HMMT November 2018
November 10, 2018

General Round

1. What is the largest factor of 130000 that does not contain the digit 0 or 5?

2. Twenty-seven players are randomly split into three teams of nine. Given that Zack is on a different team from Mihir and Mihir is on a different team from Andrew, what is the probability that Zack and Andrew are on the same team?

3. A square in the xy-plane has area A, and three of its vertices have x-coordinates $2, 0$, and 18 in some order. Find the sum of all possible values of A.

4. Find the number of eight-digit positive integers that are multiples of 9 and have all distinct digits.

5. Compute the smallest positive integer n for which

$$\sqrt{100 + \sqrt{n}} + \sqrt{100 - \sqrt{n}}$$

   is an integer.

6. Call a polygon *normal* if it can be inscribed in a unit circle. How many non-congruent normal polygons are there such that the square of each side length is a positive integer?

7. Anders is solving a math problem, and he encounters the expression $\sqrt{15!}$. He attempts to simplify this radical by expressing it as $a\sqrt{b}$ where a and b are positive integers. The sum of all possible distinct values of ab can be expressed in the form $q \cdot 15!$ for some rational number q. Find q.

8. Equilateral triangle ABC has circumcircle Ω. Points D and E are chosen on minor arcs AB and AC of Ω respectively such that $BC = DE$. Given that triangle ABE has area 3 and triangle ACD has area 4, find the area of triangle ABC.

9. 20 players are playing in a Super Smash Bros. Melee tournament. They are ranked $1 - 20$, and player n will always beat player m if $n < m$. Out of all possible tournaments where each player plays 18 distinct other players exactly once, one is chosen uniformly at random. Find the expected number of pairs of players that win the same number of games.

10. Real numbers x, y, and z are chosen from the interval $[-1, 1]$ independently and uniformly at random. What is the probability that

$$|x| + |y| + |z| + |x + y + z| = |x + y| + |y + z| + |z + x|?$$