UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2023

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1. **On his birthday, Darius received a $250 gift card from Academy Sports in Groom. He purchased Pegasus running shoes for $128.00, a Nike running shirt for $24.95, Nike running shorts for $33.85, and Nike socks for $6.95. If the tax rate is 8.25%, how much is left on his gift card?**

   (A) $40.15 **(B) $40.19** **(C) $40.23** **(D) $40.27** **(E) $40.31**

2. **Consider line $\overleftrightarrow{AB}$ with points $A(8, 6)$ and $B(-6, -10)$. If the point $(1, b)$ lies on $\overleftrightarrow{AB}$, then b = ___.**

   (A) -2 **(B) $-\frac{12}{7}$** **(C) $-\frac{10}{7}$** **(D) $-\frac{8}{7}$** **(E) $-\frac{6}{7}$**

3. **Chay has a jar full of nickels, dimes and quarters. The jar has a total of 264 coins with a value of $37.20. There are 20 more quarters than nickels in the jar. How many dimes are in the jar?**

   (A) 83 **(B) 88** **(C) 93** **(D) 98** **(E) 103**

4. **Which of the following values is not in the solution to $3 \le x + 7 < 12$?**

   (A) -4 **(B) -1** **(C) 1** **(D) 4** **(E) 5**

5. **Samuel is solving the quadratic equation $x^2 + 10x - 6 = 0$ by completing the square. His second step is $x^2 + 10x + c = 6 + c$. The value of c is _____.**

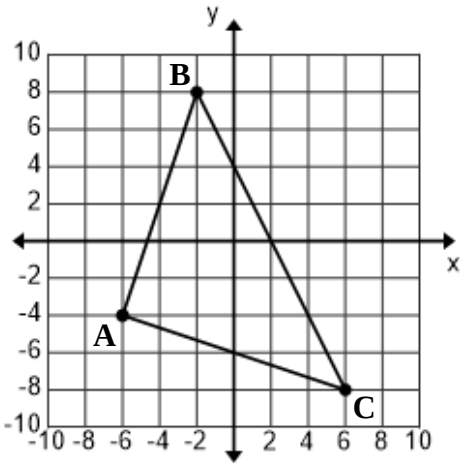
   (A) 5 **(B) 6** **(C) 10** **(D) 12** **(E) 25**

6. **The perimeter of ΔABC shown on the right is ____. (nearest tenth)**

   (A) 42.6 **(B) 42.9** **(C) 43.2** **(D) 43.5** **(E) 43.8**

7. **The coordinates of the midpoint of $\overline{BC}$ are (a, b).**
   $a + b$ = _____. (nearest tenth)

   (A) 2.0 **(B) 2.2** **(C) 2.4** **(D) 2.6** **(E) 2.8**



Problems 6, 7, 8, 9

8. **The area of ΔABC is _____. (nearest tenth)**

   (A) 78.8 **(B) 79.2** **(C) 79.6** **(D) 80.0** **(E) 80.4**

9. **Point D (not shown) lies on $\overline{AC}$ such that $\overrightarrow{BD}$ bisects $\angle ABC$. DC = _____. (nearest tenth)**

   (A) 6.6 **(B) 6.8** **(C) 7.0** **(D) 7.2** **(E) 7.4**

10. Four times the complement of $\angle A$ is 36° greater than the supplement of $\angle A$. $m\angle A =$ _____.

(A) 44° (B) 46° (C) 48° (D) 50° (E) 52°

Problems 11-12. The base of a pyramid is a square with each side equal to 14 cm. The height is 10 cm.

11. The volume of the pyramid is _____. (nearest whole number)

(A) 650 cm^3 (B) 653 cm^3 (C) 656 cm^3 (D) 659 cm^3 (E) 662 cm^3

12. The total surface area of the pyramid is _____. (nearest whole number)

(A) 522 cm^2 (B) 526 cm^2 (C) 530 cm^2 (D) 534 cm^2 (E) 538 cm^2

13. The volume of a cone is 768π cm^3 and the height of the cone is 12 cm. What is the diameter of the circular base? (nearest whole number)

(A) 26.9 cm (B) 27.3 cm (C) 27.7 cm (D) 28.1 cm (E) 28.5 cm

14. The hypotenuse of an isosceles right triangle is 24.0416. The area of the triangle is _____. (nearest tenth)

(A) 144.5 (B) 145.6 (C) 146.7 (D) 147.8 (E) 148.9

15. Jen can mow 3 large yards in 8 hr. Tom can mow 5 large yards in 16 hr. If they work together, how long would it take them to mow 12 large yards? (nearest minute)

(A) 16 hr 35 min (B) 16 hr 48 min (C) 17 hr 1 min (D) 17 hr 14 min (E) 17 hr 27 min

16. Find the number that is $\frac{5}{12}$ of the way from $-3\frac{1}{3}$ to $8\frac{1}{2}$.

(A) $\frac{37}{24}$ (B) $\frac{113}{72}$ (C) $\frac{115}{72}$ (D) $\frac{13}{8}$ (E) $\frac{119}{72}$

17. If $f(x) = x^2$, then $\frac{f(x+h)-f(x)}{h} =$ _____.

(A) 2x (B) 2x + h (C) $\frac{2x}{h}$ (D) $x^2 + h$ (E) 2x + 2h

18. Dad's age is two more than three times Abe's age and Dad's age is eight more than twice Connie's age. Connie is six years older than Abe. What is the sum of their ages?

(A) 90 (B) 92 (C) 94 (D) 96 (E) 98

19. Find the total number of distinct diagonals that can be drawn from the vertices of a regular decagon?

(A) 24 **(B) 32** **(C) 35** **(D) 36** **(E) 42**

20. Consider $\triangle ABC$ with point D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. If $m\angle ABC = 90°$, $AD = 10.8$ and $DC = 19.2$, then $BD =$ _____. (nearest tenth)

(A) 14.0 **(B) 14.2** **(C) 14.4** **(D) 14.6** **(E) 14.8**

21. Find the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 25}}{|x - 6|}$.

(A) $(-\infty, -5] \cup [5, \infty)$ **(B)** $(-\infty, -5) \cup (5, 6) \cup (6, \infty)$ **(C)** $(-\infty, -5) \cup (5, \infty)$

(D) $(-\infty, \infty)$ **(E)** $(-\infty, -5] \cup [5, 6) \cup (6, \infty)$

22. How many distinguishable permutations can be formed using the letters from the word Massachusetts?

(A) 64,864,800 **(B) 129,729,600** **(C) 259,459,200** **(D) 518,918,400** **(E) 1,037,836,800**

23. Consider the circle $x^2 + y^2 - 8x + 10y - 8 = 0$. The area of the circle is _____. (nearest whole number)

(A) 154 **(B) 156** **(C) 158** **(D) 160** **(E) 162**

24. $\sin(x)\tan\left(\dfrac{\pi}{2} - x\right) =$ _____.

(A) $\cos(x)$ **(B)** $\sin(2x)$ **(C)** $\tan(x)$ **(D)** $-\tan(x)$ **(E)** $-\cos(x)$

25. Consider a geometric sequence in which the first term is $22\frac{1}{2}$ and the fifth term is $4\frac{4}{9}$. What is the seventh term of the sequence?

(A) $\frac{52}{27}$ **(B)** $\frac{158}{81}$ **(C)** $\frac{160}{81}$ **(D) 2** **(E)** $\frac{164}{81}$

26. Find the eccentricity of the ellipse. $16x^2 + 25y^2 - 128x + 150y + 81 = 0$. (nearest hundredth)

(A) 0.60 **(B) 0.67** **(C) 0.75** **(D) 0.80** **(E) 0.83**

27. The pressure of an ideal gas varies directly with the temperature and inversely with the volume. The values of the initial state were 2.0 atm, 6.0 L, and 300 K. What will the pressure be in the final state if the volume is reduced to 2.0 L and the temperature is increased to 500 K? (nearest tenth)

(A) 9.8 atm (B) 10.0 atm (C) 10.2 atm (D) 10.4 atm (E) 10.6 atm

28. The value of my RAV4 Prime (plug-in hybrid) depreciates exponentially. I originally paid $55,200 on July 8, 2022. On July 8, 2025, the value had fallen to $51,500. If the value continues to depreciate exponentially, what is the expected value of my RAV4 Prime on July 8, 2030?

(A) $45,416.16 (B) $45,531.17 (C) $45,646.18 (D) $45,761.19 (E) $45,876.20

29. Consider the baseball diamond at SHS. Home plate and the three bases are located at the vertices of a square with the length of each side of the square being 90 ft. The pitcher's mound lies on a straight line from home plate to second base and it is located 60 ft 6 in from home plate. How far is it from the pitcher's mound to first base? (nearest tenth)

(A) 63.7 ft (B) 63.9 ft (C) 64.1 ft (D) 64.3 ft (E) 64.5 ft

30. At the Moulton Fall Festival, cash prizes were awarded for the top twenty places in the peach pie contest. First place received $500, second place received $475, third place received $450, fourth placed received $425 and so on. What was the total amount of prize money awarded?

(A) $5200 (B) $5225 (C) $5250 (D) $5275 (E) $5300

31. Six couples plan to attend a concert and sit in the same row. Each row has 12 seats. If the two members of each couple are to sit together, how many different seating arrangements are possible?

(A) 24,060 (B) 36,060 (C) 42,060 (D) 46,080 (E) 48,040

32. The point $(-7, b)$ lies of the curve defined by the parametric equations $\begin{matrix} x = t - 5 \\ y = t^2 \end{matrix}$. b = _____.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

33. Convert the rectangular equation $2x - 3y - 5 = 0$ to polar form.

(A) $r = \dfrac{5}{\cos^2\theta - \sin^3\theta}$ (B) $r = \dfrac{5}{2\cos\theta - 3\sin\theta}$ (C) $r = \dfrac{5}{2\sin\theta - 3\cos\theta}$

(D) $r = 5\sec(\theta)$ (E) $r = 5\csc(\theta)$

34. Find the perimeter of the triangle with vertices $(4, -5, 6)$, $(3, -2, -1)$ and $(-5, 7, -9)$. (nearest tenth)

(A) 43.4 (B) 43.6 (C) 43.8 (D) 44.0 (E) 44.2

35. Three of the zeros of $f(x) = x^4 + bx^3 + cx^2 + dx + 12 = 0$ are $-2, 3, 1+\sqrt{3}$. $f(2) =$ _____.

(A) –8 (B) –4 (C) 0 (D) 4 (E) 8

36. Consider the graph of $f(x) = 3 - 2\cot\left(\frac{4\pi x}{3} + \frac{\pi}{6}\right)$. The period of the graph is _____.

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

37. Assume that the earth rotates about its axis once every 23 hr 56 min 3.72 sec. Find the linear speed of a person sitting in a chair on the equator. The radius of the earth is 3960 miles. (nearest tenth)

(A) 1037.4 mph (B) 1038.5 mph (C) 1039.6 mph (D) 1040.7 mph (E) 1041.8 mph

38. The graph of $r = 12\cos(8\theta)$ has _____ rose petals.

(A) 6 (B) 8 (C) 12 (D) 16 (E) 24

39. On January 1, 2020, Lily placed $15,000 into account A where it earns 7% annual interest compounded semiannually. On the same day, she also placed $14,500 into account B that earns 7.5% annual interest compounded monthly. Of the following choices, which is the earliest day in which the balance in account B exceeds the balance in account A?

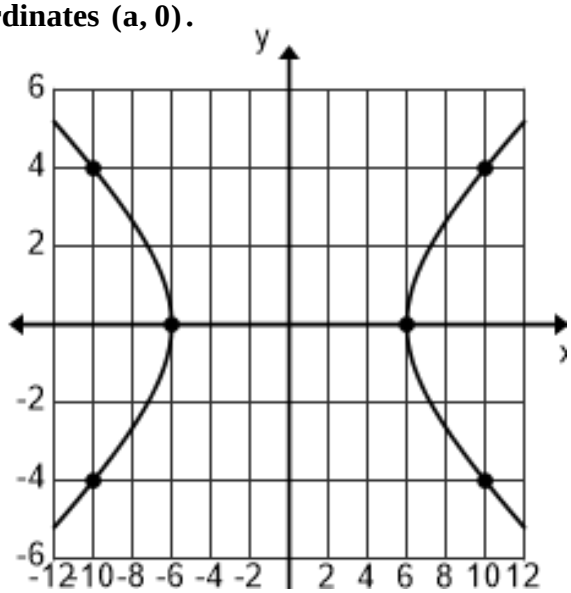
(A) July 10, 2025 (B) Sept 10, 2025 (C) Nov 10, 2025 (D) Jan 10, 2026 (E) March 10, 2026

40. One of the foci of the hyperbola shown on the right has coordinates (a, 0). a = _____. (nearest tenth)

(A) 6.5
(B) 6.7
(C) 6.9
(D) 7.1
(E) 7.3

41. Find the area bounded by the graph of the right branch of the hyperbola and the line x = 10. (nearest tenth)

(A) 20.2
(B) 20.4
(C) 20.6
(D) 20.8
(E) 21.0



Problems 40, 41

42. A 50-foot ladder is leaning against a building. If the base of the ladder is being pulled away from the building at a rate of 4 feet per second, then the top of the ladder will move down the wall at a rate of _____ feet per second at the moment the base of the ladder is 14 feet from the base of the building.

(A) $\frac{7}{6}$ **(B)** $\frac{5}{4}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{5}{3}$

43. Consider the graph of $f(x) = 3\sin(x) + 2\cos(2x)$. How many values of x exist in the interval $(6, 10)$ such that there is a horizontal tangent at x?

(A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) 5**

44. Find the average value of $f(x) = 2\sin(x) + 5\cos(x)$ on the interval $[6, 8]$. (nearest tenth)

(A) 4.3 **(B) 4.5** **(C) 4.7** **(D) 4.9** **(E) 5.1**

45-46. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \frac{x}{2y}$ with the initial condition $f(4) = 3$.

45. Use Euler's method to approximate $y(5)$ using two steps of equal size starting at x = 4. (nearest ten-thousandth)

(A) 3.6648 **(B) 3.6668** **(C) 3.6688** **(D) 3.6708** **(E) 3.6728**

46. Find the exact value of y(5). (nearest thousandth)

(A) 3.6702 **(B) 3.6722** **(C) 3.6742** **(D) 3.6762** **(E) 3.6782**

47-48. Consider the region bounded by the graphs of $y_1 = .5x^2$ and $y_2 = 4 - x$.

47. Find the area of the specified region. (nearest tenth)

(A) 18.0 **(B) 18.4** **(C) 18.8** **(D) 19.2** **(E) 19.6**

48. Find the volume of the solid generated by revolving the specified region about the x-axis. (nearest whole number)

(A) 350 **(B) 353** **(C) 356** **(D) 359** **(E) 362**

49. If $P_3(x)$ is the third degree Maclaurin polynomial for $f(x) = e^x$, then $f(.5) - P_3(.5) =$ _____. (nearest ten-thousandth)

(A) 0.0021 **(B) 0.0023** **(C) 0.0025** **(D) 0.0027** **(E) 0.0029**

50. The position of a particle is given by the parametric equations $x(t) = 3e^{4t}$ and $y(t) = \ln(t^3 + 3t - 2)$. Find the velocity vector when t = 3.

(A) $\left\langle 12e^{12}, \frac{30}{39} \right\rangle$ **(B)** $\left\langle 12e^{6}, \frac{15}{17} \right\rangle$ **(C)** $\left\langle 12e^{12}, \frac{30}{37} \right\rangle$ **(D)** $\left\langle 12e^{4}, \frac{15}{17} \right\rangle$ **(E)** $\left\langle 12e^{12}, \frac{15}{17} \right\rangle$

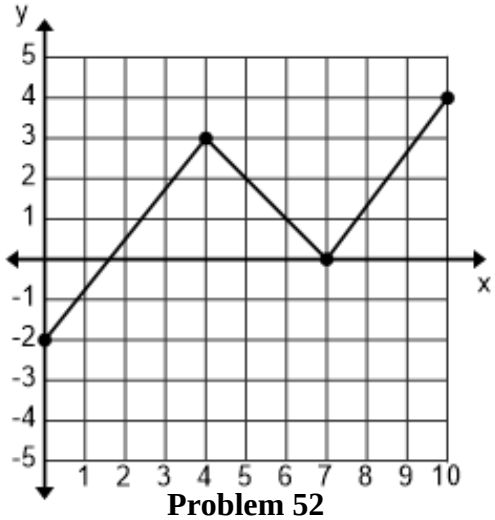
51. The second derivative of a function h(x) is given by $h''(x) = x^2(x-2)^3(x-8)^5$. Of the following values of x, which of these does the graph of h(x) have a point of inflection? {0, 2, 8}

(A) 2 only **(B) 2, 8 only** **(C) 0, 8 only** **(D) 0, 2 only** **(E) 0, 2, 8**

52. The graph of the piecewise function f(x) is shown on the right. Find the value of $\int_0^{10} f'(x)dx$.

(nearest hundredth)

(A) 6.00
(B) 7.75
(C) 9.50
(D) 11.25
(E) 13.00



Problem 52

Use the table below and the information below the table for problems 53 and 54.

	1	2	3	4	5	6	7	8	9
Score	**166**	**172**	**154**	**205**	**221**	**198**	**144**	**172**	**188**

Phoenix entered the Irion County Bowling Marathon last week. Each contestant is required to bowl nine games in six hours. The table above show the results.

53. What is the difference in the mean score and the median score, a positive number?

(A) 2 **(B) 4** **(C) 6** **(D) 8** **(E) 10**

54. Find the interquartile range of the scores.

(A) 6 **(B) 23.75** **(C) 41.5** **(D) 59.25** **(E) 77**

55. Alessandra flipped a fair coin 6 times and it came up heads all 6 times. She decided to flip the coin one more time. What is the probability that the seventh flip will produce a tails? (nearest ten-thousandth)

(A) 0.0078 **(B) 0.1309** **(C) 0.2500** **(D) 0.3772** **(E) 0.5000**

Use the table below and the information below the table for problems 56-58.

M.S. Time	12.25	11.88	11.36	10.75	10.33	9.91
H.S. Score	139	148	153	164	171	178

Christopher decided that he would warm up for his number sense competitions in 2023 by taking a middle school number sense test 30 minutes before the high school competition started. The table above shows the time, in minutes, it took him to complete the middle school warm up tests and his scores at the six high school meets he has attended so far this year.

56. Christopher's coach plotted the data and observed a strong, negative, linear relationship. His coach analyzed the data and his statistical software generated a LSRL with a correlation of r = _____. (nearest ten-thousandth)

(A) -0.9967 (B) -0.9922 (C) -0.9885 (D) -0.9843 (E) -0.9812

57. Use the data in the table above to predict his score at the next competition if he took 9.66 minutes to complete a middle school warm up test. (nearest whole number)

(A) 180 (B) 182 (C) 184 (D) 186 (E) 188

58. Find the value of the residual for the data point $(11.88, 148)$. (nearest tenth)

(A) 1.5 (B) 1.7 (C) 1.9 (D) 2.1 (E) 2.3

59-60. A researcher is testing the claim that 62% of Americans believe that the economy is the most important issue in America today. In a survey involving a simple random sample of 1200 Americans, 782 responded by placing the economy as their top concern. The researcher performed an appropriate test with $H_0 : p = 0.62$ and, $H_a : p \neq 0.62$. He used an $\alpha = 0.05$ level as his standard.

59. What is the value of the test statistic? (nearest hundredth)

(A) 2.02 (B) 2.14 (C) 2.26 (D) 2.38 (E) 2.50

60. What was the conclusion of the researcher after studying the test results?

(A) Based on a p-value of 0.06, he failed to reject H_0.
(B) Based on a p-value of 0.06, he rejected H_0.
(C) Based on a p-value of 0.02, he failed to reject H_0.
(D) Based on a p-value of 0.02, he rejected H_0.
(E) Based on a p-value of 0.01, he failed to reject H_0.

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2023
Answer Key

1. D	**21. E**	**41. A**
2. A	**22. A**	**42. A**
3. B	**23. A**	**43. C**
4. E	**24. A**	**44. A**
5. E	**25. C**	**45. D**
6. C	**26. A**	**46. C**
7. A	**27. B**	**47. A**
8. D	**28. E**	**48. E**
9. E	**29. A**	**49. E**
10. C	**30. C**	**50. E**
11. B	**31. D**	**51. B**
12. E	**32. C**	**52. A**
13. C	**33. B**	**53. D**
14. A	**34. A**	**54. C**
15. E	**35. E**	**55. E**
16. C	**36. A**	**56. A**
17. B	**37. C**	**57. B**
18. E	**38. D**	**58. D**
19. C	**39. B**	**59. C**
20. C	**40. B**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2023

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1. **The equation for kinetic energy is $K = \frac{1}{2}mv^2$, where K is kinetic energy in joules, m is mass in kilograms, and v is velocity in meters/second. Find the mass of an object with a kinetic energy of 184 joules and a velocity of 18.8 meters/second. (nearest hundredth)**

   (A) 1.04 kg **(B) 1.25 kg** **(C) 1.46 kg** **(D) 1.67 kg** **(E) 1.88 kg**

2. **Consider a line, L_1, that contains the points $A(3, 8)$ and $B(-4, -6)$. If the point $C(a, 14)$ lies on L_1, what is the value of a?**

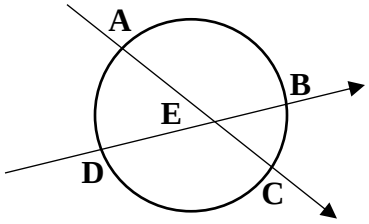
   (A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

3. **Consider three consecutive prime numbers such that the sum of the numbers is 143. Find the product of the three numbers.**

   (A) 103,071 **(B) 107,113** **(C) 111,155** **(D) 115,197** **(E) 119,239**

4. **The measure of minor arc AB is $128°$ and the measure of minor arc CD is $112°$. Find the measure of $\angle BEC$.**

   (A) 54° **(B) 56°** **(C) 58°** **(D) 60°** **(E) 62°**



Problem 4

5. **At Pizza Heaven in Rankin, a large pizza cost $12.00 plus $1.25 per topping. A coke cost $2.25. The Tubbs family ordered a large pizza with 3 toppings and 4 cokes. The Wyatt family ordered a large Pizza with 4 toppings and 5 cokes. Mr. Wyatt also ordered a salad. If the total cost for both families was $62.37 after an 8.00% tax was added, what was the cost of the salad?**

   (A) $3.75 **(B) $4.00** **(C) $4.25** **(D) $4.50** **(E) $4.75**

6. **Mr. Clanton plans to hire Alexis, Skylee and Arthur to paint the 40 classrooms at the new elementary school during the summer. Alexis can paint a classroom in 8 hours, Skylee can paint a classroom in 9 hours and Arthur can paint a classroom in 10 hours. If they work together, how many hours are required to paint the 40 classrooms? (nearest whole number)**

   (A) 115 hr **(B) 117 hr** **(C) 119 hr** **(D) 121 hr** **(E) 123 hr**

7. **My final grade is determined by four tests and the final exam, which counts twice. I scored 86, 82, 89 and 95 on my tests. If I need to have an average of 89.5 or higher to make an A, what is the minimum score that I need to make on the final exam in order to make an A?**

   (A) 91 **(B) 92** **(C) 93** **(D) 94** **(E) 95**

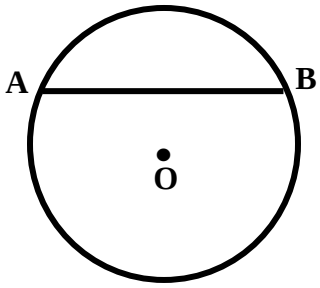
8. **Siep is driving his 2022 Dodge Ram 3500 Pickup on I-10 at a speed of 75 mph. If each tire has a radius of 16 inches, what is the rotational speed of the tires? (nearest whole number)**

   (A) 764 rpm **(B) 770 rpm** **(C) 776 rpm** **(D) 782 rpm** **(E) 788 rpm**

9. An equilateral triangle has a side length of 14. Find the circumference of a circle that has the same area as the triangle. (nearest tenth)

(A) 32.4 **(B) 32.7** **(C) 33.0** **(D) 33.3** **(E) 33.6**

10-11. Consider the circle with center O shown on the right. The length of chord $\overline{AB}$ is 24. The area of the circle is 530.929.



Problems 10, 11

10. Find the measure of minor arc AB. (nearest tenth)

(A) 131.5
(B) 132.6
(C) 133.7
(D) 134.8
(E) 135.9

11. Find the area bounded by minor arc AB and chord $\overline{AB}$. (nearest tenth)

(A) 138.7 **(B) 139.1** **(C) 139.5** **(D) 139.9** **(E) 140.3**

12. If $\Delta ABC \approx \Delta DEF$, $AB = 10$, $AC = 8$ and $DE = 7$, then DF = _____. (nearest tenth)

(A) 5.6 **(B) 5.8** **(C) 6.0** **(D) 6.2** **(E) 6.4**

13. Consider the Fibonacci type series $12+9+21+30+51+...+558+903$. The sum of the series is ____.

(A) 2339 **(B) 2343** **(C) 2347** **(D) 2351** **(E) 2355**

14. Consider an arithmetic sequence whose 4^{th} term is 33 and 9^{th} term is 73. Find the sum of the first 13 terms of the sequence.

(A) 738 **(B) 741** **(C) 744** **(D) 747** **(E) 750**

15-16. Consider the geometric sequence $48, a, b, c, 23\frac{4}{27}, d, ...$ with $a > 0$.

15. b = _____.

(A) $32.\overline{3}$ **(B) $32.\overline{6}$** **(C) 33** **(D) $33.\overline{3}$** **(E) $33.\overline{6}$**

16. The sum of the first 8 terms of the sequence is _____. (nearest whole number)

(A) 209 **(B) 213** **(C) 217** **(D) 221** **(E) 225**

17. A right circular cone has a diameter of 12 and a slant height of 10. Find the volume of the cone. (nearest tenth)

(A) 300.8 **(B) 301.2** **(C) 301.6** **(D) 302.0** **(E) 302.4**

18. Madison plans to take 4 days to drive from Rankin, TX to Bellingham, WA, a distance of 1964 miles. Madison drove 485 miles at an average speed of 59 mph on Monday, 611 miles at an average speed of 67 mph on Tuesday, and 447 miles at an average speed of 62 mph on Wednesday. What average speed must Madison travel at on Thursday in order to have an overall average speed of 63 mph for the entire trip? (nearest tenth)

(A) 62.9 mph (B) 63.2 mph (C) 63.5 mph (D) 63.8 mph (E) 64.1 mph

19. Coach Perkins has 3 posts, 8 wings and 4 guards on his team. His starting lineup must consist of one post, two wings and two guards. How many starting lineups are possible?

(A) 96 (B) 232 (C) 368 (D) 504 (E) 640

20. If $f(x) = 17x^{\left(\frac{3}{2}\right)}$ and $h(x) = \sqrt[3]{x-22}$, then $f\left(h(86)\right) =$ _____. (nearest whole number)

(A) 136 (B) 138 (C) 140 (D) 142 (E) 144

21. Find the remainder when $x^4 - 3$ is divided by $x^2 - 1$.

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

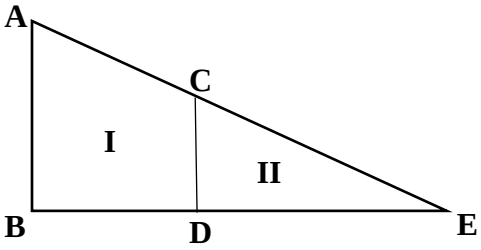
22. When the vector $v = -18i + 5j$ is converted to polar coordinates, one correct answer, rounded to the nearest tenth is $v = \langle 18.7, \theta° \rangle$. θ could be _____°

(A) -254.5 (B) -195.5 (C) 74.5 (D) 166.5 (E) 195.5

23-24. $\overline{AB}$ is parallel to $\overline{CD}$ and $\overline{CD} \perp \overline{BE}$.
$AB = 8.00$, $CD = 4.75$, $CE = 10.50$

23. AC = _____. (nearest hundredth)

(A) 6.94 (B) 7.06 (C) 7.18
(D) 7.30 (E) 7.42



Problems 23, 24

24. Find the area of region I. (nearest hundredth)

(A) 36.40 (B) 37.51 (C) 38.62 (D) 39.73 (E) 40.84

25. On Rihot's farm, the number of cows varies inversely as the number of pigs and directly as the number of chickens squared. When there was 400 cows, he had 20 chickens and 40 pigs. How many cows will he have when there are only 4 pigs and 16 chickens?

(A) 256 (B) 832 (C) 1408 (D) 1984 (E) 2560

26. If $f(x) = 6x - 1$ and $h(x) = x^2 + 8$, then $(h \circ f)(2) =$ _____.

(A) 127 (B) 129 (C) 131 (D) 133 (E) 135

27. The roots of the equation $12x^2 + bx + c = 0$ are $-\frac{1}{3}$ and $\frac{3}{4}$. $b + c =$ _____.

(A) –8 **(B)** –6 **(C)** –4 **(D)** –2 **(E)** 2

28. The six members of the Elkins math team were seated in a circle around a large circular table along with their coach in yesterday's math practice. If there were seven seats in the room, how many seating arrangements were possible?

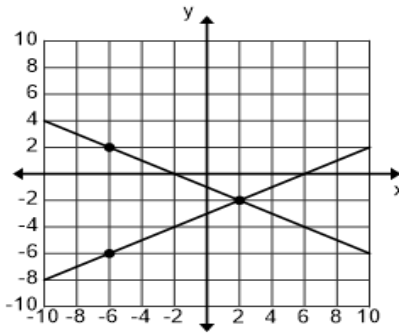
(A) 24 **(B)** 120 **(C)** 720 **(D)** 5040 **(E)** 40320

29. $x^2 + y^2 - 8x - 12y - 36 = 0$ is the equation of a circle with an area of _____. (nearest hundredth)

(A) 276.46 **(B)** 278.56 **(C)** 280.66 **(D)** 282.76 **(E)** 284.86

30. Find the acute angle formed by the two intersecting lines shown on the right. (nearest hundredth)

(A) 53.13° **(B)** 54.24° **(C)** 55.35° **(D)** 56.46° **(E)** 57.57°



Problems 30, 31

31. The asymptotes of a hyperbola are graphed on the right. If the equation of the hyperbola centered at (h, k) is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{4} = 1$, and one of the vertices is the point (m, n) then $m + n =$ _____. (nearest tenth)

(A) –4.4 **(B)** –4.2 **(C)** –4.0 **(D)** –3.8 **(E)** –3.6

32. The perimeter of a regular octagon is 32. What is the area of the octagon? (nearest whole number)

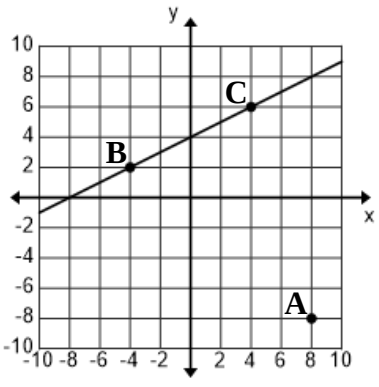
(A) 75 **(B)** 77 **(C)** 79 **(D)** 81 **(E)** 83

33. Find the distance from point A to the line $\overleftrightarrow{BC}$ shown on the right. (nearest tenth)

(A) 13.5 **(B)** 13.7 **(C)** 13.9 **(D)** 14.1 **(E)** 14.3

34. Draw auxiliary lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$. Find the perimeter of ΔABC. (nearest tenth)

(A) 38.8 **(B)** 39.1 **(C)** 39.4 **(D)** 39.7 **(E)** 40.0



Problems 33, 34, 35

35. Find the area of ΔABC. (nearest whole number)

(A) 64 **(B)** 66 **(C)** 68 **(D)** 70 **(E)** 71

36. At 6:00 AM, the hour hand and the minute hand of my clock point in opposite directions. How long will it be until the hands point in the same direction? (nearest tenth)

(A) 32.1 min (B) 32.3 min (C) 32.5 min (D) 32.7 min (E) 32.9 min

37. Allison Engineering has three open positions. Twelve A&M grads apply and ten UT grads apply. If the first position must be filled by an A&M grad, then how many different ways can these positions be filled?

(A) 840 (B) 2940 (C) 5040 (D) 7140 (E) 9240

38. Ivan wants to find a set of parametric equations to represent the graph of $y = -4x^2 + 6$.

If $x = -t + 3$, and $y = at^2 + bt + c$, then $a + b + c =$ _____.

(A) -14 (B) -12 (C) -10 (D) -8 (E) -6

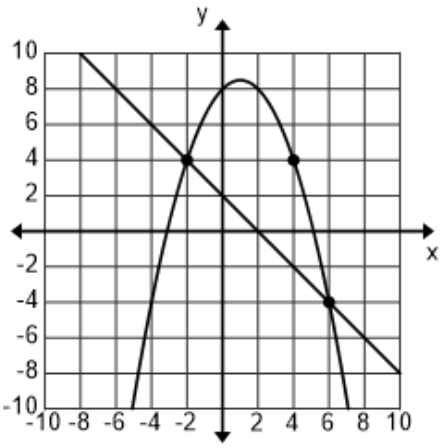
39. Find the area of the quadrilateral with vertices A(6, 4, −2), B(−4, 4, −6), C(−4, 10, −8) and D(6, 10, -4).

(nearest tenth)

(A) 61.3 (B) 63.4 (C) 65.5 (D) 67.6 (E) 69.7

40. If the parabola shown on the right intersects the line at points A and B, then AB = _____. (nearest tenth)

(A) 11.3
(B) 11.5
(C) 11.7
(D) 11.9
(E) 12.1



Problems 40, 41

41. The area bounded by the graphs of the parabola and the line shown on the right equals _____. (nearest tenth)

(A) 41.5 (B) 41.8 (C) 42.1 (D) 42.4 (E) 42.7

42. Consider the graph of $f(x) = \dfrac{x^2 + 3x - 28}{2x^2 + 9x - 35}$. Which of the following are true?

I. $f(x)$ is not continuous at $x = -7$ and $x = 4$.
II. The vertical lines $x = -7$ and $x = 2.5$ are vertical asymptotes.
III. The horizontal line $y = 0.5$ is a horizontal asymptote.

(A) I only (B) III only (C) I, II only (D) I, III only (E) I, II, III

43. If $y = \sec(x)$, then $\frac{d^2y}{dx^2} =$ _____.

(A) $\sec(x)\tan^2(x)+\sec^3(x)$ **(B)** $\sec(x)\tan(x)+\sec^2(x)$ **(C)** $\sec^2(x)\tan^2(x)+\sec^3(x)$
(D) $\sec(x)\tan(x)+\sec(x)$ **(E)** $\sec^2(x)\tan^2(x)+\sec^2(x)$

44. The first floor of the student fitness building at SIU consists of a rectangle with a semicircle on each end. A 200-meter track runs around the outside. If the designers of the building wanted to maximize the rectangular area of the first floor, how long should the radius of each semicircle be? (nearest tenth)

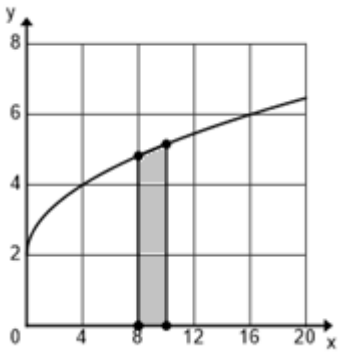
(A) 15.5 m **(B) 15.7 m** **(C) 15.9 m** **(D) 16.1 m** **(E) 16.3 m**

45. Find the area of one petal of the rose curve $r = 6\cos 3\theta$. (nearest tenth)

(A) 8.8 **(B) 9.0** **(C) 9.2** **(D) 9.4** **(E) 9.6**

46. Consider the graph shown on the right. Use the trapezoidal approximation method with six subintervals of equal width to approximate the area bounded by the curves $y_1 = \sqrt{x}+2$, $y_2 = 0$, $x_1 = 4$, $x_2 = 16$. One of the trapezoids is shown on the right. (nearest hundredth)

(A) 59.25
(B) 60.27
(C) 61.29
(D) 62.31
(E) 63.33



Problems 46, 47

47. Find the exact area of the region bounded by the curves $y_1 = \sqrt{x}+2$, $y_2 = 0$, $x_1 = 4$, $x_2 = 16$.

(A) $60\frac{5}{6}$ **(B) 61** **(C)** $61\frac{1}{6}$ **(D)** $61\frac{1}{3}$ **(E)** $61\frac{1}{2}$

48. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Consider an object placed in a room kept at a constant temperature of 55°. At t = 0, the temperature of the object is 120°. At t = 15 min, the temperature of the object is 105°. The temperature of the object should reach 90° at t = _____. (nearest tenth)

(A) 35.4 min **(B) 36.5 min** **(C) 37.6 min** **(D) 38.7 min** **(E) 39.8 min**

49. Naveed is evaluating $\int_0^1 \arcsin(x)dx$ using the method of integration by parts. The best choice for dv is _____.

(A) arcsin(x) (B) x (C) sin(x) (D) cos(x) (E) dx

50. Which of the following tests will show that the series $\sum_{n=1}^{\infty}\left(\frac{n}{(n^2+1)^3}\right)$ converges?

(A) nth Term test (B) p-Series test (C) Integral test
(D) Geometric Series test (E) Telescoping Series Test

51-52. Consider the function $f(x)=\sin(x^2)$

51. For x > 0, the first local minimum occurs when x = _____. (nearest hundredth)

(A) 2.05 (B) 2.09 (C) 2.13 (D) 2.17 (E) 2.21

52. Use the first three non-zero terms of the McLaurin series for f(x) to approximate $f(.25)$. (6 decimal places)

(A) 0.062455 (B) 0.062457 (C) 0.062459 (D) 0.062461 (E) 0.062463

53-54. Eric was initially disappointed when he learned he had made 72 on his Honors Cal III test. His U.T. professor, Dr Hookem, revealed that the test scores were approximately normal with a mean of 60 with a standard deviation of 5.

53. Eric felt better after he calculated his z-score, which was _____.

(A) 1.8 (B) 2.0 (C) 2.2 (D) 2.4 (E) 2.6

54. He felt much better after his calculations placed him at the _____ percentile, based on his z-score.

(A) 91st (B) 93rd (C) 95th (D) 97th (E) 99th

Season	1981	1982	1983	1984	1985	1986	1987	1988	1988	1989
Homeruns	22	36	44	46	52	56	50	74	48	46

The table above shows the home run production for Steve Strutz of the Portland Beavers. Use this table for problems 55, 56, and 57.

55. The mode of the data is _____ homeruns.

(A) 44 (B) 46 (C) 48 (D) 50 (E) 52

56. The mean of the data is _____ homeruns. (nearest tenth)

(A) 47.0 (B) 47.2 (C) 47.4 (D) 47.6 (E) 47.8

57. Which values are considered outliers?

(A) none (B) 22 only (C) 74 only (D) 22, 74 only (E) 22, 36, 74 only

Event	Probability
Obese	0.28
Obese and has heart disease	0.16
Not obese and does not have heart disease	0.65

Use the table above for problem 58.

58. Researchers are interested in the relationship between obesity and heart disease. Suppose a person is randomly selected from a large population of males in the 50-59 age group. The table above shows the probabilities of some events related to this chance process. If two males from this population are selected at random, what is the probability that at least one of them has heart disease. (nearest thousandth)

(A) 0.387 (B) 0.407 (C) 0.427 (D) 0.447 (E) 0.467

School	Mean Math ACT Score	Standard Deviation
School A	23	4
School B	19	3

59-60. State officials wanted to know if an experimental math curriculum implemented at a school in southeast Idaho was helping students score higher on the math section of the ACT Test. Two schools which are located ten miles apart and which had similar scores using the state curriculum in 2014 were compared after one school began using an experimental math curriculum in 2015. The results of the 2020 ACT Test are given in the table above. School A was using the new curriculum while School B continued to use the standard state curriculum. One student from School A is randomly selected and one student from School B is randomly selected.

59. Find the expected value for the difference in their scores.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

60. Find the standard deviation of the difference in their scores.

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2023
Answer Key

1. A	**21. B**	**41. E**
2. C	**22. B**	**42. B**
3. B	**23. C**	**43. A**
4. D	**24. E**	**44. C**
5. E	**25. E**	**45. D**
6. C	**26. B**	**46. C**
7. C	**27. A**	**47. D**
8. E	**28. C**	**48. A**
9. B	**29. A**	**49. E**
10. D	**30. A**	**50. C**
11. A	**31. C**	**51. D**
12. A	**32. B**	**52. C**
13. E	**33. E**	**53. D**
14. B	**34. B**	**54. E**
15. D	**35. A**	**55. B**
16. D	**36. D**	**56. C**
17. C	**37. C**	**57. D**
18. C	**38. C**	**58. B**
19. D	**39. D**	**59. C**
20. A	**40. A**	**60. C**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2023

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. After math practice on Thursday, the Holliday math team drove to the Whataburger in Wichita Falls for supper. The principal gave them $50 to spend. They ordered 5 cheeseburger combos. A combo cost $7.85 plus tax. If the tax rate is 8.25% and if an apple pie cost $1.25 plus tax, how many apple pies could they order?

(A) 2 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

2. To pay for some new HP Prime G2 calculators for Mr. C's math team, Crosby, Stills, and Nash agreed to perform at SHS with all proceeds going to the math department. Student tickets cost $15.00 and adult tickets cost $25.00. If they raised $7,700 by selling 372 tickets, how many adult tickets were sold?

(A) 208 **(B) 210** **(C) 212** **(D) 214** **(E) 216**

3. Line L_1 contains the points $(-8, 6)$ and $(4, -10)$. Line L_2 is parallel to L_1 and contains the point $(-6, -12)$. The y-intercept of L_2 is $(0, b)$. The value of b is _____.

(A) –22 **(B) –20** **(C) –18** **(D) –16** **(E) –14**

4. Tal rented a car at the airport when his plane landed in Boise. The city of Boise charges an upfront fee of $20 to rent a car at the airport. He was also charged $25 per day and $0.55 per mile. If Tal used his car for five days and his final bill was $241.25, how many miles did he drive during his stay?

(A) 175 **(B) 177** **(C) 179** **(D) 181** **(E) 183**

5. Anthony can wash and wax a car in 45 minutes while Jacob needs one hour to wash and wax a car. If Anthony works by himself for two hours before being joined by Jacob, how much time will it take for them to wash and wax 16 cars? (nearest minute)

(A) 7 hr 31 min **(B) 7 hr 34 min** **(C) 7 hr 37 min** **(D) 7 hr 40 min** **(E) 7 hr 43 min**

6. The value of Warith's house is increasing by 9.35 percent each year. If his house is worth $378,000 on January 1st, 2023, what should it be worth on January 1st, 2035? (nearest dollar)

(A) $1,100,441 **(B) $1,101,552** **(C) $1,102,663** **(D) $1,103,774** **(E) $1,104,885**

7. If the values of the roots of the function $f(x) = 7x^2 + 14x - 105$ are a and b, then $\frac{a+b}{ab}$ = _____.

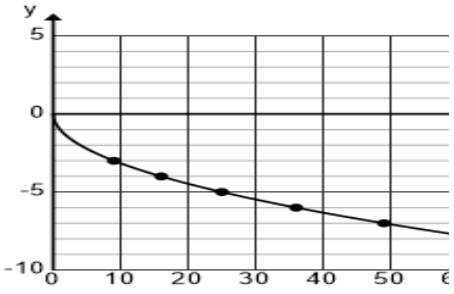
(A) 0.125 **(B) $0.1\overline{3}$** **(C) $0.14\overline{2}$** **(D) 0.15** **(E) $0.1\overline{6}$**

8. The graph of y = h(x) begins at the point $(0, 0)$. h(121) = ___.

(A) –9 **(B) –10** **(C) –11** **(D) –12** **(E) –13**

9. If g(x) is the inverse function of h(x), find the domain of g(x).

(A) $[0, 121]$ **(B) $[-121, \infty)$** **(C) $[0, \infty)$**
(D) $(-\infty, \infty)$ **(E) $(-\infty, 0]$**

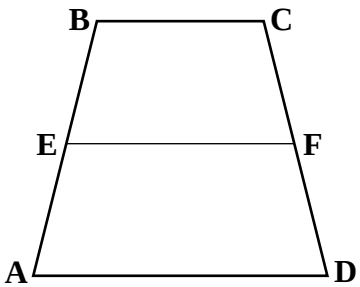


Problems 8, 9

10. The number of 2-liter cokes sold at Walmart each month varies inversely as the price. In a month when the price was $1.80, they sold 3448 2-liter cokes. If the price is reduced to $1.20 the next month, what is the expected number of cokes that will be sold?

(A) 5166 **(B) 5168** **(C) 5170** **(D) 5172** **(E) 5174**

11-12. Consider isosceles trapezoid ABCD. $\overline{EF}$ is the median.
BC = BE = 12. $m\angle BAD = 80°$.



Problems 11, 12

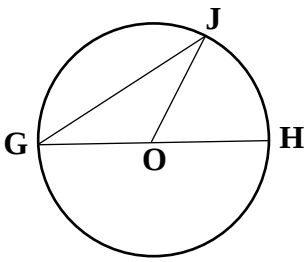
11. Draw auxiliary line segment $\overline{EC}$. Find the area of triangle EBC. (nearest tenth)

(A) 70.3 **(B) 70.6** **(C) 70.9** **(D) 71.2** **(E) 71.5**

12. Find the area of trapezoid ABCD. (nearest whole number)

(A) 382 **(B) 385** **(C) 388** **(D) 391** **(E) 394**

13-14. Consider the circle with center O and diameter $\overline{GH}$. The measure of minor arc GJ = 110° and GH = 18.



Problems 13, 14

13. The area of sector JOH is _____. (nearest tenth)

(A) 48.7 **(B) 49.1** **(C) 49.5** **(D) 49.9** **(E) 50.3**

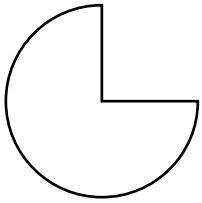
14. The perimeter of triangle GOJ is _____. (nearest tenth)

(A) 32.5 **(B) 32.7** **(C) 32.9** **(D) 33.1** **(E) 33.3**

15. A regular hexagon is inscribed in a circle. If the area of the circle is 452, then the perimeter of the hexagon is _____. (nearest whole number)

(A) 68 **(B) 70** **(C) 72** **(D) 74** **(E) 76**

16. The area of the three-quarter circle is 530. Find the perimeter of the three-quarter circle. (nearest whole number)


Problem 16

(A) 97 **(B) 99** **(C) 101** **(D) 103** **(E) 105**

17. A right circular cone has a radius of 7.75 and a total surface area of 462. Find the volume of the cone. (nearest whole number)

(A) 495 **(B) 499** **(C) 503** **(D) 507** **(E) 511**

18. The center of a circle is in quadrant IV and the circumference of the circle is 16π. The equation of the circle is $x^2 + y^2 + ax + 8y - 39 = 0$. The value of a is _____.

(A) –8 **(B) –6** **(C) –4** **(D) –2** **(E) –1**

19. Consider right triangle ABC with $m\angle C = 90°$. Point D lies on $\overline{AB}$, $\overline{CD} \perp \overline{AB}$, AC = 6 and AB = 10. Find the area of triangle ACD. (nearest hundredth)

(A) 8.53 **(B) 8.64** **(C) 8.75** **(D) 8.86** **(E) 8.97**

20. Consider $f(x) = 2x^3 + bx^2 + 4x - 8$. If $f(3) = 31$, then b = _____.

(A) –3 **(B) –2** **(C) –1** **(D) 1** **(E) 2**

21. Consider four consecutive even negative integers (in increasing order) such that the product of the first and third is 122 greater than the product of -25 and the fourth. Find the sum of the four integers.

(A) –108 **(B) –104** **(C) –100** **(D) –96** **(E) –92**

22. How many even four-digit counting numbers are less than 7000?

(A) 1500 **(B) 2000** **(C) 2500** **(D) 3000** **(E) 3500**

23. If the distance from the point $(e, -12)$ to the line $y = \frac{3}{5}x + 6$ is $\frac{66}{\sqrt{34}}$, then e = ____. $(e > -15)$

(A) –11 **(B) –10** **(C) –9** **(D) –8** **(E) –7**

24. Consider the conic with equation $9x^2 - 4y^2 - 36x - 24y - 36 = 0$. If the coordinates of the foci are (a, b) and (c, b) then $a + b + c =$ _____. (nearest tenth)

(A) 1.0 **(B) 1.2** **(C) 1.4** **(D) 1.6** **(E) 1.8**

25. Two cables are attached to a vertical tower from a point on the ground. The angle between the cables is 20°. The longer cable is 270 feet long and is attached to the top of the tower. The shorter cable is attached to the tower 105 feet below the top of the tower. Find the length of the shorter cable. (nearest whole number)

(A) 195 ft **(B) 198 ft** **(C) 201 ft** **(D) 204 ft** **(E) 207 ft**

26. Consider an arithmetic sequence in which the sixth term is 47 and the twelfth term is 95. What is the product of the eighteenth and nineteenth terms?

(A) 21,579 **(B) 21,586** **(C) 21,593** **(D) 21,600** **(E) 21,607**

27. If $f(x) = \frac{3x-4}{5x-6}$ and $h(x) = \frac{-2x+5}{-3x-8}$, then $\left(h^{-1} \circ f^{-1}\right)(1) =$

(A) –13 **(B) –10** **(C) –7** **(D) –4** **(E) –1**

28. Three of the roots of the fourth-degree polynomial $x^4+bx^3+cx^2+dx+e$ are $-2,\ 3,$ and $1-\sqrt{5}$. If b, c, d, and e are rational numbers, then $b+c+d+e=$ _____.

(A) 17 **(B) 20** **(C) 23** **(D) 26** **(E) 29**

29. The sound level in decibels, β, is given by $\beta=10\log\left(\frac{I}{10^{-12}}\right)$, where I is the intensity of sound in W/m^2. Andrew is playing his trumpet, producing a sound level of 88 dB. If twelve other musicians join him and they all play their trumpets at the same intensity as Andrew, what is the sound level of all of the trumpets playing together? (nearest whole number)

(A) 88 dB **(B) 99 dB** **(C) 110 dB** **(D) 121 dB** **(E) 132 dB**

30. Assume the temperature on a typical day in January in Idaho Falls varies sinusoidally with a low of $12°\,F$ at 5:00 AM and a high of $29°\,F$ at 5:00 PM. What is the expected temperature at midnight? (nearest tenth)

(A) $17.7°\,F$ **(B) $18.0°\,F$** **(C) $18.3°\,F$** **(D) $18.6°\,F$** **(E) $18.9°\,F$**

31. Angle A is in quadrant II and angle B is in quadrant III. If $\sin A=\frac{3}{5}$ and $\cos B=-\frac{5}{13}$, then $\tan(A+B)=$ _____. (nearest hundredth)

(A) 0.56 **(B) 0.59** **(C) 0.62** **(D) 0.65** **(E) 0.68**

32. The partial fraction decomposition of $\frac{x+8}{x^2+x-6}$ is $\frac{A}{x-2}+\frac{B}{x+3}$. $A+B=$ _____.

(A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) 5**

33. Find the angle between the vectors $u=\langle 4,-6\rangle$ and $v=\langle 12,8\rangle$ is _____ rad. (nearest hundredth)

(A) 1.13 **(B) 1.24** **(C) 1.35** **(D) 1.46** **(E) 1.57**

34. Consider the sequence $1, 5, 12\frac{1}{2}, 20\frac{5}{6}, 26\frac{1}{24}, \ldots$. Find the eighth term in the sequence. (nearest hundredth)

(A) 15.02 **(B) 15.14** **(C) 15.26** **(D) 15.38** **(E) 15.50**

35. A ball is dropped from a height of six feet and begins bouncing. Each bounce is three-fourths of the height of the previous bounce. Which bounce is the first bounce in which the height of the bounce is less than one foot?

(A) 5 **(B) 6** **(C) 7** **(D) 8** **(E) 9**

36. For the final exam in calculus, Mrs. Wilcox gave her class a list of 18 study problems. Of these, 10 will be on the exam. If Emmy knows how to correctly solve 16 of these, find the probability that she will correctly solve all 10 problems on the final exam. (nearest thousandth)

(A) 0.141 **(B) 0.162** **(C) 0.183** **(D) 0.204** **(E) 0.225**

37. Consider an ellipse in which the vertices are $(0, 4)$ and $(10, 4)$ and the endpoints of the minor axis are $(5, 2)$ and $(5, 6)$. What is the eccentricity of the ellipse? (nearest hundredth)

(A) 0.84 **(B) 0.86** **(C) 0.88** **(D) 0.90** **(E) 0.92**

38. Find the area of a triangle with vertices $A(6,4,2)$, $B(8,6,10)$, and $C(6,2,8)$. (nearest tenth)

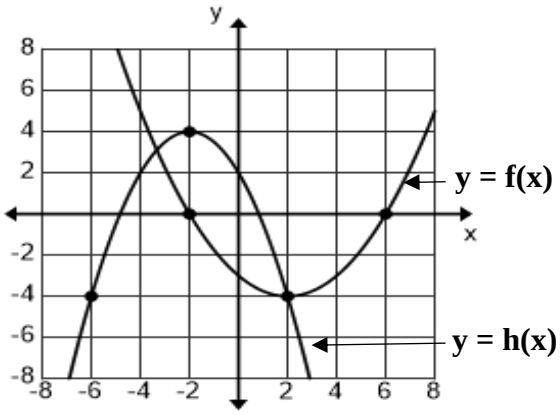
(A) 14.5 **(B) 14.8** **(C) 15.1** **(D) 15.4** **(E) 15.7**

39. The directrix of the graph of y = h(x) is the line y = c. The value of c is _____.

(A) 4.125 **(B) 4.25** **(C) 4.375**
(D) 4.5 **(E) 4.625**

40. If the parabolas intersect at points A and B, then AB = _____. (nearest tenth)

(A) 8.9 **(B) 9.0** **(C) 9.1**
(D) 9.2 **(E) 9.3**



Problems 39, 40, 41, 42

41. The area of the region bounded by the graphs of the parabolas is _____. (nearest tenth)

(A) 18.4 **(B) 18.6** **(C) 18.8** **(D) 19.0** **(E) 19.2**

42. Find the arc length of the graph of y = f(x) on the interval [0, 8]. (nearest tenth)

(A) 13.2 **(B) 13.4** **(C) 13.6** **(D) 13.8** **(E) 14.0**

43. A rectangle is to be inscribed between the graph of $y = 16 - x^2$ and the x-axis with its base on the x-axis. What is the maximum area of such a rectangle? (nearest tenth)

(A) 44.9 **(B) 46.0** **(C) 47.1** **(D) 48.2** **(E) 49.3**

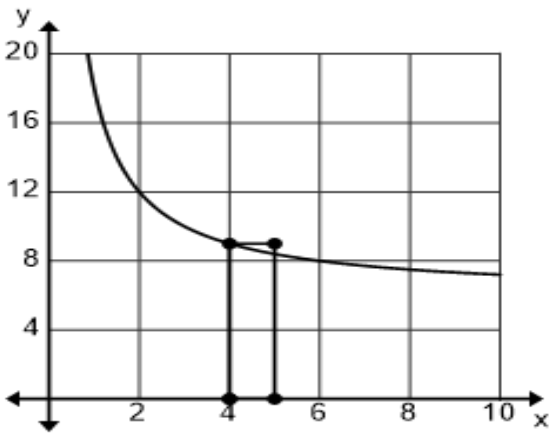
44. Find the sum of the series. $2 - \frac{4}{3} + \frac{4}{15} - \frac{8}{315} + \ldots$

(A) sin(2) **(B) cos(2)** **(C) e^2** **(D) ln(2)** **(E) 2sin(1)**

45. Find the area in the second quadrant bounded by the x-axis, the y-axis, and the graph of $r(\theta) = 2\theta + 3\sin(\theta),\ 0 \le \theta \le 2\pi$. (nearest tenth)

(A) 33.9 **(B) 34.2** **(C) 34.5** **(D) 34.8** **(E) 35.1**

46. Consider the graph of $y_1 = \frac{12}{x} + 6$. Use the Left Rectangle Approximation Method with six rectangles of equal width to approximate the area bounded by the curves $y_1 = \frac{12}{x} + 6,\ y_2 = 0$, $x_1 = 2$, and $x_2 = 8$. One of the rectangles is shown on the right. (nearest hundredth)



Problems 46, 47

(A) 55.11 **(B) 55.22** **(C) 55.33**
(D) 55.44 **(E) 55.55**

47. Find the volume of the solid generated when the region bounded by the curves $y_1 = \frac{12}{x} + 6,\ y_2 = 0,\ x_1 = 2,\ x_2 = 8$ is revolved around the line x = 12. (nearest whole number)

(A) 2369 **(B) 2373** **(C) 2377** **(D) 2381** **(E) 2385**

48. The Panhandle Coffee Shop keeps their dining area at a constant 72°. They serve their famous coffee at exactly 175° and the temperature of the coffee changes at the rate $r(t) = -5.89e^{-.0766t}$ degrees per minute. Darius received a phone call at the moment his coffee was served and his coffee cooled for exactly five minutes before he was able to take his first sip. What was the temperature of the coffee when he took his first sip? (nearest whole number)

(A) 139° **(B) 143°** **(C) 147°** **(D) 151°** **(E) 155°**

49. (rad) The derivative of the function f is given by $f'(x) = 2x^3 - 6\sin\left(x^2\right) + 2$. On the interval $(-2, 2)$, at which of the following values does f have a relative minimum? (nearest thousandth)

I. –0.535 **II. 0.669** **III. 1.260**

(A) I only **(B) I, III only** **(C) II only** **(D) III only** **(E) I, II, III**

50-51. (rad) The position of an object moving in the xy-plane is given by $(x(t), y(t))$, $0 \le t \le \frac{5\pi}{12}$, with $\frac{dx}{dt} = 4t\sin(t)$ cm/s and $\frac{dy}{dt} = 4t\cos(t)$ cm/s. At t = 0, the position of the object is $(4, 8)$.

50. Find the speed of the object at t = $\frac{\pi}{6}$. (nearest hundredth)

(A) 1.87 cm/s (B) 1.98 cm/s (C) 2.09 cm/s (D) 2.20 cm/s (E) 2.31 cm/s

51. The position of the object at t = $\frac{\pi}{3}$ is (a, b). b = _____. (nearest hundredth)

(A) 5.19 cm (B) 6.30 cm (C) 7.41 cm (D) 8.52 cm (E) 9.63 cm

52. Consider the first quadrant region bounded by the y-axis, the line x = 4, the line y = 10, and the curve $y = 2\ln(5 - x)$. This region is the base of a solid by cross sections in which each cross section is a square perpendicular to the x-axis. What is the volume of the solid? (nearest whole number)

(A) 250 (B) 254 (C) 258 (D) 262 (E) 266

Year	1911	1931	1951	1971	1995
Distance	50 ft 11 in	51 ft 1.25 in	52 ft 6.25 in	57 ft 1in	60 ft

The progression of the world record in the men's triple jump is shown in the table above. Use this table for problems 53 and 54.

53. Professor Stat instructed his students to find the LSRL for the data. The linear regression model overestimates the true value of the 1951 distance by _____. (nearest hundredth)

(A) 1.35 ft (B) 1.44 ft (C) 1.53 ft (D) 1.62 ft (E) 1.71 ft

54. Use the LSRL for the data and predict what the world record should be in 2022. (nearest inch)

(A) 62 ft 3 in (B) 62 ft 6 in (C) 62 ft 9 in (D) 63 ft (E) 63 ft 3 in

55. Assume the mean hang time of a punt for all NFL punters over the 2022 season was 4.40 seconds with a standard deviation of 0.25 seconds. If Jordan Stout had a mean hang time of 4.82 seconds for the 2022 season, what percentile did that place him at?

(A) 89th (B) 91st (C) 93rd (D) 95th (E) 97th

56. Consider a random variable X that is normally distributed with a mean of 75 and a standard deviation of 16. The approximate interquartile range for this distribution is _____. (nearest tenth)

(A) 20.5 (B) 21.6 (C) 22.7 (D) 23.8 (E) 24.9

57. A random sample of 500 Texas high school students is used to estimate the proportion of Texas high school students who participate in UIL academics. What is the maximum margin of error if a 96 percent confidence interval is to be constructed? (nearest thousandth)

(A) 0.028 **(B) 0.034** **(C) 0.040** **(D) 0.046** **(E) 0.052**

University	Texas	A&M	Tech	TCU
Students	96	82	112	70

58. A random sample of 360 high school seniors in the Texas Panhandle were asked which university they hoped to attend. Students were asked to choose between Texas, A&M, Tech, and TCU. The results are in the table above. Researchers had expected a ratio of 3:3:4:2 for their choices. An appropriate test at the $\alpha = 0.05$ level was performed to see if the observed values differ from what was expected. Based on a P-value of _____, researchers concluded that there was insufficient evidence to show that student choices differ from what was expected.

(A) 0.262 **(B) 0.283** **(C) 0.304** **(D) 0.325** **(E) 0.346**

59. Ninety-five percent of the Olympic athletes who have been using steroids will test positive using a new test just developed. Ninety-eight percent of Olympic athletes who have not been using steroids will test negative using the new test. If ten percent of Olympic athletes have been using steroids, what percent of Olympic athletes will test positive using the new test? (nearest tenth)

(A) 9.1% **(B) 10.2%** **(C) 11.3%** **(D) 12.4%** **(E) 13.5%**

60. In the Fort Bend school district, 16 out of 88 randomly selected high school seniors plan to study computer science in college, while 21 out of 72 juniors plan to study computer science in college. A 96% confidence interval for the difference between the proportion of high school seniors who plan to study computer science in college and the proportion of high school juniors who plan to study computer science is to be calculated. What is the standard error of the difference? (nearest ten-thousandth)

(A) 0.0591 **(B) 0.0612** **(C) 0.0633** **(D) 0.0654** **(E) 0.0675**

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University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2023
Answer Key

1. D	**21. C**	**41. D**
2. C	**22. D**	**42. C**
3. B	**23. D**	**43. E**
4. A	**24. A**	**44. A**
5. E	**25. D**	**45. C**
6. E	**26. C**	**46. A**
7. B	**27. A**	**47. E**
8. C	**28. E**	**48. D**
9. E	**29. B**	**49. B**
10. D	**30. C**	**50. C**
11. C	**31. B**	**51. E**
12. A	**32. A**	**52. C**
13. C	**33. E**	**53. E**
14. B	**34. E**	**54. B**
15. C	**35. C**	**55. D**
16. C	**36. C**	**56. B**
17. E	**37. E**	**57. D**
18. B	**38. D**	**58. E**
19. B	**39. D**	**59. C**
20. A	**40. A**	**60. E**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2023

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Joe and Arlene took their three children to Fuzzy's on Tuesday. Joe ordered a burrito for $6.95, Arlene ordered a chicken fajita salad for $7.95, and each child ordered a breakfast taco. If a breakfast taco costs $2.95 and the tax rate is 8.25%, what is the total cost?

(A) $25.68 **(B) $25.71** **(C) $25.74** **(D) $25.77** **(E) $25.80**

2. The total energy of a sphere rolling down an incline without slipping is $E = .7mv^2 + mgh$, where E is the energy (J), m is the mass (kg), v is the velocity (m/s), g is the acceleration of gravity (m/s^2), and h is the height (m). Find v if E = 27.6 J, m = 1.72 kg, g = 9.807 m/s^2, and h = 1.53 m. (nearest hundredth)

(A) 1.16 m/s **(B) 1.18 m/s** **(C) 1.20 m/s** **(D) 1.22 m/s** **(E) 1.24 m/s**

3-4. Given the following points with coordinates: $A(-4, 8)$, $B(8, 2)$, $C(2, -2)$, $D(6, 6)$, and $E(r, 3)$.

3. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point $P(a, b)$. $a + b =$ _____. (nearest tenth)

(A) 7.8 **(B) 8.0** **(C) 8.2** **(D) 8.4** **(E) 8.6**

4. If point E lies on the perpendicular bisector of $\overline{AB}$, then r = _____. (nearest tenth)

(A) 1.0 **(B) 1.2** **(C) 1.4** **(D) 1.6** **(E) 1.8**

5. A good approximation for a person's maximum heart rate is given by $MHR = 220 - x$, where MHR = maximum heart rate in beats per minute and x = a person's age. Randy recently ran a 3-mile run in 30 minutes. He ran with his heart rate at 85% of his MHR and his heart beat 4080 times during the run. How old is Randy?

(A) 58 **(B) 60** **(C) 62** **(D) 64** **(E) 66**

6. Salem left Canadian and traveled north on State Highway 60 on his new bicycle at 24 mph for 10 minutes. Then he turned east and headed toward Lake Marvin on FM 2266. He traveled at 28 mph, arriving at the lake 22 minutes after turning east. What is the straight-line distance from Canadian to Lake Marvin? (nearest tenth)

(A) 11.0 mi **(B) 11.2 mi** **(C) 11.4 mi** **(D) 11.6 mi** **(E) 11.8 mi**

7-8. Given: $h(x) = \dfrac{\sqrt{x}}{\sqrt{16 - x^2}}$.

7. Find the domain of $h(x)$.

(A) $[0, \infty)$ **(B) $[0, 4]$** **(C) $(0, 4)$** **(D) $[0, 4)$** **(E) $(0, \infty)$**

8. Find the range of $h(x)$.

(A) $[0, \infty)$ **(B) $[0, 4]$** **(C) $(0, 4)$** **(D) $[0, 4)$** **(E) $(0, \infty)$**

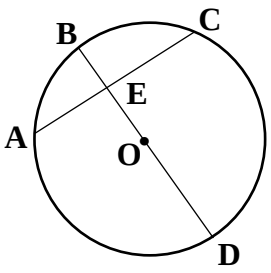
9. Mary trimmed a large square picture so that it would fit into a frame. She trimmed 10 inches from the length and 6 inches from the width. The area of the trimmed picture is 780 square inches. What was the perimeter of the original picture before she trimmed it? (nearest inch)

(A) 132 in **(B) 136 in** **(C) 140 in** **(D) 144 in** **(E) 148 in**

10. In Southeast Idaho, the number of cougars in a protected area varies directly as the number of acres the area covers and inversely as the square of the number of grizzly bears in the area. The protected area in Teton County covers 600 acres, has 88 cougars and has 12 grizzly bears. If the protected area in Fremont County covers 964 acres and has 22 grizzly bears, how many cougars live in this area?

(A) 42 **(B) 62** **(C) 82** **(D) 102** **(E) 122**

11-12. Consider the circle on the right with center O.
Chord $\overline{AC}$ intersects diameter $\overline{BD}$ at point E.
$\overline{AC} \perp \overline{BD}$, $BD = 20$, and $AC = 16$.



Problems 11, 12

11. BE = _____. (nearest tenth)

(A) 3.9 **(B) 4.0** **(C) 4.1**
(D) 4.2 **(E) 4.3**

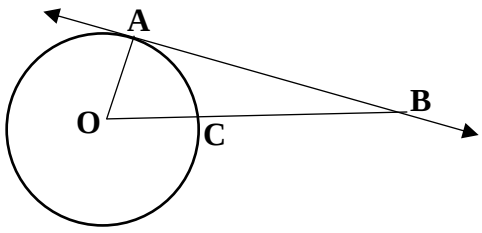
12. Find the area of sector AOD. (nearest tenth)

(A) 107.4 **(B) 108.5** **(C) 109.6** **(D) 110.7** **(E) 111.8**

13. Triangle ABC is similar to triangle DEF. AB = 24, BC = 18, AC = 20, and EF = 15. Find the perimeter of triangle DEF. (nearest tenth)

(A) 51.7 **(B) 51.9** **(C) 52.1** **(D) 52.3** **(E) 52.5**

14-15. Line $\overleftrightarrow{AB}$ is tangent to the circle at point A.
Point O is the center of the circle.
$AO = x$, $AB = 5x - 5$, and $BC = 4x - 4$.



Problems 14, 15

14. Find the perimeter of triangle AOB. (nearest tenth)

(A) 86.4 **(B) 87.6** **(C) 88.8**
(D) 90.0 **(E) 91.2**

15. Find the area of the region inside triangle AOB but outside sector AOC. (nearest whole number)

(A) 122 **(B) 125** **(C) 128** **(D) 131** **(E) 134**

16. The following points are the vertices of a right triangle. $A(-6, -2)$, $B(4, b)$, and $C(6, -2)$.
If $m\angle ABC = 90°$ and $b > 0$, then b = _____. (nearest hundredth)

(A) 2.47 **(B) 2.59** **(C) 2.71** **(D) 2.83** **(E) 2.94**

17. Consider triangle DEF with DE = 13, EF = 15, and DF = 17. Find the length of the longest median of the triangle. (nearest tenth)

(A) 11.5 (B) 12.3 (C) 13.1 (D) 13.9 (E) 14.7

18. Consider triangle GHI with a point J that lies on side $\overline{GI}$. GH = 24, HI = 36, IJ = 24 and ray $\overrightarrow{HJ}$ bisects $\angle GHI$. GJ = _____. (nearest tenth)

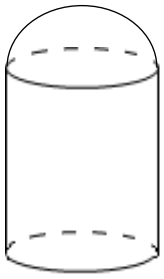
(A) 15.6 (B) 15.8 (C) 16.0 (D) 16.2 (E) 16.4

19. Consider triangle KLM with vertices $K(-8,-3)$, $L(2,7)$, and $M(5,-6)$. Triangle KLM is a/an _____ triangle.

(A) scalene (B) isosceles (C) equilateral (D) right (E) obtuse

20. A hemisphere sits on top of a cylinder as shown on the right. The total surface area of the figure shown is 1520 cm^2. The height of the cylinder is 3 times the radius. Find the volume of the figure shown on the right. (nearest whole number)

(A) 4536 cm^3 (B) 4540 cm^3 (C) 4544 cm^3
(D) 4548 cm^3 (E) 4552 cm^3



Problem 20

21. Dwayne "the Rock" Johnson weighs 21 stones. There are 8 stones in a hundredweight, 20 hundredweight in a ton, and 9.81 newtons in a kilogram. He weighs _____ newtons. (nearest whole number)

(A) 1160 (B) 1164 (C) 1168 (D) 1172 (E) 1176

22. The amount of a radioactive substance present is decreasing exponentially. At t = 0, there was 100 mg of the substance present. At t = 27 years, there was 59 mg of the substance present. Predict the amount present at t =144 years. (nearest hundredth)

(A) 5.88 mg (B) 5.92 mg (C) 5.96 mg (D) 6.00 mg (E) 6.04 mg

23. The graph of a cubic function has a local minimum at $A(7,-4)$ and a point of symmetry at $B(0,6)$. If a local maximum occurs at point $C(a,b)$, then $a+b=$ _____. (nearest whole number)

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

24. Consider a regular pentagon with a perimeter of 23. Find the area of a circle inscribed in the pentagon. (nearest tenth)

(A) 30.3 (B) 30.6 (C) 30.9 (D) 31.2 (E) 31.5

25. Given: $f(x)=\frac{8-3x}{9-2x}$. If $g(x)$ is the inverse function of $f(x)$, then $g(7)=$ _____. (nearest tenth)

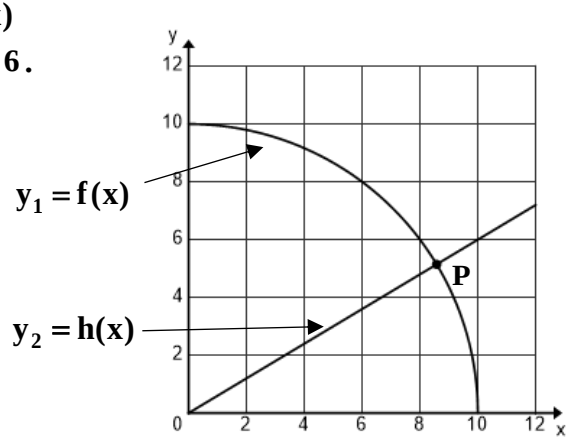
(A) 4.6 (B) 4.8 (C) 5.0 (D) 5.2 (E) 5.4

26. The graphs of quarter circle $y_1 = f(x)$ and line $y_2 = h(x)$ intersect in the first quadrant at point $P(a, b)$. $h(10) = 6$. $a + b =$ _____. (nearest hundredth)

(A) 13.60 (B) 13.63 (C) 13.66
(D) 13.69 (E) 13.72

27. Find the area of the first quadrant region bounded by the curves $y_1 = f(x)$, $y_2 = h(x)$, and $x = 0$. (nearest tenth)

(A) 51.5 (B) 51.7 (C) 51.9
(D) 52.1 (E) 52.3



Problems 26, 27

28. Erica left Muenster High School at 12:00 PM and cycled at 22 mph on a bearing of 15°. At 1:30 PM, Halle left Muenster High School and cycled at 25 mph on a bearing of 215°. How far apart were they at 4:30 PM? (nearest whole number)

(A) 163 mi (B) 165 mi (C) 167 mi (D) 169 mi (E) 171 mi

29. Consider the circle $x^2 + y^2 = 25$ and the point $A(10, 0)$. The length of a tangent line segment from point A to the circle is _____. (nearest hundredth)

(A) 8.44 (B) 8.55 (C) 8.66 (D) 8.77 (E) 8.88

30. Consider the ellipse $4x^2 + 3y^2 - 40x - 30y + 127 = 0$. The coordinates of the foci are (a, b) and (a, c). $b + c =$ _____. (nearest tenth)

(A) 9.6 (B) 9.8 (C) 10.0 (D) 10.2 (E) 10.4

31. Consider a hyperbola centered at the origin with a vertex at $(0, -2)$ and an asymptote with equation $y = x$. The coordinates of the foci are (a, b) and (a, c). $|b - c| =$ _____. (nearest tenth)

(A) 5.1 (B) 5.3 (C) 5.5 (D) 5.7 (E) 5.9

32. Which of the following is one of the four fourth roots of $-8 - 8\sqrt{3}\,i$?

(A) $1 - \sqrt{3}\,i$ (B) $-\sqrt{3} - i$ (C) $-1 - \sqrt{3}\,i$ (D) $-1 + \sqrt{3}\,i$ (E) $\sqrt{3} + i$

33. There are 830 seniors at Pokie High. Two hundred are taking BC Calculus, 196 are taking AB Calculus, and 192 are taking Statistics. Forty-four are taking all three courses, 80 are only taking BC Calculus, 94 are only taking AB Calculus, and 66 are only taking Statistics. How many seniors are not taking any of these three courses?

(A) 432 (B) 434 (C) 436 (D) 438 (E) 440

34. The parametric equations $x(t) = (v_0 \cos\theta)t$ and $y(t) = y_0 + (v_0 \sin\theta)t - 16.087t^2$ can be used to model the path of a projectile. If $v_0 = 110$ ft/s, $\theta = 36°$, and $y_0 = 120$ ft, how far will the projectile travel horizontally from the moment it is launched until the instant it hits the ground? (nearest whole number)

(A) 469 ft (B) 472 ft (C) 475 ft (D) 478 ft (E) 481 ft

35. Find the distance from the point $(6, 2, 10)$ to the plane $3x + 3y + z = 12$. (nearest hundredth)

(A) 5.05 (B) 5.16 (C) 5.27 (D) 5.38 (E) 5.49

36. Rick's backyard pool is shaped like a regular pentagon with each side equal to 20 feet. The pool has a constant depth of 4 feet. If the pipe filling the pool delivers 40 gallons of water per minute, how long will it take to completely fill the pool? (nearest tenth)

(A) 7.8 hr (B) 8.0 hr (C) 8.2 hr (D) 8.4 hr (E) 8.6 hr

37. Consider the sequence $3, 7, 12, 18, 25, 33, \ldots$ Find the sum of the first 20 terms of the sequence.

(A) 1958 (B) 1960 (C) 1962 (D) 1964 (E) 1966

38. A circle with center P has a radius of 6, and it is externally tangent to a circle with center O, which has a radius of 9. Points A and B lie on circle O. From P, tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn to the circle with center O. Find the area that is inside kite PAOB, but outside both circles. (nearest tenth)

(A) 9.7 (B) 9.9 (C) 10.1 (D) 10.3 (E) 10.5

39. The graph of $r = \dfrac{4\sin\theta + \sqrt{16\cos^2\theta + 16\sin^2\theta}}{16\cos^2\theta}$ is a parabola of the form $y = ax^2 + bx + c$.

$a + b + c =$ _____.

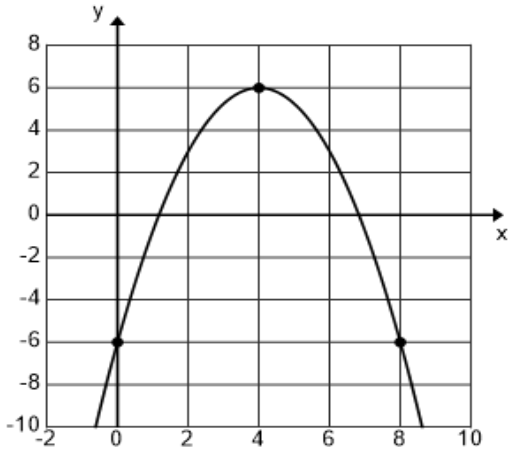
(A) $1\frac{1}{2}$ (B) $1\frac{3}{4}$ (C) $1\frac{7}{8}$ (D) 2 (E) $2\frac{1}{8}$

40. The focus of the graph shown on the right is the point $F(a, b)$. $b =$ _____. (nearest hundredth)

(A) 5.25 (B) 5.33 (C) 5.50
(D) 5.67 (E) 5.75

41. Consider the line tangent to the graph of the parabola at x = 6. The x-intercept of this tangent line is the point $H(c, 0)$. $c =$ _____. (nearest tenth)

(A) 6.8 (B) 7.0 (C) 7.2
(D) 7.4 (E) 7.6



Problems 40, 41

42-44. Consider the functions $f(x) = -.2(x+1)^2 + 10$ and $g(x) = 0.4(x-1)^2 - 4$.

42. Find the value c that satisfies the conclusion of the Mean Value Theorem for the function f(x) on $[-12, -2]$? (nearest hundredth)

(A) –7.24 **(B) –7.16** **(C) –7.08** **(D) –7.00** **(E) –6.92**

43. Find the area bounded by the graphs of $y_1 = f(x)$ and $y_2 = g(x)$. (nearest whole number)

(A) 85 **(B) 87** **(C) 89** **(D) 91** **(E) 93**

44. Find the volume of the solid of revolution generated when the region bounded by the graphs of $y_1 = f(x)$ and $y_2 = g(x)$ is revolved around the line $x = -6$. (nearest whole number)

(A) 3379 **(B) 3382** **(C) 3385** **(D) 3388** **(E) 3391**

x	f	f′	g	g′
1	–3.75	1.5	3	–4
2	–2	2	–2	–6
3	0.25	2.5	–9	–8
4	3	3	–18	–10

45. The table above shows values of differentiable functions f and g. If $h(x) = f\big(g(x)\big)$, then $h'(1) =$ _____.

(A) –16 **(B) –14** **(C) –12** **(D) –10** **(E) –8**

46. Consider the curve defined by the parametric equations $x(t) = 5t^{(1.5)} + 4$ and $y(t) = 3t^2 - 2$. The length of the curve from t = 1 to t = 3 is given by the expression _____.

(A) $\int_1^3 \sqrt{1 + \frac{144t}{225}}\, dt$ **(B)** $\int_1^3 \sqrt{\left(5t^{1.5} + 4\right)^2 + \left(3t^2 - 2\right)^2}\, dt$ **(C)** $\int_1^3 \sqrt{\frac{225t^2}{4} + 36t^2}\, dt$

(D) $\int_1^3 \sqrt{\frac{225t}{4} + 36t^2}\, dt$ **(E)** $\int_1^3 \sqrt{1 + \frac{144t^2}{225}}\, dt$

47. Consider the line tangent to the curve $3x^2 - y^3 = 2$ when x = 1. The y-intercept of this line is the point $(0, b)$. b = _____. (nearest tenth)

(A) –1.2 **(B) –1.0** **(C) –0.8** **(D) –0.6** **(E) –0.4**

48. Let x and y be functions of time t that are related by the equation $3x^2 + 2y^2 - 6xy = 99$. At time t = 2, x = 3, $y < 0$, and $\frac{dx}{dt} = 6$. Find the value of $\frac{dy}{dt}$ at t = 2. (nearest tenth)

(A) 7.0 **(B) 7.2** **(C) 7.4** **(D) 7.6** **(E) 7.8**

t (hr)	0	4	8	12	16	20	24
T (deg)	2°	10°	20°	24°	18°	7°	2°

49-51. The table above shows the temperature, T, at 4-hour intervals, on January 4th at a cabin in the mountains south of Pocatello, Idaho. Assume that $T'(t)$ is a differentiable function of time.

49. Use the table to find the approximate value of $T'(8)$ in _____ degrees per hour. (nearest hundredth)

(A) 1.69 **(B) 1.72** **(C) 1.75** **(D) 1.78** **(E) 1.81**

50. Using the table above, estimate the average temperature over the 24-hour time interval by using a trapezoidal approximation. The length of each subinterval is 4 hours. (nearest tenth)

(A) 12.9° **(B) 13.1°** **(C) 13.3°** **(D) 13.5°** **(E) 13.7°**

51. A professor at ISU modeled the temperature with the function $T(t) = 13 - 11\cos\left(\frac{t+1}{4}\right)$. Use his model to find the average temperature over the time interval $0 \le t \le 24$ hours. (nearest thousandth)

(A) 13.403° **(B) 13.514°** **(C) 13.625°** **(D) 13.736°** **(E) 13.847°**

52-53. The distribution of IQ scores is approximately normal with a mean of 100 and a standard deviation of 15.

52. It has been reported that President Biden has an IQ of 115. What percentile does that place him at?

(A) 80th **(B) 82nd** **(C) 84th** **(D) 86th** **(E) 88th**

53. Find the interquartile range for the distribution of IQ scores. (nearest tenth)

(A) 19.8 **(B) 20.2** **(C) 20.6** **(D) 21.0** **(E) 21.4**

Event	Probability
Obese	.33
Obese and has diabetes	.12
Is not obese and does not have diabetes	.59

54. A study of a large population of adult men in Montana was done in which researchers were interested in the relationship between obesity and diabetes. The results of the study are in the table above. If four adult men are selected at random, find the probability that at least two of them have diabetes. (nearest hundredth)

(A) 0.14 **(B) 0.16** **(C) 0.18** **(D) 0.20** **(E) 0.22**

55. The office manager at Aire Texas conducts follow-up phone calls with customers one week after work was done on their AC units. She reviews her notes on the phone calls to evaluate customer satisfaction. She wants to estimate the proportion of customers who are satisfied with the work done by her workers. Of the following, which is the smallest sample size needed to estimate the proportion of satisfied customers within five percent at a 96% confidence level.

(A) 320 **(B) 354** **(C) 388** **(D) 422** **(E) 456**

Runner	Mean 440 yd time in seconds	Standard Deviation in seconds
Bruce	48.8	0.4
Keenan	50.2	1.1
Kelly	49.7	0.6
Joe	48.2	0.8

56. Coronado High School has had the same four runners on the mile relay the last two years. The times, in seconds, for all four runners over the last two years are approximately normal with means and standard deviations shown in the table above. Assuming each runner's individual times are independent, find the probability that the total team time at a randomly selected track meet is less than 195 seconds. (nearest hundredth)

(A) 0.11 (B) 0.13 (C) 0.15 (D) 0.17 (E) 0.19

57. Events A and B are independent, P(A) = 0.5, and P(A and B) = 0.15. P(A or B) = _____. (nearest hundredth)

(A) 0.55 (B) 0.60 (C) 0.65 (D) 0.70 (E) 0.75

58. In Idaho, all high school junior boys are required to see how far they can run in 12 minutes. A distance of 1.632 miles is at the 30^{th} percentile for the distribution of distances for all junior boys. The distribution is approximately normal with a standard deviation of 0.225 miles. Find the mean distance run in 12 minutes for Idaho junior boys. (nearest hundredth)

(A) 1.75 mi (B) 1.78 mi (C) 1.81 mi (D) 1.84 mi (E) 1.87 mi

Days	0-2	3-5	6-7	Total
Heart Disease	566	288	146	1000
No Heart Disease	162	314	438	914
Total	728	602	584	1914

59-60. Researchers wanted to know if there is an association between exercise and heart disease. A large random sample of men in their 70s were studied. The average number of days each man exercised each week and his heart condition were recorded. Results of the study are in the table above. A Chi-Square Test for Association/Independence was performed.

59. Find the expected count for the 6-7 days/Heart Disease cell. (nearest whole number)

(A) 269 (B) 278 (C) 287 (D) 296 (E) 305

60. The contribution of the 0-2 days/No Heart Disease cell to the Chi-Square statistic is _____. (nearest whole number)

(A) 75 (B) 81 (C) 87 (D) 93 (E) 99

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Region ● 2023
Answer Key

1. B	**21. C**	**41. B**
2. D	**22. D**	**42. D**
3. D	**23. A**	**43. A**
4. A	**24. E**	**44. C**
5. B	**25. C**	**45. D**
6. A	**26. E**	**46. D**
7. D	**27. A**	**47. B**
8. A	**28. E**	**48. B**
9. D	**29. C**	**49. C**
10. A	**30. C**	**50. D**
11. B	**31. D**	**51. B**
12. D	**32. C**	**52. C**
13. A	**33. D**	**53. B**
14. D	**34. E**	**54. C**
15. B	**35. A**	**55. D**
16. A	**36. E**	**56. A**
17. E	**37. B**	**57. C**
18. C	**38. A**	**58. A**
19. B	**39. C**	**59. E**
20. B	**40. D**	**60. E**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2023

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. The absolute pressure P, in pascals (Pa), on an object at a given depth h, in meters (m), is given by $P = P_0 + \rho gh$, where P_0 is the atmospheric pressure at the surface of the water, ρ is the density of the water, and g is the acceleration of gravity. Find the absolute pressure on an object 60 m below the surface of the ocean. $\left(P_0 = 1.013 \times 10^5 \text{ Pa}, \rho = 1023 \text{ kg/m}^3, g = 9.807 \text{ m/s}^2\right)$ (nearest whole number)

(A) 703,238 Pa (B) 703,242 Pa (C) 703,246 Pa (D) 703,250 Pa (E) 703,254 Pa

2. Ted met Terry at The Sportsman for breakfast. Terry ordered the deluxe omelet for $8.95 and orange juice for $2.35. Ted ordered eggs and pancakes for $9.75 and coffee for $1.95. The tax rate is 8.25%. If they added a $5.00 tip, what was the total cost?

(A) $29.75 (B) $29.80 (C) $29.85 (D) $29.90 (E) $29.95

3. Given: points $A(-6,-3)$, $B(6,-9)$, and $C(12,9)$. Line L_1 contains point C and is parallel to $\overleftrightarrow{AB}$. If point $D(-9,b)$ lies on line L_1, then $b =$ _____. (nearest tenth)

(A) 18.6 (B) 18.9 (C) 19.2 (D) 19.5 (E) 19.8

4. Pam's Pizza offers small, medium and large pizzas. On Friday night, they sold 138 pizzas. They sold 5 more small pizzas than large pizzas and they sold 11 more medium pizzas than small pizzas. How many medium pizzas did they sell?

(A) 54 (B) 55 (C) 56 (D) 57 (E) 58

5. Diane took a trip from Plano to Billings, a distance of 1424 miles. Her total drive time was 20 hr 30 min. She drove 512 miles in 7 hr 45 min on day 1. She drove 488 miles in 6 hr 36 min on day 2. What was her average speed on day 3? (nearest tenth)

(A) 68.6 mph (B) 68.9 mph (C) 69.2 mph (D) 69.5 mph (E) 69.8 mph

6. Mom is 28 years older than Cathy. Grandma's age is 3 more than 6 times Cathy's age. Twice Mom's age is 5 more than Grandma's age. How old is Cathy?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

7. Lance left Lubbock and cycled 10 miles west. Then he turned south and cycled 15 miles. Next, he turned east and cycled 25 miles. Finally, he turned south and cycled 10 miles. How far was Lance from where he started? (nearest tenth)

(A) 28.6 mi (B) 28.8 mi (C) 29.0 mi (D) 29.2 mi (E) 29.4 mi

8. Given: points $A(-6,-8)$, $B(4,10)$, and $C(-6,-2)$. Find the distance from point C to the midpoint of $\overline{AB}$. (nearest tenth)

(A) 5.8 (B) 6.0 (C) 6.2 (D) 6.4 (E) 6.6

9. Consider four consecutive positive even numbers in ascending order. Four times the sum of the first and second is 42 greater than three times the sum of the third and fourth. What is the largest number?

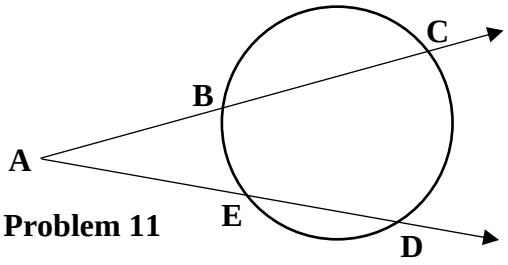
(A) 36 **(B) 38** **(C) 40** **(D) 42** **(E) 44**

10. The resistance of a length of copper wire is directly proportional to the length of the wire and inversely proportional to the cross-sectional area of the wire. The resistance of a 20.0-m length of copper wire with a cross sectional area of 3.46×10^{-6} m^2 is $0.100\ \Omega$. If the resistance of a 32.0-m length of copper wire is $0.626\ \Omega$, what is the diameter of the 32.0-m length of wire? (nearest hundredth)

(A) 1.02 mm **(B) 1.04 mm** **(C) 1.06 mm** **(D) 1.08 mm** **(E) 1.10 mm**

11. Consider the circle on the right. The measure of minor arc CD is 114° and the measure of minor arc EB is 48°. The measure of $\angle CAD$ is _____.

(A) 32° **(B) 33°** **(C) 34°**
(D) 35° **(E) 36°**



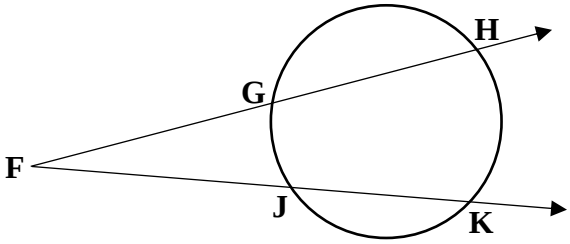
Problem 11

12. Consider equilateral triangle PQR with an inscribed circle. If the area of triangle PQR is 77.8, the area of the inscribed circle is _____. (nearest tenth)

(A) 45.8 **(B) 46.1** **(C) 46.4** **(D) 46.7** **(E) 47.0**

13. Consider the circle on the right. If FG = 18, GH = 16 and JK = 8, then FJ = _____. (nearest tenth)

(A) 20.7
(B) 20.9
(C) 21.1
(D) 21.3
(E) 21.5



Problem 13

14. Point C(4, 6) is rotated 90° counterclockwise about the origin to point E. Point E is reflected across the y-axis to point F. Point F is translated horizontally 15 units to the right to point G. If the coordinates of point G are (e, f), then $e+f =$ _____.

(A) 22 **(B) 23** **(C) 24** **(D) 25** **(E) 26**

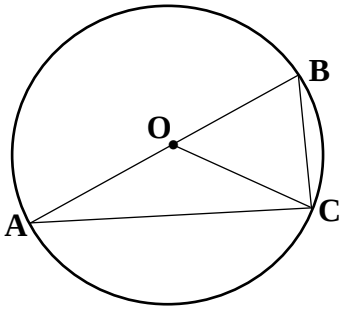
15. Consider kite ABCD. AB = BC = 12. CD = DA = 20. Diagonal AC = 12. Find the area of kite ABCD. (nearest whole number)

(A) 165 **(B) 168** **(C) 171** **(D) 174** **(E) 177**

16. The base of a pyramid is a regular hexagon with a perimeter of 72. The height of the pyramid is 18. Find the volume of the pyramid. (nearest whole number)

(A) 2236 **(B) 2239** **(C) 2242** **(D) 2245** **(E) 2248**

17-18. The circle shown on the right has an area of 452.
The measure of $\angle BAC$ is 30°.
Point O is the center of the circle.



Problems 17, 18

17. Find the area of triangle ABC. (nearest whole number)

(A) 122 **(B) 125** **(C) 128**
(D) 131 **(E) 134**

18. Find the area of sector BOC. (nearest tenth)

(A) 74.1 **(B) 74.4** **(C) 74.7** **(D) 75.0** **(E) 75.3**

19-20. The vertices of triangle DEF are $D(0, 8)$, $E(16, -4)$, and $F(14, 10)$.

19. Triangle DEF is a/an _____ triangle. Choose two of the given choices.

I. scalene II. isosceles III. equilateral IV. acute V. obtuse VI. right

(A) I, V **(B) II, IV** **(C) III, IV** **(D) I, VI** **(E) II, VI**

20. The area of triangle DEF is _____. (nearest whole number)

(A) 96 **(B) 98** **(C) 100** **(D) 102** **(E) 104**

21. A circle with center O has a diameter of 30. Chord $\overline{AB}$ is perpendicular to diameter $\overline{CD}$. If AB = 18, find the area bounded by chord $\overline{AB}$ and minor arc AB. (nearest tenth)

(A) 35.6 **(B) 35.9** **(C) 36.2** **(D) 36.5** **(E) 36.8**

22. The average price for a loaf of bread was \$0.22 on January 1, 1962. On January 1, 2022, the average price was up to \$2.50. If the price of bread is used to calculate the rate of inflation, what was the average annual rate of inflation from 1962 to 2022? (nearest hundredth)

(A) 4.09% **(B) 4.13%** **(C) 4.17%** **(D) 4.21%** **(E) 4.25%**

23. Consider an arithmetic sequence in which the fourth term is 31 and the twelfth term is 103. Find the sum of the first twenty-seven terms.

(A) 3261 **(B) 3263** **(C) 3265** **(D) 3267** **(E) 3269**

24. A math consultant earned \$80,000 his first year with Raytheon. Each year his salary increased by 8%. What was the total amount earned over the first 12 years? (nearest dollar)

(A) \$1,518,134 **(B) \$1,518,143** **(C) \$1,518,152** **(D) \$1,518,161** **(E) \$1,518,170**

25. A hawk is positioned on top of a tall building looking down at a mouse on the ground that is heading straight toward the base of the building at a constant speed of 6 inches per second. At t = 0, the angle of elevation from the mouse to the hawk is 15°. At t = 240 sec, the angle of elevation from the mouse to the hawk is 25°. How tall is the building? (nearest tenth)

(A) 75.2 ft (B) 75.4 ft (C) 75.6 ft (D) 75.8 ft (E) 76.0 ft

26. Consider a location on the earth at sea level and at a latitude of $36° 20' 10''$ north. If the radius of Earth is 3960 miles, then the linear speed of a point on the earth's surface at this location is _____. (nearest tenth)

(A) 832.9 mph (B) 834.0 mph (C) 835.1 mph (D) 836.2 mph (E) 837.3 mph

27. On Monday, there was heavy traffic at 7:30 AM and Kay only averaged 56 mph on her commute to work. She arrived 16 minutes late. On Tuesday, Kay left for work at 6:30 AM. Traffic was better and she averaged 70 mph and arrived on time. How far is her commute to work? (nearest tenth)

(A) 73.8 mi (B) 74.1 mi (C) 74.4 mi (D) 74.7 mi (E) 75.0 mi

28. Three of the roots of $f(x)$, a fourth-degree polynomial, are $-4, 5,$ and $1-\sqrt{3}$. If $f(x) = x^4 + bx^3 + cx^2 + dx + e$ and b, c, d and e are rational numbers, then $b+c+d+e =$ _____.

(A) 55 (B) 57 (C) 59 (D) 61 (E) 63

29. The range of the function $f(x) = \text{Cos}^{-1}(x)$ is _____.

(A) $[-1, 1]$ (B) $(-1, 1)$ (C) $[-\pi, \pi]$ (D) $(0, \pi)$ (E) $[0, \pi]$

30. If $\text{Sin}^{-1}(x) + \text{Sin}^{-1}(y) = \frac{\pi}{2}$, then $x^2 + y^2 =$ _____.

(A) 0.50 (B) 0.75 (C) 1.00 (D) 1.25 (E) 1.50

31. The coordinates of the foci of the hyperbola $4y^2 - x^2 + 8y - 4x - 4 = 0$ are (a, b) and (a, c). $|c - b| + a =$ _____. (nearest hundredth)

(A) 2.41 (B) 2.44 (C) 2.47 (D) 2.50 (E) 2.53

32. The perihelion of Earth's orbit about the sun is 147.095×10^6 km and the aphelion is 152.100×10^6 km. Find the eccentricity of Earth's orbit. (nearest ten-thousandth)

(A) 0.0167 (B) 0.0191 (C) 0.0215 (D) 0.0239 (E) 0.0263

33. Consider the sequence 5, 7, 11, 19, 35, 67,… The ninth term of the sequence is _____.

(A) 511 (B) 512 (C) 513 (D) 514 (E) 515

34. The graph of $x^2 + 4xy + 4y^2 - 1 = 0$ is _____.

(A) an ellipse (B) a hyperbola (C) a parabola (D) a line (E) two parallel lines

35. Planet X has 36-hour days and nine 40-day months. In month 5, the average temperature of the city of Xanadu varies sinusoidally each day with an average low temperature of 10° at t = 0 and an average high temperature of 90° at t = 18 hours. On average, the temperature is 60° or higher for _____ hours each day in month 5. (nearest tenth)

(A) 14.7 (B) 14.9 (C) 15.1 (D) 15.3 (E) 15.5

36. Consider the curve defined by the parametric equations $x(t) = e^{t+1}$ and $y(t) = e^{3t}$. The rectangular equation that represents the curve is $y = f(x)$. $f(10) =$ _____. (nearest tenth)

(A) 49.4 (B) 49.6 (C) 49.8 (D) 50.0 (E) 50.2

37. Consider the curve defined by the polar equation $r = \cot^2\theta\csc\theta$. The rectangular equation that represents the curve is $y = h(x)$. $h(8) =$ _____. (nearest tenth)

(A) 4.0 (B) 4.3 (C) 4.6 (D) 4.9 (E) 5.2

38. A 1.00-meter-long string is cut into two pieces. One piece is formed into a circle and the other piece is formed into a square. If the area of the circle is equal to the area of the square, what is the diameter of the circle? (nearest tenth)

(A) 15.0 cm (B) 15.2 cm (C) 15.4 cm (D) 15.6 cm (E) 15.8 cm

39. The general form of the equation of the plane passing through the point $P(2, 3, 5)$ and perpendicular to the vector $n = 7i - 4j + 6k$ is $ax + by + cz + d = 0$. $a + b + c + d =$ _____.

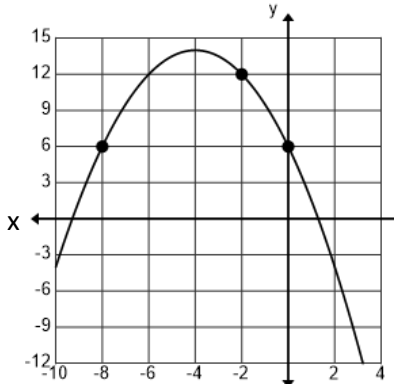
(A) –25 (B) –23 (C) –21 (D) –19 (E) –17

40. The equation of the directrix of the graph shown on the right is y = k. k = _____.

(A) 14.25 (B) 14.5 (C) 14.75
(D) 15 (E) 15.25

41. The shortest distance from the origin to the graph of the parabola is _____. (nearest hundredth)

(A) 1.21 (B) 1.23 (C) 1.25
(D) 1.27 (E) 1.29



Problems 40, 41

x	0	2	4	6	8	10
f(x)	6	11	20	21	2	-49

42. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = -9.5$?

(A) (0, 2) **(B) (2, 4)** **(C) (4, 6)** **(D) (6, 8)** **(E) (8, 10)**

43. The line normal to the curve $y = .25x^3 - x^2 + .75x - 1$ at x = 3 intersects the x-axis at the point (a, 0). a = _____. (nearest hundredth)

(A) 1.17 **(B) 1.28** **(C) 1.39** **(D) 1.50** **(E) 1.61**

44. Let f be a continuous function such that $\int_0^{15} f(x)dx = 12$, $\int_{15}^{20} f(x)dx = -4$, and $\int_{10}^{20} f(x)dx = 6$.

What is the value of $\int_0^{10} f(x)dx$?

(A) 2 **(B) 4** **(C) 6** **(D) 8** **(E) 10**

45. A particle moves along the x-axis so that at time $t \geq 0$ its acceleration is given by $a(t) = 6t$. At t = 0, the velocity of the particle is -6 and the position of the particle is 3. How far did the particle travel from t = 0 to t = 4? (nearest whole number)

(A) 45 **(B) 46** **(C) 47** **(D) 49** **(E) 51**

46. Consider the first quadrant region bounded above by the curve $y = 6 + (0.01)e^{3x}$, below by the x-axis, on the left by the y-axis and on the right by the line $x = 2$. Find the periphery of this region. (nearest tenth)

(A) 23.1 **(B) 23.4** **(C) 23.7** **(D) 24.0** **(E) 24.3**

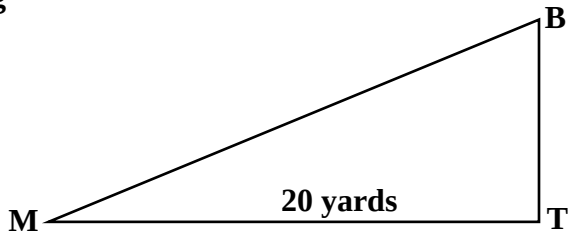
47. A north-south jogging trail intersects an east-west road at point D. At t = 0, a runner leaves point D traveling south at 7.5 mph. At the same time, a car is one mile east of point D traveling west at 30 mph. The distance between the runner and the car is a minimum at t = _____. (nearest tenth)

(A) 111.8 sec **(B) 112.9 sec** **(C) 114.0 sec** **(D) 115.1 sec** **(E) 116.2 sec**

48. Given: the parametric equations $x(t) = 2\sqrt{t}$ and $y(t) = \frac{t^2 - 1}{2}$. Find the value of $\frac{d^2y}{dx^2}$ when $t = 4$.

(A) –4 **(B) 2** **(C) 4** **(D) 6** **(E) 8**

49. Teresa (T) was 20 yards from a mouse (M) when she released a balloon (B) from the ground that began rising vertically at a constant rate of 10 feet per second. The rate of change of $\angle BMT$ at the instant the balloon is 80 feet above the ground is _____ rad/s. (nearest thousandth)



Problem 49

(A) 0.056 **(B) 0.060** **(C) 0.064**
(D) 0.068 **(E) 0.072**

50. The length of the curve $y = 3\cos\left(\frac{x}{4}\right)$ from $x = \pi$ to $x = 2\pi$ is given by

(A) $\int_{\pi}^{2\pi} \sqrt{1+\frac{9}{16}\cos^2\left(\frac{x}{4}\right)}\,dx$ **(B)** $\int_{\pi}^{2\pi} \sqrt{1+3\cos\left(\frac{x}{4}\right)}\,dx$ **(C)** $\int_{\pi}^{2\pi} \sqrt{1+12\sin^2\left(\frac{x}{4}\right)}\,dx$

(D) $\int_{\pi}^{2\pi} \sqrt{\left(1-\frac{3}{4}\sin\left(\frac{x}{4}\right)\right)}\,dx$ **(E)** $\int_{\pi}^{2\pi} \sqrt{1+\frac{9}{16}\sin^2\left(\frac{x}{4}\right)}\,dx$

51. The function $y = f(t)$ models the amount of a substance present at time t, in years. On March 1, 2022, (t = 0), there was 100 g present. The function satisfies the differential equation $\frac{dy}{dt} = -0.005y^2$. Find $f(t)$ and determine the amount present on March 1, 2040. (nearest gram)

(A) 10 g **(B) 20 g** **(C) 25 g** **(D) 50 g** **(E) 75 g**

52. The following list gives the five-number summary for the maximum bench press for a large group of Texas high school football players. 184, 220, 240, 320, 480
About what percent of the players had a maximum bench press between 220 and 480 pounds?

(A) 50% **(B) 67%** **(C) 75%** **(D) 86%** **(E) 95%**

53. The Potter County Sheriff's Department reports that its response time to emergency calls is approximately normally distributed with a mean of 18 minutes and a standard deviation of 6.5 minutes. Approximately what proportion of response times are over 25 minutes? (nearest hundredth)

(A) 0.14 **(B) 0.16** **(C) 0.18** **(D) 0.20** **(E) 0.22**

	Mean	Standard Deviation
# of workouts per year	150	70
Annual medical costs	$2000	$600

54. An insurance company did a study of a large group of females aged 30 to 35. The results of the study are in the table above. The correlation for these two variables is –0.91. Find the equation of the least-squares regression line and find the predicted annual medical cost for a female in this age group who works out 300 days per year. (nearest dollar)

(A) $812 **(B) $818** **(C) $824** **(D) $830** **(E) $836**

55. Over 2000 high school seniors in Ada County took the ACT test this school year. The scores closely followed the Normal distribution with a mean of 21.6 and a standard deviation of 4.6. Two students were selected at random from the group. Find the probability that the difference in their scores was greater than 7. (nearest thousandth)

(A) 0.141 (B) 0.176 (C) 0.212 (D) 0.247 (E) 0.282

56. Cindy performed 12 independent tests of the form $H_0 : \mu = 36$ versus $H_a : \mu < 36$, each at the $\alpha = 0.05$ significance level. What is the probability of committing a Type I error with at least 3 of the 12 tests? (nearest hundredth)

(A) 0.01 (B) 0.02 (C) 0.03 (D) 0.04 (E) 0.05

Type	Apple	Cherry	Apricot	Lemon
Frequency	28	20	36	16

57-58. A researcher in Big Timber, Montana wanted to test the claim that an equal proportion of people preferred each of the four types of pies listed in the table above. He surveyed a random sample of 100 people and asked each person to identify their favorite type of pie. The results are in the table above. Assume all conditions have been met to perform a chi-square test with the null hypothesis, H_0: favorite types of pies are evenly distributed across the four types, and with $\alpha = 0.05$.

57. Based on a p-value of _____, he rejected H_0 at the $\alpha = 0.05$ level. (nearest thousandth)

(A) 0.012 (B) 0.018 (C) 0.024 (D) 0.030 (E) 0.036

58. The apricot cell contributed _____ to the chi-square statistic. (nearest hundredth)

(A) 4.40 (B) 4.51 (C) 4.62 (D) 4.73 (E) 4.84

59. Give the correct order of the following from least to greatest in a normal curve.
I. 1st quartile II. value of the 30th percentile III. value of a z-score of -1

(A) I, II, III (B) I, III, II (C) III, II, I (D) III, I, II (E) II, I, III

Return	–5.0%	5.0%	15.0%	25.0%	35.0%
Probability	0.12	0.14	0.22	0.24	0.28

60. Cantu Investments handles accounts for people agreeing to invest a minimum of two million dollars. The average annual return on investments over the last twenty years is given in the table above. How much should Audrey expect to earn in one year if she invests $3,000,000?

(A) $565,000 (B) $576,000 (C) $587,000 (D) $598,000 (E) $609,000

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University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2023
Answer Key

1. E	**21. E**	**41. D**
2. D	**22. B**	**42. D**
3. D	**23. D**	**43. D**
4. B	**24. E**	**44. A**
5. B	**25. C**	**45. E**
6. C	**26. C**	**46. A**
7. D	**27. D**	**47. B**
8. A	**28. C**	**48. D**
9. B	**29. E**	**49. B**
10. C	**30. C**	**50. E**
11. B	**31. C**	**51. A**
12. E	**32. A**	**52. C**
13. C	**33. E**	**53. A**
14. D	**34. E**	**54. D**
15. E	**35. C**	**55. E**
16. D	**36. C**	**56. B**
17. B	**37. A**	**57. C**
18. E	**38. A**	**58. E**
19. E	**39. B**	**59. D**
20. C	**40. B**	**60. B**