians is noted for his work with polynomial equations that allow variables to be integers only?

- (A) Eratosthenes (B) Leibniz (C) Descartes  
(D) Diophantus (E) Archimedes

35. How much 64% solution does a chemist need to add to 18 ounces of 80% solution to produce a 75% solution? (nearest ounce)

- (A) 6 oz. (B) 8 oz. (C) 10 oz. (D) 11 oz. (E) 12 oz.

36. Find  $\lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{4\theta}$

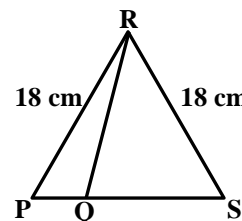
- (A) 0.25 (B) 1.25 (C) 0.8 (D) 0.5 (E) Does not exist

37. Given the sequence  $-11, -5, -3, 1, 13, 39, \dots$ , find the 20<sup>th</sup> term.

- (A) 4381 (B) 5233 (C) 6189 (D) 824 (E) 918

38. Given  $PQ = 5$  cm, find  $m\angle PRQ$  on the equilateral triangle shown below. (nearest degree)

- (A)  $13^\circ$  (B)  $12^\circ$  (C)  $17^\circ$  (D)  $16^\circ$  (E)  $19^\circ$



39. Let  $f(x) = \frac{x^5 - 3x^4}{x^4 - x^3 - 5}$  and  $s(x)$  be the slant asymptote of  $f(x)$ . Find the value of  $s(-1)$

- (A) 1 (B) -3 (C) -4 (D) -1 (E) -2

40. Let  $f(x) = \begin{cases} 7x - 3, & \text{if } x \leq 2 \\ kx + 1, & \text{if } x > 2 \end{cases}$ . Find the value of  $k$  that makes  $f(x)$  continuous on  $(-\infty, \infty)$ .

- (A) 0.7 (B) 5 (C) 0 (D) -2 (E) 1.4

41. Given  $f(x) = 2x - 5$  and  $g(x) = x^2 - 1$  find  $g(f(x))$ .

- (A)  $4x^2 + 24$  (B)  $2x^2 - 1$  (C)  $4x^2 - 20x + 26$  (D)  $4x^2 - 20x + 24$  (E)  $4x^2 - 1$

42.  $(-2 - 3\sqrt{-10})(4\sqrt{-8})$

- (A)  $48\sqrt{5} + 16i\sqrt{2}$  (B)  $112\sqrt{10}$  (C)  $80\sqrt{3}$   
(D)  $48\sqrt{5} - 16i\sqrt{2}$  (E)  $80\sqrt{10}$

43. Simplify:  $\sin(2\theta)\cot(\theta) - \cos(2\theta)$

- (A)  $\cot(2\theta)$  (B) 1 (C)  $2\cos^2(\theta)$  (D) -1 (E)  $\sin^2(\theta)$

44. If  $f(x) = \sin(2x)$ , then  $\lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{3} + h\right) - f\left(\frac{\pi}{3}\right)}{h}$  is

- (A)  $\sqrt{3}$  (B) -1 (C) -2 (D)  $-\sqrt{3}$  (E) undefined

45. Which of the following statements is a false statement for  $f(x) = \begin{cases} 3x^2 - 7 & \text{if } x < 3 \\ 20 & \text{if } x = 3 \\ 6x + 2 & \text{if } x > 3 \end{cases}$ .

- (A)  $f(3)$  exists (B)  $f$  is continuous at 3 (C)  $\lim_{x \rightarrow 3} f(x)$  exists  
(D)  $f'(3)$  exists (E) none of these

46. The repeating decimal 0.113113113... in base 6 can be written as which of the following fractions in base 6 in simplified form?

- (A)  $\frac{2}{43_6}$  (B)  $\frac{45}{215_6}$  (C)  $\frac{13}{111_6}$  (D)  $\frac{11}{111_6}$  (E)  $\frac{9}{43_6}$

47. If  $x - \frac{1}{x} = 27$ , then  $x^3 - \frac{1}{x^3} = ?$

- (A) 19,683 (B) 19,737 (C) 19,710 (D) 19,764 (E) 19,791

48. A fair coin is flipped 8 times. What is the probability that the coin will come up heads exactly five times in a row?

- (A)  $\frac{3}{64}$  (B)  $\frac{21}{128}$  (C)  $\frac{1}{32}$  (D)  $\frac{3}{128}$  (E)  $\frac{5}{128}$

49. The Real value solution set for  $11 < |7 + 5x| - 9$  is?

- (A)  $\{x | \{x < -5.4\} \cup \{x > 2.6\}\}$  (B)  $\{x | 2.6 < x < 5.4\}$  (C)  $\{x | \{x < -2.6\} \cup \{x > 5.4\}\}$   
(D)  $\{x | -5.4 < x < -2.6\}$  (E)  $\{x | -5.4 < x < 2.6\}$

50. Let P and Q be the roots of  $0 = 2x^2 - 3x - 21$ . Find  $P^3 + 3P^2Q + 3PQ^2 + Q^3$ .

- (A)  $-\frac{3}{2}$  (B) 8 (C)  $-\frac{27}{8}$  (D)  $-\frac{1}{8}$  (E)  $\frac{27}{8}$

51. If  $(a + bi)^4 = -119 - 120i$  and  $(a + bi)^5 = -597 - 122i$  then  $a + b =$

- (A) 3 (B) -1 (C) -2 (D) 1 (E) 0

52.  $B89_{12} - 123_{12} = \underline{\hspace{1cm}}_{10}$

- (A) 1860 (B) 1854 (C) 1518 (D) 1066 (E) AB6

53. If  $f''(x) = 18x - 16$ ,  $f(2) = -6$  and  $f(1) = -10$  then  $f(-1) =$

- (A) 32 (B) -34 (C) 30 (D) -30 (E) 10

54. If  $a_1 = 2$ ,  $a_2 = \frac{1}{3}$ ,  $a_n = (a_{n-2}) \div (a_{n-1})$  for  $n \geq 3$  and  $a_6$  is written as a simplified fraction, then the denominator is?

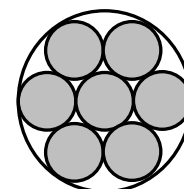
(A) 1 (B) 72 (C) 1244 (D) 648 (E) 288

55. An investor invested \$1200 for 5 years. The chart below shows the percentage growth each year. What was the average percentage growth for the 5-year period? (nearest hundredth percent)

Year	1	2	3	4	5
Growth %	3.7%	1.85%	2.31%	-6.25%	0.75%

(A) 0.41% (B) 2.59% (C) 4.10% (D) 1.27% (E) 0.34%

56. The outer six small circles are tangent to the big circle and to the inner small circle in the center. All seven small circles are congruent. A dart lands somewhere in the large circle. What are the odds that the dart will land in the shaded region?



(A) 7:2 (B) 9:7 (C) 7:9 (D) 7:4 (E) 4:11

57. The line  $\overline{AB}$  passes through the point  $(2,6)$  and bounds a right triangle in the first quadrant along with the lines  $x = 0$  and  $y = 0$ . What is the smallest possible area for such a triangle?

(A) 10 (B) 20 (C) 4 (D) 16 (E) 24

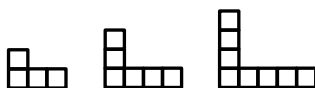
58. What is the digit in the millionths place in the sum of  $6 + \frac{5^2}{2} + \frac{5^3}{6} + \frac{5^4}{24} + \dots$ ?

(A) 5 (B) 0 (C) 7 (D) 6 (E) 9

59. The lengths of the sides of triangle PQR are the roots of  $f(x) = x^3 - 11x^2 + 37x - 35$ . Find the area of triangle PQR. (nearest tenth square unit)

(A) 12.1 (B) 7.9 (C) 11.7 (D) 3.4 (E) 3.6

60. The three shapes below are made up of one-unit squares. If the pattern continues, the area of the figure with a perimeter of 102 units is \_\_\_\_\_ square units.



(A) 50 (B) 52 (C) 54 (D) 48 (E) 46

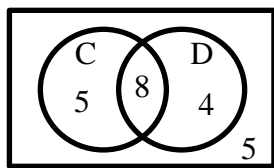


**2017-2018 TMSCA Mathematics Test Five Answers**

<b>1. E</b>	<b>21. D</b>	<b>41. D</b>
<b>2. D</b>	<b>22. B</b>	<b>42. D</b>
<b>3. B</b>	<b>23. E</b>	<b>43. B</b>
<b>4. A</b>	<b>24. A</b>	<b>44. B</b>
<b>5. B</b>	<b>25. C</b>	<b>45. D</b>
<b>6. B</b>	<b>26. C</b>	<b>46. C</b>
<b>7. D</b>	<b>27. E</b>	<b>47. D</b>
<b>8. B</b>	<b>28. D</b>	<b>48. A</b>
<b>9. D</b>	<b>29. C</b>	<b>49. A</b>
<b>10. E</b>	<b>30. A</b>	<b>50. E</b>
<b>11. C</b>	<b>31. C</b>	<b>51. D</b>
<b>12. A</b>	<b>32. D</b>	<b>52. C</b>
<b>13. D</b>	<b>33. C</b>	<b>53. D</b>
<b>14. B</b>	<b>34. D</b>	<b>54. C</b>
<b>15. B</b>	<b>35. B</b>	<b>55. A</b>
<b>16. C</b>	<b>36. B</b>	<b>56. A</b>
<b>17. C</b>	<b>37. B</b>	<b>57. E</b>
<b>18. D</b>	<b>38. D</b>	<b>58. E</b>
<b>19. E</b>	<b>39. B</b>	<b>59. D</b>
<b>20. A</b>	<b>40. B</b>	<b>60. A</b>

## 2017-2018 TMSCA Mathematics Test Five Selected Solutions

5.



$$8. V = \frac{36\pi}{3}(9)(0.75)ft^3 \cdot \left(\frac{12in}{1ft}\right)^3 \cdot \frac{1gal}{231in^3} \approx 5710 gal$$

$$13. \text{Solve } 42^2 + (r-16)^2 = r^2 \text{ for } r \approx 63$$

$$(3)^x (2)^{4x+2} = (3)^{x+2} (2)^{x+2}$$

$$16. 2^{3x} = 3^2 \quad \text{for } x = \frac{\ln 9}{\ln 8}$$

$$x \ln 8 = \ln 9$$

$$20. 12 \text{ letters, with 3-E's and 2-L's for } \frac{12!}{(3!)(2!)} = 39,916,800$$

$$21. \text{The frequency is always } \frac{b}{2\pi}, \text{ where } b \text{ is the coefficient of } x,$$

$$\text{so here the frequency is } \frac{2}{2\pi} = \frac{1}{\pi}$$

$$26. \text{Let } \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ then perform the multiplication for } \begin{bmatrix} \sqrt{3}/2 \\ -1/2 \end{bmatrix}$$

which is a rotation of 30 degrees clockwise.

29. 6 in 100's, 7 in 200's, 8 in 300's, 9 in 400's, 10 in 500's, 9 in 600's, 8 in 700's, 7 in 800's and 6 in 900's for a total of 70.

$$32. -2(45) + 7(9+6) = 15$$

$$36. \text{Use the fact that } \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \text{ for } \lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{4\theta} =$$

$$\lim_{\theta \rightarrow 0} \frac{5 \sin(5\theta)}{4(5\theta)} = 1.25$$

$$38. (RQ)^2 = 18^2 + 5^2 - 2(18)(5)\cos 60 \text{ for } RQ = \sqrt{259} \text{ then}$$

$$\frac{\sin 60}{\sqrt{259}} = \frac{\sin \theta}{5} \text{ for } \theta \approx 16^\circ$$

$$39. \text{Divide for } s(x) = x - 2 \text{ and } s(-1) = -3$$

$$40. \text{Solve } 7(2) - 3 = 2k + 1 \text{ for } k = 5$$

$$43. 2 \sin \theta \cos \theta \cdot \frac{\cos \theta}{\sin \theta} - (\cos^2 \theta - \sin^2 \theta) = \cos^2 \theta + \sin^2 \theta = 1$$

$$44. \text{This is the definition } f'\left(\frac{\pi}{3}\right) = -1$$

$$48. \text{Let H be the 5 in a row. The possibilities are: tttH, Httt, ttHt, tHtt, httH, Htth, hhtH, Hthh, Htht, thtH, hhtH and Hthh for}$$

$$\frac{12}{2^8} = \frac{3}{64}$$

$$51. \frac{-597-122i}{-119-120i} = 3-2i \text{ for } a+b=1$$

$$55. \sqrt[3]{(1.037)(1.0185)(1.0231)(0.9375)(0.0075)} \approx 0.41\%$$

$$56. \text{Let the large circle have a radius of 6, then the smaller circles have a radius of 1 for odds of } 7\pi : (9\pi - 7\pi) = 7 : 2$$

$$57. \text{Let } (0,b) \text{ and } (a,0) \text{ be the y-intercept and x-intercept.}$$

$$\text{The area of the triangle will be } A = \frac{ab}{2}. \text{ Then the slope of any}$$

$$\text{line through } (2,6) \text{ will be } \frac{6-b}{2} = \frac{-b}{a} \text{ for } a = \frac{-2b}{6-b} \text{ and an area}$$

$$\text{in terms of } b \text{ only of } A = \frac{1}{2} \left( \frac{-2b}{6-b} \right) (b) \text{ for a minimum of 24.}$$

$$58. \text{This is the McClaurin Series expansion of the function } f(x) = e^x \text{ evaluated at } x=5 \text{ for a millionths digit 9.}$$