T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 3 ©

N O V E M B E R 4 , 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2017-2018 TMSCA Mathematics Test Three

1. **Evaluate:** $1-3\times 9+30\div(6-1)\times 1-5$.

   (A) -25 **(B)** -10 **(C)** -12 **(D)** -79 **(E)** -27

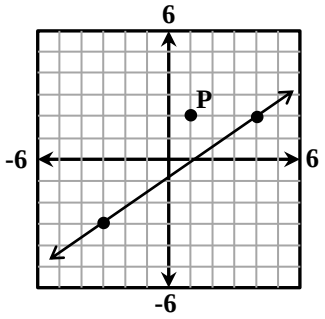
2. **Carl had \$120 to spend. He spent 60% on games, then 30% of what he had left on lunch. If he spent \$11.25 on a movie, how much did he have to take home?**

   (A) \$10.35 **(B)** \$8.50 **(C)** \$12.50 **(D)** \$22.35 **(E)** \$3.15

3. **The *y*-intercept of the line through point P that is perpendicular to the line shown is** (x,y). **Find** $x+y$.

   (A) -2.4 **(B)** 4.4 **(C)** 3.4

   (D) 5.2 **(E)** 3.8



4. **A rhombus has a perimeter of 48 cm and one diagonal length of 16.2 cm. What is the area of the rhombus? (nearest square centimeter)**

   (A) 72 cm^2 **(B)** 96 cm^2 **(C)** 143 cm^2 **(D)** 83 cm^2 **(E)** 288 cm^2

5. **Which of the following relations describes a 1-1 function?**

   (A) $\{(2,4),(-2,-4),(3,5),(-3,-5)\}$ **(B)** $\{(2,4),(-2,4),(3,5),(-3,5)\}$ **(C)** $\{(2,4),(2,-4),(3,5),(3,-5)\}$

   (D) $\{(-2,4),(-2,-4),(-3,5),(-3,-5)\}$ **(E)** **none of these**

6. **Find the numeric value of** $\frac{1+1}{1}\div\frac{2}{1+2}\times\frac{1+3}{3}\div\frac{4}{1+4}\times\ldots\div\frac{14}{1+14}\times\frac{1+15}{15}\div\frac{16}{1+16}$.

   (A) $\frac{17}{16}$ **(B)** $\frac{16}{15}$ **(C)** $\frac{17}{2}$ **(D)** 17 **(E)** 16

7. **If** $18x^2-x-39=(ax+b)(cx+d)$ **then** $ab+cd=$ **?**

   (A) 34 **(B)** 111 **(C)** 78 **(D)** 123 **(E)** 30

8. **75 mph is the same as _____ inches per second.**

   (A) 110 **(B)** 6600 **(C)** 2112 **(D)** 3960 **(E)** 1320

9. **Find the greatest common divisor of** $2^3\times 7^2$, $3^2\times 2^7$ **and** $3^7\times 2$.

   (A) 1 **(B)** 21 **(C)** 42 **(D)** 14 **(E)** 2

10. **What is the measure of a central angle in a regular dodecagon?**

    (A) 30° **(B)** 120° **(C)** 36° **(D)** 150° **(E)** 60°

11. Events *A* and *B* are independent such that $p(B)=4p(A)$ and $p(A\cup B)=\frac{21}{25}$. Find $p(A)$.

(A) $\frac{4}{5}$ **(B)** $\frac{9}{25}$ **(C)** $\frac{4}{25}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{3}{20}$

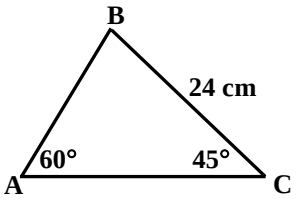
12. Lily has 9 non-fiction books to shelve. She wants to keep her 4 math books together, but otherwise any order is fine. In how many different ways can Lily organize her shelf?

(A) 17280 **(B)** 2880 **(C)** 3024 **(D)** 720 **(E)** 2160

13. Find the perimeter of triangle ABC. (nearest centimeter)

(A) 57 cm **(B)** 73 cm **(C)** 70 cm

(D) 63 cm **(E)** 64 cm



14. Simplify $\left(b^3\right)\times\frac{\left(b^5\right)^{-3}\left(\sqrt{b}\right)}{b^{-4}}$.

(A) $\left(\sqrt{b}\right)^{-7}$ **(B)** $\left(\sqrt{b}\right)^{-23}$ **(C)** $\left(\sqrt{b}\right)^{-11}$ **(D)** $\left(\sqrt{b}\right)^{-7}$ **(E)** $\left(\sqrt{b}\right)^{-15}$

15. Given that $\angle P$ is supplementary to $\angle Q$; $m\angle R=41^\circ$; and $\angle Q$ is complementary to $\angle R$, find $m\angle P$.

(A) 143° **(B)** 131° **(C)** 139° **(D)** 49° **(E)** 135°

16. Determine the range of $f(\theta)=4+5\sin\left(\frac{4\pi}{3}\theta-2\right)$.

(A) $[-12,8]$ **(B)** $[-7,3]$ **(C)** $[-1,9]$ **(D)** $[-2,8]$ **(E)** $[-6,4]$

17. Two fair standard six-sided dice are tossed and the top numbers are added. What are the odds that the sum is an abundant number?

(A) 1:17 **(B)** 5:13 **(C)** 1:8 **(D)** 1:35 **(E)** 5:31

18. Given the sequence 1, 12, 25, 40, 57, 76, *k*, 120, … find *k*.

(A) 91 **(B)** 95 **(C)** 98 **(D)** 99 **(E)** 97

19. If $\frac{5x-4}{x-2}+\frac{x+2}{3x-2}=\frac{ax^2+bx+c}{px^2+qx+r}$ **then** $\frac{a+b+c}{p+q+r}=$

(A) –7 **(B)** –6 **(C)** 2 **(D)** –2 **(E)** 7

20. How many cups of water are needed to fill a pitcher in the shape of a frustum with base diameters 6 inches and 8 inches and a height of 10 inches? (nearest cup)

(A) 27 cups **(B)** 14 cups **(C)** 13 cups **(D)** 12 cups **(E)** 28 cups

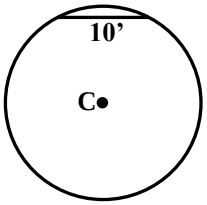
21. How many distinguishable arrangements can be made from the letters "HARRIS"?

(A) 240 **(B)** 144 **(C)** 360 **(D)** 540 **(E)** 720

22. What is the constant term in the expansion of $\left(3x^2-\frac{3}{x}\right)^9$?

(A) $-1,653,372$ **(B)** 61,236 **(C)** 1,653,372 **(D)** 551,124 **(E)** $-551,124$

23. The circle C shown has a diameter of 26 feet. How far is the chord shown from the center of the circle?



(A) 16 ft **(B)** 12 ft **(C)** 9 ft **(D)** 13 ft **(E)** $13\sqrt{2}$ ft

24. If $\frac{A}{x+8}+\frac{B}{2x+5}=\frac{3x+46}{2x^2+21x+40}$, then $A+B=$:

(A) -6 **(B)** 5 **(C)** -2 **(D)** 4 **(E)** -5

25. The first term of an infinite geometric sequence is 729, while the fifth term is 9. What is the smallest possible value for the sum of all the terms in the sequence?

(A) 1093.5 **(B)** 1089 **(C)** 549 **(D)** 369 **(E)** 546.75

26. Using the following array, determine the value of the median of the 15th row.

1					(row 1)
3	5				(row 2)
7	9	11			(row 3)
13	15	17	19		(row 4)
21	23	25	27	29	(row 5)
…					(…)

(A) 169 **(B)** 289 **(C)** 198 **(D)** 225 **(E)** 256

27. Let $f(x)=x^2-x$ and $g(x)=x^3$. Calculate $f(g(-1))$.

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

28. Which of the following is the reference angle for 225°?

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{6}$ **(E)** $\frac{\pi}{7}$

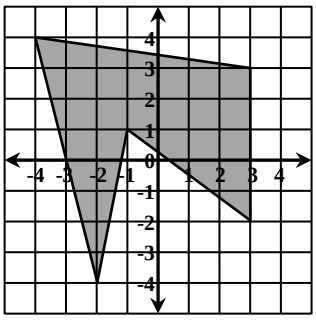
29. The ADA recommends that wheelchair ramps have no more than a 5° angle of elevation. Paul needs to build a ramp up to a porch that stands 3 feet 2 inches off the ground. How long should Paul make the ramp itself? (nearest inch)

(A) 36 inches **(B)** 432 inches **(C)** 436 inches **(D)** 320 inches **(E)** 328inches

30. What is the probability that a factor of 225 is a multiple of 3?

(A) $\frac{3}{7}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{2}{7}$

31. The coordinates of the vertices of the pentagon shown are all integers. What is the area of the pentagon?



(A) 37 units^2 **(B)** 31 units^2 **(C)** 35 units^2

(D) 26 units^2 **(E)** 28 units^2

32. $1111_5 + 1111_4 + 1111_3 =$ _____$_6$.

(A) 1100 **(B)** 1030 **(C)** 1003 **(D)** 1010 **(E)** 1145

33. Which of the following are the side lengths of a scalene acute triangle?

(A) 10, 12, 12 **(B)** 8, 10, 12 **(C)** 10, 12, 16 **(D)** 12, 16, 20 **(E)** 16, 20, 26

34. A square has a perimeter of 18 cm. A right triangle with one leg length of 16 cm has the same area as the square. What is the perimeter of the triangle? (nearest tenth)

(A) 34.7 cm **(B)** 36.4 cm **(C)** 34.2 cm **(D)** 34.5 cm **(E)** 30.2cm

35. Let P and Q be the roots of $2x^2 + 17x - 19$ **. Find** $P^4 + 4P^3Q + 6P^2Q^2 + 4PQ^3 + Q^4$ **.**

(A) $\frac{130,321}{16}$ **(B)** $\frac{83,521}{256}$ **(C)** $-\frac{83,521}{16}$ **(D)** $\frac{83,521}{16}$ **(E)** $\frac{130,321}{256}$

36. Find the mean value of the function $f(x) = 2x^3 - 8x$ **over the interval** $[-1, 9]$ **.**

(A) 48 **(B)** 478 **(C)** 296 **(D)** 238 **(E)** 370

37. The quantity *y* varies inversely with the quantity *x*. Which of the following equations could represent the relationship?

I. $y = \frac{\pi}{5x}$ **II.** $9x - 8y = 0$ **III.** $2xy = 11$ **IV.** $3x + 7y = 9$

(A) II only **(B)** II & IV **(C)** I, II & IV **(D)** I and III **(E)** III and IV

38. What is the angle between the minute and hour hands on a circular clock at 8:17 pm?

(A) 142° **(B)** 146.5° **(C)** 155.85° **(D)** 138° **(E)** 134°

39. The intersection of the three altitudes in a triangle is called the_____.

(A) Incenter **(B)** Orthocenter **(C)** Center **(D)** Centroid **(E)** Circumcenter

40. A polyhedron has 20 faces and 30 edges, how many vertices does it have?

(A) 12 **(B)** 52 **(C)** 14 **(D)** 48 **(E)** 10

41. Given that the binomial $x-3$ is a factor of $2x^3-Ax^2+Ax-24$, calculate the value of *A*.

(A) -5 **(B)** 3 **(C)** 5 **(D)** 0 **(E)** -3

42. Brothers Barry, Carl and Dave can each roof a house alone in 60 hours, 64 hours and 68 hours respectively. How long will it take the brothers to roof a house if Barry and Carl start together and Dave joins them for the last two-thirds of the job? (nearest hour)

(A) 22 hours **(B)** 27 hours **(C)** 26 hours **(D)** 24 hours **(E)** 25 hours

43. Let $f_0=0$, $f_1=1$, $f_2=1$, $f_3=2$, $f_4=3$ be the terms of the Fibonacci sequence. Find f_{19}.

(A) 2584 **(B)** 6765 **(C)** 3610 **(D)** 4181 **(E)** 5473

44. If $a_0=-2$, $a_1=3$, $a_2=5$ and $a_n=\left(a_{n-3}\right)\left(a_{n-2}\right)+a_{n-1}$ for $n\geq 3$, then $a_6=$?

(A) 9 **(B)** 121 **(C)** -23 **(D)** -5 **(E)** 23

45. The function $f\left(x\right)=2x^4-3x^2$ is decreasing over which of the following intervals?

(A) $\left(-\infty,0\right]$ **(B)** $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$ **(C)** $\left[0,\infty\right)$ **(D)** $\left[0,\frac{\sqrt{6}}{2}\right]$ **(E)** $\left[0,\frac{\sqrt{3}}{2}\right]$

46. How many 3-digit numbers exist such that the sum of their digits equals 9?

(A) 44 **(B)** 42 **(C)** 46 **(D)** 38 **(E)** 45

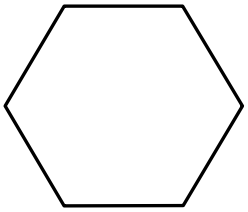
47. If $z=a+bi$, where $a,b\in\mathbb{R}$, $z^4=1241+2520i$ and $z^3=-286+259i$ find the value of $a+b$.

(A) -5 **(B)** 7 **(C)** 2 **(D)** 5 **(E)** -7

48. Zoe has bins of apple, cherry, grape, watermelon and strawberry lollipops. In how many ways can she package them in bags of six for a fundraiser?

(A) 210 **(B)** 126 **(C)** 120 **(D)** 462 **(E)** 504

49. Find the sum of the length of all the diagonals of a regular hexagon if the length of each side is 12 ft. (nearest foot)



(A) 185 ft **(B)** 174 ft **(C)** 151 ft

(D) 197 ft **(E)** 144 ft

50. Given $y=\ln\left(4x-3\right)$, find the value of *x* for which $\frac{dy}{dx}=\frac{dx}{dy}$.

(A) 1.5 **(B)** 1 **(C)** 2.5 **(D)** 0 **(E)** 1.75

51. If $P=\begin{bmatrix}-2 & -4\\ 5 & 1\end{bmatrix}$ and $P^3=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $ad-bc=$?

(A) 7992 **(B)** 324 **(C)** 5832 **(D)** 6912 **(E)** 484

52. The inequality $x+2>\frac{12}{x-2}$ is true for what percentage of the real number values on the interval $[-5,5]$?

(A) 30% **(B) 60%** **(C) 80%** **(D) 70%** **(E) 40%**

53. If $\frac{x+9}{x-9}+\frac{x-9}{x+9}=2+\frac{B}{(x-9)(x+9)}$ where $B\in\mathbb{Z}^+$ then $B=$?

(A) 27 **(B) 324** **(C) 162** **(D) 81** **(E) 54**

54. If $x-\frac{1}{x}=17$, then $x^3-\frac{1}{x^3}=$?

(A) 4913 **(B) 4930** **(C) 4947** **(D) 4964** **(E) 4981**

55. Given $f(x)=ax^5+bx^3+cx-17$ and $f(4)=11$, calculate $f(-4)$.

(A) −28 **(B) −34** **(C) 62** **(D) −62** **(E) −45**

56. If $f(x)=3x^2-5x-12$ and $g(x)=x-2$, and $S(x)$ be the slant asymptote of $\frac{f(x)}{g(x)}$, then $S(2)=$?

(A) 7 **(B) −5** **(C) −7** **(D) 13** **(E) −9**

57. A fair coin is flipped 6 times. What is the probability that the coin will come up heads exactly four times in a row?

(A) $\frac{3}{64}$ **(B) $\frac{5}{64}$** **(C) $\frac{3}{32}$** **(D) $\frac{7}{64}$** **(E) $\frac{5}{32}$**

58. The lengths of the sides of ΔPQR are the roots of $f(x)=x^3-15x^2+68x-84$. Find the area of ΔPQR. (nearest tenth)

(A) 5.6 **(B) 11.1** **(C) 1.9** **(D) 1.6** **(E) 3.2**

59. Find the shortest distance between the *x*-intercept of the line $2x+7y=14$ to the line $5x-6y=-48$. (nearest tenth)

(A) 4.6 **(B) 10.6** **(C) 7.7** **(D) 4.5** **(E) 8.1**

60. $A378C_{13}+6B927_{13}=$ _____$_{13}$

(A) $1523B6_{13}$ **(B) $1513C6_{13}$** **(C) $C13B6_{13}$** **(D) $1423B6_{13}$** **(E) $1513C4_{13}$**

2017-2018 TMSCA Mathematics Test Three Answers

1. A	**21. C**	**41. C**
2. D	**22. C**	**42. E**
3. C	**23. B**	**43. D**
4. C	**24. B**	**44. D**
5. A	**25. E**	**45. E**
6. D	**26. D**	**46. E**
7. B	**27. E**	**47. A**
8. E	**28. B**	**48. A**
9. E	**29. C**	**49. D**
10. A	**30. C**	**50. E**
11. D	**31. E**	**51. C**
12. A	**32. E**	**52. D**
13. C	**33. B**	**53. B**
14. E	**34. A**	**54. D**
15. B	**35. D**	**55. E**
16. C	**36. C**	**56. A**
17. D	**37. D**	**57. B**
18. E	**38. B**	**58. A**
19. C	**39. B**	**59. B**
20. A	**40. A**	**60. D**

2017-2018 TMSCA Mathematics Test Three Select Solutions

4. Find the length of the other diagonal by $d_2 = 2\sqrt{12^2 - 8.1^2} \approx 17.7$ then the area is $\frac{1}{2}(16.2)(17.7) \approx 143$

11. $A \cup B = P(A) + P(B) - P(A) \times P(B)$ solve $\frac{21}{25} = 4P(A) + P(A) - 4\left(P(A)\right)^2$ for $P(A) = \frac{1}{5}$

12. Group the 4 together as one object, then $6! = 720$ arrangements, but there are also $4! = 24$ arrangements within the group for 17,280 possible arrangements.

16. The range of the parent function is $[-1,1]$. The coefficient 5 would lead to a range of $[-5,5]$ then the 4 moves the whole graph up for a range of $[-1,9]$.

17. The only abundant number 1 – 12 is 12, so the odds of rolling a sum that is an abundant number is 1:35.

20. $V = \frac{10\pi}{3}\left(4^2 + 4 \bullet 3 + 3^2\right) \approx 387 in^3$ then $387 in^3 \bullet \frac{1 gal}{231 in^3} \bullet \frac{16 c}{1 gal} \approx 27 c$

22. $\left({}_9C_3\right)\left(3x^2\right)^3\left(-\frac{3}{x}\right)^6 = 1{,}653{,}372$

23. The diameter perpendicular to the 10' chord will bisect the chord, then use the right triangle formed with the 13' radius for $d = \sqrt{13^2 - 5^2} = 12$

25. $r = \pm\frac{1}{3}$ for sums of $\frac{729}{1 - \frac{1}{3}} = 1093.5$ and $\frac{729}{1 + \frac{1}{3}} = 546.75$

31. Find the area of any polygon using coordinates of vertices by $A = \left|\frac{(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) \ldots + (x_ny_1 - y_nx_1)}{2}\right|$ where the points are arranged in either clockwise or counter-clockwise order. So here $A = 28$ square units

33. The side lengths are all different and $a^2 + b^2 > c^2$ for the triangle with sides 8, 10, 12.

40. $V + F - E = 2$ for $V + 20 - 30 = 2$ and $V = 12$

41. $2(3)^3 - A(3)^2 + A(3) - 24 = 0$ for $A = 5$

42. $\frac{\frac{1}{3}}{\frac{1}{60} + \frac{1}{64}} + \frac{\frac{2}{3}}{\frac{1}{60} + \frac{1}{64} + \frac{1}{68}} \approx 25h$

47. $\frac{1241 + 2520i}{-286 + 259i} = 2 - 7i$ for $a + b = -5$

48. ${}_{(5+6-1)}C_6 = 210$

50. $\frac{4}{4x - 3} = \pm 1$ only the positive solution is in the domain of the original function, for $x = 1.75$

52. The expression is true in the intervals $(-4,2)$ and $(4,5]$ which is 7 out of 10 values or 70%.

54. $x^3 - \frac{1}{x^3} = (17)\left(17^2 + 3\right) = 4964$

57. The arrangements are hhhhtt, thhhht, tthhhh, hthhhh and hhhhth, for probability of $5\left(\frac{1}{2}\right)^6 = \frac{3}{64}$

58. $s = \frac{15}{2} = 7.5$, then the area is $\sqrt{7.5 \bullet f(7.5)} \approx 5.6$

59. $d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ when the line is in the form $ax + by + c = 0$ for $d = \frac{|7(5) + 0(-6) + 48|}{\sqrt{25 + 36}} \approx 10.6$