T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 4 ©

N O VE M B E R 1 0 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Four

1. **Evaluate:** $(6-8)-8^2+4\div(-2)\times\left(5^2-8\right)$.

   (A) -100 **(B)** $-64\frac{15}{16}$ **(C)** -96 **(D)** $-60\frac{15}{16}$ **(E)** -32

2. $(5x+8)(9-2x)=(9-2x)(5x+8)$ **is an example of _____property of equality.**

   (A) Associative (B) Symmetric (C) Substitution (D) Commutative (E) Distributive

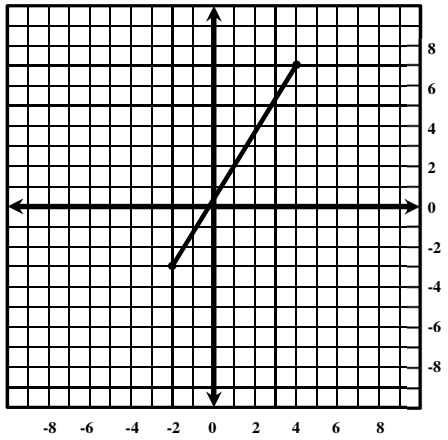
3. **Libby made a cake that called for 3 cups of flour, 2 cups of sugar, 1 cup of butter, 1 tablespoon of baking powder, 1 cup of milk and 1 teaspoon of vanilla. Calculate the price of the cake if flour costs $2.89 for 20 cups; sugar costs $1.79 for 12 cups; butter costs $3.99 for 2 cups; baking powder costs $1.42 for 30 tablespoons; milk costs $2.69 per gallon and vanilla costs $13.99 for 1.5 cups. All of the ingredients in Libby's cake are tax exempt groceries.**

   (A) $2.38 (B) $3.27 (C) $4.32 (D) $3.14 (E) $3.46

4. **485 feet per second = _____ kilometers per hour. (nearest whole number)**

   (A) 82 (B) 532 (C) 825 (D) 887 (E) 349

5. **The coordinates of the endpoints of the line segment shown are all integers. Which of the following is an equation of the line that includes the segment?**

   (A) $3x+5y=47$ **(B)** $3x+5y=-21$ **(C)** $3x+5y=40$

   (D) $5x-3y=-2$ **(E)** $5x-3y=-1$

6. **Find the greatest common factor of** x^4-625 **and** $x^3-5x^2+8x-40$.

   (A) $x+5$ **(B) 1** **(C)** $x-5$ **(D)** x^2+25 **(E)** x^2-25

7. **The total volume of a cylinder is 3400 cm^3, and the radius is 12 cm. The height of the cylinder is _____cm. (nearest tenth)**

   (A) 7.2 (B) 7.5 (C) 30.0 (D) 22.5 (E) 5.6

8. **The point of concurrency of the _____s of a triangle divides each _____ into segments with lengths in a ratio of 2:1.**

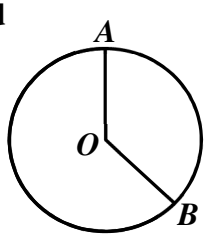
   (A) angle bisector (B) perpendicular bisector (C) median

   (D) altitude (E) Side

9. **The graph of** $9x^2+25y^2-54x+100y=44$ **is an ellipse with a center** (h,k). **Find** $h+k$.

   (A) 1 (B) –1 (C) –5 (D) 5 (E) 8

10. Given the circle with center O shown, OB = 9" and the arclength of $\overset{\frown}{AB}$ is 21", find $m\angle AOB$. (nearest degree)



(A) 122° (B) 130° (C) 141°

(D) 129° (E) 134°

11. Find the shortest distance from the line $y = -\frac{3}{5}x + 1$ to the point $(5,-8)$. (nearest whole number)

A) 6 (B) 4 (C) 7 (D) 5 (E) 8

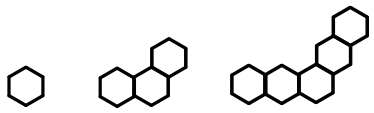
12. $\int_{-a}^{a}\left(x^2 + \cos x\right)dx$.

(A) $\frac{2}{3}a^3$ (B) $\frac{2}{3}a^3 + 2\sin a$ (C) $\frac{1}{3}a^3$ (D) $\frac{1}{3}a^3 - 2\sin a$ (E) $\frac{2}{3}a^3 - 2\sin a$

13. Which of the following are the side lengths of a scalene, acute triangle? (All lengths are centimeters.)

(A) 16, 7, 12 (B) 12, 9, 14 (C) 16, 18, 25 (D) 27, 38, 22 (E) 30, 24, 18

14. The three shapes below are made up of regular hexagons with side lengths of 1 unit. If the pattern continues, the perimeter of the shape with 87 hexagons will is _____units.

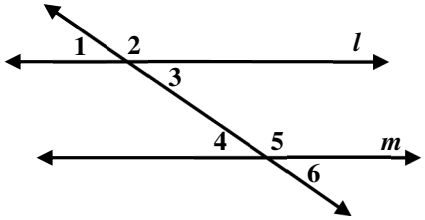


(A) 342 (B) 348 (C) 344 (D) 352 (E) 350

15. The length of a rectangular movie screen is 2 feet less than double the width, and the length of the diagonal is 2 feet more than double the width. The area of the movie screen is _____ft^2.

(A) 544 (B) 480 (C) 272 (D) 92 (E) 240

16. The three lines in the figure are coplanar with $m \,//\, l$. Which of the following are false?



I. $\angle 2$ and $\angle 3$ are supplementary II. $\angle 5$ and $\angle 6$ are vertical angles

III. $\angle 4$ and $\angle 5$ are a linear pair IV. $\angle 2$ and $\angle 4$ are corresponding angles

(A) II only (B) III only (C) II and IV (D) IV only (E) I and IV

17. The letters "ARCHIMEDES" are arranged in a line. How many distinct arrangements are possible if all of the vowels occur together?

(A) 8,640 (B) 151,200 (C) 75,600 (D) 25,200 (E) 60,480

18. If $(3x-1)^2+4=ax^2+bx+c$ then $a+b+c=$ _____.

(A) 14 **(B)** 5 **(C)** 8 **(D)** 9 **(E)** 4

19. If $\cos\theta=-0.96$ and θ is in QIII, then $\tan\theta=$?

(A) $\frac{24}{7}$ **(B)** $\frac{7}{24}$ **(C)** $\frac{7}{18}$ **(D)** $-\frac{7}{24}$ **(E)** $-\frac{24}{7}$

20. What is the constant term in the expansion of $\left(2x-\frac{1}{x^4}\right)^{10}$?

(A) 1024 **(B)** 3360 **(C)** 6561 **(D)** 11,520 **(E)** 720

21. $\angle A$ and $\angle B$ are complementary and $m\angle B$ is 22° less than three times $m\angle A$. Find the measure of the supplement of $\angle A$.

(A) 146° **(B)** 152° **(C)** 101° **(D)** 56° **(E)** 50.5°

22. Given that $\int_0^4\left(3x^3+kx-22\right)dx=168$, find the value of k.

(A) 16 **(B)** 4 **(C)** 12 **(D)** 8 **(E)** 18

23. Let $f(x)=x^2-4x-6$ and $g(x)=x^2+2x-1$. Find $g\left(f'(2)\right)$.

(A) –1 **(B)** 1 **(C)** –6 **(D)** 79 **(E)** 6

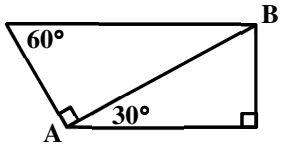
24. Two dice are rolled. What is the probability that the product of the top faces is a perfect number?

(A) $\frac{5}{36}$ **(B)** $\frac{1}{17}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{18}$ **(E)** $\frac{1}{9}$

25. The 3rd term of an infinite geometric sequence is 12 and the 6th term is $3\frac{5}{9}$. What is the sum of the sequence?

(A) 27 **(B)** 18 **(C)** 81 **(D)** 54 **(E)** 72

26. The two triangles shown together form a trapezoid. If AB = 12 feet, the area of the trapezoid is _____ft².



(A) $42\sqrt{3}$ **(B)** 42 **(C)** 21 **(D)** $14\sqrt{6}$ **(E)** $21\sqrt{6}$

27. Simplify: $\sqrt{5a^3b}\cdot\sqrt[4]{125ab^2}$

(A) $5ab\sqrt{5a^3}$ **(B)** $5ab\sqrt[4]{5a^3}$ **(C)** $5ab\sqrt{5b^3}$ **(D)** $5ab\sqrt[4]{5b^3}$ **(E)** $5ab$

28. Which of the following is defined by $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$?

(A) ε **(B)** φ **(C)** π **(D)** e **(E)** $\frac{\pi}{2}$

29. The graph of the function $f(x)=ax^3-bx^2$ where a and b are both positive integers has a point of inflection when $x=$ _____.

(A) $\frac{2b}{3a}$ **(B)** $-\frac{b}{a}$ **(C)** $\frac{b}{3a}$ **(D)** $-\frac{2b}{3a}$ **(E)** $-\frac{b}{3a}$

30. Repeating decimal 0.545454… in base 7 can be written as which of the following fraction in base 7?

(A) $\frac{9}{11_7}$ **(B)** $\frac{16}{22_7}$ **(C)** $\frac{6}{11_7}$ **(D)** $\frac{9}{22_7}$ **(E)** $\frac{36}{101_7}$

31. A 20-foot rope is cut into two pieces and used to form a rectangle and triangle, both with side lengths that are positive integers. What is the maximum possible combined area of the triangle and rectangle? (nearest square foot)

(A) 11.8 sq. ft. (B) 10.5 sq. ft. (C) 12.1 sq. ft. (D) 13.7 sq. ft. (E) 25 sq. ft.

32. Determine the range of $f(x)=7\cos\left[-3\left(x-\frac{\pi}{2}\right)\right]-5$.

(A) $[-12,2]$ **(B)** $[-10,4]$ **(C)** $[-2,12]$ **(D)** $[2,12]$ **(E)** $[-4,10]$

33. Find *K* if the triangular pattern continues:

						1						
					3		1					
				9		6		1				
			27		27		9		1			
		81		108		*K*		12		1		

(A) 108 (B) 27 (C) 81 (D) 25 (E) 54

34. Which of the following mathematicians is one of a small group of American Indians who earned a Ph.D. in mathematics and is doing work applying mathematical models to the study of groundwater contamination?

(A) Emmy Noether (B) Ada Byron (C) Hypatia (D) Freda Porter (E) Alicia Scott

35. $D93_{14}+2AC_{14}=$ _____$_{14}$

(A) 2261 (B) 1661 (C) 1262 (D) 1261 (E) 2251

36. Let $f(x)=ax^3-bx-18$ where a and b are constants. If $f(9)=48$ then $f(-9)=$?

(A) –66 (B) –84 (C) –48 (D) 48 (E) 66

37. If $a+b=32$ and $a\times b=48$ then $|a-b|=$?

(A) $8\sqrt{13}$ **(B)** $4\sqrt{26}$ **(C)** $4\sqrt{58}$ **(D)** $4\sqrt{70}$ **(E)** $4\sqrt{76}$

38. Point $P(-3,2)$ lies in the *x*-*y* plane. Point P is reflected across the line $y=-x$ to point Q. Point Q is reflected across the *x*-axis to point R. Point R is reflected across the line $y=2$ to point $S(x,y)$. Find $x+y$.

(A) 1 (B) 5 (C) 7 (D) –2 (E) 3

39. Let $f(x)=\dfrac{2-x}{3x+5}$. Find $f^{-1}(3)$.

(A) $-\dfrac{11}{196}$ **(B)** $-\dfrac{1}{14}$ **(C)** $-\dfrac{3}{10}$ **(D)** $-\dfrac{13}{10}$ **(E)** $-\dfrac{17}{10}$

40. Using a polynomial function that fits this data, find $f(12)$.

x	–2	–1	0	1	2	3	4
$f(x)$	140	–7	–102	–181	–208	–75	398

(A) 61,782 **(B)** 43,496 **(C)** 85,295 **(D)** 21,303 **(E)** 27,191

41. The polynomial $x^2-2x-15$ is a factor of $x^3+ax^2+bx-30$. Find the value of the constant a.

(A) –19 **(B)** 19 **(C)** 0 **(D)** 29 **(E)** 38

42. Evaluate $\lim\limits_{h\to 0}\dfrac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}-h\right)}{2h}$.

(A) $\dfrac{4}{3}$ **(B)** $\dfrac{3}{4}$ **(C)** 2 **(D)** $\dfrac{2\sqrt{3}}{3}$ **(E)** does not exist

43. How many asymptotes does $f(x)=\dfrac{8x^3-27}{6x^2-13x+6}$ have?

(A) 4 **(B)** 2 **(C)** 3 **(D)** 0 **(E)** 1

44. The secant to a circle contains the center. The secant intersects a tangent outside the circle. Find the measure of the angle formed by the secant and tangent if the smaller intercepted arc measures 42°.

(A) 48° **(B)** 42° **(C)** 52° **(D)** 69° **(E)** 21°

45. Express $\ln\left(x^6\right)-3\ln\sqrt[3]{x^5}$ as a single, simplified logarithm.

(A) $\ln x^2$ **(B)** $\ln\sqrt{x}$ **(C)** $\ln\left(x\sqrt{x}\right)$ **(D)** $-\ln x$ **(E)** $\ln x$

46. $\dfrac{1}{\sin x}-\sin x=$

(A) $\sec x\cot x$ **(B)** $\cot x$ **(C)** $\csc x$ **(D)** $\cos x\cot x$ **(E)** $\sec x$

47. Three cards are drawn from a standard 52-card deck *without replacement*. What is the probability that the cards drawn all hearts?

(A) $\dfrac{33}{2704}$ **(B)** $\dfrac{1}{64}$ **(C)** $\dfrac{11}{850}$ **(D)** $\dfrac{1}{13}$ **(E)** $\dfrac{3}{64}$

48. Find the Real number solution set of $\left|\frac{1}{2}+5x\right| \geq 20$.

(A) $\left(-\infty,-\frac{39}{10}\right]\cup\left[\frac{41}{10},\infty\right)$ **(B)** $\left(-\infty,-\frac{41}{10}\right]\cup\left[\frac{39}{10},\infty\right)$ **(C)** $\left[-\frac{41}{10},\frac{39}{10}\right]$

(D) $\left(-\frac{41}{10},\frac{39}{10}\right)$ **(E)** $\left[-\frac{39}{10},\frac{41}{10}\right]$

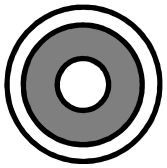
49. If $x-\frac{1}{x}=12$, then $x^2+\frac{1}{x^2}$

(A) 144 **(B)** 146 **(C)** 142 **(D)** 148 **(E)** 150

50. What is the angle between the vectors $\langle 3,7\rangle$ and $\langle -4,9\rangle$. (nearest degree)

(A) 50° **(B)** 47° **(C)** 48° **(D)** 52° **(E)** 46°

51. The diameter of the concentric circles on the right are 8 cm, 7 cm and 5 cm. If a dart hits the figure at random, what is the probability that the dart will land in the shaded region?



(A) $\frac{25}{64}$ **(B)** $\frac{49}{64}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{25}{39}$

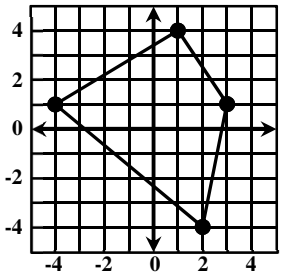
52. If $\frac{3}{8}$ of A is $\frac{2}{5}$ of B, then A is what percent of B? (nearest whole percent)

(A) 94% **(B)** 15% **(C)** 107% **(D)** 67% **(E)** 96%

53. Carol drove an average speed of 32 mph on her way to work. On the way home, she averaged 12 mph due to heavy traffic. If Carol's time on the road was a total of 2 hours and 45 minutes, how far was a one-way trip?

(A) 27 mi **(B)** 36 mi **(C)** 32 mi **(D)** 20 mi **(E)** 24 mi

54. The coordinates of each of the vertices of the quadrilateral shown are all integers. Calculate the area of the quadrilateral?



(A) 28 **(B)** 32 **(C)** 25 **(D)** 31 **(E)** 30

55. How many perfect cubes are factors of $(4!)(5!)(6!)$?

(A) 8 **(B)** 9 **(C)** 5 **(D)** 7 **(E)** 4

56. According to Descartes' Rule of Signs, how many possible negative real roots could $f(x) = 7x^5 + 13x^4 - 3x^3 - 4x - 12$ have?

(A) 0 (B) 0 or 1 (C) 0 or 2 (D) 1 or 3 (E) 1 only

57. Given that x varies inversely with $y - 3$, and $x = 9$ when $y = 6$. Find the value of y when $x = 36$.

(A) $\frac{5}{6}$ (B) 12 (C) $3\frac{3}{4}$ (D) $-2\frac{1}{4}$ (E) $2\frac{1}{4}$

58. Lindsay's final homework average in math class is 94% which counts for 15% of her final grade. Her test average is 87% which is 65% of her final grade. Her final exam makes up the rest of her final grade. What is the lowest score Caroline can earn on her final to earn a 90% grade in the class? Assume that only integers are possible, and that her teacher will not round up.

(A) 97% (B) 89% (C) 94% (D) 99% (E) 92%

59. If $\frac{x-11}{x+5} + \frac{x+5}{x-11}$ is equal to the mixed number $A + \frac{B}{(x+5)(x-11)}$, then B = ?

(A) 32 (B) 64 (C) 256 (D) 36 (E) 72

60. $(\ln x^2)^3 + (\ln x^3)^3 + (\ln x^4)^3 + \ldots + (\ln x^{12})^3 =$

(A) $6084(\ln x)^3$ (B) $6083 \ln x^3$ (C) $\ln x^{231}$ (D) $6083(\ln x)^3$ (E) $(\ln x)^{231}$

2018-2019 TMSCA Mathematics Test Four Answers

1. A	**21. B**	**41. C**
2. D	**22. D**	**42. A**
3. D	**23. A**	**43. B**
4. B	**24. E**	**44. A**
5. E	**25. C**	**45. E**
6. C	**26. A**	**46. D**
7. B	**27. B**	**47. C**
8. C	**28. D**	**48. B**
9. A	**29. C**	**49. B**
10. E	**30. B**	**50. B**
11. D	**31. D**	**51. D**
12. B	**32. A**	**52. C**
13. B	**33. E**	**53. E**
14. E	**34. D**	**54. A**
15. B	**35. D**	**55. A**
16. C	**36. B**	**56. C**
17. E	**37. A**	**57. C**
18. C	**38. B**	**58. A**
19. B	**39. D**	**59. C**
20. D	**40. A**	**60. D**

2018-2019 TMSCA Mathematics Test Four Solutions

9. Complete the squares for $9\left(x^2-6x+9\right)+25\left(y^2+4y+4\right)=44+81+100$ **then** $\frac{(x-3)^2}{25}+\frac{(y+2)^2}{9}=1$**, a center of** $(3,-2)$ **and a sum of 1.**

12. $\int_{-a}^{a}\left(x^2+\cos x\right)dx=\frac{x^3}{3}+\sin x\Big]_{-a}^{a}=$

$\frac{2a^3}{3}+\sin a-\sin(-a)=\frac{2a^3}{3}+2\sin a$

17. The combinations of the group of vowels along with the seven distinct consonants is 7! = 5040. The arrangements of the vowels within their group is $\frac{4!}{2!}=12$ **due to the repeating E's for a total of** $(5040)(12)=60{,}480$**.**

19. $\pm\sqrt{1-0.96^2}=\pm0.28$ **, so** $\sin\theta=-0.28$**. It is negative because the angle lies in QIII. Then** $\tan\theta=\frac{-0.28}{-0.96}=\frac{7}{24}$ **.**

20. ${}_{10}C_2(2x)^8\left(-\frac{1}{x^4}\right)^2=11{,}520$

22. $\left[\frac{3x^4}{4}+\frac{kx^2}{2}-22x\right]_0^4=192+8k-88=168$ **for** $k=8$**.**

24. The only perfect number that is a possible product is 6 and there are four rolls that result in a product of 6: $(1\times6),(6\times1),(2\times3),(3\times2)$ **and probability** $\frac{4}{36}=\frac{1}{9}$

25. $12r^3=\frac{32}{9}$ **and** $r=\frac{2}{3}$**, then solve** $a\left(\frac{2}{3}\right)^2=12$ **for** $a=27$**.**

Finally, evaluate $S=\frac{27}{1-\frac{2}{3}}=81$**.**

26. Use the special triangle relationships to get bases $6\sqrt{3}$ **and** $8\sqrt{3}$ **and height 6, then** $A=\frac{6}{2}\left(6\sqrt{3}+8\sqrt{3}\right)=42\sqrt{3}$

31. The largest possible area is 3 by 4 rectangle and an equilateral triangle with side lengths 2.

33. Each row is the coefficients of the expansion of $(3x+2)^n$ **, where** n **is the row number and for the top "1",** $n=0$**. The coefficients of** $(3x+1)^4$ **are 81, 108, 54, 12 and 1.**

37. $|a-b|=\sqrt{32^2-4(48)}=\sqrt{832}=\sqrt{64(13)}=8\sqrt{13}$

40. Use quartic regression.

43. $f(x)=\frac{(2x-3)\left(4x^2+6x+9\right)}{(2x-3)(2x+3)}$ **. The matching factors in the numerator and denominator create a hole. The other factor in the denominator creates a vertical asymptote, and because the degree of the numerator is one higher than the denominator, the graph has a slant asymptote.**

44. The intercepted arcs together form a semicircle, so the other intercepted arc is $138°$ **and** $\frac{138°-42°}{2}=48°$**.**

49. $x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+4=12^2+4=148$

50. $\cos\theta=\frac{3(-4)+7(9)}{\sqrt{9+49}\bullet\sqrt{16+81}}$ **for** $\theta\approx47°$

53. $\frac{d}{32}+\frac{d}{12}=2.75$ **for** $d=24$ **mi.**

57. Solve $9=\frac{k}{3}$ **for** $k=27$ **and** $x=\frac{27}{y-3}$ **and** $y=3\frac{3}{4}$ **when** $x=36$**.**

60.

$(2\ln x)^3+(3\ln x)^3+(4\ln x)^3\ldots=8(\ln x)^3+27(\ln x)^3+64(\ln x)^3\ldots$

, so the sum of the coefficients will be the sum of cubes from 2^3 **to** 12^3 **will be** $\left(\frac{12\times13}{2}\right)^2-1=6083$ **and** $6083(\ln x)^3$