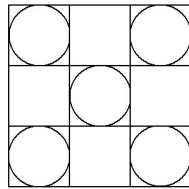# Problem 1

Last month a pet store sold three times as many cats as dogs. If the store had sold the same number of cats but eight more dogs, it would have sold twice as many cats as dogs. How many cats did the pet store sell last month?

Problem 2

What is the greatest three-digit divisor of 111777?

Problem 3

The diagram below shows a large square divided into nine congruent smaller squares. There are circles inscribed in five of the smaller squares. The total area covered by all the five circles is 20π. Find the area of the large square.

Problem 4

The following diagram shows an equilateral triangle and two squares that share common edges. The area of each square is 75. Find the distance from point A to point B.
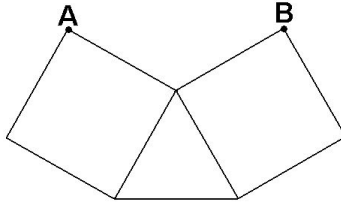
Problem 5

Find the sum of the squares of the values x that satisfy $\frac{1}{x} + \frac{2}{x+3} + \frac{3}{x+6} = 1$.

Problem 6

Find the least positive integer n so that both n and $n+1$ have prime factorizations with exactly four (not necessarily distinct) prime factors.

Problem 7

Two convex polygons have a total of 33 sides and 243 diagonals. Find the number of diagonals in the polygon with the greater number of sides.

Problem 8

In the tribe of Zimmer, being able to hike long distances and knowing the roads through the forest are both extremely important, so a boy who reaches the age of manhood is not designated as a man by the tribe until he completes an interesting rite of passage. The man must go on a sequence of hikes. The first hike is a 5 kilometer hike down the main road. The second hike is a $5\frac{1}{4}$ kilometer hike down a secondary road. Each hike goes down a different road and is a quarter kilometer longer than the previous hike. The rite of passage is completed at the end of the hike where the cumulative distance walked by the man on all his hikes exceeds 1000 kilometers. So in the tribe of Zimmer, how many roads must a man walk down, before you call him a man?

Problem 9

Find the value of x that satisfies $\log_3(\log_9 x) = \log_9(\log_3 x)$.

Problem 10

Consider a sequence of eleven squares that have side lengths $3, 6, 9, 12, \ldots, 33$. Eleven copies of a single square each with area A have the same total area as the total area of the eleven squares of the sequence. Find A.

Problem 11

Define $f(x) = 2x + 3$ and suppose that $g(x + 2) = f\left(f(x - 1) \cdot f(x + 1) + f(x)\right)$. Find $g(6)$.

Problem 12

Ted flips seven fair coins. There are relatively prime positive integers m and n so that $\frac{m}{n}$ is the probability that Ted flips at least two heads given that he flips at least three tails. Find $m + n$.

Problem 13

Find the least n for which $n!(n + 1)!(2n + 1)! - 1$ ends in thirty digits that are all 9's.

Problem 14

A circle in the first quadrant with center on the curve $y = 2x^2 - 27$ is tangent to the y-axis and the line $4x = 3y$. The radius of the circle is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Problem 15

Let N be a positive integer whose digits add up to 23. What is the greatest possible product the digits of N can have?

Problem 16

Let a, b, and c be non-zero real numbers such that $\frac{ab}{a+b} = 3$, $\frac{bc}{b+c} = 4$, and $\frac{ca}{c+a} = 5$. There are relatively prime positive integers m and n so that $\frac{abc}{ab+bc+ca} = \frac{m}{n}$. Find $m + n$.

Problem 17

How many positive integer solutions are there to $w + x + y + z = 20$ where $w + x \geq 5$ and $y + z \geq 5$?

Problem 18

Find the number of three-digit numbers such that its first two digits are each divisible by its third digit.

Problem 19

Find the remainder when $2^{5^9} + 5^{9^2} + 9^{2^5}$ is divided by 11.

Problem 20

Square $ABCD$ has side length 68. Let E be the midpoint of segment $\overline{CD}$, and let F be the point on segment $\overline{AB}$ a distance 17 from point A. Point G is on segment $\overline{EF}$ so that $\overline{EF}$ is perpendicular to segment $\overline{GD}$. The length of segment $\overline{BG}$ can be written as $m\sqrt{n}$ where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.

Problem 21

Each time you click a toggle switch, the switch either turns from *off* to *on* or from *on* to *off*. Suppose that you start with three toggle switches with one of them *on* and two of them *off*. On each move you randomly select one of the three switches and click it. Let m and n be relatively prime positive integers so that $\frac{m}{n}$ is the probability that after four such clicks, one switch will be *on* and two of them will be *off*. Find $m + n$.

Problem 22

The diagram below shows circles radius 1 and 2 externally tangent to each other and internally tangent to a circle radius 3. There are relatively prime positive integers m and n so that a circle radius $\frac{m}{n}$ is internally tangent to the circle radius 3 and externally tangent to the other two circles as shown. Find $m + n$.
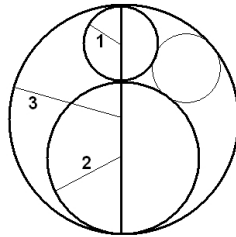
Problem 23

Find the greatest seven-digit integer divisible by 132 whose digits, in order, are 2, 0, x, y, 1, 2, z where x, y, and z are single digits.
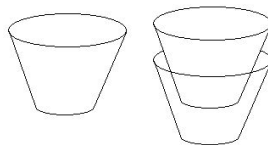
Problem 24

There are positive integers m and n so that $x = m + \sqrt{n}$ is a solution to the equation $x^2 - 10x + 1 = \sqrt{x}(x + 1)$. Find $m + n$.

Problem 25

Find the largest prime that divides $1 \cdot 2 \cdot 3 \ + \ 2 \cdot 3 \cdot 4 \ + \ ... \ + \ 44 \cdot 45 \cdot 46$.

Problem 26

A paper cup has a base that is a circle with radius r, a top that is a circle with radius $2r$, and sides that connect the two circles with straight line segments as shown below. This cup has height h and volume V. A second cup that is exactly the same shape as the first is held upright inside the first cup so that its base is a distance of $\frac{h}{2}$ from the base of the first cup. The volume of liquid that will fit inside the first cup and outside the second cup can be written $\frac{m}{n} \cdot V$ where m and n are relatively prime positive integers. Find $m + n$.

Problem 27

You have some white one-by-one tiles and some black and white two-by-one tiles as shown below. There are four different color patterns that can be generated when using these tiles to cover a three-by-one rectangle by laying these tiles side by side (WWW, BWW, WBW, WWB). How many different color patterns can be generated when using these tiles to cover a ten-by-one rectangle?

Problem 28

A bag contains 8 green candies and 4 red candies. You randomly select one candy at a time to eat. If you eat five candies, there are relatively prime positive integers m and n so that $\frac{m}{n}$ is the probability that you do not eat a green candy after you eat a red candy. Find $m + n$.

Problem 29

Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$. Let f be a randomly chosen function from the set $A \cup B$ into itself. There are relatively prime positive integers m and n such that $\frac{m}{n}$ is the probability that f is a one-to-one function on $A \cup B$ given that it maps A one-to-one into $A \cup B$ and it maps B one-to-one into $A \cup B$. Find $m + n$.

Problem 30

The diagram below shows four regular hexagons each with side length 1 meter attached to the sides of a square. This figure is drawn onto a thin sheet of metal and cut out. The hexagons are then bent upward along the sides of the square so that A_1 meets A_2, B_1 meets B_2, C_1 meets C_2, and D_1 meets D_2. The resulting dish is set on a table with the square lying flat on the table. If this dish is filled with water, the water will rise to the height of the corner where the A_1 and A_2 meet. There are relatively prime positive integers m and n so that the number of cubic meters of water the dish will hold is $\sqrt{\frac{m}{n}}$. Find $m + n$.