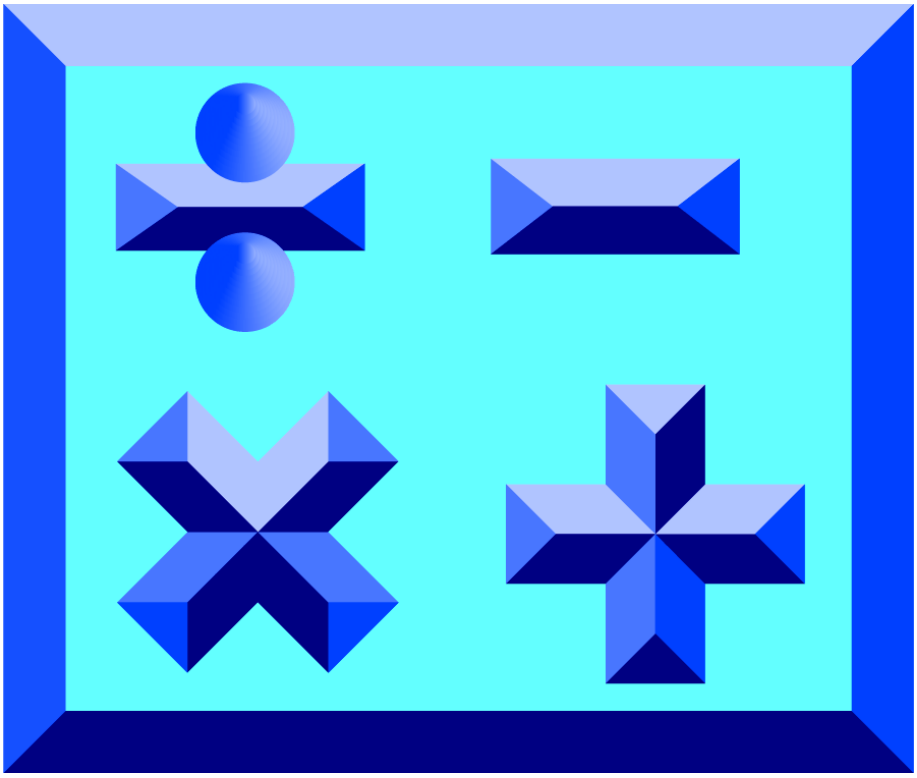
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2018



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $4! \times (4)^{-2} + (4^2)^{\frac{1}{4}} - 4 \div 2$

   (A) -1.25 **(B)** -0.25 **(C)** 1.5 **(D)** 3.5 **(E)** 6

2. **Lotta Cash received a \$50.00 gift card for graduation. She went shopping at the *Cheap Shoppe*. She bought 2 pair of shorts at \$7.99 each, 3 pair of flip-flop sandals at \$4.50 each, a bottle of suntan lotion at \$8.25, a sun hat at \$9.89, and 2 bottles of water at 75¢ each. She got 15% off for using a gift card instead of a credit card. How much does she have left on her gift card if the tax rate was 7.5%?**

   (A) \$8.25 **(B)** \$5.12 **(C)** \$4.35 **(D)** \$3.68 **(E)** \$2.80

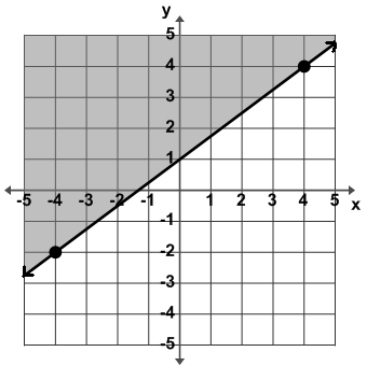
3. **If 45% of A is $4\frac{1}{5}$ of B, then B is what per cent of A?**

   (A) 3% **(B)** $4\frac{2}{7}\%$ **(C)** $7\frac{6}{7}\%$ **(D)** $9\frac{3}{7}\%$ **(E)** $10\frac{5}{7}\%$

4. **$2 \times 4 \times 8 = 8 \times 8 = 64$ and $2 \times 4 \times 8 = 2 \times 32 = 64$ are examples of the ? property of equality.**

   (A) associative **(B)** commutative **(C)** distributive **(D)** identity **(E)** inverse

5. **Which of the inequalities is best represented by the graph below?**



   (A) $4x - 3y \geq 4$ **(B)** $3x - 4y \geq -4$ **(C)** $x + 4y \leq 4$
   (D) $3x - 4y \leq -4$ **(E)** $x - 4y \leq -4$

6. **If $2(3 + 5) = 16$ and $16 = 4^2$ then $2(3 + 5) = 4^2$. Which of the following properties does this example illustrate ?**

   (A) associative **(B)** commutative **(C)** distributive **(D)** symmetric **(E)** transitive

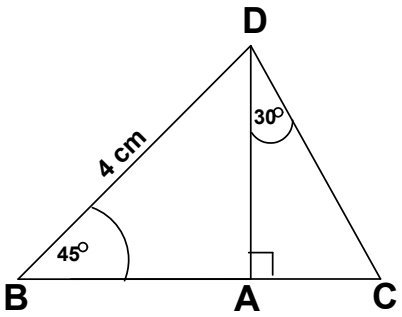
7. **Simplify:** $\left(\frac{2x^2 - 7x + 5}{4x^2 + 8x - 12}\right) \div \left(\frac{4x^2 - 8x - 5}{2x^2 + 7x + 3}\right)$

   (A) $\frac{1}{4}$ **(B)** $\frac{x+3}{4(x-1)}$ **(C)** $\frac{2x+1}{2x-5}$ **(D)** $\frac{2x+5}{4(2x-1)}$ **(E)** $\frac{4(x+3)}{x-1}$

8. **If $4x^2 - x + c = (ax + b)(x + 1)$ then $a + b + c =$ _____.**

   (A) -4 **(B)** -6 **(C)** 1 **(D)** 5 **(E)** 14

9. Find the perimeter of $\triangle BCD$. (nearest tenth).



(A) 11.7 cm (B) 12.5 cm (C) 13.8 cm (D) 10.1 cm (E) 8.9 cm

10. The line $y = mx + b$ contains the point $(-5, -2)$ and has a slope of $-\frac{3}{4}$. The y-intercept is:

(A) $(0, 2\frac{1}{3})$ (B) $(0, 7\frac{2}{3})$ (C) $(0, \frac{3}{4})$ (D) $(0, -1\frac{3}{4})$ (E) $(0, -5\frac{3}{4})$

11. The circles $(x-3)^2 + (y+1)^2 = 16$ and $(x-4)^2 + (y-2)^2 = 9$ intersect in two points. The slope of the line through the two points of intersection is:

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$ (C) $-\frac{1}{7}$ (D) $-\frac{1}{3}$ (E) -3

12. A rectangular swimming pool is twice as long as it is wide and has a 10 foot-wide concrete border around it. If the border has an area of 2800 sq. ft., find the perimeter of the pool.

(A) 210 ft (B) 240 ft (C) 280 ft (D) 300 ft (E) 320 ft

13. If $27^{(k)} = 9^{(k+1)}$, then $3^{(k+2)} = ?$

(A) 243 (B) 81 (C) 27 (D) 9 (E) 3

14. Let $f(x) = x - 2$, $g(x) = 2x - 1$, $h(x) = 3x$, and $g(f(x)) + f(h(x)) = -4$. Find x.

(A) 1 (B) $-2\frac{1}{5}$ (C) $-\frac{1}{5}$ (D) $1\frac{2}{5}$ (E) $\frac{3}{5}$

15. Which of the following functions does not have an inverse function?

(A) $y = 2x - 4$ (B) $y = \frac{1}{4}x + 2$ (C) $y = -x^2 + 4$ (D) $y = \ln(x+4) - 2$ (E) $y = \sqrt{2x - 4}$

16. Phil Dewallit got a \$20.00 allowance for mowing his parent's lawn this week. They agreed to increase his previous week' s allowance 80¢ each week for the next 24 weeks. Phil decides to put half of his allowance in his piggy bank each week. How much will he have in the bank at the end of the 25 week period?

(A) \$370.00 (B) \$351.00 (C) \$333.25 (D) \$266.50 (E) \$257.50

17. Determine the range of $f(x) = 2 - 4\cos(x + 3)$.

(A) $[-4, 4]$ (B) $[-2, 4]$ (C) $[-2, 6]$ (D) $[-4, 2]$ (E) $[4, 12]$

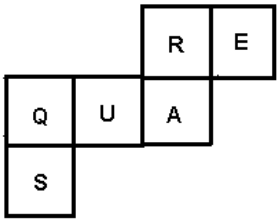
18. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ is equivalent to:

(A) 1 (B) $\frac{1}{\sin\theta}$ (C) $\frac{\cos\theta}{2\sin\theta}$ (D) $2\sec\theta$ (E) $2\csc\theta$

19. Captain Ed Inberg went sailing on Lake Falcon. He sailed his scow from the dock 8 km on a bearing of 40°. Then he changed course and sailed 5 km on a bearing of 120°. Then he decided to return to the dock. What bearing will Captain Ed have to sail to go straight back to the dock? (nearest degree)

(A) 249° (B) 231° (C) 219° (D) 151° (E) 111°

20. Paulie Gone folds the net shown into a cube. What letter will be on the opposite face of face E?



(A) U (B) S (C) R (D) Q (E) A

21. In the expansion of $(3x - 2)^5$, the sum of the coefficients of the 3rd and the 4th term is:

(A) 1,320 (B) 360 (C) −1,520 (D) 480 (E) 1,800

22. Find $a + b + c + d$ given the Fibonacci characteristic sequence: 3, a, b, 17, c, d, 71, ...

(A) 179 (B) 91 (C) 159 (D) 88 (E) 105

23. $\sum_{k=1}^{3} (-1)^k (kx - (k+1)y - k) = ?$

(A) $-6x + 9y + 6$ (B) $2x - 3y - 2$ (C) $-(2x - 3y - 2)$
(D) $6x - 3y + 2$ (E) $6x - 9y - 6$

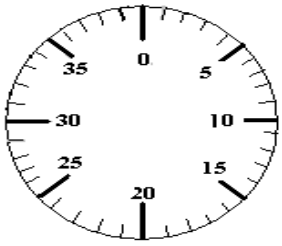
24. Sameer, Anisha, and Ian worked a total of 125 problems on the number sense test at the math camp. Sameer worked 28% of the total problems, Anisha worked 40 less problems than Ian did. What percent of the problems did Ian work?

(A) 36% (B) 48% (C) 52% (D) 65% (E) 68%

25. Find the area of the region bounded by the graphs of $x = 4 - y^2$ and $x = 4 - 4y$.

(A) $10\frac{2}{3}$ (B) 10 (C) $9\frac{2}{3}$ (D) $9\frac{1}{3}$ (E) 9

26. P-Q-R is the combination needed to open the safe with the combination dial shown below. How many distinct combinations exist if P is a triangular number, Q is a square number greater than 0, R is a pentagonal number.



(A) 240 (B) 225 (C) 175 (D) 128 (E) 19

27. If $f'(x) = 3x^2 - 5$ and $f(-1) = 4$, find $f(1)$.

(A) -5 (B) -4 (C) -3 (D) -2 (E) -1

28. Max Space has a rectangular sheet of cardboard that is 4 feet by 6 feet. He is going to cut out a 5 inch square from each of the four corners, then fold up the sides, tape edges, and make a rectangular box without a top. What is the volume of the box? (nearest tenth)

(A) 9.7 cu. ft. (B) 8.3 cu. ft. (C) 6.8 cu. ft. (D) 6 cu. ft. (E) 3.8 cu. ft.

29. Roland Bones rolls a pair of dice. What are the odds that the sum of top faces he rolls is a 7 or an 11?

(A) $\frac{2}{9}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{17}$ (E) $\frac{2}{7}$

30. Which of the following mathematicians is known for developing a "machine" that uses a system of rules, states, and transitions used to decide a language or to solve mathematical functions? It is a powerful tool used in computer science and code breaking.

(A) Eratosthenes (B) Charles Babbage (C) John Napier
(D) George Boole (E) Alan Turing

31. Arnie has a bag with 3 white golf balls and 2 yellow golf balls. Jack has a bag with 4 yellow golf balls and 2 white golf balls. Tiger picks a bag and a ball at random. The probability that the ball will be white is: (nearest whole percent)

(A) 47% (B) 10% (C) 23% (D) 45% (E) 20%

32. Find $a + b + c + d$ given the arithmetic sequence: $-11, a, b, c, 3, d, ...$

(A) 6.5 (B) 3.25 (C) -2.25 (D) -2.5 (E) -5.5

33. Let $f(x) = ax^3 - bx + 3$ where a and b are integers. If $f(2) = -4$, then $f(-2) = ?$

(A) 4 (B) 7 (C) 10 (D) -3 (E) -4

34. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the sum of the numbers in the 8^{th} row.

1				1				2				3				5		
			2				3				5				8			
		3				5				8				13				21
	5				8				13				21				34	
8				13				21				34				55		
			21				34				55				89			
		34				55				89				144				233

(A) 932 (B) 665 (C) 864 (D) 521 (E) 898

35. Coach Ball has 22 students in his PE class. 9 of the students play football, 10 play basketball, 5 play tennis and basketball but not football, 5 play basketball and football but not tennis, and 2 play tennis only. How many students do not play any of these 3 sports?

(A) 1 (B) 3 (C) 5 (D) 6 (E) not enough information

36. I. Cee and U. Saul used a 2 in. x 12 in. x 16 ft. board to make a teeter-totter with the center being on a fulcrum. Cee weighs 85 pounds and is sitting 8 feet from the center of the teeter-totter. Saul weighs 100 pounds and is sitting on the opposite end. How far from the center should Saul sit if the teeter-totter has a slope of zero? (nearest inch)

(A) 4' 1" (B) 5' 8" (C) 6' 10" (D) 7' 9" (E) 9' 5"

37. Twenty-five seniors took the state math test last year. Fifteen of them were boys and ten were girls. All of them had an equal chance to win one of the top three medals. What was the probability that one girl and two boys won one of the top three medals? (nearest whole percent)

(A) 21% (B) 25% (C) 42% (D) 29% (E) 46%

38. If the probability that a student in a Statistics class studies for an exam is 70%, and the probability that a student who studies passes the test is 85%, then the probability that a student both studies and passes the test is: (nearest whole percent)

(A) 75 % (B) 60% (C) 55 % (D) 50% (E) 45%

39. Given: $\triangle ABE$ is similar to $\triangle DON$; $\angle A \cong \angle N$; $\angle B \cong \angle D$; AB = 30 cm; DN = 24 cm; and NO = 16 cm. Find AE.

(A) 45 cm (B) 12.8 cm (C) 22 cm (D) 16 cm (E) 20 cm

40. If $\log_6(16) - \log_6(4x) = \log_6(x+2)$, then x equals _____.

(A) $\sqrt{5}+1$ (B) 2 (C) $1\frac{1}{2}$ (D) $\sqrt{5}-1$ (E) $\frac{2}{3}$

41. $\triangle ABC$ is a scalene triangle. Point P lies on segment AB such that segment CP is the altitude of the triangle, $m\angle CBP = 65°$, AP = 12", BP = 15". Find $m\angle ACP$. (nearest degree)

(A) 15 ° **(B)** 20 ° **(C)** 25 ° **(D)** 32 ° **(E)** 35 °

42. Given: $f(x) = 2 - 4\sin(x + 3)$. What quadrant(s) would the graph of f(x) be in if the amplitude is cut in half, the vertical displacement is decreased by 5 and the phase shift is increased by 1?

(A) I & II **(B)** I & IV **(C)** II & III **(D)** III & IV **(E)** I, II, III, & IV

43. The harmonic mean of the real roots of $3x^3 + 2x^2 + 5x + 4 = 0$ is _____.

(A) — 0.333... **(B)** — 0.8 **(C)** — 1.333... **(D)** — 2.4 **(E)** — 3

44. Let $g(x) = 3x^2 - 2x + 1$. Find k if $g(k - 1) - g(k) = 11$.

(A) $2\frac{2}{3}$ **(B)** $\frac{3}{5}$ **(C)** 0 **(D)** $-\frac{2}{3}$ **(E)** — 1

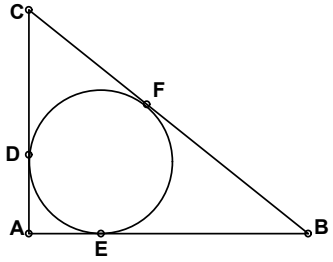
45. Let $\frac{1}{x} + \frac{1}{y} = 1$. Find $D_x y$.

(A) $\frac{y-1}{1-x}$ **(B)** $\frac{x-1}{1-y}$ **(C)** $\frac{1-y}{x}$ **(D)** $\frac{x}{y-1}$ **(E)** $\frac{y-1}{x-1}$

46. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \frac{x^2-2}{x-1}$

(A) — 2 **(B)** 0.1666... **(C)** 0.5 **(D)** 2 **(E)** 6

47. Find the radius of the circle inscribed in $\triangle ABC$ with AC = 3", AB = 4", and BC = 5".



(A) $\frac{1}{2}$ " **(B)** $\frac{2}{3}$ " **(C)** 1" **(D)** 2" **(E)** 6"

48. Let $f(x) = \begin{cases} 3 + x & \text{if } x \leq 1 \\ 3 - x & \text{if } 1 < x \end{cases}$. Which of the following is/are true?

1. $\lim_{x \to 1^+} f(x)$ exists 2. $\lim_{x \to 1^-} f(x)$ exists 3. f(x) is continuos

(A) none of these **(B)** 1 & 2 but not 3 **(C)** 1 only **(D)** 2 only **(E)** 1, 2, & 3

49. Which of the following pairs of numbers are considered to be 'fangs' of a 'vampire' number?
I. (35, 41) II. (21, 87) III. (72, 27) IV. (51, 63)

(A) I & III (B) I, II, & III (C) II & IV (D) I & II (E) I only

50. Let $4022_b - k_b = 1665_b$, where k_b is a four digit number. Find k_b in base 10.

(A) 949 (B) 1,117 (C) 1,263 (D) 2,066 (E) 2,135

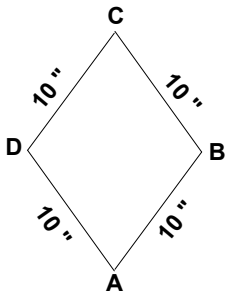
51. 7,158,AB3 ÷ 9 has a remainder of 7. Find A + B.

(A) 3 (B) 6 (C) 9 (D) 10 (E) 12

52. How many ordered pairs of positive integers (a, b) with $a + b \leq 50$, satisfy the equation: $(a + b^{-1}) \div (a^{-1} + b) = 13$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

53. Find the area of the rhombus shown given that AC — BD = 4",



(A) 100 in^2 (B) 98 in^2 (C) 96 in^2 (D) 94 in^2 (E) 92 in^2

54. If $x < y$ and $x < 0$, which of the following is never greater than any of the others?

(A) $x + y$ (B) $x - y$ (C) $x + |y|$ (D) $x - |y|$ (E) $-|x + y|$

55. The *Hole-In-One* golf shop has periodic sales given by the function $G(m) = 5 + 5\cos((\frac{\pi}{3})(m + 3))$ where m is the number of months and G(m) is the number of golf sets sold. If the store opened on Jan. 1, when did the maximum sales first occur?

(A) 3 months (B) 4 months (C) 5 months (D) 6 months (E) 7 months

56. A *square-free semiprime* is a composite number that is the product of two different primes. How many composite numbers less than 20 are considered *square-free semiprimes*?

(A) 8 (B) 6 (C) 4 (D) 2 (E) 0

57. The function $f(x) = \begin{cases} nx^3 - x & \text{if } x \leq 1 \\ mx^2 + 5 & \text{if } 1 < x \end{cases}$ is differentiable everywhere. Find n.

(A) 13 **(B)** -17 **(C)** -14 **(D)** -11 **(E)** -9

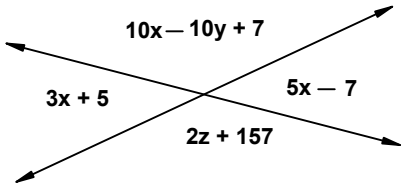
58. In how many ways can the letters of the word 'DIVIDE' be arranged in such a way that the vowels always come together?

(A) 18 **(B)** 36 **(C)** 72 **(D)** 144 **(E)** 180

59. Given the sequence, $\frac{7}{(1\times1+1)} - \frac{7}{(2\times2-1)} + \frac{7}{(3\times3+1)} - \frac{7}{(5\times5-1)} + \frac{7}{(8\times8+1)} - \ldots$, find the digit in the ten-thousandths place.

(A) 6 **(B)** 5 **(C)** 4 **(D)** 2 **(E)** 1

60. Find the sum of x, y, and z, given the degree measures of the angles shown.



(A) -3 **(B)** -1 **(C)** 0 **(D)** 12 **(E)** 15

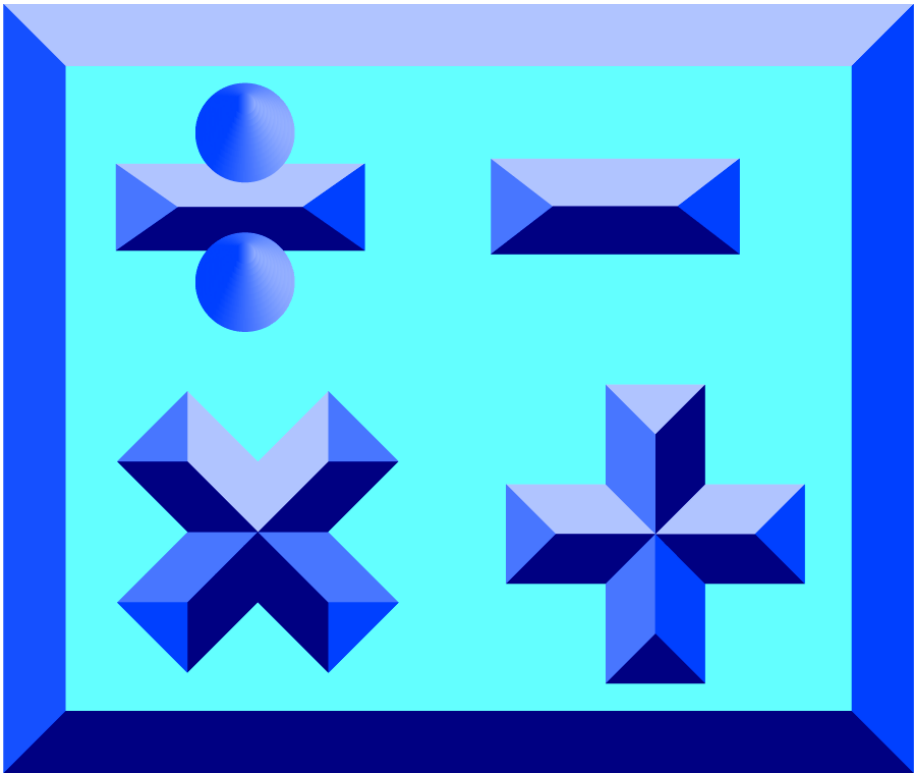
University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2018
Answer Key

1. C	**21. B**	**41. B**
2. B	**22. D**	**42. D**
3. E	**23. C**	**43. D**
4. A	**24. C**	**44. E**
5. D	**25. A**	**45. A**
6. E	**26. A**	**46. D**
7. A	**27. B**	**47. C**
8. B	**28. C**	**48. B**
9. A	**29. E**	**49. D**
10. E	**30. E**	**50. B**
11. D	**31. A**	**51. D**
12. B	**32. E**	**52. B**
13. B	**33. C**	**53. C**
14. E	**34. E**	**54. D**
15. C	**35. D**	**55. A**
16. A	**36. C**	**56. C**
17. C	**37. E**	**57. D**
18. E	**38. B**	**58. B**
19. A	**39. E**	**59. C**
20. A	**40. D**	**60. A**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2018



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $\sqrt[3]{1728} \div (16)^{\frac{1}{2}} + 8 \times (2)^{-1} - 4$

(A) 8 **(B) 3** **(C) 1.5** **(D) — 2** **(E) — 19**

2. **Two and one-fourth million is added to three hundred twenty thousand five hundred. One million one thousand one hundred is subtracted from the sum. The difference is divided by eleven. The quotient is truncated to the units place. Which digit appears the most in the final results?**

(A) 7 **(B) 6** **(C) 4** **(D) 2** **(E) 1**

3. **Find the average of the arithmetic mean, the median, and the mode of these quiz grades: 75, 95, 75, 100, 95, 80, 75, & 70. (nearest whole number)**

(A) 79 **(B) 80** **(C) 81** **(D) 83** **(E) 85**

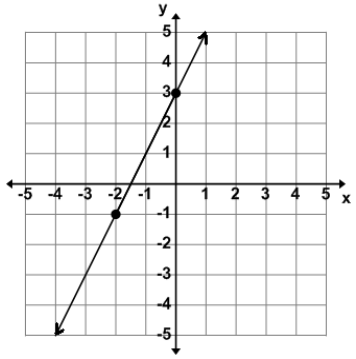
4. **Let U (universal set) = {u, i, l, m, a, t, h, b}, B = {b, u, i, l, t}, and T = {t, h, u, m, b}. Let $I = (B \cap T)^C$. Set I contains how many distinct elements?**

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

5. **If $(3x + 1)(x - 3)(2x) = ax^3 + bx^2 + cx + d$ then $a + b + c + d =$ _____.**

(A) — 20 **(B) — 16** **(C) — 4** **(D) 3** **(E) 6**

6. **A line parallel to the line shown through the point (1, — 1) has x-intercept at point (a, b) and y-intercept at point (c, d). Find $a + b + c + d$.**



(A) — 1.5 **(B) 2** **(C) 0.5** **(D) — 4.5** **(E) 4**

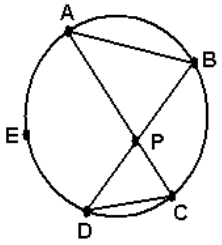
7. **Which of the following sets of numbers is closed under multiplication and addition?**
I. Primes II. Integers III. Wholes IV. Rationals

(A) I, II, & III (B) II, III, & IV (C) I, III, & IV (D) all of them (E) None of them

8. **Max Whale likes to mix his regular blend coffee with a boost blend coffee at a ratio of 3 to 1. The regular blend sells for $11.00 per pound and the boost blend sells for $8.00 per pound. Find the cost per pound of Max's special mixture of regular blend and boost blend. (nearest cent)**

(A) $14.25 **(B) $6.67** **(C) $11.67** **(D) $4.75** **(E) $10.25**

9. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\overset{\frown}{AED}$ given $m\angle BPC = 95°$ and $m\angle BAP = 25°$.



(A) 140° (B) 70° (C) 72.5° (D) 120 ° (E) 165°

10. $\angle A$ and $\angle B$ are supplementary angles with m $\angle A = 5x - 4$ and m $\angle B = 3x + 2$. Find the absolute value difference in the measures of $\angle A$ and $\angle B$.

(A) 22.75° (B) 87° (C) 17° (D) 43.5° (E) 39.5°

11. Les Square increased the length of two opposite sides of a square by 20%, and decreased the other two opposite sides by 50%. What percent of the area of the original square is the area of the new rectangle?

(A) 30% (B) 40% (C) 60% (D) 70% (E) 80%

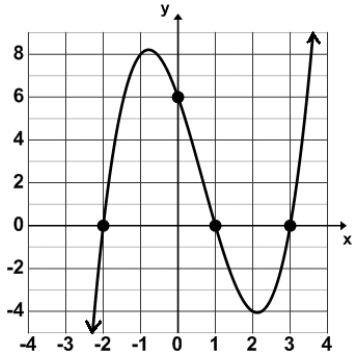
12. If $\frac{x+5}{2x-1} + \frac{Ax+B}{3x+2} = \frac{-7x^2+30x+6}{6x^2+x-2}$, where A and B are constants, then $A + B$ equals:

(A) 91 (B) 5 (C) 4 (D) 1 (E) -1

13. Let f(x) = 2x — 1 and g(x) = 2 — 3x and h(x) = x + 3. Find $g\Big(h\big(f(1-x)\big)\Big)$.

(A) $6x + 2$ (B) $6 - 10x$ (C) $5 - 6x$ (D) 4 (E) $6x - 10$

14. The graph of $f(x) = Ax^3 + Bx^2 + Cx + D$ is shown here. Find $A + B + C + D$.



(A) -1 (B) 0 (C) 1 (D) 4 (E) 10

15. Les Qwik and Lotta Speed worked together to finish their research project in 12.5 hours. Lotta works 2.5 times faster than Les. How long would it have taken Lotta to do the project alone?

(A) 17.5 hrs (B) 15 hrs (C) 10 hrs (D) 7.5 hrs (E) 5 hrs

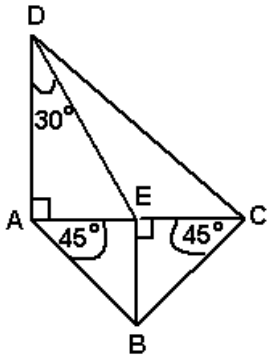
16. If you start at $(\frac{7\pi}{2}, 0)$ on the x-axis and travel horizontally 15.7 radians to the left, how many times will you cross the graph of $y = 2\sin(3x)$?

(A) 8 (B) 11 (C) 14 (D) 15 (E) 16

17. Given: $f(x) = 3\cos[4\pi(x + 1)] - 2$. Find the sum of the numeric values of the period and the vertical displacement.

(A) -1.5 (B) -1 (C) 0 (D) 2 (E) 3.5

18. Find DC if AE = 3".



(A) $3\sqrt{7}$ in (B) $2\sqrt{10}$ in (C) $3\sqrt{8}$ in (D) $4\sqrt{3}$ in (E) $3\sqrt{13}$ in

19. How many negative real roots will $x^5 + x^4 - 2x^3 + x^2 - 1 = 0$ have?

(A) 3 or 1 (B) 4, 2, or 0 (C) 1 (D) 0 (E) 2 or 0

20. Which of the following is true about the function $f(x) = \frac{x^2 + 4}{x^3 - 3}$?

I. f(x) is odd II. f(x) is even III. f(x) has 3 asymptotes.

(A) I & III (B) II & III (C) I only (D) III only (E) none of these

21. Meagan Money invested some money in the stock market. Her investment increased 8% by the end of the first year, decreased 2% by the end of the second year, and increased 12% by the end of the third year. What was Meagan's average rate of return over the three year period? (nearest tenth)

(A) 6.2% (B) 6.0% (C) 5.8% (D) 5.6% (E) 5.5%

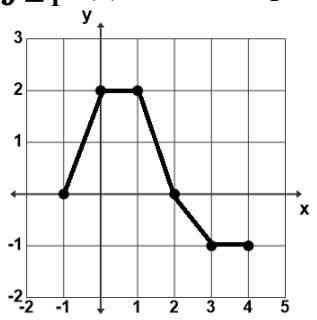
22. The vertex of a parabola is located at (3, 1) and the focus is located at (3, 3). Find the directrix of the parabola.

(A) $y = 5$ (B) $y = 1$ (C) $y = 0$ (D) $y = -\frac{1}{3}$ (E) $y = -1$

23. Let $f(x) = \frac{5x - 2}{4 + 3x}$. Find $f'(-2)$.

(A) 6.5 (B) 6 (C) 6 (D) 14 (E) 13

24. Find the value of $\int_{-1}^{4} f(x)\,dx$ for the piecewise-linear function f, $-1 \le x \le 4$, shown below?



(A) 1 **(B) 2.5** **(C) 4** **(D) 5.5** **(E) 8**

25. In a triple play game, Willie When performs three tasks. He flips a quarter, and success would be heads. He rolls a single die, and success would be a six. He picks a card from a standard deck of cards, and success would be picking a heart. If any of these task are successful, He will win the game. What is the probability he will win? (nearest whole percent)

(A) 2% **(B) 31%** **(C) 92%** **(D) 69%** **(E) 48%**

26. If two dice are tossed, what is the probability that the sum of the faces is a prime number?

(A) $\frac{5}{36}$ **(B)** $\frac{13}{36}$ **(C)** $\frac{7}{9}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{5}{12}$

27. The *Blow Upp* balloon company package 6 balloons per pack. The company has red, blue, white, pink, yellow, green, and magenta colored balloons. How many different packs of 6 balloons can they package?

(A) 12,012 **(B) 5,040** **(C) 924** **(D) 720** **(E) 42**

28. Leonardo Pisano Bigollo was an Italian mathematician who referenced and made known which of the following special sequences of numbers to Western mathematics?

(A) {1,1,2,3,5,8,...} **(B) {1,3,6,10,15,21,...}** **(C) {0,2,4,6,8,10,...}**
(D) {2,3,5,7,11,13,...} **(E) {1,5,25,125,625,3125,...}**

29. Which of the following numbers is an abundant, happy, and lucky number?

(A) 28 **(B) 31** **(C) 44** **(D) all of these** **(E) none of these**

30. Find k if GCF(48, k) = 8 and LCM(48, k) = 336.

(A) 56 **(B) 64** **(C) 72** **(D) 80** **(E) 88**

31. $\{(x, y) \mid x,y \in \{\text{Integers}\}, -10 \le x \le 10, \text{ and } -10 \le y \le 10\}$ is the solution set of $2x + 5y = 10$. How many such ordered pairs exist?

(A) 2 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

32. Which of the following points of concurrency are always on the exterior of an obtuse triangle?
(1) circumcenter (2) centroid (3) incenter (4) orthocenter

(A) 1 only (B) 1, 3, & 4 (C) 1 & 4 (D) 2 & 3 (E) 4 only

33. An elongated square pyramid is a nonahedron. It has 9 faces and 9 vertices. How many edges does it have?

(A) 18 (B) 16 (C) 20 (D) 9 (E) 11

34. Find C if the remainder when $(3x^3 + 2x^2 - x + C) \div (x + 1)$ is 4.

(A) -1 (B) 0 (C) 1 (D) 2 (E) 4

35. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, 2, b, c, 20, d, 51, ...

(A) 58 (B) 80 (C) 52 (D) 47 (E) 73

36. Given the function f(x) = sin x, find the slope of the secant line between x = 0 and $x = \frac{\pi}{2}$.

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{2}{\pi}$ (D) π (E) no slope

37. Ester Bunnee had a box of chocolate eggs. She hid half of them in the yard for the big hunt. Then she put two of the remaining eggs in her room for a late night snack. The remaining six eggs were put in the refrigerator for a later day. How many chocolate eggs were in the original box?

(A) 32 (B) 28 (C) 24 (D) 20 (E) 16

38. Sir Benjamin Hall was looking at the circular face of the famous *Big Ben* clock. He noted that the time was 5:43 pm. What was the measure of the acute angle formed by the big hand and the little hand at that time?

(A) 65° (B) 75.5° (C) 85° (D) 86.5° (E) 89.5°

39. Chip Shought hit his golf ball over a pond onto the edge of the green. He had to walk around the pond to his ball. He walked 70 yards on a bearing of 250° from the tee. Then he walked 90 yards on a bearing of 50° to his ball. What was the straight line distance from the tee to his ball? (nearest yard)

(A) 29 yds (B) 34 yds (C) 30 yds (D) 35 yds (E) 44 yds

40. I. C. Itt spotted a plane flying over his house. He noted that the angle of elevation from him to the plane was 32°40' and he was 1,530 meters from his house. Using this information I. C. was able to determine the altitude of the plane. What was the altitude of the plane? (nearest meter)

(A) 2,386 meters (B) 1,287 meters (C) 981 meters (D) 971 meters (E) 826 meters

41. Mei Chado is 5' 4" tall. She is walking at a rate of 3 ft/sec toward a street light that is 16 feet tall. At what rate is the tip of her shadow moving? (nearest tenth)

(A) 4.5 ft/sec (B) 4.0 ft/sec (C) 3.5 ft/sec (D) 3.0 ft/sec (E) 1.5 ft/sec

42. 14 out of 17 Millersviewites have spouses. 4 out of 6 Millerviewites own at least 3 acres and a travel trailer. What is the probability that a Millersviewite has a travel trailer given that a Millersviewite has a spouse? (nearest whole percent)

(A) 83% (B) 81% (C) 78% (D) 67% (E) 24%

43. Anthony and Chuck take three number sense tests. Anthony is twice as likely to score higher than Chuck. What are the odds that Anthony scores higher on all three tests? Due to an unknown tiebreaker, there are no ties.

(A) $\frac{8}{11}$ (B) $\frac{11}{27}$ (C) $\frac{8}{19}$ (D) $\frac{1}{3}$ (E) $\frac{1}{9}$

44. If $12x^2 + ax - 5 = (bx - 5)(2x + c)$ then abc = _____.

(A) 11 (B) 3 (C) 1 (D) — 15 (E) — 24

45. Let $e^{(2x-3)} = 4e^{(5x+6)}$. Find $e^{(x)}$. (nearest hundredth)

(A) .03 (B) 0.22 (C) 0.42 (D) — 2.23 (E) — 3.46

46. The set of Lucas numbers is {1,3,4,7,11, ...}, where $L_1 = 1$. The set of Fibonacci numbers is {1,1,2,3,5, ...}, where $F_1 = 1 = F_2$. If $L_{10} = F_x + F_y$, where $y > x$, then y is _____.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

47. Let f(x) = ax + 4 and g(x) = bx — 1, where a and b are positive integers. Find a + b if f(g(x)) = g(f(x)).

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

48. Let $f(x) = 4x^2 - 4x + 1$. The tangent to f(x) at (x, y) is parallel to y = 4x — 2. Find x + y.

(A) 4 (B) 2 (C) 1 (D) 0 (E) — 1

49. In honor of Valentines day, let $x = 2 + \cfrac{14}{2 + \cfrac{14}{2 + \cfrac{14}{2 + \cfrac{14}{2 + \ldots}}}}$. Find x. (nearest tenth)

(A) 4.9 (B) 4.7 (C) 3.9 (D) 2.7 (E) 2.1

50. If the following patten continues, determine which of the following numbers will be in row 10.

1	row 0
1 1	row 1
1 2 1	row 2
1 3 3 1	row 3
1 4 6 4 1	row 4
1 5 10 10 5 1	row 5
...	...

(A) 84 (B) 110 (C) 126 (D) 215 (E) 252

51. The fraction $\frac{30}{\sqrt{3}+\sqrt{5}+\sqrt{8}}$ can be written as $a\sqrt{30}+b\sqrt{3}+c\sqrt{5}+d\sqrt{8}$. Find $a+b+c+d$.

(A) 15 (B) 10 (C) 8 (D) 6 (E) 3

52. Let $f(x)=\sqrt{6-\sqrt{2x+7}}$. The domain of f(x) is $\{x \mid p \le x \le q\}$. Find $\frac{P+Q}{2}$.

(A) -3.5 (B) -3 (C) 5.5 (D) 6 (E) 9

53. Points P (— 1, 1), Q(3, 5), R(17, 1), and S(x, y) are the coordinates of the vertices of a parallelogram. How many possible coordinates of S exist for the fourth vertex?

(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

54. Given: $9x-6y=21$ and $6x-4y=k$. Find the value of k such that this system of equations has an infinite number of solutions.

(A) 31.5 (B) 24 (C) 21 (D) 14 (E) 13.5

55. If x is in QIII then $\frac{1-\cos(2x)}{\sin(2x)} = \tan kx$ and k equals:

(A) $1\frac{1}{2}$ (B) 1 (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

56. An "*emirp*" number is a prime number that becomes a new prime number when the digits are reversed. Single digit primes and palindromic primes cannot be *emirp* numbers. How many prime numbers less than 20 are considered to be *emirp* numbers?

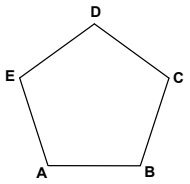
(A) 8 (B) 4 (C) 3 (D) 2 (E) 0

57. Let $f(x)=\begin{cases} -x+5 & x<-2 \\ x^2+1 & -2 \le x \text{ and } x \le 1 \\ 2x^3-1 & 1 \le x \end{cases}$. Which of the following is/are true?

1. f is continuous at — 2 2. f is differentiable at x = 1 3. f has a local minimum at x = 0

(A) 1 & 3 (B) 2 & 3 (C) 2 only (D) 3 only (E) 1, 2, & 3

58. Given the regular pentagon shown, find BC with AC + AD + BE + BD + CE = 44.5". (nearest tenth)



(A) 8.9" **(B) 5.2"** **(C) 8.1"** **(D) 4.5"** **(E) 5.5"**

59. Let $(131_b) \times 3_b = k_b$, where k_b is a 3-digit number. Find b if $k_b = 1323_4$.

(A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

60. If P, Q, and R are different digits, then the largest possible three-digit sum for PPP + QP + P = ? has which of the following forms?

(A) PPQ **(B) PQR** **(C) QQP** **(D) QQR** **(E) RRQ**

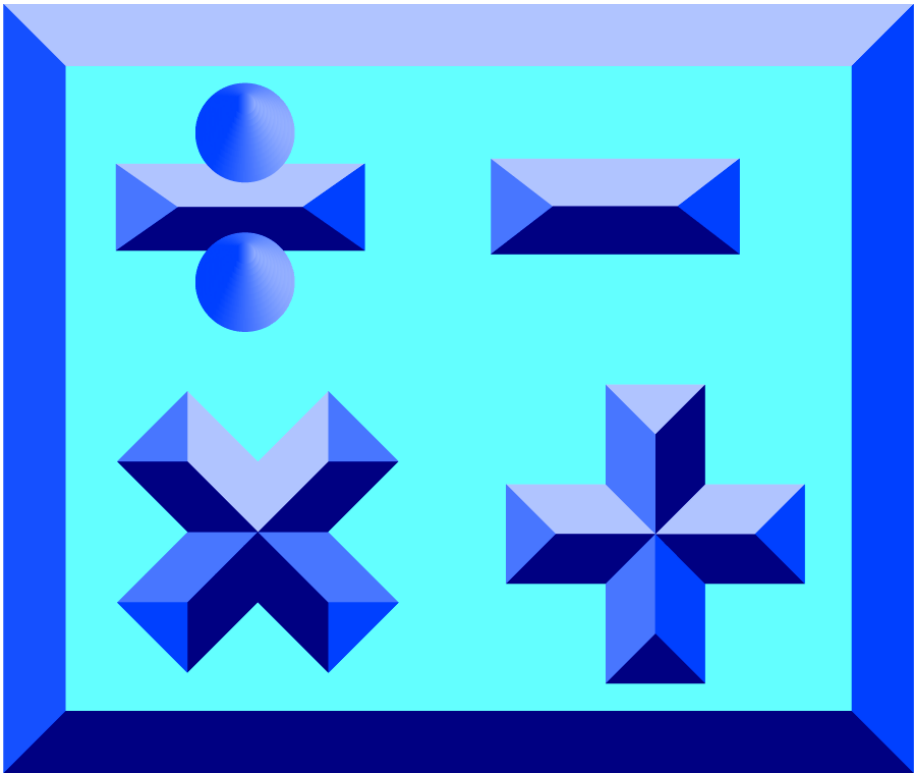
University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2018
Answer Key

1. B	**21. C**	**41. A**
2. D	**22. E**	**42. B**
3. A	**23. A**	**43. C**
4. C	**24. B**	**44. E**
5. B	**25. D**	**45. A**
6. A	**26. E**	**46. D**
7. B	**27. C**	**47. D**
8. E	**28. A**	**48. B**
9. A	**29. E**	**49. A**
10. E	**30. A**	**50. E**
11. C	**31. D**	**51. D**
12. E	**32. C**	**52. C**
13. E	**33. B**	**53. C**
14. B	**34. E**	**54. D**
15. A	**35. A**	**55. B**
16. D	**36. C**	**56. D**
17. A	**37. E**	**57. D**
18. A	**38. D**	**58. E**
19. E	**39. B**	**59. B**
20. E	**40. C**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2018



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $1 - (1 + 2^3 - 5) \div 8 \times (1 - 3^2) + 1$

   (A) -3 **(B)** 0 **(C)** 3.25 **(D)** 4 **(E)** 6

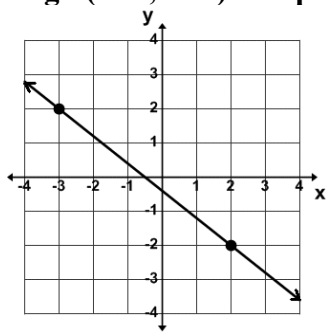
2. **Three million nine hundred thousand nine hundred twenty is added to one million eight thousand three hundred twenty four. The sum is multiplied by eleven. The digits in the product are added together. What is the sum of the digits?**

   (A) 28 **(B)** 30 **(C)** 32 **(D)** 38 **(E)** 41

3. **Les Tred is shopping for a new set of 4 tires at the local tire store. The regular price is $64.98. He can buy the 1st tire at the regular price. The 2nd tire is half off the regular price. The 3rd tire is discounted $33\frac{1}{3}$%. And, $10.98 is taken off the regular price for the 4th tire. What would it cost Les for the 4 tires before taxes? (nearest cent)**

   (A) \$194.79 **(B)** \$183.61 **(C)** \$173.13 **(D)** \$184.98 **(E)** \$195.77

4. **Find an equation of the line through $(-1, -3)$ and perpendicular to the line shown.**



   (A) $4x + 5y = -7$ **(B)** $5x - 4y = 7$ **(C)** $4x + 5y = -11$
   (D) $5x - 4y = -7$ **(E)** $5x + 4y = -17$

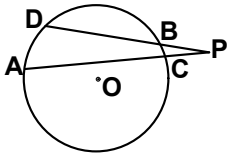
5. **Simplify:** $\left(\frac{2x^2 + 7x + 3}{x^2 - 9}\right)\left(\frac{x^2 - 3x}{2x^2 + 11x + 5}\right)$

   (A) $\frac{x}{x+5}$ **(B)** $\frac{x+3}{x-4}$ **(C)** $x^2 + 5x$ **(D)** $\frac{4x+3}{11x-4}$ **(E)** $\frac{x^2}{x^2+5}$

6. **Penni Les has 4 times as many dimes as nickels and half as many pennies as dimes. She has $4.70. How much would she have left if she spent all of her nickels?**

   (A) \$0.50 **(B)** \$0.70 **(C)** \$4.00 **(D)** \$4.20 **(E)** \$4.50

7. **Given the circle with center O shown with DP = 10 cm, BP = 3 cm, and AP = 12 cm. Find AC.**



   (A) 7.2 cm **(B)** 8.4 cm **(C)** 9.5 cm **(D)** 10.5 **(E)** 10.8

8. **Find the sum of the measure of an exterior angle of a regular pentagon, the measure of a central angle of a regular hexagon, and the measure of an interior angle of a regular heptagon. (nearest degree)**

(A) 261° (B) 180° (C) 297° (D) 183° (E) 321°

9. **Les Space increased the length of two opposite sides of a rectangle by 40%, and decreased the other two opposite sides by 10%. What percent of the area of the original rectangle is the area of the new rectangle?**

(A) 74% (B) 50% (C) 230% (D) 4% (E) 126%

10. **If $\frac{Ax+B}{4x+1} - \frac{2x+3}{3x-2} = \frac{7x^2-36x+5}{12x^2-5x-2}$, where A and B are constants, then A — B equals:**

(A) 1 (B) 4 (C) 5 (D) 9 (E) 10

11. **The graph of $x^2 + y^2 - 10x + 12y + 57 = 0$ is a circle with a center (h, k) and a radius r. Find $h \times k - r$.**

(A) — 32 (B) — 26 (C) — 3 (D) 46 (E) 55

12. **If $8^{(k+1)} = 16^{(k-1)}$, then $2^{(k)} = ?$**

(A) 512 (B) 128 (C) 64 (D) 1,024 (E) 4

13. **Determine the range of $f(x) = 4\sin(3x - \pi) - 2$.**

(A) [— 2, 6] (B) [4, — 2] (C) [— 1, 3] (D) [— 6, 2] (E) [3, — 1]

14. **Which of the following is an identity for $\frac{\csc\theta - \cot\theta}{1 - \cos\theta}$?**

(A) $\csc\theta$ (B) $\cot\theta$ (C) $\cos\theta$ (D) $\sec\theta$ (E) $\tan\theta$

15. **Given: $f(x) = 5\cos(2x - 1) - 3$. What quadrant(s) would the graph of f(x) be in if the amplitude is decreased by 2, the vertical displacement was increased by 2 and the phase shift was multiplied by 2?**

(A) I & II (B) I & IV (C) II & III (D) III & IV (E) I, II, III, & IV

16. **In the expansion of $(2x + 1)^6$, the sum of the coefficients of the 2^{nd}, 3^{rd}, 5^{th} and 6^{th} term is:**

(A) 2,688 (B) 524 (C) 672 (D) 504 (E) 1,344

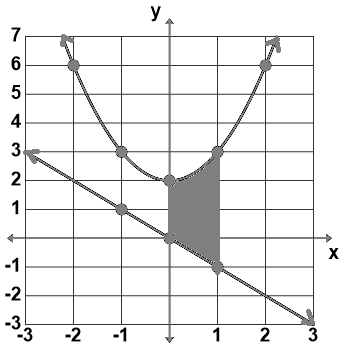
17. **Find $a + b + c + d$ given the Fibonacci characteristic sequence: 2, a, b, 12, c, d, 50, ...**

(A) 56 (B) 57 (C) 59 (D) 62 (E) 64

18. The 4th term of a geometric sequence is $\frac{1}{8}$. The 7th term is $\frac{1}{64}$. Find the sum of the first 6 terms of this geometric sequence.

(A) $\frac{63}{64}$ **(B)** $\frac{31}{32}$ **(C)** $1\frac{63}{64}$ **(D)** $1\frac{15}{16}$ **(E)** $1\frac{31}{32}$

19. Find the area of the shaded region.



(A) 2.666... **(B)** 2.75 **(C)** 2.8333... **(D)** 2.875 **(E)** 3

20. Let $f(x) = \frac{x^2 - 4x - 5}{x + 1}$. A *removable discontinuity* exists at x = ?

(A) -4 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 5

21. Let $f(x) = 3x^2 - 4x - 5$ and $g(x) = 4x + 5$. Find $g(f'(1))$

(A) 13 **(B)** 2 **(C)** -3 **(D)** -19 **(E)** 27

22. Al Fahbett randomly selected a letter from the set {L, E, T, T, E, R}. What are the odds that he selected E?

(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{1}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{3}$

23. The Millersview Dunkers have 4 centers, 6 guards, and 7 forwards. How many different teams consisting of 1 guard, 2 forwards, and 2 centers could be formed?

(A) 6,188 **(B)** 756 **(C)** 1,237 **(D)** 126 **(E)** 1,260

24. Given the equation: $4^x = 7$. Which of the following mathematicians would be the best one to ask for help to solve for x?

(A) Aryabhata **(B)** Charles Babbage **(C)** John Napier **(D)** George Boole **(E)** Alan Turing

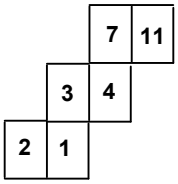
25. If $x^2 - 3x + b = (x + a)(x - 7)$, where a and b are integers then $a + b =$ _____.

(A) -34 **(B)** -28 **(C)** -24 **(D)** 32 **(E)** 36

26. If $x + y = -3$ and $xy = 6$ then $x^3 + y^3 = ?$

(A) -19 **(B)** -18 **(C)** 21 **(D)** 27 **(E)** 33

27. Poly Gawn folds the net shown into a cube. She lets the face with the 11 on it be the base of the cube. What is the sum of the numbers on the lateral faces?



(A) 10 (B) 11 (C) 13 (D) 14 (E) 16

28. How many non-negative proper fractions in lowest terms have a denominator of 24?

(A) 12 (B) 11 (C) 10 (D) 9 (E) 8

29. If $a_1 = -3$, $a_2 = -2$, $a_3 = 1$ and $a_n = [(a_{n-1}) + (a_{n-3})] \times (a_{n-2})$ for $n \geq 4$, then a_6 equals:

(A) 32 (B) 12 (C) 6 (D) 4 (E) 2

30. Find the sum of the x-values in $\{x \mid \cos(2x) + \sin(x) = 0, x \in [0, 2\pi)\}$? (nearest tenth)

(A) 12.6 (B) 11.0 (C) 9.4 (D) 7.3 (E) 5.2

31. How many asymptotes does $f(x) = \frac{x^2 + 4x + 3}{x + 2}$ have?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

32. A function, $g(x) = x^2 + bx + c$, exists such that $g(1) = 2$ and $g(2) + g(3) = 7$. Find $g(-4)$.

(A) $20\frac{3}{4}$ (B) $31\frac{2}{3}$ (C) 21 (D) 25 (E) $30\frac{1}{3}$

33. Given the function $f(x) = 2\cos(x) + 1$, find the slope of the secant line between $x = 0$ and $x = \frac{\pi}{2}$.

(A) 0 (B) -2π (C) $\frac{2}{\pi}$ (D) $-\frac{4}{\pi}$ (E) no slope

34. Find the 20^{th} tetrahedral number.

(A) 35 (B) 210 (C) 1,330 (D) 1,540 (E) 1,771

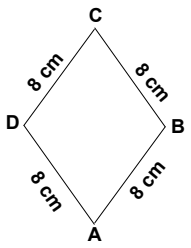
35. The fraction $\frac{11}{14}$ base 6 can be written as which of the following decimals in base 6?

(A) $0.333..._6$ (B) $0.1444..._6$ (C) $0.3222..._6$ (D) $0.1414..._6$ (E) $0.4111..._6$

36. Let $424_b + 332_b - 241_b = 1020_b$. Find 113_b in base 10.

(A) 45 (B) 33 (C) 59 (D) 81 (E) 41

37. Find the area of the rhombus shown given that AC — BD = 2 cm,



(A) 32 cm^2 (B) 15.75 cm^2 (C) 64 cm^2 (D) 72 cm^2 (E) 63 cm^2

38. The sum of the digits of a certain three-digit number is 18. The sum of the hundreds digit and the tens digit is 10. And, the tens digit is one less than half the units digit. How many of the digits are prime numbers?

(A) 0 (B) 1 (C) 2 (D) 3 (E) not enough information

39. The average of Seymore Anser's first five quiz grades is 87. He made 75 on his sixth quiz. What does he have to make on his seventh quiz to have a quiz average of 84?

(A) 68 (B) 75 (C) 78 (D) 85 (E) 88

40. Jose Canyusee is standing 20 feet from the base of a flag pole. The angle of depression from his eyes to the base of the pole is 16°. The angle of elevation from his eyes to the top of the pole is 60°. What is the height of the flag pole? (nearest foot)

(A) 104 ft (B) 81 ft (C) 40 ft (D) 52 ft (E) 80 ft

41. Every morning, Johnny Jogger covers 9 miles on a trail near Lake Ray Roberts. He walks the first 3 miles at a speed of 4 mph, he runs the next 3 miles at a speed of 7 mph, and he jogs the last 3 miles at a speed of 5 mph. Find the mean speed for his 9 mile trek. (nearest tenth)

(A) 5.1 mph (B) 5.2 mph (C) 5.3 mph (D) 5.4 mph (E) 5.5 mph

42. Twenty class 2A seniors took the TMSCA State math test this year. Twelve of them were boys and eight were girls. All of them had an equal chance to win one of the top three trophies. What was the probability that all three of the top trophies were won by girls? (nearest whole percent)

(A) 5% (B) 7% (C) 13% (D) 19% (E) 25%

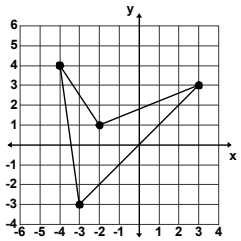
43. Cookie Baykur baked chocolate chip cookies, peanut butter cookies, raisin cookies, and snickerdoodles. She put six cookies per zip lock bag to sell at the bake sale. How many different bags of 6 cookies could she make?

(A) 126 (B) 24 (C) 696 (D) 90 (E) 84

44. How many distinct combinations exist for a 4-digit combination padlock so that the first digit is a prime number, the second digit is a factor of 10, the third digit is a positive Fibonacci number, and the fourth digit is divisible by 5?

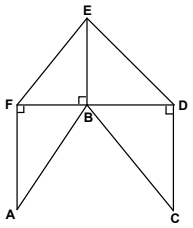
(A) 14 **(B) 60** **(C) 80** **(D) 120** **(E) 160**

45. Rene Dezkartez drew the quadrilateral shown, whose vertices are integers. What is the area of Rene's quadrilateral?



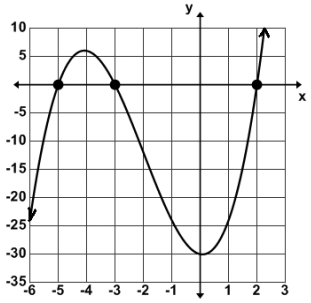
(A) 15 units^2 **(B) 14.5 units^2** **(C) 14 units^2** **(D) 13.5 units^2** **(E) 13 units^2**

46. Given: m∠BED = 45°, m∠ABF = 30°, m∠EFB = 60°, m∠BCD = 45°,and EF = 4". Find the perimeter of pentagon ABDEF. (nearest tenth).



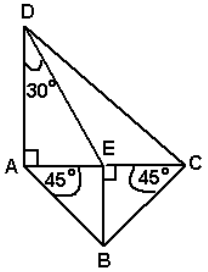
(A) 12.0" **(B) 12.9"** **(C) 15.8"** **(D) 20.2"** **(E) 24.2"**

47. The graph of $f(x) = Ax^3 + Bx^2 + Cx + D$ is shown here. Find $A + B + C + D$.



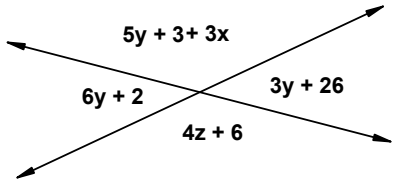
(A) 37 **(B) 6** **(C) — 23** **(D) — 24** **(E) — 37**

48. Find the perimeter of the quadrilateral ABCD if AE = 4". (nearest inch)



(A) 24 in **(B) 25 in** **(C) 29 in** **(D) 31 in** **(E) 32 in**

49. Find the sum of x, y, and z, given the degree measures of the angles shown.



(A) 29 (B) 50 (C) 68 (D) 70 (E) 80

50. Write this expression as a simplified proper fraction. $0 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{5 + \frac{1}{7}}}}$

(A) $\frac{37}{86}$ **(B)** $\frac{115}{266}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{58}{133}$ **(E)** $\frac{36}{115}$

51. $\left\{(x, y) \middle| x,y \in \{\text{Integers}\}, -7 \leq x \leq 11, \text{and} -11 \leq y \leq 7\right\}$ **is the solution set of $3x - 2y = 5$. How many such ordered pairs exist?**

(A) 5 (B) 6 (C) 7 (D) 10 (E) 11

52. How many ordered pairs of positive integers (a, b) with $a + b \leq 91$, satisfy the equation: $(a + b^{-1}) \div (a^{-1} + b) = 19$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

53. Points P (0, 5), Q(4, — 7), R(7, — 3), and S(x, y) are the coordinates of the vertices of a parallelogram, where S(x,y) is in quadrant II. Find $x + y$.

(A) — 2 (B) — 1 (C) 2 (D) 4 (E) 6

54. Let function f = {(1, 1), (3, 4), (2, 5)} and function g = {(3, 1), (1, 3), (2, 2)}. Which of the following is a member of the function f o g ?

(A) (1, 1) (B) (3, 3) (C) (2, 1) (D) (3, 1) (E) (3, 4)

55. P, Q, & R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, & R is — 9 and C is — 16. Find D.

(A) — 72 (B) — 48 (C) — 37 (D) — 32 (E) — 25

56. Let $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$ **, for all real numbers x. Which of the following must be true?**

I. f(x) is continuous everywhere.
II. f(x) is differentiable everywhere
III. f(x) has a local minimum at x = 1

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, and III

57. Which of the following polar equations will produce the graph of a lemniscate that is symmetric to the polar axis?

(A) $r^2 = 2\sin(4\theta)$ **(B)** $r = 2\cos(\theta)$ **(C)** $r^2 = \sin(\theta)$ **(D)** $r = 4\cos(\theta)$ **(E)** $r^2 = 4\cos(2\theta)$

58. Pennie Flipper is going to toss a fair penny 6 times. What is the probability that she will get at least two tails? (nearest whole percent)

(A) 67% **(B) 64%** **(C) 89%** **(D) 28%** **(E) 25%**

59. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the numbers in row 10.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 1,729 **(B) 2,030** **(C) 1,638** **(D) 1,748** **(E) 1820**

60. Let $f(x) = (2x - 1)^2$. The tangent to f(x) at (x, y) is perpendicular to $x = 4 - 2y$. Find $x + y$.

(A) 3.25 **(B) 2.375** **(C) 2** **(D) 1** **(E) 0**

DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST

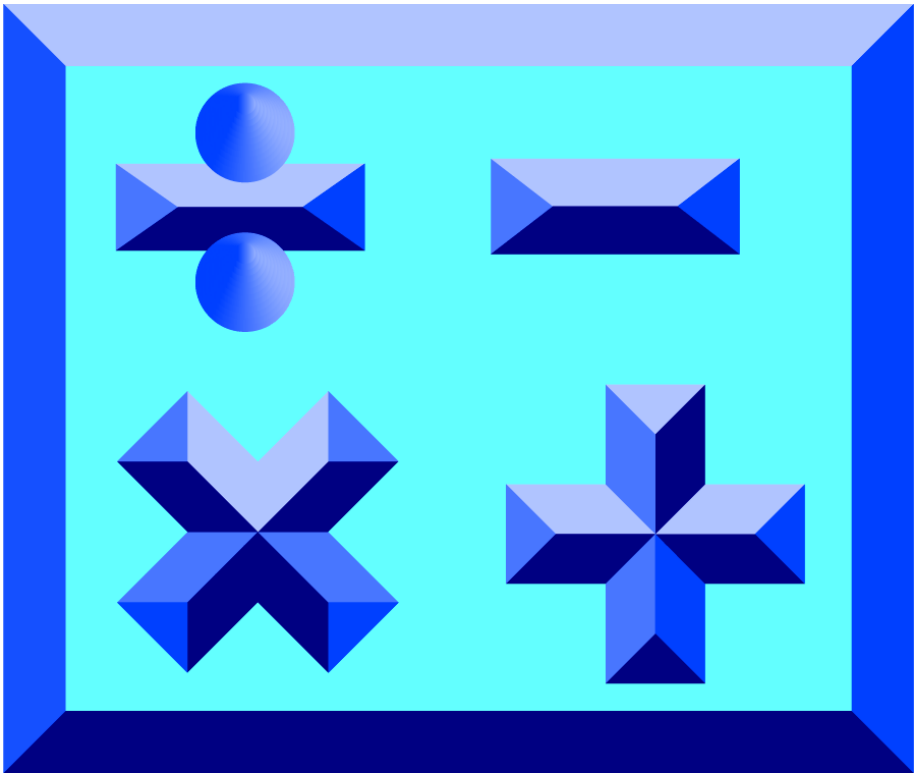
University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2018
Answer Key

1. E	**21. A**	**41. A**
2. A	**22. A**	**42. A**
3. A	**23. B**	**43. E**
4. B	**24. C**	**44. D**
5. A	**25. C**	**45. B**
6. D	**26. D**	**46. C**
7. C	**27. D**	**47. D**
8. A	**28. E**	**48. C**
9. E	**29. B**	**49. C**
10. D	**30. B**	**50. B**
11. A	**31. C**	**51. B**
12. B	**32. E**	**52. D**
13. D	**33. D**	**53. A**
14. A	**34. D**	**54. D**
15. E	**35. E**	**55. B**
16. D	**36. B**	**56. B**
17. D	**37. E**	**57. E**
18. E	**38. C**	**58. C**
19. C	**39. C**	**59. A**
20. B	**40. C**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2018



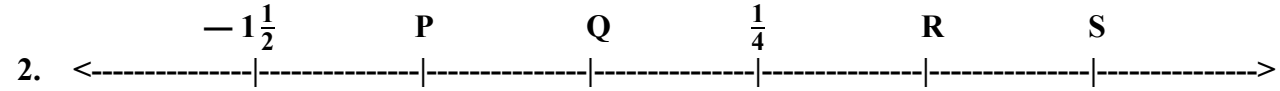
DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $5! \div (4)^3 + (2 - 1)^{\frac{2}{3}} - 4 \times 5$

(A) -18.203125 **(B)** -17.125 **(C)** -9 **(D)** -5.625 **(E)** 35

2.



The distances between the hash marks (|) are equal. Find $P + Q + R - S$.

(A) $-3\frac{1}{12}$ **(B)** 1 **(C)** $-1\frac{1}{4}$ **(D)** $\frac{7}{12}$ **(E)** $-1\frac{5}{6}$

3. **The universal set U = {r,e,g,i,o,n,a,l,s}. Subset A = {s,i,g,n,a,l} and subset B = {l,e,g,i,o,n}. How many elements are in the complement set of $A \cap B$?**

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

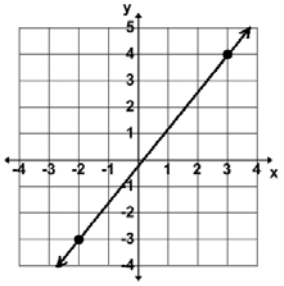
4. **Little Richie bid on 4 old HP11C calculators at the EZPay auction site. He got two of the calculators each at half the original price, one at 20% off the original price, and got the last one for $40.00 less than the original price. How much did he pay for the 4 calculators if the original price of an HP11C was $89.99? (tax not included) (nearest cent)**

(A) $201.98 **(B)** $211.97 **(C)** $157.98 **(D)** $152.98 **(E)** $215.97

5. **If** $(4x - 1)(3x + 2)(5 - x) = ax^3 + bx^2 + cx + d$ **then** $a + b + c + d =$ _____.

(A) 78 **(B)** 70 **(C)** 60 **(D)** 40 **(E)** 6

6. **Find an equation of the line shown.**



(A) $5x - 7y = 11$ **(B)** $7x - 5y = 1$ **(C)** $x - y = 1$ **(D)** $7x - 5y = 13$ **(E)** $x - 5y = 13$

7. **A line contains the points $(-3, 5)$ and $(6, -1)$. The line intersects the y-axis at y = ?**

(A) -2 **(B)** 0 **(C)** 3 **(D)** 3.5 **(E)** 5.5

8. **The line $4x - 2y = 7$ is perpendicular to the line $ax + 2y = 3$. The point of intersection is (p, q). Find $p + q$.**

(A) -1.5 **(B)** -1 **(C)** 2 **(D)** 2.5 **(E)** 3

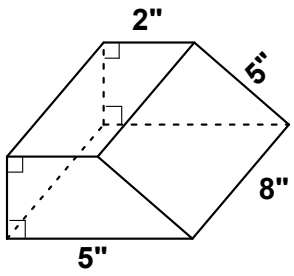
9. **How many integers are NOT in the solution set for** $|2x - 5| \geq 3$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

10. Papa John is sitting on one side of a seesaw and his son and daughter are sitting on the other side. He weighs 200 lbs, his son weighs 75 lbs, and his daughter weighs 50 lbs. How far from the fulcrum will Papa John have to sit if his son is sitting 5 feet from the fulcrum and his daughter is sitting 4 feet from the fulcrum in order to balance the seesaw?

(A) 2' 10.5" **(B) 2' 9.75"** **(C) 2' 9"** **(D) 2' 7.5"** **(E) 2' 5"**

11. Find the volume of the trapezoidal prism shown. (nearest cu. in)



(A) 112 cu. in **(B) 160 cu. in** **(C) 106 cu. in** **(D) 80 cu. in** **(E) 200 cu. in**

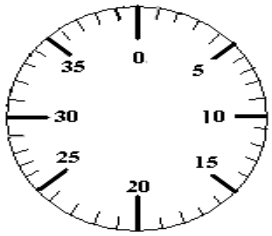
12. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, b, 9, c, d, 31, ...

(A) 31 **(B) 40** **(C) 50** **(D) 62** **(E) 80**

13. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \ldots + \frac{1}{120} + \frac{1}{136} = ?$

(A) $\frac{15}{17}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{13}{15}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{9}{10}$

14. A (right) - B (left) - C (right) is the combination needed to open the safe with the dial shown below. How many distinct combinations exist if A is a positive multiple of 4, B is a factor of 38, and C is a nonnegative power of 3.



(A) 180 **(B) 144** **(C) 17** **(D) 108** **(E) 200**

15. If two dice are tossed, what are the odds that the sum of the faces is 7 or an 11?

(A) $\frac{1}{2}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{7}{29}$ **(E)** $\frac{2}{9}$

16. Which of the following is/are Aryabhata's contribution(s)?

I. concept of sine called a "half chord"
II. a place value system using letters to represent numbers
III. an early approximation of pi

(A) I only **(B) I & II** **(C) II & III** **(D) I, II, & III** **(E) none of them**

17. How many positive proper fractions in lowest terms have a denominator of 18?

(A) 32 (B) 5 (C) 6 (D) 7 (E) 9

18. If the roots of $2x^3 + bx^2 + cx + d = 0$ are -2, 1, and 4, then $b + c + d$ equals:

(A) -2 (B) -1 (C) 1 (D) 3 (E) 5

19. Les Area cut off 30% of the length of his rectangle and 40% off the width of his rectangle. What percent of the area of his original rectangle is the area of his new rectangle?

(A) 12% (B) 58% (C) 88% (D) 35% (E) 42%

20. If $\frac{x+5}{Ax+B} - \frac{2x-3}{4x+1} = \frac{2x^2+36x-13}{4x^2-23x-6}$, where A and B are constants, then $A + B$ equals:

(A) -5 (B) -3 (C) 1 (D) 6 (E) 7

21. Which of the following is a looped limacon?

(A) $r = 2 + \cos(\theta)$ (B) $r = 1.5 + \cos(\theta)$ (C) $r = 1 + \cos(\theta)$
(D) $r = 0.5 + \cos(\theta)$ (E) $r = \cos(\theta)$

22. Determine the range of $f(x) = 3\sin(2x + 1) - 4$.

(A) $[-7, -1]$ (B) $[-6.5, -2.5]$ (C) $[-8, -2]$ (D) $[-6, 0]$ (E) $[-8, 1]$

23. Find $P + Q$ if $\begin{bmatrix} -1 & 3 \\ 6 & -10 \end{bmatrix} \cdot \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

24. Let $f(x) = 3x^2 - 4x - 5$ and $g(x) = 2x^2 + x$. Find $g'(f'(-1))$

(A) -39 (B) -22 (C) 7 (D) 10 (E) 210

25. The function $f(x) = x^3 - 6x^2 + 12x - 7$ has an inflection point at (x, y). Find $x + y$.

(A) -2 (B) -1 (C) 1 (D) 3 (E) 4

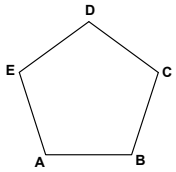
26. Let set P = {2, 1, 3, 4, 7}, set Q = {1, 3, 6, 10, 15}, and set R = {2, 3, 4, 5, 6, 7}. Exactly one digit is chosen from each set. What is the probability of picking at least one prime number? (nearest %)

(A) 50% (B) 53% (C) 75% (D) 83% (E) 89%

27. The ordered pair (x, y) are the fangs of the vampire number, 1395. Find $x + y$.

(A) 18 (B) 36 (C) 54 (D) 72 (E) 108

28. Given the regular pentagon shown, find the length of a side with AC + AD + BE + BD + CE = 72". (nearest tenth)



(A) 8.9" (B) 4.7" (C) 6.4" (D) 9.1" (E) 8.1"

29. Lotta Bucks had a pack of one-dollar bills. She put 25% of them in her safe for future spending. Then she put 20% of what was left in an envelope to send to her brother. Then she gave her sister five one-dollar bills. How many dollar bills did she put in the envelope for her brother if she had 7 one-dollar bills left?

(A) 3 (B) 5 (C) 7 (D) 12 (E) 15

30. Tu Yung is three years younger than her cousin Soh Ohd. In two years the sum of their ages will be twenty-one. What will Tu Yung's age be in five years?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

31. Point P (3, 3) lies on the x-y plane. P is reflected across the origin to point Q. Point Q is translated vertically + 4 units to point R. Point R is reflected across the y-axis to point S. Point S is translated horizontally — 4 units to point T (x, y). Find x + y

(A) — 14 (B) — 8 (C) 0 (D) 6 (E) 12

32. A large pipe can fill an empty tank in 3 hours. A smaller pipe can fill the same empty tank in 5 hours. The drain pipe in the tank can empty the full tank in 4 hours. How long would it take to fill the tank if both of the fill pipes and the drain tank are open? (nearest minute)

(A) 1 hr 3 min (B) 2 hrs 2 min (C) 3 hrs 3 min (D) 3 hrs 32 min (E) 4 hrs

33. The sum of the digits of a certain 3-digit number is 17. The sum of the squares of the digits is 109. The absolute value difference between the 3-digit number and 495 is the 3-digit number written in reverse. How many of the digits are divisible by 3?

(A) 0 (B) 1 (C) 2 (D) 3 (E) not enough information

34. Mary Goround rode the Ferris Wheel at the county fair. When Mary gets on the cart, the wheel is 3 ft off the ground. The diameter of the wheel is 40 ft. The ride makes 5 revolutions over a 2 minute time span. How high off the ground was Mary's cart after 40 seconds? (nearest foot)

(A) 40 ft (B) 31 ft (C) 30 ft (D) 35 ft (E) 33 ft

35. Betty Luzes invested some of her money in some mutual funds. Her investment earned 3% the first year, lost 6% the second year, and lost 5% the third year. What was her mean loss over the 3 year period? (nearest tenth)

(A) 2.1% (B) 2.4% (C) 2.7% (D) 2.8% (E) 3.1%

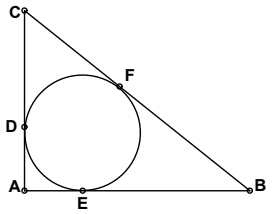
36. Rancher O. K. Corale needs to fence in a rectangular pen for his horses and mules. The pen is divided by a fence to separate the horses from the mules into two equal smaller pens. Find the maximum area of the pen that he can fence in if he has 300 feet of fencing. (nearest ft^2)

(A) 5,625 ft^2 (B) 3,750 ft^2 (C) 4,500 ft^2 (D) 3,450 ft^2 (E) 1,875 ft^2

37. Eight girls, twelve boys, four men coaches and six women coaches will be attending the Teckie math camp. A special council consisting of four girls, three boys, and two coaches, one man and one women will be formed. How many different councils are possible?

(A) 300 (B) 30,800 (C) 693,000 (D) 2,700 (E) 369,600

38. The circle shown is inscribed in △ABC. The radius of the circle is 29.5 cm and the perimeter of △ABC is 310 cm. Find the area of △ABC. (nearest cm^2)

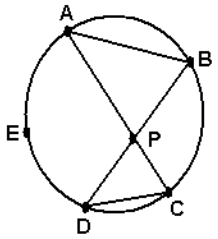


(A) 2,286 cm^2 (B) 3,048 cm^2 (C) 4,573 cm^2 (D) 6,097 cm^2 (E) not enough information

39. Find the digit in the units place of the integer 7^{654}.

(A) 1 (B) 3 (C) 7 (D) 9 (E) cannot be determined

40. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\overset{\frown}{AED}$ given $m\angle DPC = 80°$ and $m\angle PAB = 30°$.



(A) 160° (B) 130° (C) 150° (D) 140 ° (E) 110°

41. Solve for x: $\log_3(x + 10) = \log_3(x - 2) + \log_3(x)$.

(A) -2 (B) -1 (C) 2 (D) 3 (E) 5

42. Let $f(x) = 1 - x$ and $g(x) = 2x - 3$. Find $f\Big(g\big(-f(x)\big)\Big)$.

(A) $-2x$ (B) $2x - 6$ (C) $2x + 6$ (D) 6 (E) $6 - 2x$

43. If the following pattern continues, find the sum of the 3rd term and the 19th term in row 20.

1	row 0
1 1	row 1
1 2 1	row 2
1 3 3 1	row 3
1 4 6 4 1	row 4
1 5 10 10 5 1	row 5
...	...

(A) 272 (B) 306 (C) 342 (D) 380 (E) 420

44. Which of the following is an identity for $\frac{\cos(\theta)}{1 - \sin(\theta)} - \tan(\theta)$

(A) $\csc^2(\theta)$ (B) $\cot(\theta)$ (C) $\tan^2(\theta)$ (D) $\sec(\theta)$ (E) $\sec^2(\theta)$

45. Let $f(x) = |2x + 1| - |x - 3|$. Find the minimum value of f(x).

(A) -4 (B) -3.5 (C) -0.5 (D) 2.5 (E) 3

46. The area (in square units) of the region bounded by $x = y^2 - 2$ and $y = -x$ is:

(A) $2\frac{2}{3}$ (B) $3\frac{1}{4}$ (C) $3\frac{2}{3}$ (D) $4\frac{1}{3}$ (E) $4\frac{1}{2}$

47. A number N in base 5 is 159 in base 10. If $34P_5 + 2Q3_5 = N$ in base 5, where P and Q are digits, then $P + Q$ is:

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

48. The ratio of the measures of the non-adjacent angles of a quadrilateral is 2:1. The measure of the smallest acute angle is 55°. The sum of the two acute angles is 120°. Find the measure of the largest obtuse angle.

(A) 135° (B) 130° (C) 125° (D) 120° (E) 115°

49. Which of the following equations can be obtained from the graph of the parent function $y = \cos(x)$ by applying a vertical stretch of -2 units, a vertical shift of -3 units, and a phase shift of 5? y = :

(A) $-3\cos(x - 5) - 2$ (B) $2\cos(x + 5) + 3$ (C) $-2\cos(x + 5) - 3$
(D) $5\cos(x + 2) - 3$ (E) $-2\cos(x - 5) - 3$

50. How many composite numbers less than or equal to 55 and divisible by 5 are considered to be *square-free semiprimes*?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

51. Let $A + B = 24$ and $A \times B = 40$. Find $A - B$, where $A < B$.

(A) $4 - \sqrt{26}$ (B) $-4\sqrt{26}$ (C) 24 (D) 12 (E) $2 - \sqrt{26}$

52. Let $f(x) = \begin{cases} nx^4 + 5x & \text{if } x \leq 2 \\ mx^2 - 3x & \text{if } 2 < x \end{cases}$ be continuous and differentiable everywhere. Find m.

(A) 7.25 (B) 6 (C) 4 (D) 3.25 (E) 3.75

53. If the probability that a student in a remedial math class studies for an exam is 50%, and the probability that a student who doesn't study fails the test is 75%, then the probability that a student both passes the test and doesn't study is: (nearest whole percent)

(A) 63% (B) 38% (C) 25% (D) 13% (E) 7%

54. If $a_1 = -2$, $a_2 = -1$, $a_3 = 0$ and $a_n = [(a_{n-3}) - (a_{n-1})] \times (a_{n-2})$ for $n \geq 4$, then a_7 equals:

(A) -3 (B) 0 (C) 1 (D) 2 (E) 3

55. If you start at $(-\frac{5\pi}{4}, 0)$ on the x-axis and travel horizontally 24 radians to the right, how many times will you cross the graph of $y = 3\cos(2x)$?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

56. Let x and y be integers such that $xy(x - y) = 96$ and $xy + x - y = 22$. Calculate $x^2 + y^2$.

(A) 68 (B) 69 (C) 71 (D) 72 (E) 74

57. The focus of the parabola $y = -x^2 + 6x - 11$ is at (x, y). Find $x + y$.

(A) 0.75 (B) 3 (C) 5.25 (D) -0.25 (E) -1.75

58. Given the function $f(x) = x^3 - x$, find the slope of the secant line between $x = 1$ and $x = 5$.

(A) 15 (B) 24 (C) 30 (D) 40 (E) no slope

59. Let $x = 4 + \cfrac{14}{4 + \cfrac{14}{4 + \cfrac{14}{4 + \cfrac{14}{4 + \ldots}}}}$. Find x. (nearest tenth)

(A) 4.5 (B) 6.2 (C) 8.5 (D) 5.6 (E) 7.5

60. The figure shown is reflected over its positive diagonal. Then it is reflected over it vertical axis. Finally, it is rotated 180° counterclockwise. Which of the following figures is the result of these three transformations?

(A)  (B)  (C)  (D)  (E) 

DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST

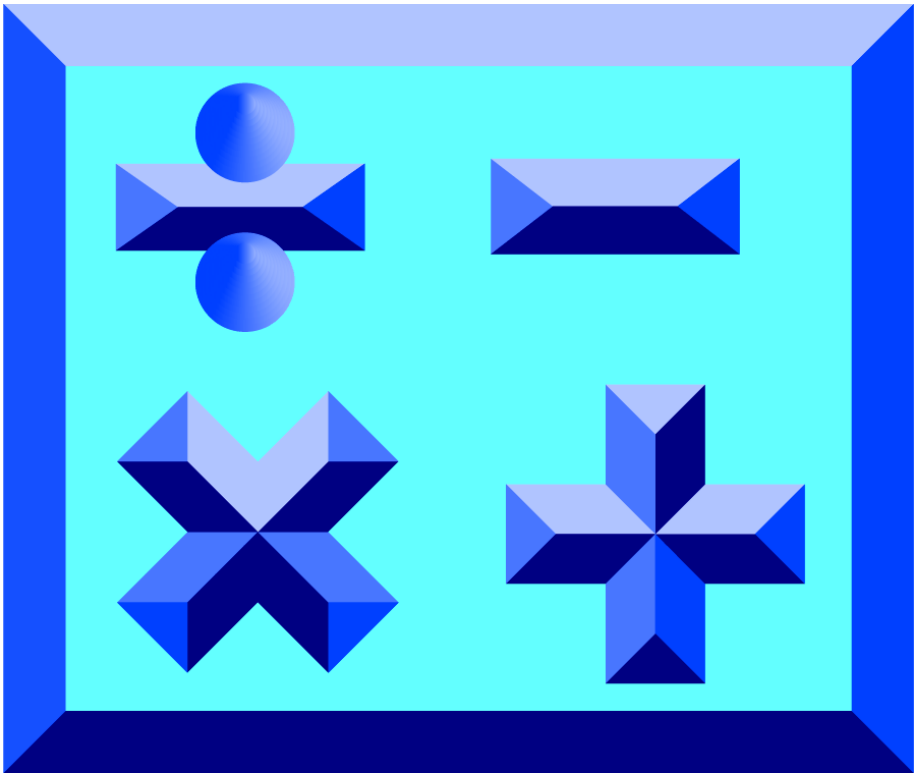
University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2018
Answer Key

1. B	**21. D**	**41. E**
2. E	**22. A**	**42. E**
3. D	**23. E**	**43. D**
4. B	**24. A**	**44. D**
5. C	**25. D**	**45. B**
6. B	**26. E**	**46. E**
7. C	**27. E**	**47. C**
8. D	**28. A**	**48. B**
9. C	**29. A**	**49. E**
10. A	**30. E**	**50. C**
11. A	**31. C**	**51. B**
12. B	**32. D**	**52. B**
13. A	**33. C**	**53. D**
14. B	**34. E**	**54. B**
15. C	**35. C**	**55. C**
16. D	**36. B**	**56. A**
17. C	**37. E**	**57. A**
18. A	**38. C**	**58. C**
19. E	**39. D**	**59. B**
20. A	**40. D**	**60. A**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2018



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $2 + 3 \times (5 - 7) \div (1 - 4) \times 6 + 8$

(A) 28 (B) 22 (C) $10\frac{1}{3}$ (D) -2 (E) -6

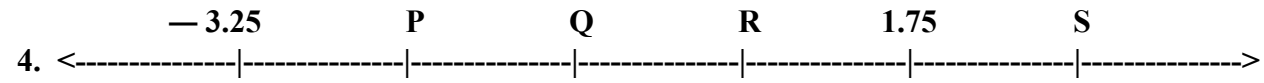
2. One million forty thousand six hundred eighty is subtracted from two million three hundred thousand five hundred seven. The difference is multiplied by nine. The digits in the product are added together. What is the sum of the digits?

(A) 9 (B) 16 (C) 18 (D) 22 (E) 27

3. Find the arithmetic mean of the median, the mode and the range of these numbers: 5, 5, 20, 18, 4, 3, 60, 81, 2, & 55. (nearest whole number)

(A) 10 (B) 12 (C) 26 (D) 32 (E) 34

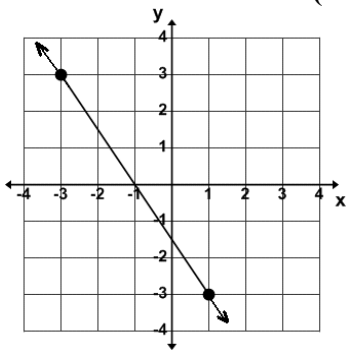
4.



The distances between the hash marks (|) are equal. Find $P + Q + R + S$.

(A) 0.75 (B) .5 (C) 0.25 (D) -0.75 (E) -1.5

5. A line perpendicular to the line shown at (— 1, 0) contains which of the following points.



(A) (— 13, — 8) (B) (— 15, — 11) (C) (— 14, — 7) (D) (13, 11) (E) (11, 7)

6. Which of the following set(s) of numbers is/are closed under multiplication but not subtraction?
I. Integers II. Naturals III. Primes IV. Rationals

(A) I only (B) II only (C) I, II & IV (D) II & IV (E) None of them

7. If $6x^2 - 7x + c = (ax - 5)(bx + 1)$, where a and b are integers then $a + b + c =$ _____.

(A) -5 (B) -1 (C) 0 (D) 5 (E) 10

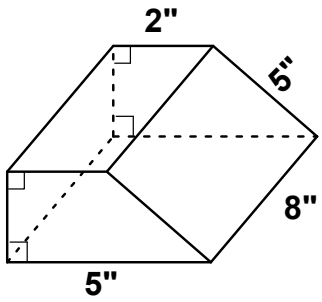
8. $\angle A$ and $\angle B$ are complementary angles with m $\angle A = 2x - 1$ and m $\angle B = 3x + 4$. $\angle C$ is supplementary to $\angle B$. Find m$\angle C$.

(A) 146.2° (B) 120.2° (C) 143.8° (D) 123.8° (E) 162.6°

9. **The distance from Austin, TX to Ft. Worth, TX by way of I-35 is 190 miles and the distance from Austin, TX to Laredo, TX by way of I-35 is 235 miles. Willie Kann leaves Ft. Worth on I-35 at 7:00 a.m. driving toward Laredo at an average speed of 55 mph due to construction and I-35 traffic. Betty Wheel leaves Laredo on I-35 at 8:00 a.m. driving toward Ft. Worth at an average speed of 70 mph. How far will Betty have driven when they meet? (nearest mile)**

(A) 277 miles (B) 218 miles (C) 163 miles (D) 248 miles (E) 207 miles

10. **Find the total surface area of the trapezoidal prism shown. (nearest sq. in)**



(A) 160 sq. in (B) 156 sq. in (C) 142 sq. in (D) 141 sq. in (E) 134 sq. in

11. **Rose Gardner built a square wooden deck with a side length of 18 ft. She cut out a circle in the center of the deck with a diameter of 10 ft. to put in a pool. What is the area of the deck that she will have to stain? (nearest sq. ft)**

(A) 292 sq. ft (B) 64 sq. ft (C) 10 sq. ft (D) 224 sq. ft (E) 245 sq. ft

12. **The graph of $x^2 + y^2 + 10x - 2y + 10 = 0$ is a circle with a center (h, k) and a radius r. Find $h + k - r$.**

(A) 0 (B) -5 (C) -8 (D) -4 (E) 10

13. **The set of Lucas numbers is {2,1,3,4,7,11, ...}, where $L_0 = 2$ and $L_1 = 1$. The set of Fibonacci numbers is {0,1,1,2,3,5, ...}, where $F_0 = 0$ and $F_1 = 1 = F_2$. $L_{15} = F_x + F_y$. $x + y = ?$**

(A) 32 (B) 31 (C) 30 (D) 28 (E) 26

14. **An "*emirp*" number is a prime number that becomes a new prime number when the digits are reversed. Single digit primes and palindromic primes cannot be *emirp* numbers. The sum of the prime numbers less than 40 which are considered to be *emirp* numbers is ?**

(A) 80 (B) 117 (C) 98 (D) 61 (E) 85

15. **Speedy randomly selected a letter from the words NASCAR RACE. What is the probability he selected a consonant?**

(A) $33\frac{1}{3}\%$ (B) 40% (C) 50% (D) 60% (E) $66\frac{2}{3}\%$

16. If two dice are tossed, what are the odds that the sum of the top faces is 2, 3, 7, or 12? (nearest whole %)

(A) 28% **(B) 20%** **(C) 38%** **(D) 13%** **(E) 67%**

17. Polynomial equations with integer coefficients and only integer solutions are known as _____ equations.

(A) Archimedian **(B) Boolean** **(C) Mersenne** **(D) Mandelbrot** **(E) Diophantine**

18. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the number in column 14 row 12.

1				1				2				3				5		
			2				3				5				8			
		3				5				8				13				21
	5				8				13				21				34	
8				13				21				34				55		
			21				34				55				89			
		34				55				89				144				233

(A) 1,595 **(B) 1,974** **(C) 2,584** **(D) 2,843** **(E) 3,571**

19. Find the sum of the x-values in $\left\{ x \mid \sin(2x) - \cos(x) = 0, x \in [\frac{\pi}{2}, 2\pi) \right\}$? (nearest tenth)

(A) 9.4 **(B) 4.1** **(C) 5.2** **(D) 4.8** **(E) 8.9**

20. Larry and Moe live on the river bank on one side of the river, and Curly lives on the river bank on the other side. The distance across the river is 80 yards. Curly rows his canoe to Larry's house on a bearing of 125°, then walks due north to Moe's house, then rows Moe's boat on a bearing of 210° back to his house. How far did Curly travel? (nearest yard)

(A) 313 yds **(B) 370 yds** **(C) 415 yds** **(D) 452 yds** **(E) 532 yds**

21. Which of the following functions are even functions? f(x) =
I. $x^2 - 1$ II. $x^5 - x^3 - x$ III. $x^2 - 2x - 1$

(A) I only **(B) II only** **(C) I & II but not III** **(D) all of them** **(E) none of them**

22. The 3rd term of a geometric sequence is $\frac{2}{27}$. The 4th term is $\frac{4}{81}$. Find the sum of the first 5 terms of this geometric sequence.

(A) $\frac{211}{486}$ **(B) $\frac{200}{243}$** **(C) $\frac{64}{81}$** **(D) $\frac{422}{2187}$** **(E) $\frac{227}{243}$**

23. Let f(x) = 3x — 1, g(x) = 2x + 4, and h(x) = 4x — 1. Find $g\left(f\left(h(-x)+1\right)-1\right)$

(A) — 24x **(B) — 24x — 8** **(C) — 24x — 4** **(D) — 24x + 4** **(E) — 24x + 8**

24. Find the digit in the ten-thousandth place of the series $\frac{3^1}{1!} - \frac{3^3}{3!} + \frac{3^5}{5!} - \frac{3^7}{7!} + \frac{3^9}{9!} - \ldots$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

25. How many non-negative proper fractions in lowest terms have a denominator of 42?

(A) 17 **(B)** 14 **(C)** 12 **(D)** 10 **(E)** 7

26. Find the digit in the units place of the integer 8^{2018}.

(A) 8 **(B)** 6 **(C)** 4 **(D)** 2 **(E)** cannot be determined

27. How many ordered pairs (A, B) exist such that 1,468,AB9 ÷ 9 has a remainder of 7?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 11 **(E)** 15

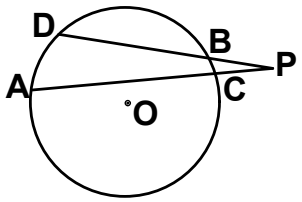
28. If the roots of $x^3 + bx^2 + cx + d = 0$ are -6, -3, and -1, then $b + c + d$ equals:

(A) -10 **(B)** 55 **(C)** 19 **(D)** 45 **(E)** -1

29. How many integers are in the solution set for $|3x + 4| - 7 \leq 11$?

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 18

30. Given the circle with center O shown with DB = 5 cm, BP = 2 cm, and CP = 1.5 cm. Find AP.



(A) 9.333... cm **(B)** 5.5 cm **(C)** 8.5 cm **(D)** 7.8333... **(E)** 13.333... cm

31. Let $A + B = 18$ and $A \times B = 32$. Find $A - B$, where $A < B$.

(A) -14 **(B)** -25 **(C)** 7 **(D)** 14 **(E)** 25

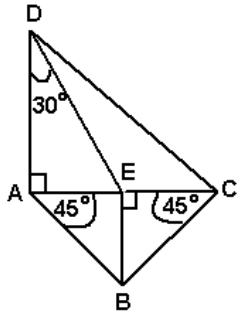
32. The roots of $x^3 + 4x^2 + x - 1 = 0$ are d, e, and f. Find $(d + e)(e + f)(f + d)$.

(A) -5 **(B)** -3 **(C)** -2 **(D)** -1 **(E)** 4

33. Let $f(x) = |2 - 3x| - |2x + 3|$. Find the minimum value of f(x).

(A) -4.5 **(B)** $-4.333...$ **(C)** -4.25 **(D)** -4.2 **(E)** $-4.111...$

34. Find the perimeter of the quadrilateral BCDE if BE = 3". (nearest inch)



(A) 21 in (B) 28 in (C) 18 in (D) 22 in (E) 24 in

35. Find P + Q if $\begin{bmatrix} 1 & P \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} Q \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

(A) — 4 (B) — 3 (C) 3 (D) 8 (E) 9

36. The polar equation $r = 2\csc(\theta)\cot(\theta)$ written in rectangular form is:

(A) $x^2 + y^2 = \sqrt{2}$ **(B)** $x + y = 2$ **(C)** $x^2 = \sqrt{2y}$ **(D)** $y^2 = 2x$ **(E)** $y = \sqrt{2x}$

37. Let $f(x) = (3x + 1)^2$. The tangent to f(x) at (a, b) is perpendicular to $y = 2 + x$. Find $a + b$.

(A) $-\frac{7}{18}$ **(B)** $-\frac{13}{36}$ **(C)** $-\frac{5}{12}$ **(D)** $-\frac{2}{9}$ **(E)** $-\frac{4}{9}$

38. Let $f(x) = x^3 - 5x^2 + 8x - 4$ and $g(x) = (2x + 1)^2$. Find $f'(g'(-1))$

(A) 0 (B) 172 (C) 96 (D) 180 (E) 140

39. If the probability of scoring 150 or more on this test is 55%, what are the odds of scoring less than 150 on this test?

(A) $\frac{9}{20}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{11}{30}$ **(E)** $\frac{9}{11}$

40. Which of the sets of numbers does 55 belong to?

I. Unhappy II. Unlucky III. Evil

(A) II only (B) I & II only (C) I & III only (D) all of these (E) none of these

41. Let $f(x) = \sqrt{3 - \sqrt{5x + 7}}$ **. The domain of f(x) is** $\{x \mid p \le x \le q\}$**. Find** $\frac{P + Q}{2}$**.**

(A) 0.4 (B) 2 (C) — 1 (D) — 0.5 (E) — 1.4

42. Find the sum of the measure of an interior angle of a regular heptagon, the measure of a central angle of a regular octagon, and the measure of an exterior angle of a regular nonagon. (nearest degree)

(A) 236° (B) 226° (C) 214° (D) 202° (E) 136°

43. Point P (— 2, 5) lies on the x-y plane. P is reflected across the line y = — x to point Q. Point Q is rotated 90° counter clockwise about the origin to point R. Point R is translated vertically — 3 units to point S(x, y). Find $x + y$

(A) — 10 (B) — 7 (C) — 3 (D) 4 (E) 7

44. Phil Witwater has a tank that uses two pipes to input water and fill the tank. It has another pipe that is used to output water and drain the tank. One of the input pipes can fill the empty tank in 8 hours by itself with the output pipe shut off. The tank can be fully drained in 18 hours with the input pipes shut off. The tank can be filled with both input pipes open and the output pipe open in 6 hours. How long would the other input pipe take to fill the tank by itself with the output pipe shut off? (nearest min)

(A) 14 hrs 24 min (B) 4 hrs 30 min (C) 9 hrs (D) 5 hrs 32 min (E) 10 hrs 17 min

45. The attendance rate last year at the Millersview's *4-Act Play Theater* is modeled by the function $A(m) = 100 + 40\cos((\frac{\pi}{4})(m + 4))$, $0 \leq m \leq 12$, where m is in months and A(m) is in people per month. If the theater opened on Jan. 1, when did the theater's attendance rate first reach a maximum?

(A) 12 months (B) 8 months (C) 6 months (D) 4 months (E) 2 months

46. Lotta Koins is putting the coins from her bank in stacks of 5 coins each. She has pennies, nickels, dimes, quarters, and half dollars. How many different stacks of 5 coins can she make if each stack has to have at least one half dollar?

(A) 126 (B) 35 (C) 124 (D) 54 (E) 70

47. Ucandoette High School has three administrators, five teachers, seven female students and nine male students. A special council consisting of one administrator, two teachers, three female students and four male students is formed. How many different ways can the council be formed?

(A) 174 (B) 728 (C) 4,440 (D) 132,300 (E) 1,961,256

48. Willie When put two blue balls, three white balls, and five red balls in a bucket. He draws out one ball. If the ball is blue, he gets $3.00. If it is white he gets nothing. If it is red he has to pay $1.00. What is the mathematical expectation value if he does this many times? (nearest cent)

(A) 10¢ (B) 30¢ (C) 40¢ (D) $1.00 (E) $1.10

49. Find $f(-1) - f(2) + f(-3)$ if $f(x) = \begin{cases} 3x - 2 & \text{if } x < -2 \\ -2x + 1 & \text{if } -2 \le x \le 1 \\ -1 - 3x & \text{if } x > 1 \end{cases}$

(A) -17 **(B)** -1 **(C)** 1 **(D)** 7 **(E)** 12

50. How many distinct triangles can be made using three sticks at a time from six sticks measuring 12 inches, 10 inches, 8 inches, 6 inches, 6 inches, and 5 inches?

(A) 20 **(B)** 16 **(C)** 14 **(D)** 12 **(E)** 10

51. Let function f = {(9, 6), (3, 9), (12, 5), (6, 7)} and function g = {(5, 6), (9, 12), (7, 4), (1,9)}. Find $(f \circ g)(9) + (g \circ f)(6)$?

(A) 5 **(B)** 9 **(C)** 11 **(D)** 15 **(E)** 16

52. Which of the following do not have an inverse function without restricting the domain?

I. $y = x^2 + 4$ II. $y = 2\cos(x) + 4$ III. $y = e^{(\sin(x))}$ IV. $y = 2^x$

(A) I only **(B)** III only **(C)** I, II, III but not IV **(D)** II & IV **(E)** I, II, III, & IV

53. Which of the following equations can be obtained from the graph of the function $y = 2 + \sin(x - 3)$ by applying a vertical stretch of 4 units, a vertical shift down 5 units, and a phase shift left 6 units ? y = :

(A) $\sin(4x + 6) - 5$ **(B)** $4\sin(x - 8) + 8$ **(C)** $4\cos(x - 8) - 3$
(D) $4\sin(x + 3) - 3$ **(E)** $2\sin(x + 3) - 5$

54. Which of the follow ordered pairs (P, Q) of numbers would make the graph of the polar equation $r = P + Q\cos(\theta)$ a dimpled limacon.

(A) (2, 3) **(B)** (5, 3) **(C)** (4, 4) **(D)** (1, 3) **(E)** (3, 1)

55. Integers P, Q, and R are the roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is 2 and C is 27. Find B.

(A) 18 **(B)** 8 **(C)** 6 **(D)** -12 **(E)** -10

56. The function $f(x) = \begin{cases} nx^3 + 2x & \text{if } x \le 1 \\ mx^2 - x & \text{if } 1 < x \end{cases}$ is continuous and differentiable everywhere. Find m.

(A) 1.2 **(B)** 2 **(C)** 2.4 **(D)** 6 **(E)** 10

57. The area (in square units) of the region bounded by $y = x^2$, $y = x + 6$, $x = 0$ and $x = 5$ is:

(A) $23\frac{2}{3}$ **(B)** $24\frac{5}{6}$ **(C)** $24\frac{1}{3}$ **(D)** $25\frac{5}{6}$ **(E)** $26\frac{1}{6}$

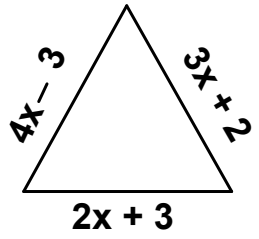
58. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \frac{x^2 - 5}{x - 3}$

(A) — 5 **(B) — 3** **(C) 0.1.666...** **(D) 2** **(E) 3.5**

59. If $(3212_b) \div 11_b = 232_b$, then $2123_b \div 11_b = N_b$, where N_b is a 3-digit number. Find the sum of the 3 digits.

(A) 4 **(B) 5** **(C) 7** **(D) 8** **(E) 9**

60. Find the least possible perimeter of this triangle given that it is isosceles but not equilateral.



(A) 47 **(B) 43** **(C) 35** **(D) 29** **(E) 11**

DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST

University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2018
Answer Key

1. B	**21. A**	**41. D**
2. E	**22. A**	**42. C**
3. D	**23. A**	**43. A**
4. A	**24. B**	**44. E**
5. A	**25. C**	**45. D**
6. B	**26. C**	**46. E**
7. C	**27. D**	**47. D**
8. D	**28. B**	**48. A**
9. E	**29. D**	**49. B**
10. B	**30. A**	**50. D**
11. E	**31. A**	**51. B**
12. C	**32. A**	**52. C**
13. C	**33. B**	**53. D**
14. C	**34. A**	**54. B**
15. D	**35. A**	**55. E**
16. C	**36. D**	**56. D**
17. E	**37. B**	**57. E**
18. C	**38. C**	**58. B**
19. E	**39. E**	**59. C**
20. D	**40. B**	**60. E**