

3rd Annual Harvard-MIT November Tournament

Sunday 7 November 2010

Theme Round

StarCraft

1. [3] 16 progamers are playing in a single elimination tournament. Each player has a different skill level and when two play against each other the one with the higher skill level will always win. Each round, each progamer plays a match against another and the loser is eliminated. This continues until only one remains. How many different progamers can reach the round that has 2 players remaining?
2. [4] 16 progamers are playing in another single elimination tournament. Each round, each of the remaining progamers plays against another and the loser is eliminated. Additionally, each time a progamer wins, he will have a ceremony to celebrate. A player's first ceremony is ten seconds long, and afterward each ceremony is ten seconds longer than the last. What is the total length in seconds of all the ceremonies over the entire tournament?
3. [5] Dragoons take up 1×1 squares in the plane with sides parallel to the coordinate axes such that the interiors of the squares do not intersect. A dragoon can fire at another dragoon if the difference in the x -coordinates of their centers and the difference in the y -coordinates of their centers are both at most 6, regardless of any dragoons in between. For example, a dragoon centered at $(4, 5)$ can fire at a dragoon centered at the origin, but a dragoon centered at $(7, 0)$ can not. A dragoon cannot fire at itself. What is the maximum number of dragoons that can fire at a single dragoon simultaneously?
4. [5] A zerg player can produce one zergling every minute and a protoss player can produce one zealot every 2.1 minutes. Both players begin building their respective units immediately from the beginning of the game. In a fight, a zergling army overpowers a zealot army if the ratio of zerglings to zealots is more than 3. What is the total amount of time (in minutes) during the game such that at that time the zergling army would overpower the zealot army?
5. [7] There are 111 StarCraft progamers. The StarCraft team SKT starts with a given set of eleven progamers on it, and at the end of each season, it drops a progamer and adds a progamer (possibly the same one). At the start of the second season, SKT has to field a team of five progamers to play the opening match. How many different lineups of five players could be fielded if the order of players on the lineup matters?

Unfair Coins

6. [4] When flipped, a coin has a probability p of landing heads. When flipped twice, it is twice as likely to land on the same side both times as it is to land on each side once. What is the larger possible value of p ?
7. [4] George has two coins, one of which is fair and the other of which always comes up heads. Jacob takes one of them at random and flips it twice. Given that it came up heads both times, what is the probability that it is the coin that always comes up heads?
8. [5] Allison has a coin which comes up heads $\frac{2}{3}$ of the time. She flips it 5 times. What is the probability that she sees more heads than tails?
9. [6] Newton and Leibniz are playing a game with a coin that comes up heads with probability p . They take turns flipping the coin until one of them wins with Newton going first. Newton wins if he flips a heads and Leibniz wins if he flips a tails. Given that Newton and Leibniz each win the game half of the time, what is the probability p ?
10. [7] Justine has a coin which will come up the same as the last flip $\frac{2}{3}$ of the time and the other side $\frac{1}{3}$ of the time. She flips it and it comes up heads. She then flips it 2010 more times. What is the probability that the last flip is heads?