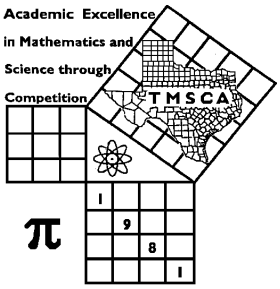

TMSCA HIGH SCHOOL MATHEMATICS TEST #5 ©

NOVEMBER 16, 2019

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2019-2020 TMSCA High School Math Test # 5

1. **Evaluate:** $125 \div 5 \times \left(\frac{1}{.2}\right) - (5)! + 5^3$

   (A) 25 **(B) 75** **(C) 120** **(D) 130** **(E) 150**

2. **$A = 80\%$ of B and $B = \frac{2}{3}$ of C. A = _____ % of C.**

   (A) 40 **(B) $53.\overline{3}$** **(C) $66.\overline{6}$** **(D) 75** **(E) 8**

3. **Jill made a batch of double chocolate cookies. She gave half of them to Joe. Then she gave 2 cookies to Mary. Next she gave one-sixth of what remained to Bill. Then she gave 20% of what was left to Suzy. If she had 24 cookies left for herself, how many cookies did she start with?**

   (A) 68 **(B) 72** **(C) 76** **(D) 80** **(E) 84**

4. **Line L_1 contains the points $(2, -7)$ and $(-6, 1)$. Line L_2 is perpendicular to L_1 and contains the point $(6, -2)$. Which of the following points lies on L_2 ?**

   (A) $(-4, -13)$ **(B) $(-2, -10)$** **(C) $(3, -6)$** **(D) $(5, -2)$** **(E) $(9, 2)$**

5. **Let p and q be the roots of $f(x) = 2x^2 - 13x + 6$. $p^3 + 3p^2q + 3pq^2 + q^3$ = _____.**

   (A) 266.125 **(B) 268.5** **(C) 270.875** **(D) 272.375** **(E) 274.625**

6. **What is the sum of the prime numbers between 190 and 200?**

   (A) 581 **(B) 583** **(C) 585** **(D) 777** **(E) 780**

7. **The set of rational numbers is closed under which of the following operations?**

   I. Addition **II. Subtraction** **III. Multiplication** **IV. Division**

   (A) I only **(B) I, II only** **(C) I, III only** **(D) I, II, III only** **(E) I, II, III, IV**

8. **Tim can mow and trim a lawn in 6 hours. Business was good in June, so he hired Bill who can mow and trim a lawn in 8 hours. Business was still good in July, so he hired Monique. Working together on Tuesday, they were able to mow and trim four lawns in 7.784 hours. How long does it take Monique to mow and trim a lawn by herself? (nearest hundredth of an hour)**

   (A) 4.50 hr **(B) 4.75 hr** **(C) 5.00 hr** **(D) 5.25 hr** **(E) 5.50 hr**

9. **If $\frac{x-3}{x+5} + \frac{x+5}{x-3}$ is written as a mixed number $A\frac{B}{C}$, then B = _____.**

   (A) 25 **(B) 36** **(C) 49** **(D) 64** **(E) 81**

10. The area of a regular heptagon is 210 in^2. Find the perimeter of the heptagon. (nearest tenth of an inch)

(A) 48.8 in (B) 51.0 in (C) 53.2 in (D) 55.4 in (E) 57.6 in

11. Ted's boat traveled 60 miles downstream on the Snake River in four hours. The return trip against the current took six hours. Find the speed of the current in the Snake River. (nearest tenth)

(A) 2.3 mph (B) 2.5 mph (C) 2.7 mph (D) 2.9 mph (E) 3.1 mph

12. $\frac{Ax+B}{3x-4}-\frac{2x+1}{5x-2}=\frac{14x^2+12x-2}{15x^2-26x+8}$ $A+B=$ _____.

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

13. Coach Taylor purchased a new SUV at Kilgore Chevrolet for $28,000. His down payment was $7,500 and he sent in payments of $446.30 each month for four years. How much interest did Coach Taylor pay over the four years?

(A) $922.40 (B) $926.60 (C) $930.80 (D) $935.00 (E) $939.20

14. Find the area of the region bounded by the curves $y_1 = x+6$ and $y_2 = x^2+6x+4$. (nearest tenth)

(A) 28.3 (B) 29.4 (C) 30.5 (D) 31.6 (E) 32.7

15. Consider a circle with center O and points A and B on the circle. The distance from O to chord $\overline{AB}$ is 5 in and the radius of the circle is 13 in. Find $m\angle AOB$. (nearest degree)

(A) 123° (B) 126° (C) 129° (D) 132° (E) 135°

16. If $\log_3\left(\frac{4+2x}{5-x}\right)=1.465$, then x = _____. (nearest hundredth)

(A) 3.00 (B) 3.14 (C) 3.28 (D) 3.42 (E) 3.60

17. If $f(x)=ax^5+bx^3+cx+12$ and $f(2)=22$, then $f(-2)=$ _____.

(A) −22 (B) −12 (C) −6 (D) 0 (E) 2

18. The fraction $\frac{23}{34}$ base 8 can be written as this decimal in base 8.

(A) 0.4777… (B) 0.5111… (C) 0.5333… (D) 0.555… (E) 0.5777…

19. This mathematician, "the father of analytic geometry", is remembered for his law of sign changes.

(A) Georg Cantor (B) Rene Descartes (C) Euclid (D) John Napier (E) Zeno of Elea

20. Consider the graph of the ellipse $4x^2 + 9y^2 + 54y - 16x + 61 = 0$. Which of these points is a vertex of the ellipse?

(A) $(-4, 3)$ (B) $(-1, 3)$ (C) $(1, -3)$ (D) $(3, -3)$ (E) $(5, -3)$

21. Jim flipped a fair coin six times and recorded the outcomes. Find the probability that he got at least four tails.

(A) $\frac{11}{32}$ (B) $\frac{23}{64}$ (C) $\frac{3}{8}$ (D) $\frac{25}{64}$ (E) $\frac{13}{32}$

22. $A = \begin{bmatrix} 3 & 5 \\ -2 & c \end{bmatrix}$ and $B = \begin{bmatrix} -3 & c \\ -1 & 6 \end{bmatrix}$. If $c < 10$ and the determinant of AB is -308, then $c =$ _____.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

23. Cindy rode her bicycle at a speed of 22 mph on a recent training session. On leg one of her ride, she pedaled for six hours due north. On leg two, she pedaled for four hours due east. On leg three, she pedaled due south for two hours before having to stop and fix her flat front tire. How far was she from the starting point when she stopped to fix the flat tire? (nearest mile)

(A) 120 mi (B) 124 mi (C) 128 mi (D) 132 mi (E) 136 mi

24. $3333_4 + 4444_5 + 5555_6 =$ _____$_9$

(A) 2845 (B) 2855 (C) 2865 (D) 2875 (E) 2885

25. Find the sum of the arithmetic sequence. $9 + 13 + 17 + 21 + 25 + ... + 121 + 125$

(A) 2002 (B) 2006 (C) 2010 (D) 2014 (E) 2018

26. In the expansion of $(2x - 5y)^6$, the sum of the x^3y^3, x^2y^4, and xy^5 terms is _____.

(A) -20000 (B) -37500 (C) 0 (D) 20000 (E) 55000

27. Let $f(x) = \sqrt{x+2}$, $g(x) = 3x - 7$, and $h(x) = x^2 - 4$. Find $h\big(f(g(10))\big)$.

(A) 0 (B) 5 (C) 12 (D) 21 (E) 32

28. Rancher Rick wants to know how many turkeys are on his ranch. He owns 3.3 labors of land and the turkey density for this part of Texas is 2.5 turkeys per acre. How many turkeys are on his ranch?

(A) 1409 (B) 1422 (C) 1435 (D) 1448 (E) 1461

29. Larry wanted to evaluate $\int_3^9 \frac{2x}{4x^2+3}dx$ using the method of u-substitution. The best choice for u is

(A) $2x$ (B) $2xdx$ (C) $8x$ (D) $4x^2+3$ (E) $\frac{1}{4x^2+3}$

30. Consider triangle ABC. D is the midpoint of $\overline{AC}$ and E is the centroid of the triangle. If BE = 12, then DB = _____.

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

31. Bowler Bob went bowling on his birthday. His scores were on his first 5 games were 182, 192, 177, 213 and 246. What must he score in game 6 to have an average of 207?

(A) 220 (B) 223 (C) 226 (D) 229 (E) 232

32. If $f(x) = 2x^3 + cx^2 + 4x - 5$ is divided by $x-1$, then the remainder is 4. Find the value of c.

(A) -6 (B) -3 (C) -1 (D) 1 (E) 3

33. Consider the geometric sequence $15, 10, 6\frac{2}{3}, 4\frac{4}{9}, \ldots$ Find the sum of the first ten terms. (nearest hundredth)

(A) 43.94 (B) 44.08 (C) 44.22 (D) 44.36 (E) 44.50

34. The Brock Ice Cream Shop has chocolate, vanilla, strawberry, rocky road, and butter pecan flavors for customers to choose from. How many different ways can Jim order a 4-scoop bowl of ice cream?

(A) 24 (B) 70 (C) 126 (D) 256 (E) 625

35. Two of the roots of the equation $x^3 + bx^2 + cx + d = 0$ are 2 and 4i. $b + c + d =$ _____.

(A) -24 (B) -21 (C) -18 (D) -15 (E) -12

36. If $f(x) = x^4 + \frac{8}{3}x^3 - 46x^2 - 240x + 112$, then the sum of the critical values of f(x) is _____.

(A) -4 (B) -2 (C) 1 (D) 3 (E) 5

37. Which of the following expressions is equal to $\sec(\theta) + \tan(\theta)$?

(A) $\frac{\cos(\theta)}{1-\sin(\theta)}$ (B) $\frac{\sin(\theta)}{1-\cos(\theta)}$ (C) $\frac{1-\sin(\theta)}{\cos(\theta)}$ (D) $\frac{1+\cos(\theta)}{\sin(\theta)}$ (E) $\frac{1-\cos(\theta)}{\sin(\theta)}$

38. Consider the Fibonacci characteristic sequence $a, b, 10, c, 27, d$. $a+b+c+d =$ _____.

(A) 63 (B) 65 (C) 67 (D) 69 (E) 71

39. $\left(444_8 + 555_8\right) \times 11_8 =$ _____$_8$

(A) 13375 (B) 13404 (C) 13413 (D) 13422 (E) 13431

40. An empty water storage tank in the shape of a right circular cylinder has a radius of 6 feet. If 6159 gallons of water are pumped into the tank, how high is the water level? (nearest inch)

(A) 7 ft 1 in (B) 7 ft 3 in (C) 7 ft 5 in (D) 7 ft 7 in (E) 7 ft 9 in

41. Find the digit in the hundred-thousandths place of the sum of the series

$$.3 - \frac{.027}{6} + \frac{.00243}{120} - \frac{.0002187}{5040} + \ldots$$

(A) 0 (B) 2 (C) 5 (D) 6 (E) 9

42. The number 124 is a member of which of the following sets?

I. Lucky II. Happy III. Abundant IV. Odious

(A) IV only (B) III, IV only (C) II, III, IV only (D) II, III only (E) I, II only

43. Joe is 6 ft tall and he is walking away from a 24 ft tall street light at a speed of 8 ft/s. At what rate is the length of his shadow increasing at the moment Joe is 30 ft from the street light?

(A) $2.\overline{3}$ ft/s (B) 2.5 ft/s (C) $2.\overline{6}$ ft/s (D) 2.75 ft/s (E) $2.8\overline{3}$ ft/s

44. Karen took an IQ test recently and her results show that she had an IQ of 130. This IQ test has a mean of 100 and a standard deviation of 15. What percentile does Karen's score place her? (nearest whole percentile)

(A) 90^{th} (B) 92^{nd} (C) 94^{th} (D) 96^{th} (E) 98^{th}

45. Point P $(1, 3)$ is the midpoint of line segment $\overline{AB}$. If point A has coordinates $(-4, e)$ and point B has coordinates $(f, 12)$. $e+f =$ _____.

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

46. The slant asymptote and vertical asymptote of $f(x) = \dfrac{x^2+4x+6}{x-2}$ intersect at the point (a, b).

$a+b =$ _____.

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

47. Let $f(x)=\frac{2x+5}{3-4x}$, $x\neq\frac{3}{4}$. Find $f^{-1}(2)=$ _____.

(A) $\frac{1}{10}$ (B) $\frac{2}{7}$ (C) $\frac{4}{5}$ (D) $\frac{9}{7}$ (E) $\frac{9}{5}$

48. Consider the harmonic progression $\frac{1}{4},\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{16}$. $a+b+c=$ _____.

(A) 24 (B) 26 (C) 28 (D) 30 (E) 32

49. If the three numbers 156, 211, and 288 are each divided by the number D, each of the quotients will will have the same remainder R. Find D.

(A) 7 (B) 8 (C) 9 (D) 11 (E) 12

50. The roots of $f(x)=x^3+bx^2+cx+12=0$ are 2, 3 and f. $b+c=$ _____.

(A) -7 (B) -5 (C) -3 (D) -1 (E) 1

51. How many distinct 4-letter code words can be made using the letters in the word SOONERS?

(A) 144 (B) 188 (C) 216 (D) 270 (E) 324

52. Sue is thinking of a number between 20 and 50. Find the probability that she is thinking of a number that is an emirp prime?

(A) $\frac{1}{30}$ (B) $\frac{2}{29}$ (C) $\frac{3}{29}$ (D) $\frac{4}{29}$ (E) $\frac{1}{6}$

53. Find the sum of the first six pentagonal numbers.

(A) 120 (B) 122 (C) 124 (D) 126 (E) 128

54. Given the table of values for f(x), find f(2).

x	-3	-0.5	0.5	3
f(x)	-32	1.125	0.375	16

(A) $2.\overline{6}$ (B) $2.8\overline{3}$ (C) 3 (D) $3.1\overline{6}$ (E) $3.\overline{3}$

55. Evaluate: $\prod_{n=3}^{4} 4^{n-3}-n^2$

(A) 84 (B) 90 (C) 96 (D) 102 (E) 108

56. The focus of the parabola $x^2 + 12y + 6x - 3 = 0$ is (a, b). b = _____.

(A) –2 **(B) –1** **(C) 0** **(D) 0.5** **(E) 0.75**

57. The point of intersection of two lines with equations $5x + 3y = 8$ and $2y - 3x = 18$ is (a, b). $a + b =$ _____.

(A) 2 **(B) 4** **(C) 6** **(D) 8** **(E) 10**

58. Thomas rolled two fair dice and recorded the sum of the top faces. Find the probability the sum was either 5 or 8.

(A) $\frac{7}{36}$ **(B) $\frac{2}{9}$** **(C) $\frac{1}{4}$** **(D) $\frac{5}{18}$** **(E) $\frac{11}{36}$**

59. Consider $f(x) = \frac{1}{4}x^2 + 2$ over the interval $[1, 4]$. Find the value of x in this interval that satisfies the mean value theorem.

(A) 1.5 **(B) 2** **(C) 2.5** **(D) 3** **(E) 3.5**

60. Andy's math team is composed of 6 freshmen, 5 sophomores, 4 juniors and 3 seniors. If he must select one student from each grade level for his 4-person team, how many different ways can he select his team?

(A) 180 **(B) 240** **(C) 300** **(D) 360** **(E) 420**

2019 – 2020 TMSCA High School Mathematics Test # 5
Answer Key

1. D	**21. A**	**41. B**
2. B	**22. C**	**42. A**
3. C	**23. B**	**43. C**
4. B	**24. D**	**44. E**
5. E	**25. C**	**45. C**
6. E	**26. A**	**46. D**
7. D	**27. D**	**47. A**
8. A	**28. E**	**48. D**
9. D	**29. D**	**49. D**
10. C	**30. B**	**50. A**
11. B	**31. E**	**51. D**
12. C	**32. E**	**52. B**
13. A	**33. C**	**53. D**
14. D	**34. B**	**54. C**
15. E	**35. C**	**55. C**
16. A	**36. B**	**56. A**
17. E	**37. A**	**57. B**
18. C	**38. E**	**58. C**
19. B	**39. E**	**59. C**
20. E	**40. B**	**60. D**

19-20 TMSCA HSMA Test 5 Selected Solutions

3. $\left(x(.5)-2\right)\left(\frac{5}{6}\right)(.8)=24$

$x=76$

$m=\frac{1--7}{-6-2}=-1$

$m_p=1$

4. $y+2=x-6$

$y=x-8$

$-10=-2-8$

5. $\left(\frac{13}{2}\right)^3=274.625$

6. $191+193+197+199=780$

$\frac{360}{14}=25\frac{5}{7}$

$\tan\left(25\frac{5}{7}\right)=\frac{x}{a}$

8. $\frac{7.784}{6}+\frac{7.784}{8}+\frac{7.784}{t}=4$

$t=4.50$

9. $5--3=8$

$8^2=64$

10. $210=\left(\frac{1}{2}\right)\left(\frac{x}{\tan\left(25\frac{5}{7}\right)}\right)(14x)$

$x=3.8$

$14x=53.2$

11. $(B+C)(4)=60$

$(B-C)(6)=60$

$C=2.5$

$5A-6=14$

$A=4$

12. $-2B+4=-2$

$B=3$

$4+3=7$

13. $48(446.3)-(28000-7500)=922.40$

14. $\int_{-5.37228}^{.37228}(y1-y2)\,dx=31.6$

15. $\tan\theta=\frac{12}{5}$

$2\theta=135$

17. $22-12=10$

$-10+12=2$

20. $4(x^2-4x+4)+9(y^2+6y+9)=-61+16+81$

$4(x-2)^2+9(y+3)^2=36$

$\frac{(x-2)^2}{9}+\frac{(y+3)^2}{4}=1$

center $(2,-3)$

vertices $(-1,-3)$ and $(5,-3)$

21. $\text{binomialCdf}(6,.5,4,6)=\frac{11}{32}$

23. $88^2+88^2=d^2$

$d=124$

24. $255+624+1295=2174_{10}$

2875_9

25. $125=9+(n-1)(4)$

$S=\frac{30}{2}(9+125)=2010$

26. $-20,000+37,500-37,500$

$-20,000$

27. $g(10)=23$

$f(23)=5$

$h(5)=21$

28. $(3.3)(177.136)(2.5)$

1461

30. $12+6=18$

31. $182+192+177+213+246+x=6(207)$

$x=232$

33. $r = \frac{2}{3}$

$S = \frac{15\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}} = 44.22$

34. ${}_8C_4 = 70$

35. $(x-2)(x-4i)(x+4i)$

$x^3 - 2x^2 + 16x - 32$

$-2 + 16 - 32 = -18$

36. $f'(x) = 0$

$x = -4, -3, 5$

$-4 - 3 + 5 = -2$

37. $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos\theta(1+\sin\theta)}{\cos\theta \cdot \cos\theta}$

$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$\frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$\frac{\cos\theta}{1-\sin\theta}$

38. $a = 3$

$b = 7$

$c = 17$

$d = 44$

$sum = 71$

39. $\frac{1221 \times 11}{13431}$

40. $\pi(72)^2 h = (6159)(231)$

$h = 7\text{ ft }3\text{ in}$

41. $\sin(.3) = .295520207$

43. $\frac{24}{x+y} = \frac{6}{y}$

$6x + 6y = 24y$

$x = 3y$

$\frac{dx}{dt} = 3\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{1}{3} \cdot 8 = 2.\overline{6}$

44. $\text{normalCdf}(-\infty, 130, 100, 15)$

.9772499

98th percentile

45. $\frac{-4+f}{2} = 1 \rightarrow f = 6$

$\frac{e+12}{2} = 3 \rightarrow e = -6$

$-6 + 6 = 0$

46. $y = x + 6$

$x = 2$

$(2, 8)$

$2 + 8 = 10$

47. $f^{-1}(x) = \frac{3x-5}{4x+2}$

$f^{-1}(2) = \frac{1}{10}$

48. $a = 7$

$b = 10$

$c = 13$

$sum = 30$

49. $D = 11$

$R = 2$

50. $f = -2$

$(x-2)(x-3)(x+2)$

$x^3 - 3x^2 - 4x + 12$

$-3 - 4 = -7$

52. 31, 37

$\frac{2}{29}$

53. $1 + 5 + 12 + 22 + 35 + 51 = 126$

54. $f(x) = x^3 - x^2 - x + 1$

$f(2) = 3$

56. $12y = -x^2 - 6x + 3$

$12y = -\left(x^2 + 6x + 9\right) + 3 + 9$

$12y = -(x+3)^2 + 12$

$y = -\frac{1}{12}(x+3)^2 + 1$

$p = -3$

vertex $(-3, 1)$

focus $(-3, -2)$