TMSCA HIGH SCHOOL MATHEMATICS TEST #9 ©

JANUARY 26, 2019

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Nine

1. **Evaluate:** $(8-4)!-5^2-3\div(-1)\times\left(3^2+2\right)-6$.

   (A) 26 **(B)** 14 **(C)** 6 **(D)** $-26\frac{8}{11}$ **(E)** $-6\frac{8}{11}$

2. **The city of Austin, TX grew from a population of 629 in 1850 to 790,390 in 2010. What was the average annual growth rate for the population of Denton during that 160-year period? (nearest tenth of a percent)**

   (A) 2.2% **(B)** 4.6% **(C)** 4.8% **(D)** 4.2% **(E)** 5.1%

3. **2111A837B is divisible by 5 and has a remainder of 2 when it is divided by 9. What is the smallest possible value of A + B?**

   (A) 0 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 7

4. **Find an equation of the line that is perpendicular to** $3x+7y=9$ **and has a** x**-intercept of** $(-2,0)$.

   (A) $7x-3y=14$ **(B)** $3x+7y=-4$ **(C)** $7x-3y=-2$

   (D) $3x+7y=6$ **(E)** $7x-3y=-14$

5. **1,125 feet per second = _____ kilometers per hour. (nearest whole number)**

   (A) 1324 **(B)** 1234 **(C)** 1350 **(D)** 1372 **(E)** 1264

6. **Let** $9x^3+18x^2-x-2=(ax+b)(ax-b)(x+c)$ **where** $a,b,c\in\mathbb{Z}$. **Find** $a+b+c$.

   (A) 2 **(B)** 0 **(C)** 6 **(D)** 3 **(E)** –5

7. **A polyhedral die has 12 faces and 30 vertices. How many edges does it have?**

   (A) 14 **(B)** 40 **(C)** 18 **(D)** 16 **(E)** 22

8. **Let** $O=\{1,3,5,7,...29\}$, $T=\{1,3,6,10,15,...66\}$ **and** $F=\{1,2,3,5,8,13,...89\}$. **How many elements are in** $(O\cup T)\cap F$ **?**

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. **Simplify:** $\frac{n!}{(n-2)!}\div\frac{(n+1)!}{n(n-3)!}$.

   (A) $\frac{1}{n-2}$ **(B)** $\frac{1}{n^2-n-2}$ **(C)** $\frac{n}{n+1}$

   (D) n^2-3 **(E)** $\frac{n}{n^2-n-2}$

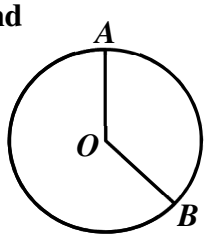
10. **If** $\frac{x-5}{x+8}+\frac{x+8}{x-5}$ **is written as the mixed number** $A\frac{B}{C}$, **then** $B=$ **?**

    (A) 52 **(B)** 104 **(C)** 9 **(D)** 3 **(E)** 169

11. Find the shortest distance from the line $y = \frac{3}{5}x - 7$ to the point $(-1,5)$. (nearest whole number)

(A) 11 (B) 1 (C) 9 (D) 21 (E) 8

12. Given the circle with center O shown, OB = 12" and the arclength of $\overset{\frown}{AB}$ is 30", find $m\angle AOB$. (nearest degree)



(A) 138° (B) 130° (C) 143°

(D) 140° (E) 134°

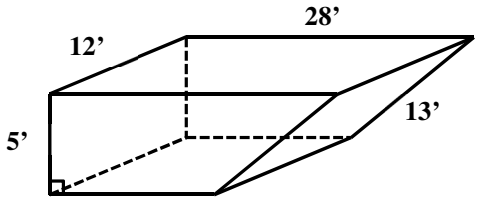
13. If $\int_{-3}^{7} f(x)dx = 32$, evaluate $\int_{-3}^{7}[3f(x)+5x]dx$.

(A) 146 (B) 196 (C) 101 (D) 151 (E) 200

14. A square has side lengths of 8 cm. If the square's width is tripled and the length is quartered, what is the percent change in the area of the shape?

(A) −33% (B) 75% (C) 125% (D) 200% (E) −25%

15. Wade's pool is shaped like a trapezoidal prism as shown. How many gallons (nearest gallon) will fill the pool completely? (not drawn to scale)



(A) 543 gal (B) 7106 gal (C) 9874 gal

(D) 767gal (E) 8452 gal

16. An art installation includes a huge rectangular digital display with dimensions such that the diagonal is one foot longer than the length, and the length is six feet shorter than six times the width. The area of the screen is _____ft^2. (nearest square foot)

(A) 726 (B) 792 (C) 671 (D) 660 (E) 732

17. Using the following pattern of numbers, determine the fifth term in row 25.

				1						(row 0)
			1		1					(row 1)
		1		2		1				(row 2)
	1		3		3		1			(row 3)
1		4		6		4		1		(row 4)
1	5		10		10		5		1	(row 5)
			…							…

(A) 12,650 (B) 53,130 (C) 2,300 (D) 10,626 (E) 45.504

18. If p is a prime number and $2p+1$ is a prime number then $2p+1$ is called a *safe prime* and p is called a _____prime.

(A) Germain (B) Pythagorean (C) Mersenne (D) Hypatian (E) Euclidean

19. $321_4 + 4321_5 + 54321_6 =$ _____$_7$

(A) 33,232 (B) 32,432 (C) 32,322 (D) 55,050 (E) 55,066

20. The mathematical statement $(x-3)(4x-7)=4x^2-7x-12x+21$ is an example of _____property.

(A) Associative **(B) Commutative** **(C) Identity**

(D) Transitive **(E) Distributive**

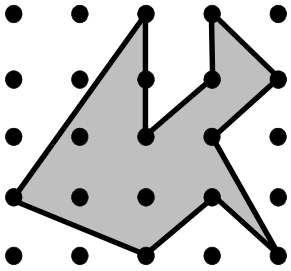
21. What is the constant term in the expansion of $\left(3x-\frac{1}{x^4}\right)^{10}$?

(A) 61,236 **(B) 6,561** **(C) 243** **(D) 20,412** **(E) 295,245**

22. If $\sin\theta=0.6$ and θ is in QII, then $\tan\theta=$?

(A) $\frac{3}{4}$ **(B) $-\frac{4}{5}$** **(C) $\frac{4}{5}$** **(D) $-\frac{4}{3}$** **(E) $-\frac{3}{4}$**

23. If the dots on the grid shown below are 4 cm apart both vertically and horizontally, find the area of the shaded region.



(A) 7.5 cm^2 **(B) 30 cm^2** **(C) 60 cm^2**

(D) 40 cm^2 **(E) 120 cm^2**

24. If $\log_5(4x+8)-\log_5(x+1)=1$, then x = ?

(A) 5 **(B) 6** **(C) 3** **(D) 2** **(E) No solution**

25. Find $m+n$ if $\begin{bmatrix}5 & -3\\ 2 & -1\end{bmatrix}\begin{bmatrix}m\\ n\end{bmatrix}=\begin{bmatrix}36\\ 14\end{bmatrix}$.

(A) 7.2 **(B) –2** **(C) 3** **(D) 4** **(E) 16**

26. Which of the following is defined by $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$?

(A) ε **(B) φ** **(C) π** **(D) e** **(E) $\frac{\pi}{2}$**

27. The 10th term of an arithmetic sequence is 42 and the common difference of the terms is 4. Find the sum of the first 20 terms.

(A) 880 **(B) 820** **(C) 1760** **(D) 860** **(E) 1720**

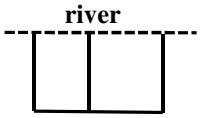
28. How many three-digit numbers have all three digits as different odd numbers?

(A) 125 **(B) 48** **(C) 100** **(D) 60** **(E) 24**

29. $(7x^4+8x^3-19x^2+x-3)\div(3x+7)$ has a remainder of_____.

(A) $\frac{16606}{81}$ **(B) $-\frac{1782}{343}$** **(C) $-\frac{236}{81}$** **(D) $-\frac{2508}{343}$** **(E) $\frac{250}{81}$**

30. Carl has 64 meters of fencing. He wants to create a rectangular livestock enclosure divided into two separate sections. He has a river that will serve as one side of the enclosure. The maximum area he can fence is _____m^2.



(A) $\frac{1000}{3}$ **(B)** $\frac{1024}{3}$ **(C)** $\frac{512}{3}$ **(D)** 720 **(E)** $\frac{640}{3}$

31. The odds of drawing a pink raffle ticket at random from a bucket containing 300 tickets is 5:1. How many pink tickets would have to be removed from the bucket to reduce the odds to 12:25?

(A) 226 **(B)** 50 **(C)** 250 **(D)** 125 **(E)** 24

32. The graph of the function $f(x)=ax^3-bx^2$ where a and b are both positive integers has a point of inflection when x = _____.

(A) $\frac{b}{3a}$ **(B)** $-\frac{b}{a}$ **(C)** $\frac{2b}{3a}$ **(D)** $-\frac{2b}{3a}$ **(E)** $-\frac{b}{3a}$

33. A fair, octahedral die with sides labelled 1 through 8 is thrown. What is the expected value of a single roll?

(A) 4 **(B)** 4.25 **(C)** 3.75 **(D)** 4.5 **(E)** 4.75

34. The finite region in the first quadrant enclosed by the curve $f(x)=2\cos(2x)$, the x-axis and the y-axis is rotated around the line $y=-3$. Calculate the volume of the solid generated. (nearest cubic unit)

(A) 44 **(B)** 40 **(C)** 24 **(D)** 46 **(E)** 15

35. The slope of the line going through the points $(-10,y)$, $(15,-7)$ and $(x,20)$ is $-\frac{3}{5}$. Find $x+y$.

(A) 38 **(B)** −22 **(C)** −30 **(D)** −8 **(E)** 8

36. $\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\ldots+\frac{1}{171}+\frac{1}{190}=?$

(A) $\frac{32}{57}$ **(B)** $\frac{9}{10}$ **(C)** $\frac{10}{11}$ **(D)** $\frac{17}{30}$ **(E)** $\frac{13}{15}$

37. If A and B be the roots of $f(x)=3x^2+13x-10$. Find the value of $A^4-4A^3B+6A^2B^2-4AB^3+B^4$.

(A) $\frac{2401}{16}$ **(B)** $\frac{28561}{16}$ **(C)** $\frac{28561}{81}$ **(D)** $\frac{83521}{81}$ **(E)** $\frac{2401}{81}$

38. May's age is three times her son's age. Four years from now, her age will be eight more than twice her son's age. What is the sum of their current ages?

(A) 36 **(B)** 56 **(C)** 48 **(D)** 52 **(E)** 50

39. Using a polynomial function that fits this data, find $f(12)$.

x	−2	−1	0	1	2	3	4
$f(x)$	151	46	17	−2	−5	86	421

(A) 37,318 **(B)** 53,645 **(C)** 74,806 **(D)** 13,166 **(E)** 16,702

40. Which of the following mathematicians is an American Indian who is doing work applying mathematical models to the study of groundwater contamination?

(A) Noether (B) Scott (C) Hypatia (D) Porter (E) Byron

41. How many asymptotes does $f(x)=\dfrac{x^3-25}{x^2-25}$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

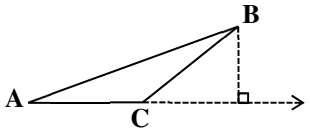
42. Given $f(x)=2x-3$ and $g(x)=3-x$ find $(f/g)^{-1}(x)$.

(A) $\dfrac{x-3}{2-x}$ (B) $\dfrac{3x-3}{2+x}$ (C) $\dfrac{2x-3}{3-x}$ (D) $\dfrac{x+3}{2-x}$ (E) $\dfrac{3x+3}{2+x}$

43. How many distinct solutions exist for $2\sin^2\theta=2+\cos\theta$, where $0\le\theta\le3\pi$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

44. On the triangle ABC shown, $m\angle BAC=\dfrac{\pi}{6}$ radians, AB = 36 and AC = 27. Find the area of triangle ABC.



(A) $243\sqrt{3}$ (B) 243 (C) 486

(D) $486\sqrt{3}$ (E) 121.5

45. Evaluate: $\prod_{k=0}^{2}(2k+n)$

(A) n^2+6n+8 (B) n^2+6n (C) n^3+6n^2 (D) n^3+6n^2+8n (E) $6n^3+5n^2+6n$

46. How many distinct 4-letter code words can be made with the letters in the word EINSTEIN?

(A) 168 (B) 354 (C) 120 (D) 138 (E) 336

47. Let $f(x)=\dfrac{3x^4-5x^2+8}{x^3+8x}$ and $s(x)$ be the slant asymptote of f. Find the value of $s(-2)$.

(A) 2 (B) 4 (C) –6 (D) –4 (E) 6

48. If $x-\dfrac{1}{x}=18$, then $x^3-\dfrac{1}{x^3}=?$

(A) 5868 (B) 5850 (C) 5886 (D) 5814 (E) 5832

49. The repeating decimal 0.5222… in base 6 can be written as which of the following fractions in base 6 in simplified terms?

(A) $\dfrac{13}{14_6}$ (B) $\dfrac{24}{30_6}$ (C) $\dfrac{14}{15_6}$ (D) $\dfrac{22}{23_6}$ (E) $\dfrac{22}{30_6}$

50. Determine the range of $f(x)=5-4\cos(2\pi x)$.

(A) $[-4,4]$ **(B)** $[-9,-1]$ **(C)** $[-9,1]$ **(D)** $[1,9]$ **(E)** $[-1,9]$

51. The measure of one exterior angle of a regular dodecagon is_____°.

(A) 150 **(B)** 30 **(C)** 36 **(D)** 144 **(E)** 60

52. What is the angle between the vectors $\langle 3,7\rangle$ and $\langle -4,11\rangle$. (nearest degree)

(A) 50° **(B)** 47° **(C)** 48° **(D)** 52° **(E)** 43°

53. Let $f''(x)=12x-8$, $f(-2)=-49$ and $f(2)=3$. Find $f(1)$.

(A) −18 **(B)** −7 **(C)** 3 **(D)** −4 **(E)** 26

54. $f(x)=1+x-\frac{x^2}{2}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}-\frac{x^6}{6!}\ldots$ **. Find the 10^{-8} digit for $f\left(\frac{\pi}{3}\right)$.**

(A) 1 **(B)** 0 **(C)** 6 **(D)** 5 **(E)** 2

55. Let $S=\{6,12,24\}$. The arithmetic mean of *S* is A. The geometric mean of *S* is G. The harmonic mean of *S* is H. Which of the following is the correct order of the means of A, G and H from least to greatest?

(A) A, H, G **(B)** H, G, A **(C)** G, A, H **(D)** A, G, H **(E)** H, A, G

56. How many perfect cubes are factors of $(4!)(5!)(8!)$?

(A) 8 **(B)** 9 **(C)** 5 **(D)** 7 **(E)** 4

57. Find $f(-2)+f(0)+f(2)$ if $f(x)=\begin{cases}3x+2, & x\le -1\\ 2x, & -1<x<1\\ 3-4x, & x\ge 1\end{cases}$

(A) −14 **(B)** 3 **(C)** 7 **(D)** 4 **(E)** −9

58. Which of the following words has exactly 181,440 unique permutations of its letters?

(A) Rectangle **(B)** Square **(C)** Trapezoid **(D)** Parallelogram **(E)** Kite

59. There are two values of *k* for which $\det\begin{bmatrix}k+1 & -5\\ -2 & -k\end{bmatrix}=-22$**. The sum of those two values is**

(A) 0 **(B)** 1 **(C)** 10 **(D)** −1 **(E)** −10

60. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$ **(B)** $\sum_{n=0}^{\infty}\frac{n+1}{2n+1}$ **(C)** $\sum_{n=1}^{\infty}\frac{n}{1000(n+1)}$ **(D)** $\sum_{n=0}^{\infty}\frac{3}{2^n}$ **(E)** $\sum_{n=1}^{\infty}\log n$

2018-2019 TMSCA Mathematics Test Nine Answers

1. A	**21. E**	**41. C**
2. B	**22. E**	**42. E**
3. D	**23. E**	**43. D**
4. E	**24. C**	**44. B**
5. B	**25. D**	**45. D**
6. C	**26. D**	**46. B**
7. B	**27. A**	**47. C**
8. C	**28. D**	**48. C**
9. E	**29. C**	**49. A**
10. E	**30. B**	**50. D**
11. A	**31. A**	**51. B**
12. C	**32. A**	**52. E**
13. B	**33. D**	**53. D**
14. E	**34. C**	**54. B**
15. C	**35. B**	**55. B**
16. D	**36. D**	**56. B**
17. A	**37. D**	**57. E**
18. A	**38. C**	**58. A**
19. B	**39. B**	**59. D**
20. E	**40. D**	**60. D**

2018-2019 TMSCA Mathematics Test Nine Solutions

3. B must be either 0 or 5, and the sum of the digits must be 2 more than a multiple of 9, so the smallest possible value of A + B is either 6 + 0 or 1 + 5 both of which have a sum of 6.

11. Distance between $3x-5y-35=0$ and $(-1,5)$ is $\frac{|3(-1)+(-5)(5)-35|}{\sqrt{3^2+(-5)^2}} \approx 11$

12. Solve $30=2\pi(12)\left(\frac{\theta}{360}\right)$ for $\theta \approx 143°$

13. $3(32)+\int_{-3}^{7}(5x)dx=196$

17. The numbers in the nth row of Pascal's triangle are the coefficients of $(a+b)^n$, so the 5th term in the 25th row is $_{25}C_{21}=12{,}650$

21. Use binomial theorem for $_{10}C_8(3x)^8\left(-\frac{1}{x^4}\right)^2=295{,}245$

23. Let P be the number of perimeter grid points and I be the number of interior grid points. The area will be $\frac{P+2I-2}{2}(4^2)=\frac{11+6-2}{2}(4^2)=120$

25. Solve the system: $\begin{matrix}5m-3n=36\\2m-n=14\end{matrix}$ for $m=6, n=-2$ and $m+n=4$

33. Let x be the width of the field and $64-3x$ be the length then find the maximum of $A=(x)(64-3x)$ or $\frac{1024}{3}$

34. $V=\pi\int_0^{\pi/4}\left[(f(x)+3)^2-3^2\right]dx \approx 24$

36. The denominators are the 3rd through the 19th triangular numbers, so evaluate $\sum_{k=3}^{19}\frac{1}{k(k+1)/2}$. Use summation function of calculator.

37. The roots are $\frac{2}{3}$ and -5, and the expression can be can be evaluated by $\left(-5-\frac{2}{3}\right)^4=\frac{83521}{81}$

41. This factors to $\frac{(x-5)(x^2+5x+25)}{(x-5)(x+5)}$ which indicates a hole at $x=5$ and a vertical asymptote at $x=-5$. Also because the degree of the numerator is 1 more than the degree of the denominator, this function will have a slant asymptote for a total of 2 asymptotes.

45. This is a product: $(n)(2+n)(4+n)=n^3+6n^2+8n$

48. $x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)=$

$\left(x-\frac{1}{x}\right)\left[\left(x-\frac{1}{x}\right)^2+3\right]=18(18^2+3)=5886$

52. $\cos\theta=\frac{3(-4)+7(11)}{\sqrt{9+49}\times\sqrt{16+121}}$ for $\theta \approx 43°$

53. Take the anti-derivative twice for $f(x)=2x^3-4x^2+Ax+B$, then solve the system $-49=2(-2)^3-4(-2)^2+A(-2)+B$ and $3=2(2)^3-4(2)^2+A(2)+B$ to find $f(x)$ and $f(1)=-4$

54. $f(x)$ is the McClaurin series representation of $f(x)=\sin x+\cos x$, so evaluate $f\left(\frac{\pi}{3}\right)$.

58. The letters in RECTANGLE have $\frac{9!}{2!}=181{,}440$ possible unique permutations. The 2! comes from the E's repeating twice.

59. $-k(k+1)-(-2)(-5)=-22$ then the sum of the roots for $-k^2-k-10=-22$ is $-\frac{b}{a}=-1$