TMSCA HIGH SCHOOL
MATHEMATICS
TEST #8 ©
JANUARY 19, 2019

GENERAL DIRECTIONS

1. About this test:
  A. You will be given 40 minutes to take this test.
  B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate $\dfrac{11 \cdot 7! \div 8}{7 + 2^3}$.

   (A) 149   (B) 66   (C) 770   (D) 462   (E) 998

2. Holly borrowed \$2400 at 6.5% simple interest for 18 months. What will her monthly payments be?

   (A) \$213.00   (B) \$130.00   (C) \$146.33   (D) \$131.67   (E) \$142.17

3. Jay's current age is ¼ of his mother's age. In 10 years, Jay's age will be 2 years less than ½ his mother's age. What is the sum of their current ages?

   (A) 35   (B) 55   (C) 45   (D) 38   (E) 28

4. The full cost of a bunch of flowers is \$12.99. Lesley bought 5 bunches with a 60% sale discount. The sales clerk then applied a 15% student discount to Lesley's bill. How much did Lesley pay for the flowers if the sales tax rate was 8.25%?

   (A) \$35.86   (B) \$17.58   (C) \$33.12   (D) \$22.08   (E) \$23.90

5. Find the point of inflection for the graph of the function $f(x) = \dfrac{1}{3}x^3 - x^2$.

   (A) $(2,0)$ (B) $\left(1, -\dfrac{2}{3}\right)$ (C) $\left(2, -\dfrac{4}{3}\right)$ (D) $(0,0)$ (E) $(1,0)$

6. Which of the following is an equation of the line through $(-5,11)$ perpendicular to $2x + 7y = 42$?

   (A) $7x - 2y = -57$ (B) $2x + 7y = 1$ (C) $7x - 2y = 24$

   (D) $2x + 7y = 67$ (E) $7x - 2y = -46$

7. The medians of a triangle all intersect at the _____.

   (A) Orthocenter   (B) Centroid   (C) Incenter   (D) Vertex   (E) Circumcenter

8. What is the sum of the arithmetic sequence 16, 18.4, 20.8, …, 119.2?

   (A) 2974.4   (B) 5125.6   (C) 2622.4   (D) 2906.8   (E) 5678.2

9. Mr. Clements finished teaching his history class at 3:07 PM. What was the smaller angle formed by the hour and minute hands of his clock at that time?

   (A) 42°   (B) 48.5°   (C) 38.5°   (D) 51.5°   (E) 55°

10. Given that $f(x) = 2x + 5$ and $g(x) = x^2 - 1$, find $g(f(x))$.

   (A) $2x^2 + 4$ (B) $4x^3 + 10x^2 + 6x + 15$ (C) $2x^2 + 3$

   (D) $4x^2 + 24$ (E) $4x^2 + 20x + 24$

11. The total surface area of a right cone with a radius at the base of 12 ft and a vertex angle of 36° is ____ft². (nearest square foot)

   (A) 843   (B) 1916   (C) 718   (D) 1222   (E) 592

12. A right triangular prism has a height of $4\sqrt{2}$ inches. The base is a 30°- 60°- 90° triangle with a hypotenuse of **3.2 inches.** Find the volume.

(A) $\dfrac{512\sqrt{2}}{25}$ in³   (B) $\dfrac{512\sqrt{6}}{25}$ in³   (C) $\dfrac{256\sqrt{2}}{25}$ in³   (D) $\dfrac{128\sqrt{6}}{25}$ in³   (E) $\dfrac{128\sqrt{5}}{25}$

13. The function $f(x) = $ _____ will produce the graph shown.
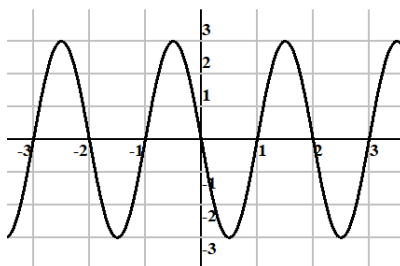
(A) $3\sin(\pi x + 1)$ (B) $3\sin(x + \pi)$ (C) $\sin(\pi(x-1)) + 3$ (D) $3\sin(\pi x - 1)$ (E) $3\sin(\pi(x-1))$

14. A belt joins two pulleys. The smaller pulley with a radius of 24 inches and rotates at 45 rpm. The larger has a radius of 30 inches and rotates at _____rpm.

(A) 42   (B) 56   (C) 48   (D) 36   (E) 30

15. Adam and Brad started together at the starting line of a 500-m circular track. Adam ran clockwise at an average rate of 5 m/s. Brad ran counter clockwise at a rate of 7.5 m/s. How far did Adam run before he met Brad?

(A) 200 m   (B) 240 m   (C) 320 m   (D) 250 m   (E) 300

16. Evaluate $\displaystyle\sum_{k=0}^{\infty} \frac{2}{3}\left(-\frac{3}{5}\right)^{k+2}$.

(A) $\dfrac{5}{3}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{3}{5}$ (D) $-\dfrac{1}{4}$ (E) $\dfrac{3}{20}$

17. Let $s(x)$ be the slant asymptote of $f(x) = \dfrac{3x^2 + 9x - 7}{x+3}$. Evaluate $s(2)$.

(A) 3.3   (B) 24   (C) 6   (D) 4.6   (E) –1

18. If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c) = k$. This is _____.

(A) Sandwich Theorem   (B) Rolle's Theorem   (C) Fundamental Theorem of Calculus

(D) Intermediate Value Theorem   (E) Fundamental Theorem of Algebra

19. Evaluate $\displaystyle\lim_{x\to 0} \frac{\tan x}{x}$

(A) ∞ (B) $-\infty$ (C) 1   (D) –1   (E) does not exist

20. Mr. Meredith set up a scavenger hunt for his pre-calculus class. Group A travelled 200 yards on a bearing of 212°, then 350 yards on a bearing of 97°, then 275 yards on a bearing of 325° to retrieve their clues. How far were they from their point of origin? (nearest yard)

   (A) 507 yd   (B) 85 yd   (C) 306 yd   (D) 97 yd   (E) 716 yd

21. $\lim\limits_{x \to 0.5} \left(\dfrac{6x^2 - 17x + 7}{2x^3 - 7x^2 + 3x} \right) =$

   (A) $\dfrac{13}{4}$ (B) $\dfrac{22}{5}$ (C) 3   (D) $\dfrac{1}{2}$ (E) does not exist

22. If $f(x) = ax^8 + bx^6 + cx^4 + dx^2 + x$ and $f(-3) = 74$ then $f(3) = ?$

   (A) 82   (B) −74   (C) 74   (D) 98   (E) 80

23. $A = \begin{bmatrix} 3 & 5 \\ 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$. Calculate $\det(2A + B)$

   (A) −119   (B) −20   (C) −126   (D) 133   (E) 9

24. Find the value of k for which the system of equations $\begin{array}{l} 2x - ky = 2 \\ 3x + (k+1)y = 4 \end{array}$ has no solutions.
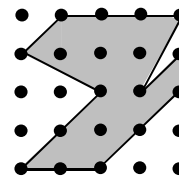
   (A) 0   (B) $\dfrac{2}{5}$ (C) $-\dfrac{5}{2}$ (D) $-\dfrac{2}{5}$ (E) $\dfrac{5}{2}$

25. Two lines that are in the same plane but never intersect are _____.

   (A) Skew   (B) Parallel   (C) Perpendicular   (D) Concurrent   (E) Bisectors

26. If the dots on the diagram shown are 2 inches apart both vertically and horizontally, then the area of the shaded region is ____in².

   (A) 36   (B) 44   (C) 48   (D) 40   (E) 64

27. What is the distance between the point $(3,7)$ and the line $12x + 5y = 27$?

   (A) $\dfrac{98}{169}$ (B) $\dfrac{44}{169}$ (C) $7\dfrac{7}{13}$ (D) $3\dfrac{5}{13}$ (E) $4\dfrac{7}{12}$

28. If $\int_0^a \cos x\, dx = C$ then $\int_{-a}^a 3\cos x\, dx =$

   (A) $12C$ (B) $6C$ (C) $3C$ (D) $2C$ (E) 0

29. A pizza shop has 8 choices of toppings and two types of crust. The run a spring special offering a large, 3-topping pizza for $12.00. How many possible pizza orders are there for the special if toppings can be repeated?

   (A) 330   (B) 120   (C) 112   (D) 240   (E) 165

30. Eight circles in a rectangle are congruent. Each circle is tangent to the adjacent circles and the outer six circles are tangent to the rectangle. A dart lands somewhere inside the rectangle. What is the probability that the dart lands in a circle? (nearest whole percent)
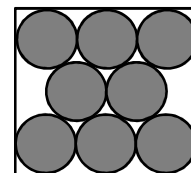
   (A) 72%   (B) 65%   (C) 70%   (D) 67%   (E) 77%

31. $1331_b =$ ____$_{b+1}$, where $b > 5$.

   (A) 4000   (B) 401   (C) 1220   (D) 1000   (E) 1001

32. The measure of one interior angle of a regular decagon is _____°.

   (A) 162   (B) 150   (C) 144   (D) 120   (E) 108

33. In the diagram, $l // m$. Find the value of x.

   (A) 7   (B) 1   (C) 4   (D) 10   (E) 12.5

   m

   $(2x^2)°$
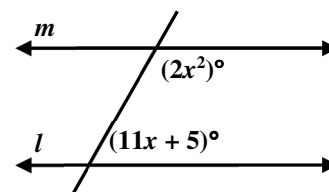
   l $(11x + 5)°$

34. A 15-oz bag of ground coffee is made up of 30% decaf grounds. How many ounces of decaf grounds need to be added to create a 50% mixture?

   (A) 10.5   (B) 6   (C) 5   (D) 7.5   (E) 7

35. What is the length of side a on the quadrilateral?
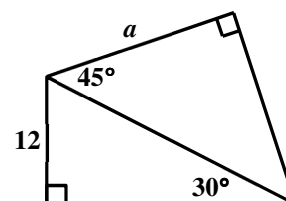
   (A) $12\sqrt{3}$ (B) 24   (C) $6\sqrt{6}$ (D) $12\sqrt{2}$ (E) $6\sqrt{5}$

   a

   $45°$

   12

   $30°$

36. The slope of the tangent to $4x^2 - 9y^2 = 19$ at $(-5,3)$ is:

   (A) $\dfrac{59}{54}$ (B) $-\dfrac{20}{27}$ (C) $-\dfrac{59}{54}$ (D) $\dfrac{20}{27}$ (E) $-\dfrac{20}{9}$

37. A hose on full force can fill a small tank in 20 minutes. How long will it take 6 similar hoses to fill a tank that is 3 times as deep, 3 times as long and 3 times as wide?

   (A) 240 min   (B) 120 min   (C) 90 min   (D) 30 min   (E) 10 min

38. If y varies directly with x and $y = 28$ when $x = 200$, calculate y when $x = 117$. (nearest tenth)

   (A) 14.0   (B) 26.9   (C) 24.6   (D) 16.4   (E) 18.7

39. What is the angle between the vectors $\langle -8,17 \rangle$ and $\langle 11,5 \rangle$?

   (A) 96°   (B) 132°   (C) 91°   (D) 89°   (E) 84°

40. Ranger Bob walks out 70 ft from the base of a large tree. If the angle of elevation from the ground where Bob stands to the top of the tree is 76°, how tall is the tree? (nearest foot)

(A)   103 ft   (B)   90 ft   (C)   68 ft   (D)   66 ft   (E)   281 ft

41. Let $a_1 = 12$, $a_2 = 5$ and $a_n = 2a_{n-1} - 3a_{n-2}$. $a_5 = ?$

(A)   −33   (B)   −159   (C)   −339   (D)   −56   (E)   147

42. $a^{-4} \times b^5 \times a^2 \div b^{-3} \times a^{-5} \div b^{-2} =$
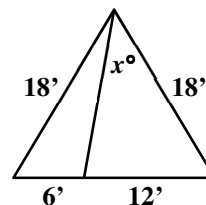
(A) $\dfrac{1}{a^3}$ (B) $\dfrac{a^7}{b^{10}}$ (C) $\dfrac{1}{a^3 b^2}$ (D) $\dfrac{b^4}{a^7}$ (E) $\dfrac{b^{10}}{a^7}$

43. Given $(3+2i)^4 + (3-2i)^3 = a + bi$, calculate $a + b$.

(A)   −54   (B)   −202   (C)   276   (D)   56   (E)   202

44. Find the value of x in the triangle diagram. (nearest degree)

(A)   40°   (B)   41°   (C)   43°   (D)   39°   (E)   42°

45. How many distinct arrangements can be made from the letters "PARALLEL".

(A)   20,160   (B)   40,320   (C)   6,720   (D)   3,360   (E)   720

46. Find the sum of the solutions of the equation $\left| x^2 - 3x \right| = -4x + 6$.

(A)   −1   (B)   −2   (C)   4   (D)   0   (E)   6

47. Given the Fibonacci-type sequence 3, a, b, 15, c, …, find the value of a + b + c.

(A)   12   (B)   48   (C)   30   (D)   33   (E)   39

48. When $f(x) = \dfrac{x^2 + 1}{2 - x}$ evaluate $f'(3)$.

(A)   3   (B)   4   (C)   7   (D)   5   (E)   2

49. Find the sum of all two-digit numbers such that reversing the digits results in another two-digit number that is $58\dfrac{2}{31}\%$ less than the original.

(A)   154   (B)   164   (C)   186   (D)   93   (E)   204

50. Given GCF(36, k) = 12 and LCM(36, k) = 252, find the value of k.

(A)   84   (B)   63   (C)   42   (D)   48   (E)   126

51. Find the area of the region defined by the inequalities $x \geq 0$, $y \geq 0$ and $y \leq 3\cos(2x)$.

(A) 1.5   (B) 3   (C) 6   (D) 0   (E) 4.5

52. $\csc x < 0$ and $\tan x < 0$. Where will θ terminate?

(A) QI   (B) QII   (C) QIII   (D) QIV   (E) y-axis

53. The chord $\overline{AB}$ has a length of 14 cm is 24 cm from the center of the circle. The area of the circle is _____ cm²

(A) 576π (B) 500π (C) 625π (D) 380π (E) 418π

54. If $\dfrac{x-7}{x+4} + \dfrac{x+4}{x-7}$ is equal to the mixed number $A + \dfrac{B}{(x+4)(x-7)}$, then $B = ?$

(A) 9   (B) 121   (C) 24   (D) 8   (E) 64

55. $\int_{-2}^{4}\left(kx^2 + 5x + 2\right)dx = 114$. What is the value of k?

(A) 1   (B) –1   (C) –3   (D) 2   (E) 3

56. Find the area of the convex quadrilateral with the vertices $(2,7)$, $(4,1)$, $(1,-5)$ and $(-6,2)$.

(A) 30.5   (B) 59.5   (C) 27.5   (D) 60.5   (E) 24

57. If $\ln 2 = a$, $\ln 5 = b$ and $\ln 7 = c$ then $\ln 5.6 = ?$

(A) $\dfrac{2a+c}{b}$ (B) $a^2 - b + c$ (C) $2a - b + c$ (D) $\dfrac{a^2+c}{b}$ (E) $\dfrac{a^2 c}{b}$

58. The set of integers in ascending order {a, b, c, d, e} has a median of 14, a mean of 16.4, a mode of 11 and a range of 16. Find the value of d.

(A) 14   (B) 19   (C) 17   (D) 21   (E) 27

59. The repeating decimal 0.363636…in base 7 can be written as which of these fractions in base 7?

(A) $\dfrac{6}{66_7}$ (B) $\dfrac{12}{343_7}$ (C) $\dfrac{12}{22_7}$ (D) $\dfrac{36}{343_7}$ (E) $\dfrac{6}{11_7}$

60. How many solutions exist such that $6x + 8y = 218$, where $x, y \in \mathbb{Z}^+$.

(A) 9   (B) 36   (C) 7   (D) 27   (E) 8

2018-2019 TMSCA Mathematics Test Eight Answers

1. D	21. B	41. D
2. C	22. E	42. E
3. A	23. C	43. A
4. E	24. D	44. B
5. B	25. B	45. D
6. A	26. D	46. B
7. B	27. D	47. E
8. A	28. B	48. B
9. D	29. D	49. C
10. E	30. E	50. A
11. B	31. D	51. A
12. D	32. C	52. D
13. E	33. A	53. C
14. D	34. B	54. B
15. A	35. D	55. E
16. E	36. B	56. D
17. C	37. C	57. C
18. D	38. D	58. B
19. C	39. C	59. C
20. B	40. E	60. A