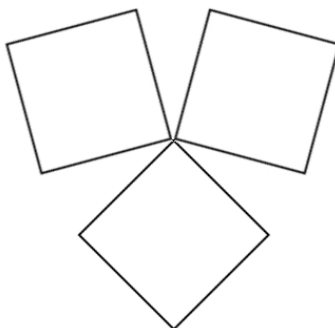**MIDDLE SCHOOL - PROBLEMS**

Problem 1

There are relatively prime positive integers m and n so that

$$\frac{\frac{1}{2}}{\frac{\frac{1}{3}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} + \frac{\frac{1}{3}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}} = \frac{m}{n}.$$

Find $m + 2n$.

Problem 2

The diagram below shows a 12-sided figure made up of three congruent squares. The figure has total perimeter 60. Find its area.

Problem 3

Find the sum of all two-digit integers which are both prime and are 1 more than a multiple of 10.

Problem 4

Jerry buys a bottle of 150 pills. Using a standard 12 hour clock, he sees that the clock reads exactly 12 when he takes the first pill. If he takes one pill every five hours, what hour will the clock read when he takes the last pill in the bottle?

Problem 5

Given that $\frac{6}{11} - \frac{10}{19} = \frac{9}{19} - \frac{n}{11}$, find n.

Problem 6

The following addition problem is not correct if the numbers are interpreted as base 10 numbers. In what number base is the problem correct?
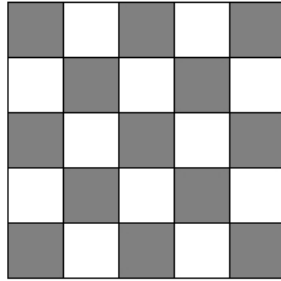
$$
\begin{array}{r}
6\,6 \\
8\,7 \\
8\,5 \\
+\,4\,8 \\
\hline
1\,3\,2
\end{array}
$$

Problem 7

When 12^{18} is divided by 18^{12}, the result is $\left(\frac{m}{n}\right)^3$, where m and n are relatively prime positive integers. Find $m - n$.

Problem 8

A square measuring 15 by 15 is partitioned into five rows of five congruent squares as shown below. The small squares are alternately colored black and white as shown. Find the total area of the part colored black.

Problem 9

A jar contains one quarter red jelly beans and three quarters blue jelly beans. If Chris removes three quarters of the red jelly beans and one quarter of the blue jelly beans, what percent of the jelly beans remaining in the jar will be red?

Problem 10

Five rays $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$, and $\overrightarrow{OE}$ radiate in a clockwise order from O forming four non-overlapping angles such that $\angle EOD = 2\angle COB$, $\angle COB = 2\angle BOA$, while $\angle DOC = 3\angle BOA$. If E, O, A are collinear with O between A and E, what is the degree measure of $\angle DOB$?

Problem 11

How many numbers are there that appear both in the arithmetic sequence 10, 16, 22, 28, ... 1000 and the arithmetic sequence 10, 21, 32, 43, ..., 1000?
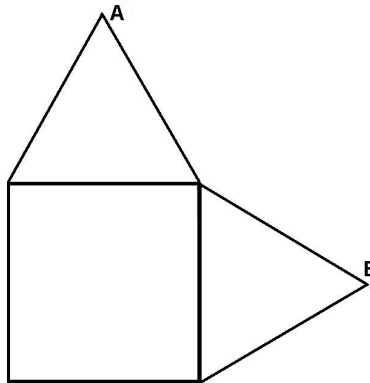
Problem 12

When Troy writes his digits, his 0, 1, and 8 look the same right-side-up and upside-down as seen in the figure below. His 6 and 9 look like upside-down images of each other. None of his other digits look like digits when they are inverted. How many different five-digit numbers (which do not begin with the digit zero) can Troy write which read the same right-side-up and upside-down?

$$0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9$$
$$6\ 8\ L\ 9\ S\ h\ \varepsilon\ 2\ 1\ 0$$

Problem 13

The diagram shows two equilateral triangles with side length 4 mounted on two adjacent sides of a square also with side length 4. The distance between the two vertices marked A and B can be written as $\sqrt{m} + \sqrt{n}$ for two positive integers m and n. Find $m + n$.
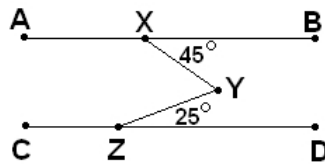
Problem 14

The five-digit number 12110 is divisible by the sum of its digits
$1 + 2 + 1 + 1 + 0 = 5$. Find the greatest five-digit number which is divisible by
the sum of its digits.

Problem 15

In the diagram below, $\overline{AB}$ and $\overline{CD}$ are parallel, $\angle BXY = 45°$, $\angle DZY = 25°$,
and $XY = YZ$. What is the degree measure of $\angle YXZ$?

Problem 16

Let a and b be nonzero real numbers such that $\frac{1}{3a} + \frac{1}{b} = 2011$ and $\frac{1}{a} + \frac{1}{3b} = 1$.
What is the quotient when $a + b$ is divided by ab?

Problem 17

Find the number of ordered quadruples (a, b, c, d) where each of a, b, c, and d
are (not necessarily distinct) elements of $\{1, 2, 3, 4, 5, 6, 7\}$ and
$3abc + 4abd + 5bcd$ is even. For example, $(2, 2, 5, 1)$ and $(3, 1, 4, 6)$ satisfy the
conditions.

Problem 18

Find the positive integer n so that n^2 is the perfect square closest to
$8 + 16 + 24 + \cdots + 8040$.

Problem 19

How many ordered pairs of sets (A, B) have the properties:

1. $A \subseteq \{1, 2, 3, 4, 5, 6\}$

2. $B \subseteq \{2, 3, 4, 5, 6, 7, 8\}$

3. $A \cap B$ has exactly 3 elements.

Problem 20

Let V be the set of vertices of a regular 25 sided polygon with center at point C. How many triangles have vertices in V and contain the point C in the interior of the triangle?