HMMT February 2023

February 18, 2023

Geometry Round

1. Let $ABCDEF$ be a regular hexagon, and let P be a point inside quadrilateral $ABCD$. If the area of triangle PBC is 20, and the area of triangle PAD is 23, compute the area of hexagon $ABCDEF$.

2. Points X, Y, and Z lie on a circle with center O such that $XY = 12$. Points A and B lie on segment XY such that $OA = AZ = ZB = BO = 5$. Compute AB.

3. Suppose $ABCD$ is a rectangle whose diagonals meet at E. The perimeter of triangle ABE is 10π and the perimeter of triangle ADE is n. Compute the number of possible integer values of n.

4. Let $ABCD$ be a square, and let M be the midpoint of side BC. Points P and Q lie on segment AM such that $\angle BPD = \angle BQD = 135°$. Given that $AP < AQ$, compute $\frac{AQ}{AP}$.

5. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. Suppose $PQRS$ is a square such that P and R lie on line BC, Q lies on line CA, and S lies on line AB. Compute the side length of this square.

6. Convex quadrilateral $ABCD$ satisfies $\angle CAB = \angle ADB = 30°$, $\angle ABD = 77°$, $BC = CD$, and $\angle BCD = n°$ for some positive integer n. Compute n.

7. Quadrilateral $ABCD$ is inscribed in circle Γ. Segments AC and BD intersect at E. Circle γ passes through E and is tangent to Γ at A. Suppose that the circumcircle of triangle BCE is tangent to γ at E and is tangent to line CD at C. Suppose that Γ has radius 3 and γ has radius 2. Compute BD.

8. Triangle ABC with $\angle BAC > 90°$ has $AB = 5$ and $AC = 7$. Points D and E lie on segment BC such that $BD = DE = EC$. If $\angle BAC + \angle DAE = 180°$, compute BC.

9. Point Y lies on line segment XZ such that $XY = 5$ and $YZ = 3$. Point G lies on line XZ such that there exists a triangle ABC with centroid G such that X lies on line BC, Y lies on line AC, and Z lies on line AB. Compute the largest possible value of XG.

10. Triangle ABC has incenter I. Let D be the foot of the perpendicular from A to side BC. Let X be a point such that segment AX is a diameter of the circumcircle of triangle ABC. Given that $ID = 2$, $IA = 3$, and $IX = 4$, compute the inradius of triangle ABC.