T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 2 ©

O C T O B E R 2 7 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Two

1. **Evaluate:** $\sqrt[3]{729} \div (81)^{3/2} + 8 \times 2^{-1} - 8$.

   (A) $\frac{649}{81}$ **(B)** 6557 **(C)** $-\frac{34}{9}$ **(D)** $-\frac{323}{81}$ **(E)** $\frac{217}{27}$

2. **Caroline had a rope that was 20 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 6:7:8 with 30 inches of string left over. How long was the longest piece?**

   (A) 6 ft 8 in **(B)** 5 ft 10 in **(C)** 8 ft **(D)** 2 ft 6 in **(E)** 5 ft

3. **Find the sum of the multiples of 9 that are greater than 0 and less than 384.**

   (A) 8192 **(B)** 8127 **(C)** 8056 **(D)** 8060 **(E)** 8068

4. **1200 feet per second = _____ miles per hour. (nearest whole number)**

   (A) 1760 **(B)** 880 **(C)** 14 **(D)** 2455 **(E)** 818

5. **Larry, Mark and Norman can each tile a floor in 6 hours, 8 hours and 9 hours respectively. How long would it take them to tile a floor that is twice as long and twice as wide together? (nearest minute)**

   (A) 2 hr 29 min **(B)** 9 hr 56 min **(C)** 4 hr 58 min **(D)** 8 hr 46 min **(E)** 9 hr 38 min

6. **If** $S = \{s,q,u,a,r,e\}$, $H = \{h,e,x,a,g,o,n\}$ **and** $D = \{d,e,c,a,g,o,n\}$ **then** $(S \cup H) \cap (H \cup D)$ **contains how many distinct elements?**

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 9

7. $\angle P$ **and** $\angle Q$ **are complementary.** $\angle Q$ **and** $\angle R$ **are supplementary. If** $m\angle R = 148°$, **then** $m\angle P =$ **?**

   (A) 58° **(B)** 42° **(C)** 18° **(D)** 32° **(E)** 48°

8. **Simplify:** $\frac{n!}{(n-2)!} \div \frac{(n+2)!}{n(n-3)!}$.

   (A) $\frac{n^2-3n}{n^2-n-2}$ **(B)** $\frac{n-3}{n^2-n-2}$ **(C)** $\frac{1}{n^2-4}$

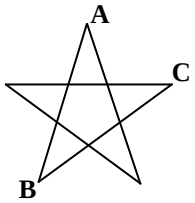
   (D) $\frac{n^2+2n-3}{n-1}$ **(E)** $\frac{n}{n^3+n^2-4n-4}$

9. **Find the least common multiple of** $2^3 \times 7^2$, $3^2 \times 2^7$ **and** $3^7 \times 2$.

   (A) 1,959,552 **(B)** 2,286,144 **(C)** 326,592 **(D)** 653,184 **(E)** 13,716,864

10. **The angles at each point on the star shown are congruent. What is** $m\angle ABC$ **?**

    (A) 30° **(B)** 32° **(C)** 36° **(D)** 42° **(E)** 54°



11. A moving sidewalk moves at a rate of 1.9 ft/s. A person walking on the moving sidewalk travels 102 ft forward in the time it takes them to travel 51 ft in the opposite direction. How fast would the person walk on a nonmoving sidewalk?

A) 3.8 ft/s (B) 1.9 ft/s (C) 2.1 mph (D) 5.7 ft/s (E) 2.4 mph

12. The four cousins Eleanor, Sarah, Kyle and Naomi wanted to go on a road trip, but Naomi had no money. Eleanor, Sarah and Kyle each gave Naomi one-fourth, one-fifth and one-eighth of her money respectively. If each gave Naomi the same amount, what fraction of the money did Naomi possess after the exchange?

(A) $\frac{23}{40}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{67}{210}$ **(E)** $\frac{3}{17}$

13. Let $f(x)=3x+2$, $g(x)=4x-7$, $h(x)=6-x$, **and** $f(g(h(a)))=29$. **Find** $6a-7$.

(A) 2 (B) −3 (C) 5 (D) 2 (E) 1

14. A square has side lengths of 8 cm. If the square's width is tripled and the length is halved, what is the percent change in the area of the shape?

(A) 75% (B) 50% (C) 125% (D) 200% (E) $133\frac{1}{3}$ **%**

15. A particle's movement along the number line is defined by $f(t)=t^4-4t^3-32t^2+30t+20$. **At which of the following times is the particle moving to the left?**

(A) −2 (B) −1 (C) 2 (D) 6 (E) 9

16. The line $ax+7y=13$ **is perpendicular to the line** $25x+21y=32$. **Find the value of *a*.**

(A) $-\frac{147}{3}$ **(B)** $\frac{3}{25}$ **(C)** $-\frac{25}{3}$ **(D)** $-\frac{147}{25}$ **(E)** $\frac{25}{3}$

17. Simplify: $\frac{\sqrt[5]{a^3b^2}}{\sqrt[4]{ab}}$.

(A) $\sqrt[20]{a^{17}b^{13}}$ **(B)** a^2b **(C)** $\sqrt[20]{a^7b}$ **(D)** $\sqrt[20]{ab^3}$ **(E)** $\sqrt[20]{a^7b^3}$

18. The mathematical statement $(2x-3)(4x-7)=8x^2-14x-12x+21$ **is an example of _____property.**

(A) Associative (B) Commutative (C) Identity

(D) Transitive (E) Distributive

19. If $\frac{5x-5}{x+2}+\frac{x+2}{x-3}=\frac{ax^2+bx+c}{px^2+qx+r}$ **then** $\frac{a+b+c}{p+q+r}=$?

(A) $-\frac{3}{2}$ **(B)** $-\frac{7}{4}$ **(C) −1 (D)** $-\frac{1}{2}$ **(E)** $-\frac{7}{3}$

20. Find the mean value of $f(x)=4x^3-6x^2+2x-1$ **for** $[-2,8]$.

(A) 374 (B) 162 (C) 809 (D) 59 (E) 309

21. How many distinct 4-letter arrangements can be formed using the letters in "ALGEBRA"?

(A) 480 (B) 720 (C) 840 (D) 960 (E) 360

22. Let $f''(x)=6x+6$, $f'(2)=12$ and $f(2)=3$. Find $f(-1)$.

(A) 32 (B) 18 (C) 21 (D) 24 (E) –15

23. If $\log_5(3x-8)-\log_5(x-4)=1$, then x = ?

(A) 5 (B) 6 (C) 9 (D) 2 (E) No solution

24. What is the reference angle of $\frac{32\pi}{3}$?

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) π (E) $\frac{\pi}{2}$

25. Find the area of the region that is completely bounded by the two functions $f(x)=-x^2-3x+54$ and $g(x)=-6x+36$. (nearest square unit)

(A) 608 (B) 41 (C) 32 (D) 122 (E) 996

26. Jerri drives the same route to work every weekday. Last week, her average speed on her commute to work was 28 mph. On the first 4 days of the week, her speeds were 22.2 mph, 31.8 mph, 26.3 mph and 25.7 mph respectively. What was her speed on Friday? (nearest tenth)

(A) 28.7 mph (B) 37.2 mph (C) 33.7 mph (D) 39.8 mph (E) 35.8 mph

27. Use the Fibonacci characteristic sequence, $-6, p, q, 4, r, s, \ldots$ to find $pq-rs$.

(A) –12 (B) –16 (C) –18 (D) 16 (E) –26

28. If $3-2i$ is one of the zeros the polynomial $f(x)=3x^3-13x^2+9x+65$, then another of its zeros is:

(A) $-\frac{5}{3}$ (B) $-\frac{3}{2}$ (C) $-\frac{3}{5}$ (D) –1 (E) $\frac{2}{5}$

29. The ADA recommends that wheelchair ramps have no more than a 5° angle of elevation. Paul needs to build a ramp up to a porch that stands 4 feet 3 inches off the ground. How long should Paul make the ramp itself? (nearest inch)

(A) 592 inches (B) 585 inches (C) 49 inches (D) 51 inches (E) 482 inches

30. If *P*, *Q* and *R* are real numbers such that $P+Q+R=12$, $R^2=P^2+Q^2$ and $PQ=6$, then $R=?$

(A) –5.5 (B) 6.5 (C) –6.5 (D) 11 (E) 5.5

31. Carl's Pizza offered a 3-topping medium pizza for $9.99 as a weekly special. There are 12 toppings on the menu and 3 different crust options. How many distinct pizzas can be ordered with the special?

(A) 364 (B) 660 (C) 220 (D) 1092 (E) 1365

32. Which of the following equations in rectangular form can be written as $r-10\sin\theta=0$ in polar form?

(A) $x^2+y^2=25$ (B) $x^2+y^2=5$ (C) $x^2+y^2-10y=0$

(D) $x^2-y^2+10y=0$ (E) $x^2+y^2=10$

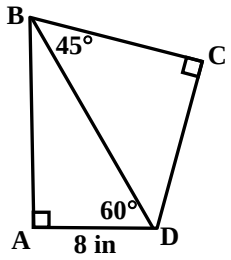
33. The slope of the line going through the points $(-6, y)$, $(-9, 10)$ and $(x, 30)$ is $\frac{4}{3}$. Find $x + y$.

(A) 11 (B) –5 (C) 20 (D) –8 (E) 8

34. Which of the following mathematicians developed a simple ancient algorithm for finding all prime numbers to a given limit?

(A) Gauss (B) Germain (C) Mersenne (D) Euler (E) Eratosthenes

35. What is the perimeter of the quadrilateral ABCD? (nearest inch)



(A) 44 in (B) 46 in (C) 54 in (D) 52 in (E) 39 in

36. Evaluate $\lim_{\theta \to 0} \frac{\sin\theta}{3\theta}$.

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) does not exist

37. Given the sequence 8, 6, 10, 20, 36, 58, … find the 20th term.

(A) 890 (B) 1108 (C) 790 (D) 824 (E) 996

38. Let *A* and *B* be the roots of $f(x) = 3x^2 + 13x - 10$. Find the value of $A + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$.

(A) $\frac{2401}{16}$ (B) $\frac{28561}{16}$ (C) $\frac{28561}{81}$ (D) $\frac{83521}{81}$ (E) $\frac{2401}{81}$

39. What is the obtuse angle between the minute and hour hands on a circular clock at 2:45 pm?

(A) 162° (B) 172.5° (C) 150° (D) 168.5° (E) 175.5°

40. The ratio of length to width of a rectangle is 17:4 and the perimeter is 546 ft. What is the area of the rectangle?

(A) 11492 ft^2 (B) 2873 ft^2 (C) 5746 ft^2 (D) 8619 ft^2 (E) 9,984 ft^2

41. The number 777 in base 8 is equivalent to the number *k* in base 4. Find the sum of the digits in the number *k*.

(A) 12 (B) 13 (C) 16 (D) 8 (E) 21

42. How many distinct solutions exist for $2\sin^2\theta = 2 + \cos\theta$, where $0 \le \theta \le 2\pi$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

43. Given $f(x) = ax^5 + bx^3 + cx - 12$ and $f(5) = 25$, calculate $f(-5)$.

(A) –37 (B) –39 (C) –52 (D) –49 (E) –28

44. The repeating decimal 0.3222… in base 5 can be written as which of the following fractions in base 5 in simplified terms?

(A) $\frac{13}{20_5}$ (B) $\frac{24}{120_5}$ (C) $\frac{12}{20_5}$ (D) $\frac{2}{10_5}$ (E) $\frac{13}{20_5}$

45. Let $f(x)$ be continuous on $[a,b]$. If $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$. This theorem is known as:

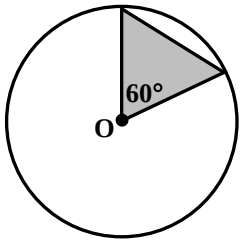
(A) Intermediate Value Theorem (B) Sandwich Theorem (C) Mean Value Theorem

(D) Fundamental Theorem of Calculus (E) Fundamental Theorem of Algebra

46. If $f(x) = \tan(2x)$, then $\lim_{h\to 0} \dfrac{f\left(\frac{\pi}{3}+h\right) - f\left(\frac{\pi}{3}\right)}{h}$ is

(A) 8 (B) –2 (C) 4 (D) 1 (E) undefined

47. Kyle throws a dart that hits the circle with center O and having a diameter of 8'. What is the probability the dart hits in the shaded area? (nearest whole percent)

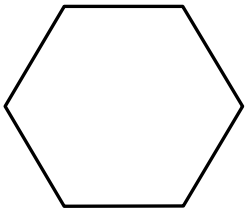


(A) 14% (B) 17% (C) 18% (D) 16% (E) 19%

48. If $x - \dfrac{1}{x} = 14$, then $x^3 - \dfrac{1}{x^3} = ?$

(A) 2702 (B) 2772 (C) 2758 (D) 2786 (E) 2730

49. Find the sum of the length of all the diagonals of a regular hexagon if the length of each side is 2 cm. (nearest centimeter)



(A) 42 cm (B) 12 cm (C) 33 cm

(D) 44 cm (E) 54 cm

50. $\dfrac{1}{10} + \dfrac{1}{15} + \dfrac{1}{21} + \dfrac{1}{28} + \ldots + \dfrac{1}{136} =$ _____.

(A) $\dfrac{28}{51}$ (B) $\dfrac{35}{136}$ (C) $\dfrac{271}{1020}$ (D) $\dfrac{13}{34}$ (E) $\dfrac{165}{476}$

51. Thirty-seven percent of homes in the U.S. use electric heating. If 8 homes are chosen at random, what is the probability that at least 4 of them will use electric heating? (nearest percent)

A. 34% B. 37% C. 28% D. 13% E. 23%

52. The real solution set to $2|3x-7|-7 \le 11$ is:

(A) $\left\{x \middle| \dfrac{2}{3} \le x \le \dfrac{16}{3}\right\}$ (B) $\left\{x \middle| -\dfrac{2}{3} \le x \le \dfrac{16}{3}\right\}$ (C) $\left\{x \middle| \left(x \le -\dfrac{16}{3}\right) \cup \left(x \ge \dfrac{2}{3}\right)\right\}$

(D) $\left\{x \middle| \left(x \le -\dfrac{2}{3}\right) \cup \left(x \ge \dfrac{16}{3}\right)\right\}$ (E) $\left\{x \middle| -\dfrac{16}{3} \le x \le \dfrac{2}{3}\right\}$

53. The measure of one interior angle of a regular decagon is_____°.

(A) 150 (B) 162 (C) 144 (D) 108 (E) 120

54. If $y^2 = -5 - 12i$ and $y^3 = -46 - 9i$ where $y = a + bi$ then $a + b =$?

(A) -1 (B) -7 (C) 1 (D) 5 (E) 6

55. Find the *y*-intercept of the tangent line to $3x^2 + 4y^3 = 35$ at the point $(-1, 2)$.

(A) $\left(0, \frac{17}{8}\right)$ (B) $(0, 17)$ (C) $\left(0, \frac{17}{4}\right)$ (D) $\left(0, \frac{1}{8}\right)$ (E) $\left(0, \frac{17}{4}\right)$

56. Consider the point (a, b) in the Cartesian plane. The transformation $\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ results in a rotation of _____about the origin.

(A) 45° counter-clockwise (B) 90° clockwise (C) 45° clockwise

(D) 90° counter-clockwise (E) 60° clockwise

57. There are two possible triangles ABC such that $m\angle A = 30°$, AB = 24 cm and BC = 16 cm. The area of the smaller triangle is _____cm^2. (nearest square centimeter)

(A) 188 (B) 166 (C) 55 (D) 222 (E) 61

58. The lengths of the sides of triangle PQR are the roots of $f(x) = 2x^3 - 33x^2 + 175x - 294$. The perimeter of triangle PQR is 16.5. Find the area of triangle PQR. (nearest tenth)

(A) 9.1 (B) 6.4 (C) 10.5 (D) 6.2 (E) 7.8

59. What is the distance between the point $(3, 7)$ and the line $12x + 5y = 27$?

(A) $4\frac{7}{12}$ (B) $7\frac{7}{13}$ (C) $\frac{44}{169}$ (D) $3\frac{5}{13}$ (E) $\frac{98}{169}$

60. The function *f* is such that $\int_{-1}^{8} f(x)dx = 12$. What is the value of $\int_{-1}^{8} [3f(x) + 5]dx$?

(A) 81 (B) 57 (C) 41 (D) 32 (E) 71

2018-2019 TMSCA Mathematics Test Two Answers

1. D	**21. A**	**41. B**
2. A	**22. C**	**42. E**
3. B	**23. B**	**43. D**
4. E	**24. C**	**44. C**
5. B	**25. D**	**45. D**
6. D	**26. D**	**46. A**
7. A	**27. E**	**47. A**
8. E	**28. A**	**48. D**
9. E	**29. B**	**49. C**
10. C	**30. E**	**50. D**
11. D	**31. D**	**51. A**
12. E	**32. C**	**52. B**
13. C	**33. C**	**53. C**
14. B	**34. E**	**54. A**
15. C	**35. A**	**55. A**
16. D	**36. B**	**56. C**
17. E	**37. E**	**57. E**
18. E	**38. C**	**58. C**
19. A	**39. B**	**59. D**
20. E	**40. A**	**60. A**

2018-2019 TMSCA Mathematics Test Two Solutions

5. $\dfrac{4}{\frac{1}{6}+\frac{1}{8}+\frac{1}{9}} \approx 9.93$ **or 9 hours and 56 minutes. The 4 is in the numerator because doubling the length and width makes the area of the job 4 times the original.**

10. Imagine that the pentagon is inscribed in a circle. Then $m\overset{\frown}{AC} = 72°$ **and the inscribed angle with that intercepted arc is 36°.**

20. Use the mean value theorem for
$\dfrac{1}{8-(-2)}\int_{-2}^{8} f(x)dx = 309$

21. There are two cases:
No repeating letters: $({}_6C_4)(4!) = 360$
2-A's are in the word: $({}_5C_2)\left(\dfrac{4!}{2!}\right) = 120$
For a total of 480 distinct arrangements.

22. $f'(x) = 3x^2 + 6x + A$ **then use** $f'(2) = 12$ **to obtain** $A = -12$ **then** $f(x) = x^3 + 3x^2 - 12x + B$ **and use** $f(2) = 3$ **to solve for** $B = 7$ **and** $f(-1) = 21$

24. $\dfrac{32\pi}{3} = 10\pi + \dfrac{2}{3}\pi$ **where the** 10π **is 5 complete revolutions and the reference angle for** $\dfrac{2\pi}{3}$ **is** $\dfrac{\pi}{3}$ **.**

26. Let the each trip have a distance of 1 mi, then the sum of the individual trips will be equal to the total time to make 5 trips at the average speed:
$\dfrac{1}{22.2}+\dfrac{1}{31.8}+\dfrac{1}{26.3}+\dfrac{1}{25.7}+\dfrac{1}{x}=\dfrac{5}{28}$ **for** $x \approx 39.8$

28. The sum of the roots is $\dfrac{13}{3}$ **and the pair of conjugates are known roots so the remaining root is**
$\dfrac{13}{3} - (3-2i+3+2i) = -\dfrac{5}{3}$.

30. $P+Q = 12-R$ **and** $P^2+2PQ+Q^2-144-24PQ+R^2$ **then** $R^2+2(6) = 144-24R+R^2$ **solve for** $R = 5.5$

31. ${}_{12+3-1}C_3 \times 3 = 1092$

36. $\dfrac{1}{3}\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta} = \dfrac{1}{3}(1) = \dfrac{1}{3}$

37. Use the method of finite differences to establish that this sequence can be modelled with a quadratic, then use a quadratic regression to get a model.

38. (sum of the roots)^4 = $\left(-\dfrac{13}{3}\right)^4 = \dfrac{28561}{81}$

46. This is the definition of derivative at $x = \dfrac{\pi}{3}$ **and**
$f'\left(\dfrac{\pi}{3}\right) = 8$

49. There are 3 diagonals with lengths of 4 cm each for a total of 12 cm. There are 6 more diagonals with lengths of $2\sqrt{3}$ **cm each for a total of** $12\sqrt{3}$ **cm and a total of all 9 diagonals of about 33 cm.**

51. Use a cumulative binomial distribution with 8 trials and a probability of 0.37 for 0-3 homes, then subtract from 1 for 34%.

58. $A = \sqrt{\dfrac{8.25gf(8.25)}{2}} \approx 10.5$

59. Use the formula for distance between a line and point for $d = \dfrac{|36+35-27|}{\sqrt{12^2+5^2}} = \dfrac{44}{13} = 3\dfrac{5}{13}$