



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

SAC • 2013



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1. Evaluate: $2 \times (1 - 3) + 4 \div 7^0 \times 11 + 18$

- (A) 66 (B) 58 (C) 29 (D) 18 (E) 14.363636...

2. If $\frac{5}{8}$ of P has the same value as 2.5Q, then Q is what % of P?

- (A) $1\frac{9}{16}\%$ (B) 6.4% (C) 15.625% (D) 25% (E) 64 %

3. Doug Upp rented a front end loader to dig up mesquite trees. The base rental fee was \$230.00 plus \$25.00 per hour. Doug rented the loader for 7 hours. He got 10% off the rental fee for cleaning the loader before returning it. What was Doug's total cost if the tax rate was 8%?

- (A) \$247.86 (B) \$393.66 (C) \$426.60 (D) \$412.56 (E) \$437.40

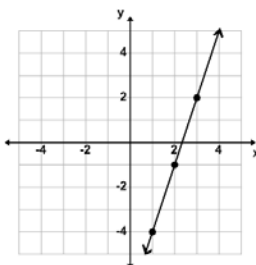
4. Let K be a two-digit number such that neither digit is zero. Reverse the digits and add the results to K. Divide the sum by the sum of the digits. What is the quotient?

- (A) 1 (B) 2 (C) 10 (D) 11 (E) P

5. Simplify: $\left(\frac{x^2 - 4}{x^2 + 4x + 4} \right) \left(\frac{x^2 - 3x - 10}{x^2 + 3x - 10} \right)$

- (A) $\frac{x-5}{x+5}$ (B) $\frac{x-6}{x+6}$ (C) -1 (D) $\frac{x+6}{x-6}$ (E) $\frac{x+5}{x-5}$

6. What is the slope of the line parallel to the line shown?



- (A) -3 (B) -2 (C) $-\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 3

7. Dee Loper and Les Speed start at the starting line of the 400 meter oval track. Dee runs clockwise around the track at an average rate of 5 meters per second and Les runs counter clockwise around the track at an average rate of 7 meters per second. How far will Dee have run when the two meet?

- (A) $233\frac{1}{3}$ meters (B) 200 meters (C) $166\frac{2}{3}$ meters (D) 80 meters (E) 300 meters

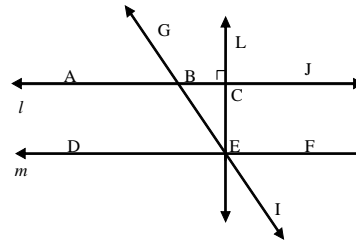
8. Let $x + y = 2$, $3x - 5y = 8$, and $13x + ky = 34$. Find the value of k for the 3 intersecting lines.

- (A) -19 (B) $-15\frac{13}{16}$ (C) -2.8125 (D) 18.1875 (E) 21

9. Which of the following numbers will appear in row 11 of Pascal's triangle?

- (A) 210 (B) 333 (C) 357 (D) 462 (E) 468

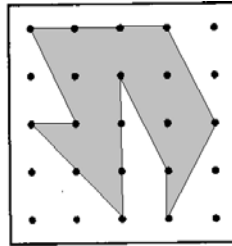
10. The lines in the figure are coplanar with $m \parallel l$. Which of the following are true statements?



1. $\angle ABG$ & $\angle FEI$ are congruent
2. $m\angle DEB + m\angle CEB = 90^\circ$
3. $\angle JBI$ & $\angle BEF$ are vertical angles
4. $\angle EBC$ & $\angle DEB$ are complementary angles

- (A) 1, 2, 3, & 4 (B) 1 & 2 (C) 1, 2, & 4 (D) 3 & 4 (E) 1 & 4

11. The adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.



- (A) 8 cm^2 (B) 8.5 cm^2 (C) 9 cm^2 (D) 9.5 cm^2 (E) 10 cm^2

12. Points A, B, and C lie on circle P with point P being the center of the circle. \overline{AB} is a diameter and point X lies on the chord \overline{AC} . Find XC if $\overline{PX} \parallel \overline{BC}$, $AB = 10 \text{ cm}$ and $AC = 8 \text{ cm}$.

- (A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm (E) 6 cm

13. Let $f(x) = \frac{3x-2}{4}$. Find $f^{-1}(f(2))$.

- (A) 0.25 (B) 0.5 (C) 1 (D) 2 (E) 4

14. If $a_1 = 3$, $a_2 = -1$ and $a_n = a_{n-2} + a_{n-1}$, where $n \geq 3$, then a_7 equals:

- (A) 7 (B) 4 (C) 11 (D) 6 (E) 11

15. Captain Saul T. Water leaves port Izzy and sails 30 miles on a bearing of 75° . Then he changes course and sails 30 miles on a bearing of 150° to port Hugh. How far will Saul have to sail to go directly back to port Izzy? (nearest tenth mile)

- (A) 40.0 miles (B) 42.4 miles (C) 45.0 miles (D) 45.9 miles (E) 47.6 miles

16. Simplify: $\frac{\sin 2x}{2 - 2\cos^2 x}$

- (A) $\tan x$ (B) $\tan 2x$ (C) $2\tan x$ (D) $\cot 2x$ (E) $\cot x$

17. Use the Fibonacci characteristic sequence ... 3, p, q, 5, r, ... to Find $p + q + r$.

- (A) 13 (B) 14 (C) 18 (D) 21 (E) 22

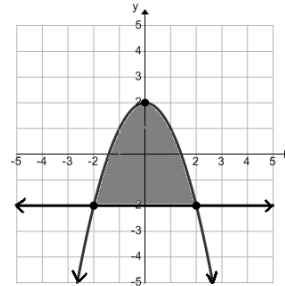
18. Willie Taasette pitches for the Millersview Mudhens. He gets \$25.00 for his first win, \$50.00 for his second win, \$100.00 for his third, and so on. How much will he receive for his 10th win?

- (A) \$3,200.00 (B) \$8,000.00 (C) \$12,800.00 (D) \$25,600.00 (E) \$38,400.00

19. Let $f(x) = 3x^2 - 4x - 5$ and $g(x) = x + 2$. Find $g(f'(2x - 1))$

- (A) $12x - 8$ (B) $24x + 4$ (C) $6x + 1$ (D) $10x - 14$ (E) $12x + 2$

20. Find the area of the shaded region in square units.



- (A) 9.333... (B) 11.0 (C) 10.666... (D) 9.5 (E) 10.5

21. Lynn Kahn tossed a penny five times and recorded the results. What is the probability of at least three consecutive heads? (nearest per cent)

- (A) 50% (B) 33% (C) 28% (D) 25% (E) 13%

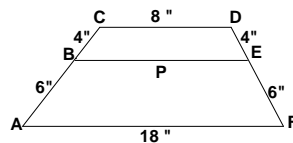
22. How many distinguishable arrangements can be made from the letters "TEXASMATH"?

- (A) 15,120 (B) 105,840 (C) 90,720 (D) 30,240 (E) 60,480

23. How many integers x , where $1 \leq x \leq 2013$, are divisible by neither 3 nor 7?

- (A) 863 (B) 958 (C) 1,053 (D) 1,055 (E) 1,150

24. Find BE.



- (A) 26" (B) 13" (C) 12" (D) 10" (E) 9"

25. The repeating decimal $0.515151\ldots$ in base 7 can be written as which of the following fractions in base 7?

- (A) $\frac{3}{4}_7$ (B) $\frac{51}{343}_7$ (C) $\frac{17}{22}_7$ (D) $\frac{5}{6}_7$ (E) $\frac{1}{5}_7$

Mathematicians (One new mathematician this year)**

Agnesi	Archimedes	Boole, George	Byron, Ada (Lady Lovelace)
Cantor, Georg	Descartes, Rene	Diophantus	Erastosthenes
Euclid	Euler, Leonard	Germain, Sophie	Goldbach, Christian
Hypatia	Kovalevsky, Sonya	Leibniz, Gottfried	Mandelbrot, Benoit
Napier, John	Noether, Emmy	Porter, Freda	Ptolemy, Claudius
Smith, Karen E.	Stott, Alicia	Theano	Venn, John
Williams, Grace	** Zeno of Elea		

Types of Numbers (One new number this year)**

Complex	Real	Imaginary	Rational	Irrational
Transcendental	Integer	Whole	Natural	Even
Odd	Prime	Composite	Unit	Deficient
Frugal	Economical	Perfect	Equidigital	Abundant
Extravagant	Wasteful	Fibonacci	Lucas	Happy
Unhappy	Lucky	Unlucky	Evil	Odious
Polite	Primeval	** Harmonic		

2013-14 Special Emphasis Concepts: number theory problems and the 7 trapezoidal "means".

Possible questions (but not limited to) might include:

1. The product of a two-digit number and the same number with its digits reversed is 1944. What is the sum of the two numbers? _____.
2. How many 3-digit numbers can be made from the digits 1, 1, 2, 3, and 3? _____.
3. Find the smallest positive integer with 25 divisors. _____.
4. Which "trapezoidal mean" can be used to find the volume of a frustrum of a cone? _____.
5. *** See #23, 24, and 25 on the 2013SAC test.

**University Interscholastic League
MATHEMATICS CONTEST
HS • SAC • 2013
Answer Key**

- | | |
|-------|-------|
| 1. B | 21. D |
| 2. D | 22. C |
| 3. D | 23. E |
| 4. D | 24. C |
| 5. A | 25. A |
| 6. E | |
| 7. C | |
| 8. A | |
| 9. D | |
| 10. B | |
| 11. C | |
| 12. C | |
| 13. D | |
| 14. A | |
| 15. E | |
| 16. E | |
| 17. B | |
| 18. C | |
| 19. A | |
| 20. C | |



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Mathematics

Invitational A • 2014



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1. Evaluate: $5 \times 3 - (1 + 2 \div 4) \times 6$

- (A) 6 (B) 7.5 (C) 10.5 (D) 24 (E) 81

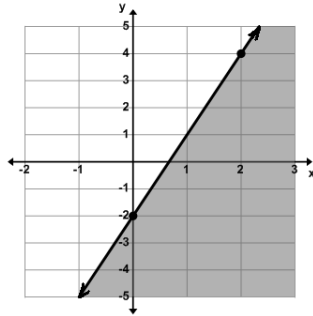
2. Len Meecash borrowed \$300.00 from the credit union at a simple interest rate of 6% for 9 months. How much interest will he have to pay? (nearest cent)

- (A) \$12.00 (B) \$13.50 (C) \$15.00 (D) \$16.20 (E) \$18.00

3. What is $3\frac{1}{5}$ of 0.375 minus 11% of 0.090909... ?

- (A) 0.012 (B) 0.099 (C) 1.19 (D) 1.21 (E) 1.1526

4. The shaded area is best represented by which of the following inequalities?



- (A) $3x + 2y \geq 1$ (B) $2 \leq x - 3y$ (C) $3x - y \geq -2$ (D) $2 \leq 3x - y$ (E) $1 \geq 3x + 2y$

5. Simplify: $\frac{4x+1}{3x-2} - \frac{2x-3}{x+4}$

- (A) $\frac{-2x^2+4x-10}{3x^2+5x-8}$ (B) $\frac{10x^2-4x+10}{3x^2+10x-8}$ (C) $\frac{-2x^2-30x+2}{3x^2+10x-6}$
(D) $\frac{10x^2+30x-2}{3x^2+5x-8}$ (E) $\frac{-2x^2+30x-2}{3x^2+10x-8}$

6. Flo Tilla is rowing her canoe on a lake. She starts rowing back toward the dock which is 500 yards away. She rows at a rate of 3 miles per hour. How long will it take her to reach the dock? (nearest second)

- (A) 3 min 31 sec (B) 4 min 19 sec (C) 5 min 41 sec (D) 6 min (E) 6 min 40 sec

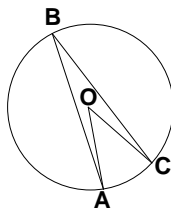
7. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle R = 42^\circ$ then $m\angle Q = ?$

- (A) 122° (B) 132° (C) 58° (D) 48° (E) 138°

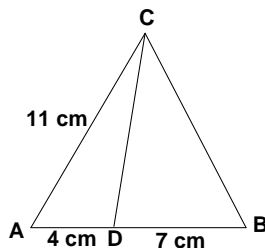
8. The numbers in row 9 of Pascal's triangle are 1, 9, k, 84, 126, 126, 84, k, 9, 1. Find k?

- (A) 18 (B) 27 (C) 36 (D) 45 (E) 54

9. Points A, B, and C lie on the circle with center O and $\widehat{AC} = 54^\circ$. Find $m\angle AOC + m\angle ABC$.



- (A) 40.5° (B) 54° (C) 71.5° (D) 81° (E) 108°
10. A cylindrical water tank is 12 feet high and has a diameter 8 feet long. The tank contains 480 gallons of water. What per cent of the tanks maximum capacity is water? (nearest %)
- (A) 80% (B) 53% (C) 39% (D) 11% (E) 3%
11. Find the circumference of the circle, $x^2 + y^2 - 6x - 10y + 30 = 0$. (nearest tenth)
- (A) 17.1 (B) 12.6 (C) 19.3 (D) 14.7 (E) 25.1
12. How many 5 character passwords are possible if the first character must be a positive digit, the second must be a letter, the third must be a vowel (excluding y and w), the fourth must be 0, and the fifth one must be a special character from this set { @, #, \$, &, + } ? Repetition is allowed.
- (A) 5,850 (B) 46 (C) 5,625 (D) 55 (E) 6500
13. Find the sum of the coefficients of the quotient: $(4x^3 + 3x^2 + 2x - 9) \div (x - 1)$
- (A) 7 (B) 9 (C) 12 (D) 20 (E) 22
14. The Sue Purr Deer tractor can plow a field in 4 hours 20 minutes. The Anne Teek Deer tractor takes three times as long to plow the same field. How long would it take them if they work together? (nearest minute)
- (A) 2 hrs 10 min (B) 2 hrs 53 min (C) 3 hrs (D) 3 hrs 15 min (E) 3 hrs 40 min
15. $\triangle ABC$ is an equilateral triangle. Find $m\angle BCD$. (nearest degree)



- (A) 28° (B) 35° (C) 39° (D) 40° (E) 50°

16. For all values of θ , which of the following has the same value as $\cos(\frac{3\pi}{4} - \theta)$?

- (A) $\sin(\theta + \frac{\pi}{4})$ (B) $\cos(\theta + \frac{3\pi}{4})$ (C) $\sin(\theta - \frac{5\pi}{4})$ (D) $\cos(\theta + \frac{3\pi}{4})$ (E) $\sin(\theta - \frac{\pi}{4})$

17. Hi Stepper, a drum major, is marking his path on the football field for the band's performance at half time. He walks 200 feet on a bearing of 110° . Then he turns and walks 180 feet on a bearing of 25° . The he turns to return to his starting point. What bearing will he need to take to return directly back to his starting point? (nearest degree)

- (A) 250° (B) 245° (C) 230° (D) 225° (E) 205°

18. Use the Fibonacci characteristic sequence ... 1, p, 3, q, r, ... to Find $p + q + r$.

- (A) 19 (B) 15 (C) 13 (D) 11 (E) 9

19. If the set of numbers {1, 2, 3, 4, 5, ...} continue in the triangular pattern shown below, the arithmetic mean of the numbers in the 8th row would be?

			1			
		2	3	4		
	5	6	7	8	9	
10	11	12	13	14	15	16
			...			

- (A) 51 (B) 53 (C) 57 (D) 61 (E) 63

20. In the binomial expansion of $(3x - 2)^4$, the coefficient of the 3rd term is:

- (A) 216 (B) 180 (C) 144 (D) 125 (E) 118

21. Let $f(x) = 5x^2 - 3x - 2$ and $g(x) = 2x - 1$. Find $g(f'(x + 1))$.

- (A) $10x - 3$ (B) $20x + 13$ (C) $10x - 7$ (D) $20x - 4$ (E) $20x + 5$

22. The polar graph of $r^2 = 36\cos(2\theta)$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$

- (A) 1 only (B) 2 only (C) 3 only (D) 1 & 3 (E) 1, 2, & 3

23. A water tank in the shape of an inverted cone has an altitude of 16 feet and a base diameter of 8 feet. Water is flowing into the tank at a rate of 2 cubic feet per minute. How fast is the water rising when the water is 5 feet deep? (nearest tenth)

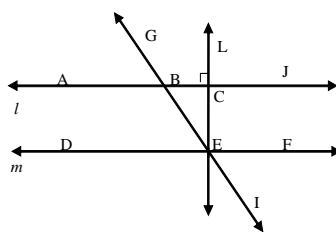
- (A) 2.4 in/min (B) 3.7 in/min (C) 4.9 in/min (D) 7.4 in/min (E) 9.8 in/min

24. The current Texas Supreme Court consists of 2 women and seven men. Special committees consist of 4 members. In how many ways can a special committee be formed such that at least one woman is on the committee?

- (A) 1,470 (B) 42 (C) 70 (D) 126 (E) 91

25. Willie Rollette rolls a pair of standard dice. What is the probability that the sum of the top faces is a Fibonacci number?
- (A) $11\frac{1}{9}\%$ (B) 25% (C) 30% (D) $33\frac{1}{3}\%$ (E) $44\frac{4}{9}\%$
26. How many 3-digit numbers can be made using the digits 1, 1, 2, 3, and 5?
- (A) 30 (B) 33 (C) 36 (D) 39 (E) 42
27. The repeating decimal 0.4222... in base 8 can be written as which of the following fractions in base 8?
- (A) $\frac{15}{28}_8$ (B) $\frac{25}{40}_8$ (C) $\frac{17}{34}_8$ (D) $\frac{21}{32}_8$ (E) $\frac{1}{2}_8$
28. Find the sum of all of the two-digit numbers whose units digit is divisible by 5.
- (A) 1,050 (B) 945 (C) 900 (D) 855 (E) 765
29. $1001_2 + 301_4 + 53_8 = \underline{\hspace{2cm}}_{10}$
- (A) 1,354 (B) 248 (C) 84 (D) 91 (E) 101
30. A cash register drawer contains \$1 bills, \$5 bills, and \$10 bills. The ratio of \$1 bills to \$5 bills is 5 to 6 and the ratio of \$5 bills to \$10 bills is 3 to 2. The total amount of money in the cash register drawer is \$300.00. How many \$5 bills are in the drawer?
- (A) 12 (B) 16 (C) 20 (D) 24 (E) 30
31. Four years ago Tu Yung was half as old as Soh Yung was. In four years Tu's age will be two-thirds of the age Soh will be. What is the sum of their ages now?
- (A) 12 (B) 20 (C) 32 (D) 36 (E) 40
32. Line m passing through $(2, -1)$ and $(1, -3)$ is perpendicular to line n passing through $(2, -3)$. Line n intersects the x-axis at (x, y) . Find x .
- (A) -4 (B) -3 (C) 1 (D) 2 (E) 8
33. The solution set of $3x + 4y = 25$ is $\{(x, y) \mid -5 \leq x \leq 7, y \geq 0, \text{ and } x, y \in \{\text{Integers}\}\}$. How many such ordered pairs exist?
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
34. Which of the following are the side lengths of a scalene acute triangle?
- (A) 7, 24, 25 (B) 7, 7, 11 (C) 9, 10, 11 (D) 6, 9, 12 (E) 8, 13, 21

35. The four lines in the figure are coplanar with $m \parallel l$. Which of the following are true statements?



- | | |
|--|---|
| 1. $\angle BEC$ & $\angle GBA$ are congruent | 2. $m\angle FEI + m\angle CEB = 90^\circ$ |
| 3. $\angle FEI$ & $\angle ABG$ are vertical angles | 4. $\angle CBE$ & $\angle GEF$ are supplementary angles |

(A) 2 & 4 (B) 1 & 2 (C) 1, 2, 3, & 4 (D) 3 & 4 (E) 1 & 4

36. Point P $(-4, 0)$ is translated 3 units up vertically to point Q. Point Q is reflected across the y-axis to point R. Point R is rotated clockwise $\frac{\pi}{2}$ radians about the origin to point S. Point S is translated 3 units left horizontally to point T (x, y) . Find $x + y$.

(A) -8 (B) -4 (C) -2 (D) 0 (E) 4

37. If $a_1 = 1$, $a_2 = -3$, $a_3 = 2$ and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_8 equals:

(A) -2 (B) -1 (C) 0 (D) 3 (E) 6

38. If $xy - \frac{1}{4} = y - x = 4 - x - y$, what is the value of $x + y$?

(A) 2.125 (B) 2.75 (C) 3.375 (D) 3.75 (E) 4.25

39. Betty Whens spins the spinner. It will land on a \$1.00 bill, a \$5.00 bill, or \$10.00 bill. The probability of landing on a \$1.00 bill is 60%, landing on a \$5.00 bill is 25%, and landing on a \$10.00 bill is 15%. What is the mathematical expectation on any one spin?

(A) \$1.17 (B) \$2.15 (C) \$2.75 (D) \$3.25 (E) \$3.35

40. Let $f(x) = x^2 - 2x - 5$ and $g(x) = 4x + 3$. Find $g^{-1}(g(f(-1)))$.

(A) 2.5 (B) 0.75 (C) -1 (D) -1.25 (E) -2

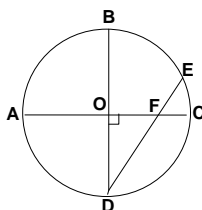
41. The graph of the polar equation $r = 3 + 4\sin(\theta)$ is a:

- | | | |
|----------------------|---------------------|---------------------|
| (A) dimpled cardioid | (B) looped cardioid | (C) dimpled limaçon |
| (D) looped limaçon | (E) convex limaçon | |

42. Which of the following is a reference angle for 2014° ?

(A) 214° (B) 146° (C) 56° (D) 34° (E) 17°

43. Given the circle O with perpendicular diameters and a chord, find the area of circle O if $EF = 1$ inch and $DF = 6$ inches. (nearest square inch)



- (A) 66 sq. in. (B) 77 sq. in. (C) 44 sq. in. (D) 57 sq. in. (E) 79 sq. in.
44. If $\begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix} \times \begin{bmatrix} 4 & -3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a \times d - b \times c = ?$
- (A) 11 (B) 22 (C) 25 (D) 55 (E) 121
45. The harmonic mean of the real roots of $x^3 - 7\frac{1}{2}x^2 + 15\frac{1}{2}x + k = 0$ is $1\frac{5}{31}$. Find k .
- (A) $-1\frac{14}{31}$ (B) -2 (C) $-2\frac{28}{31}$ (D) -6 (E) -8
46. Solve for x : $\log_2[\log_3(\log_4(x^{3x}))] = 0$
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
47. Find the area, in square units, of the figure bounded by $y = x^2 - 4$ and $y = 5$.
- (A) 40 (B) 36 (C) 32 (D) 30 (E) 27
48. If $3x^2 - 4y = 5$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$
- (A) 0.75 (B) 0.6 (C) 0.8 (D) 0.666... (E) 0.444...
49. Les Scents flipped a penny four times and recorded the results. What are the odds of only two consecutive heads or only two consecutive tails?
- (A) 6 (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) $\frac{3}{5}$ (E) 1
50. A package of 25 DVD discs contains some good discs and some bad discs. The probability of selecting a good disc is 92%. How many bad discs can be expected to be in the package?
- (A) 1 (B) 2 (C) 3 (D) 5 (E) 8
51. The *Achilles* is one of the four famous paradoxes that is credited to?
- (A) Parmenidas (B) Archimedes (C) Diophantus (D) Theano (E) Zeno

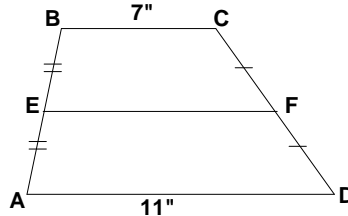
52. The fourth *harmonic number* is:

- (A) 0.25 (B) 1.5 (C) 2.08333... (D) 2.28333... (E) 5.8333...

53. Let $f(x) = x^3 - 6x^2 + 9x + 5$. Find the sum of the x -values of the critical points of $f(x)$.

- (A) 4 (B) 3 (C) 2 (D) 0 (E) -5

54. Given the trapezoid shown where $AD \parallel EF \parallel BC$, find EF . (nearest hundredth)



- (A) 9.22 " (B) 9.00 " (C) 8.92 " (D) 8.77 " (E) 8.56 "

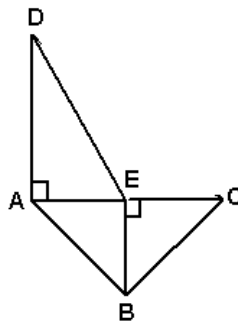
55. The altitude of $\triangle ABC$ forms two right triangles, $\triangle ABD$ and $\triangle CBD$. Find AD if $CD = 8$ cm, $AB = 12$ cm, and $m\angle CBD = 50^\circ$. (nearest tenth)

- (A) 8.9 cm (B) 7.3 cm (C) 9.9 cm (D) 6.7 cm (E) 11.8 cm

56. The roots of the equation $x^3 + kx^2 - 18x + 40 = 0$ are 2, 5, and R . Find k .

- (A) 7 (B) 3 (C) -3 (D) -4 (E) -11

57. Given: $m\angle DAB = 135^\circ$, $m\angle BED = 150^\circ$, $m\angle ECB = 45^\circ$ and $BE = 4$ cm. Find the perimeter of the figure shown. (nearest tenth).






- (A) 25.7 cm (B) 26.9 cm (C) 27.3 cm (D) 29.2 cm (E) 30.2 cm

58. A coordinate is plotted on the Cartesian plane. It's abscissa is less than zero and its ordinate is greater than zero. Where is the coordinate located on the plane?

- (A) Quadrant I (B) Quadrant II (C) Quadrant III (D) Quadrant IV (E) Origin

59. Let $A = \{t, r, i, p, o, d\}$, $B = \{t, r, a, p, e, z, o, i, d\}$, and $C = \{t, r, o, p, h, y\}$. The number of distinct elements in $A \cup (B \cap C)$ is _____.

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

60. The Mayan number system consists of three symbols,   . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number would this Mayan number represent?



- (A) 6 (B) 15 (C) 25 (D) 51 (E) 101

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation A • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. B | 41. D |
| 2. B | 22. E | 42. D |
| 3. C | 23. C | 43. A |
| 4. D | 24. E | 44. E |
| 5. E | 25. D | 45. D |
| 6. C | 26. B | 46. C |
| 7. B | 27. C | 47. B |
| 8. C | 28. B | 48. D |
| 9. D | 29. E | 49. E |
| 10. D | 30. D | 50. B |
| 11. B | 31. C | 51. E |
| 12. A | 32. A | 52. C |
| 13. D | 33. C | 53. A |
| 14. D | 34. C | 54. B |
| 15. C | 35. A | 55. C |
| 16. E | 36. B | 56. C |
| 17. A | 37. E | 57. E |
| 18. B | 38. B | 58. B |
| 19. C | 39. E | 59. D |
| 20. A | 40. E | 60. C |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

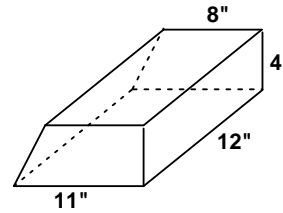
Invitational B • 2014



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1. Evaluate: $20 + 7 \times [(14 - 30) \div 8] - 14$
 (A) -8 (B) 77.75 (C) 100.25 (D) 20 (E) -68
2. Find the number of positive integral divisors of 360.
 (A) 3 (B) 6 (C) 15 (D) 21 (E) 24
3. Les Dough borrowed \$500.00 from his bank at a compound interest rate of 6%. The loan was for 1 year and interest was compounded monthly. How much interest will he have paid at the end of the year? (nearest cent)
 (A) \$30.00 (B) \$30.60 (C) \$30.84 (D) \$36.16 (E) \$60.00
4. A set of 5 positive integers has a mean of 48, a median of 42, a mode of 22, and a range of 66. If A, B, C, D, and E are the integers arranged from least to greatest, then D is?
 (A) 49 (B) 55 (C) 66 (D) 70 (E) 77
5. Simplify: $\left(\frac{16x^2 + 8x + 1}{16x^2 - 1} \right) \left(\frac{16x^2 - 8x + 1}{4x - 1} \right)$
 (A) $x - 4$ (B) $x^2 + 16$ (C) $4x + 1$ (D) $x + 4$ (E) $4x - 1$
6. The set of all integers is closed under which of the following standard operations :
 I. addition II. subtraction III. multiplication IV. division
 (A) I & II (B) I, II & III (C) II & III (D) I, II, III & IV (E) none of these
7. Find k when $2 \times 7 + 14 \div 3 \div (k - 8) = 20$
 (A) $7\frac{2}{9}$ (B) -16 (C) $8\frac{7}{9}$ (D) $-7\frac{5}{8}$ (E) $8\frac{3}{8}$
8. Teeter and Totter were riding on a seesaw. Teeter weighs 80 pounds and is sitting 4 feet from the center of the seesaw. Totter weighs 95 pounds and is sitting on the opposite end of the seesaw. How far from the center is Totter sitting if the seesaw is balanced? (nearest inch)
 (A) 3' 4" (B) 3' 6" (C) 3' 9" (D) 4' 3" (E) 4' 9"
9. Point A lies on the x-y plane. Point A (3,0) is rotated 120° clockwise about the origin to point B. Point B is reflected across the line $y = -x$ to point C. Point C is translated horizontally 6 units to the left to point D. Point D is reflected across the y-axis to point E. Point E is located in which quadrant or on which axis?
 (A) QI (B) QII (C) QIII (D) QIV (E) y-axis

10. Find the lateral area of the figure shown. Drawing is not to scale.

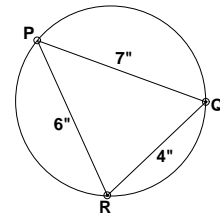


- (A) 324 sq. in (B) 336 sq. in (C) 374 sq. in (D) 412 sq. in (E) 424 sq. in

11. M. T. Tank has an empty right cylinder-shaped water trough. The trough is 2 feet high and has an inside diameter of 12 feet. What percent of the tank's capacity will contain water if he puts in 400 gallons? (nearest percent)

- (A) 6% (B) 33% (C) 12% (D) 17% (E) 24%

12. Find the diameter of the circle circumscribed about $\triangle PQR$. (nearest inch)



- (A) 8" (B) 7.5" (C) 7.2" (D) 7" (E) 6.8"

13. If $\frac{5}{x^2 - x - 6} - \frac{5}{x^2 - 2x - 3} = \frac{k}{x^3 - 7x - 6}$, then k equals:

- (A) $10x + 15$ (B) $5x - 10$ (C) -5 (D) $10x - 5$ (E) 0

14. Let $A = \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -6 \\ -3 & 10 \end{bmatrix}$. Find $|B - A|$.

- (A) 30 (B) 10 (C) 6 (D) -2 (E) -4

15. The sum of the numbers greater than 1 in row 8 of Pascal's Triangle is:

- (A) 510 (B) 254 (C) 165 (D) 126 (E) 123

16. If $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_n = (a_{n-3} + a_{n-1}) - a_{n-2}$, where $n \geq 4$, then a_6 equals:

- (A) 18 (B) 12 (C) 8 (D) 13 (E) 24

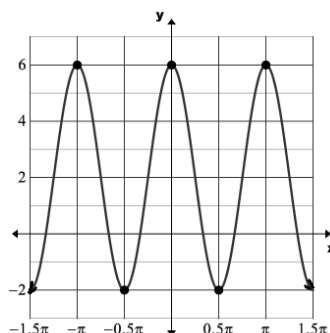
17. The Upp brothers, Doug, Sid, and Stan can wash all of the windows in their house in 4, 3, and 6 hours respectively. How long would it take them when they work together?

- (A) 1 hr 20 min (B) 2 hrs 15 min (C) 3 hrs 30 min (D) 4 hrs 20 min (E) 4 hrs 45 min

18. Simplify: $\sqrt{9 - 9\cos^2\theta}$

- (A) 3 (B) $3 + 3|\sin\theta|$ (C) $3 - |\sin^2\theta|$ (D) $3 + |\cos\theta|$ (E) $3|\sin\theta|$

19. The equation $y = D + A\cos(Bx + C\pi)$, will produce this graph. If A, B, C and $D > 0$ then $A + B + C + D$ is:

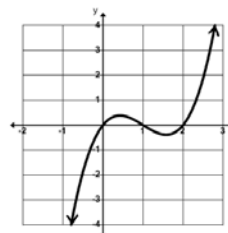


- (A) 11 (B) 10 (C) 9 (D) 8 (E) 7

20. Which of the following is not a solution for $2\cos^2(\theta) + 3\cos(\theta) = 2$?

- (A) $-\frac{17\pi}{3}$ (B) $-\frac{5\pi}{3}$ (C) $\frac{9\pi}{3}$ (D) $\frac{23\pi}{3}$ (E) $\frac{35\pi}{3}$

21. Which of the following is true about the relation graphed below?



- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
(D) It is neither an even nor an odd function (E) It is a one-to-one function.

22. Given the arithmetic sequence $-4, a, b, c, 8, d, \dots$, find $d(a + b + c)$.

- (A) 66 (B) 110 (C) 48 (D) 96 (E) 21

23. The directrix of the parabola $(y + 1)^2 = -8(x + 5)$ is:

- (A) $x = -1$ (B) $x = -2$ (C) $x = -3$ (D) $x = -5$ (E) $x = -7$

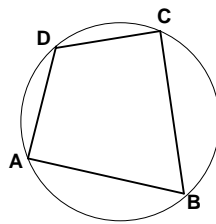
24. Let $f(x) = \sqrt[3]{x^2}$. Find $f'(8)$.

- (A) $\frac{3}{4}$ (B) $2\sqrt{2}$ (C) $3\sqrt{2}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

25. The function $f(x) = x^3 + 5x^2 + 3$ has an inflection point at $x = ?$

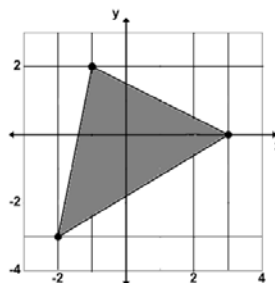
- (A) $-3\frac{1}{3}$ (B) $-1\frac{2}{3}$ (C) -1 (D) 0 (E) does not have one

26. A ladder 24 feet long is leaning against a building. The base of the ladder is slipping horizontally away from the wall at a rate of 5 feet per second. How fast is the top of the ladder sliding down the side of the building when the bottom is 10 feet from the building? (nearest tenth)
- (A) 2.1 ft/sec (B) 2.3 ft/sec (C) 2.5 ft/sec (D) 3.0 ft/sec (E) 3.2 ft/sec
27. How many ways can four letters from the letters of the word GEOMETRIC be written such that the first two letters are consonants and the last two are vowels?
- (A) 180 (B) 60 (C) 30 (D) 240 (E) 120
28. Twenty-four golf balls are in a bag. One-fourth of them are Bridgestones, three-eighths of them are Calloways, one-sixth of them are Titleists, and the rest are Pinnacles. Mr. White randomly draws out one ball. What is the probability that it is his favorite, a Pinnacle?
- (A) $20\frac{5}{6}\%$ (B) 25% (C) $26\frac{6}{19}\%$ (D) $37\frac{1}{2}\%$ (E) $62\frac{1}{2}\%$
29. I. C. Kreme has 8 different buckets of ice cream. He uses 4 scoops of ice cream to make a special sundae. How many different types of sundaes can he make?
- (A) 70 (B) 330 (C) 1,680 (D) 490 (E) 165
30. Which of the following is considered to be an evil number?
- (A) 13 (B) 57 (C) 7 (D) 117 (E) 25
31. Points A, B, C, and D lie on the circle shown. If $\widehat{AB} = 116^\circ$, $\widehat{BC} = 100^\circ$, and $\widehat{CD} = 60^\circ$, then $m\angle ABC = ?$



- (A) 36° (B) 54° (C) 72° (D) 84° (E) 108°
32. The contraharmonic mean of 6 and 12 is: (nearest tenth)
- (A) 8.0 (B) 8.5 (C) 8.8 (D) 9.3 (E) 10.0
33. Let P be a two-digit prime number such that neither digit is prime. What is the sum of all such numbers, P?
- (A) 221 (B) 28 (C) 270 (D) 141 (E) 246

34. The number 10110111 in base 2 is equivalent to the number X in base 4. Find the sum of the digits in the number X.
- (A) 12 (B) 8 (C) 6 (D) 15 (E) 9
35. Cookie Baykur made a batch of cookies. She gave $\frac{2}{3}$ of the cookies to her son for his scout meeting. She kept 80% of the cookies she had left for her family for afternoon snacks. She ate 2 of the remaining cookies. There were 8 cookies left for her bridge club. How many dozens of cookies did she bake originally?
- (A) $8\frac{1}{3}$ doz (B) 10 doz (C) $11\frac{1}{2}$ doz (D) $12\frac{1}{2}$ doz (E) 15 doz
36. The Real value solution set for $|2 - 3x| + 5 < 7$ is?
- (A) $\{x \mid \{-4 < x\} \cup \{x > 0\}\}$ (B) $\{x \mid 0 < x < 1\frac{1}{3}\}$ (C) $\{x \mid \{x > 1\frac{1}{3}\} \cup \{x < 0\}\}$
- (D) $\{x \mid -1\frac{1}{3} < x < 0\}$ (E) $\{x \mid \{0 < x\} \cup \{x < 1\frac{1}{3}\}\}$
37. A gas station wants to mix grade A gasoline worth \$2.70 per gallon with some grade B gasoline worth \$2.25 per gallon to make 500 gallons of gasoline worth \$2.40 per gallon. How many gallons of grade A will the station need? (nearest gallon)
- (A) 250 gal (B) 83 gal (C) 167 gal (D) 417 gal (E) 333 gal
38. Determine the greatest integer value of k so that $2x^2 - 9x + k = 0$ has two positive real roots.
- (A) -7 (B) -4 (C) 6 (D) 10 (E) 11
39. What kind of triangle will the segment lengths of 7", 11", and 13" form?
- (A) Acute (B) Equiangular (C) Isosceles (D) Obtuse (E) Right
40. Rene D'Kart plotted this triangle on the coordinate plane below. The coordinates of the vertices are integers. What is the area of his triangle?



- (A) 10 units² (B) 10.5 units² (C) 10.75 units² (D) 11 units² (E) 11.5 units²
41. Find the shortest distance from the origin to the line $3x + 4y = 12$.
- (A) 2.5 (B) 2.4 (C) 2 (D) $1\frac{5}{7}$ (E) $1\frac{1}{3}$

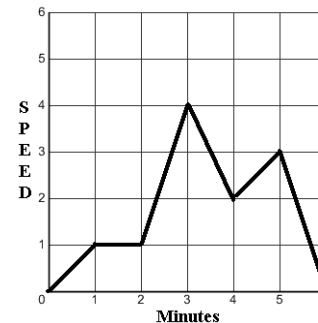
42. Let $f(x) = ax^5 - bx^3 + cx - 1$ where a , b , and c are integers. If $f(2) = -5$ then $f(-2) = ?$

- (A) 5 (B) 3 (C) -3 (D) -6 (E) -7

43. If the digits of the three-digit number, $95k$, are reversed and that number is subtracted from the original number, the results would contain the same three digits in a different order. Find k .

- (A) 9 (B) 8 (C) 5 (D) 4 (E) 1

44. Joy Ryder goes on a roller coaster ride at the state fair. The graph best depicts the 6 minute ride. During the time interval of 4 minutes to 5 minutes Joy's roller coaster car is _____. (Speed is not truly linear in this case.)



- (A) climbing uphill (B) turning on a curve (C) going downhill
(D) decreasing speed (E) increasing speed

45. Let $f(x + 1) = 2f(x) + 1$. Find $f(3)$ if $f(1) = 3$.

- (A) 5 (B) 7 (C) 15 (D) 17 (E) 23

46. If $\sin x - \cos x = \frac{\sqrt{2}}{2}$, find the numerical value of $\cos 3x$, where $180^\circ < x < 270^\circ$.

- (A) $-\frac{3\sqrt{2}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) $\frac{\sqrt{2}}{2}$

47. The direction angles of vectors v and u are 60° and 150° , respectively. Find the direction angle of vector $v + u$ if $\|v\| = 9$, $\|u\| = 8$. (nearest degree)

- (A) 90° (B) 93° (C) 98° (D) 102° (E) 108°

48. The diameter of a circle has length D . The length of a chord is C and the length of the arc cut off by the chord is A . Find $\sin(\frac{A}{D})$ if the chord and the diameter share a common endpoint.

- (A) $\frac{2C}{D}$ (B) $2C$ (C) $\frac{D}{C}$ (D) $C + D$ (E) $\frac{C}{D}$

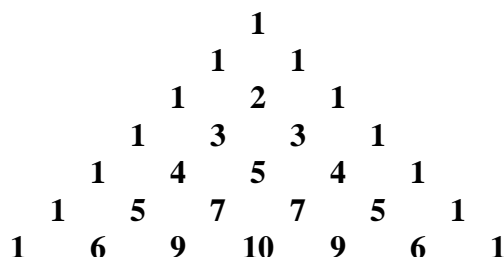
49. Hipparchus converted the polar equation $r = 2\sin(\theta)$ to a rectangular equation to determine the center of the circle to be (x, y) . Find $x + y$.

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 3

50. How many solutions are there for the equation $4x + 5y = 2014$ such that both x and y are non-negative integers?

- (A) 101 (B) 100 (C) 180 (D) 200 (E) 223

51. If the set of natural numbers continue in the triangular pattern shown below, the median of the row beginning with 1 10 ... would be?

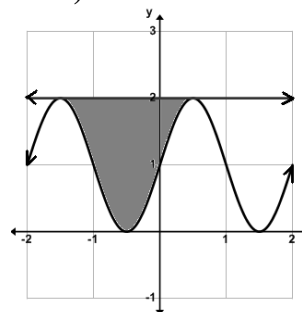


- (A) 25 (B) 26 (C) 27 (D) 30 (E) 37

52. Let $n \log_n(n^n) = 9$. Find 9^n .

- (A) 81 (B) $9\sqrt{3}$ (C) 343 (D) $9\sqrt{2}$ (E) 729

53. Find the area in square units of the shaded region. (nearest hundredth)



- (A) 3.14 (B) 2.50 (C) 2.00 (D) 1.85 (E) 1.58

54. Which of the following sequences is divergent?





- (A) $\left\{ \frac{2n}{3n+1} \right\}$ (B) $\left\{ \frac{1}{2^n} \right\}$ (C) $\left\{ \frac{\ln(n)}{n^2} \right\}$ (D) $\left\{ \frac{n^2+2}{2n^2-1} \right\}$ (E) $\left\{ \ln\left(\frac{1}{n-1}\right) \right\}$

55. How many kinds of symmetry does the graph of the polar equation, $r = 5 - \sin(2\theta)$, have?

- (A) none (B) 1 (C) 2 (D) 3 (E) 4

56. The Quik Pik lock company uses 3 randomly picked numbers for each of their lock's combination. The first digit is randomly selected from the set of odd numbers, the second digit is from the set of composite numbers, and the third digit is from the set of factors of 10. How many unique combinations fit this criteria?

- (A) 9 (B) 12 (C) 45 (D) 60 (E) 80

57. The Rangers and the Astros are playing a three game series. The Ranges are twice as likely to win any of these games as the Astros. What is the probability that the Rangers will win all 3 games? (nearest percent)
- (A) 78% (B) 67% (C) 33% (D) 30% (E) 22%
58. What are the odds of randomly choosing the letter A from a bag containing the letters in the word MATHEMATICS?
- (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{2}{11}$ (D) $\frac{1}{8}$ (E) $\frac{2}{9}$
59. Change the base 10 proper fraction $\frac{3}{8}$ to a repeating decimal in base 5.
- (A) 0.232323...₅ (B) 0.2666...₅ (C) 0.141414...₅ (D) 0.5333...₅ (E) 0.1767676...₅
60. The Mayan number system consists of three symbols,   . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. How many 's would be needed to write 150 base 10 as a Mayan number?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. D | 41. B |
| 2. E | 22. A | 42. B |
| 3. C | 23. C | 43. D |
| 4. C | 24. D | 44. E |
| 5. C | 25. B | 45. C |
| 6. B | 26. B | 46. B |
| 7. C | 27. E | 47. D |
| 8. A | 28. A | 48. E |
| 9. A | 29. B | 49. D |
| 10. B | 30. B | 50. A |
| 11. E | 31. C | 51. B |
| 12. D | 32. E | 52. E |
| 13. C | 33. A | 53. C |
| 14. D | 34. E | 54. E |
| 15. B | 35. D | 55. B |
| 16. A | 36. B | 56. D |
| 17. A | 37. C | 57. D |
| 18. E | 38. D | 58. E |
| 19. B | 39. A | 59. C |
| 20. C | 40. D | 60. C |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 1 • 2014



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1. Evaluate: $[2 - 3(5 - 7)] + 11 \times (13 + 17 - 19)$

- (A) 123 (B) 129 (C) 132 (D) 149 (E) 167

2. Let $S = \{s, e, c, o, n, d\}$, $M = \{m, i, n, u, t, e\}$, $H = \{h, o, u, r\}$, and $T = \{t, i, m, e\}$. The number of distinct elements in $(S \cap H) \cup (M \cap T)$ is _____.

- (A) 2 (B) 6 (C) 10 (D) 4 (E) 5

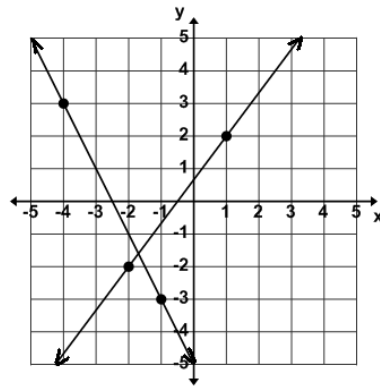
3. Penni Lesse got a \$2000.00 student loan to help pay her dorm rent. She has to pay it back in 24 equal monthly payments. What will her monthly payments be if the annual interest rate of 3.5% is compounded monthly? (nearest cent)

- (A) \$84.80 (B) \$85.63 (C) \$86.30 (D) \$89.37 (E) \$90.17

4. If P is 130% of Q and R is 70% of P, then R is what percent of Q?

- (A) 100% (B) 60% (C) 91% (D) 86% (E) 90%

5. The two lines shown intersect at the point (x, y) . Find $x + y$.



- (A) $-3.777\ldots$ (B) -3.5 (C) $-3.222\ldots$ (D) -3.3 (E) -3.275

6. Let $\frac{x+2}{3x-4} - \frac{4x-1}{2x+3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$. Find $\frac{A+B+C}{P+Q+R}$.

- (A) 2.4 (B) 0.4 (C) -1 (D) -2.4 (E) -3.6

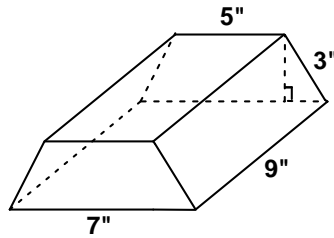
7. Which of the sets of numbers given is not closed under the operation of addition?

- (A) Even numbers (B) Integers (C) Irrationals (D) Whole numbers (E) Rationals

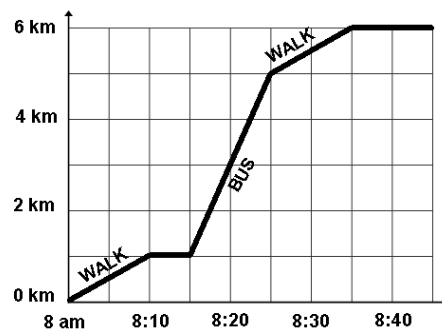
8. Saul T. Baye is fishing in his kayak at Baffin Bay. He starts paddling towards the shore at a rate of 4 miles per hour. How far from shore was he if he makes it to shore in 12 minutes?

- (A) 1,184 yds (B) 1,408 yds (C) 1,420.8 yds (D) 1,440.2 yds (E) 1,712 yds

9. Find the base area of the isosceles trapezoid prism. (nearest sq. in). Drawing is not to scale.

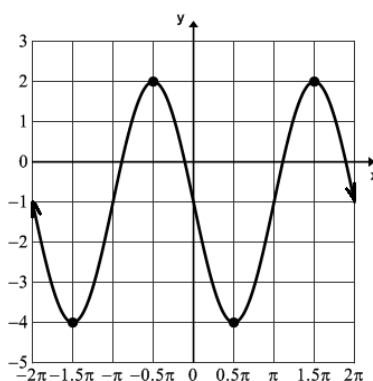


- (A) 17 sq. in (B) 18 sq. in (C) 24 sq. in (D) 34 sq. in (E) 63 sq. in
10. Find the shortest distance from the point (1, 1) to the line $5x + 12y = 60$.
- (A) $1\frac{7}{13}$ (B) $2\frac{2}{5}$ (C) $3\frac{4}{13}$ (D) 5 (E) $5\frac{12}{13}$
11. A pentagonal rotunda has 17 faces and 35 edges. How many vertices does it have?
- (A) 15 (B) 20 (C) 33 (D) 37 (E) 50
12. Phil Whitwhatter has an empty rectangular water tank that is 10' long, 8' wide, and 6' deep. If Phil puts 2500 gallons of water in the tank, what percent of the tank's capacity contains water? (nearest percent)
- (A) 48% (B) 52% (C) 60% (D) 63% (E) 70%
13. The heronian mean of 8 and 18 is: (nearest tenth)
- (A) 12.7 (B) 13 (C) 11.1 (D) 13.6 (E) 12
14. One of the three most famous and well known Zeno paradoxes involves trying to reach a specific point by covering only half of the remaining distance each time. Hence not ever reaching that specific point. This paradox is known as the:
- (A) Achilles and Tortoise (B) Arrow flight (C) Dichotomy (D) Stadium (E) Grain of Millet
15. Willie Makette has to walk and ride a bus to get to school each day. Using the graph below determine the average speed of the bus.



- (A) 14 kmph (B) 24 kmph (C) 32 kmph (D) 40 kmph (E) 48 kmph

16. The probability of scoring at least 120 on the district 1 test in class 1A is 64%. Based on this probability, if 500 take this test what are the odds of scoring less than 120?
- (A) $\frac{9}{25}$ (B) $\frac{3}{10}$ (C) $\frac{8}{15}$ (D) $\frac{9}{16}$ (E) $\frac{6}{25}$
17. The Millersview *Old-Goats* football team consists of three 25¢ backs, five lopers, seven kneelers, four pass-outers, and a kicker of the bucket. In how many ways can a group of eleven Old Goats be formed such that there is one 25¢ back, three lopers, five kneelers, and two pass-outers?
- (A) 40 (B) 2,943 (C) 3,780 (D) 12,600 (E) 59
18. The first five numbers in row 11 of Pascal's triangle are 1, 11, k, 165, 330, Find k?
- (A) 121 (B) 110 (C) 88 (D) 55 (E) 44
19. If $3^{(3x-y)} = 27$ and $5^{(x+5y)} = 3,125$ then $4^{(2x+4y)} =$ _____.
- (A) 4,096 (B) 256 (C) 512 (D) 2,048 (E) 65,536
20. Tye Purr and Ed Ittur need to read and edit a 500 page book before it is published. Tye can do it alone in 6 hours 50 minutes. Ed can do it alone in 5 hours 20 minutes. How long would it take them to complete the edits working together? (nearest minute)
- (A) 2 hrs 53 min (B) 3 hrs (C) 5 hrs 45 min (D) 6 hrs 2 min (E) 6 hrs 5 min
21. How many integral values of n exist such that $n > 1$ and $\frac{(n+1)!}{(n-1)!} \leq 110$
- (A) 22 (B) 20 (C) 11 (D) 10 (E) 9
22. Let $y = D + A\sin(Bx - C\pi)$ will produce this graph. If $C > 0$ then $A + B + C + D = ?$



- (A) 6 (B) 4 (C) 2 (D) -2 (E) -3
23. $\sin(2\theta) \times \tan(\theta) + \cos(2\theta)$ is equivalent to:
- (A) $\sec(\frac{\pi}{4})$ (B) $\tan(\frac{\pi}{4})$ (C) $\cos(\frac{\pi}{2})$ (D) $\sin(\frac{\pi}{4})$ (E) $\cot(\frac{\pi}{2})$

24. Tim Burr needs to know how tall a tree is before cutting it down to be sure it doesn't hit a shed. The angle of elevation to the top of the tree is 29° from where is standing. From a point 50 feet closer, the angle of elevation is 34° . How tall is the tree? (nearest inch)
- (A) 189' 5" (B) 121' 1" (C) 155' 6" (D) 180' 2" (E) 127' 10"
25. The sum of the coefficients of the 2nd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 5th term of $(x + 1)^5$, and 7th term of $(x + 1)^7$ is:
- (A) 10 (B) 14 (C) 17 (D) 18 (E) 22
26. Find C + D if the the triangular pattern shown below continues.
- | | | | | | | | | |
|----|---|----|-----|----|---|---|---|---|
| | | | 1 | | | | | |
| | | 2 | | 1 | | | | |
| | | 4 | | 4 | | 1 | | |
| | 8 | | 12 | | 6 | | 1 | |
| 16 | | 32 | | 24 | | 8 | | 1 |
| A | B | C | D | E | F | | | |
| | | | ... | | | | | |
- (A) 160 (B) 112 (C) 50 (D) 72 (E) 120
27. Let $f(x) = 4x^2 - 8x + 1$ and $g(x) = 2x + 1$. Find $g(f'(x - 1))$
- (A) $16x - 31$ (B) $4x^2 - 16x + 13$ (C) $8x - 7$ (D) $8(x^2 - 2x + 1)$ (E) $16(x - 1)$
28. Evaluate: $\int_{-n}^n (3 - 2x^3) dx$
- (A) n^4 (B) $6n$ (C) $6n + n^4$ (D) $6n - n^4$ (E) $n^4 - 6n$
29. I. C. Cleerly has 25 students in his Algebra 1 class. Each student is assigned a letter of the alphabet, except for the letter "I" which is reserved for Mr. Cleerly. Two of the student's letters are drawn at random without replacement. If the probability that both of the letters drawn are students that wear glasses is 22%, how many of the 25 students do not wear glasses?
- (A) 10 (B) 11 (C) 13 (D) 14 (E) 15
30. The number 90 is considered to be a *polite number*. What is it's *politeness*?
- (A) 12 (B) 9 (C) 6 (D) 5 (E) 3
31. $111_2 + 222_4 + 333_8 = \underline{\hspace{2cm}}_{10}$
- (A) 1,934 (B) 268 (C) 222 (D) 206 (E) 108
32. Which of the following lines is concurrent to the lines $2x + 3y = -1$ and $x - 2y = 3$?
- (A) $4x + 5y = 1$ (B) $3x + 2y = -1$ (C) $3x - 4y = -7$ (D) $2x + y = -3$ (E) $5x - 6y = 11$

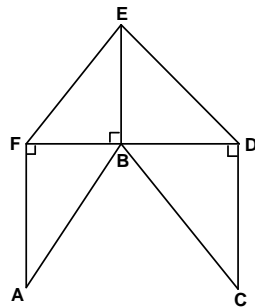
33. I. M. Cheep wants to buy four shirts for his Spring Break trip. There is a "No-Tax Discount" sale going on. Which of the following is the cheapest deal?

- (A) \$13.90 each (B) \$14.50 each and he has a 10% off coupon
 (C) buy 3 at \$17.85 each and get 1 free (D) buy 2 at \$19.95 each and get 2 half off
 (E) buy 3 at \$15.60 and get 25% off of the 4th one

34. The Real value solution set for $|4 - 5x| + 6 > 10$ is?

- (A) $\{x \mid \{x < 0\} \cup \{x > 1.6\}\}$ (B) $\{x \mid \{x < 0\} \cup \{x > 4\}\}$ (C) $\{x \mid \{x > 0\} \cup \{x < -1.6\}\}$
 (D) $\{x \mid 0 < x < 4\}$ (E) $\{x \mid 0 < x < 1.6\}$

35. Given: $m\angle BFE = 45^\circ$, $m\angle BED = 60^\circ$, $m\angle CBD = 30^\circ$, $m\angle ABF = 60^\circ$, and $BE = 4$ cm. Find the perimeter of the hexagon shown. (nearest tenth).



- (A) 55.5 cm (B) 45.9 cm (C) 42.2 cm (D) 40.6 cm (E) 38.4 cm

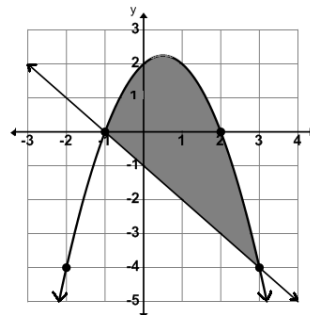
36. The roots of the equation $x^3 + kx^2 - 23x = 60$ are R , 5, and -3 . Find k .

- (A) 2 (B) 1 (C) -12 (D) -6 (E) -4

37. If $a_1 = -2$, $a_2 = 1$, $a_3 = -3$, and $a_n = a_{n-3}(a_{n-2} + a_{n-1})$, where $n \geq 4$, then a_7 equals:

- (A) -15 (B) -4 (C) 64 (D) 72 (E) -56

38. Find the area of the shaded region in square units.

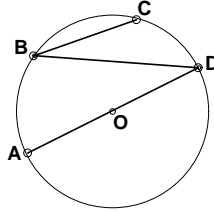


- (A) 10 (B) $10\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 11 (E) $11\frac{1}{3}$

39. The probability that statement P is true is 48%, and the probability that statement Q is true is 75%. Determine the probability that $P \rightarrow Q$ is false.

(A) 88% (B) 64% (C) 36% (D) 27% (E) 12%


40. Given the circle with center O. Find $m\widehat{CD} + m\angle ADB + m\angle CBD$, if $m\widehat{AB} = 70^\circ$ and $m\angle BOC = 60^\circ$. (Drawing not to scale.)



(A) 180° (B) 135° (C) 110° (D) 105° (E) 90°

41. How many 3-digit numbers can be made using the digits 0, 2, 2, 4, and 6?

(A) 39 (B) 36 (C) 30 (D) 26 (E) 24

42. The Mayan number system consists of three symbols, \bullet — . The total number of these symbols needed to write 2014 in base 10 as a Mayan number is:

(A) 12 (B) 10 (C) 8 (D) 7 (E) 5

43. Let K be a two-digit number such that the tens digit is divisible by 3 and units digit is a positive multiple of 4. What is the sum of all of these two-digit numbers?

(A) 396 (B) 72 (C) 384 (D) 184 (E) 408

44. A box contains five rods whose lengths are 4 in, 7 in, 10 in, 1 ft 1 in and 1 ft 6 in. How many different acute triangles can be made using only three rods at a time.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

45. Use the Fibonacci characteristic sequence $\dots - 8, p, q, r, - 5, \dots$ to Find $p + q + r$.

(A) $-1\frac{1}{3}$ (B) -1 (C) $-\frac{2}{3}$ (D) $1\frac{2}{3}$ (E) $2\frac{1}{3}$

46. If $(1 - 2i) - (2 + 3i) \times (3 - 4i) = a + bi$ then $a + b$ is:

(A) 14 (B) 4 (C) -34 (D) -22 (E) -20

47. The eccentricity of the ellipse $16x^2 + 25y^2 = 400$ is: (nearest hundredth)

(A) $\frac{25}{3}$ (B) $\frac{9}{16}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$ (E) $\frac{3}{4}$

48. Find the slope of the line tangent to the curve $y = 7 - 6x - x^2$ at the point $(-1, 12)$.

- (A) $-4\frac{1}{3}$ (B) -4 (C) $-3\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) $-\frac{1}{12}$

49. Willis A. Nutt bought 5 pounds of walnuts at \$4.50 per pound and 7 pounds of filberts at \$3.75 per pound. He mixed them together. The cost per pound of the mixture would be? (nearest cent)

- (A) \$3.81 (B) \$3.94 (C) \$4.06 (D) \$4.13 (E) \$4.16

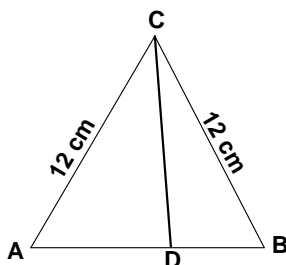
50. Points P, Q, and R lie on a circle with center C and chord PQ is the diameter. Point X is on chord PR. Find the perimeter of $\triangle CPX$ if $\overline{CX} \parallel \overline{QR}$, $PQ = 13$ cm and $QR = 5$ cm.

- (A) 32.5 cm (B) 30 cm (C) 22.5 cm (D) 16.25 cm (E) 15 cm

51. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 3 & 6 \end{bmatrix}$. Find $|A|$.

- (A) -4 (B) -6 (C) 3 (D) 2 (E) 1

52. $\triangle ABC$ is an equilateral triangle. The ratio of AD to DB is 2 to 1. Find $m\angle CDB$. (nearest degree)



- (A) 101° (B) 104° (C) 108° (D) 112° (E) 120°

53. $(3\text{cis}(\frac{\pi}{3}))^3$ equals:

- (A) $9 - 9i$ (B) $-6.25 + 4.5i$ (C) $3 - i\sqrt{3}$ (D) $1 - i$ (E) -27

54. Rusty Yaht sets sail from his personal dock at 8:00 am on a bearing of 65° . After sailing 5 km, he changes course and sails 10 km on a bearing of 120° . How far away from his personal dock is he at the end of the 15 km voyage? (nearest tenth)

- (A) 11.6 km (B) 12.5 km (C) 13.2 km (D) 13.5 km (E) 14.4 km

55. Given the geometric sequence $-4, a, b, -\frac{27}{128}, c, \dots$, find $a + b + c$.

- (A) $-2\frac{181}{512}$ (B) $-2\frac{145}{1024}$ (C) $-4\frac{81}{256}$ (D) $-6\frac{35}{128}$ (E) $-6\frac{361}{1024}$

56. If $\log(x + 1) + \log(x + 2) = \log(2x + 22)$, solve for x .

- (A) 2 (B) 4 (C) 7 (D) 11 (E) no solution

57. Which of the following series are divergent? 1. $\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$ 2. $\sum_{n=0}^{\infty} \cos(n\pi)$ 3. $\sum_{n=0}^{\infty} \frac{n!}{1000^n}$

- (A) 1 only (B) 2 only (C) 3 only (D) 1 & 2 (E) 2 & 3

58. A light bulb is hung 15 feet above a straight horizontal path. A man is walking away from the light at a rate of 5 feet per second. If the man is 6 feet tall, at what rate is the tip of the man's shadow moving? (nearest tenth)

- (A) 8.3 ft/sec (B) 8.0 ft/sec (C) 6.0 ft/sec (D) 2.5 ft/sec (E) 1.2 ft/sec

59. Cookie Baykur baked chocolate chip cookies, peanut butter cookies, sugar cookies, carrot cake cookies, and pecan cookies. She packages them in sets of three per package. How many different packages can she make?

- (A) 35 (B) 21 (C) 720 (D) 60 (E) 15

60. Change the base 10 proper fraction $\frac{5}{12}$ to a repeating decimal in base 11.

- (A) 0.464646...₁₁ (B) 0.363636...₁₁ (C) 0.2181818...₁₁ (D) 0.636363...₁₁ (E) 0.8333...₁₁

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. B | 21. E | 41. D |
| 2. E | 22. B | 42. C |
| 3. D | 23. B | 43. A |
| 4. C | 24. C | 44. A |
| 5. D | 25. C | 45. A |
| 6. E | 26. E | 46. E |
| 7. C | 27. A | 47. D |
| 8. B | 28. B | 48. B |
| 9. A | 29. C | 49. C |
| 10. C | 30. D | 50. E |
| 11. B | 31. B | 51. D |
| 12. E | 32. E | 52. A |
| 13. A | 33. B | 53. E |
| 14. C | 34. A | 54. D |
| 15. B | 35. D | 55. B |
| 16. D | 36. A | 56. B |
| 17. C | 37. E | 57. E |
| 18. D | 38. C | 58. A |
| 19. D | 39. E | 59. A |
| 20. B | 40. C | 60. A |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 2 • 2014



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[1 - 4 \times (6 + 8)] - 9 \div (10 - 12) + 14$

- (A) -36.5 (B) -28.4 (C) -23.5 (D) 0.4 (E) 26

2. Les Cash had \$225.00 in cash to spend on Spring break. He spent 30% of his cash on food, one-fourth on lodging, 15% on travel, and \$40.00 on entertainment. How much did he have left?

- (A) \$60.40 (B) \$27.50 (C) \$24.75 (D) \$22.50 (E) \$15.00

3. Find the number of positive integral divisors of 464.

- (A) 10 (B) 16 (C) 12 (D) 8 (E) 14

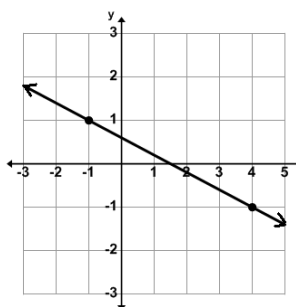
4. On a map legend, $\frac{3}{4}$ inch represents 75 miles. How far is it on the map, from Slippery Slope, Colorado to Busted Buns, Utah, if the distance in miles is 637.5?

- (A) $8\frac{1}{2}$ " (B) $8\frac{1}{4}$ " (C) $7\frac{3}{4}$ " (D) $7\frac{1}{8}$ " (E) $6\frac{3}{8}$ "

5. Let $\frac{3x-2}{2x-3} - \frac{4x+5}{5x+4} = \frac{Ax^2+Bx+C}{Dx^2+Ex+F}$. Find $\frac{A+B+C}{D+E+F}$.

- (A) -2 (B) $-1.333...$ (C) $-0.888...$ (D) 0.5 (E) 9

6. Line m is perpendicular to the line shown and contains the point $(-5, 2)$. Which of the following points is on line m ?



- (A) $(0, 4)$ (B) $(-7, 3)$ (C) $(7.5, -3)$ (D) $(5, -2)$ (E) $(-1, 12)$

7. A smaller pulley with a radius of 4" is connected with a fan belt to a larger pulley with a radius of 6". Find the speed of the larger pulley if the speeds of the pulleys are in inverse proportion to their diameters and the smaller pulley runs at 1,452 rpm.

- (A) 2,420 rpm (B) 2,178 rpm (C) 1,210 rpm (D) 968 rpm (E) 645 rpm

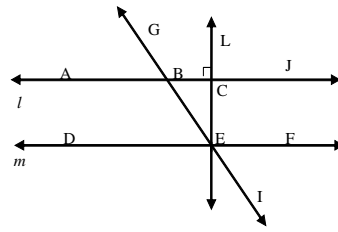
8. These three lines, $x + 2y = 3$, $3x - 2y = 1$, and $7x + 8y = k$, are concurrent. Find k .

- (A) -1 (B) 1 (C) 6 (D) 9 (E) 15

9. How many distinguishable arrangements can be made from the letters "STATEMEET"?

- (A) 120 (B) 3,024 (C) 10,080 (D) 90,720 (E) 181,440

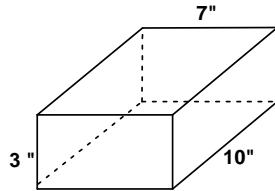
10. The four lines in the figure are coplanar with $m \parallel l$. Which of the following are true statements?



1. $\angle CEG$ & $\angle FEI$ are congruent
2. $\angle GBA$ & $\angle CEB$ are alternate exterior angles
3. $m\angle JBG = m\angle BEF$
4. $\angle FEG$ & $\angle BED$ are supplementary angles

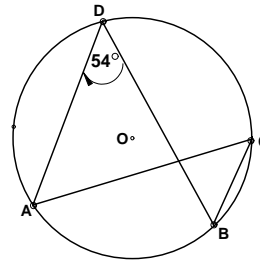
- (A) 1, 2, 3, & 4 (B) 2 & 3 (C) 1, 2, & 4 (D) 3 & 4 (E) 1 & 4

11. Find the total surface area of the rectangle shown. (nearest sq. in).



- (A) 200 sq. in (B) 210 sq. in (C) 221 sq. in (D) 234 sq. in (E) 242 sq. in

12. Given the circle with center O and $m\angle DAC = 54^\circ$, find $m\angle DBC$. (Drawing not to scale.)



- (A) 72° (B) 54° (C) 46° (D) 36° (E) 27°

13. The ratio of the distance from the center to a focus to the distance that focus is to the endpoint of the semiminor axis of an ellipse is the _____.

- (A) asymptote (B) directrix (C) eccentricity (D) focal distance (E) locus

14. Let $f(x) = 2x^2 - 3x - 4$ and $g(x) = 5x + 1$. Find $g\left(f\left(g^{-1}(6)\right)\right)$.

- (A) -24 (B) -9 (C) -6 (D) 3 (E) 6

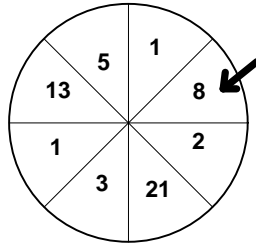
15. If $a_1 = -1$, $a_2 = 0$, $a_3 = 1$, and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_{11} equals:

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

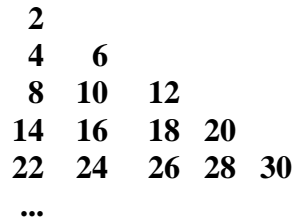
16. The graph of the polar equation $r = 3 - 3\cos(\theta)$ is a:

- (A) cardioid (B) lemniscate (C) dimpled limaçon (D) looped limaçon (E) convex limaçon


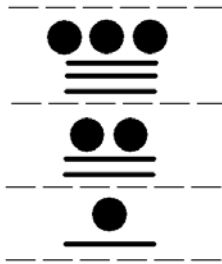
17. Willie When spins the spinner shown, where all sectors are the same size. If it lands on a prime number he gets \$5.00 and if it lands on a composite number he gets \$3.00. If it doesn't land on a prime or composite number, he loses \$2.00. Assuming it does not land on a line, what is the mathematical expectation on any one spin?



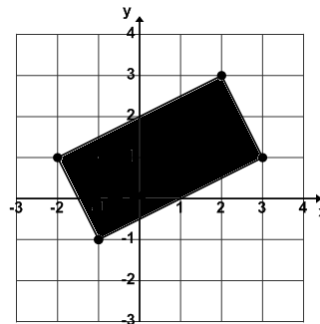
- (A) \$2.25 (B) \$2.50 (C) \$2.75 (D) \$3.75 (E) \$4.00
18. Frank Lynn's kite is flying 30 feet above the ground. He anchors the kite's string to the ground. Find the angle of elevation of the kite string if the string is 50 feet long. (nearest degree)
- (A) 31° (B) 37° (C) 40° (D) 53° (E) 59°
19. Simplify: $\frac{\sec x}{\sin x} - \frac{\cos x}{\sin x}$
- (A) 1 (B) $\csc x$ (C) $\tan x$ (D) $\sin x \cos x$ (E) $\cot x$
20. Simplify to the form $a + bi$: $(2 - \sqrt{-98})(2 + \sqrt{-72})$
- (A) $88 - 2\sqrt{2}i$ (B) $46 - 13\sqrt{2}i$ (C) $46 - 2\sqrt{2}i$ (D) $88 - 13\sqrt{2}i$ (E) $-80 - 2\sqrt{2}i$
21. If the set of even numbers continue in the triangular array shown below, the sum of the numbers in the 9th row would be?



- (A) 729 (B) 810 (C) 720 (D) 819 (E) 738
22. If $\begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} \times \begin{bmatrix} -\frac{2}{3} & -4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a \times d + b \times c = ?$
- (A) 1 (B) 2 (C) 3 (D) 10 (E) 18
23. Let $f(x) = x^2 - 3x - 5$ and $g(x) = 2x^2 + 3x - 5$. Find $f'(g'(x + 1))$
- (A) $4(x + 2)$ (B) $8x - 1$ (C) $4(2x + 1)$ (D) $6(x + 1)$ (E) $8x + 11$

24. Lotta Space is building a rectangular parking lot for her *Little Lots* store. The lot will be bordered on one side by a road. She has 1500 feet of fence to enclose the lot. What is the maximum area of her parking lot?
- (A) 140,625 ft.² (B) 281,250 ft.² (C) 421,875 ft.² (D) 500,000 ft.² (E) 1,125,000 ft.²
25. Integrate: $\int \sin(\frac{\theta}{2})\cos(\frac{\theta}{2}) d\theta$
- (A) $\cos(2\theta) + C$ (B) $2\sin(\theta) + C$ (C) $-\frac{\cos \theta}{2} + C$ (D) $-\frac{\sin \theta}{2} + C$ (E) $2\cos(\frac{\theta}{2}) + C$
26. The Millersview Swatters and the Fifi Flies are playing a five games series. The Swatters are three times as likely to beat the Flies. What is the probability that the Swatters will win all 5 games? (nearest percent)
- (A) 85% (B) 75% (C) 33% (D) 30% (E) 24%
27. The state math test result sheets show that 70 of the 125 students taking the test scored above 120. Based on these results, what were the odds that a student scored 120 or less?
- (A) $\frac{11}{14}$ (B) $\frac{3}{5}$ (C) $\frac{11}{24}$ (D) $\frac{7}{12}$ (E) $\frac{11}{25}$
28. The Mayan number system consists of three symbols, \bullet — . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number would the following Mayan number be?
- 
- (A) 2,656 (B) 1,296 (C) 7,446 (D) 13,126 (E) 966
29. Which of these mathematicians proved that the infinite set of real numbers are "more numerous" than the infinite set of natural numbers?
- (A) Georg Cantor (B) John Napier (C) George Boole (D) Theano (E) Benoit Mandelbrot
30. Find k when $4 - 5(3 + k) \div 2 = 6 - k$
- (A) $-14\frac{1}{4}$ (B) $-11\frac{1}{3}$ (C) $-6\frac{1}{3}$ (D) $-1\frac{3}{4}$ (E) $-1\frac{2}{3}$

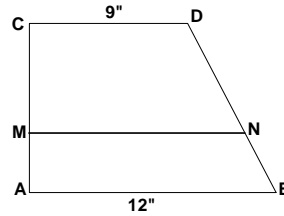
31. Let X be a two-digit number such that when the sum of the digits is subtracted from X , the difference is 63. Let Y be the two digit number when the digits of X are reversed. When the sum of the digits of Y is subtracted from Y , the difference is 9. Find $X + Y$.
- (A) 36 (B) 72 (C) 88 (D) 104 (E) 132
32. Find the sum of all of the two-digit numbers whose units digit is divisible by 3 or 9.
- (A) 1,008 (B) 1,369 (C) 1,458 (D) 1,962 (E) 3,069
33. Find the average of the arithmetic mean, the median, the mode, and the range of these quiz grades: 85, 90, 75, 100, 85, 90, 70, & 85.
- (A) 66.666... (B) 68.75 (C) 70.5 (D) 71.25 (E) 71.875
34. $10001_2 + 3003_4 + 707_8 = \underline{\hspace{2cm}}_{10}$
- (A) 13,711 (B) 99 (C) 667 (D) 226 (E) 3,592
35. The Real value solution set for $|1 + 2x| - 3 < 4$ is?
- (A) $\{x \mid \{x < -4\} \cup \{x > 3\}\}$ (B) $\{x \mid -3 < x < 4\}$ (C) $\{x \mid -4 < x < -3\}$
- (D) $\{x \mid -4 < x < 3\}$ (E) $\{x \mid \{x > 4\} \cup \{x < -3\}\}$
36. Kandy Kane wants to mix 5 pounds of peppermint bits with some mint chips for a special blend called K&K's. If peppermint bits sell for 85¢ a pound and mint chips sell for \$1.12 a pound, how many pounds of mint chips will she need to make K&K's that will sell for \$1.00 per pound?
- (A) 6.6 lbs (B) 6.25 lbs (C) 5.27 lbs (D) 4.5 lbs (E) 3.8 lbs
37. Desi Cortez drew the rectangle on his Cartesian Coordinate plane below. The vertices of the rectangle have integral coordinates. What is the area of his rectangle? (nearest sq. unit)



- (A) 10 units² (B) 10.25 units² (C) 10.5 units² (D) 11.2 units² (E) 12.2 units²
38. If $\frac{3x-1}{2x+3} + \frac{3x+1}{2x-3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $(A + B + C) - (P + Q + R)$ equals:
- (A) 10 (B) 13 (C) 17 (D) 23 (E) 27

39. Let O be the origin on a Cartesian Coordinate plane. Point O is reflected across the line $y = x + 4$ to point P . Point P is translated vertically 3 units up to point Q . Point Q is reflected back across the line $y = x + 4$ to point R . Point R is translated vertically 3 units down to point S . If the coordinates of S are (x, y) then $x + y$ equals:
- (A) -4 (B) -2 (C) 0 (D) 3 (E) 4
40. How many integral values of n exist such that $n > 3$ and $\frac{(n-1)!}{(n-3)!} \leq 12$
- (A) 7 (B) 5 (C) 2 (D) 1 (E) none
41. Let $f(x-1) = 1 - 2f(x)$. Find $f(-2)$ if $f(1) = -1$.
- (A) -5 (B) -3 (C) 2 (D) 4 (E) 11
42. Let vector $u = (1, -4)$ and vector $v = (5, -3)$. Find the measure of the angle with initial side u and terminal side v . (nearest second)
- (A) $14^\circ 2' 10''$ (B) $30^\circ 57' 50''$ (C) $45^\circ 0' 0''$ (D) $59^\circ 2' 10''$ (E) $75^\circ 57' 50''$
43. The altitude of $\triangle ABC$ forms two right triangles, $\triangle ABD$ and $\triangle CBD$. Find BC if $AB = 15''$, $m\angle BCD = 55^\circ$, and $m\angle BAD = 45^\circ$. (nearest tenth)
- (A) $18.5''$ (B) $15.0''$ (C) $10.6''$ (D) $12.9''$ (E) $7.4''$
44. The sum of the coefficients of the 3rd term in the expansion of $(x+1)^2$, the 3rd term of $(x+1)^3$, the 3rd term of $(x+1)^4$, and 3rd term of $(x+1)^5$ is:
- (A) 12 (B) 14 (C) 15 (D) 17 (E) 20
45. If $\log_2(15) = x$ and If $\log_{15}(32) = y$, then $xy = ?$
- (A) 0.0625 (B) 0.2 (C) 4 (D) 5 (E) 16
46. Cliff D. Kemist has 20 ounces of a 15% solution of argyle in water. How much water should he add to reduce his solution to a solution that is 90% water?
- (A) 5 oz (B) 10 oz (C) 15 oz (D) 20 oz (E) 25 oz
47. Nick Ohl tossed a nickel four times and recorded the results. What is the probability of at least two consecutive heads? (nearest per cent)
- (A) 50% (B) 33% (C) 28% (D) 25% (E) 13%
48. $\triangle ABC$ and $\triangle PQR$ exist such that $\angle BCA \cong \angle PQR$, $\angle CBA \cong \angle QPR$, $AB = 20 \text{ cm}$, $BC = 25 \text{ cm}$, $PQ = 75 \text{ cm}$, and $RQ = 48 \text{ cm}$. Find the ratio of AC to PR .
- (A) $\frac{25}{48}$ (B) $\frac{4}{15}$ (C) $\frac{4}{5}$ (D) $\frac{5}{12}$ (E) $\frac{1}{3}$

49. Given the trapezoid shown where the area of trapezoid ABNM is equal to the area of trapezoid CDN M, find MN. (nearest hundredth)



- (A) 10.71 (B) 10.61 (C) 10.57 (D) 10.50 (E) 10.39
50. The coordinates of the vertices of $\triangle RST$ are $(-2, 6)$, $(5, -3)$ and (x, y) . The coordinates of the centroid of $\triangle PQR$ are $(2, 2)$. Find xy .
- (A) $2\frac{7}{9}$ (B) 9 (C) $2\frac{1}{4}$ (D) 6 (E) 25
51. The harmonic mean of the real roots of $3x^3 - 10x^2 + kx - 2 = 0$ is $\frac{2}{3}$. Find k .
- (A) 12 (B) 9 (C) 6 (D) 3 (E) 2
52. Let $f(x) = \frac{x-1}{x+1}$. Which of the following is true about $f(x)$?
- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
(D) It is neither an even nor an odd function (E) It is a logarithmic function.
53. If $3x^2 + 6y = 10$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$
- (A) 0.5 (B) 1 (C) 1.5 (D) 1.666... (E) 3.333...
54. Find the area of the region bounded by $x = 10 - y^2$ and $x = y^2 - 4y + 4$.
- (A) 10.666... (B) 14.666... (C) 18.333... (D) 21.333... (E) 24.0
55. Les Speed's car is traveling at a rate of 30 ft/sec approaching an intersection. When Les is 120 feet from the intersection, Joy Ryder's truck traveling at a rate of 40 ft/sec crosses the intersection. Both vehicles are on roads that are at right angles to each other. How fast are the vehicles separating 2 seconds after Joy leaves the intersection? (nearest tenth)
- (A) 27.7 ft/sec (B) 7.1 ft/sec (C) 13.8 ft/sec (D) 12.0 ft/sec (E) 14.0 ft/sec
56. The probability that Larry will ask Shemp to go to the movies is 25%, that Moe will ask him is 50%, and that Curley will ask him is 87.5%. What is the probability that Moe and Curley will ask Shemp, but Larry will not? (nearest percent)
- (A) 33% (B) 11% (C) 78% (D) 44% (E) 16%

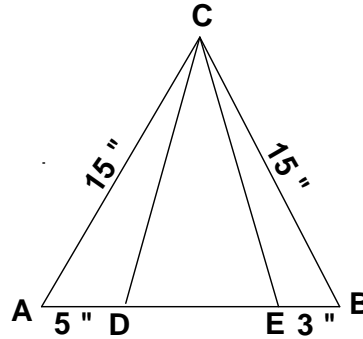
57. The repeating decimal $0.\overline{676767}$ in base 8 can be written as which of the following fractions in base 8?

- (A) $\frac{15}{17}_8$ (B) $\frac{45}{51}_8$ (C) $\frac{61}{64}_8$ (D) $\frac{67}{77}_8$ (E) $\frac{55}{63}_8$

58. Find k , given: $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, k, \frac{49}{20}, \dots$

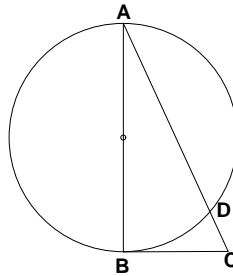
- (A) $\frac{35}{18}$ (B) $\frac{47}{24}$ (C) $\frac{75}{48}$ (D) $\frac{137}{60}$ (E) $\frac{145}{72}$

59. $\triangle ABC$ is an equilateral triangle. Find the perimeter of $\triangle CDE$. (nearest tenth)



- (A) 34.6" (B) 34.0" (C) 33.0" (D) 32.4" (E) 29.8"

60. Given: \overline{AB} is a diameter of the circle shown, \overline{BC} is tangent to the circle, $m\angle BAC = 30^\circ$, and $CD = \sqrt{3}$ cm. Find the perimeter of $\triangle ABC$. (nearest tenth)



- (A) 21.6 cm (B) 15.6 cm (C) 10.4 cm (D) 20.8 cm (E) 16.4 cm

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 2 • 2014
Answer Key**

1. A	21. E	41. E
2. B	22. C	42. C
3. A	23. E	43. D
4. E	24. B	44. E
5. A	25. C	45. D
6. E	26. E	46. B
7. D	27. A	47. A
8. E	28. C	48. B
9. C	29. A	49. B
10. D	30. C	50. B
11. E	31. C	51. B
12. B	32. D	52. D
13. C	33. D	53. B
14. A	34. C	54. D
15. D	35. D	55. E
16. A	36. B	56. A
17. C	37. A	57. D
18. B	38. D	58. D
19. C	39. C	59. B
20. A	40. C	60. E



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Region • 2014



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $128 \div (64)^{\frac{1}{3}} + 16 \times (8)^{-1} - 4! \times 2^0$

- (A) 34 (B) 18 (C) 10 (D) -6 (E) -18

2. If P is 125% more than Q and R is $\frac{3}{4}$ of Q, then R is what percent of P?

- (A) 25% (B) $33\frac{1}{3}\%$ (C) $44\frac{1}{4}\%$ (D) 50% (E) 60%

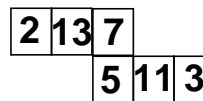
3. Let $F = \{f, r, a, c, t, i, o, n\}$, $R = \{r, a, t, i, o\}$, $P = \{p, a, r, t, o, f\}$, and $W = \{w, h, o, l, e\}$. The number of distinct elements in $(F \cup W) \cap (R \cup P)$ is _____.

- (A) 6 (B) 1 (C) 4 (D) 7 (E) 13

4. Bill Meelator went shopping for a new smart phone. The regular price of the phone he wanted was \$172.80. A sale was going on, so he got one-fourth off of the regular price. When he checked out he was given 10% off of the sale price for being a good customer. How much money did Bill save because of the two discounts? (tax not included)

- (A) \$35.00 (B) \$47.52 (C) \$54.86 (D) \$56.16 (E) \$60.38

5. The net shown is used to fold into a cube. Find the sum of each face and its opposite face. What is the difference between the largest sum and the smallest sum?

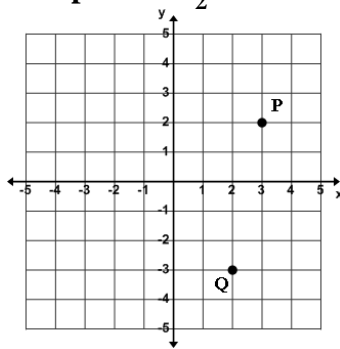


- (A) 17 (B) 16 (C) 15 (D) 9 (E) 7

6. Simplify: $\frac{4x^2 - 4x - 3}{4x^2 + 12x + 9} \div \frac{2x - 3}{8x + 12} \times \frac{4x^2 - 9}{8x^2 - 12x - 8}$

- (A) $2x - 3$ (B) $\frac{x-2}{2x+3}$ (C) $2x^2 - 7x + 6$ (D) $\frac{2x-3}{x-2}$ (E) $2x^2 - x - 6$

7. The coordinates of points P and Q have integral values. A line through point P has a slope of $\frac{2}{3}$. A line through point Q has a slope of $-\frac{3}{2}$. The lines intersect at the point (x, y). Find x + y.



- (A) -5 (B) $-2\frac{1}{6}$ (C) 0 (D) $\frac{5}{6}$ (E) 2

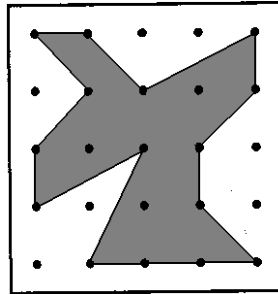
8. Find $f(-6) + f(-4) + f(-2)$ if $f(x) = \begin{cases} 3x + 2 & \text{if } x \leq -5 \\ 4x - 1 & \text{if } -5 < x < -1 \\ 5x & \text{if } x \geq -1 \end{cases}$

- (A) -12 (B) -27 (C) -34 (D) -42 (E) -43

9. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle Q = 114^\circ$ then $m\angle R = ?$

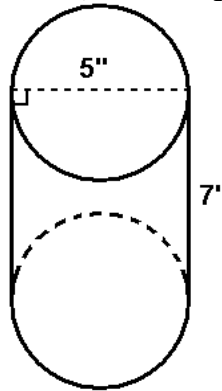
- (A) 44° (B) 66° (C) 34° (D) 56° (E) 24°

10. Dottie Pick determined that the adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. She computed the area of the shaded figure shown to be ____.



- (A) 11.5 cm^2 (B) 10.5 cm^2 (C) 10 cm^2 (D) 9.5 cm^2 (E) 9 cm^2

11. Find the total surface area of the figure shown. (nearest sq. in).



- (A) 149 sq. in (B) 175 sq. in (C) 110 sq. in (D) 94 sq. in (E) 165 sq. in

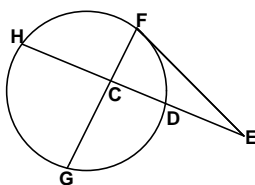
12. A two-digit number exists such that the square of the number is equal to four times the sum of the number and twenty-four. The sum of digits of the two-digit number is:

- (A) 15 (B) 12 (C) 8 (D) 6 (E) 3

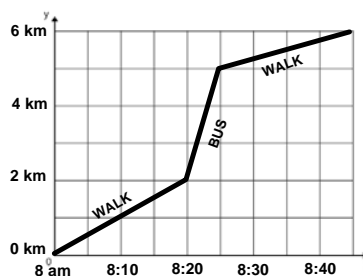
13. Willie Lawkette uses a 4 digit combination to chain his bicycle to the bicycle rack. The first digit has to be a positive even number, the second a prime number, the third a Fibonacci number, and the fourth a triangular number. How many unique sets of 4 combinations can Willie select from if digits can be repeated?

- (A) 625 (B) 375 (C) 300 (D) 256 (E) 240

14. Given the circle with center O, $m\widehat{DF} = 50^\circ$, $m\widehat{HF} = 90^\circ$, and $m\angle HCG = 85^\circ$. Find $m\angle EFG$.
(Drawing not to scale.)

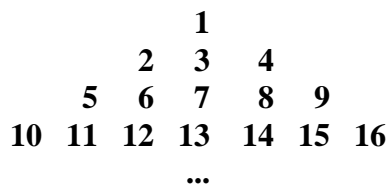


- (A) 70° (B) 75° (C) 80° (D) 85° (E) 90°
15. If $\frac{4x+3}{2x-1} - \frac{x+2}{3x-4} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{ABC}{PQR}$ equals:
- (A) $-1.515151\dots$ (B) $-2.121212\dots$ (C) $-3.787878\dots$ (D) $-4.444\dots$ (E) $-5.232323\dots$
16. Ima Nutt sells peanuts for \$1.50 per pound and pecans for \$4.00 per pound. How many pounds of pecans will she need to mix with the 60 pounds of peanuts that will sell for \$2.50 per pound of mixed nuts?
- (A) 140 lbs (B) 100 lbs (C) 60 lbs (D) 40 lbs (E) 20 lbs
17. Willis Fast walks from his house to the bus stop, gets on the school bus, then gets off and walks the rest of the way to school. Using the graph below determine his average walking speeds.



- (A) 4 kmph (B) 4.5 kmph (C) 5 kmph (D) 6.7 kmph (E) 8 kmph
18. Simplify: $\frac{\cos x - \csc x}{\sin x - \sec x}$
- (A) $\sec x$ (B) $\csc x$ (C) $\tan x$ (D) $\cot x$ (E) 1
19. If $\sin B = -\frac{\sqrt{3}}{2}$, where $\frac{3\pi}{2} < B < 2\pi$, and $\cos A = -\frac{1}{2}$, where $\pi < A < \frac{3\pi}{2}$, then $\sin(A+B) - \sin(A-B) = ?$
- (A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$ (E) $\frac{1}{2}$
20. Use the Fibonacci characteristic sequence ... L, -3, U, 5, C, A, S, ... to find A + S.
- (A) 21 (B) 31 (C) 43 (D) 49 (E) 83

21. If the set of numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, the arithmetic mean of the numbers in the 13th row would be?



- (A) 133 (B) 145 (C) 157 (D) 168 (E) 170
22. The sum of the coefficients of the 2nd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 5th term of $(x + 1)^5$, the 2nd term of $(x + 1)^7$ and 11th term of $(x + 1)^{11}$ is:
- (A) 31 (B) 28 (C) 25 (D) 23 (E) 16
23. Which of these mathematicians and his work was important to the development of digital computers?

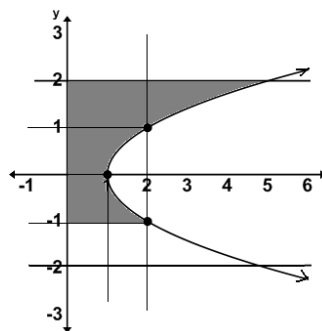
(A) Erastosthenes (B) John Venn (C) Rene Descartes (D) Agnesi (E) Gottfried Leibniz

24. Which of the following series is/are convergent?

$$1. \sum_{n=1}^{+\infty} e^{-n} \quad 2. \sum_{n=1}^{+\infty} \left(\frac{2}{3}\right)^n \quad 3. \sum_{n=1}^{+\infty} \left(\frac{1}{2n} - \frac{1}{3n}\right)$$

- (A) 1 only (B) 2 only (C) 3 only (D) 1 & 2 (E) 2 & 3

25. Find the area of the shaded region in square units.




- (A) $6\frac{2}{3}$ (B) 6 (C) $5\frac{2}{3}$ (D) $5\frac{1}{3}$ (E) 5
26. Let $f(x) = \frac{5x+9}{5-9x}$. Find $f'(1)$.
- (A) $-11\frac{7}{9}$ (B) $-4\frac{5}{9}$ (C) $3\frac{1}{2}$ (D) $6\frac{5}{8}$ (E) $12\frac{1}{4}$
27. A bucket of balls at the Quicksand golf course driving range contains thirty balls. One-third of them are yellow, two-fifths of them are white, and the rest are pink. What is the probability that a ball randomly selected is pink?

- (A) 40% (B) 25% (C) $26\frac{2}{3}\%$ (D) $37\frac{1}{2}\%$ (E) $33\frac{1}{3}\%$

28. The current United States Supreme Court consists of three women and six men. How many ways can a five member committee be formed consisting of three women and two men or two women and three men?

(A) 90 (B) 15 (C) 60 (D) 45 (E) 75

29. The Mayan number system consists of three symbols, \bullet — . The total number of these symbols needed to write 5314 base 10 as a Mayan number is:

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

30. The three-digit number, 707, is considered to be which of the following type numbers?

(A) evil (B) unhappy (C) lucky (D) economical (E) harmonic

31. Penni Les had a jar of new pennies. She gave 30% of them to her little sister. Then she gave $\frac{1}{4}$ of what was left to her favorite niece. Then she gave her little brother 21 pennies leaving 42 pennies in her jar. How many were in her jar originally?

(A) 260 (B) 70 (C) 110 (D) 200 (E) 120

32. Find k when $2 + 3(k - 5) \div 7 = 11 - 3k$

(A) $2\frac{2}{3}$ (B) 3.25 (C) 2 (D) 3.5 (E) $4\frac{5}{12}$

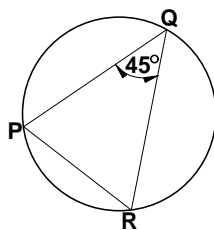
33. Soh Yung is twice as old as her sister, Tu Yung. Soh's age now is 10 years older than Tu was last year. What will the sum of their ages be in 6 years?

(A) 39 (B) 36 (C) 34 (D) 33 (E) 30

34. If a trip takes 7 hours at 60 miles per hour, then how long would the same trip take at 75 miles per hour? (nearest minute)

(A) 6 hrs (B) 5 hrs 24 min (C) 5 hrs 15 min (D) 5 hrs 12 min (E) 5 hrs 36 min

35. Points P, Q, and R lie on the circle shown. Find the length of the chord PR if the radius of the circle is 4". (nearest tenth)



(A) 2.8" (B) 4.8" (C) 5.7" (D) 8.5" (E) 11.3"

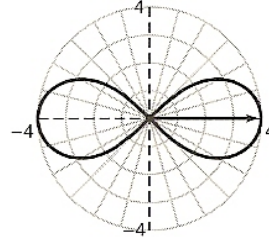
36. The measure of an external angle of an n -sided regular polygon is 30° . What is the sum of the interior angles of this polygon?
- (A) 720° (B) 900° (C) 1080° (D) 1440° (E) 1800°
37. Let $A = \begin{bmatrix} 1 & 4 & 9 \\ -1 & 0 & 1 \\ 2 & 3 & 5 \end{bmatrix}$. Find $|A|$.
- (A) -58 (B) -52 (C) -2 (D) 4 (E) 17
38. Points $(1, 3)$ and $(-1, 7)$ are members of the function $\{(x, y) \mid y = 3ax + b\}$. Find $a + b$.
- (A) $5\frac{2}{3}$ (B) $4\frac{1}{3}$ (C) $3\frac{1}{2}$ (D) $2\frac{1}{3}$ (E) $1\frac{1}{2}$
39. The ranger at station 1 saw smoke in the distance. He used his transit to determine the smoke was on a bearing of 70° . Ranger station 2 is 10 miles from station 1 on a bearing of 120° . The ranger at station 2 used his transit to determine the smoke was on a bearing of 20° . How far from station 1 was the smoke? (nearest tenth)
- (A) 12.9 mi (B) 11.3 mi (C) 10.5 mi (D) 12.2 mi (E) 13.1 mi
40. Let $A = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 6 & 10 \end{bmatrix}$. Find $|A^T - B|$.
- (A) -18 (B) -13 (C) -13 (D) 26 (E) 44
41. Tye Guhr drops a golf ball from his balcony which is 12 feet from the ground. If the golf ball rebounds up 75% of the distance it falls each time, how far will the golf ball have traveled when it hits the ground the 4th time?
- (A) $57' 5\frac{1}{16}"$ (B) $53' 7\frac{1}{2}"$ (C) $48' 6\frac{1}{4}"$ (D) $41' 7\frac{1}{2}"$ (E) $32' 9\frac{3}{4}"$
42. Leo Lyan and Elsa Lyaness start at the same time and place running to their designated spots. Leo runs due West at 12 mph and Elsa runs due South at 10 mph. How fast is the distance between them changing after 30 minutes of running? (nearest tenth)
- (A) 15.6 mph (B) 30.0 mph (C) 15.4 mph (D) 10.7 mph (E) 31.2 mph
43. If $2x^2 - 3y = 5$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$
- (A) 0.6 (B) 0.666... (C) 0.75 (D) 1.333... (E) 3.333...
44. Find the area of an obtuse scalene triangle whose side lengths are 5", 8" and 12". (nearest sq. inch)
- (A) 13 in^2 (B) 20 in^2 (C) 30 in^2 (D) 23 in^2 (E) 15 in^2

45. The *Phil Upp* gas station mixes 50 gallons of a special grade gasoline worth \$3.05 per gallon with some regular grade gasoline worth \$2.80 per gallon. The mixture sells for \$2.95 per gallon. How many gallons of the regular grade did the station mix with the special grade? (nearest gallon)
- (A) 25 gal (B) 30 gal (C) 33 gal (D) 38 gal (E) 40 gal
46. A box contains five rods whose lengths are 1", 3", 6", 10" and 15". How many different obtuse triangles can be made using only three rods at a time.
- (A) 1 (B) 3 (C) 6 (D) 10 (E) 15
47. Let $f(x + 1) = 2 - f(x)$. Find $f(2)$ if $f(-1) = 3$.
- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3
48. How many integral values of n exist such that $n > 4$ and $\frac{(n-2)!}{(n-4)!} \leq 20$
- (A) none (B) 2 (C) 3 (D) 4 (E) 7
49. Let $(2\text{cis}(\frac{\pi}{6}))^5 = a + bi$. Find $a + b$. (nearest tenth):
- (A) -11.7 (B) -7.3 (C) -5.9 (D) 3.7 (E) 43.7
50. Convert the polar equation $r = 2\sin \theta$ to a rectangular equation.
- (A) $x^2 + y^2 = 4$ (B) $x^2 + (y - 2)^2 = 4$ (C) $x^2 + y^2 = 2$
 (D) $(x - 1)^2 + 2y = 1$ (E) $x^2 + (y - 1)^2 = 1$
51. How many solutions are there for the equation $5x + 3y = 2014$ such that both x and y are non-negative integers?
- (A) 72 (B) 81 (C) 101 (D) 115 (E) 134
52. If the three numbers 425, 356, and 264 are each divided by the number D , each of their quotients will have the same remainder R . Find R where $R > 1$.
- (A) 11 (B) 7 (C) 5 (D) 3 (E) 2
53. Find a real value of x such that $\log_2(\log_2 x) = \log_4(\log_4 x)$?
- (A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$ (E) 4
54. The polar graph of $r^2 = 9\sin(2\theta)$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$
- (A) 2 only (B) 3 only (C) 1 & 2 (D) 2 & 3 (E) 1, 2, & 3

55. The probability that Larry will ask Shemp to go to the movies is 30%, that Moe will ask him is 25%, and that Curley will ask him is 12.5%. What is the probability that at least two of them will ask Shemp to go to the movies? (nearest tenth)

- (A) 23.1% (B) 15.3% (C) 14.4% (D) 12.5% (E) 11.6%

56. Which of the following polar equations will produce the graph shown?



- (A) $r^2 = 16\cos 2\theta$ (B) $r = 4\sin 2\theta$ (C) $r^2 = 2\sin 2\theta$ (D) $r = 4\cos 2\theta$ (E) $r^2 = 16\sin 2\theta$

57. The odds of scoring less than 150 on this test is $\frac{2}{5}$. Based on these odds, if 126 take this test how many would be expected to score less than 150?

- (A) 64 (B) 60 (C) 50 (D) 40 (E) 36

58. The repeating decimal 0.3666... in base 8 can be written as which of the following fractions in base 8?

- (A) $\frac{27}{56}_8$ (B) $\frac{9}{25}_8$ (C) $\frac{33}{70}_8$ (D) $\frac{39}{100}_8$ (E) $\frac{13}{31}_8$

59. The centroidal mean of 20 and 30 is: (nearest tenth)

- (A) 24.8 (B) 25.0 (C) 25.3 (D) 25.5 (E) 26.0

60. How many positive integers less than or equal to 2014 are divisible by neither 3 nor 5?

- (A) 939 (B) 941 (C) 1,007 (D) 1,073 (E) 1,075

**University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. C | 41. B |
| 2. B | 22. B | 42. A |
| 3. A | 23. E | 43. C |
| 4. D | 24. D | 44. E |
| 5. B | 25. B | 45. C |
| 6. D | 26. D | 46. A |
| 7. C | 27. C | 47. A |
| 8. D | 28. E | 48. C |
| 9. E | 29. D | 49. A |
| 10. D | 30. B | 50. E |
| 11. A | 31. E | 51. E |
| 12. E | 32. B | 52. A |
| 13. E | 33. A | 53. B |
| 14. B | 34. E | 54. A |
| 15. C | 35. C | 55. D |
| 16. D | 36. E | 56. A |
| 17. B | 37. C | 57. E |
| 18. D | 38. B | 58. C |
| 19. C | 39. A | 59. C |
| 20. D | 40. D | 60. E |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

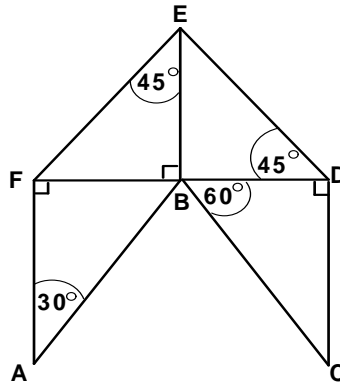
State • 2014



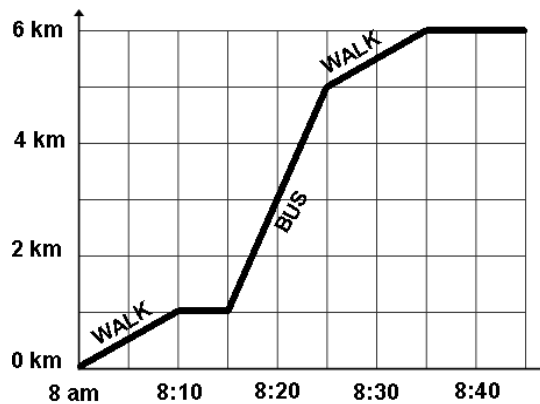
DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

- Evaluate: $(64)^{\frac{1}{3}} + 32 \div (4! - 2^4) \times 8^{(-1)}$
 (A) 0.5625 (B) 1 (C) 4.5 (D) 6 (E) 21.8333...
- Rose Thorn went to the *Fruits and Nuts* Nursery to buy some trees. She paid \$14.75 for each fruit tree and \$12.50 for each nut tree. She bought three fruit trees and two nut trees. Since Rose belongs to the *Sav-A-Tree* club she received a 20% discount on fruit trees and a 15% discount on nut trees. How much did the five trees cost her before taxes? (nearest cent)
 (A) \$42.13 (B) \$45.01 (C) \$55.08 (D) \$55.48 (E) \$56.65
- Find the number of positive integral divisors of 2,400.
 (A) 3 (B) 12 (C) 24 (D) 36 (E) 48
- If P is 45% of Q and Q is $\frac{4}{5}$ of R, then P is what percent less than R?
 (A) 80% (B) 64% (C) 55% (D) 36% (E) 35%
- Simplify: $\frac{x^2 - 4x - 5}{x^2 - 2x - 15} \times \frac{x^2 + 2x - 3}{x^3 + x^2 - x - 1}$
 (A) $\frac{5}{3x^4 + 3x^3}$ (B) $x - 1$ (C) $\frac{1}{x + 1}$ (D) $x + 1$ (E) $\frac{3}{x - 1}$
- Line m passing through (2, 3) has a slope of $\frac{1}{2}$. Line n is perpendicular to line m and passes through (3, -4). Lines m and n intersect at (x, y). Find y.
 (A) -1 (B) $-\frac{1}{2}$ (C) 2 (D) 4 (E) 5
- Three years ago, Ima Oldie was one year older than twice Ura Goodie's age. Six years from now Ima's age will be ten years more than half of Ura's age. What is the sum of their ages now?
 (A) 28 (B) 21 (C) 16 (D) 13 (E) 10
- Let p and q be the roots of $x^2 + 2x - 15 = 0$. Find $|p^3 - 3p^2q + 3pq^2 - q^3|$.
 (A) 8 (B) 64 (C) 90 (D) 169 (E) 512
- Leon D. Oiler folded a geometry net into a cuboctahedron. It has 24 edges, 12 vertices, 8 triangular faces and some square faces. How many square faces does it have?
 (A) 2 (B) 4 (C) 6 (D) 8 (E) 10
- If you slice a right circular cone with a plane having the same slope as the slant of the cone, the cross section is a(n) _____ .
 (A) line (B) circle (C) elongated ellipse (D) parabola (E) hyperbola

11. Find the perimeter of the hexagon ABCDEF if $AB = 10$ cm. (nearest tenth).



- (A) 54.5 cm (B) 51.5 cm (C) 49.1 cm (D) 44.4 cm (E) 39.9 cm
12. Point A $(-2, -2)$ lying on the x-y plane is rotated $\frac{3\pi}{2}$ radians counter-clockwise about the origin to point B. Point B is translated horizontally 3 units to the right to point C. Point C is reflected across the line $y = x$ to point D. Point D is translated vertically down 3 units to point E. Point E is rotated $\frac{\pi}{2}$ radians clockwise about the origin to point F (x, y) . Find $x + y$.
- (A) -4 (B) -2 (C) 0 (D) 1 (E) 3
13. These four examples are examples of one of the best-known conjectures of number theory.
 1) $8 = 3 + 5$ 2) $20 = 7 + 13$ 3) $66 = 29 + 37$ 4) $100 = 11 + 89$
 Which of these mathematicians is the conjecture attributed to?
- (A) Zeno of Elea (B) John Napier (C) Theano (D) Lady Lovelace (E) Christian Goldbach
14. The sixth *harmonic number* is:
- (A) 0.0476190 (B) $2.5928...$ (C) 1.5 (D) 2.45 (E) $0.408333...$
15. Willie Makette walks to the bus stop, gets on the bus and rides to the next bus stop, then walks the rest of the way to school. Using the graph of Willie's trip to school, determine the difference in time Willie spent walking and the time he spent riding the bus.



- (A) 25 min (B) 20 min (C) 15 min (D) 10 min (E) 5 min

16. How many positive integral values of n exist such that $\frac{(n+3)!}{(n+1)!} < 30$.

- (A) none (B) 2 (C) 3 (D) 5 (E) 6

17. Determine the range of $f(\theta) = 5\sin(3\theta + 1.5\pi) - 2$.

- (A) $[-4.5, 1.5]$ (B) $[-3.5, 3.5]$ (C) $[3, 7]$ (D) $[-3, 2]$ (E) $[-7, 3]$

18. Simplify: $(1 - \sin^2 x)(1 + \tan^2 x)$

- (A) 2 (B) 1 (C) $\cos x$ (D) $\cos^2 x$ (E) $1 - \cos^2 x$

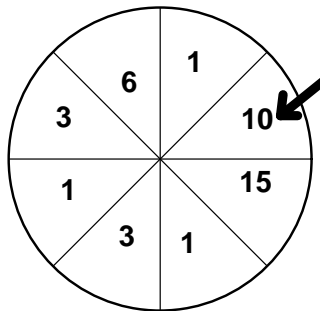
19. Find C if the remainder of $8x^4 + 4x^3 + 7x^2 + 5x + C$ divided by $x + 1$ is 9.

- (A) -3 (B) -1 (C) 3 (D) 2 (E) 1

20. In the binomial expansion of $(2x + 1)^6$, the sum of the coefficients of the 2nd, 4th, and 6th terms is:

- (A) 729 (B) 96 (C) 556 (D) 720 (E) 364

21. Betty Luzes spins the spinner shown, where all sectors are the same size. If it lands on a prime number she gets that number of points tripled. If it lands on a unit she gets that number of points doubled. If it lands on any other number, she loses 5 points. Assuming it does not land on a line, what is the mathematical expectation on any one spin?



- (A) $+2.75$ pts (B) $+1.125$ pts (C) -0.375 pts (D) -0.75 pts (E) -4.0 pts

22. How many distinguishable ways can the letters in the word ARRANGEMENT be arranged?

- (A) 2,494,800 (B) 1,260 (C) 226,800 (D) 1,247,800 (E) 5,040

23. A group of 75 northern Aleuts took a survey about ice cream. The results showed the following: 39 liked chocolate; 47 liked vanilla; 38 liked strawberry; 1 did not like ice cream; 11 liked chocolate and strawberry, but not vanilla; 14 liked vanilla and strawberry, but not chocolate; 7 liked chocolate and vanilla, but not strawberry; and 9 liked all three flavors. How many of the Aleuts liked only strawberry?

- (A) 1 (B) 4 (C) 8 (D) 12 (E) 17

24. Let $\frac{2x+1}{3x-2} - \frac{4x+3}{5x-3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$. Find $\frac{A+B+C}{P+Q+R}$.

- (A) $-5\frac{1}{2}$ (B) $-\frac{1}{9}$ (C) $-\frac{1}{20}$ (D) $-\frac{1}{10}$ (E) $-\frac{1}{2}$

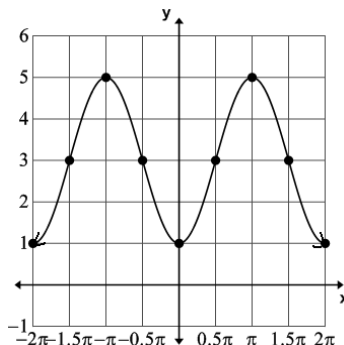
25. A water tank in the shape of a regular hexagonal based prism with side lengths of 4 feet and a height of 5 feet is 80% full of water. How many gallons of water would it take to fill the tank? (nearest gallon)

- (A) 389 gal (B) 363 gal (C) 346 gal (D) 324 gal (E) 311 gal

26. If $a_1 = -1$, $a_2 = 3$, $a_3 = -2$ and $a_n = (a_{n-1} - a_{n-3})(a_{n-2})$, where $n \geq 4$, then a_7 equals:

- (A) -468 (B) -42 (C) 0 (D) 265 (E) $12,800$

27. The equation $y = D + A\sin(Bx - C\pi)$, where $-2\pi \leq x \leq 2\pi$, will produce this graph. If A, B, C, and D > 0, which of the following is the sum of A, B, C, and D?



- (A) 8.5 (B) 7 (C) 6.5 (D) 5 (E) 4.5

28. If $(5 + 3i)(4 + 2i)(7 - i) = a + bi$ then $a + b$ is:

- (A) 260 (B) 140 (C) 76 (D) -20 (E) -12

29. How many kinds of symmetry does the graph of the polar equation, $r = 5\cos(4\theta)$, have?

- (A) none (B) 1 (C) 2 (D) 3 (E) 4

30. Find the slope of the line tangent to the curve $y = 3x^2 + 4x - 5$ at the point $(-2, -1)$.

- (A) -21 (B) -8 (C) -6 (D) -2 (E) -1

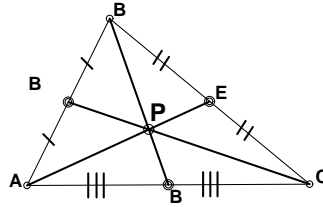
31. A freshman, a sophomore, and a junior are working on this problem. The probability that the freshman will solve this problem is $\frac{1}{4}$, that the sophomore will solve it is $\frac{1}{3}$, and that the junior will solve it is $\frac{3}{5}$. What is the probability that the freshman and the sophomore will solve it, but the junior will not? (nearest tenth)

- (A) 3.3% (B) 16.7% (C) 18.3% (D) 31.7% (E) 33.3%

32. P varies jointly as Q and R and inversely as the square of Z . If $P = 150$, $Q = 15$, $R = 18$, and $Z = 9$, find P when $Q = 21$, $R = 12$, and $Z = 6$.

(A) 337.5 (B) 315 (C) $71\frac{3}{7}$ (D) 45 (E) 28.4

33. Find AP if $EP = 16$ cm.



(A) 32 cm (B) 48 cm (C) 40 (D) 24 cm (E) 8 cm

34. If $(-1, 1)$, $(2, -3)$, and $(5, k)$ are members of the function $\{(x, y) \mid y = 4a + bx\}$, then $k = ?$

(A) -8 (B) -7 (C) -6 (D) -5 (E) -4

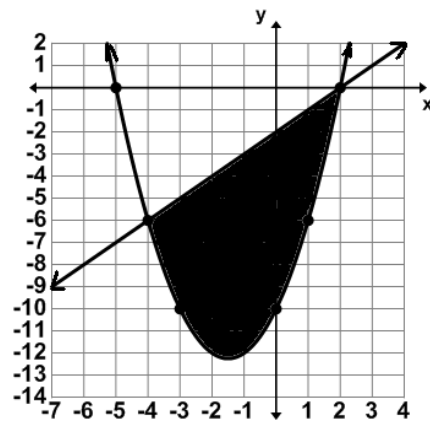
35. Let vector $u = (-2, 4)$ and vector $v = (-1, -3)$. Find the measure of the angle with initial side u and terminal side v . (nearest minute)

(A) $135^\circ 0'$ (B) $132^\circ 16'$ (C) $124^\circ 0'$ (D) $116^\circ 34'$ (E) $108^\circ 26'$

36. The directrix of the parabola $12x + y^2 + 6y = 3$ is:

(A) $x = 9$ (B) $x = 6$ (C) $x = 4$ (D) $x = -2$ (E) $x = -3$

37. Find the area of the shaded region in square units.



(A) $34\frac{2}{3}$ (B) 35 (C) $35\frac{1}{3}$ (D) 36 (E) $36\frac{2}{3}$

38. The probability that statement P is true is 35%, and the probability that statement Q is true is 70%. Determine the probability that $P \rightarrow Q$ is false.

(A) 10.5% (B) 24.5% (C) 27% (D) 50% (E) 52.5%

39. Eratosthenes numbered 20 ping pong balls from 1 to 20 and mixed them up in a bag. He drew out three balls, without replacement, and recorded their numbers. What is the probability all three of them were prime numbers? (nearest tenth)

(A) 3.1 % (B) 4.9 % (C) 6.7 % (D) 25.5% (E) 40.0 %

40. Chef Sal Add wants to make a salad dressing using two types of vegetable oil. One type of oil contains 8% saturated fats and a second oil contains 25% saturated fats. How many ounces of the second oil will Chef Sal have to add to 10 ounces of the first oil if the per cent of saturated fats is to be 15%?

(A) 17 oz (B) 15 oz (C) 10 oz (D) 8 oz (E) 7 oz

41. Ray Diaz waters his crops using a long pipe with multiple sprinkler heads on it. One end of the pipe is connected to a pivot on his well. The well is 400 feet from the corner of his field on a bearing of 55° . The other end of the irrigation pipe is 300 feet from the same corner of his field on a bearing of 100° . The pipe rotates in a circular motion about the well when it is turned on. How many acres of land will his sprinkler system water when it completes one revolution if 1 acre equals 43,560 square feet. (nearest tenth)

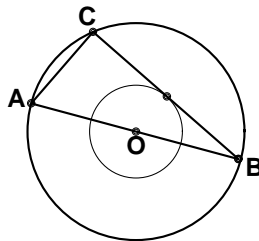
(A) 3.9 acres (B) 4.5 acres (C) 5.8 acres (D) 6.7 acres (E) 8.1 acres

42. If the set of numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, in what row would you find the number 14,878?

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 148 (B) 122 (C) 130 (D) 154 (E) 112

43. Given the two concentric circles with center O: the ratio of the radii is 1 to 3; segment AB is the diameter of the larger circle; chord BC is tangent to the smaller circle; and $AC = 12$ ". Find the radius of the larger circle.



(A) 8 " (B) 15 " (C) 16 " (D) 18 " (E) 24 "

44. Let $f(x - 1) = 1 + 2f(x)$. Find $f(-2)$ if $f(1) = -1$.

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2

45. Find $\lim_{x \rightarrow -1} f(x)$, if it exists. Let $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1, \\ 3 & \text{if } -1 < x < 1, \\ 2x + 1 & \text{if } x > 1. \end{cases}$

- (A) -1 (B) 1 (C) 2 (D) 3 (E) does not exist

46. Phil Whitwater is putting water in his small cylindrical shaped pool. The diameter of the pool is 12 feet and the height of the pool is 4 feet. The water is flowing into the tank at a rate of 3 cubic feet per minute. How fast is the water rising when the water is 2 feet deep? (nearest tenth)

- (A) 0.7 ft/hr (B) 1.1 ft/hr (C) 1.6 ft/hr (D) 2.1 ft/hr (E) 2.4 ft/hr

47. Which of the following is NOT a member of the solution set for $\frac{|4x + 1|}{3} - 2 \geq 5$?

- (A) 4.875 (B) $\frac{21}{4}$ (C) 8 (D) $-\frac{11}{2}$ (E) -6.125

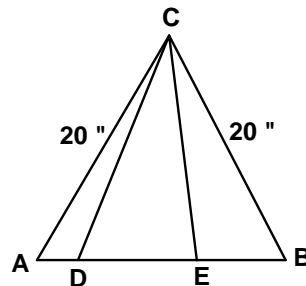
48. Find the total surface area of a right cone with a height of 7 inches and a slant height of 9 inches. (nearest sq. in)

- (A) 187 sq. in (B) 243 sq. in (C) 260 sq. in (D) 278 sq. in (E) 335 sq. in

49. Find the sum of the coefficients of the quotient: $(10x^3 + 13x^2 + 10x + 3) \div (2x + 1)$

- (A) 29 (B) 17 (C) 14 (D) 12 (E) 6

50. $\triangle ABC$ is an equilateral triangle. Find the sum of the perimeters of $\triangle CAD$ and $\triangle BEC$ if $AD = 2''$ and $BE = 8''$ (nearest tenth)



- (A) 74.6" (B) 78.0" (C) 82.7" (D) 84.4" (E) 86.5"

51. $\sum_{k=-2}^{-1} k^k(x + k) + \sum_{k=1}^2 k^k(x - k) = ?$

- (A) $\frac{17(x-2)}{4}$ (B) $\frac{19x-34}{4}$ (C) $\frac{19x}{4}$ (D) 0 (E) $\frac{17x-42}{4}$

52. If $f''(x) = 20x^3 - 24x^2 + 18x - 8$ and $f'(1) = 3$ and $f(-1) = -21$, then $f(1) = \underline{\hspace{2cm}}$.

- (A) 21 (B) 6 (C) -3 (D) -7 (E) -18

53. Penni Tass flipped a penny five times and recorded the results. The odds of only three consecutive heads happening is A:B. Find $A \div B$ to the nearest hundredths.
- (A) 0.25 (B) 0.14 (C) 0.28 (D) 0.16 (E) 0.19
54. Let K be a two-digit number such that the units digit is divisible by 4. What is the sum of these numbers?
- (A) 1,458 (B) 1,378 (C) 1,368 (D) 1,098 (E) 888
55. McDonald's mc nuggets comes in a 4-pack, a 6-pack, a 9-pack and a 20-pack. They do not sell them in any other amounts and will not break up a pack. What is the largest number of mc nuggets less than 40 can you NOT get?
- (A) 39 (B) 32 (C) 19 (D) 14 (E) 11
56. $(62_8 - 47_8 + 13_8) \times 3_8 = \underline{\hspace{2cm}}_8$
- (A) 102 (B) 52 (C) 132 (D) 24 (E) 104
57. Mei Yhun wants to change 5212014 from base 10 to base 20. Which of the following is the correct base 20 number for 5212014.
- (A) 1CBA0E (B) 13B107 (C) 578ABF (D) 52C014 (E) A4040A
58. Change the base 10 proper fraction $\frac{6}{7}$ to a repeating decimal in base 4.
- (A) 0.213213213...₄ (B) 0.323232...₄ (C) 0.312312312...₄ (D) 0.232323...₄ (E) 0.123123123...₄
59. Let P and Q be positive integers such that the harmonic mean of P and Q is 6.875. Find the contraharmonic mean of P and Q.
- (A) 7.0 (B) 7.75 (C) 8.375 (D) 8.625 (E) 9.125
60. Solve the system of equation for x: $(\log_7 16)x + (\log_5 3)y = \log_{49} 8$
 $(\log_3 25)x - (\log_2 7)y = \log_9 125$
- (A) 3 (B) 1.5 (C) 1 (D) 0.5 (E) 0.25

**University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. B | 41. C |
| 2. E | 22. A | 42. B |
| 3. D | 23. B | 43. D |
| 4. B | 24. E | 44. D |
| 5. C | 25. E | 45. E |
| 6. C | 26. A | 46. C |
| 7. C | 27. C | 47. A |
| 8. E | 28. A | 48. C |
| 9. C | 29. D | 49. D |
| 10. D | 30. B | 50. E |
| 11. B | 31. A | 51. A |
| 12. A | 32. B | 52. C |
| 13. E | 33. A | 53. E |
| 14. D | 34. B | 54. A |
| 15. D | 35. A | 55. E |
| 16. B | 36. C | 56. A |
| 17. E | 37. D | 57. A |
| 18. B | 38. A | 58. C |
| 19. C | 39. B | 59. E |
| 20. E | 40. E | 60. D |