



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

SAC • 2012



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1. Evaluate: $1 + 1 \times 2 - (3 + 5) \div 8 \times 13 + 21$

- (A) -14.5 (B) -10 (C) 5 (D) 11 (E) 30.75

2. The Retread Tire store has several package deals of sets of 4 tires. Which of the following is the best deal, assuming all of the tires are of the same quality?

- (A) \$45 each (B) buy 3 at \$65 each, get the 4th one free
(C) \$55 each, but get 20% off for buying 4 (D) \$50 each for 3 and \$25 for the 4th one
(E) \$60 for the 1st, \$50 for the 2nd, \$40 for the 3rd, and \$30 for the 4th

3. Noah Dough has 9 canisters. Five canisters contain dimes, three contain nickels, and one contains both nickels and dimes. How many canisters contain neither nickels nor dimes?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. $\begin{array}{ccccccc} & P & & -2 & & Q & & 3 & & R \\ & | & & | & & | & & | & & | \\ <-----|-----|-----|-----|-----|-----> \end{array}$

The distances between the hash marks (|) are equal. Find $P + Q + R$.

- (A) 0.5 (B) 1.5 (C) 2.5 (D) 3.0 (E) 4.5

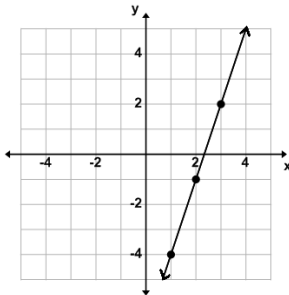
5. If the sum of the proper divisors of a number N is greater than N then N is a(n) _____ number.

- (A) abundant (B) deficient (C) odious (D) perfect (E) prime

6. The Real value solution set for $|5x - 3| < 2$ is?

- (A) $\{x \mid \frac{1}{5} < x < 1\}$ (B) $\{x \mid -1 < x < -\frac{1}{5}\}$ (C) $\{x \mid \{x > \frac{1}{5}\} \cup \{x < 1\}\}$
(D) $\{x \mid 1 < x < 5\}$ (E) $\{x \mid \{x < \frac{1}{5}\} \cup \{x > 1\}\}$

7. What is the slope of the line perpendicular to the line shown?



- (A) -3 (B) -2 (C) $-\frac{1}{3}$ (D) 1 (E) 3

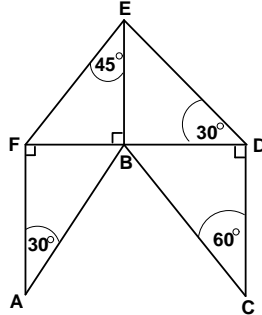
8. Simplify: $\left(\frac{x^2 + 4x + 4}{x^2 - 4} \right) \div \left(\frac{x^2 + x}{x^2 - x - 2} \right)$

- (A) $\frac{x+2}{x}$ (B) $x+2$ (C) $x^2 + 2x$ (D) $\frac{x(x-1)}{x-2}$ (E) $\frac{x+2}{x(x+1)}$

9. Juana Cook knows that the number of cups she needs for her recipe is directly proportional to the number of servings needs. The recipe calls for 2 cups of sugar for 5 servings. How many cups of sugar will she need if she needs 25 servings for her family party?

(A) 3 cups (B) 4 cups (C) 8 cups (D) 10 cups (E) 13 cups

10. Find the perimeter of the hexagon if $DE = 2$ cm. (nearest cm).

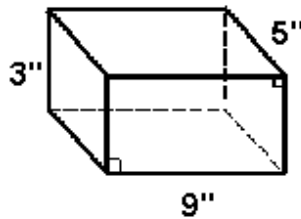


(A) 14 cm (B) 13 cm (C) 12 cm (D) 11 cm (E) 10 cm

11. Two lines that do not lie in the same plane and do not intersect are:

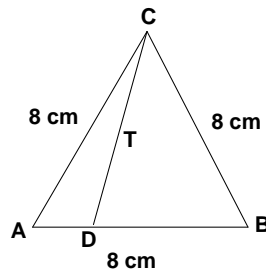
(A) collinear (B) concurrent (C) parallel (D) perpendicular (E) skew

12. Find the lateral surface area of the rectangular prism shown.



(A) 174 " (B) 135 " (C) 90 " (D) 84 " (E) 34"

13. Find T if $BD = 6$ cm. (nearest tenth)

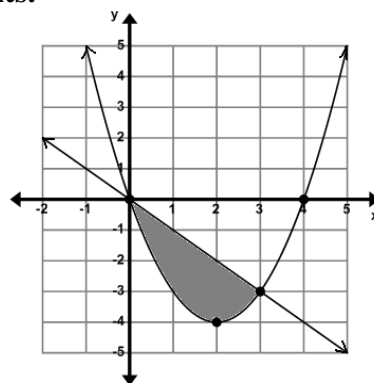


(A) 6.8 cm (B) 7.0 cm (C) 7.2 cm (D) 7.4 cm (E) 7.6 cm

14. If $\frac{x-2}{2x+3} + \frac{3x+4}{4x-5} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

(A) 1.44 (B) 1.2 (C) -1 (D) -7.2 (E) -9.4

15. Find the average of all two-digit numbers such that reversing the digits results in another two-digit number that is 75% larger than the original two-digit number.
- (A) 42 (B) 36 (C) 30 (D) 24 (E) 18
16. Mark Deefield is roping off a triangle using main posts at three points. After putting in the first stake he walks 60 yards on a bearing of 100° . Then he walks 40 yards on a bearing of 200° . What will be the perimeter of Mark's triangle? (nearest yard)
- (A) 172 yds (B) 166 yds (C) 160 yds (D) 154 yds (E) 148 yds
17. Ima Round draws a circle with a radius of 3 cm and the center at the origin O, in the x-y plane. Then she draws a line tangent to the circle at point P that intersects the x-axis at Q. Find PQ if $m\angle POQ = \frac{\pi}{6}$.
- (A) $2\sqrt{3}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{3\sqrt{3}}{2}$ (E) $\sqrt{3}$
18. Use the Fibonacci characteristic sequence ... p, -2, q, 5, r, ... to Find $p + q + r$.
- (A) 28 (B) 12 (C) 13 (D) 10 (E) 31?
19. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}$. Find the sum of the elements of $A(A + B)$.
- (A) 0 (B) 3 (C) 4 (D) 5 (E) 6
20. Let $f(x) = 5x^2 - 14x - 8$ and $g(x) = x - 1$. Find $g(f'(x + 1))$
- (A) $9x - 4$ (B) $10x - 5$ (C) $9x - 10$ (D) $10x - 14$ (E) $5x - 1$
21. Find the area of the shaded region in square units.

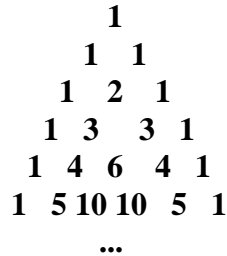


- (A) 3.75 (B) 4 (C) 4.25 (D) 4.5 (E) 4.75
22. How many distinguishable arrangements can be made from the letters "MATHTEST"?
- (A) 20,160 (B) 6,720 (C) 40,320 (D) 3,360 (E) 13,440

23. Willie Drawette has five blue markers, two green markers, four red markers and three black markers. Willie selects two markers at random. What is the probability that both of them are blue? (nearest per cent)

(A) 36% (B) 31% (C) 27% (D) 11% (E) 7%

24. Using Blaise Pascal's triangle and letting the 1 at the top be row 1, determine which of the following numbers will be in the 9th row.



(A) 28 (B) 45 (C) 55 (D) 72 (E) 81

25. $8A_{16} + 42_8 + 21_4 = \underline{\hspace{2cm}}_2$

(A) 10101010 (B) 11001101 (C) 11011011 (D) 10111011 (E) 10110101

26. Find the next four-digit number in the following sequence:

9861 — 1986 9681 — 1896 8961 — 1968 8691 ?????

(A) — 6918 (B) — 1698 (C) — 9816 (D) — 6198 (E) — 1869

Mathematicians (No new ones this year.)

Agnesi	Archimedes	Boole, George	Byron, Ada (Lady Lovelace)
Cantor, Georg	Descartes, Rene	Diophantus	Erastosthenes
Euclid	Euler, Leonard	Germain, Sophie	Goldbach, Christian
Hypatia	Kovalevsky, Sonya	Leibniz, Gottfried	Mandelbrot, Benoit
Napier, John	Noether, Emmy	Porter, Freda	Ptolemy, Claudius
Smith, Karen E.	Stott, Alicia	Theano	Venn, John
Williams, Grace			

Types of Numbers (No new ones this year.)

Complex	Real	Imaginary	Rational	Irrational
Transcendental	Integer	Whole	Natural	Even
Odd	Prime	Composite	Unit	Deficient
Frugal	Economical	Perfect	Equidigital	Abundant
Extravagant	Wasteful	Fibonacci	Lucas	Happy
Unhappy	Lucky	Unlucky	Evil	Odious
Polite	Primeval			

2012-13 Special Emphasis Concepts: Platonic Solids - Pascal's Triangle - Binomial Theorem

Possible questions (but not limited to) might include:

1. The faces of a regular icosahedron are _____.
2. The dihedral angle on a dodecahedron with an edge of 3 cm is _____.
3. The simplified coefficient of the x^3y term in the expansion of $(3x + 2y)^4$ is _____.
4. Set $A = \{1, 2, 3, 4, 5\}$. How many 3-element subsets of set A are there?
5. If $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$, then $f(4) =$ _____.
6. Using Blaise Pascal's triangle and letting the 1 at the top be row 1, the sum of the elements in the 12th row.
7. *** See #24 on the 2012SAC test.

**University Interscholastic League
MATHEMATICS CONTEST
HS • SAC • 2012
Answer Key**

- | | |
|-------|-------|
| 1. D | 21. D |
| 2. D | 22. B |
| 3. C | 23. D |
| 4. B | 24. A |
| 5. A | 25. E |
| 6. A | 26. B |
| 7. C | |
| 8. A | |
| 9. D | |
| 10. E | |
| 11. E | |
| 12. D | |
| 13. C | |
| 14. D | |
| 15. C | |
| 16. B | |
| 17. E | |
| 18. A | |
| 19. E | |
| 20. B | |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2013



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1. Evaluate: $(1 + 3 + 6) \div (10 - 15) \times 21 \div 28$

- (A) $-10\frac{1}{2}$ (B) $-2\frac{2}{3}$ (C) $-1\frac{1}{2}$ (D) $-\frac{3}{8}$ (E) $-\frac{1}{294}$

2. Lotta Dough lends her brother, Les, \$500.00 to go to summer camp. Les promises to pay back the \$500.00 plus simple interest at a rate of 5% in 9 equal monthly payments. How much will Les Dough's monthly payments be? (nearest cent)

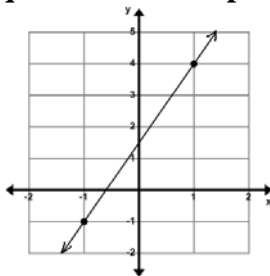
- (A) \$55.56 (B) \$57.12 (C) \$57.64 (D) \$59.11 (E) \$61.11

3. $\begin{array}{ccccccc} & -4 & & P & & Q & & R & & 5 \\ & | & & | & & | & & | & & | \\ <-----&-----&-----&-----&-----&-----&-----&-----&-----&-----> \end{array}$

The distances between the hash marks (|) are equal. Find $P + Q + R$.

- (A) 1 (B) 1.5 (C) 2.25 (D) 4.5 (E) 5

4. Which of the following is not an equation of a line perpendicular to the line shown?



- (A) $2x + 5y = 3$ (B) $4x + 10y = 5$ (C) $-6x - 15y = 9$ (D) $10x - 4y = 5$ (E) $-2x - 5y = 3$

5. Simplify: $\left(\frac{2x^2 + 3x - 2}{2x^2 - 3x + 1} \right) \left(\frac{x^2 - 2x + 1}{x^2 - 1} \right)$

- (A) $2 - 2x$ (B) $\frac{x+2}{x-1}$ (C) $-2x - 3$ (D) $\frac{x+2}{x+1}$ (E) $\frac{x+1}{x-2}$

6. Roland Uppwater's boat travels up a river for 2 hours 15 minutes. The return trip only takes 1 hour 30 minutes. The river's current flows at 5 miles per hour. Find the speed of Roland's boat in still water?

- (A) $3\frac{3}{4}$ mph (B) $14\frac{1}{16}$ mph (C) $18\frac{3}{4}$ mph (D) 20 mph (E) 25 mph

7. How many different regular polygons are faces of Platonic solids?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. Point P(− 1, 1) lies in the x-y plane. Point P is translated horizontally 3 units in a positive direction to point Q. Point Q is reflected across the y-axis to point R. Point R translated vertically 2 units in a negative direction to point S. Find the coordinates of the point S.

- (A) (2, − 3) (B) (1, − 1) (C) (− 2, − 1) (D) (4, 3) (E) (− 2, 1)

9. If two coplanar lines are perpendicular to the same line then the two lines are:

- (A) intersecting (B) skew (C) concurrent (D) transversal (E) parallel

10. Two numbers are in the ratio 6:11. If the first number is decreased by 4 and the second increased by 6, the resulting numbers are in the ratio 4:9. Find the sum of the numbers.

- (A) 123 (B) 102 (C) 62.8 (D) 30 (E) 6.8

11. If $a_1 = 2$, $a_2 = -1$ and $a_n = (a_{n-1}) \div (a_{n-2})$ for $n \geq 3$, then a_6 equals:

- (A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) -1 (E) -2

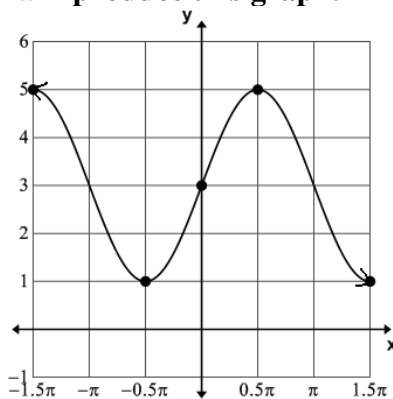
12. The eccentricity of the ellipse $2x^2 + 6y^2 = 12$ is:

- (A) $\frac{\sqrt{6}}{3}$ (B) $\frac{\sqrt{2}}{6}$ (C) 2 (D) $\frac{\sqrt{6}}{2}$ (E) $\frac{\sqrt{2}}{3}$

13. Roland Upp built a skateboard ramp that is 20 feet long and the end of the ramp is 3 feet from the ground. What is the angle of inclination of the ramp? (nearest second)

- (A) $10^\circ 9' 51''$ (B) $4^\circ 2' 11''$ (C) $8^\circ 31' 51''$ (D) $8^\circ 37' 37''$ (E) $10^\circ 29''$

14. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $2 - 3\cos(x - \pi)$ (B) $3 - 2\sin(\frac{1}{2}(2x + \pi))$ (C) $3 + 2\sin(x - \pi)$
 (D) $3 + 2\cos(x - \pi)$ (E) $3 - 2\sin(x + \pi)$

15. Simplify: $\frac{\sin(\theta)}{1 + \cos(\theta)} + \cot(\theta)$

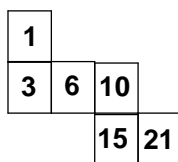
- (A) $\csc(\theta)$ (B) $\sin(\theta)$ (C) 1 (D) $\sin(\theta)\cos(\theta)$ (E) $1 + \csc(\theta)$

16. In the expansion of $(2x + 3)^5$, the sum of the coefficients of the 3rd and the 4th term is:

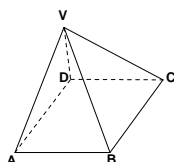
- (A) 1,800 (B) 1,260 (C) 1,035 (D) 275 (E) 180

17. Let $f(x) = \frac{3x^2 - 4x + 5}{x - 2}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(-2)$.
- (A) -4 (B) -6.25 (C) 9 (D) 5 (E) -2
18. $\int \left(\frac{x+3}{2}\right) dx = \text{_____} + C$, where C is some arbitrary constant.
- (A) $2x + 6$ (B) $\frac{x(x+6)}{4}$ (C) $3x + \frac{1}{2}$ (D) $\frac{x^2 + 3x}{2}$ (E) $\frac{x(2x+3)}{4}$
19. Let f be a function such that it is continuous on $[a,b]$ and it is differentiable on (a,b) . Then there exists at least one number c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. This theorem is known as:
- (A) Intermediate-value Theorem (B) Mean-value Theorem (C) Rolle's Theorem
- (D) Fundamental Theorem of Algebra (E) Fundamental Theorem of Calculus
20. Polygon South college employs 5 math teachers, 3 science teachers, and 4 art teachers. How many ways can they form a committee consisting of 2 teachers from each of the three subject areas?
- (A) 19 (B) 38 (C) 924 (D) 120 (E) 180
21. Ima Deeller picks 4 spades, 4 hearts, 8 diamonds, and 12 clubs from a standard deck of cards. She shuffles these cards and deals the top two cards face up on the table. What is the probability that top card is a spade and next card is a club? (nearest %)
- (A) 75% (B) 59% (C) 33% (D) 13% (E) 6%
22. Each of four integers, -2 , -1 , 0 , and 1 , are written on a separate post card. Two cards are randomly selected. What are the odds that the product of the integers selected is negative?
- (A) 1 to 6 (B) 1 to 5 (C) 1 to 1 (D) 1 to 2 (E) 1 to 3
23. Using Blaise Pascal's triangle, determine the sum of all of the numbers in the first 8 rows.
- (A) 255 (B) 128 (C) 511 (D) 256 (E) 127
24. Which of the following mathematicians was considered to be the author of a treatise on the "Golden Mean"?
- (A) Theano (B) Hypatia (C) Euclid (D) Erastosthenes (E) Agnesi
25. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x \leq 1, \\ 8 - 3x & \text{if } 1 < x < 2. \\ x + 3 & \text{if } 2 \leq x. \end{cases}$ At what value(s) of x is the function discontinuous?
- (A) 0 & 1 (B) 1 & 2 (C) 0 (D) 1 (E) 2

26. Plato folds the net shown into a cube. He adds the numbers on each pair of opposite sides. What is the largest sum possible?

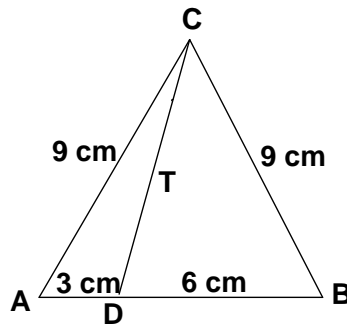


- (A) 16 (B) 18 (C) 24 (D) 27 (E) 31
27. If the sum of the divisors of a number N is twice the number N then N is a(n) ____ number.
- (A) abundant (B) deficient (C) odious (D) perfect (E) prime
28. Points $(4, -5)$ and $(k, -3)$ lie on line l and line l is parallel to $2x - 3y = 5$. Find k .
- (A) $2\frac{2}{3}$ (B) 7 (C) $1\frac{1}{2}$ (D) $-1\frac{1}{3}$ (E) 6
29. If F varies inversely as the square of G , and $G = 3$ when $F = 2$, find F if $G = 5$.
- (A) 0.69444... (B) 0.72 (C) 1.44 (D) 3.6 (E) 7.5
30. Let $AB = 16$ cm. If the height of the square based pyramid shown is 6 cm, then the lateral surface area of the pyramid is _____.



- (A) 320 cm^2 (B) 256 cm^2 (C) 192 cm^2 (D) 480 cm^2 (E) 384 cm^2
31. Evaluate: $\log_3(18) - \log_3(12) + \log_3(6)$
- (A) 36 (B) 15 (C) 12 (D) 9 (E) 2
32. How many elements are in $\{x \mid \sec(x) - 1 = \tan(x), x \in (0, 2\pi]\}$?
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
33. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$. Find the sum of the elements of AB .
- (A) -30 (B) -17 (C) -7 (D) 5 (E) 7
34. The volume, nearest cubic inch, of a regular tetrahedron with edge length 6" is:
- (A) 12 cu. in. (B) 21 cu. in. (C) 25 cu. in. (D) 31 cu. in. (E) 34 cu. in.

35. A pack of crayons contains 64 different colored crayons. The probability of randomly selecting a shade of green crayon is 25%. What are the odds of randomly selecting a crayon that is not a shade of green?
- (A) 1 to 3 (B) 1 to 4 (C) 1 to 1 (D) 4 to 1 (E) 3 to 1
36. Find the sum of the mean, median, mode, and range of 1, 1, 2, 3, 5, 8, 13, & 21.
- (A) 31.75 (B) 32.125 (C) 32.625 (D) 33.25 (E) 33.75
37. Fran Sysco had a box of unsharpened colored pencils. She gave 20% of the pencils to her friend, Van Essay. Then she gave $\frac{1}{4}$ of what she had left to her other friend, Me Gell. She sharpened and used 5 of the remaining pencils on her school project. How many pencils were in the box originally if there are only 10 unused pencils left in the box?
- (A) 24 (B) 25 (C) 36 (D) 40 (E) 48
38. Find $m\angle BCD$. (nearest degree)



- (A) 41° (B) 43° (C) 45° (D) 47° (E) 49°
39. Find the sum of all two-digit numbers whose digits differ by 3 and reversing the digits results in a square number.
- (A) 61 (B) 74 (C) 115 (D) 121 (E) 176
40. Find the slope of the tangent line to $16x^2 - 9y^2 = 144$ at point $P(3\frac{3}{4}, 3)$.
- (A) $\frac{15}{16}$ (B) $1\frac{19}{45}$ (C) $1\frac{2}{3}$ (D) $1\frac{7}{9}$ (E) $2\frac{2}{9}$
41. Bill E. Goat put up a fence to protect part of his lawn from his kids. He drove in the first fence post then measured 12 feet on a bearing of 50° to put in the second post. The third post is 8 feet from the second post on a bearing of 130° . What is the perimeter of the part of his lawn he fenced in? (nearest inch)
- (A) 34' 5" (B) 35' 6" (C) 35' 10" (D) 36' 0" (E) 36' 4"

42. Let $f(x) = \frac{3x-4}{2x-5}$. Find $f'(-1)$.

- (A) $\frac{5}{7}$ (B) $\frac{19}{49}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{9}$ (E) undefined

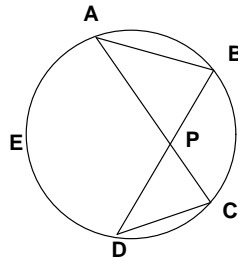
43. In how many ways can a class of fifteen students be divided into three sets, so that three students are in the first set, five in the second set, and seven in the third set?

- (A) 455 (B) 1,248 (C) 360,360 (D) 2,730 (E) 24,024

44. $BAD_{16} - ABC_{16} = \underline{\hspace{2cm}}_{16}$

- (A) 0 (B) 50E (C) 3D (D) F0F (E) 1515

45. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O. Find $m\widehat{CD}$ if $m\widehat{AB} = 70^\circ$ and $m\angle APB = 50^\circ$.



- (A) 60° (B) 50° (C) 40° (D) 30° (E) 20°

46. Simplify: $a^{-2} \div b^{-3} \times a^{-5} \div b^5 \times a^3 \div b^2$

- (A) $(ab)^{-4}$ (B) b^{-4} (C) $(ab)^{30}$ (D) ab (E) $a^{-4} b^4$

47. The Real value solution set for $2 + |3x - 4| < 5$ is?

- (A) $\{x \mid \{\frac{1}{3} < x\} \cup \{x > 2\frac{1}{3}\}\}$ (B) $\{x \mid -2\frac{1}{3} < x < -\frac{1}{3}\}$ (C) $\{x \mid -\frac{1}{3} < x < 2\frac{1}{3}\}$
 (D) $\{x \mid \{-\frac{1}{3} < x\} \cup \{x > 2\frac{1}{3}\}\}$ (E) $\{x \mid \frac{1}{3} < x < 2\frac{1}{3}\}$

48. Let P and Q be the roots of $2x^2 + 11x + 15 = 0$. Find $P^3 + 3P^2Q + 3PQ^2 + Q^3$.

- (A) -0.125 (B) -421.975 (C) -166.375 (D) -2.536 (E) -39.304

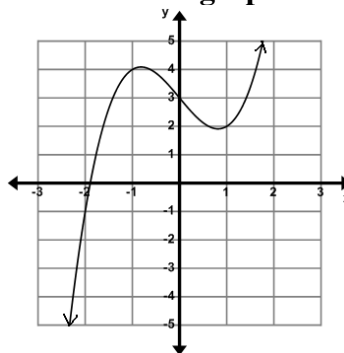
49. Find $(1 - i)^3$ and express the answer in standard form.

- (A) $1 + 8i$ (B) $2 - 2i$ (C) $-1 - 2i$ (D) $-2 - 2i$ (E) $-7 + 3i$

50. In $\triangle PQR$ the 3 medians intersect at point M. If $PM = 6$ cm then the length of the median from point P is:

- (A) 2 cm (B) 3 cm (C) 6 cm (D) 9 cm (E) 12 cm

51. Which of the following is true about the relation graphed below?



- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
(D) It is neither an even nor an odd function (E) It is a one-to-one function.

52. The I. B. Ritch bank offers a 5 year deal on money market accounts. The following chart shows the expected percentage growth of the account. What is the expected average percentage growth for the 5 year period? (nearest hundredth)

<u>Year</u>	<u>Percentage Growth</u>
1	increased 2.5%
2	increased 3%
3	decreased 1.75%
4	decreased 1%
5	increased 4%

- (A) 1.32% (B) 1.35% (C) 2.50% (D) 3.48% (E) 6.25%

53. Let $f(x) = ax^5 - bx^3 - cx - 5$ where a , b , and c are constants. If $f(5) = 10$ then $f(-5) = ?$

- (A) 2 (B) 5 (C) -5 (D) -10 (E) -20

54. Sir Kumfrance draws a circle having an 8" diameter with the center at the origin O , in the xy plane. He draws a line tangent to the circle at point P that intersects the x -axis at Q . Find PQ if $m\angle PQO = 40^\circ$. (nearest quarter inch)

- (A) $3\frac{1}{2}$ " (B) $9\frac{1}{2}$ " (C) $6\frac{1}{4}$ " (D) $4\frac{3}{4}$ " (E) 4"

55. If $\sec \theta < 0$ and $\csc \theta > 0$ then where will θ terminate?

- (A) QI (B) QII (C) QIII (D) QIV (E) x -axis

56. The complex numbers $w = 3 - i$ and $z = 4i + 3$ are graphed as vectors. What is the measure of the angle formed between them? (nearest tenth)

- (A) 161.6° (B) 108.4° (C) 71.6° (D) 60.0° (E) 36.9°

57. Find the remainder when $5x^4 + 4x^3 - 3x^2 + 2x - 1$ is divided by $x + 1$.

- (A) 28 (B) 7 (C) 6 (D) -1 (E) -5

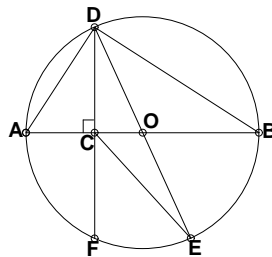
58. The function $f(x) = x^3 - x^2 - 3x + 2$ has an inflection point at $x = ?$

- (A) -3 (B) $-1\frac{2}{27}$ (C) $\frac{1}{3}$ (D) 2 (E) does not have one

59. Let $y = x^{(\frac{1}{\ln x})}$. Find $D_x y$.

- (A) 1 (B) $x \ln x$ (C) $x^{\ln x}$ (D) 0 (E) $x(\ln(\frac{1}{x}))$

60. G. I. Amaree drew a circle with center O with $BC = 2 \cdot AC$. He needs help in determining the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$. What is the ratio of the areas?



- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation A • 2013
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. E | 41. B |
| 2. C | 22. D | 42. C |
| 3. B | 23. A | 43. C |
| 4. D | 24. A | 44. D |
| 5. D | 25. E | 45. D |
| 6. E | 26. D | 46. A |
| 7. C | 27. D | 47. E |
| 8. C | 28. B | 48. C |
| 9. E | 29. B | 49. D |
| 10. B | 30. A | 50. D |
| 11. E | 31. E | 51. D |
| 12. A | 32. D | 52. A |
| 13. D | 33. B | 53. E |
| 14. E | 34. C | 54. D |
| 15. A | 35. E | 55. B |
| 16. A | 36. A | 56. C |
| 17. A | 37. B | 57. E |
| 18. B | 38. A | 58. C |
| 19. B | 39. C | 59. D |
| 20. E | 40. E | 60. C |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

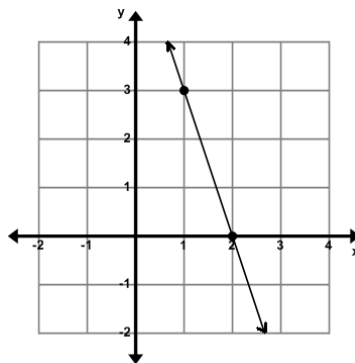
Mathematics

Invitational B • 2013



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1. Evaluate: $\frac{1}{3} \div 0.1666... \times \frac{1}{9} + 0.08333... - \frac{1}{15}$
- (A) $\frac{7}{30}$ (B) 0.22777... (C) $\frac{1}{18}$ (D) 0.23888... (E) $\frac{1}{44}$
2. A waitress received a \$4.50 tip from one customer whose bill was \$25.00 and a \$6.00 tip from another customer whose bill was \$30.00. What was her average tip percentage?
- (A) 20% (B) 19% (C) 18% (D) 16% (E) 15%
3. Venn Circle Estates has 25 houses. Three houses have dogs, but not cats. Ten houses have cats and dogs. Eight houses have cats, but no dogs. How many house do not have a cat or a dog?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. A bag contains 1 white chip, 2 blue chips, and 3 red chips. What is the probability of randomly selecting two chips (without replacement) that are different colors?
- (A) $73\frac{1}{3}\%$ (B) 66% (C) 48% (D) $36\frac{2}{3}\%$ (E) $10\frac{2}{3}\%$
5. Two thousand thirteen plus thirty-one thousand two is subtracted from one million one hundred twenty-three thousand five hundred eight. The sum of the digits in the difference is?
- (A) 32 (B) 27 (C) 26 (D) 20 (E) 13
6. Line k contains point (1, 3) and is perpendicular to the line shown. Which of the following points lies on line k ?



- (A) (11, 7) (B) (10, 6) (C) (9, 3) (D) (13, 8) (E) (−7, 0)
7. The set $\{-1, 0, 1\}$ is closed under how many of these operations:
 + addition − subtraction \times multiplication \div division $\sqrt{\quad}$ square root
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
8. Simplify: $\left(\frac{x^2 - 9}{x^2 + 6x + 9} \right) \div \left(\frac{3x^2 + 10x + 3}{3x^2 - 8x - 3} \right)$
- (A) −1 (B) $\frac{x+3}{x-3}$ (C) 1 (D) $\frac{x-3}{x+3}$ (E) $\frac{x^2 - 6x + 9}{x^2 + 6x + 9}$

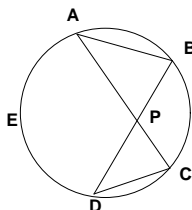
9. If S varies directly as the square of r and $S = \pi$ when $r = 0.5$, find r^2 when $S = 36\pi$.

- (A) 3 (B) 6 (C) 9 (D) $\frac{1}{3}$ (E) $3\sqrt{2}$

10. If a quadrilateral is inscribed in a circle, then its opposite angles are _____ angles.

- (A) right (B) supplementary (C) base (D) congruent (E) complementary

11. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O . Find $m\angle APB$ if $m\widehat{AED} = 190^\circ$ and $m\widehat{BC} = 30^\circ$.

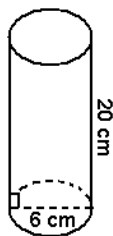


- (A) 70° (B) 80° (C) 90° (D) 100° (E) 110°

12. How many vertices does a Platonic solid have if it has 20 faces and 30 edges?

- (A) 2 (B) 10 (C) 12 (D) 18 (E) 20

13. Given the height and diameter find the lateral surface area of the cylinder. (nearest sq. cm.)



- (A) 754 cm^2 (B) 565 cm^2 (C) 502 cm^2 (D) 377 cm^2 (E) 314 cm^2

14. Willie Mixit has a 15-ounce solution of alcohol and water that is 75% alcohol. He wants to add some 67% alcohol solution to his solution to make a 72% alcohol solution. How many ounces of the 67% solution must he add?

- (A) 3 oz (B) 5 oz (C) 9 oz (D) 10 oz (E) 15 oz

15. Let $f(x) = \frac{2-5x}{2x+5}$. Find $f^{-1}(3)$.

- (A) $7\frac{1}{2}$ (B) $\frac{11}{13}$ (C) $\frac{1}{17}$ (D) $-1\frac{2}{11}$ (E) $-3\frac{2}{5}$

16. $ACE_{16} \times 8_{16} = \underline{\hspace{2cm}}_{16}$

- (A) 5670 (B) 456C (C) 765 (D) 72F (E) 158C

17. If $A + B = 15$ and $A \times B = 27$ then $|B - A| = \underline{\hspace{2cm}}$.

- (A) $3 + \sqrt{13}$ (B) $3\sqrt{13}$ (C) $2 - \sqrt{3}$ (D) $2\sqrt{3}$ (E) 3

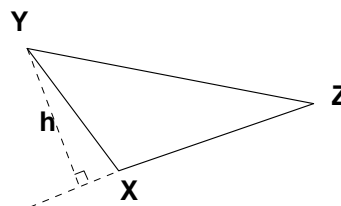
18. Let $f(x) = ax^3 - bx - 11$ where a , b , and c are constants. If $f(7) = -4$ then $f(-7) = ?$

- (A) 15 (B) 7 (C) -7 (D) -18 (E) -26

19. How many degrees are there in $6\frac{3}{4}\pi$ radians?

- (A) $2,250^\circ$ (B) 607.5° (C) $1,080^\circ$ (D) $1,215^\circ$ (E) 911.5°

20. Given the triangle shown, find the height, h , if $m\angle XZY = \frac{\pi}{6}$, $YZ = 22$ cm, and $XZ = 16$ cm.



- (A) 11 cm (B) 12.375 cm (C) 13.125 cm (D) 13.75 cm (E) 14 cm

21. If the $\sin \theta = 0.28$ and θ is in QII, then $\sec \theta$ is :

- (A) -0.68 (B) -0.96 (C) $-1\frac{1}{24}$ (D) $-3\frac{3}{7}$ (E) $-3\frac{4}{7}$

22. In the expansion of $(x - y)^6$, the sum of the coefficients of the 2nd, 4th, and 6th terms is:

- (A) -41 (B) -32 (C) -27 (D) -20 (E) -8

23. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$. Find the sum of the elements of AB .

- (A) 24 (B) 18 (C) 12 (D) 6 (E) 0

24. Simplify: $3\log_a b - 2\log_a 2b + \log_a 3b$

- (A) $2\log_a(\frac{3b}{4})$ (B) $\log_a(2b)$ (C) $\log_a(\frac{3b^2}{4})$ (D) $2\log_a(\frac{2b}{3})$ (E) $3\log_a(\frac{b}{2})$

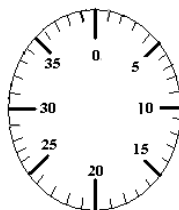
25. Point P has polar coordinates of $(5, \frac{\pi}{6})$ and rectangular coordinates of (x, y) . Where does point P lie on the Cartesian coordinate plane?

- (A) Q1 (B) QII (C) QIII (D) x-axis (E) y-axis

26. If $f'(x) = 2x^3 + 4x - 5$ and $f(2) = 0$, find $f(-1)$.

- (A) 5.5 (B) 1.5 (C) 1 (D) -1 (E) -3.5

27. U. R. SAFE combination locks have three number combinations. The first number of each combination is a prime number, the second is a Fibonacci number, and third digit is a triangular number. How many different combinations are possible?



- (A) 768 (B) 678 (C) 616 (D) 28 (E) 25
28. Evaluate: $\int_{-a}^a (2x^2 - 1) dx$
- (A) $\frac{4a^3}{3}$ (B) $2a^2 - a$ (C) $\frac{4a^3 - 6a}{3}$ (D) $\frac{2a^3 - 2a}{3}$ (E) $-2a$
29. Let $x^2 + y^2 = 16$. Find D_{xy} .
- (A) 4 (B) $x + y$ (C) $-\frac{x}{y}$ (D) $\frac{y}{x}$ (E) $\frac{x+y}{8}$
30. Lotta Scents has 2 quarters, 3 dimes, and 4 nickels. Each of the coins are distinguishable since they all have a different date on them. How many ways can she give her brother Noah Scents \$0.50?
- (A) 144 (B) 120 (C) 50 (D) 26 (E) 25
31. All of the elements of $\{14, 16, 19, 21, 25\}$ are considered to be __ numbers.
- (A) abundant (B) happy (C) lucky (D) odious (E) wasteful
32. The cardinality of the set of real numbers is strictly larger than the cardinality of the set of natural numbers is a major theorem proven by _____.
- (A) John Napier (B) Georg Cantor (C) Christian Goldbach (D) Freda Porter (E) Euclid
33. The midradius (nearest tenth) of a dodecahedron with edge length of 10 cm is:
- (A) 2.6 cm (B) 6.5 cm (C) 8.1 cm (D) 11.6 cm (E) 13.1 cm
34. Using Blaise Pascal's triangle and letting the 1 at the top be row 0, determine which of the following numbers will be in row 11.
- (A) 136 (B) 222 (C) 330 (D) 451 (E) 505
35. Find the number of positive integral divisors of 1,620.
- (A) 30 (B) 25 (C) 20 (D) 15 (E) 10

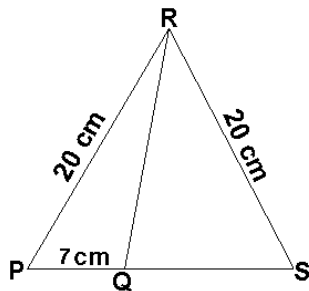
36. If R, S, and T represent digits then $RST_2 - ST_4 + R_8$ has a numeric value in base 10 of:

- (A) $9R - 2S$ (B) $5R - 2S$ (C) $5R + 6S$ (D) $9R - 2S - 2T$ (E) $10R - 2S$

37. Matt E. Mattiks wants to score 194 on this mathematics test. He gets 6 points for each correct answer and loses 2 points for each incorrect answer. Skipped problems are not counted. What is the least number of problems Matt has to work?

- (A) 33 (B) 35 (C) 37 (D) 39 (E) 41

38. Find $m\angle PQR$ of the equilateral triangle shown. (nearest degree)



- (A) 112° (B) 73° (C) 80° (D) 107° (E) 100°

39. Three years ago, Ima Old's age was one year more than twice Ura Young's age. Six years from now, Ima will be ten years more than half Ura's age. What is the difference in their ages now?

- (A) 4 years (B) 10 years (C) 5 years (D) 3 years (E) 6 years

40. Let P and Q be the roots of $3x^2 + 6x - 10 = 0$.
Find $P^5 + 5P^4Q + 10P^3Q^2 + 10P^2Q^3 + 5PQ^4 + Q^5$.

- (A) -12 (B) -15 (C) -19 (D) -21 (E) -32

41. If $\frac{2x}{5} - \frac{1}{6y} = \frac{3x}{4}$, then y equals _____.

- (A) $-\frac{11}{10x}$ (B) $-\frac{10}{21x}$ (C) $-\frac{21}{13x}$ (D) $\frac{13}{21x}$ (E) $\frac{20}{42x}$

42. If $a_1 = 5$, $a_2 = 4$ and $a_n = [(a_{n-1}) - (a_{n-2})]^{(n-3)}$ for $n \geq 3$, then a_5 equals:

- (A) 25 (B) 16 (C) 9 (D) -1 (E) -3

43. The circumcenter, centroid, and orthocenter of a scalene triangle are always:

- (A) concurrent (B) on the interior (C) collinear (D) equidistant (E) on the exterior

44. The Tic Toc Shop's circular clock stopped working at 11:20 pm. What was the measure of the smaller angle between the big hand and the little hand at that time?

- (A) 72° (B) 108° (C) 112° (D) 140° (E) 150°

45. The length of the latus rectum of $9x^2 - 4y^2 = 36$ is:

- (A) $2\frac{2}{3}$ (B) 4 (C) $5\frac{1}{3}$ (D) 9 (E) 18

46. Find the slope of the tangent line to $9x^2 + 4y^2 = 36$ at point $P(1\frac{1}{3}, -\sqrt{5})$. (nearest tenth)

- (A) 1.3 (B) 1.7 (C) 2.3 (D) 3.0 (E) 3.1

47. Captain Saul T. Water leaves port and sails his barge on a bearing of 40° for 20 miles. Then he changes course and sails 12 miles on a bearing of 75° . How far from port is the barge? (nearest tenth)

- (A) 27.3 mi (B) 21.9 mi (C) 32.0 mi (D) 35.0 mi (E) 30.6 mi

48. Find $(2 + 3i)^4$ and express the answer in standard form.

- (A) $61 - 30i$ (B) $-313 - 312i$ (C) $-119 - 120i$ (D) $-1 + 0i$ (E) $0 - 239i$

49. Let $\|V_1\| = 12$, $\|V_2\| = 15$, where the direction angles of V_1 and V_2 are 61° and 331° , respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

- (A) 10° (B) 13° (C) 22° (D) 24° (E) 27°

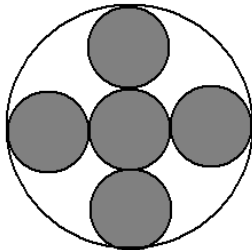
50. Use the Fibonacci characteristic sequence $\dots p, q, -3, r, 4 \dots$ to Find $p + q + r$.

- (A) -5 (B) -14 (C) -1 (D) 1 (E) 4

51. Let $f(x) = 3x - 4$ and $g(x) = 5x + 2$. Find $g(f(x)) + f(g(x))$.

- (A) $2x + 6$ (B) $30x - 16$ (C) $8x - 2$ (D) $-2x - 6$ (E) -20

52. The outer four small circles are tangent to the big circle and to the inner small circle in the center. All five small circles are congruent. Will Amtel shoots an arrow and hits somewhere in the large circle. What are the odds that the arrow hit the shaded section?



- (A) 5:4 (B) 3:2 (C) 5:2 (D) 1:1 (E) 2:1

53. Given the function $f(x) = 2x^4 - 3x^3 - 2x^2 - x + 6$ there is a possibility of how many real zeroes?

- (A) 4, 2, or 0 (B) 3 or 1 (C) 2 or 0 (D) 2 (E) 0

54. Let $f(x) = \frac{1}{\sqrt{3 + 2x - x^2}}$. At which of these intervals is function f continuous?

- (A) $[-1, 3]$ (B) $(-3, 1]$ (C) $[-1, 3)$ (D) $(-1, 3)$ (E) $(-3, 1)$

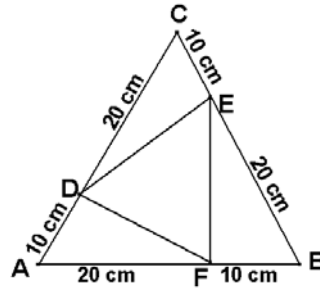
55. The polar graph of $r = 2 + 4\cos \theta$ is symmetric to the:

- (A) polar axis (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{4}$ (E) line $\theta = \frac{3\pi}{4}$

56. Determine the number of non-negative integer solutions to the equation: $p + q + r = 24$.

- (A) 2,024 (B) 1,012 (C) 775 (D) 325 (E) 276

57. Find DE to the nearest tenth.



- (A) 22.4 cm (B) 10 cm (C) 16.4 cm (D) 17.3 cm (E) 14.1 cm

58. The series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$ converges to _____.

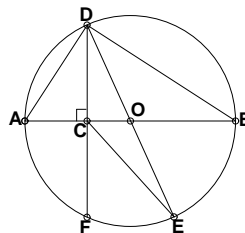
- (A) ∞ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) 0 (E) $\frac{\pi}{4}$

59. Dee Sprinter ran 4 warm-up laps before practice. The following chart shows his lap speed. What was Dee's average speed? (nearest hundredth)

Lap	1	2	3	4
Speed	22.5 km/hr	20.6 km/hr	20.0 km/hr	24.0 km/hr

- (A) 21.83 km/hr (B) 21.78 km/hr (C) 21.72 km/hr (D) 21.66 km/hr (E) 21.12 km/hr

60. G. I. Amaree drew a circle with center O with $BC = 2 \cdot AC$. After doing some measuring he calculated the area of $\triangle ABD$ to be 48 sq. cm. Based on his calculations what would the area of $\triangle DCO$ be?



- (A) 6 sq. cm. (B) 8 sq. cm. (C) 9.6 sq. cm. (D) 12 sq. cm. (E) 14.4 sq. cm.

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2013
Answer Key**

- | | | |
|-------|-------|-------|
| 1. D | 21. C | 41. B |
| 2. B | 22. B | 42. B |
| 3. E | 23. A | 43. C |
| 4. A | 24. C | 44. D |
| 5. C | 25. A | 45. D |
| 6. B | 26. B | 46. A |
| 7. A | 27. A | 47. E |
| 8. E | 28. C | 48. C |
| 9. C | 29. C | 49. A |
| 10. B | 30. C | 50. E |
| 11. A | 31. D | 51. B |
| 12. C | 32. B | 52. A |
| 13. D | 33. E | 53. A |
| 14. C | 34. C | 54. D |
| 15. D | 35. A | 55. A |
| 16. A | 36. B | 56. D |
| 17. B | 37. B | 57. D |
| 18. D | 38. E | 58. E |
| 19. D | 39. A | 59. D |
| 20. A | 40. E | 60. B |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 1 • 2013



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1. Evaluate the following: $4! + (8)^{\frac{2}{3}} \div (12^{-1}) \times \sqrt{16} - 20$

- (A) $-29\frac{1}{3}$ (B) $-3\frac{11}{12}$ (C) 16 (D) 115 (E) 196

2. Clara Nett owns a music store. She has guitars on a "tax free" sale this weekend. If Vi Ollinn buys two guitars she gets 25% off the price for both guitars. And if Vi buys three more guitars she gets 40% off of the price for the three guitars. If the price of a guitar is \$150.00, how much will it cost Vi if she buys all five guitars?

- (A) \$510.00 (B) \$495.00 (C) \$487.50 (D) \$450.00 (E) \$255.00

3. 440 inches per second = _____ miles per hour.

- (A) 88 (B) 70 (C) 55 (D) 40 (E) 25

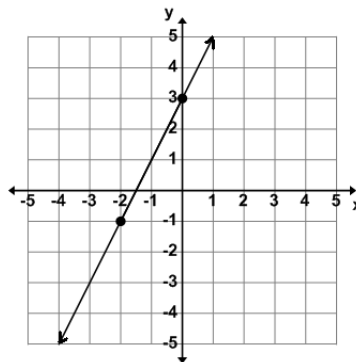
4. $3 \times 5 \times 7 = 15 \times 7 = 105$ and $3 \times 5 \times 7 = 3 \times 35 = 105$ are examples of the _____ properties of equality.

- (A) associative (B) commutative (C) distributive (D) identity (E) inverse

5. Simplify: $\left(\frac{x^2 + 6x + 9}{x - 3} \right) \left(\frac{x^2 - 6x + 9}{x^2 - 9} \right)$

- (A) $x - 3$ (B) $x + 3$ (C) $x^2 + 9$ (D) $x^2 - 9$ (E) $x^2 + 3x - 6$

6. A line perpendicular to the line shown intersects the line at point $(-1, 1)$ and has a y-intercept at point (x, y) . Find y.



- (A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

7. Truth tables are mathematical tables used in an area of math best developed by which of the following mathematicians?

- (A) Goldbach (B) Boole (C) Venn (D) Cantor (E) Agnesi

8. One year ago, L. Paso was two and a half times as old as his daughter, K. Paso. In five years her father will be twice as old as her. What is the sum of their ages now?

- (A) 31 years (B) 36 years (C) 42 years (D) 44 years (E) 54 years

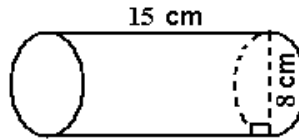
9. If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is always a(n) _____ triangle.

- (A) acute (B) isosceles (C) equilateral (D) obtuse (E) scalene

10. A regular icosahedron has F faces, E edges, and V vertices. Find $F + E + V$.

- (A) 70 (B) 62 (C) 52 (D) 26 (E) 14

11. Find the total surface area of the right cylinder shown. (nearest cm^2)



- (A) 377 cm^2 (B) 478 cm^2 (C) 528 cm^2 (D) 578 cm^2 (E) 754 cm^2

12. Which of the following points of concurrency are always on the interior of any triangle?

- (1) circumcenter (2) centroid (3) orthocenter (4) incenter

- (A) 2 only (B) 2, 3, & 4 (C) 1 & 3 (D) 2 & 4 (E) all of them

13. Harry Face uses an after-shave lotion that contains 50% ethyl alcohol. How much water does Harry need to add to 6 fluid ounces of his lotion to reduce it to 25% ethyl alcohol?

- (A) 3 fl. oz. (B) 4.5 fl. oz. (C) 6 fl. oz. (D) 6.75 fl. oz. (E) 18 fl. oz.

14. If $\frac{3x+4}{2x-3} + \frac{x+2}{4x-5} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

- (A) $-\frac{1}{3}$ (B) $-\frac{10}{29}$ (C) -4 (D) -10 (E) -14

15. The length of the latus rectum of $16x^2 - 25y^2 = 400$ is:

- (A) 6.4 (B) 12.5 (C) 16 (D) 20 (E) 25

16. Find the area of the circle, $x^2 + y^2 - 10x + 6y - 18 = 0$. (nearest tenth)

- (A) 45.3 (B) 56.6 (C) 106.8 (D) 163.4 (E) 171.1

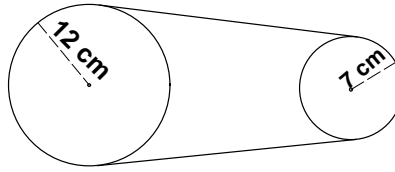
17. Smokie D. Bare spots two camp fires from his ranger station. One camp fire is 3 miles on a bearing of 75° and the other is 5 miles on a bearing of 225° . How far apart are the two camp fires? (nearest tenth)

- (A) 8.0 mi (B) 7.7 mi (C) 7.0 mi (D) 5.5 mi (E) 4.4 mi

18. Let $(2 + 3i)^4 = a + bi$. Find $a - b$.

- (A) -239 (B) -193 (C) 625 (D) 31 (E) 1

19. A belt joins the two pulleys shown. If the larger pulley is rotating at 10 rpm, then the smaller pulley is rotating at _____ rpm . (nearest tenth)

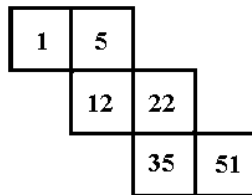


- (A) 17.1 (B) 18.4 (C) 19.0 (D) 5.0 (E) 5.8
20. Let $f(x) = 4x - 3$ and $g(x) = 2x + 5$. Find $f(f(x)) - g(g(x))$.
- (A) $12x - 22$ (B) $2x - 8$ (C) $20x - 2$ (D) $12x - 30$ (E) $20x$
21. Perri Mitter drew this figure on dot paper. The dots are 1 cm apart from each other both vertically and horizontally. Find the perimeter of the shaded region. (nearest tenth)
-
- (A) 6 cm (B) 15 cm (C) 17.7 cm (D) 19.3 cm (E) 37.8 cm
22. Seymore Sand can fill Pebble's sand box in 3 hours. If Saul T. Water helps Seymore they can fill the sand box in 2.25 hours working together. How long would it take Saul to fill it by himself?
- (A) 2 hrs 37.5 min (B) 3 hrs 22.5 min (C) 5 hrs 15 min (D) 6 hrs 45 min (E) 9 hrs 0 min
23. If $f'(x) = 6x - 4$ and $f(-1) = 12$, find $f(2)$.
- (A) 6 (B) 8 (C) 9 (D) 11 (E) 24
24. The series $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \dots$ converges to _____.
- (A) ∞ (B) 2 (C) $\frac{\sqrt{5}+1}{2}$ (D) e (E) $\frac{\pi^2}{6}$
25. The polar graph of $r = 3 - 3\sin \theta$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$
- (A) 1 only (B) 2 only (C) 3 only (D) 2 & 3 (E) 1, 2, & 3
26. The Texannas and the Calgirls play two games during the season. The Texannas are three times as likely to win any game as is the Calgirls. What is the probability that the Texannas will win both games?
- (A) 75% (B) $66\frac{2}{3}\%$ (C) $62\frac{1}{2}\%$ (D) $56\frac{1}{4}\%$ (E) 50 %

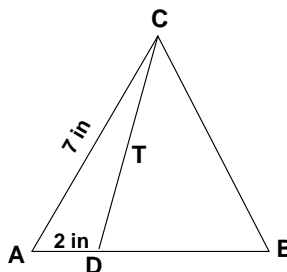
27. April Showers charted the Spring rainfall data as follows. What was the average rainfall for the 4 Spring months? (nearest hundredth)

<u>Month</u>	<u>Inches of rain</u>
March	4.5
April	6
May	4.6
June	3.1

- (A) 4.27" (B) 4.31" (C) 4.43" (D) 4.55" (E) 4.66"
28. Using Blaise Pascal's triangle, determine the sum of all of the numbers in the first 12 rows.
- (A) 1,023 (B) 2,048 (C) 4,095 (D) 6,145 (E) 8,192
29. Plato folds the net shown into a cube. He adds the numbers on each pair of opposite sides. What is the largest sum possible?



- (A) 52 (B) 56 (C) 63 (D) 73 (E) 86
30. $111_4 + 1111_2 = \underline{\hspace{2cm}}_2$
- (A) 10110 (B) 1010 (C) 11001 (D) 100100 (E) 1001110
31. Juan Prop flies his small plane 120 miles with the wind in 1 hour 20 minutes. The return trip against the wind takes 1 hours 40 minutes. What was the speed of the wind?
- (A) 4.5 mph (B) 6 mph (C) 9 mph (D) 10 mph (E) 13.5 mph
32. If $\frac{x-15}{x+16} + \frac{x+16}{x-15}$ is written as the mixed number $A \frac{B}{C}$, then B is?
- (A) 1 (B) 31 (C) 32 (D) 62 (E) 961
33. Given the equilateral triangle shown, find T^2 .



- (A) 21 (B) 22.5 (C) 39 (D) 45 (E) 63.5

34. If $a_1 = -3$, $a_2 = 3$ and $a_n = (a_{n-2}) \div (a_{n-1})$ for $n \geq 3$, then a_7 equals:

- (A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $-\frac{1}{27}$ (D) -3 (E) -9

35. If $A + B = 20$ and $A \times B = 16$ then $|B - A| = \underline{\hspace{2cm}}$.

- (A) $4 + 2\sqrt{5}$ (B) $5\sqrt{42}$ (C) $2\sqrt{5} - 4$ (D) $4\sqrt{11}$ (E) $4\sqrt{21}$

36. Let $f(x) = \begin{cases} kx - 1 & \text{if } x < 2 \\ kx^2 & \text{if } 2 \leq x. \end{cases}$ Find the value of k that makes $f(x)$ continuous on $(-\infty, +\infty)$.

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) 4 (D) 2 (E) $\frac{1}{6}$

37. The Texas Senate consists of 31 senators. Twelve of them are selected to serve on three special committees. In how many ways can the 12 senators be divided, so that 5 senators are on the budget committee, 3 on the governor's committee, and 4 on the highways committee?

- (A) 50,388 (B) 27,720 (C) 13,860 (D) 7,440 (E) 828

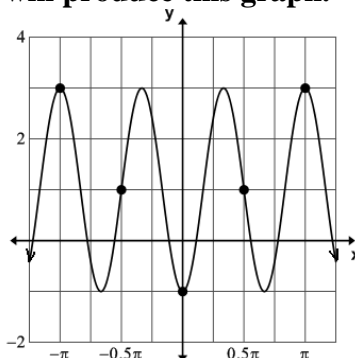
38. $FACF_{16} \div D_{16} = \underline{\hspace{2cm}} 16$

- (A) 1012 (B) 1234 (C) 1347 (D) 13A5 (E) 134B

39. Which of the following is not in the solution set of $2|3 + 5x| - 7 < 11$?

- (A) $-2\frac{2}{3}$ (B) $-2\frac{1}{7}$ (C) $-1\frac{7}{11}$ (D) $\frac{1}{2}$ (E) $1\frac{1}{6}$

40. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $2\cos(3x) - 1$ (B) $2\sin(x) - 3$ (C) $-2\cos(3x) + 1$ (D) $2\cos(3x) + 1$ (E) $2\sin(3x) - 1$

41. Use the Fibonacci characteristic sequence ... $p, q, -3, r, s, 3, \dots$ to find $(p + q) - (r + s)$.

- (A) 12 (B) 9 (C) 0 (D) -3 (E) -6

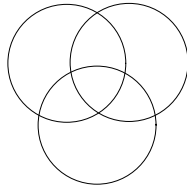
42. Let $A = \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix}$. Find $\det(A + A^T)$.

- (A) 3 (B) 2 (C) 1 (D) -2 (E) -17

43. How many distinguishable arrangements could Plato make from the letters DODECAHEDRA?

- (A) 1,663,200 (B) 1,200,000 (C) 483,840 (D) 40,320 (E) 7,920

44. M. T. Pockets surveys 26 men and finds out that 5 have dimes, 14 have nickels, and 10 have pennies. One has only dimes and pennies, 3 have only nickels and dimes, and 4 have only pennies and nickels. If no one has all three coins, how many men do not have any of these coins?
(This diagram might help.)



- (A) 0 (B) 2 (C) 5 (D) 6 (E) 8

45. Which of the following is considered to be an *equidigital* number?

- (A) 21 (B) 22 (C) 33 (D) 36 (E) 44

46. If the roots of $x^3 + bx^2 + cx + d = 0$ are -5 , 3 , and 8 , then $b + c + d$ equals:

- (A) -145 (B) -95 (C) 0 (D) 6 (E) 83

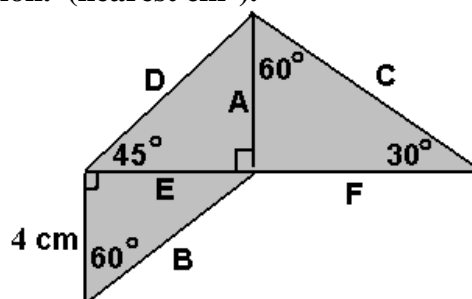
47. Simplify: $\frac{\cos^2(\theta)}{(1 - \sin^2(\theta))^2}$

- (A) $\sec^2(\theta)$ (B) $\csc^2(\theta)$ (C) 1 (D) $\cos(\theta)$ (E) $\sec(\theta)$

48. Dee Kart connected the points $A(1, 3)$, $B(4, -4)$, and $D(-3, -2)$ forming an acute, scalene triangle. What is the area of this triangle?

- (A) 20 sq. units (B) 21.5 sq. units (C) 22 sq. units (D) 22.5 sq. units (E) 23 sq. units

49. Find the area of the shaded region. (nearest cm^2).



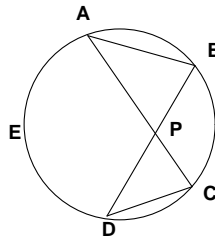
- (A) 79.4 cm^2 (B) 69.3 cm^2 (C) 94 cm^2 (D) 83.1 cm^2 (E) 112 cm^2

50. Find the slope of the tangent line to $25x^2 - 16y^2 = 400$ at point $P(-5, 3.75)$.

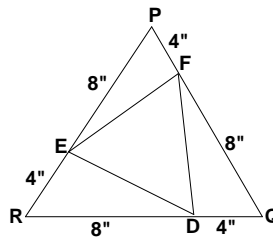
- (A) $-2\frac{1}{12}$ (B) $-1\frac{2}{3}$ (C) $-1\frac{5}{12}$ (D) $-1\frac{1}{3}$ (E) $-1\frac{11}{64}$

51. Let $\|V_1\| = 18$, $\|V_2\| = 10$, where the direction angles of V_1 and V_2 are 20° and 80° , respectively. Find the direction angle of $V_1 + V_2$. (nearest degree)
- (A) 21° (B) 41° (C) 50° (D) 60° (E) 100°
52. Each of the first five "happy" numbers, 1, 7, 10, 13, and 19 are written on blank sheets of paper. Three pieces of paper are randomly selected. What are the odds that the sum of the "happy" numbers selected is an even number?
- (A) 1 to 4 (B) 3 to 2 (C) 1 to 2 (D) 2 to 3 (E) 2 to 1
53. The volume, nearest cubic inch, of a regular octahedron with edge length 4" is:
- (A) 6 cu. in. (B) 8 cu. in. (C) 23 cu. in. (D) 30 cu. in. (E) 32 cu. in.

54. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O. If $m\widehat{CD} = 42^\circ$ and $m\widehat{AB} = 68^\circ$ then $m\angle BPC = ?$



- (A) 55° (B) 62.5° (C) 70° (D) 110° (E) 125°
55. Find the area of $\triangle DEF$ to the nearest inch.

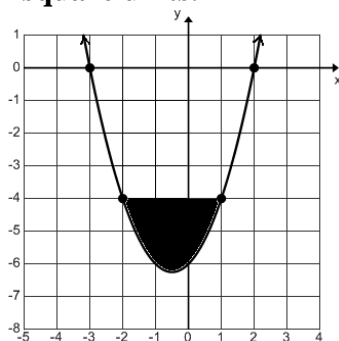


- (A) 62" (B) 35" (C) 27" (D) 21" (E) 12"
56. Let $f(x) = \frac{x^3 + 7x + 1}{x^2 - x + 1}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(-1)$.
- (A) $-2\frac{1}{3}$ (B) -2 (C) 0 (D) $2\frac{2}{3}$ (E) 3
57. An infinite geometric sequence has a common ratio of $\frac{3}{4}$ and a sum of $26\frac{2}{3}$. What is the first term of the sequence?
- (A) $35\frac{5}{9}$ (B) $6\frac{2}{3}$ (C) 20 (D) $3\frac{2}{9}$ (E) $6\frac{1}{12}$

58. Determine the concavity of the graph of $f(x) = 2\sin(x) + 3\cos(x)$ at $x = \frac{\pi}{4}$.

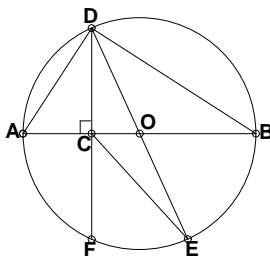
- (A) $-\frac{\sqrt{2}}{2}$ (B) $-\frac{5\sqrt{2}}{2}$ (C) -1 (D) $\sqrt{2}$ (E) $\frac{3\sqrt{2}}{2}$

59. Find the area of the shaded region in square units.



- (A) 3 (B) 3.25 (C) 4 (D) 4.5 (E) 4.75

60. G. I. Amaree drew a circle with center O with $BC = 2AC$, $CD = 3\sqrt{2}$ ", and $AC = 3$ ". Based on the information and the drawing what is the area of $\triangle CDE$? (nearest tenth)



- (A) 2.1 sq. in. (B) 3.2 sq. in. (C) 6.4 sq. in. (D) 12.7 sq. in. (E) 19.1 sq. in.

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2013
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. C | 41. E |
| 2. B | 22. E | 42. A |
| 3. E | 23. C | 43. A |
| 4. A | 24. B | 44. C |
| 5. B | 25. C | 45. A |
| 6. E | 26. D | 46. E |
| 7. B | 27. D | 47. A |
| 8. D | 28. C | 48. B |
| 9. A | 29. C | 49. A |
| 10. B | 30. D | 50. A |
| 11. B | 31. C | 51. B |
| 12. D | 32. E | 52. B |
| 13. C | 33. C | 53. D |
| 14. D | 34. C | 54. E |
| 15. A | 35. E | 55. D |
| 16. D | 36. A | 56. C |
| 17. B | 37. B | 57. B |
| 18. E | 38. E | 58. B |
| 19. A | 39. A | 59. D |
| 20. D | 40. C | 60. C |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 2 • 2013



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $0.333... \div 0.1666... + 0.142857142857148527... - 0.111... \div 0.090909...$

- (A) $22\frac{22}{63}$ (B) $1\frac{23}{63}$ (C) $\frac{58}{63}$ (D) $-\frac{41}{42}$ (E) $-1\frac{1}{42}$

2. Cal Q. Lait is buying scientific calculators for his math team. The cost of each calculator is \$125.00. If he buys a set of 6 he gets 25% off of the regular price for 4 and 40% off the regular price for the other 2. How much money will he save if he buys a set of 6?

- (A) \$81.25 (B) \$150.00 (C) \$187.25 (D) \$212.50 (E) \$225.00

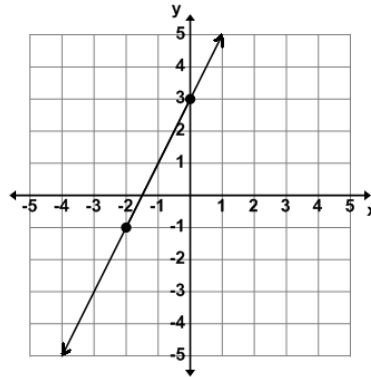
3. $3 \times (6 + 10) = 3 \times 16 = 48$ and $3 \times 6 + 3 \times 10 = 18 + 30 = 48$ are examples of the _____ property of equality.

- (A) commutative (B) closure (C) associative (D) transitive (E) distributive

4. All triangular numbers and all odd numbers are considered to be _____ numbers?

- (A) perfect (B) polite (C) pretty (D) prime (E) primeval

5. A line perpendicular to the line shown intersects the line at point $(-2, -1)$ and has an x-intercept at point (x, y) and a y-intercept at point (x_1, y_1) . Find $x + x_1 + y + y_1$.



- (A) -9 (B) -8 (C) -6 (D) -3 (E) -2

6. Simplify: $\left(\frac{6x^3 + x^2 - 2x}{9x^2 + 12x + 4}\right) \div \left(\frac{2x^2 - x}{6x^2 + 7x + 2}\right)$

- (A) $2x + 1$ (B) $3x - 1$ (C) $4x^2 - 1$ (D) $4x^2 - 4x + 1$ (E) $x^2 + 3x - 6$

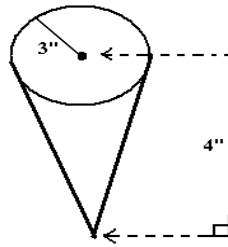
7. The number of hours it takes to pick up an orchard of pecans varies inversely to the number of pecan pickers. If it takes 30 hours for 5 pickers to pickup all of the pecans, how long would it take 8 pickers to pickup all of them? (nearest minute)

- (A) 22 hrs 30 min (B) 40 hrs (C) 18 hrs (D) 10 hrs 15 min (E) 18 hrs 45 min

8. If $y = x - 5$ and $xy = 2$ then $x^3 - y^3 = ?$

- (A) 155 (B) 133 (C) 117 (D) 106 (E) 95

9. Find the total surface area of the right circular cone shown. (nearest sq. in.)



- (A) 38 sq. in. (B) 47 sq. in. (C) 66 sq. in. (D) 75 sq. in. (E) 113 sq. in.

10. Corresponding sides of two similar triangles are proportional to the corresponding:

(1) altitudes (2) medians (3) perimeters

- (A) 1 only (B) 1 & 2 (C) 1 & 3 (D) 1, 2, & 3 (E) 2 only

11. A tourist looked up at the Big Ben circular clock in Westminster and saw the time to be 8:40 a.m. What was the measure of the smaller angle between the big hand and the little hand at that time?

- (A) 24° (B) 20° (C) 16° (D) 5° (E) 0°

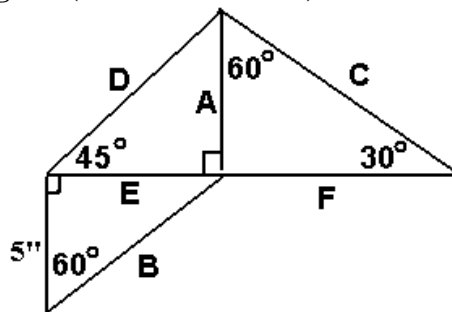
12. If you connect the centers of all the faces of an octahedron you will form a(n):

- (A) tetrahedron (B) cube (C) icosahedron (D) square pyramid (E) decahedron

13. Find the circumference of the circle, $x^2 + y^2 - 10x - 12y = 60$. (nearest tenth)

- (A) 71.0 (B) 69.1 (C) 59.6 (D) 52.9 (E) 48.7

14. Find the perimeter of the pentagon. (nearest half inch).



- (A) $4'8''$ (B) $4'10''$ (C) $4'11\frac{1}{2}''$ (D) $5'\frac{1}{2}''$ (E) $5'2''$

15. The graph of $10x - 4y^2 + 12y = 21$ is a(n):

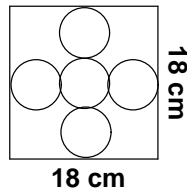
- (A) circle (B) cardioid (C) ellipse (D) hyperbola (E) parabola

16. Let $f(x) = 3x - 1$ and $g(x) = 3 - 2x$ and $h(x) = 4x + 5$. Find $g(f(h(-x)))$.

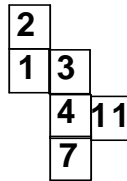
- (A) $24x + 35$ (B) $24x - 21$ (C) $24x - 25$ (D) $24x - 9$ (E) $24x + 7$

17. Admiral C. Weed leaves port and sails his ship 3 hours on a bearing of 60° at 15 knots. Then he changes course and sails on a bearing of 150° for 2 hours at 12 knots. Then he turns to sail back to port. What bearing should he set his course for to sail directly back to port? (nearest degree)
- (A) 298° (B) 300° (C) 332° (D) 268° (E) 272°
18. Which of the following equations will have a graph with an amplitude of 4, a frequency of $\frac{1}{2}$, a phase shift of -2 , and a displacement of $\frac{1}{4}$?
- (A) $\frac{1}{4} + 4\cos(\pi x + 2\pi)$ (B) $4 + \frac{1}{4}\cos(\frac{1}{2}\pi x - 2)$ (C) $\frac{1}{4} + 4\cos(\pi x - 2\pi)$
 (D) $\frac{1}{2} + 4\cos(4\pi x + 8\pi)$ (E) $\frac{1}{4} + 4\cos(2\pi x + \frac{\pi}{2})$
19. Simplify: $\sin(\theta) + \sin(\theta)\cot^2(\theta)$
- (A) $\sec(\theta)$ (B) $\sin^3(\theta)$ (C) $\tan^2(\theta)$ (D) $\csc(\theta)$ (E) $1 - \sin(\theta)$
20. In the expansion of $(2x + 3)^6$, the sum of the coefficients of the 3rd and the 6th term is:
- (A) 900 (B) 5,076 (C) 5,113 (D) 5,436 (E) 15,625
21. Suhn Wi Noh and Wen Wi Kan solve all of the advanced math test questions in 3 hours and 45 minutes. Suhn can solve all of them in 10 hours by himself. How long would it take Wen to solve all of them by himself?
- (A) 6 hrs (B) 6 hrs 15 min (C) 6 hrs 52 min (D) 7 hrs 30 min (E) 7 hrs
22. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$. Find the sum of the elements of $A(A - B)$.
- (A) 7 (B) 5 (C) 3 (D) 1 (E) 0
23. Determine the concavity of the graph of $f(x) = \sin(x) + \cos(x)$ at $x = \frac{4\pi}{3}$.
- (A) $\frac{\sqrt{3}-1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{-1-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1+\sqrt{3}}{2}$
24. Let $f(x) = \frac{2}{x+5}$. At which of these intervals is function f not continuous?
- (A) $(-\infty, 0)$ (B) $[-10, -5)$ (C) $(-5, \infty)$ (D) $(3, 7)$ (E) all of these
25. Five ping pong balls are numbered using the first five "lucky" numbers, 1, 3, 7, 9, and 13. Two balls are randomly selected. What is the probability that the sum of the "lucky" numbers selected is an "unlucky" number?
- (A) 100% (B) 60% (C) 40% (D) 20% (E) 10%

26. The five circles in the quadrilateral are congruent. The four outer circles are tangent to the quadrilateral and to the center circle. Amos Goode throws a dart that hits somewhere in the quadrilateral. What is the probability that the dart hits inside a circle? (nearest whole percent)

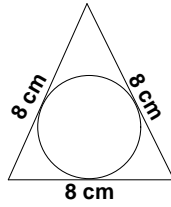


- (A) 65% (B) 57% (C) 51% (D) 44% (E) 29%
27. An icosahedron has an edge length of 2". The length of its midradius is: (nearest hundredth)
- (A) 2.62" (B) 0.81" (C) 3.24" (D) 2.29" (E) 1.62"
28. Which of the following numbers will be in the 18th row of Pascal's triangle?
- (A) 1,820 (B) 3,003 (C) 5,239 (D) 6,188 (E) 19,455
29. A women mathematician credited with writing the first book discussing both differential and integral calculus had a special curve named after her. Her name is:
- (A) Agnesi (B) Hypatia (C) Ada Byron (D) Freda Porter (E) Alecia Stott
30. Lucas folds the net shown into a cube. He multiplies the numbers on each pair of opposite sides. What is the sum of the three products?

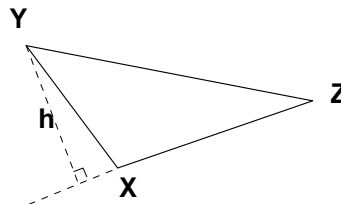


- (A) 28 (B) 29 (C) 40 (D) 47 (E) 51
31. $456_9 - 1221_3 = \underline{\hspace{2cm}}_3$
- (A) 102222 (B) 112110 (C) 121012 (D) 120222 (E) 122210
32. P and Q are the real roots of $4x^2 - 8x - 5 = 0$. Find $P^4 + 4P^3Q + 6P^2Q^2 + 4PQ^3 + Q^4$.
- (A) 44 (B) 28.125 (C) 16 (D) 12.5 (E) 9
33. $\{(x, y) \mid x, y \in \{\text{Integers}\}, -7 \leq x \leq 7, \text{ and } -11 \leq y \leq 11\}$ is the solution set of $5x - 3y = 2$. How many such ordered pairs exist?
- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2

34. Find the circumference of the inscribed circle. (nearest tenth)



- (A) 25.1 cm (B) 12.6 cm (C) 13.9 cm (D) 16.8 cm (E) 14.5 cm
35. The eccentricity of $18x^2 - 9y^2 = 162$ is:
- (A) $\frac{3\sqrt{2}}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{6}}{2}$ (E) $\sqrt{3}$
36. Otto Mobill has 16 quarts of a 20% solution of antifreeze in his radiator. How much will he have to drain out of his radiator and replace with pure antifreeze to make a 25% solution?
- (A) 1 qt (B) 1.25 qts (C) 2 qts (D) 2.5 qts (E) 5 qts
37. Jack Black throws out all of the cards from a standard deck except for the 4 aces and the face cards, 4 jacks, 4 queens, and 4 kings. He shuffles these cards and deals the top two cards face up on the table. What is the probability that the top card is a face card and the next card is an ace?
- (A) 15% (B) 16.666...% (C) 18.75% (D) 20% (E) 22.333...%
38. A triangle is drawn as shown. Find the $m\angle XZY$, if area of $\triangle XZY = 66 \text{ cm}^2$, $h = 11 \text{ cm}$ and $YZ = 22 \text{ cm}$.



- (A) 45° (B) 15° (C) 22.5° (D) 26.6° (E) 30°
39. Willie Spotette looks down from his deer blind and spots a deer feeding at his corn feeder. The altitude from the ground to Willie's eyes is 20 feet and the distance from his blind to his feeder is 60 yards. What is the angle of depression from Willie to the deer? (nearest minute)
- (A) $18^\circ 26'$ (B) $1^\circ 26'$ (C) $6^\circ 20'$ (D) $6^\circ 37'$ (E) $19^\circ 28'$
40. How many elements are in $\left\{ x \mid 4\sin(x)\cos(x) = \sqrt{3}, x \in (-\pi, \pi) \right\}$?
- (A) 1 (B) 2 (C) 4 (D) 6 (E) 8

41. If $\cos(\theta - \pi) < 0$ and $\sin(\theta + \pi) > 0$ then where will θ terminate?

- (A) origin (B) QI (C) QII (D) QIII (E) QIV

42. Simplify: $\log_b 3x + 2\log_b 2x - 3\log_b x$, if $x > 0$

- (A) $\log_b 12$ (B) $2\log_b 4x$ (C) $-\log_b 6x$ (D) $\frac{2}{3}\log_b 4x$ (E) $\log_b 6$

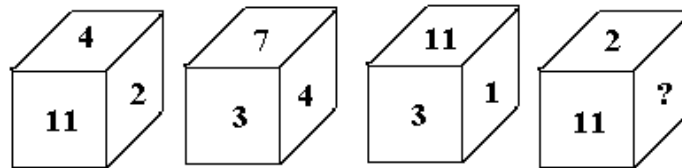
43. The polar graph of $r = 3 \cos 2\theta$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$

- (A) 1 only (B) 3 only (C) 1 & 3 (D) 2 & 3 (E) 1, 2, & 3

44. The series $\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots$ converges to _____.

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1+\sqrt{5}}{2}$ (D) ∞ (E) 2

45. Each face of a cube is labeled with the Lucas digits, 2, 1, 3, 4, 7, 11. Four views of the cube are shown. What is the sum of the numbers on the faces containing the "?" and its opposite face? Ignore orientation of the numbers.



- (A) 4 (B) 5 (C) 7 (D) 8 (E) 10

46. Determine the number of non-negative integer solutions to the equation: $p + q + r = 12$.

- (A) 36 (B) 91 (C) 105 (D) 455 (E) 1,728

47. Bunny Wabbit dyed a basket of hard boiled eggs for the big egg hunt. She hid 50% of the eggs in open sight. Then she hid $\frac{1}{4}$ of the remaining eggs under bushes. She gave each of her 3 helpers an egg for helping and kept the last 3 eggs for her morning breakfast. How many eggs were in the basket originally?

- (A) 12 (B) 16 (C) 18 (D) 24 (E) 30

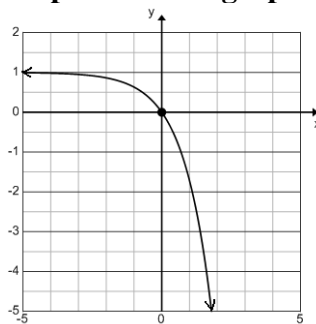
48. Find the sum, nearest whole number, of the mean, median, mode, and range of 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, & 31.

- (A) 101 (B) 98 (C) 96 (D) 95 (E) 93

49. The coordinates of the vertices of $\triangle PQR$ are $(-5, -2)$, $(-2, 2)$, and $(1, -2)$. The coordinates of the incenter is (x, y) . Find y .

- (A) -2 (B) -1.5 (C) -1 (D) -0.5 (E) -0.25

50. Which of the following equations will produce the graph shown here?



- (A) $y = \ln(-x) - 1$ (B) $y = 1 + \ln(x)$ (C) $y = 1 + e^{\ln(x)}$ (D) $y = 1 - e^{(-x)}$ (E) $y = 1 - e^{(x)}$

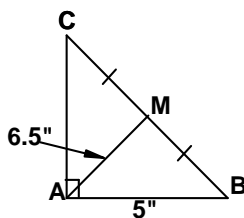
51. Dim Whitt and his brother, Nit Whitt can paint 6 houses in 32 hours. How long would it take them to paint 10 houses if their other brother, Half Whitt, helps them and they all work at the same rate as the Dim and Nit? (nearest minute)

- (A) 16 hrs (B) 17 hrs 47 min (C) 26 hrs 40 min (D) 35 hrs 33 min (E) 53 hrs 20 min

52. Find the sum of all two-digit numbers such that reversing the digits results in another two-digit number that is $42\frac{6}{7}\%$ less than the original two-digit number.

- (A) 120 (B) 360 (C) 420 (D) 240 (E) 210

53. $\triangle ABC$ exists as shown. Let $m\angle AMB = \theta$. Find $\cos(\theta)$. (nearest hundredth)



- (A) 0.38 (B) 0.92 (C) 0.56 (D) 0.70 (E) 0.77

54. How many solutions are there for the equation $4x + 5y = 123$ such that both x and y are positive integers?

- (A) 14 (B) 13 (C) 12 (D) 6 (E) 5

55. The polar coordinates of point P is $(-9, \frac{5\pi}{6})$. If point P is converted to rectangular coordinates, where would point P lie on the Cartesian coordinate plane?

- (A) QII (B) QIII (C) QIV (D) x-axis (E) y-axis

56. Find the area of the region bounded between the graphs of $y = -1$ and $y = x^3$ for $-1 \leq x \leq 0$.

- (A) 1.5 (B) 1.25 (C) 0.75 (D) 0.666... (E) 0.333...

57. $F(x) = \frac{x+1}{3-x}$ has an inflection point at $x = ?$
- (A) -1 (B) 3 (C) 1 (D) -3 (E) does not exist
58. The probability that statement P is true is $\frac{2}{5}$, and the probability that statement Q is true is $\frac{3}{8}$. Determine the probability that $P \rightarrow Q$ is false.
- (A) $\frac{3}{4}$ (B) $\frac{15}{16}$ (C) $\frac{1}{40}$ (D) $\frac{1}{4}$ (E) $\frac{3}{20}$
59. $110010101111_2 + 303_4 + 1E3_{16} = \underline{\hspace{2cm}}_{16}$
- (A) EC5 (B) 103E (C) BAD1 (D) 3E5 (E) ABC
60. P, Q, and R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is $\frac{18}{23}$ and $PQR = 1$. Find C.
- (A) $4\frac{1}{6}$ (B) $3\frac{5}{6}$ (C) $2\frac{5}{9}$ (D) $1\frac{5}{18}$ (E) 1

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 2 • 2013
Answer Key**

1. C	21. A	41. E
2. E	22. A	42. A
3. E	23. E	43. E
4. B	24. A	44. B
5. C	25. A	45. B
6. A	26. D	46. B
7. E	27. E	47. B
8. A	28. D	48. D
9. D	29. A	49. D
10. D	30. C	50. E
11. B	31. A	51. D
12. B	32. C	52. E
13. B	33. B	53. D
14. C	34. E	54. D
15. E	35. E	55. C
16. C	36. A	56. C
17. D	37. D	57. E
18. A	38. E	58. D
19. D	39. C	59. A
20. B	40. C	60. B



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Region • 2013



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[(2 + 4!) \div (8)^{\frac{1}{3}} - \sqrt{16} \times 32^{(-1)}] \div 0.125$

- (A) 105 (B) 103 (C) 38 (D) $2\frac{43}{64}$ (E) 2.25

2. *Bear Foot* Shoes is having a sale. Harry Paw needs to buy 4 pair of shoes. Which of the following is the least expensive deal?

- (A) \$64.00 a pair (B) buy 2 pair at \$75 a pair and 2 pair at \$50 a pair
(C) buy 3 pair at \$70 a pair and 1 pair at \$42 a pair (D) \$72 a pair and get 15% off
(E) \$80 for the 1st pair, \$70 for the 2nd, \$60 for the 3rd, and \$50 for the 4th

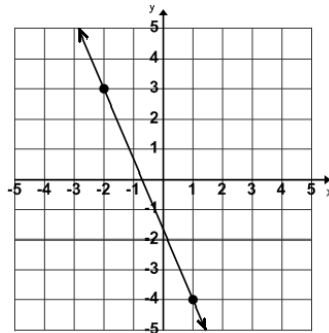
3. Find the arithmetic mean of the median, mode, and range of 2, 1, 3, 4, 7, 1, 1, 2, 3, & 5.

- (A) $2\frac{9}{10}$ (B) $3\frac{1}{3}$ (C) $3\frac{5}{6}$ (D) $2\frac{4}{9}$ (E) $3\frac{1}{6}$

4. Let $N = \{n,a,p,i,e,r\}$, $E = \{e,u,c,l,i,d\}$, $A = \{a,g,n,e,s,i\}$, and $T = \{t,h,e,a,n,o\}$. The number of distinct elements in $(A \cap N) \cup (T \cap E)$ is _____.

- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2

5. Which of the following points lies on the line shown?



- (A) $(-6, 9)$ (B) $(4, -7)$ (C) $(-8, 17)$ (D) $(-3, 7)$ (E) $(8, -17)$

6. The set $\{-1, 1\}$ is closed under which of these operations:

+ addition - subtraction \times multiplication \div division $\sqrt{\quad}$ square root

- (A) + & \times (B) \times & \div (C) +, \times , & $\sqrt{\quad}$ (D) \times only (E) none of these

7. Lotta Dough Bakery knows that the amount of flour needed to make bread is directly proportional to the number of loaves of bread needed. Their recipe calls for 5 cups of flour for each loaf. How many pounds of flower will be needed to bake 5 dozen loaves of bread if, according to the Farmer's Almanac, it takes 1.5 cups of flour to equal 1 pound of flour?

- (A) 450 lbs (B) 312.5 lbs (C) 300 lbs (D) 200 lbs (E) 187.5 lbs

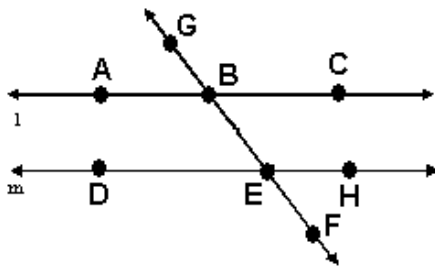
8. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle R = 37^\circ$ then $m\angle Q = ?$

- (A) 53° (B) 87° (C) 113° (D) 127° (E) 143°

9. If two inscribed angles of a circle intercept the same arc then the angles are _____.

- (A) complementary (B) right (C) congruent (D) vertical (E) supplementary

10. The three lines in the figure are coplanar with $m \parallel l$. How many of the following are true statements?



1. $\angle DEG$ & $\angle HEB$ are vertical angles 2. $m\angle ABG = \frac{1}{3}m\angle DEF$
 3. $\angle HEF \cong \angle CBG$ 4. $\angle BED$ and $\angle BEH$ are supplementary

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

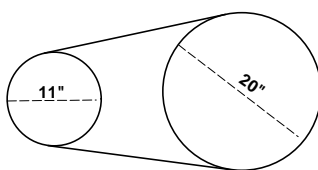
11. Let $f(x) = 2x - 5$ and $g(x) = 3 - 4x$ and $h(x) = x + 5$. Find $h(f(g(1-x)))$.

- (A) $8x - 2$ (B) $1 - 48x$ (C) $8x + 11$ (D) $4 - 3x$ (E) $3x + 4$

12. Find the area of the circle, $2x^2 + 2y^2 + 4x - 5y - 2 = 0$. (nearest tenth)

- (A) 14.3 (B) 10.2 (C) 5.9 (D) 5.1 (E) 11.2

13. A belt joins the two pulleys shown with the given diameters. If the smaller pulley is rotating at 75 rpm, then the larger pulley is rotating at _____ rpm.



- (A) $1\frac{7}{11}$ (B) $37\frac{1}{2}$ (C) $26\frac{9}{11}$ (D) $41\frac{1}{4}$ (E) $20\frac{5}{8}$

14. If the $\sec \theta = -2\frac{1}{3}$ and θ is in QIII, then $\sin \theta$ is : (nearest tenth)

- (A) -0.5 (B) -0.9 (C) -0.4 (D) 0.4 (E) 0.9

15. Let $(-2 + 2i)^4 = a + bi$. Find $a + b$.

- (A) -224 (B) -172 (C) -64 (D) -32 (E) 0

16. Mathis Grate placed a tennis ball, a soccer ball, and a golf ball on the football field, each in a different place. He gave his math class the following information. The bearing of the tennis ball from the soccer ball is 45° and they are 24 yards apart. The bearing of the golf ball from the tennis ball is 135° and they are 18 yards apart. What should the students calculate the bearing to be of the soccer ball from the golf ball? (nearest whole degree)

(A) 188° (B) 225° (C) 262° (D) 278° (E) 315°

17. An infinite geometric sequence has a common ratio of 0.75 and a sum of 120. What is third term of the sequence?

(A) 30 (B) 22.5 (C) 16.875 (D) 15 (E) 12.5625

18. Simplify: $\log_b 16x - 2\log_b 8x + \log_b x^{-4}$, if $x > 0$

(A) $-\log_b 4x^5$ (B) $\log_b 3x$ (C) $-\log_b x^4$ (D) $\log_b x$ (E) $-5\log_b x$

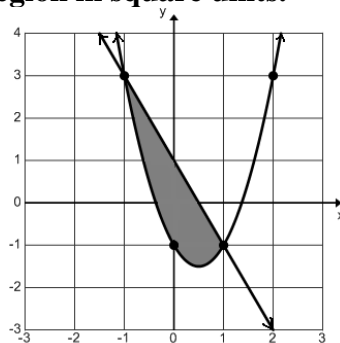
19. Let $f(x) = \frac{x^2 - 3}{2x - 4}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(5)$.

(A) 3.666... (B) 3.5 (C) 3.333... (D) 3.25 (E) 3.1666...

20. Let $f(x) = x^2 - 4x + 1$ and $g(x) = x^2 + 5x - 2$. Find $g(f'(x + 1))$

(A) $4x^2 + 20x + 37$ (B) $2x^2 + 3x$ (C) $4x^2 + 2x - 8$ (D) $4x^2 + 24x + 50$ (E) $x^2 + 7x - 1$

21. Find the area of the shaded region in square units.



(A) $3\frac{1}{2}$ (B) $3\frac{1}{3}$ (C) $3\frac{1}{6}$ (D) 2 (E) $2\frac{2}{3}$

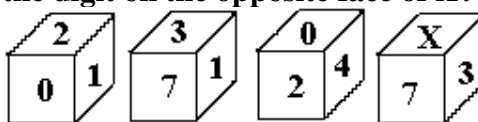
22. How many distinguishable arrangements can be made from the letters "CALCULUS"?

(A) 5,040 (B) 120 (C) 13,440 (D) 6,720 (E) 40,320

23. Ura Trechee picks 2 spades, 3 hearts, 4 diamonds, and 5 clubs from a standard deck of cards. She shuffles the cards and deals the top three cards face up on the table. What is the probability that the top card is a heart or a diamond and the next two cards are either a spade or a club? (nearest %)

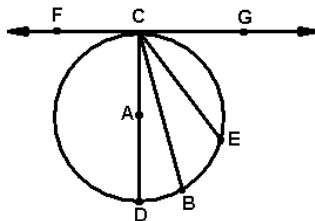
(A) 51% (B) 27% (C) 13% (D) 11% (E) 7%

24. Each face of a cube is labeled with the digits, 0, 2, 1, 3, 4, 7. Four views of the cube are shown. What is the product of X and the digit on the opposite face of X? Ignore orientation.



- (A) 8 (B) 6 (C) 4 (D) 2 (E) 0
25. All of the elements of $\{125, 343, 729, 1331, 1728, 2197, 4913\}$ are considered to be ____ numbers.
- (A) equidigital (B) even (C) extravagant (D) evil (E) economical
26. Simplify: $\left(\frac{x^2 + 14x + 49}{x + 7}\right)\left(\frac{x^2 - 14x + 49}{x^2 - 49}\right)$
- (A) $x - 7$ (B) $x + 7$ (C) $x^2 + 49$ (D) $x^2 - 49$ (E) $x^2 + 7x - 7$
27. The dihedral angles of a tetrahedron are supplementary to the dihedral angles of a(n):
- (A) icosahedron (B) dodecahedron (C) octahedron (D) hexahedron (E) tetrahedron
28. If $a_1 = -1$, $a_2 = 0$, $a_3 = 1$ and $a_n = (a_{n-3})(a_{n-2}) + (a_{n-1})$ for $n \geq 4$, then a_9 equals:
- (A) 21 (B) 16 (C) 13 (D) 11 (E) 8
29. Using Pascal's triangle, determine the sum of the 3rd term and the 14th term of the 16th row.
- (A) 182 (B) 210 (C) 238 (D) 240 (E) 272
30. A(n) _____ number is an even positive integer greater than 2 can be expressed as a sum of two primes, e.g. $8 = 3 + 5$, $44 = 13 + 31$, $100 = 11 + 89$, ...
- (A) Archimedean (B) Porter (C) Goldbach (D) Germain (E) Euclidean
31. How many points of intersection occur when $r = 2\cos(\theta) + 1$ and $r = 2\sin(\theta)$ are graphed on a polar coordinate system where $0 \leq \theta \leq 2\pi$.
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
32. Justin Time can stuff 100 envelopes in 30 minutes working at a constant rate. He has to stuff 5000 envelopes in 1 hour. He hires enough people to complete the task on time. Since they all are working at the same constant rate, how many envelopes will each one stuff?
- (A) 50 (B) 75 (C) 125 (D) 200 (E) 300
33. P, Q, and R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is $2\frac{10}{13}$ and $PQR = -24$. Find C.
- (A) -26 (B) $-22\frac{2}{13}$ (C) $-21\frac{3}{13}$ (D) -9 (E) $-7\frac{1}{13}$

34. \overleftrightarrow{FG} is tangent to $\odot A$ at point C. Find $m\angle BCE$ if $m\angle DCB = 20^\circ$ and $\widehat{CE} = 80^\circ$.

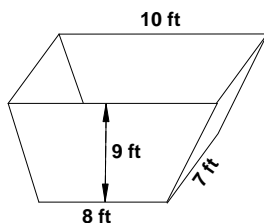


- (A) 30° (B) 35° (C) 40° (D) 45° (E) 50°
35. Which of the following is a true statement associated with the function $f(x) = \frac{\sin(x)}{x^2}$?
- (A) The function does not have a horizontal asymptote. (B) The graph is always concave up.
 (C) The graph is symmetric about the axis. (D) The function has a vertical asymptote at 0.
 (E) None of the above are true.
36. A tee bag contains 35 colored tees. The probability of randomly selecting a white tee is 60%. What is the least number of white tees needed to be added to the bag to change the probability of randomly selecting a white tee to 75%?

- (A) 5 (B) 14 (C) 15 (D) 21 (E) 28
37. Find k if $\text{GCF}(63, k) = 9$ and $\text{LCM}(63, k) = 252$.

- (A) 36 (B) 28 (C) 22 (D) 7 (E) 4
38. $\begin{array}{ccccccc} & P & -4 & Q & -1.125 & R & \\ & | & & | & & | & \\ <-----|-----|-----|-----|-----|-----> \end{array}$
- The distances between the hash marks (|) are equal. Find $P + Q + R$.
- (A) -12.8125 (B) -8.3125 (C) -7.6875 (D) -6.25 (E) -2.5625

39. Find the volume of this trapezoid bin.

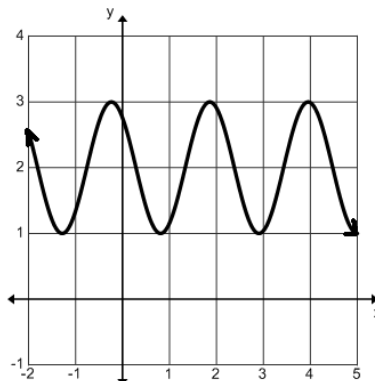


- (A) 504 cu. ft. (B) 567 cu. ft. (C) 587 cu. ft. (D) 630 cu. ft. (E) 729 cu. ft.
40. Which of the following is not a member of solution set of $|5x - 4| - 3 < 2$?
- (A) $-\frac{1}{9}$ (B) $\frac{2}{3}$ (C) 1.444... (D) $1\frac{8}{9}$ (E) 1.777...

41. The length of the latus rectum of $13x^2 - 3y^2 = 39$ is: (nearest tenth)

- (A) 15.0 (B) 8.7 (C) 2.9 (D) 0.9 (E) 0.3

42. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $2 + \cos(3x + 4)$ (B) $2 - \sin(3x + 4)$ (C) $2 - \cos(4x - 3)$
 (D) $2 + \cos(4x - 2)$ (E) $2 + \sin(3x - 4)$

43. Let $(2 + i)(3 + 4i) \div (7 + 11i) = a + bi$. Find $a + b$.

- (A) $\frac{17}{18}$ (B) $1\frac{2}{17}$ (C) $1\frac{5}{34}$ (D) $1\frac{9}{17}$ (E) $2\frac{1}{36}$

44. The series $\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots$ converges to _____.

- (A) ∞ (B) $\ln(2)$ (C) $\frac{\sqrt{5}+1}{2}$ (D) e (E) ψ

45. Three trillion five million two hundred-thousand four hundred one take away six billion forty million five hundred and three thousand seventy-eight equals K. The sum of the digits of K is?

- (A) 62 (B) 72 (C) 110 (D) 117 (E) 147

46. If $y = 1 - x$ and $xy = -2$ then $x^3 + y^3 = ?$

- (A) 17 (B) 11 (C) 7 (D) 5 (E) 1

47. Les Ismoor had 15 fluid ounces of a 75% solution of Argyrol in water. What is the greatest amount of a 67% solution of Argyrol in water can he add to make a final solution of at least 72% Argyrol?

- (A) 3 oz (B) 5 oz (C) 7.25 oz (D) 8 oz (E) 9 oz

48. The sum of the lengths of the edges of regular octahedron is 60 inches. The surface area of this octahedron is: (nearest square inch)

- (A) 390 sq. in. (B) 195 sq. in. (C) 173 sq. in. (D) 115 sq. in. (E) 87 sq. in.

49. Let $x^2 - xy + 2y^2 = 4$. Find $D_x y$.

- (A) $\frac{y-2x}{4y-x}$ (B) $2x + 4y$ (C) $\frac{4y+x}{y-2x}$ (D) $4x - 2y$ (E) $\frac{4y-x}{2x-y}$

50. The probability that statement P is true is $83\frac{1}{3}\%$, and the probability that statement Q is true is 25%. Determine the probability that $P \rightarrow Q$ is true. Statements P and Q are independent.

- (A) $12\frac{1}{2}\%$ (B) $20\frac{5}{6}\%$ (C) $37\frac{1}{2}\%$ (D) $41\frac{2}{3}\%$ (E) $62\frac{1}{2}\%$

51. Point P(2, 2) lies in the x-y plane. Point P is reflected across the origin to point Q. Point Q is translated vertically up 2 units and horizontally 3 units to the right to point R. Point R is rotated 270° clockwise about the origin to point S (x,y). Find $x + y$.

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

52. Let $P = 82$, $Q = 75$, and $R = 100$. Find the arithmetic mean of the geometric mean and the harmonic mean of P, Q, and R. (nearest hundredth)

- (A) 84.27 (B) 84.44 (C) 84.74 (D) 85.04 (E) 85.67

53. $789B410_{16} \div 5_{16} = \underline{\hspace{2cm}}_{16}$

- (A) 1573010 (B) 181F0D0 (C) 157F082 (D) 181B082 (E) 1578280

54. Let $f(x) = ax^5 - bx^3 + cx - 4$ where a, b, and c are constants. If $f(-4) = 2$ then $f(4) = ?$

- (A) 0 (B) -2 (C) -4 (D) -6 (E) -10

55. Determine the concavity of the graph of $f(x) = 3\sin(x) - 2\cos(x)$ at $x = \frac{5\pi}{4}$.

- (A) $-\frac{\sqrt{2}}{2}$ (B) $-\frac{5\sqrt{2}}{2}$ (C) $-\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{3\sqrt{2}}{2}$

56. Shirley Knott gave the same pop quiz to her three algebra classes. The first class averaged 80%, the second 85%, and the third 89%. Together the grades of the first two classes averaged 82%, while the grades of the second and third class averaged 87%. What was the average of the grades from all three classes?

- (A) 83% (B) 84% (C) 85% (D) 86% (E) 87%

57. Which of the following is equivalent to $\frac{1}{\sin(\theta)} - \frac{1}{\tan(\theta)}$

- (A) $\tan(\frac{\theta}{2})$ (B) $\cos(\frac{\theta}{2})$ (C) $\cot(\frac{\theta}{2})$ (D) $2\sin(\theta)\tan(\theta)$ (E) $\cos(\theta)\cot(\theta)$

58. Find the slope of the tangent line to $13x^2 - 3y^2 = 39$ at $x = 3$ and $y > 0$. (nearest hundredth)
- (A) 0.50 (B) 0.67 (C) 1.18 (D) 1.50 (E) 2.55
59. In the expansion of $(2x + 1)^6$, the sum of the coefficients of the 2nd, 4th, and 6th term is:
- (A) 382 (B) 729 (C) 225 (D) 198 (E) 364
60. Sam D. Seenyor and Willis A. Fressmann are taking a number sense test, a math test, and a calculator test at the TMSCA State meet. Sam is three times as likely to score higher on each test than Willis. What are the odds that Willis will score higher on at least one of the tests?
- (A) $\frac{2}{19}$ (B) $\frac{7}{57}$ (C) $\frac{27}{64}$ (D) $\frac{37}{27}$ (E) $\frac{16}{7}$

**University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2013
Answer Key**

- | | | |
|-------|-------|-------|
| 1. B | 21. E | 41. A |
| 2. D | 22. A | 42. E |
| 3. E | 23. C | 43. B |
| 4. C | 24. C | 44. D |
| 5. C | 25. E | 45. B |
| 6. B | 26. A | 46. C |
| 7. D | 27. C | 47. E |
| 8. D | 28. D | 48. E |
| 9. C | 29. B | 49. A |
| 10. B | 30. C | 50. C |
| 11. A | 31. D | 51. B |
| 12. E | 32. D | 52. C |
| 13. D | 33. A | 53. B |
| 14. B | 34. A | 54. E |
| 15. C | 35. D | 55. D |
| 16. C | 36. D | 56. B |
| 17. C | 37. A | 57. A |
| 18. A | 38. C | 58. D |
| 19. B | 39. B | 59. E |
| 20. C | 40. D | 60. D |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

State • 2013



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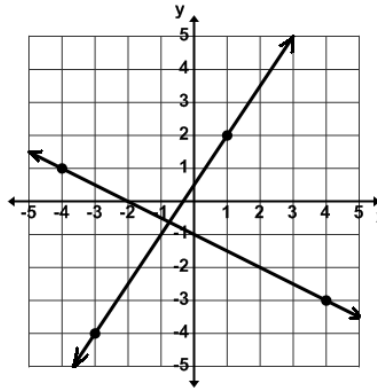
1. $(42 + 30) \div 12 - (18 - 6) \times 12 \div (-18) + (30 - 48)$

- (A) -14 (B) $-10.333...$ (C) -6 (D) -4 (E) $-0.777...$

2. Robin Banks teaches an accounting class. She has 30 students in the class. 22 students use scientific calculators. 11 use scientific calculators and graphing calculators. 5 don't use scientific or graphing calculators. How many students use just one of the two types of calculators?

- (A) 17 (B) 16 (C) 14 (D) 11 (E) 3

3. The two lines shown intersect at the point (x,y) . Find $x + y$.



- (A) -1.375 (B) -1.5 (C) -1.625 (D) -1.25 (E) -1.125

4. Simplify: $\left(\frac{3 + 4x - 4x^2}{9 - 4x^2}\right) \left(\frac{9 + 3x - 2x^2}{2x^2 - 5x - 3}\right)$

- (A) $x + 3$ (B) $\frac{3+x}{3-x}$ (C) -1 (D) $\frac{-12x}{5}$ (E) $3 - x$

5. If $2x + 3 = 7$ and $7 = 5y - 3$ then $2x + 3 = 5y - 3$ is an example of the ____ property of equality.

- (A) algebraic (B) transitive (C) identity (D) distributive (E) symmetric

6. Simplify: $a^{-2} \times b^{-1} \div a^{-3} \times b^3 \div a \times b^2$

- (A) $a^{-4}b^{-2}$ (B) ab^4 (C) $a^{-5}b^4$ (D) ab^{-6} (E) b^4

7. If y varies directly to $x + 2$, and $y = 14$ when $x = 3$, find x if $y = 21$.

- (A) 1.5 (B) 3 (C) 3.5 (D) 5.5 (E) 7

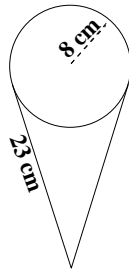
8. If $\frac{7}{4x} - \frac{5y}{6} = \frac{8}{3x}$, then y^{-1} equals _____.

- (A) $-\frac{11}{10x}$ (B) $-4.8x$ (C) $-6\frac{6}{55x}$ (D) $-2.1x$ (E) $-\frac{10x}{11}$

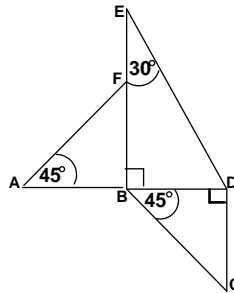
9. If $a_1 = -2$, $a_2 = -1$, $a_3 = 1$ and $a_n = (a_{n-3})^{(a_{n-2})} - (a_{n-1})$ for $n \geq 4$, then a_6 equals:

- (A) $1\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) $-\frac{2}{3}$

10. Given the radius shown find the volume of the right circular cone. (nearest cu. cm.)



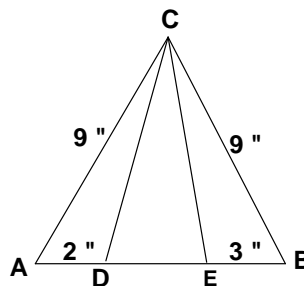
- (A) $1,734 \text{ cm}^3$ (B) $1,541 \text{ cm}^3$ (C) $1,445 \text{ cm}^3$ (D) $1,156 \text{ cm}^3$ (E) $1,084 \text{ cm}^3$
11. A rectangular prism water tank is 3 feet high, 2 feet wide, and 4 feet long. It is 75% full. How many gallons of water would it take to fill the tank to the top? (nearest gallon)
- (A) 18 gal (B) 22 gal (C) 36 gal (D) 45 gal (E) 67
12. Let $f(x) = 6x - 5$ and $g(x) = 4x - 3$ and $h(x) = 2x - 1$. Find the constant term of $h(f(g(x + 1)))$.
- (A) -9 (B) -7 (C) -3 (D) -1 (E) 1
13. Find the perimeter of the object shown if $EB = 6\sqrt{3}$ ", and $AF = 4\sqrt{2}$ ". (nearest half inch).



- (A) 2 ft 11 in (B) 3 ft 3.5 in (C) 3 ft 5.5 in (D) 3 ft 6.5 in (E) 4 ft 1.5 in
14. The incenter, circumcenter, orthocenter, and centroid of a triangle are collinear only if the triangle is a(n) _____ triangle.
- (A) isosceles (B) equilateral (C) scalene right (D) scalene obtuse (E) scalene acute
15. Let $f(x) = \frac{3+4x}{4x-3}$. Find $f^{-1}(\frac{1}{2})$.
- (A) -0.375 (B) -2.25 (C) -5 (D) 3.5 (E) 2.666...
16. Two numbers are in the ratio 2:3. If the smaller number is increased by 8 and the larger number is decreased by 4, the resulting numbers are in the ratio 10:7. Find the product of the numbers.
- (A) 120 (B) 132 (C) 192 (D) 216 (E) 252

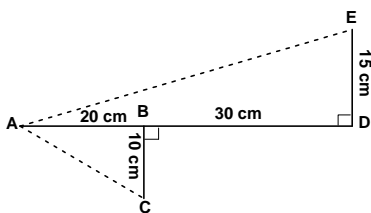
17. Captain I. C. Delite looks at his navigation chart and determines that lighthouse A is 60 miles from lighthouse B and the bearing of B from A is 100° . He calculates the bearing of lighthouse A from the ship to be 50° and the bearing of lighthouse B from the ship to be 75° . How much further is the ship from lighthouse B than from lighthouse A? (nearest tenth)
- (A) 42.9 mi (B) 48.8 mi (C) 55.0 mi (D) 71.2 mi (E) 77.1 mi
18. Simplify: $\frac{\cos(\theta) - \csc(\theta)}{\sin(\theta) - \sec(\theta)}$
- (A) $\tan(\theta)$ (B) $\sin(2\theta)$ (C) -1 (D) $\sin(\theta)\cos(\theta)$ (E) $\cot(\theta)$
19. Given the polynomial, $8x^6 - 3x^5 - 2x^3 + x^2 - 3x + 8$, how many negative real zeros are possible?
- (A) 4, 2, or 0 (B) 3 or 1 (C) 5, 3, or 1 (D) 2 (E) 0
20. Which of the following functions is neither an even nor an odd function? $f(x) =$
- (A) $2|x|$ (B) $\frac{x^3 - x}{x^5 + x}$ (C) $4x^4 + 2x^2 + x$ (D) $\cos(x)$ (E) $\sqrt{3 - x^2}$
21. If $f'(x) = 6(x^2 - x - 1)$ and $f(1) = -16$, find $f(-1)$.
- (A) -14 (B) -10 (C) -9 (D) -8 (E) -6
22. The series $\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$ converges to _____.
- (A) ϕ (B) $\frac{\pi^2}{15}$ (C) $\ln(2)$ (D) ∞ (E) e
23. Let $f(x) = \frac{x^2 - 2x + 1}{x^2 + 2x + 1}$. Find $f'(3)$.
- (A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$ (E) $\frac{3}{8}$
24. Betty Luzes puts nine face cards, four aces, two jokers, and a blank card in a container. She shakes the container then randomly selects three cards placing them face up in a row in the order they were drawn. What is the probability that all three cards are face cards? (nearest %)
- (A) 15% (B) 56% (C) 19% (D) 53% (E) 12%
25. The probability of selecting a prime number from a set containing 2-digit numbers is 16%. What are the odds of selecting a composite number from the set?
- (A) 4 to 25 (B) 21 to 25 (C) 4 to 21 (D) 25 to 21 (E) 21 to 4
26. Using Pascal's triangle, determine which of the following is true about all of the numbers greater than P in row P, where P is a prime number greater than 3? All are _____.
- (A) even (B) odd (C) prime (D) divisible by P (E) triangular numbers

27. A special abacus using an index rod and a set of ten rods corresponding to the digits 0 to 9 was developed for calculating products and quotients. This special abacus is based on lattice multiplication and is known as _____.
- (A) Napier's Bones (B) Erastosthenes' sieve (C) Venn's diagrams
- (D) Descartes' Cartesian System (E) Archimedes' spiral
28. Let $Y = X + X + 1 + X + 2 + \dots + X + N$, where X and N are positive integers. Y is considered to be a _____ number.
- (A) complex (B) Lucas (C) polite (D) lucky (E) Fibonacci
29. The *Lookin Gud* dress shop sold two special prom dresses for \$75.00 each. One dress was sold for a 25% profit and the other sold for a 20% loss. What was the shop's net gain or loss for the sale of the two dresses?
- (A) gained 7.5¢ (B) gained \$1.58 (C) broke even (D) lost \$3.75 (E) lost \$7.50
30. The average age of the contestants and coaches at the state math contest is 25. If the average age of the contestants is 17 and the average age of the coaches is 39, what is the ratio of coaches to students?
- (A) 4:11 (B) 4:3 (C) 7:11 (D) 3:4 (E) 4:7
31. $\triangle ABC$ is an equilateral triangle. Find the perimeter of $\triangle CDE$. (nearest inch)



- (A) 18 " (B) 19 " (C) 20 " (D) 21 " (E) 22 "
32. Saul T. Bryne has 75 grams of a 4% salt solution. How many grams of salt would he have to add to change his solution to a 10% salt solution?
- (A) 3 grams (B) 3.5 grams (C) 5 grams (D) 6 grams (E) 7.5 grams
33. Which of the following equations will have a graph with an amplitude of 3, a frequency of 3, a phase shift of 3, and a displacement of -3 ?
- (A) $y = 3 - 3\sin(6\pi x - 3)$ (B) $y = 3\sin(\frac{3\pi}{2} - 18) - 3$ (C) $y = 3\sin(6\pi x + 18) + 3$
- (D) $y = 3\sin(3\pi x - 3) - 3$ (E) $y = 3\sin(6\pi x - 18) - 3$

34. Find $m\angle CAE$. (nearest degree).



- (A) 29° (B) 30° (C) 43° (D) 47° (E) 53°

35. Three workers can wash 5 PT Cruisers in 2 hours 15 minutes. If 5 more workers are hired how long would it take the 8 workers to wash 30 PT Cruisers if they all work at the same rate as the original 3 workers? (nearest minute)

- (A) 5 hrs 4 min (B) 5 hrs 21 min (C) 5 hrs 38 min (D) 6 hrs (E) 6 hrs 6 min

36. Let $f(x) = \begin{cases} 2 & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ -3 & \text{if } 1 < x. \end{cases}$ Which of the following is/are true?

1. $\lim_{x \rightarrow 1^+} f(x) = -3$ 2. $\lim_{x \rightarrow 1^-} f(x) = 2$ 3. $\lim_{x \rightarrow 1} f(x)$ does not exist

- (A) none of these (B) 1 & 2 (C) 3 only (D) 1 & 3 (E) 1, 2, & 3

37. The *I. C. Delite* packaging company has 40-watt bulbs, 60-watt bulbs, 75-watt bulbs and 100-watt bulbs. In how many ways can they package a 12-pack of bulbs if each 12-pack must contain at least one of each size bulb?

- (A) 70 (B) 140 (C) 165 (D) 275 (E) 495

38. The enrollment at Millersview Tech consists of 12 seniors, 11 juniors, 10 sophomores, and 9 freshmen. How many ways can Master Wyte form a Super Nerd committee consisting of 4 seniors, 3 juniors, 2 sophomores and 1 freshman?

- (A) 33,078,375 (B) 420 (C) 1,471,442,973 (D) 12,969 (E) 714

39. Find the harmonic mean of the real roots of $x^3 - 16x^2 + 73x - 90 = 0$. (nearest tenth)

- (A) 6.1 (B) 5.3 (C) 4.5 (D) 4.0 (E) 3.7

40. If the roots of $x^3 + bx^2 + cx + d = 0$ are -2 , 3 , and 5 , then $c - b - d$ equals:

- (A) -25 (B) -23 (C) 0 (D) 35 (E) 37

41. A regular octahedron has F faces, E edges, and V vertices. Find $F + E + V$.

- (A) 14 (B) 24 (C) 26 (D) 32 (E) 36

42. The Real value solution set for $|6x + 2| > 4$ is?

- (A) $\{x \mid -\frac{1}{3} < x < 1\}$ (B) $\{x \mid \{x < -1\} \cup \{x > \frac{1}{3}\}\}$ (C) $\{x \mid \{x > \frac{1}{3}\} \cup \{x < 1\}\}$
(D) $\{x \mid -1 < x < \frac{1}{3}\}$ (E) $\{x \mid \{x < -\frac{1}{3}\} \cup \{x > 1\}\}$

43. If $8^{(x+2y)} = 64$ and $9^{(2x-y)} = 81$ then $x + y$ equals _____.

- (A) 1.6 (B) 0.48 (C) 2.0 (D) 2.4 (E) 0.8

44. $(2 + 3i)^2 - (3 + 2i)^2 = a + bi$. Find $(a + b)^2$.

- (A) 100 (B) 121 (C) 125 (D) 144 (E) 169

45. Find the sum of the first three terms of the arithmetic sequence $x^2, 2x + 5, x, \dots$, where $x > 0$.

- (A) 45 (B) 40 (C) 30 (D) 25 (E) 15

46. Use the Fibonacci characteristic sequence $\dots -4, p, q, r, 13 \dots$ to Find $p + q + r$.

- (A) 21 (B) 20 (C) 19 (D) 18 (E) 17

47. Find the surface area of an icosahedron with an edge length of 5". (nearest sq. in)

- (A) 87 sq. in (B) 130 sq. in (C) 173 sq. in (D) 217 sq. in (E) 344 sq. in

48. $11_{16} \times (ABC_{16} - 321_{16}) = \underline{\hspace{2cm}}_{16}$

- (A) 8811 (B) 7F4B (C) 814B (D) 8181 (E) 884B

49. Determine the number of non-negative integer solutions to the equation: $p + q + r = 9$.

- (A) 27 (B) 36 (C) 45 (D) 55 (E) 66

50. $F(x) = 2x^2 - x^4$ has an inflection point at (x, y) . Find y .

- (A) $\frac{\sqrt{3}}{3}$ (B) 1 (C) 0 (D) $\frac{5}{9}$ (E) $\sqrt{3}$

51. Find the area of the region bounded between the graphs of $y = 0$ and $y = x(x^2 - 1)$ for $-1 \leq x \leq 0$, and between the graphs of $y = 0$ and $y = x(x^2 - 1)$ for $0 \leq x \leq 1$.

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) $\frac{1}{4}$ (E) 0

52. The polar graph of $r = 2\cos(2\theta)$ is symmetric to: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$

- (A) 1 only (B) 2 only (C) 3 only (D) 2 & 3 (E) 1, 2, & 3

53. If R, S, and T represent digits then $TR\overline{S}_3 - STR\overline{4}_4 + RST\overline{5}_5$ has a numeric value in base 10 of:

(A) $27R - 10S + 6T$

(B) $3T - 12R + 2S$

(C) $25R - 4T + 5S$

(D) $4R + 4S + 4T$

(E) $29R - 10S + 14T$

54. \overleftrightarrow{AB} is tangent to a semicircle at point C. Points P, Q, and R lie on the semicircle such that \overline{CP} is the diameter, $m\angle PCQ = 33^\circ$, $m\angle QCR = 35^\circ$. Find \widehat{CR} .

(A) 11°

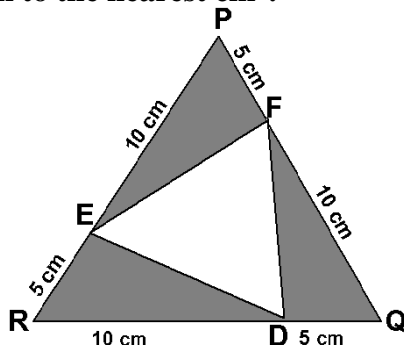
(B) 22°

(C) 32°

(D) 38°

(E) 44°

55. Find the area of the shaded region to the nearest cm^2 .



(A) 96 cm^2

(B) 84 cm^2

(C) 75 cm^2

(D) 65 cm^2

(E) 32 cm^2

56. Let $A = \begin{bmatrix} 5 & -2 \\ 6 & 3 \end{bmatrix}$. Find $\det(A + A^T)$.

(A) 6

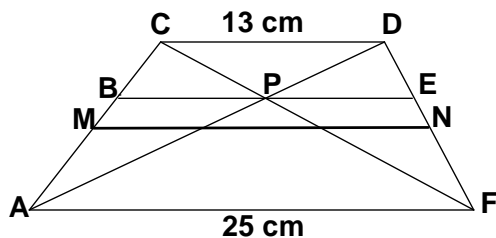
(B) 9

(C) 27

(D) 44

(E) 54

57. Drew Tumeens sketched the trapezoid shown where segments AF, MN, BE, and CD are parallel to each other, segments AD, CF, and BP are concurrent at point P, and M and N are midpoints of segments AC and DF, respectively. Find $MN - BE$. (nearest tenth)



(A) 1.3 cm

(B) 1.5

(C) 1.7 cm

(D) 1.9 cm

(E) 2.1 cm

58. The curve $y = 4x^3 - 13x^2 + 4x - 3$ has two horizontal tangents. Find the shortest distance between the two horizontal tangents.

(A) $12\frac{35}{108}$

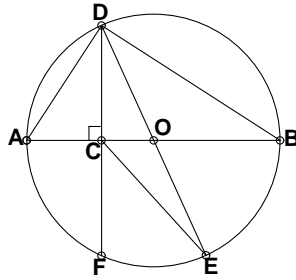
(B) 13

(C) $14\frac{145}{216}$

(D) 15

(E) $17\frac{73}{108}$

59. I. M. Daboss drew a circle with center O with $BC = 2AC$, $CD = 8\sqrt{2}$ cm, and $AC = 8$ cm. Based on the information and the drawing what is the area of $\triangle CEO$? (nearest tenth)



- (A) 11.3 sq. cm (B) 22.6 sq. cm (C) 32.0 sq. cm (D) 33.9 sq. cm (E) 45.3 sq. cm
60. The coordinates of the vertices of $\triangle DEF$ are $(-1, 1)$, $(2, -2)$, and $(3, 3)$. The coordinates of the incenter is (x, y) . Find $x + y$. (nearest tenth)
- (A) 2.5 (B) 2.0 (C) 1.8 (D) 1.5 (E) 1.2

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- | | | |
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| 5. B | 25. E | 45. A |
| 6. E | 26. D | 46. B |
| 7. D | 27. A | 47. D |
| 8. E | 28. C | 48. C |
| 9. B | 29. D | 49. D |
| 10. C | 30. E | 50. D |
| 11. D | 31. C | 51. B |
| 12. E | 32. C | 52. E |
| 13. D | 33. E | 53. A |
| 14. A | 34. C | 54. E |
| 15. B | 35. A | 55. D |
| 16. D | 36. E | 56. D |
| 17. B | 37. C | 57. D |
| 18. E | 38. A | 58. A |
| 19. E | 39. E | 59. B |
| 20. C | 40. A | 60. C |