UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics
SAC • 2008
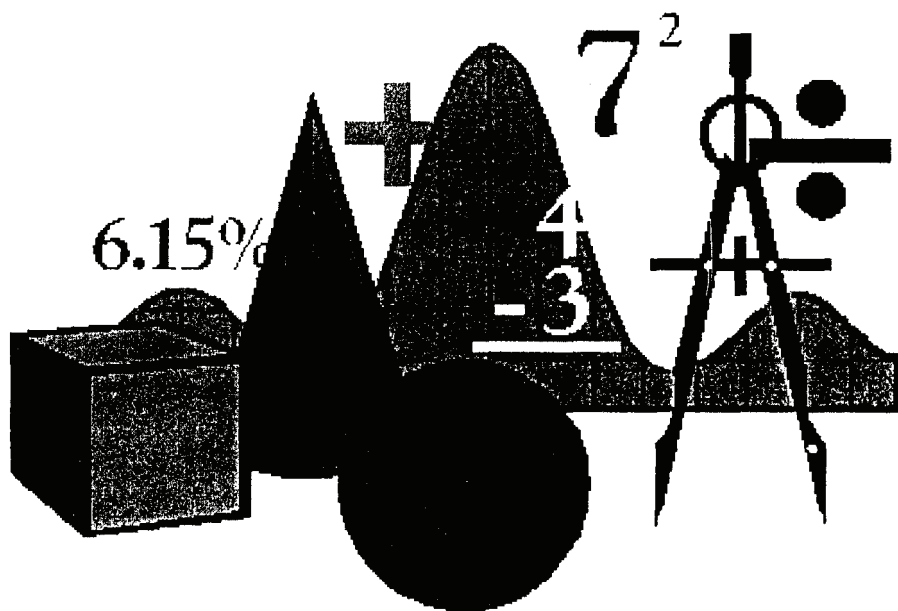
1. Evaluate: $\frac{3}{8} \div .75 \times \frac{1}{2} - .25 + \frac{1}{16}$

   (A) $-\frac{1}{8}$ (B) $-.1875$ (C) $.0625$ (D) $\frac{3}{4}$ (E) $.5$

2. A legend on a map shows 2.5 cm representing 200 miles. The distance on the map from El Paso to Texarkana is 9.8 cm. According to the map, how far is it from El Paso to Texarkana?

   (A) 735   (B) 750   (C) 763 miles   (D) 784 miles   (E) 800 miles

3. Phil Errup's car has a gas tank with a capacity of 18 gallons. The gauge shows that it is $\frac{1}{4}$ full. How many gallons will need to be added to the tank so that it is 75% full?

   (A) 4.5   (B) 6   (C) 7.5   (D) 9   (E) 13.5

4. Find an equation of the line shown.
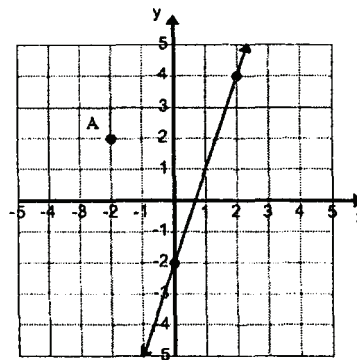
   (A) $x - 3y = 6$ (B) $3x - y = 2$ (C) $x - 3y = -2$ (D) $-3x + y = 2$ (E) $3x + y = -2$

5. Let p and q be the roots of $8x^2 + 2x - 15 = 0$. Find $p^3 + 3p^2q + 3pq^2 + q^3$.

   (A) $3\frac{1}{2}$ (B) $1\frac{5}{8}$ (C) $-\frac{1}{4}$ (D) $-\frac{15}{32}$ (E) $-\frac{1}{64}$

6. One of the factors of $x^3 - 4x^2 - 3x + 18$ is:

   (A) $x + 2$ (B) $x + 3$ (C) $x + 6$ (D) $x - 2$ (E) $x - 9$

7. Point P (3 , 3) is rotated 270 ° counterclockwise about the origin to point Q. Point Q is reflected across the y-axis to point R. Find the coordinates of point R.

   (A) $(-3, 3)$ (B) $(-3, -3)$ (C) $(3, -3\sqrt{2})$ (D) $(3\sqrt{2}, 3)$ (E) $(3, 3)$

8. Two chords, AC and BD intersect in the interior of a circle at point X such that $m\overset{\frown}{BC} = 20°$ and $m\overset{\frown}{AD} = 120°$. If points B and C are not on $\overset{\frown}{AD}$ then $m\angle AXD$ is:

   (A) 50°   (B) 70°   (C) 100°   (D) 110°   (E) 140°

9. The adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.
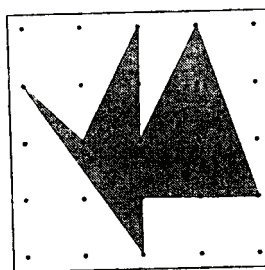
(A) 4.5 cm^2 (B) 5 cm^2 (C) 5.5 cm^2 (D) 6 cm^2 (E) 6.5 cm^2

10. The roots of the equation $x^3 - 5x^2 + cx + 24 = 0$ are $3, 4$, and R. Find c.

(A) -26 (B) -24 (C) -9 (D) -5 (E) -2

11. Let $f(x) = 2x + 5$ and $g(x) = 3x - 4$ and $h(x) = 6x$. Find $f\big(g(h(-1))\big)$.

(A) -49 (B) -39 (C) -23 (D) -21 (E) -11

12. The graph of $x^2 + y^2 + 10x - 12y - 20 = 0$ is a circle with a radius of:

(A) $2\sqrt{5}$ (B) $\sqrt{41}$ (C) 9 (D) $4\sqrt{39}$ (E) 11

13. One of the base angles of an acute isosceles triangle has a measure of $50°$ and the length of its base is 6 cm. Find the perimeter of the acute isosceles triangle. (nearest tenth)

(A) 13.0 cm (B) 13.8 cm (C) 15.3 cm (D) 16.0 cm (E) 17.7 cm

14. A cliff near a lake is 125 feet high. The angle of depression of a canoe from the top of the cliff is $30°$. How far is the canoe from the base of the cliff? (nearest foot).

(A) 63 ft (B) 72 ft (C) 108 ft (D) 188 ft (E) 217 ft

15. Simplify: $\sin \theta \tan \theta + \cos \theta$

(A) $2\sin(\frac{\pi}{2} - \theta)$ (B) $2\csc \theta$ (C) $2\tan(\frac{\pi}{2} + \theta)$ (D) $\sec \theta$ (E) $-\cos \theta$

16. Let $A = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}$. What is the determinant of AB?

(A) 5 (B) 3 (C) 2 (D) 1 (E) 0

17. The coefficient of the 2nd term of the expansion of $(3x - 4)^5$ is:

(A) -1620 (B) -1296 (C) -324 (D) -60 (E) -1

18. Which of the following is true about the relation $h(x) = 5 - x^2$?

(A) odd function   (B) even function   (C) neither even nor oddfunction

(D) not a function   (E) none of these are true

19. If $f'(x) = 6x^2 - 4x + 1$ and $f(1) = 0$, find $f(-1)$.

(A) -8 (B) -6 (C) 0   (D) 4   (E) 11

20. $f(x) = 2x^3 - 6x + 1$ has an inflection point at :

(A) $(0, 1)$ (B) $(-1, 1)$ (C) $(-1, 0)$ (D) $(1, 0)$ (E) $(-1, -1)$

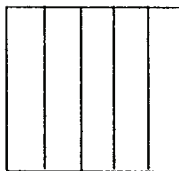21. Find the area (in square units) of the region bounded by $x = \frac{y^2 + 2}{2}$ and $x = y + 5$.

(A) 6   (B) $7\frac{1}{3}$ (C) 9   (D) $12\frac{2}{3}$ (E) 18

22. If two dice are rolled at one time, what is the probability that both dice show a prime number?

(A) $8\frac{1}{3}$ %   (B) 9 %   (C) $16\frac{2}{3}$ %   (D) 25 %   (E) $66\frac{2}{3}$%

23. The square below is divided into 5 congruent rectangles. The perimeter of each of the congruent rectangles is 30 units. What is the perimeter of the square?

(A) 60 units   (B) 50 units   (C) 45 units   (D) 35 units   (E) 25 units

24. Simplify: $\dfrac{n! + (n-1)!}{(n-2)!}$

(A) $n^2 - 1$ (B) $\dfrac{n^2 - n}{n - 1}$ (C) $\dfrac{n^2 - 1}{n - 2}$ (D) $\dfrac{n!}{(n - 2)!}$ (E) $n^2 - 2n - 1$
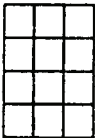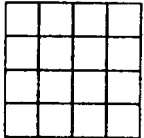
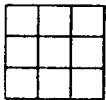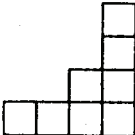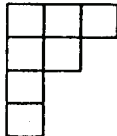Note: # 25, 26, and 27 are new approaches for 2009

25. Use the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, to transform the conic $xy = 1$ into an equation that is in standard position and does not contain an xy term. The transformed equation is:

(A) $x^2 + y^2 = 1$ (B) $x^2 = y^2$ (C) $\dfrac{x^2 + y^2}{2} = 0$ (D) $x^2 - y^2 = 2$ (E) $\dfrac{x^2 - y^2}{2} = 0$

26. Which of the following would best represent a two dimensional perspective of the top view of this figure shown?
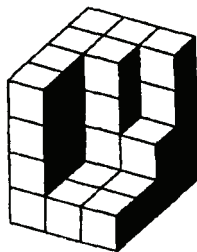
(A) (B) (C) (D) (E)

27. Let AB be the diameter of the circle with center C with CG $\perp$ AB, DE $\perp$ AB, and EF $\perp$ DC. If AE = 9 and BE = 4 then DE = ?
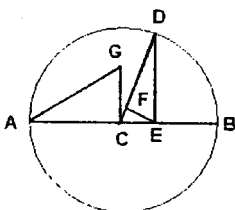
(A) 5 (B) $5\frac{7}{13}$ (C) 6 (D) $6\frac{1}{2}$ (E) $6\frac{27}{28}$

1.	C		21.	E
2.	D		22.	D
3.	D		23.	B
4.	B		24.	A
5.	E		25.	D
6.	A		26.	A
7.	B		27.	C
8.	B			
9.	D			
10.	E			
11.	B			
12.	C			
13.	C			
14.	E			
15.	D			
16.	D			
17.	A			
18.	B			
19.	B			
20.	A			

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Mathematics
Invitational A • 2009

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $\left[1.2 \div \left(\frac{3}{5}\right)^2 - (3)^{-1}\right] \times 4!$

    (A) 88   (B) 72   (C) $48\frac{9}{25}$ (D) $27\frac{2}{3}$ (E) 8

2.

               P $-3\frac{1}{4}$ R   S   2.75

    <------------|-------------|-------------|-------------|-------------|------------->

    The distances between the hash marks (|) are equal. Find P + R + S.

    (A) -5.75 (B) $-\frac{1}{2}$ (C) $2\frac{1}{2}$ (D) 4.25   (E) 6

3. Phil Upp's truck gets 17 miles per gallon of gas. He has $20.00 to spend on gas. If the cost of a gallon of is gas is $3.50, how far can Phil drive? (nearest whole mile)

    (A) 70 miles   (B) 76 miles   (C) 97 miles   (D) 100 miles   (E) 102 miles

4. Line l going through points $(-1, 3)$ and $(k, -5)$ is perpendicular to $x + 4y = 5$. Find k.

   (A) -5 (B) -3 (C) -1 (D) 2   (E) 5

5. Simplify: $\left(\dfrac{6w^2 + 7w - 3}{2w^3 + 5w^2 + 3w}\right)\left(\dfrac{w^2 - w - 2}{3w^2 - 7w + 2}\right)$

    (A) $\dfrac{w-3}{w^2-2}$ (B) $\dfrac{w+2}{3w^2+2w}$ (C) $\dfrac{1}{w}$ (D) $\dfrac{3w-2}{4+w}$ (E) $\dfrac{3w+1}{2w}$

6. Ima Whett paddles her kayak at a constant speed of 5 mph relative to the water. She paddles upstream for 1 hour 20 minutes. The return trip back only takes 1 hour 5 minutes. Which of the following is the closest approximation of the speed of the current?

    (A) $\frac{1}{2}$ mph   (B) $\frac{1}{3}$ mph   (C) $\frac{5}{6}$ mph   (D) $\frac{5}{8}$ mph   (E) $\frac{1}{4}$ mph

7. Rene drew this quadrilateral on the coordinate plane below. The coordinates of the vertices are integers. What is the area of his quadrilateral?
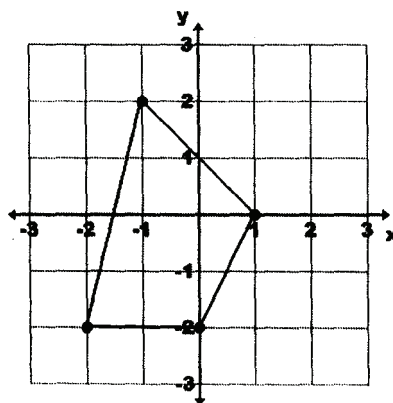
    (A) $6\frac{1}{2}$ units^2 (B) 7 units^2 (C) $7\frac{2}{3}$ units^2 (D) $8\frac{1}{4}$ units^2 (E) $8\frac{1}{2}$ units^2

8. If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle then the line is _____ to the circle.

   (A) complementary   (B) diagonal   (C) tangent   (D) adjacent   (E) secant

9. $\angle A$ and $\angle B$ are complementary . The ratio of $m\angle A$ to $m\angle B$ is 4:5. Find the ratio of $m\angle A$ to its supplement.

   (A) 2:7   (B) 4:9   (C) 5:4   (D) 6:3   (E) 5:14

10. The graph best depicts Mei Strol's daily 6 minute walk. (speed is not truly linear in this case). During the time interval of 3 minutes to 4 minutes Mei is _____.
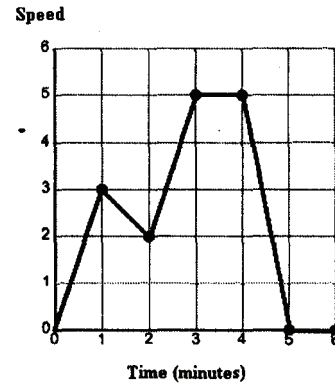
   (A) walking on flat ground   (B) walking at a constant speed   (C) standing still
   (D) decreasing speed   (E) increasing speed

11. A line perpendicular to the axis of symmetry of a parabola is called the _____.

   (A) focus   (B) eccentricity (C) directrix   (D) centroid   (E) asymptote

12. The length of the sides of each of the small cubes is 1 cm. How many of the small cubes would need to be added to this figure to make a rectangular prism that is 4 cm long, 3 cm wide, and 2 cm tall?
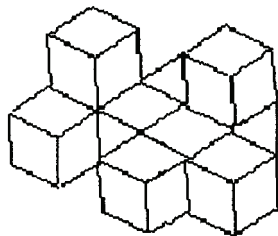
   (A) 14   (B) 13   (C) 12   (D) 11   (E) 10

13. A laser beam from the top of a 30-ft building hits an object on the ground 100 ft from the base of the building. The angle of depression of the laser to the object is: (nearest second)

   (A) 14° 24' 11"   (B) 16° 6' 9"   C) 16° 41' 57"   (D) 17° 4' 6"   (E) 17° 27' 27"

14. Find the largest value of θ if $6\cos^2\theta + \cos\theta = 2$ and $\pi \le \theta \le 2\pi$.

(A) $\frac{6\pi}{5}$ (B) $\frac{4\pi}{3}$ (C) $\frac{5\pi}{3}$ (D) $\frac{8\pi}{5}$ (E) $\frac{5\pi}{4}$

15. Simplify $\sin\theta \cot\theta \sec\theta - \cos^2\theta$.

(A) $\sin^2\theta$ (B) $\csc^2\theta$ (C) $\sin 2\theta$ (D) $\sec^2\theta$ (E) 1

16. Let $x^5 - x^4 - px^3 + qx^2 - x - 1 = 0$, where p, q > 0. According to Descartes' Rule of Signs, how many possible positive roots are there?

(A) 5, 3, or 1   (B) 3 or 1   (C) 4, 2, or 0   (D) 2 or 0   (E) 0

17. The directrix of the parabola $8y = x^2 - 4x + 12$ is:

(A) $x = 2$ (B) $y = 1$ (C) $y = \frac{1}{8}$ (D) $x = -\frac{1}{4}$ (E) $y = -1$

18. If 5 adults and 2 teenagers work together, they can do a job in 1 day. If only 2 adults work, then 6 teenagers must in order to do the job in 1 day. If no adults work and only 1 teenager works, how long will it take the teenager to do the job?

(A) $7\frac{1}{2}$ days   (B) 8 days   (C) $8\frac{1}{3}$ days   (D) $8\frac{2}{3}$ days   (E) 9 days

19. A function $y = f(x)$ is continuous on [a,b], if $f(a) < y_0 < f(b)$ then $y_0 = f(c)$ for some c in [a,b]. This theorem is the:

(A) Intermediate Value Theorem   (B) Mean Value Theorem   (C) Sandwich Theorem
(D) Max–Min Theorem   (E) Fundamental Theorem of Calculus

20. Which of the following polar equations will produce this graph on the polar grid?
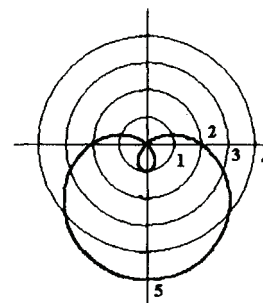
(A) $r = 1 + 5\sin\theta$ (B) $r = 2 - 4\sin\theta$ (C) $r = 1 + 2\cos\theta$

(D) $r = 2 - 3\sin\theta$ (E) $r = 2 - 5\cos\theta$

21. Let $f(x) = ax^5 - bx^4 - bx^3 + ax^2 + ax - b$. Find $f''(1)$.

(A) $22a - 18b$ (B) $18a - 6b$ (C) $22a - 19b$ (D) $18a - 6b$ (E) $22a - 7b$

22. Seymore Endelite randomly selects two socks from his drawer to wear to school. The socks are identical except for their color and are not paired up. He has 8 blue socks, 6 black socks, and 4 white socks. What is the probability that he selects two black socks? (nearest percent)

(A) 9 %   (B) 18 %   (C) 32 %   (D) 4 %   (E) 10 %

23. Lotta Dough has a bag that contains one $100 bill, two $20 bills, three $10 bills, four $5 bills, and five $1 bills. The odds of her pulling out a $10 bill is 25%. How many $10 bills would have to be added to the bag to change the odds to 50%?

(A) 1   (B) 3   (C) 6   (D) 9   (E) 12

24. How many subsets containing 4 members can be made from the set {2, 1, 3, 4, 7, 11}?

(A) 6   (B) 10   (C) 15   (D) 20   (E) 21

25. Which of the following was the first Nigerian woman to be awarded a doctorate in mathematics?

(A) Emmy Noether   (B) Freda Porter   (C) Hypatia
(D) Karen E. Smith   (E) Grace Alele Williams

26. Find the harmonic mean of the roots of $x^3 - 7x^2 + 14x - 8 = 0$.

(A) $1\frac{5}{7}$ (B) $1\frac{3}{4}$ (C) 2   (D) $2\frac{1}{3}$ (E) $2\frac{2}{5}$

27. If R, S, and T are distinct digits then $RST_2 - ST_3 - R_4$ has a numeric value in base 10 of:

(A) $2R - S + 2T$ (B) $-S$ (C) $S + T$ (D) $5R$ (E) $3R - S$

28. Find the ratio of the median to the mean of the following list of numbers.
          2, 3, 5, 2, 4, 3, 2, 0, 5, 3, 5, 2

(A) 1:1   (B) 3:2   (C) 3:5   (D) 2:5   (E) 1:2

29. Missy Klas was absent the day of the algebra exam. She took the test the next day and made a 96. Her score raised the class average from 71 to 72. How many students, including Missy, took the test?

(A) 22   (B) 24   (C) 25   (D) 26   (E) 28

30. The set {..., — 6, — 4, — 2, 0, 2, 4, 6, ...} is closed under which of the following operations :

          I. addition   II. subtraction   III. multiplication   IV. division

(A) all of these   (B) I & III only (C) I, II, & III   (D) II & IV only (E) none of these

31. If the roots of $x^3 + bx^2 + cx + d = 0$ are $-5, 1$, and 3, then $b + c + d$ equals:

(A) -1 (B) 0 (C) 3 (D) 31 (E) 33

32. Mr. White's college math class has 40 students. 75% of the students are math majors. 32 of the students passed the final exam. 75% of those who passed the final exam are math majors. What percentage of the class who were not math majors passed the final exam?

(A) 8%   (B) 20%   (C) 25%   (D) 75%   (E) 80%

33. Find the lateral area, nearest square cm, of the oblique cylinder.
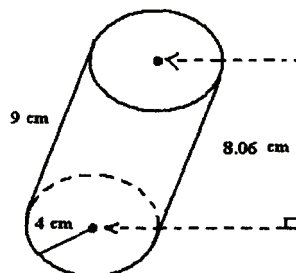

(A) 452 cm^2 (B) 352 cm^2 (C) 327 cm^2 (D) 226 cm^2 (E) 176 cm^2

34. If $a_1 = -4$, $a_3 = -9$, and $a_4 = 13.5$ are terms of a geometric sequence, then $a_2 =$ _____.

(A) -6 (B) -5 (C) 1.5 (D) 6 (E) 6.75

35. If $y^2 = -4 + 0i$ and $y^3 = 0 - 8i$ where $y = a + bi$ then $a + b$ equals:

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

36. Pop Eye takes his family sailing. They leave dock A and sail 1.5 miles on a course of 30° to buoy B. They turn and travel 1.75 miles on a bearing of 110° to buoy C. How far is it from buoy C to dock A? (nearest tenth)

(A) 1.6 miles   (B) 2.0 miles   (C) 2.3 miles   (D) 2.5 miles   (E) 2.7 miles

37. How many points of intersection occur when $r = 2\cos\theta + 1$ and $\theta = \pi$ are graphed on a polar coordinate system?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

38. $\sum_{k=0}^{2} \left(kx + (k+1)y \right) = ?$

(A) $4x + 3y$ (B) $2x + 3y$ (C) $3y$ (D) $3x + 6y$ (E) $2x + y$

39. $F_0 = 0$ and $F_1 = 1$ are the first two Fibonacci numbers. Find F_{10}.

    (A) 34   (B) 47   (C) 55   (D) 76   (E) 89

40. Find the area of the shaded region in square units.
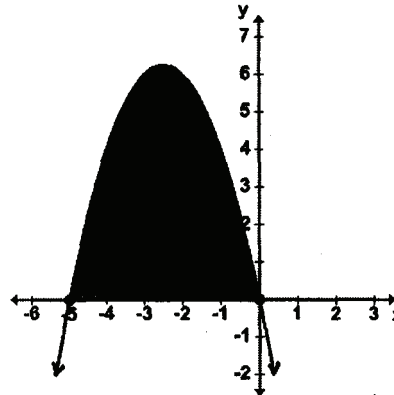
    (A) $18\frac{3}{4}$ (B) $19\frac{2}{3}$ (C) $20\frac{1}{2}$ (D) $20\frac{5}{6}$ (E) $21\frac{1}{4}$

41. Use the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, to transform the conic $x^2 + xy + y^2 = 3$ into an equation that does not contain an xy term. The equation is:

    (A) $x^2 + y^2 = 9$ (B) $3x^2 + y^2 = 6$ (C) $x^2 - 2y^2 = 3$

    (D) $x^2 + 3y^2 = 6$ (E) $3x^2 + y^2 = 3$

42. If $f(x) = \frac{2x+3}{4x-5}$, then $f'(1) =$ _____

    (A) -22 (B) -5 (C) -2 (D) $\frac{1}{2}$ (E) 2

43. Betty Luzes rolls a fair die 4 times. What is the mathematical expectation of the sum of the outcomes of the 4 rolls ?

    (A) 7   (B) 10.5   (C) 14   (D) 21   (E) 28

44. Five married couples attend the square dance planning meeting. How many committees of four people can be chosen if no committee is to include a husband-and-wife pair?

    (A) 20   (B) 25   (C) 50   (D) 80   (E) 105

45. Let $R = \{1, 3, 5\}$, $S = \{0, 2, 4\}$, and $T = \{1, 2, 3\}$. How many elements are in $(R \cup T) \cap (S \cup T)$?

    (A) 6   (B) 5   (C) 4   (D) 3   (E) 2

46. The circles $x^2 + y^2 + 3x - 6y + 5 = 0$ and $2x^2 + 2y^2 + 5x - 6y + 3 = 0$ intersect in two points. The slope of the line through the two points of intersection is:

    (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{7}{6}$ (D) $\frac{3}{5}$ (E) $\frac{5}{6}$

47. Let AB be the diameter of the circle with center C with CG $\perp$ AB, DE $\perp$ AB, and EF $\perp$ DC. If AE = 9 and BE = 4 then DF = ?
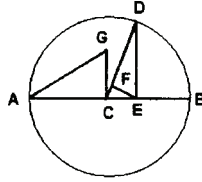
(A) 5 (B) $5\frac{7}{13}$ (C) 6 (D) $6\frac{1}{2}$ (E) 7

48. How many of the following numbers are NOT solutions to $7 - 5|3x + 1| \geq -1$?

-0.987 $-0.777...$ $.222...$ 0.3 $.12$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

49. It is precisely 2:45 pm on a circular clock. What is the measure of the smaller angle formed by the minute hand and the hour hand of the clock?

(A) $192°$ (B) 187.5 (C) $150°$ (D) $168°$ (E) $172.5°$

50. $y^2 - x^2 = 0$ is an equation of a degenerate conic. Which of the following is the best graphical representation of this equation?

(A) point (B) line (C) parallel lines (D) intersecting lines (E) no graph

51. Let $f(x) = 4 - x$ and $g(x) = 3x - 5$ and $h(x) = 2x$. Find $h\left(f\left(g(0)\right)\right)$.

(A) 7 (B) 9 (C) 14 (D) 18 (E) 19

52. Points A, B, C, and D are the vertices of a square. Point E is on the interior of the square such that points A, B, and E form an equilateral triangle. A line segment connects points D and E. Another line segment connects points C and E. Find m$\angle$CED.

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{12}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{9}$ (E) $\frac{5\pi}{6}$

53. A regular deck of 52 cards is shuffled and the top five cards are dealt face up. What is the probability, nearest $\frac{1}{1000}$%, that all 5 cards are face cards (Jacks, Queens, Kings)?

(A) $\frac{1}{40}$% (B) $\frac{3}{100}$% (C) $\frac{2}{25}$% (D) $\frac{13}{200}$% (E) $\frac{23}{500}$%

54. Vector $v = (2, 9)$ is perpendicular to vector $w = (4, k)$. Find k.

(A) $-\frac{1}{9}$ (B) $-\frac{1}{8}$ (C) $-\frac{8}{9}$ (D) $-1\frac{1}{8}$ (E) -18

55. The graph shown is the graph of which of the following equations.
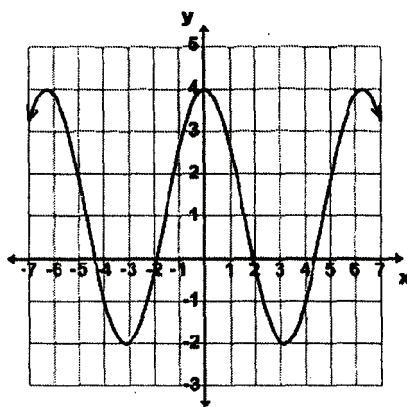
(A) $y = 1 + 3\sin(x)$

(B) $y = 3\cos(x + 1)$

(C) $y = \cos(3x + 1)$

(D) $y = 1 + 3\cos(x)$

(E) $y = 3\sin(x + 1)$

56. Point P has polar coordinates of $(4 , \frac{2\pi}{3})$ and rectangular coordinates of (x , y). Where does point P lie on the Cartesian coordinate plane?

(A) QII   (B) QIII   (C) QIV   (D) x-axis   (E) y-axis

57. How many asymptotes does $f(x) = \frac{2 - 3x^2}{x - 1}$ have?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

58. The slope of the line tangent to the curve $y = 2x^3 - 3x^2 - 5$ at $x = 2$ is 12. The point of intersection of the tangent line and the curve is:

(A) $(2, 2)$ (B) $(-1, -2)$ (C) $(-1, 2)$ (D) $(2, 1)$ (E) $(2, -1)$

59. Evaluate: $\int_{-n}^{n} (x^3 - 3x^2 - 5) \, dx$

(A) $\frac{n^4}{2}$ (B) $-10n$ (C) $-2n(n^2 + 5)$ (D) $\frac{n^4}{2} + 2n^2$ (E) $2n(n^2 - 5)$

60. The coordinates of the vertices of $\triangle ABC$ are $(-1, 2), (1, 0)$ and $(-2, -2)$. The medians of the $\triangle ABC$ intersect at (x, y). Find $x + y$.

(A) 1   (B) $\frac{3}{4}$ (C) $\frac{1}{3}$ (D) $-\frac{2}{3}$ (E) -1

1.	B	21.	A	41.	B
2.	A	22.	E	42.	A
3.	C	23.	B	43.	C
4.	B	24.	C	44.	D
5.	C	25.	E	45.	D
6.	A	26.	A	46.	A
7.	B	27.	E	47.	B
8.	C	28.	A	48.	C
9.	A	29.	C	49.	E
10.	B	30.	C	50.	D
11.	C	31.	A	51.	D
12.	A	32.	E	52.	E
13.	C	33.	D	53.	B
14.	C	34.	D	54.	C
15.	A	35.	E	55.	D
16.	B	36.	D	56.	A
17.	E	37.	B	57.	C
18.	D	38.	D	58.	E
19.	A	39.	C	59.	C
20.	D	40.	D	60.	D

UIL

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics
Invitational B • 2009

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $2(3 \times 4! \div (5-6) + 7^2 - 8)$

   (A) -132 (B) -62 (C) 9.6   (D) 63.8   (E) 66

2. What is 25% of $\frac{3}{4}$ of 50 plus 75% of $\frac{5}{8}$ of 40?

   (A) 62.5   (B) 46.875   (C) 28.125   (D) 15.625   (E) 175.78125

3. The Cheep Choppe is having a February Sale. The regular price of their special coats is $89.95. They are on sale for 30% off of the regular price. A newspaper coupon offers 10% off of the sale price. What would the selling price be if the customer brings in the coupon?

   (A) $53.97   (B) $56.67   (C) $49.95   (D) $71.98   (E) $69.95

4. Find an equation of the line shown.
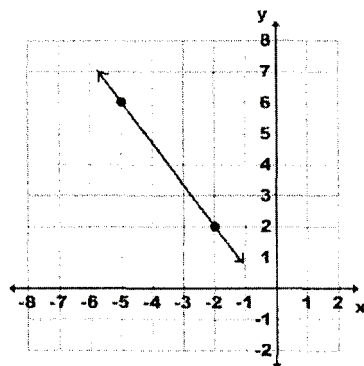
   (A) $3x + 4y = 2$ (B) $4x - 3y = 2$ (C) $3x + 3y = -2$ (D) $4x + 3y = -2$ (E) $4x + 4y = 2$

5. $(p - q) \times r = pr - qr$ is an example of which property of equality?

   (A) distributive   (B) identity   (C) inverse   (D) subtraction   (E) associative

6. Deputy Dawg is building two adjacent rectangular pens to hold his puppies. Each pen has a length 3 times longer than its width and the pens share a common side (width). He has 65 feet of fencing. What will the area of each pen be?

   (A) $64\frac{2}{3}$ sq. ft.   (B) $63\frac{3}{8}$ sq. ft.   (C) $60\frac{3}{4}$ sq. ft.   (D) $58\frac{2}{3}$ sq. ft.   (E) $56\frac{1}{3}$ sq. ft.

7. If a quadrilateral is inscribed in a circle, then its opposite angles are _____.

   (A) obtuse   (B) congruent   (C) supplementary   (D) complementary   (E) acute

8. The coordinates of the vertices of $\triangle ABC$ are $(-2, 0)$, $(1, 4)$ and $(4, 0)$. The coordinates of the incenter is:

   (A) $(\frac{13}{16}, 1\frac{1}{4})$ (B) $(1, 1\frac{3}{16})$ (C) $(1\frac{3}{16}, 1\frac{1}{4})$ (D) $(1, 1\frac{1}{2})$ (E) $(1\frac{1}{4}, 1)$

9. Shirley Knott is filling up her circular wading pool. The diameter of the pool is 6 feet and the height of the pool is 1 foot. What is the maximum number of whole gallons of water can she use and not cause the pool to overflow?

(A) 231   (B) 230   (C) 212   (D) 211   (E) 200

10. Betty Wheel rides her bicycle up and down the hilly streets from her house to school . The graph best depicts her 6 minute ride. (speed is not truly linear in this case).
During the time interval of 2 minutes to 3 minutes Betty is _____.
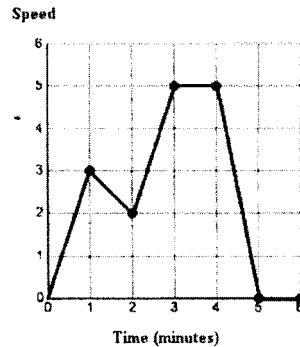
(A) going uphill   (B) getting tired   (C) going downhill

(D) decreasing speed   (E) increasing speed

11. Willie Dublett deposits $500 in a bank account with an interest rate of 2.5% compounded monthly. How many months will it take for his balance to reach $750?

(A) 195   (B) 183   (C) 250   (D) 192   (E) 236

12. The 8th Fibonacci number is 13. The 10th Fibonacci number is 34. Find the 9th Lucas number.

(A) 21   (B) 23.5   (C) 29   (D) 34   (E) 47

13. The vertex angle of an obtuse isosceles triangle has a measure of 100 ° and the length of one the sides adjacent to the vertex angle is 4 cm. Find the area of the triangle. (nearest tenth)

(A) 13.9 cm^2 (B) 12.2 cm^2 (C) 10.3 cm^2 (D) 7.9 cm^2 (E) 6.1 cm^2

14. Determine the range of $f(x) = 2 + 3\cos(4x - 5)$.

(A) $[-1, 5]$ (B) $[-2, 4]$ (C) $[-3, 3]$ (D) $[-1, 3]$ (E) $[-5, 3]$

15. A ramp is 18 ft. long and the angle of elevation of the ramp from the ground to the platform is 15° 10' 5". Find the height of the platform. (nearest approximation)

(A) 4' 7.9"   (B) 4' 8.52"   (C) 4' 8.56"   (D) 4' 8.65"   (E) 4' 8.72"

16. Find L_9 if $L_0 = 2$, $L_1 = 1$, and $L_n = L_{(n-1)} + L_{(n-2)}$, where $n \geq 2$.

    (A) 29   (B) 47   (C) 76   (D) 34   (E) 55

17. Find f(2) when $f(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$. (nearest thousandth)

    (A) — .441   (B) — .408   (C) — .416   (D) — .466   (E) — .422

18. If 5 men working 5 hours a day for 5 days can dig a tunnel 5 km in length, then how long of a tunnel can 10 men working 10 hours a day for 10 days dig?

    (A) 10 km   (B) 15 km   (C) 25 km   (D) 40 km   (E) 50 km

19. Let f be a function such that it is differentiable on (a,b) and continuous on [a,b], and f(a) = f(b) = 0. Then there is a number c in (a,b) for which $f'(c) = 0$. This theorem is known as:

    (A) Sandwich Theorem   (B) Least Limit Theorem   (C) Rolle's Theorem

    (D) Max—Min Theorem   (E) L'Hopital's Rule

20. The point P (2, 1) is rotated clockwise about the origin to point (— 1, — 2). The angle of rotation, to the nearest degree, is:

    (A) 127°   (B) 143°   (C) 153°   (D) 190°   (E) 217°

21. The function $f(x) = \frac{2}{x-1} + 18x$ is increasing at which of the following values of x?

    (A) $1\frac{2}{3}$ (B) $1\frac{1}{4}$ (C) 1   (D) $\frac{1}{12}$ (E) $\frac{1}{2}$

22. Roland Tuwin is playing a special dice game. He rolls two dice. If he rolls a double (1-1, 2-2, 3-3, etc.) he gets 20 points. If he does not roll a double and the sum of the dice is a prime number he gets 10 points. If he does not roll a double and the number is not a prime he loses 5 points. What is the mathematical expectation on any one roll?

    (A) 0 points   (B) 5 points   (C) 10 points   (D) 15 points   (E) 20 points

23. Betty Cheetz flips a fair coin and rolls a fair six-sided die. What are the odds that she will get a head and a prime number?

    (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{9}$ (D) $\frac{1}{6}$ (E) $\frac{1}{2}$

24. The universal set $U = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$. Subset L = $\{5, 7, 15, 17\}$, subset M = $\{3, 13\}$. How many elements are in the complement set of L $\cup$ M ?

    (A) 3   (B) 4   (C) 6   (D) 7   (E) 9

16. Find L_9 if $L_0 = 2$, $L_1 = 1$, and $L_n = L_{(n-1)} + L_{(n-2)}$, where $n \geq 2$.

    (A) 29   (B) 47   (C) 76   (D) 34   (E) 55

17. Find $f(2)$ when $f(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$. (nearest thousandth)

    (A) $-.441$ (B) $-.408$ (C) $-.416$ (D) $-.466$ (E) $-.422$

18. If 5 men working 5 hours a day for 5 days can dig a tunnel 5 km in length, then how long of a tunnel can 10 men working 10 hours a day for 10 days dig?

    (A) 10 km   (B) 15 km   (C) 25 km   (D) 40 km   (E) 50 km

19. Let f be a function such that it is differentiable on (a,b) and continuous on [a,b], and $f(a) = f(b) = 0$. Then there is a number c in (a,b) for which $f'(c) = 0$. This theorem is known as:

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20. The point P (2, 1) is rotated clockwise about the origin to point $(-1, -2)$. The angle of rotation, to the nearest degree, is:

    (A) $127°$ (B) $143°$ (C) $153°$ (D) $190°$ (E) $217°$

21. The function $f(x) = \frac{2}{x-1} + 18x$ is increasing at which of the following values of x?

    (A) $1\frac{2}{3}$ (B) $1\frac{1}{4}$ (C) 1 (D) $\frac{1}{12}$ (E) $\frac{1}{2}$

22. Roland Tuwin is playing a special dice game. He rolls two dice. If he rolls a double (1-1, 2-2, 3-3, etc.) he gets 20 points. If he does not roll a double and the sum of the dice is a prime number he gets 10 points. If he does not roll a double and the number is not a prime he loses 5 points. What is the mathematical expectation on any one roll?

   (A) 0 points   (B) 5 points   (C) 10 points   (D) 15 points   (E) 20 points

23. Betty Cheetz flips a fair coin and rolls a fair six-sided die. What are the odds that she will get a head and a prime number?

    (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{9}$ (D) $\frac{1}{6}$ (E) $\frac{1}{2}$

24. The universal set $U = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$. Subset $L = \{5, 7, 15, 17\}$, subset $M = \{3, 13\}$. How many elements are in the complement set of $L \cup M$?

    (A) 3   (B) 4   (C) 6   (D) 7   (E) 9

25. Given the trapezoid shown with bases a and b, the length of segment PQ is the _____ mean of a and b.
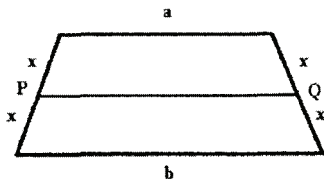

(A) arithmetic   (B) geometric   (C) harmonic   (D) half   (E) quadratic

26. Which of the following mathematicians was most remembered as the inventor of logarithms?

(A) Alicia Stott (B) Freda Porter (C) John Napier (D) Rene Descartes (E) Sophie Germain

27. If $x + y = -6$ and $xy = 9$ then $x^3 + y^3 = ?$

(A) 567   (B) 513   (C) 378   (D) -27 (E) -54

28. Adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.
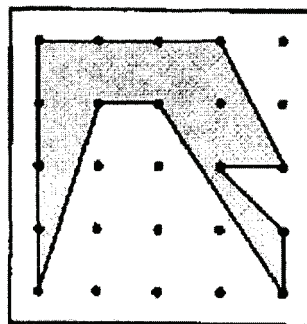
(A) 6.5 cm^2 (B) 7 cm^2 (C) 7.75 cm^2 (D) 8.5 cm^2 (E) 9 cm^2

29. The center of the circle, $x^2 + y^2 - 4x - 6y + 9 = 0$, is _____ units from the origin. (nearest tenth)

(A) 2.5   (B) 3.0   (C) 3.6   (D) 5.0   (E) 7.2

30. A Ferris wheel has a radius of 7 meters and turns at 6 revolutions per minute. The bottom of the Ferris wheel is 1 meter above the ground. The height h of a passenger above the ground varies sinusoidally with time t. Which of the following equations best describes the relationship between h and t?

(A) $h = 6 + 7\cos(\frac{\pi}{6} t)$ (B) $h = 8 + 7\sin(\frac{\pi}{5} t)$ (C) $h = 7 - 6\cos(\frac{\pi}{5} t)$

(D) $h = 7 - 8\sin(\frac{\pi}{3} t)$ (E) $h = 8 - 7\cos(\frac{\pi}{5} t)$

31. Find the area (in square units) of the region bounded by $y = -x^2$ and $y = -4$.

(A) 16   (B) $11\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 8   (E) $5\frac{1}{3}$

32. Let L = {2, 1, 3, 4, 7, 11}. Two elements of set L are selected at random without replacement. What is the probability that the median of the two numbers selected is a whole number ?

(A) $46\frac{2}{3}\%$ (B) $53\frac{1}{3}\%$ (C) 40%   (D) $26\frac{2}{3}\%$ (E) $23\frac{1}{3}\%$

33. Lesleys Kwik runs the 400 meter dash at the local track meet. She runs the first 100 meters in 15 seconds, the second 100 meters in 16 seconds, the third 100 meters in 17.2 seconds and the last 100 meters in 18.5 seconds. What was her average speed? (nearest thousandth)

(A) 5.997 m/sec   (B) 6.016 m/sec   (C) 6.034 m/sec   (D) 6.052 m/sec   (E) 6.228 m/sec

34. How many ordered pairs (x , y) are solutions to the equation $5x + 3y < 40$, where x,y are integers and $0 < y < x < 9$?

(A) none   (B) 6   (C) 10   (D) 14   (E) 21

35. Find the smallest integer k so that $4x^2 - 3x + k = 0$ has two imaginary roots.

(A) -5 (B) -2 (C) -1 (D) 1   (E) 0

36. Let $f(x) = 2x + 1$ and $g(x) = 4 - 3x$, then $f^{-1}\left[g^{-1}(-1)\right]$ equals:

(A) -7 (B) $-1\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $2\frac{1}{3}$ (E) 13

37. If $p + q = 12$ and $p \times q = 22$ then $(p - q)^2 = ?$

(A) 34   (B) 56   (C) 100   (D) 340   (E) 436

38. Noah Kanwen won 40 of 75 games. How many of the next 25 games can Noah lose in order to have won 60% overall?

(A) 23   (B) 20   (C) 15   (D) 13   (E) 5

39. Three students in Miss Woik's class were absent the day of the exam. The average of the other 12 students was 84. What would the three absent students have to average on their make-up exam in order to bring the entire class average to 86?

(A) 85   (B) 88   (C) 91   (D) 94   (E) 99

40. Two chords, WY and XZ intersect in the interior of a circle at point P such that $m\angle WPX = 70°$ and $m\overset{\frown}{WX} = 120°$. If points X and Y are not on $\overset{\frown}{WZ}$ then $m\overset{\frown}{YZ}$ is:

(A) 20°   (B) 25°   (C) 35°   (D) 60°   (E) 95°

41. Find the lateral area, nearest square cm, of the cone.
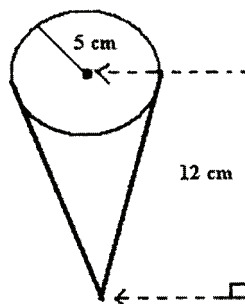
(A) 188 cm^2 (B) 204 cm^2 (C) 245 cm^2 (D) 283 cm^2 (E) 377 cm^2

42. Which of the following is true about the relation $f(x) = x^2 + 2x + 2$?

   (A) odd function   (B) even function   (C) neither even nor odd function

   (D) not a function   (E) none of these are true

43. Find the determinant: $\begin{bmatrix} -1 & 2 & 3 \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{bmatrix}$

   (A) -6 (B) -5 (C) 6   (D) 18   (E) 24

44. How many leafs are in the "rose" curve graph of the polar equation $r = 3 - 4\sin(2\theta + 5)$?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

45. Which of the following is a reference angle for $1645°$?

   (A) $45°$ (B) $25°$ (C) $22.5°$ (D) $18°$ (E) $4.5°$

46. Simplify: $\dfrac{(n-1)!\,(n+2)!}{(n+1)!\,(n-2)!}$

   (A) $(n-1)^2$ (B) $\dfrac{n+1}{n-1}$ (C) $\dfrac{n+2}{n-2}$ (D) $n^2 - 2$ (E) $n^2 + n - 2$

47. How many different ways can you select 5 bills from a cash box containing $1, $2, $5, $10, $20, $50, and $100 bills?

   (A) 330   (B) 462   (C) 792   (D) 1,584   (E) 1,980

48. Let $f(x) = \dfrac{4x+5}{3x}$. Find $f'(2)$.

   (A) $-1\frac{2}{3}$ (B) $-\frac{5}{12}$ (C) $-\frac{1}{12}$ (D) $1\frac{1}{3}$ (E) $2\frac{1}{6}$

49. Find an equation of the tangent line to the curve $y = \sqrt{9 - 4x}$ at the point $(-4, 5)$.

(A) $5x - 2y = -30$ (B) $2x - 5y = 33$ (C) $-2x + 5y = -10$

(D) $2x - 5y = -10$ (E) $2x + 5y = 17$

50. A bag contains yellow golf balls and orange golf balls. The probability of selecting a yellow ball is $\frac{2}{5}$. If 20 yellow balls are added to the bag, the probability of selecting a yellow ball becomes $\frac{4}{7}$. How many orange balls are in the bag?

(A) 10   (B) 14   (C) 30   (D) 35   (E) 42

51. Simplify: $\left((a^2 b)^{-3} \times (ab^2) \div (a^2 b^{-3}) \times (ab) \right)^{-1}$, where a, b > 0.

(A) $a^6 b^{-3}$ (B) $a^{-5} b^3$ (C) $a^8 b^3$ (D) $a^{-3} b^2$ (E) $a^{-2} b^3$

52. In 3 years Sid Upp will be twice as old as his son, Stan Upp. Five years ago Stan's age was $\frac{1}{3}$ of his father's age at that time. What is the sum of their ages now?

(A) 48   (B) 42   (C) 37   (D) 32   (E) 29

53. Point $P(2, -3)$ is reflected across the origin to point Q. Then point Q is translated horizontally 3 units to the right to point R. Point R is reflected across the origin to point S. The coordinates of point S is (x, y). Find $x + y$.
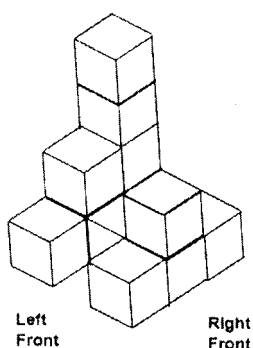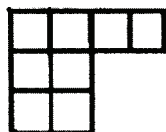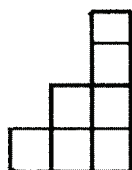
(A) 4   (B) 1   (C) 0   (D) -1 (E) -4

54. Which of the following would best represent a two dimensional perspective of the front right side view of this figure shown?
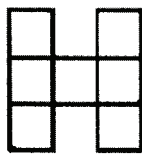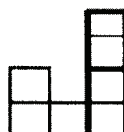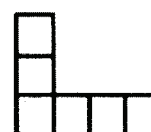

Left Front   Right Front

(A)   (B)   (C)   (D)   (E)

55. If you start at $(-1.5, 0)$ on the x-axis and travel horizontally 12 radians to the right, how many times will you cross the graph of $y = \sin(3x)$?

    (A) 11   (B) 12   (C) 13   (D) 14   (E) 15

56. If $9^{(x+2y)} = 81$ and $9^{(2x-y)} = \frac{1}{9}$, then $3^{xy} = ?$

    (A) 0   (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{9}$ (E) 1

57. Tye Guhr drops a golf ball from a height of 10 feet. It bounces back to a height of 60% of the distance it fell. How far has it traveled when it hits the ground the fourth time? (nearest inch)

    (A) 43 ' 5 "   (B) 33 ' 6 "   (C) 31 ' 4 "   (D) 25 ' 7 "   (E) 23 ' 6 "

58. The point of inflection on the graph of $f(x) = 2x^3 - 6x^2 + 6x - 6$ is (a, b). Find b.

    (A) 2   (B) 1   (C) 0   (D) -4 (E) -6

59. The Buddy System motorcycle testing company is testing a motorcycle with a side car. They hire 4 cyclists to do the testing in pairs. How many arrangements of driver and rider are possible?

    (A) 4   (B) 6   (C) 8   (D) 12   (E) 24

60. Find the lateral surface area of this prism. All angles are right angles.
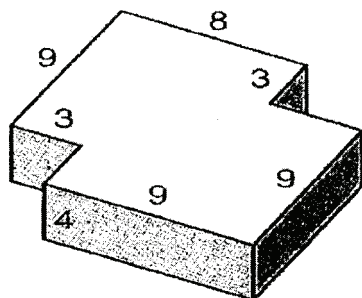
    (A) 188 units ^{2} (B) 315 units ^{2} (C) 438 units ^{3} (D) 246 units ^{2} (E) 192 units ^{2}

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2009
Answer Key

1. B	21. A	41. B
2. C	22. B	42. C
3. B	23. A	43. E
4. D	24. A	44. D
5. A	25. A	45. B
6. E	26. C	46. E
7. C	27. E	47. B
8. D	28. B	48. B
9. D	29. C	49. E
10. E	30. E	50. C
11. A	31. C	51. A
12. E	32. A	52. B
13. D	33. A	53. E
14. A	34. D	54. B
15. B	35. D	55. A
16. C	36. C	56. E
17. C	37. B	57. B
18. D	38. E	58. D
19. C	39. D	59. D
20. B	40. A	60. E

UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
District 1 • 2009
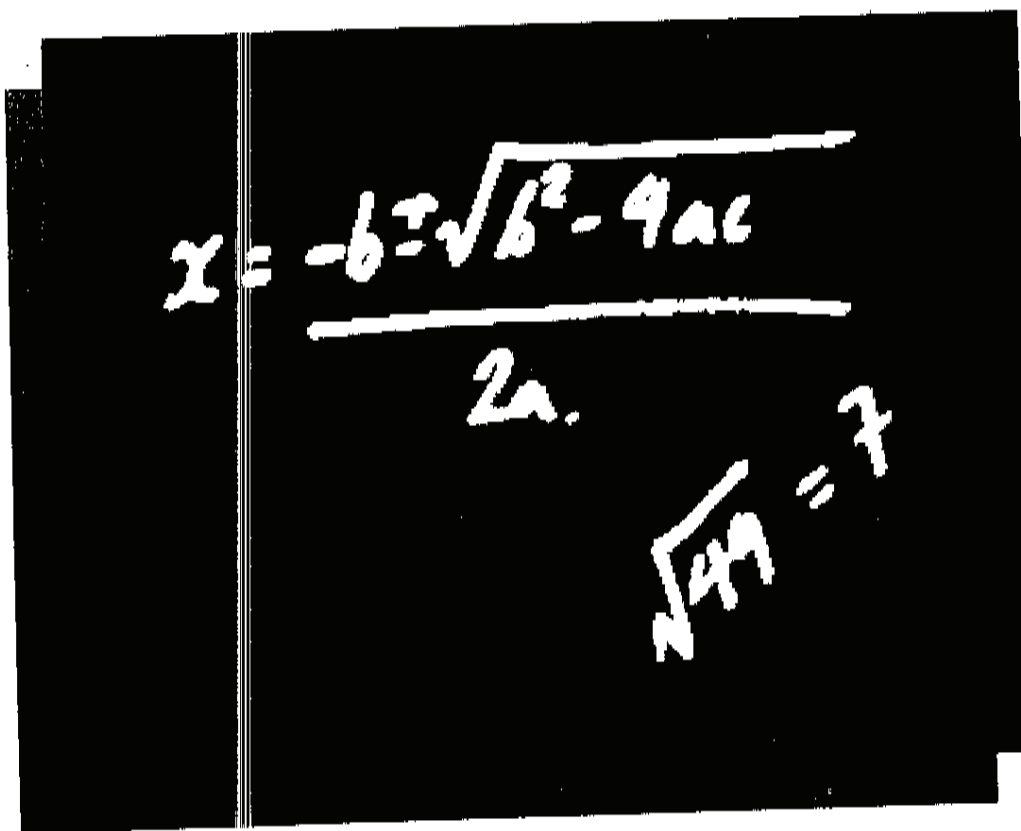
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
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1. Evaluate: $2 \div \left[(\tfrac{2}{3})^2 - (3)^{-2}\right] \times 3!$

   (A) 36   (B) $26\tfrac{1}{2}$ (C) $21\tfrac{3}{5}$ (D) 4   (E) 1

2. $3A + 0 = 3A$ and $3B \times 1 = 3B$ are examples of the _____ properties of equality.

   (A) associative   (B) commutative (C) distributive (D) identity   (E) inverse

3. In three years Odie Butgoody's age will be six times what Yung Su's age was last year. Two years ago Odie's age was five times what Yung Su's age now. What is the sum of their ages now?

   (A) 68   (B) 65   (C) 60   (D) 57   (E) 55

4. The shaded area is best represented by which of the following inequalities?
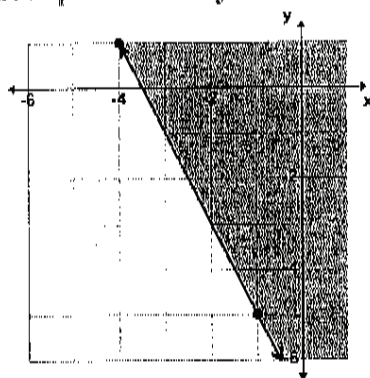
   (A) $4x + 2y \geq -3$ (B) $2x - y \leq 7$ (C) $2x + y \geq -6$

   (D) $4x + 2y \leq 1$ (E) $2x + y \geq -7$

5. Shirley Knott is filling up her circular wading pool. The radius of the pool is 4 feet. How deep will the water be if she puts 250 gallons of water in the pool? (nearest inch)

   (A) 5"   (B) 6"   (C) 7"   (D) 8"   (E) 9"

6. A segment extending from the vertex angle of an isosceles triangle to the midpoint of the opposite side is called a(n):

   (A) altitude   (B) bisector   (C) centroid   (D) directrix   (E) mean

7. The center of the circle $x^2 + y^2 - 4x + 10y + 12 = 0$ is:

   (A) $(4, -10)$ (B) $(-2, 6)$ (C) $(2, -5)$ (D) $(6, -5)$ (E) $(6, -4)$

8. Which of the following is one of the few American Indian women to earn a Ph.D. in applied mathematics and computational sciences? She is a member of the Lumbee tribe.

   (A) Emmy Noether   (B) Freda Porter   (C) Hypatia
   (D) Karen E. Smith   (E) Grace Alele Williams

9. The graph best depicts Ima Hiker's 6 hour backpack trip. During which of the following time intervals was Ima walking the fastest?

Distance (miles)

Time (hours)
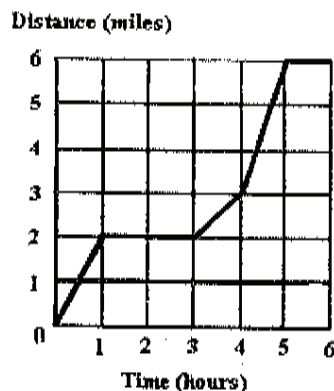
(A) 0 - 1　　　(B) 1 - 3　　　(C) 3 - 4　　　(D) 4 - 5　　　(E) 5 - 6

10. How many 2-member subsets and 4-member subsets can be made from the set $\{\clubsuit, \triangle, \heartsuit, \oplus, \blacklozenge, \square\}$?

(A) 35　　　(B) 30　　　(C) 21　　　(D) 20　　　(E) 15

11. Petunia's garden shop has three varieties of tomato plants, Big Boy, Cherry, and Porter. They have 5 Big Boy plants, 4 Cherry plants and 7 Porter plants mixed together without labels. The young plants all look alike. Petunia randomly selects three plants. What is the probability that the three she selects are the same variety?

(A) 7.97 %　　(B) 2.58 %　　(C) 8.75 %　　(D) 3.42 %　　(E) 5.62 %

12. Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = ?$

(A) $f'(a) - g'(a)$　　(B) $\dfrac{f'(a)}{g'(a)}$　　(C) 0　　(D) $[f'(a)][g'(a)]$　　(E) $\dfrac{f(a)}{g(a)}$

13. Let $f(x) = \sin x \cos x$. Find $f'(\frac{\pi}{3})$.

(A) $-\dfrac{1}{2}$　　(B) $-\sqrt{3}$　　(C) 1　　(D) $\dfrac{\sqrt{3}}{2}$　　(E) $\dfrac{3}{2}$

14. Find $x + y$ if $\begin{bmatrix} x & -2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} 7 \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(A) 5　　　(B) 4　　　(C) 3　　　(D) — 4　　　(E) — 7

15. Which of the following is true about the relation $f(x) = \dfrac{x^2 - 1}{x^4 + 1}$?

(A) odd function　　　(B) even function　　　(C) neither even nor odd function

(D) not a function　　　(E) one-to-one function

16. A belt joins the two pulleys shown. The smaller pulley with the given radius revolves at the rate of 48 rpm. Find the linear velocity of the belt in centimeters per minute. (nearest tenth)
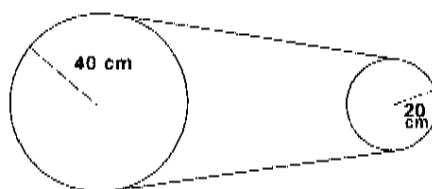
   (A) 1508.0   (B) 3055.8   (C) 4523.9   (D) 6031.9   (E) 6111.5

17. Point P has polar coordinates of ($3\sqrt{3}$, $\frac{5\pi}{3}$). What are the rectangular coordinates of point P?

   (A) ($1\frac{2}{3}$, $-4\frac{1}{2}$) (B) ($-4\frac{1}{2}$, $-\frac{\sqrt{3}}{2}$) (C) (5 , $3\sqrt{3}$) (D) ($\frac{\sqrt{3}}{3}$, 5) (E) ($\frac{3\sqrt{3}}{2}$, $-\frac{9}{2}$)

18. Last year the cost of a gallon of gas was $3.75. This year a gallon of gas costs $1.55. Phil Upp's car gets 20 miles per gallon of gas and he has $15.00 to spend on gas. How much farther can Phil drive this year than he could last year? (nearest whole mile)

   (A) 174 miles   (B) 114 miles   (C) 87 miles   (D) 80 miles   (E) 44 miles

19. The set { 0, 1, 1, 2, 3, 5, 8, 13, 21, 34 ...} is closed under which of the following operations :

                I. addition   II. subtraction   III. multiplication   IV. division

   (A) all of these   (B) I & III only   (C) I, II, & III   (D) II & IV only   (E) none of these

20. The circles $(x-2)^2 + (y+5)^2 = 17$ and $(x-4)^2 + (y+2)^2 = 28$ intersect in two points. The slope of the line through the two points of intersection is:

   (A) $-\frac{1}{5}$ (B) $-\frac{2}{3}$ (C) $-3\frac{1}{3}$ (D) 1   (E) $1\frac{1}{2}$

21. Which of the following would best represent a two dimensional perspective of the right front view of this figure shown?

Right Front
Left Front

(A)   (B)   (C)   (D)   (E)
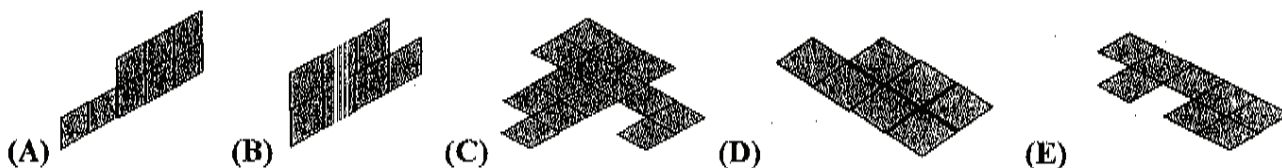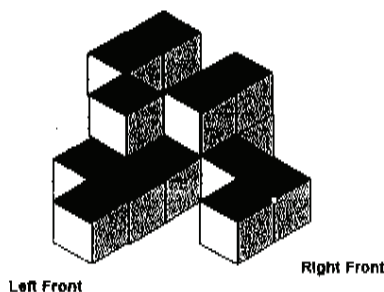
22. If $\cos x + \sin x = .5$ then the approximate value of $\tan x$ is:

   (A) -1.134 (B) -0.268 (C) -0.451 (D) -0.577 (E) -2.679

23. Dr. J drops a basketball ball from a height of 9 feet. It bounces back to a height of 40% of the distance it fell. How far has it traveled when it hits the ground the fourth time? (nearest inch)

   (A) $14'2''$ (B) $14'7''$ (C) $19'8''$ (D) $20'3''$ (E) $20'8''$

24. Find the equation of the line tangent to the curve $y = x^3 - 2x^2 + 1$ at $x = -1$.

   (A) $y = 1 - 2x$ (B) $y = -2$ (C) $y = 7x + 5$ (D) $y = 2x - 1$ (E) $y = 7x + 1$

25. Find the sum of all numbers between 0 and 250 that are multiples of 8.

   (A) 4000   (B) 3948   (C) 3,906   (D) 3,968   (E) 4,096

26. The integers greater than 1 are arranged in five columns as shown. If this pattern continues which column would contain the number 725?

A	B	C	D	E
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	

   (A) A   (B) B   (C) C   (D) D   (E) E

27. Minnie Mumm has a sheet of metal that is 3 feet by 5 feet. She is going to cut out the same size square from each of the four corners, then fold up the sides, solder them, and make a rectangular cistern that will hold the greatest amount of water. What percent of the sheet of metal will she cut out and not use? (nearest tenth of a percent)

   (A) 7.4 %   (B) 8.4 %   (C) 8.9 %   (D) 9.8 %   (E) 10.5 %

28. The function $f(x) = \frac{3}{2x-1} + 24x$ is decreasing at which of the following values of x?

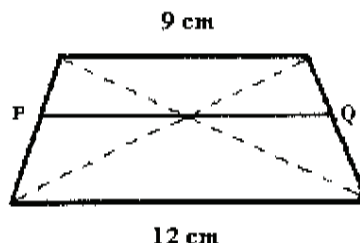   (A) $\frac{5}{6}$ (B) $\frac{5}{7}$ (C) $\frac{7}{9}$ (D) $\frac{4}{5}$ (E) $\frac{9}{11}$

29. Simplify $\left[\sin \theta + \cos \theta \times \cot \theta \right] \times \tan \theta$.

   (A) $\sin \theta - \cos \theta$ (B) $\sec \theta$ (C) $\sin \theta \cos \theta$ (D) $\csc \theta$ (E) $\sin \theta + \cos \theta$

30. Determine the range of $f(x) = 5 - 4\cos(3x + 2)$.

   (A) $[1, 9]$ (B) $[-2, 6]$ (C) $[3, 7]$ (D) $[-1, 3]$ (E) $[-4, 1]$

31. Find the length of segment PQ in the trapezoid shown. (nearest hundredth)

9 cm

12 cm

   (A) 10.00 cm   (B) 10.29 cm   (C) 10.39 cm   (D) 10.50 cm   (E) 10.61 cm

32. $\sum_{k=0}^{2} (-1)^k x - (-1)^k = ?$

   (A) $3x+1$ (B) $x-3$ (C) $2x-1$ (D) -1 (E) $x-1$

33. The universal set $L = \{1, 3, 4, 7, 11, 18, 29, 47, 76\}$. Subset $E = \{4, 18, 76\}$, subset $P = \{3, 7, 11, 29, 47\}$. How many elements are in the complement set of $E \cup P$?

   (A) 9   (B) 8   (C) 5   (D) 3   (E) 1

34. Willie Cheet flips two fair coins and rolls a fair six-sided die. What are the odds that he will get a head, a tail, and a Fibonacci number?

   (A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

35. Simplify: $\dfrac{2x-4}{x^2-4} \times \dfrac{4x^2+4x+1}{2x-2} \div \dfrac{2x^2-3x-2}{x+2}$

   (A) $\dfrac{2x^2-x-1}{x-2}$ (B) $\dfrac{2x+1}{2x^2-3x-2}$ (C) $\dfrac{x-1}{x-2}$ (D) $\dfrac{2x+1}{x^2-3x+2}$ (E) $\dfrac{x-2}{2x+1}$

36. Which of the following is NOT a solution to $\dfrac{3|2x+4|}{5} < 1$?

   (A) $-2\frac{2}{3}$ (B) -2 (C) $-1\frac{1}{8}$ (D) $-2\frac{1}{6}$ (E) $-1\frac{1}{5}$

37. Hickory, Dickory, and Doc were looking at their circular clock. The time was precisely 4:44 am. They calculated the measure of the smaller angle formed by the minute hand and the hour hand of the clock to be:

   (A) $144°$ (B) $120°$ (C) $140°$ (D) $130°$ (E) $122°$

38. A segment extending from the edge of an ellipse to the center of the ellipse and is perpendicular to the major axis is called the _____.

   (A) asymptote   (B) directrix   (C) eccentricity   (D) focus   (E) semiminor axis

39. The Fibonacci numbers (denoted by F) are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ... .
The Lucas numbers (denoted by L) are 2, 1, 3, 4, 7, ... Find L_{11}.

(A) 89   (B) 123   (C) 144   (D) 199   (E) 322

40. $4_5 \times 33_4 - 222_3 = \underline{\hspace{2in}}{}_2$

(A) 1011010   (B) 10010   (C) 1111   (D) 101101   (E) 100010

41. Trudy Frudy wants to mix some fruit ade that is 92.5% fruit with a special solution that is only 90% fruit. How many grams of fruit ade will she need to mix with the special solution to create 500 grams of 91% fruit?

(A) 200 g   (B) 225 g   (C) 250 g   (D) 275 g   (E) 300 g

42. Two chords, PS and QR intersect in the interior of a circle at point C such that $m\overset{\frown}{QS} = 40°$ and $m\angle PCR = 60°$. If points R and S are not on $\overset{\frown}{PQ}$ then $m\overset{\frown}{PR}$ is:

(A) 20°   (B) 50°   (C) 80°   (D) 100°   (E) 120°

43. Let $f(x) = 3x + 2$, $g(x) = 4 - 5x$, and $h(x) = x - 1$ then $h\left[f\left(g[1]\right)\right]$ equals:

(A) −26   (B) −2   (C) −1   (D) 0   (E) 14

44. Fuzzy Hare raises rabbits. He has 3 brown rabbits, 5 white rabbits, and 8 multicolored rabbits. How many ways can he put rabbits in a cage that will hold 1 brown rabbit, 2 white rabbits, and 3 multicolored rabbits?

(A) 8008   (B) 3360   (C) 840   (D) 7200   (E) 1680

45. Which of the following is a focus for $3x^2 - y^2 - 6x + 2y = 10$.

(A) (3, 1)   (B) (−4, −1)   (C) (5, 1)   (D) (−4, 2)   (E) (−1, 1)

46. Simplify: $\dfrac{(n-2)! \, (n+1)!}{(n-1)! \, n!}$

(A) $n^2 + n$ (B) $\dfrac{n+1}{n-1}$ (C) $\dfrac{n-2}{n+2}$ (D) $\dfrac{n+1}{n-2}$ (E) $\dfrac{n^2+n}{n-1}$

47. Hope Imakett is staking out a triangular garden in a big field. She puts a stake in the ground where she is standing. Then she walks 25 feet on a bearing of 75° where she puts a second stake. She walks another 25 feet on a bearing of 150° where she puts the third stake. What is the perimeter of Hope's garden? (nearest foot)

(A) 69 feet   (B) 80 feet   (C) 85 feet   (D) 90 feet   (E) 96 feet

48. Find the area of the shaded region in square units.
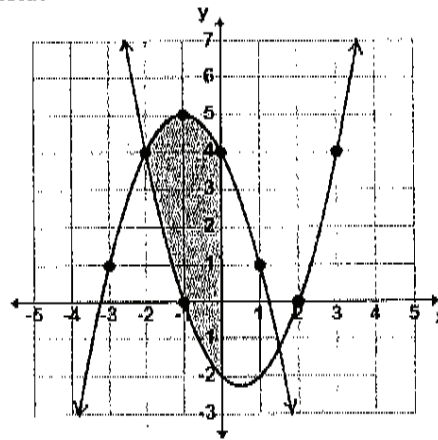
(A) $7\frac{5}{6}$ (B) $8\frac{1}{3}$ (C) $8\frac{1}{2}$ (D) $8\frac{2}{3}$ (E) $9\frac{1}{6}$

49. Which of the following is a reference angle for $-2400°$?

(A) $15°$ (B) $30°$ (C) $45°$ (D) $60°$ (E) $75°$

50. Harry Green can mow the #1 fairway in 1.5 hours. If Harry hires Sandy Trappe to help him they can mow it in 1 hour working together. How long would it take Sandy to mow it by herself?

(A) 1 hour   (B) 1.5 hours   (C) 2 hours   (D) 2.5 hours   (E) 3 hours

51. The point $(-1, -2)$ is rotated $210°$ counterclockwise about the origin to point P. Which of the following points is the closest approximation to point P.

(A) $(-0.13, 2.23)$ (B) $(1.87, -1.23)$ (C) $(-0.50, 0.87)$ (D) $(1.87, -2.23)$ (E) $(-0.13, 1.87)$

52. Snow White's 7 dwarfs are the only contestants in a beauty contest, in how many ways can She award 1st place, 2nd place and 3rd place trophies?

(A) 336   (B) 180   (C) 216   (D) 343   (E) 210

53. Ye Olde Arte has a large supply of colored poster board. The colors are blue, green, beige, white, yellow, and red. How many different ways can the store package 5 poster boards?

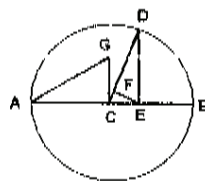(A) 126   (B) 231   (C) 252   (D) 378   (E) 462

54. Let $f(x) = 2x - 5$ and $g(x) = f(1 - x)$. Find $g^{-1}(x)$.

(A) $-(2x + 3)$ (B) $\frac{x - 2}{3}$ (C) $3x - 2$ (D) $3 + 2x$ (E) $\frac{-x - 3}{2}$

55. The coordinates of the vertices of $\triangle ABC$ are $(-3, -2)$, $(3, -1)$ and $(0, 3)$. The coordinates of the centroid is:

(A) $(\frac{1}{3}, \frac{2}{3})$ (B) $(-\frac{1}{2}, -\frac{1}{3})$ (C) $(2, 0)$ (D) $(-1\frac{2}{3}, 0)$ (E) $(0, 0)$

56. Let AB be the diameter of the circle with center C with CG $\perp$ AB, DE $\perp$ AB, and EF $\perp$ DC. If AE = 10, BE = $6\frac{2}{5}$, and CG = $1\frac{4}{5}$ then DE = ?

   (A) $3\frac{2}{5}$ (B) $4\frac{1}{2}$ (C) $4\frac{3}{5}$ (D) 8   (E) $8\frac{1}{5}$

57. The variable P varies jointly as the values of Q and R. Find the constant of proportionality if P = 88 when Q = 4 and R = $\frac{2}{5}$.

   (A) $35\frac{1}{5}$ (B) 55   (C) 64   (D) $140\frac{4}{5}$ (E) 220

58. Simplify: $(ab^2)^{-1} \times (a^2b^2)^{-3} \div (a^{-1}b^{-1}) \times a \div b$, where a, b > 0.

   (A) $\frac{1}{a^7b^6}$ (B) $\frac{1}{a^8b^{10}}$ (C) $\frac{1}{a^5b^8}$ (D) $\frac{1}{a^7b^{10}}$ (E) $\frac{1}{a^5b^6}$

59. Find the product of the arithmetic mean 3, 5, & 8, the geometric mean of 3, 5, & 8, and the harmonic mean of 3, 5, & 8. (nearest whole number)

   (A) 120   (B) 153   (C) 174   (D) 3657   (E) 3920

60. Find the lateral area, nearest square cm, of the right prism.

5 cm

6 cm

4 cm
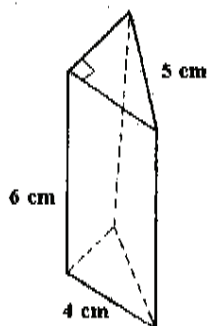
   (A) 66 cm²   (B) 72 cm²   (C) 78 cm²   (D) 84 cm²   (E) 99 cm²

University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2009
Answer Key

1. A	21. A	41. A
2. D	22. C	42. C
3. A	23. D	43. B
4. E	24. C	44. E
5. D	25. D	45. C
6. A	26. E	46. B
7. C	27. D	47. D
8. B	28. B	48. D
9. D	29. B	49. D
10. B	30. A	50. E
11. C	31. B	51. A
12. B	32. E	52. E
13. A	33. E	53. C
14. A	34. E	54. E
15. B	35. D	55. E
16. D	36. C	56. D
17. E	37. E	57. B
18. B	38. E	58. C
19. E	39. D	59. A
20. B	40. E	60. B

UIL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
District 2 • 2009
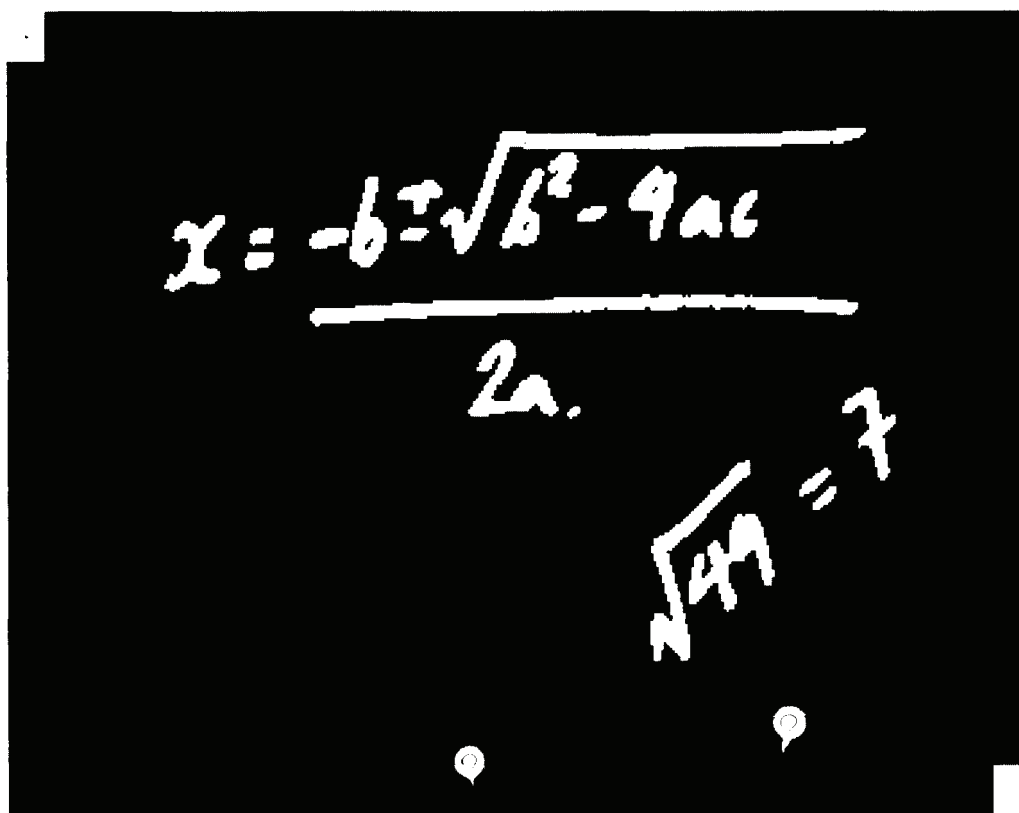
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $6 \times 5^2 \div (4-1)! + 2^0 - 3$

    (A) 148   (B) 8   (C) 42   (D) 23   (E) 33.5

2. The Bonnet Shoppe is having an Easter Bonnet sale. The regular price of their bonnets is $24.50. The sale price is $18.50. Senior women over the age of 60 get a coupon for 20% off the sale price. What percent of the regular price do the senior women pay? (nearest whole percent)
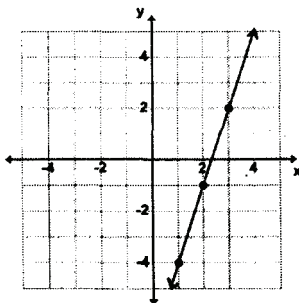
    (A) 60%   (B) 56%   (C) 55%   (D) 45%   (E) 40%

3. The distances between the hash marks (|) are equal. Find A + B + C.
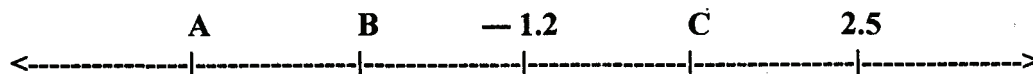
    (A) — 8.6   (B) — 7.3   (C) — 4.35   (D) 1.85   (E) 5.55

4. A line through the origin is perpendicular to the line shown. The point of intersection is (x, y). Find x + y.

    (A) 2.8   (B) 1.05   (C) 1   (D) 0.525   (E) 1.4

5. If the roots of $x^3 + bx^2 + cx + d = 0$ are — 3, — 4, and 2, then b + c + d equals:

    (A) 31   (B) 27   (C) 21   (D) — 5   (E) — 21

6. Rowan Boatright took 4 hours to travel upstream against a 3 mph current in his motor boat. It took 2.5 hours to return back to where he started. How far did Rowan travel?

    (A) 26 miles   (B) 40 miles   (C) 42.25 miles   (D) 80 miles   (E) 84.5 miles

7. Point P is on the negative x-axis. It is translated vertically up to point Q. Point Q is reflected across the y-axis to point R. Point R is rotated $\frac{3\pi}{2}$ radians about the origin to point S. Point S is reflected across the x-axis to point T. Where is point T?

   (A) y-axis   (B) QI   (C) QII   (D) QIII   (E) QIV

8. Let $p^2x^2 - q^2y^2 = 0$, where p > 0 and q < 0. Which of the following would be the best graphical representation of this equation?

    (A) point   (B) line   (C) parallel lines   (D) intersecting lines   (E) no graph

9. Two chords, WX and YZ are in the same circle and do not intersect. If WX > YZ then the measure of $\overset{\frown}{WX}$ is _____ the measure of $\overset{\frown}{YZ}$.

(A) less than   (B) one-half   (C) equal to   (D) double   (E) greater than

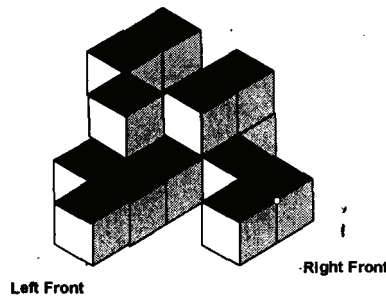10. If AD = 12 cm then DP = ?
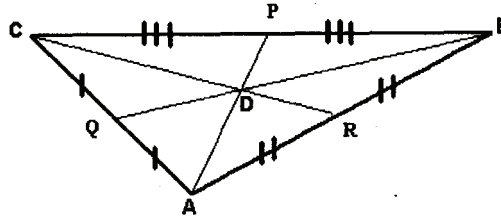
(A) 18 cm   (B) 12 cm   (C) 9 cm   (D) 6 cm   (E) 4 cm

11. Simplify: $\dfrac{a^2}{b^{-2}} \times \dfrac{b^3}{a^{-3}} \div (ab)^{-4}$

(A) $(ab)^9$ (B) $a^{-2}b^{-2}$ (C) a^9b^2 (D) a^2b^2 (E) ab

12. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the left front view of this figure has a perimeter of:

Right Front

Left Front

(A) 10 cm   (B) 12 cm   (C) 14 cm   (D) 16 cm   (E) 18 cm

13. The Stormchaser leaves port and sails 30 nautical miles due east. Then she changes direction and sails 30 nautical miles on a bearing of 120°. What bearing will she have to travel to sail straight back to port? (nearest degree)

(A) 75°   (B) 300°   (C) 15°   (D) 285°   (E) 330°

14. How many points of intersection occur when $r = \sin(3\theta)$ and $\theta = -\dfrac{\pi}{2}$ are graphed on a polar coordinate system?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

15. The sum of the coefficients of the 2nd term in the expansion of $(x - 2y)^4$ and the 4th term in the expansion of $(2x - y)^4$ is:

(A) −16   (B) −12   (C) 0   (D) 8   (E) 24

16. The sales at Honest Abe's car lot fluctuated from a high of $40,000 in January to a $25,000 in July. Which of the following equations best models the monthly sales at Honest Abe's car lot?

(A) $s(t) = 15\cos(\frac{\pi}{6}t) + 12.5$ (B) $s(t) = 7.5\cos(\frac{\pi}{6}t) + 25$ (C) $s(t) = 7.5\cos(\frac{\pi}{6}t) + 32.5$

(D) $s(t) = 15\cos(\frac{\pi}{12}t) + 32.5$ (E) $s(t) = 7.5\cos(\frac{\pi}{12}t) + 20$

17. Al G. Brah rode his bicycle from his camp to the end of the bike trail at an average speed of 3.5 mph and returned to his camp on the same bike trail at an average speed of 3.0 mph. What was Al's average speed for the round trip? (nearest thousandth)

(A) 3.225 mph   (B) 3.231 mph   (C) 3.240 mph   (D) 3.250 mph   (E) 3.260 mph

18. If $5^{(x+y)} = 625$ and $5^{(x-y)} = \frac{1}{5}$, then $5^{(x^2 - y^2)} = ?$

(A) 5   (B) $\frac{1}{125}$ (C) $\frac{1}{625}$ (D) $\frac{1}{25}$ (E) 4

19. Minnie Mumm has a rectangular sheet of cardboard that is 3 feet by 4 feet. She is going to cut out the same size square from each of the four corners, then fold up the sides and make a box with maximum volume. What will the area of the bottom of the box be? (nearest square inch)

(A) 437 sq. in.   (B) 450 sq. in.   (C) 772 sq. in.   (D) 795 sq. in.   (E) 5240 sq. in.

20. Let k be a number in the domain of the function f. If $f'(k) = 0$ or $f'(k)$ does not exist, then k is called a(n) _____ point of function f.

(A) increasing   (B) inflection   (C) tangent   (D) critical   (E) decreasing

21. The curve of a polar equation $r = 1 + k \cos \theta$, where $k \geq 2$, is called a:

(A) circle   (B) cardioid   (C) spiral   (D) lemniscate   (E) limacon

22. Evaluate: $\int_{-n}^{n} (3x^2 + 2x - 1)\, dx$

(A) $2n^2 + 2n$ (B) $2n^3 - 2n^2$ (C) $2n$ (D) $2n^3 - 2n$ (E) $2n^3$

23. Slick Willy and Fingers Dolly each have a standard deck of cards. After shuffling several times each one flips over a randomly selected card. Determine the probability that at least one card is an ace. (nearest percent)

(A) 23 %   (B) 15 %   (C) 8 %   (D) 4 %   (E) 1 %

24. Karen E. Smith is currently a mathematics professor at the University of Michigan. She is highly recognized for her thesis and outstanding work in:

(A) commutative algebra   (B) Fibonacci sequences   (C) 4-dimensional hyper solids
(D) symmetry of triangles   (E) logarithms

25. The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many distinct combinations exist if P is a multiple of 5, Q is a factor of 10, and R is a perfect square? Stopping at 0 is not allowed.
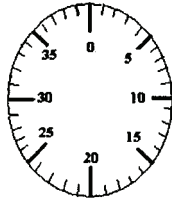

    (A) 17   (B) 192   (C) 168   (D) 105   (E) 70

26. The Mel Ting ice cream store offers 12 different flavors of ice cream. Mel makes banana splits using three scoops of ice cream. How many different three-scoop banana slips can he make?

    (A) 364   (B) 220   (C) 455   (D) 165   (E) 440

27. If $x = \sqrt{64}$ and $\sqrt{64} = 8$, then $x = 8$ is an example of the _____ property of equality.

    (A) associative   (B) commutative   (C) distributive   (D) identity   (E) transitive

28. $3333_4 - 222_3 + 11_2 =$ _____ $_5$

    (A) 1412   (B) 44442   (C) 1441   (D) 3122   (E) 4444

29. In $\triangle PRS$, $QT \parallel RS$, $QR = x$, $PQ = 12$, $QT = 4$ and $RS = 3x$. Find x.

    (A) 2   (B) 1.5   (C) 3   (D) 4.5   (E) not enough information given

30. In a plane, the locus of points equidistant from two given points is the _____ of the segment joining the points.

    (A) midpoint   (B) perpendicular bisector   (C) endpoint   (D) length   (E) angle bisector

31. Mr. White's high school math class has 30 students. 60% of the students are on his math team. 20 members of the class took a practice math test on Friday. 75% of those who took the math test were on the math team. What percentage of the class who were not on the math team took the practice math test?

    (A) $16\frac{2}{3}\%$ (B) $27\frac{7}{9}\%$ (C) $33\frac{1}{3}\%$ (D) $41\frac{2}{3}\%$ (E) 50%

32. The equation $3x^2 + 4x + k = 0$ has two imaginary roots. Which of the following is always a true statement about the value of k?

    (A) $k < 0$ (B) $k < -2$ (C) $k > 2$ (D) $k > 1$ (E) $k > 0$

33. The equation $y = $ _____ will produce this graph.
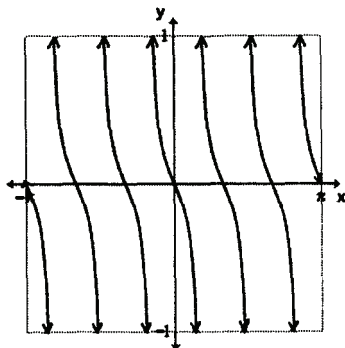
    (A) $-\frac{1}{3}\tan(4x + \pi)$ (B) $-\frac{1}{3}\cot(4x - \pi)$ (C) $\frac{1}{4}\cot(3x + \pi)$

    (D) $\frac{1}{4}\tan(3x - \pi)$ (E) $-\frac{1}{4}\tan(3x + \pi)$

34. Simplify: $\sec(\frac{\pi}{2} - x) - \tan(\frac{\pi}{2} - x)\sin(\frac{\pi}{2} - x)$

    (A) $-\cos(\frac{\pi}{2} - x)$ (B) $\sin x$ (C) $\sin 2x$ (D) $\sin(x - \frac{\pi}{2})$ (E) $\cos(\frac{\pi}{2} + x)$

35. The directrix of the parabola $y = x^2 - 6x + 5$ is:

    (A) $x = 3$ (B) $y = -4$ (C) $y = -3\frac{3}{4}$ (D) $y = -4\frac{1}{4}$ (E) $x = -4\frac{3}{4}$

36. Find the determinant: $\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

    (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

37. A bag contains 3 yellow marbles, 4 green marbles, and 7 blue marbles. All of the marbles are the same size. Three marbles are randomly drawn without replacement. What is the probability that the first one drawn is blue, second one is green, and the third one is blue? (nearest tenth)

    (A) 7.7 %   (B) 3.8 %   (C) 6.1 %   (D) 13.1 %   (E) 6.4 %

38. Coach Mallery's math/science team consists of 3 girls and 5 boys. She wants to enter at least 2 student, but no more than 4 students in the math contest. In how many ways can she enter her students in the math contest?

    (A) 162   (B) 154   (C) 112   (D) 98   (E) 66

39. Find the area (in square units) of the region bounded by $y = x^2 - 2x + 2$ and $y = 6 - x^2$.

(A) $9\frac{2}{3}$ (B) $11\frac{1}{3}$ (C) $10\frac{1}{3}$ (D) 12   (E) 9

40. Line l is a line tangent to the curve $y = x^3 - 5x + 1$ at point (x, y) and parallel to the x-axis. Which of the following could be the y-coordinate? (nearest tenth)
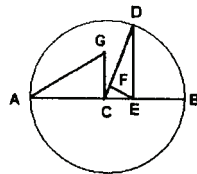
(A) -3.9 (B) 5.3 (C) -6.5 (D) 3.3 (E) 5.0

41. The integers greater than 0 are arranged in five columns as shown. If this pattern continues which column would contain the number 2009?

A	B	C	D	E
1	2	3	4	
	8	7	6	5
9	10	11	12	
	16	15	14	13

(A) A   (B) B   (C) C   (D) D   (E) E

42. Let AB be the diameter of the circle with center C with CG $\perp$ AB, DE $\perp$ AB, and EF $\perp$ DC. If AE = 10, BE = 6.4, and CG = 1.8 then AG = ? (nearest tenth)

(A) 8.2   (B) 7.8   (C) 8.0   (D) 7.2   (E) 8.4

43. Find the mean of the median, mode, and range of 88, 72, 90, 85, 92, 67, & 85.

(A) $82\frac{1}{4}$ (B) 85   (C) 65   (D) 55   (E) $82\frac{5}{7}$

44. $x + 2$ is one of the factors of $6x^3 + 5x^2 - 17x - 6$. Which of the following is also a factor?

(A) $2x + 3$ (B) $6x - 1$ (C) $3x + 1$ (D) $2x - 1$ (E) $x - 3$

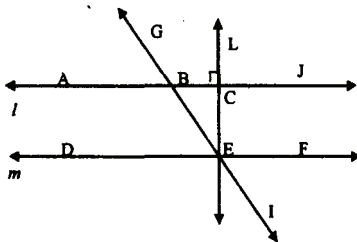45. If $x - y = 5$ and $xy = 2$ then $x^3 - y^3 = ?$

(A) 155   (B) 399   (C) 95   (D) 250   (E) 133

46. If two-thirds of A equals the sum of three-fourths of B and one-half of C and B equals one-third of C, then C is what part of A?

(A) $\frac{1}{2}$ (B) $1\frac{1}{8}$ (C) $\frac{5}{6}$ (D) $\frac{8}{9}$ (E) $\frac{1}{4}$

47. The four lines in the figure are coplanar and $m//l$. Which of the following are true statements?

1. $\angle CBE$ & $\angle DEB$ are congruent
2. $m\angle FEI + m\angle GBJ = m\angle ABG + m\angle DEI$
3. $m\angle CBG = m\angle FEI$
4. $\angle GED$ & $\angle GEF$ are complementary angles

(A) 1 & 2   (B) 3 & 4   (C) 4 only   (D) 2 & 3   (E) 1, 2, & 4

48. Willie Maykette deposits $100 in an account with an annual interest rate of 3% compounded monthly. Betty Dont deposits $100 in an account with an annual simple interest rate of 4.5%. The interest is added to their respective accounts at the end of each month. How much longer will it take Willie's original deposit to double than it will take Betty's to double? (nearest month)

(A) 5 months   (B) 9 months   (C) 11 months   (D) 18 months   (E) 23 months

49. The I. M. Dry ranch has a horse trough in the shape of a rectangular prism. It is 3 yards long, 1.5 feet wide and 14 inches high. How many whole gallons of water will it take to fill the trough without spilling over?

(A) 117   (B) 98   (C) 294   (D) 231   (E) 63

50. The 11th Fibonacci number is 55 and the 13th Fibonacci number is 144. Find the 12th Lucas number?

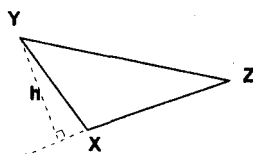(A) 89   (B) 123   (C) 144   (D) 199   (E) 322

51. If $y^2 = 8 - 6i$ and $y^3 = 18 - 26i$ where $y = a + bi$ then $a + b$ equals:

(A) -2 (B) -1 (C) 0   (D) 1   (E) 2

52. If you start at $(-3\pi, 2)$ and travel horizontally 22 radians to the right, how many times will you cross the graph of $y = \sec(x)$?
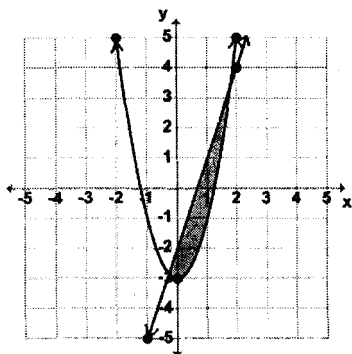
(A) 5   (B) 6   (C) 7   (D) 8   (E) 9

53. A triangle is drawn as shown. Find the height, h, if YZ = 15", $m\angle XZY = 30°$, and XZ = 10". (nearest tenth)

(A) 7.5 "   (B) 8.7 "   (C) 10.0 "   (D) 12.5 "   (E) 13.0 "

54. Which of the following system of inequalities would be best represented by the shaded region shown?

(A) $2x^2 + y \leq -3$
$3x + y \geq 2$

(B) $3x + y \geq -2$
$2x^2 - y \geq 3$

(C) $y \leq 2x^2 - 3$
$y \geq -3x - 2$

(D) $3x^2 - y \leq 2$
$2x - y \geq 3$

(E) $3x - y \geq 2$
$2x^2 - y \leq 3$

55. How many horizontal asymptotes does $f(x) = \frac{4x^2 - 5}{x - 2}$ have?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

56. Find the angle of rotation, θ (closest approximation), where $0 < \theta < \frac{\pi}{2}$, such that the conic $2x^2 + 5x + 4y^2 - 3xy + 12 = 0$ contains no xy term in its equation.

(A) 0.294   (B) 0.340   (C) 0.409   (D) 0.491   (E) 1.080

57. Lon Kutter can mow the yard by himself in 1.5 hours. His sister, Ima, can mow the yard by herself in 2.5 hours. How long would it take to mow the yard if they both mowed the lawn together? (nearest minute)

(A) 64 minutes   (B) 62 minutes   (C) 60 minutes   (D) 58 minutes   (E) 56 minutes

58. Les Sense has a jar that contains Indian Head pennies and Lincoln pennies. There are less than 100 coins in the jar. The odds of selecting a Lincoln is $\frac{3}{5}$. If 10 more Lincolns are added to the jar, the probability of selecting a Lincoln becomes 50%. How Indian Heads are in the jar?

(A) 75   (B) 50   (C) 45   (D) 30   (E) 25

59. Find the harmonic mean of the roots of $x^3 - 3x^2 + 3.25x - 1.5 = 0$.

(A) 1   (B) $1\frac{1}{4}$ (C) $1\frac{11}{76}$ (D) $1\frac{5}{13}$ (E) $1\frac{39}{56}$

60. The three-dimensional vector $(-3, -2, 1)$ is perpendicular to which of the following vectors?

(A) $(-1, 2, 3)$ (B) $(-5, -1, -2)$ (C) $(3, 2, 1)$ (D) $(1, 1, 5)$ (E) $(3, -2, 1)$

1. D	21. E	41. A
2. A	22. D	42. E
3. B	23. B	43. C
4. E	24. A	44. C
5. E	25. C	45. A
6. D	26. A	46. D
7. B	27. E	47. A
8. D	28. A	48. C
9. E	29. B	49. A
10. D	30. B	50. D
11. A	31. D	51. E
12. C	32. C	52. C
13. D	33. E	53. A
14. C	34. B	54. E
15. A	35. D	55. A
16. C	36. C	56. D
17. B	37. A	57. E
18. C	38. B	58. E
19. C	39. E	59. D
20. D	40. B	60. D

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Mathematics
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1. Evaluate: $0.4 - \frac{3}{5} \times 0.3 + \frac{1}{5} \div 0.1$

   (A) $\frac{21}{50}$ (B) $\frac{3}{5}$ (C) $1\frac{1}{5}$ (D) $1\frac{2}{5}$ (E) $2\frac{11}{50}$

2. $4a \times (4a)^{-1} = 1$ and $-4a + 4a = 0$ are examples of the _____ property of equality.

   (A) associative   (B) commutative (C) distributive (D) identity   (E) inverse

3. Simplify: $\left(\frac{4x^2 - 1}{2x^2 + 3x - 2}\right)\left(\frac{x^2 + 4x + 4}{4x^3 + 8x^2 - x - 2}\right)$

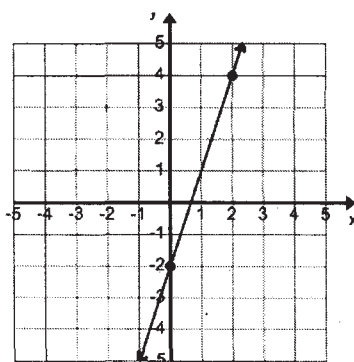   (A) $\frac{1}{2x-1}$ (B) $\frac{1}{x+2}$ (C) $2x^2 + 5x + 2$ (D) $3x + 5$ (E) $x^2 + 4x + 4$

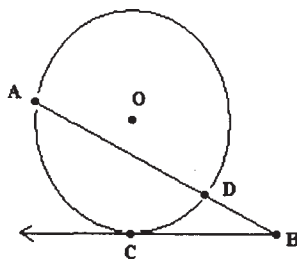4. Find an equation of the line shown using the two points shown.

   (A) $3x - 2y = 4$ (B) $4x - 3y = 10$ (C) $3y - x = 2$ (D) $3x - y = 2$ (E) $3y - 2x = 6$

5. A ray with an endpoint on the exterior of a circle extends through the circle intersecting it at two distinct points. The ray is called a/an _____.

   (A) chord   (B) diameter   (C) radius   (D) secant   (E) tangent

6. Using the figure below, if $m\angle ABC = 20°$ and $m\overarc{AC} = 70°$ then $m\overarc{CD} =$ _____.

   (A) $50°$ (B) $45°$ (C) $35°$ (D) $30°$ (E) $20°$

7. Let $f(x) = 2x - 5$ and $g(x) = x + 3$. Find $f^{-1}\left(g\left(f(-x)\right)\right)$.

   (A) $1.5 - x$ (B) $-4x - 9$ (C) $x - 3$ (D) $4 - x$ (E) $-2x - 4$

8. If $a_1 = -1$, $a_2 = -2$, $a_3 = -3$ and $a_n = (a_{n-1})(a_{n-3}) + (a_{n-2})$, where $n \geq 4$, then $a_7 =$ ___.

(A) -16 (B) -5 (C) 1 (D) 11 (E) 39

9. A vulture is perched on top of a telephone pole that is 30 feet tall. Looking down at an angle of depression of 35° he sees some "road kill" on the highway. How far from the base of the pole is the "road kill"? (nearest foot)

(A) 21 ft. (B) 25 ft. (C) 34 ft. (D) 43 ft. (E) 64 ft.

10. Let $f(x) = \frac{1}{2} + \frac{3}{2} \sin (4x + \frac{4\pi}{3})$. Which of the following has a value of $\frac{\pi}{3}$?

(A) amplitude (B) displacement (vertical) (C) frequency (D) phase shift (E) period

11. Find the determinant: $\begin{bmatrix} -1 & 2 & -3 \\ 1 & -2 & 3 \\ -1 & 2 & -3 \end{bmatrix}$

(A) -6 (B) -3 (C) -2 (D) -1 (E) 0

12. $\prod\limits_{k=1}^{3} (kx - k + 1) = ?$

(A) $6x^3 - 7x^2 + 2x$ (B) $6x - 3$ (C) $6x^2 - 7x + 2$

(D) $6x^3 - 19x^2 + 16x - 4$ (E) $6x^3 + 5x^2 - 12x + 4$

13. A point of inflection on the graph of $f(x) = -x^3 - 3x^2 - 3x - 10$ is (a, b). Find b.

(A) 3 (B) 1 (C) 0 (D) -1 (E) -9

14. Using the inequality $\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos(x)}{x^2} < \frac{1}{2}$, the value of $\lim\limits_{x \to 0} \frac{1 - \cos(x)}{x^2}$ is ?

(A) $\frac{25}{52}$ (B) $\frac{5}{12}$ (C) $\frac{11}{24}$ (D) $\frac{1}{2}$ (E) 1

15. Les Sense has 3 identical pennies and 3 identical dimes. He wants to place a single coin in each of the circles shown below. How many different ways can Les place the coins if rotation doesn't matter?
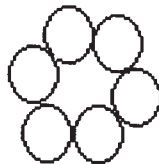

(A) 20 (B) 12 (C) 10 (D) 6 (E) 4

16. Three fair six-sided dice are tossed and the spots on the top faces are recorded. One die is red, one is white, and one is blue. The probability of one of the following is 25%. Which one is it?

    (A) All three numbers are even.
    (B) The red die is a six or the blue die is even.
    (C) Exactly two of the three numbers are equal.
    (D) The red die is even and the blue die is 4 or more.
    (E) None of the above

17. Let AB be the diameter of the circle with center C with $CG \perp AB$, $DE \perp AB$, and $EF \perp DC$. Let AE = 9, BE = 5, and CG = 2. Find DE. (nearest hundredth)
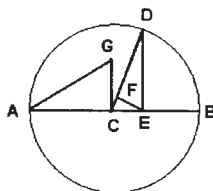
    (A) 6.71   (B) 7.28   (C) 6.43   (D) 6.25   (E) 7.00

18. Which of the following mathematicians' major contributions included a system of latitude and longitude, a measurement of the Earth's circumference, and a prime number sieve?

    (A) Emmy Noether   (B) Euclid   (C) Euler   (D) Eratosthenes   (E) Emilie du Châtelet

19. Melody Toones wants to buy 5 music CDs. The cost of a single CD is $20.00. The cost of a 2-pack set of CDs is 20% less than the cost of the 2 single CDs. The cost of the 3-pack set is 40% less than the cost of 3 single CDs. How much would Melody save if she buys the sets instead of the singles?

    (A) $16.00   (B) $20.00   (C) $25.00   (D) $32.00   (E) $44.00

20. $4444_5 + 333_4 + 22_3 + 1_2 =$ _____ $_6$.

   (A) 116   (B) 1234   (C) 3120   (D) 5432   (E) 55555

21. Five pounds of regular blend coffee sells for $6.50 per pound. Three pounds of super blend coffee sells for $8.00 per pound. Find the cost per pound of the special mixture of the regular blend and the super blend. (nearest cent)

    (A) $7.06   (B) $7.41   (C) $6.85   (D) $7.00   (E) $7.25

22. The equation $4x^2 - 3x - k = 0$ always has two positive roots when which of these is true?

    (A) $-1 < k < 0$ (B) $-\frac{9}{16} > k > -1$ (C) $k < -1$ (D) $k > -\frac{9}{16}$ (E) $-\frac{9}{16} < k < 0$

23. Find the volume of the cone. (nearest tenth)
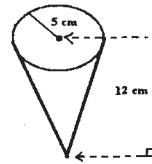
(A) 471.2 cm³   (B) 188.5 cm³   (C) 314.2 cm³   (D) 235.6 cm³   (E) 340.3 cm³

24. The coordinates of the vertices of $\triangle ABC$ are $(-2, -1)$, $(1, 2)$ and $(4, -1)$. The coordinates, nearest hundredth, of the incenter is:

(A) (2.34, 0.88)   (B) (1.00, 0.24)   (C) (1.00, -0.76)   (D) (2.33, 1.33)   (E) (1.00, 0.00)

25. If $p + q = 11$ and $p \times q = 22$ then $(p - q)^2 = ?$

(A) 11   (B) 33   (C) 121   (D) 209   (E) 242

26. Let $\left[\log_a(18k) - \log_a(2)\right] + \log_a(4k) = 2\log_a(x)$. Find x.

(A) 13k   (B) 11k − 1   (C) 6k   (D) 18k   (E) 36k²

27. The annual Lake Ivie Jetski Race starts at marker X, goes due North to marker Y, a distance of 500 yards. At marker Y, the racers travel on a bearing of 250° to marker Z, a distance of 750 yards. At marker Z they race straight back to marker X. Find the bearing they must take get from marker Z to marker X. (nearest degree)

(A) 120°   (B) 109°   (C) 78°   (D) 39°   (E) 20°

28. The expression $\frac{\sin 2A}{1 - \cos 2A}$ is equivalent to which of the following?

(A) $\cot 2A$ (B) $\tan A$ (C) $\tan(\frac{\pi}{2} - A)$ (D) $\cot(\frac{\pi}{2} + A)$ (E) $\frac{\tan 2A}{1 - \cot 2A}$

29. U. R. Kwik can make 10 ice cream sundaes in 5 minutes. I. M. Kwiker can make 10 sundaes in 3 minutes. Soh Slo can make 10 sundaes in 10 minutes. How long would it take all three of them to make 10 sundaes in they worked together? (nearest second)

(A) 1 min 40 sec   (B) 3 min 8 sec   (C) 36 sec   (D) 1 min 35 sec   (E) 1 min 18 sec

30. An infinite geometric sequence has a common ratio of $-\frac{1}{2}$ and a sum of $-\frac{4}{9}$. What is the first term of the sequence?

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $-\frac{2}{3}$ (D) $-\frac{9}{8}$ (E) $-\frac{2}{9}$

31. The curve of a polar equation $r = a + b \cos \theta$ is called a looped limacon when which of the following is true:

(A) a = b   (B) a < b < 2a   (C) a < b   (D) 2a ≤ b   (E) a > b

32. Let $f'(x) = 9x^2 + 4x - 2$ and $f(-1) = 4$. Find $f(1)$.

(A) -1 (B) 0 (C) 1 (D) 4 (E) 6

33. The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many distinct combinations exist if P is a factor of 35, Q is a perfect cube, and R is a Fibonacci number? Stopping at 0 is not allowed.
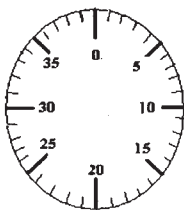
(A) 108   (B) 96   (C) 64   (D) 48   (E) 15

34. How many subsets containing at least one member, but no more than 4 members, can be made from the set $\{\clubsuit, \heartsuit, \oplus, \diamondsuit, \bigstar\}$?

(A) 30   (B) 24   (C) 2500   (D) 100   (E) 10

35. Let P = { T, M, S, C, A }, Q = { T, E, X, A, S}, and R = { M, A, T, H }. How many elements are in $(P \cap Q) \cup (P \cap R)$?

(A) 2   (B) 4   (C) 5   (D) 6   (E) 8

36. Which of the following is NOT a solution to $1 + 2|3x - 4| \leq 5$?

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$ (E) $\frac{1}{5}$

37. Point J is on the negative y-axis. It is rotated 90° counter clockwise to point K. Point K is reflected across the y-axis to point L. Point L is translated vertically up to point M. Point M is reflected across the origin to point N. Where is point N?

(A) y-axis   (B) QI   (C) QII   (D) QIII   (E) QIV

38. Which of the following is a reference angle for 4.8π ?

(A) $\frac{3\pi}{10}$ (B) 0.7π (C) $\frac{4\pi}{5}$ (D) 1.2π (E) $\frac{\pi}{5}$

39. Which of the following is true about the relation $f(x) = x^3 - 5x - 1$?

(A) odd function   (B) even function   (C) neither even nor odd function

(D) not a function   (E) none of these are true

40. Which of the following would best represent a two dimensional perspective of the right rear view of this figure shown?

left rear

right rear
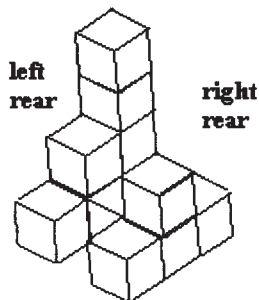
(A)   (B)   (C)   (D)   (E)

41. The function $f(x) = \frac{3}{2-x} - 5x$ is increasing at which of the following values of x?

(A) 2.4   (B) -1.4 (C) -0.4 (D) 4.4   (E) 3.4

42. Roland Bones rolls three fair six-sided dice, one die at a time. What is the probability that the he rolls a composite number on the first die, a prime number on the next die, and a unit on the last die?

(A) $\frac{1}{216}$ (B) $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) $\frac{1}{72}$ (E) $\frac{1}{3}$

43. Find the area of the bases of this prism. All angles are right angles.
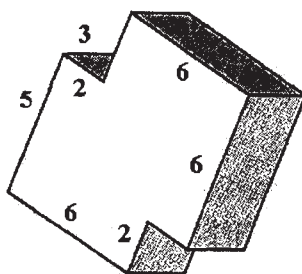
(A) 90 units ^{2} (B) 108 units ^{2} (C) 144 units ^{2} (D) 162 units ^{2} (E) 198 units ^{2}

44. Find the area of the region bounded by the graphs of $y = \ln(x + 1)$, $y = 1 - x$, and $y = 0$. (closest approximation)

(A) 0.231 units^2 (B) 0.226 units^2 (C) 0.224 units^2 (D) 0.185 units^2 (E) 0.146 units^2

45. Let $f(x) = \frac{2x^2}{1-x}$ and s(x) be the slant asymptote of f. Find the slope of s(x).

(A) -2 (B) -1 (C) -0.5 (D) 1   (E) 2

46. Use the angle of rotation, θ (nearest degree), where $0\,° < \theta < 90\,°$, to transform the conic $3xy - 1 = 0$ into an equation that is in standard position and does not contain an xy term. The transformed equation is:

(A) $x^2 - y^2 = 3$ (B) $3x^2 + 2y^2 = 1$ (C) $x^2 - 3y^2 = 2$ (D) $3x^2 + 3y^2 = 1$ (E) $3x^2 - 3y^2 = 2$

47. Phil Upp can fill his circular wading pool in 45 minutes using two hoses. He can fill the pool in 1 hour 20 minutes using the larger hose by itself. How long would it take the him to fill the pool if he uses the smaller hose by itself? (nearest minute)

(A) 1 hr 50 min   (B) 1 hr 43 min   (C) 1 hr 35 min   (D) 1 hr 30 min   (E) 1 hr 28 min

48. The depth d of water at Rod Bending's favorite fishing hole in the Gulf varies sinusoidally from 5 feet to 15 feet, depending on the time t. Last Saturday, the high tide occurred at 5:00 a.m. and the next high tide occurred at 5:00 p.m. Which of the following equations best describes the relationship between the depth of the water and the time?

(A) $d = 10 + 5\sin(\frac{\pi}{6}t - \frac{\pi}{3})$ (B) $d = 10 + 5\sin(\frac{\pi}{6}t - 2)$ (C) $d = 10 - 5\sin(\frac{\pi}{6}(t + \frac{1}{2}))$

(D) $d = 10 - 5\sin(\frac{\pi}{3}t)$ (E) $d = 10 + 5\sin(\frac{\pi}{6}t + \frac{\pi}{3})$

49. How many points of intersection occur when $r = 2\sin(\theta) - 3$ and $\theta = \frac{3\pi}{2}$ are graphed on a polar coordinate system?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

50. Joy Ryder goes on a roller coaster ride at the state fair. The graph best depicts the 6 minute ride. (speed is not truly linear in this case). During the time interval of 3 minutes to 4 minutes Joy's roller coaster car is _____.
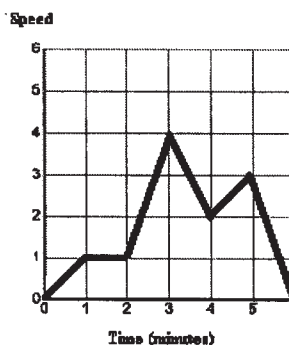
(A) climbing uphill   (B) turning on a curve   (C) going downhill

(D) decreasing speed   (E) increasing speed

51. The Fibonacci numbers (denoted by F) are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ... . The Lucas numbers (denoted by L) are 2, 1, 3, 4, 7, ... Find L_{12}.

(A) 521   (B) 377   (C) 322   (D) 233   (E) 199

52. A rectangular shaped prism water tank has a maximum capacity of 360 gallons. The tank is 21 feet long and 11 inches deep. How wide is the tank?

   (A) 30"   (B) 31.5"   (C) 33"   (D) 34.5"   (E) 36"

53. The vertices of a pentagon are (2,0), (0,1), (0,3), (-2, 1), and (0, -2). What is the area of the pentagon formed by these points?

   (A) 5.5 units^2 (B) 6 units^2 (C) $6\sqrt{2}$ units^2 (D) $7\sqrt{2}$ units^2 (E) 8 units^2

54. Rob Deebank has a bag of $1.00 bills, $5.00 bills, $10.00 bills, $20.00 bills, $50.00 bills, and $100.00 bills. He hands out 4 bills to people who walk by him and smile. How many different sets of 4 bills could a smiling person receive?

   (A) 7560   (B) 100   (C) 330   (D) 744   (E) 126

55. Find the mean of the median, mode, and range of 8, 6, 7, 8, 9, 4, 7, 6, 6, 5, 9, & 5

   (A) 5   (B) $5\frac{5}{6}$ (C) 6   (D) $6\frac{1}{2}$ (E) $8\frac{3}{4}$

56. Juan Moorwin has lost 30% of the 40 games he has bowled. If he bowls 40 more games how many of them must he win to raise his overall winning percentage to 80%?

   (A) 36   (B) 32   (C) 28   (D) 20   (E) 18

57. How many ordered pairs (x , y) are solutions to the equation $4x + 3y < 10$, where x,y are integers and $x \geq 0$ and $y \geq 0$?

   (A) 3   (B) 4   (C) 7   (D) 10   (E) 12

58. Les Kwik took a leisurely drive in the country. He drove 2 hours at a constant speed before stopping for a picnic. After the picnic he drove 3 hours at a constant rate 15 mph faster than he drove before the picnic. If the total distance he traveled was 250 miles, what was his speed before the picnic?

   (A) 56 mph   (B) 35 mph   (C) 48 mph   (D) 50 mph   (E) 41 mph

59. Antonio Pottier is organizing his math students into teams of 5. He has 8 seniors, 6 juniors, 4 sophomores, and 2 freshmen. How many different teams consisting of 2 seniors, 1 junior, 1 sophomore, and 1 freshman could be created?

   (A) 4845   (B) 40   (C) 225   (D) 1344   (E) 384

60. Find the arithmetic mean of the roots of $x^3 - 7x^2 + 14x - 8 = 0$.

   (A) $1\frac{5}{7}$ (B) $2\frac{3}{10}$ (C) $2\frac{1}{3}$ (D) $2\frac{2}{3}$ (E) $4\frac{2}{3}$

1. E	21. A	41. A
2. E	22. E	42. C
3. A	23. C	43. B
4. D	24. B	44. A
5. D	25. B	45. A
6. D	26. C	46. E
7. A	27. B	47. B
8. D	28. C	48. A
9. D	29. D	49. C
10. D	30. C	50. D
11. E	31. C	51. C
12. A	32. E	52. A
13. E	33. B	53. E
14. D	34. A	54. E
15. E	35. B	55. B
16. D	36. E	56. A
17. A	37. E	57. C
18. D	38. E	58. E
19. D	39. C	59. D
20. C	40. D	60. C

UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE

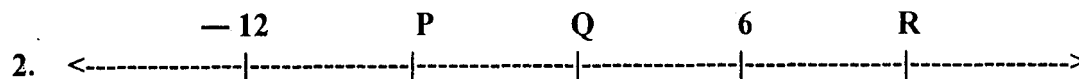Making a World of Difference

Mathematics
State • 2009

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $0.875 + (\frac{3}{4} - 0.625 \times \frac{1}{2}) \div 0.375 + \frac{1}{4} \div 0.125$

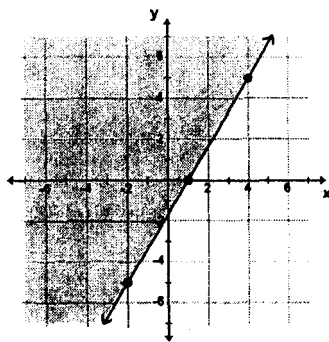   (A) $4\frac{1}{24}$ (B) 3.4375   (C) $4\frac{1}{2}$ (D) 4.375   (E) $3\frac{1}{6}$

2.
```
        — 12        P          Q          6          R
<--------------|--------------|--------------|--------------|--------------|-------------->
```

   The distances between the hash marks (|) are equal. Find $\frac{R}{P-Q}$.

   (A) -6 (B) -2 (C) $\frac{1}{2}$ (D) 1   (E) 2

3. If $2x + 3 = 5$ and $5 = 6x - 1$ then $2x + 3 = 6x - 1$ is an example of the _____ property.

   (A) reflexive   (B) commutative   (C) symmetric   (D) transitive   (E) distributive

4. The shaded area is best represented by which of the following inequalities?

   (A) $y \le \frac{5x-3}{5}$ (B) $y \ge \frac{3x-3}{5}$ (C) $y \ge \frac{5-5x}{3}$ (D) $y \le \frac{3-5x}{5}$ (E) $y \ge \frac{5x-5}{3}$

5. If $\frac{4x}{7} + \frac{3}{5y} = \frac{x}{2}$, then xy equals _____.

   (A) -4.2 (B) -5.3 (C) -6.1 (D) -8.4 (E) -10.2

6. $\angle A$ and $\angle B$ are complementary angles. If m $\angle A = 2x - 3$ and m $\angle B = 4x + 5$, the measure of the supplement to the larger angle is:

   (A) $146\frac{2}{3}°$ (B) $116\frac{1}{3}°$ (C) $127\frac{1}{3}°$ (D) $63\frac{2}{3}°$ (E) $153\frac{1}{3}°$

7. Sweetwater's water tank is in the shape of a sphere. The radius of the tank is 12 feet. How many gallons of water will the tank hold? (nearest gallon)

   (A) 7,238 gal   (B) 81,219 gal   (C) 13,536 gal   (D) 162,437 gal   (E) 54,146 gal

8. The point $(-4, 4)$ is reflected across the line $y = x$ to point P. Point P is rotated 90° clockwise about the origin to point Q. Point Q is translated 4 units horizontally to the right point R. Point R is reflected across the line $y = -x$ to point S. Where does point S lie?

   (A) x-axis   (B) QI   (C) QII   (D) QIII   (E) QIV

9. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the top view of this figure has a perimeter of:
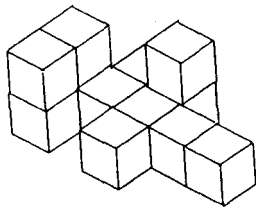
(A) 27 cm   (B) 23 cm   (C) 21 cm   (D) 18 cm   (E) 16 cm

10. The ratio of the distance between the foci to the length of the major axis of an ellipse is called the _____.

(A) asymptote   (B) directrix   (C) eccentricity   (D) focal distance   (E) semiminor axis

11. Let $f(x) = 3 - 4x$ and $g(x) = 2x + 1$. Find $f\left(g^{-1}\left(f(x+1)\right)\right)$.

(A) $32x + 7$ (B) $-2x + 3$ (C) $8x - 1$ (D) $8x + 7$ (E) $-32x - 1$

12. Let $f(x) = \frac{1}{2} + \frac{3}{2}\sin\left(4x + \frac{4\pi}{3}\right)$. Which of the following has a value of $\frac{2}{\pi}$?

(A) amplitude   (B) displacement (vertical)   (C) frequency   (D) phase shift   (E) period

13. A Cell tower is 500 feet from a TV tower. The angle of elevation from the top of the Cell tower to the top of the TV tower is 30°. The angle of depression from the top of the Cell tower to the bottom of the TV tower is 40°. How tall is the TV tower? (nearest foot)

(A) 885 ft   (B) 847 ft   (C) 789 ft   (D) 708 ft   (E) 689 ft

14. How many asymptotes does $f(x) = \frac{x^3 - 1}{x^2 - x - 2}$ have?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

15. Mr. White wants to buy 3 different pepper plants from Juanita's Hot Pepper Shop. She has 5 Jalapeno plants, 4 Hot Banana plants, and 3 Serrano plants to choose from, but she does not know which plant is which type. She randomly picks 3 plants to sell to Mr. White. What is the probability that the three she selects are all different?

(A) $4\frac{6}{11}$ %   (B) $9\frac{1}{11}$ %   (C) $20\frac{5}{11}$ %   (D) $27\frac{3}{11}$ %   (E) $30\frac{9}{11}$ %

16. It takes 7 minutes for Phil DeTubb to fill his bathtub using the hot water faucet. It only takes 5 minutes using the cold water faucet. Phil turns on the hot water faucet. After 1 minute he turns on the cold water faucet and runs both faucets together. How long will it take to fill the bathtub? (nearest second)

(A) 3 min 5 sec   (B) 3 min 30 sec   (C) 3 min 44 sec   (D) 3 min 50 sec   (E) 3 min 55 sec

17. Find $a + b + c$ given the arithmetic sequence: $a, -1.1, b, 3.3, c, ...$

(A) 0 (B) 1.1 (C) 3.3 (D) 6.6 (E) 9.9

18. Find the area of the shaded region in square units.
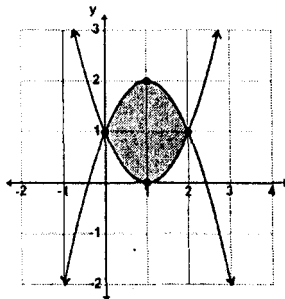
(A) $2\frac{3}{4}$ (B) $2\frac{2}{3}$ (C) $2\frac{1}{2}$ (D) $2\frac{1}{3}$ (E) 2

19. Use the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, to transform the conic $x + 2xy - y = 0$ into an equation that is in standard position and does not contain an xy term. The transformed equation is:

(A) $x^2 = y^2 + \sqrt{2}\,y$ (B) $x^2 = y^2$ (C) $x^2 = y^2 - \sqrt{2}\,y$
(D) $y^2 = \sqrt{2}\,x - x^2$ (E) $y^2 = x^2 + \sqrt{2}\,x$

20. The curve of a polar equation $r = a + b \cos \theta$ is called a dimpled limacon when which of the following is true:

(A) $a = b$ (B) $a < b < 2a$ (C) $a < b$ (D) $2a \leq b$ (E) $a > b$

21. The Austin Bats has 3 coaches, 10 infielders and 6 outfielders. A committee consisting of 1 coach, 3 infielders, and 2 outfielders is selected. How many different committees could be selected?

(A) 27,132 (B) 4,522 (C) 145 (D) 1,800 (E) 5,400

22. How many subsets containing an odd number of members can be made from the set $\{ \Omega, \Phi, \Pi, \Psi, \Sigma, \Theta \}$?

(A) 30 (B) 32 (C) 35 (D) 41 (E) 42

23. Coach Venn has 30 students on his math team. Sixteen students want to do math, 16 want to do calculator, and 11 want to do number sense. Five want to do both math and number sense. Three of those 5 want to do calculator as well. Eight students want to do calculator only and 5 want to do number sense only. How many students want to do math only?

(A) 9 (B) 8 (C) 7 (D) 4 (E) 1

24. The sum of the real solutions of $5 - \left|4x + |3 - 2x|\right| = 1$ is:

(A) $-3\frac{2}{3}$ (B) -3 (C) -2 (D) 1 (E) $1\frac{2}{3}$

25. Using the figure below, if $m\angle BAD = 50°$ and $m\overset{\frown}{CE} = 150°$ then $m\overset{\frown}{BD} =$ _____ .
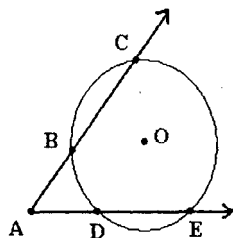
    (A) 25°   (B) 35°   (C) 50°   (D) 60°   (E) 75°

26. The integers greater than 1 are arranged in five columns as shown. If this pattern continues which column would contain the number 2010?

A	B	C	D	E
	1	2	3	4
8	7	6	5	
	9	10	11	12
16	15	14	13	

   (A) A   (B) B   (C) C   (D) D   (E) E

27. Emmy Noether was a German mathematician known for her groundbreaking contributions to:

    (A) number theory   (B) calculus   (C) Euclidean geometry
    (D) irrational numbers   (E) abstract algebra

28. The current record of the Millersview Swatters baseball team is 35 wins and 15 losses. They have 40 more games to play. What will their overall winning percentage be if they win 5 games out of every 8 games they have left to play?

    (A) $66\frac{2}{3}$ %   (B) 70 %   (C) $68\frac{2}{11}$ %   (D) $62\frac{1}{2}$ %   (E) 80%

29. Point G is called the _____ of $\triangle ABC$.
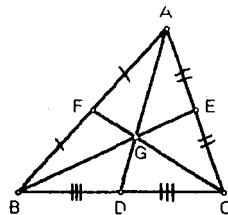
    (A) incenter   (B) circumcenter   (C) centroid   (D) orthocenter   (E) cevian

30. Approximate to the nearest tenth: $(\log_3 5)(\log_5 7)(\log_7 9)(\log_9 11)$

   (A) 2.6   (B) 2.2   (C) 1.8   (D) 1.4   (E) 1.1

31. Which of the following is a reference angle for 2009 ° ?

    (A) 29 °   (B) 71 °   (C) 80 °   (D) 151 °   (E) 209 °

32. How many points of intersection occur when $r = 3\sin(2\theta)$ and $r = 3\cos(2\theta)$ are graphed on a polar coordinate system?

(A) 5   (B) 8   (C) 9   (D) 10   (E) 12

33. The isosceles trapezoid shown has a top base of length a and a bottom base of length b. The segment PQ divides the trapezoid into two similar trapezoids. The length of segment PQ is the _____ mean of a and b.
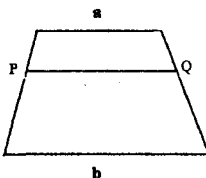
(A) arithmetic   (B) geometric   (C) harmonic   (D) quadratic   (E) cevian

34. The sum of the finite geometric sequence, $10, -20, 40, -80, ..., n$ is -3410. How many terms are there in the sequence?

(A) 30   (B) 25   (C) 20   (D) 15   (E) 10

35. Evaluate: $\int_{-n}^{n} (x^{-3} - 2x + 1)\, dx$

(A) $-2n(n+1)$ (B) $2n$ (C) $\frac{1}{2}n^2$ (D) $2n(n-1)$ (E) n^{-2}

36. If $f''(x) = 30x$ and $f'(1) = 7$ and $f(-1) = 6$, then $f(0) = $ _____.

(A) -5 (B) 0   (C) 3   (D) 5   (E) 9

37. $44_5 + 333_4 \times 2_3 - 1111_2 = $ _____ $_6$

(A) 4321   (B) 135   (C) 423   (D) 55   (E) 343

38. The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many combinations exist if P is a multiple of 7, Q is divisible by 11, and R is a power of 2? Stopping at 0 is not allowed.
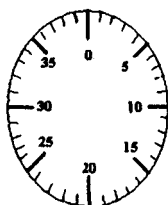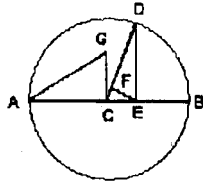

(A) 90   (B) 100   (C) 75   (D) 120   (E) 14

39. The discriminant of $(5x - 3)(x - 3) = 0$ is _____.

(A) 106   (B) 135   (C) 144   (D) 279   (E) 504

40. Let AB be the diameter of the circle with center C with CG ⊥ AB, DE ⊥ AB, and EF ⊥ DC. Let AE = x and BE = y such that the arithmetic mean of x and y is 12.5 and the geometric mean of x and y is 12. Find CE.


    (A) 12.98   (B) 11.52   (C) 7   (D) 6.25   (E) 3.5

41. Phil Upp's car gets 25 miles per gallon of gas when he drives a constant rate of 60 mph. The car gets 20 miles per gallon when he drives a constant rate of 70 mph. He takes a 250 mile trip and the cost of gas is $1.85 per gallon. How much would he save by keeping a constant rate of 60 mph instead of 70 mph?

    (A) $3.70   (B) $4.63   (C) $4.75   (D) $5.00   (E) $5.55

42. The function $f(x) = 6x^2 - 5x - 6$ crosses the x-axis at two points. Find the distance between the two points.

    (A) $\frac{5}{6}$ (B) $1\frac{5}{6}$ (C) $2\frac{1}{6}$ (D) 1   (E) $1\frac{1}{5}$

43. How many ordered pairs (x , y) are solutions to the equation $3x - 5y < 15$, where x,y are integers and $x \geq 0$ and $y \leq 0$?

    (A) 15   (B) 12   (C) 11   (D) 10   (E) 6

44. Simplify: $\left(\dfrac{2x^3 - 2xy^2 + x^2y - y^3}{2x^2 + 3xy - 2y^2}\right) \div \left(\dfrac{x^2 - y^2}{2y^2 - 3yx - 2x^2}\right)$

    (A) $2x^2 - xy - y^2$ (B) $2x + y$ (C) $x^2 - y^2$ (D) $-2x - y$ (E) $(2x - y)^{-1}$

45. Interstate 77 runs West to East. Radio station RUOK is 5 miles due North of I77. The broadcast range of the station is 9 miles. What length of the highway is in range of RUOK? (nearest tenth)

    (A) 14.5 miles   (B) 15.0 miles   (C) 16.1 miles   (D) 19.7 miles   (E) 20.6 miles

46. The circles $(x - \frac{1}{2})^2 + (y + 1)^2 = 10$ and $(x + 1)^2 + (y - 2)^2 = 5$ intersect in two points. The y-intercept of the line through the two points of intersection is (x,y). Find y.

    (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $1\frac{11}{24}$ (E) $1\frac{1}{2}$

47. Find the multiplicative inverse, if it exists, of $\begin{bmatrix} 8 & 2 \\ 16 & 4 \end{bmatrix}$.

    (A) $\begin{bmatrix} -4 & 2 \\ 16 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -16 \\ -2 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{4} & \frac{1}{16} \\ \frac{1}{2} & \frac{1}{8} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{8} & \frac{1}{16} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ (E) does not exist

48. The adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.
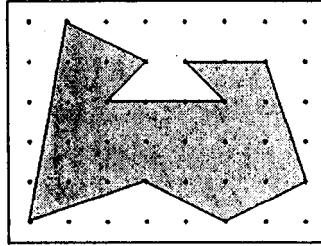
   (A) 20.5 cm^2 (B) 19 cm^2 (C) 13.5 cm^2 (D) 22 cm^2 (E) 14 cm^2

49. If $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, ..., $F_n = F_{n-2} + F_{n-1}$ and $L_0 = 2$, $L_i = 1$, $L_2 = 3$, ..., $L_n = L_{n-2} + L_{n-1}$, for $n \geq 3$, then $F_{n+6} + F_{n+8} = $?

   (A) L_{n+2} (B) L_{n+14} (C) $L_{n+7} + L_{n+9}$ (D) L_{n+7} (E) $L_{n+5} + L_{n+7}$

50. Which of the following system of inequalities is best represented by the shaded region?
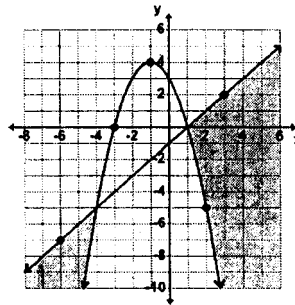
   (A) $x - y \geq 1$
       $x^2 + 2x + y \geq 3$

   (B) $x + y \leq 1$
       $x^2 + 2x - y \geq 3$

   (C) $x - y \geq 1$
       $x^2 + 2x + y \leq 3$

   (D) $x + y \leq -1$
       $x^2 - 2x - y \leq 3$

   (E) $x - y \geq -1$
       $x^2 + 2x + y \geq 3$

51. A belt joins the two pulleys shown. The smaller pulley with the given radius revolves at the rate of 60 rpm. How much greater is the angular velocity of the smaller pulley than the larger pulley? (nearest tenth)
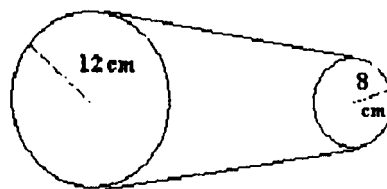
   (A) $4\pi \frac{rad}{min}$ (B) $8\pi \frac{rad}{min}$ (C) $20\pi \frac{rad}{min}$ (D) $40\pi \frac{rad}{min}$ (E) $60\pi \frac{rad}{min}$

52. An altitude of $\triangle LEG$ forms two right triangles, $\triangle LET$ and $\triangle GET$, such that GT = 6" LT = 4", and m$\angle EGT = 50°$. Find m$\angle ELG$. (nearest degree)

   (A) 29°   (B) 45°   (C) 61°   (D) 69°   (E) 130°

53. Which of the following is a false statement?

(A) $\cos \theta = \sin (\frac{\pi}{2} - \theta)$ (B) $\cos 3\theta = 4\cos^3\theta - 3\cos \theta$ (C) $\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

(D) $3\sin \theta = 4\sin^3\theta + \sin 3\theta$ (E) $\sin \theta = \cos (2\pi - \theta)$

54. Ura Gittenbigger paid $12.00 for 6 hot dogs and 4 sodas at the county fair for lunch. Her niece, Ima Gittenbigtu paid $9.00 for 5 hot dogs and 2 sodas. How much would it cost their cousin, I. B. Hungree, to buy 8 hot dogs and 5 sodas?

(A) $14.25 (B) $14.75 (C) $15.00 (D) $15.75 (E) $16.50

55. Max E. Mumm paid $2.50 per square foot for a rectangular sheet of metal that is 4' by 8'. He made a rectangular water trough out of the sheet that would hold the greatest amount of water by cutting out the same size square from each of the four corners, folding up the remaining sides, and soldering the sides. What was the cost of metal used just to form the tank?

(A) $52.42 (B) $58.57 (C) $72.85 (D) $77.14 (E) $78.21

56. Let $f(x) = \sin x \cos x$. Find $f'(\frac{\pi}{6})$. (nearest hundredth)

(A) 0.01 (B) 0.05 (C) 0.50 (D) 1.00 (E) undefined

57. Three fair six-sided dice are tossed and the spots on the top faces are recorded. One die is red, one is white, and one is blue. Which of the following has the greatest probability of occurring?

(A) All three numbers are odd. (B) The red die is a 6 and the blue die is even.
(C) All three numbers are prime (D) The white die is even and the red die is is less than 3.
(E) The red die is a 1, the white die is a 3, and the blue die is a 5.

58. The odds of scoring less than 118 on this test is $\frac{7}{20}$. Based on the odds, if 297 students take this test how many would be expected to score 118 or greater?

(A) 269 (B) 230 (C) 220 (D) 193 (E) 179

59. Let a, b, and c be positive integers where a, b, and c are not necessarily distinct. How many ordered triples (a, b, c) exist such that $a + b + c = 11$?

(A) 33 (B) 45 (C) 55 (D) 66 (E) 90

60. Find the lateral surface area of this prism. All angles are right angles.
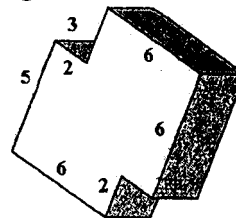
(A) 81 units^2 (B) 87 units^2 (C) 93 units^2 (D) 96 units^2 (E) 108 units^2

University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2009
Answer Key

1. A	21. E	41. B
2. B	22. B	42. C
3. D	23. C	43. C
4. E	24. B	44. D
5. D	25. C	45. B
6. B	26. C	46. D
7. E	27. E	47. E
8. A	28. A	48. A
9. D	29. C	49. D
10. C	30. B	50. A
11. D	31. A	51. D
12. C	32. C	52. C
13. D	33. B	53. E
14. D	34. E	54. D
15. D	35. B	55. C
16. B	36. C	56. C
17. C	37. E	57. D
18. B	38. A	58. C
19. A	39. C	59. B
20. E	40. E	60. D