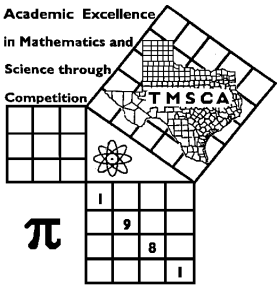


TMSCA HIGH SCHOOL MATHEMATICS TEST # 9 © JANUARY 27, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

2017-2018 TMSCA Mathematics Test Nine

1. **Evaluate** $2.7 \div \left(\frac{5}{9}\right)^{-1} - 7! + 5.8$.

   (A) -5034.14 **(B)** 5047.3 **(C)** -5029.34 **(D)** -5032.7 **(E)** -5040.94

2. **Seven billion, eight hundred seventy is subtracted from sixteen billion, four hundred sixty-one thousand, eight hundred seventy-two. What is the sum of the digits in the difference?**

   (A) 20 **(B)** 35 **(C)** 22 **(D)** 63 **(E)** 39

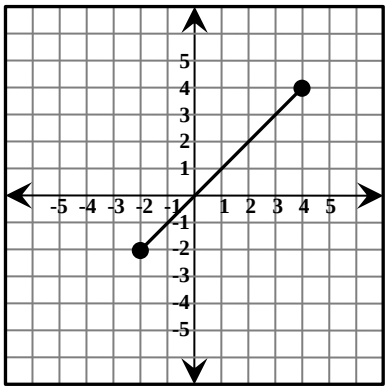
3. **The sum of the first four terms of an arithmetic sequence is 97, while the sum of the first eight terms is 234. Find the value of the first term.**

   (A) 23 **(B)** 10 **(C)** 20 **(D)** 11.5 **(E)** 20.5

4. **120 miles per hour = _____ yards per second. (nearest tenth)**

   (A) 58.7 **(B)** 176.0 **(C)** 117.3 **(D)** 88.0 **(E)** 44.0

5. **Which of the following is an equation of the perpendicular bisector of the line segment illustrated?**



   A) $x + y = 2$ **(B)** $x - y = 0$ **(C)** $x + y = 0$

   (D) $x + y = 8$ **(E)** $x + y = -4$

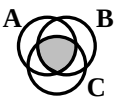
6. **Simplify:** $\frac{(n+3)!}{(n-1)!} \times \frac{1}{n^2} \div \frac{(n+2)!}{n!}$

   (A) $\frac{n+3}{n^3 - n^2}$ **(B)** $\frac{n+3}{n^2 - n}$ **(C)** $\frac{n+3}{n}$ **(D)** $n^2 + 3n$ **(E)** $\frac{n^2 + 5n + 6}{n^3}$

7. **The shaded region in which of the following Venn diagrams represents the set** $(A \cap C) \cap (B \cup C)$**?**

   (A) 

   (B) 

   (C) 

   (D) 

   (E) 

8. **Decaffeinated coffee usually contains 5% of the caffeine present in the same amount of regular coffee. How much decaffeinated coffee should Ilea add to 2 ounces of regular coffee to achieve a blend that contains 50% of the caffeine present in regular coffee alone? (nearest ounce)**

   (A) 24 oz. **(B)** 28 oz. **(C)** 36 oz. **(D)** 27 oz. **(E)** 18 oz.

9. **A box contains 6 black marbles, 8 red marbles and 8 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 red marbles?**

   (A) 2:53 **(B)** 42:1289 **(C)** 2:55 **(D)** 42:1331 **(E)** 42:79

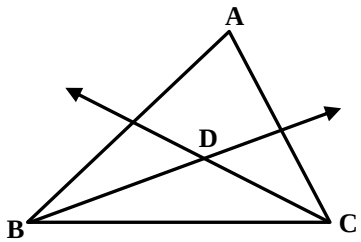
10. How many proper fractions have a denominator of 24?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

11. Point P(3, 7) lies on the *x*-*y* plane. P is translated horizontally 4 units left to point Q. Then Q is translated 9 units down to point R. Finally, R is reflected across the line $y = -x$ to point S with coordinates (x, y). Find $x + y$.

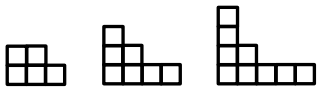
(A) 3 (B) 5 (C) 0 (D) −3 (E) 1

12. In triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet at D. Find $m\angle BDC$ if $m\angle BAC = 78°$.



(A) 102° (B) 156° (C) 132° (D) 116° (E) 129°

13. The three shapes below are made up of one-unit squares. If the pattern continues, the area of the figure with a perimeter of 110 units is _____square units.



(A) 53 (B) 48 (C) 55 (D) 57 (E) 49

14. A square has side lengths of 8 cm. If the square's width is quadrupled and the length is halved, what is the percent change in the area of the shape?

(A) 75% (B) 100% (C) 125% (D) 200% (E) $133\frac{1}{3}$ %

15. What is the Real value solution set for $7 - 3|2x - 5| > -5$?

(A) $\left\{x \left| -\frac{9}{2} < x < \frac{1}{2}\right.\right\}$ **(B)** $\left\{x \left| \left\{x < \frac{1}{2}\right\} \cup \left\{x > \frac{9}{2}\right\}\right.\right\}$ **(C)** $\left\{x \left| \left\{x < -\frac{1}{2}\right\} \cup \left\{x > \frac{9}{2}\right\}\right.\right\}$

(D) $\left\{x \left| \frac{1}{2} < x < \frac{9}{2}\right.\right\}$ **(E)** $\left\{x \left| -\frac{1}{2} < x < \frac{9}{2}\right.\right\}$

16. Find $f(g(a+2))$ when $g(x) = 2x - 3$ and $f(x) = 4 - 2x$

(A) $2 - 4a$ (B) $4 - 2a$ (C) $2 - 2a$ (D) $2 + 4a$ (E) $6 - 4a$

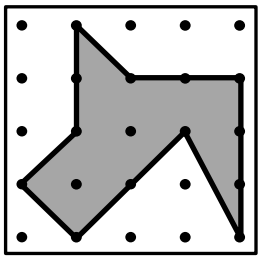
17. Let $\mathbf{A} = \begin{bmatrix} 2 & 7 \\ -13 & -9 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 9 \\ 2 & -5 \end{bmatrix}$ and $\mathbf{C} = \mathbf{AB}$. Find $\mathbf{C}_{1,2}$.

(A) −18 (B) 63 (C) −26 (D) −17 (E) −27

18. An operation "Ω" is defined by $a\Omega b = a^b - ab$. What is the value of $(-1\Omega 2)\Omega(3\Omega 2)$?

(A) −2 (B) 18 (C) −14 (D) 1 (E) 0

19. A rubber band was stretched on the geoboard to form this 9-sided figure. What is the area?



(A) 6 units^2 **(B)** 7 units^2 **(C)** 7.5 units^2

(D) 7.5 units^2 **(E)** 8 units^2

20. Determine the amplitude of $f(x)=7+4\cos\left[6\pi(x-3)\right]$.

(A) 7 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

21. Let p and q be the roots of $2x^2-5x+3=0$. **Find** $p^3+3p^2q+3pq^2+q^3$

(A) 31.25 **(B)** 15.625 **(C)** −15.375 **(D)** −7.875 **(E)** 7.375

22. $\angle A$ **and** $\angle B$ **are supplementary. If** $m\angle A=7x+6$ **and** $m\angle B=3x+4$**, the measure of the larger angle is:**

(A) 125° **(B)** 60° **(C)** 55° **(D)** 57° **(E)** 17°

23. Given the geometric sequence -48**, a, b, c,** -3**,… find the smallest possible value the sum of the sequence.**

(A) −32 **(B)** −64 **(C)** 32 **(D)** −48 **(E)** −96

24. What is the reference angle of $\frac{35\pi}{3}$ **?**

(A) $\frac{2\pi}{3}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** π **(E)** $\frac{\pi}{2}$

25. How many distinguishable 3-letter arrangements can be made with the letters "MATH MEET"?

(A) 60 **(B)** 336 **(C)** 96 **(D)** 300 **(E)** 120

26. A plane is 200 miles north and 110 miles east of an airport. What bearing should the plane take to fly directly to the airport? (nearest degree)

(A) 29° **(B)** 241° **(C)** 209° **(D)** 61° **(E)** 51°

27. Find the area of the ellipse $16x^2+100y^2=1600$.

(A) 400π **(B)** 20π **(C)** 14π **(D)** 40π **(E)** 196π

28. Which of the following functions, $f(x)=$_____ **is neither even nor odd?**

(A) $\sin(3x)$ **(B)** $|x|-5$ **(C)** $7x^4-9$ **(D)** $4x^3-8$ **(E)** $\cos(2x)$

29. The graph of $f(x)=\left(x^3-6x^2+12x-8\right)\div\left(x^2-4\right)$ **has a vertical asymptote(s) at:**

(A) $x=2$ **(B)** $x=-2$ **(C)** $x=2$ **and** $x=-2$ **(D)** $x=0$ **(E)** **no vertical asymptotes**

30. In a survey of 82 students, 32 listened to country music, 42 listened to pop music and 22 listened to both. How many students listened to neither?

(A) 20 **(B)** 30 **(C)** 22 **(D)** 6 **(E)** 16

31. The base radius of a cone is 9 cm, and the vertex angle is 42°. Find the total surface area of the cone. (nearest square centimeter)

(A) $965\ \text{cm}^2$ **(B)** $1675\ \text{cm}^2$ **(C)** $710\ \text{cm}^2$ **(D)** $1420\ \text{cm}^2$ **(E)** $333\ \text{cm}^2$

32. What is the sum of the digits in the tens place and units place of $7^{(102)}$ **?**

(A) 1 **(B)** 3 **(C)** 7 **(D)** 9 **(E)** 13

33. Let $f''(x) = 42x + 6$, $f(-1) = 3$ **and** $f(1) = 7$. **Find** $f(2)$.

(A) -32 **(B)** 91 **(C)** 67 **(D)** 60 **(E)** 22

34. Which of the following mathematicians developed a simple ancient algorithm for finding all prime numbers to a given limit?

(A) Gauss **(B)** Germain **(C)** Mersenne **(D)** Euler **(E)** Eratosthenes

35. If $25^{x-y} = 125$ **and** $4^{x+y} = 128$ **then** $y =$ **?**

(A) 0.25 **(B)** 1 **(C)** 1.75 **(D)** 2 **(E)** 2.5

36. Point $P(5,-3)$ **is the midpoint of the line segment with endpoints** $Q(-8, y)$ **and** $R(x,4)$. **Find** $x + y$

(A) 6 **(B)** 8 **(C)** -1 **(D)** 28 **(E)** 20

37. Let the "1" at the top of Pascal's triangle be row 0. Determine the fifth number in row 21.

(A) 1330 **(B)** 20349 **(C)** 5985 **(D)** 1140 **(E)** 4845

38. Coach Euclid's UIL team consists of 9 boys and 6 girls. How many different 4-member teams consisting of at least one girl could he make up?

(A) 1224 **(B)** 1260 **(C)** 504 **(D)** 1239 **(E)** 540

39. If $\frac{3x+1}{4x-3} + \frac{2x-1}{3x+5} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$ **then** $\frac{A+B+C}{P+Q+R} =$ **?**

(A) $\frac{29}{7}$ **(B)** $\frac{13}{31}$ **(C)** $\frac{33}{8}$ **(D)** $\frac{49}{31}$ **(E)** $\frac{29}{31}$

40. Given that $x + y = 9$ **and** $xy = 15$, **find** $x^3 + y^3$.

(A) 54 **(B)** 324 **(C)** 1134 **(D)** 459 **(E)** 594

41. The statement $(3x-7)(x+2) = 3x^2 + 6x - 7x - 14$ **is an example of _____property.**

(A) Distributive **(B)** Commutative **(C)** Associative

(D) Transitive **(E)** Identity

42. The length of the sides of triangle PQR are the roots of $f(x) = x^3 - 19x^2 + 113x - 207$. **Find the area of triangle PQR. (nearest tenth)**

(A) 4.7 **(B)** 18.6 **(C)** 9.3 **(D)** 14.0 **(E)** 3.1

43. Find $\lim_{x\to 0}\frac{3\sin(2x)}{x}$.

(A) 0 **(B)** 1 **(C)** 3 **(D)** 6 **(E)** undefined

44. Let $f(x)$ **be continuous on** $[a,b]$. **If** $F(x)$ **is any antiderivative of** $f(x)$, **then** $\int_a^b f(x)dx = F(b)-F(a)$. **This theorem is known as:**

(A) Intermediate Value Theorem **(B)** Sandwich Theorem **(C)** Mean Value Theorem

(D) Fundamental Theorem of Calculus **(E)** Fundamental Theorem of Algebra

45. Use the table of values to create a polynomial function and find K.

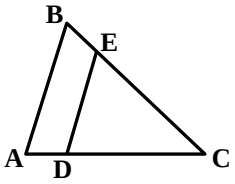
X	1	2	3	4	5	…	18
Y	12	30	70	150	288		K

(A) 13,020 **(B)** 18,390 **(C)** 5,578 **(D)** 15,550 **(E)** 6,265

46. Given $f(x)=\frac{5x-2}{3x+1}$, **find** $f^{-1}(3)$.

(A) $\frac{13}{10}$ **(B)** $-\frac{3}{4}$ **(C)** $-\frac{5}{4}$ **(D)** $-\frac{10}{13}$ **(E)** $-\frac{1}{4}$

47. Triangle ABC and triangle CDE exist such that $AB \parallel DE$, $AB = 9'$, $BC = 12'$ **and** $BE = 3'$. **Find DE.**



(A) 6.5’ **(B)** 6.75’ **(C)** 7’ **(D)** 7.25’ **(E)** Not enough information

48. $14_{16}+12_8+10_4 =$ _____$_2$

(A) 100101 **(B)** 100000 **(C)** 100001 **(D)** 100010 **(E)** 101001

49. Find the slope of the graph of $2x^2+3y^2=11$ **at the point** $(-2,1)$.

(A) $-\frac{19}{3}$ **(B)** $-\frac{4}{3}$ **(C)** $\frac{19}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{3}$

50. Simplify: $(b^3)\left(\frac{(b^{-2})(\sqrt[3]{b})}{b^{-1}}\right)$

(A) $\sqrt[3]{b}$ **(B)** $(\sqrt[3]{b})^5$ **(C)** $(\sqrt[3]{b})^7$ **(D)** $(\sqrt[3]{b})^{-1}$ **(E)** b

51. If $\int_5^{12} f(x)dx = 19.5$, **calculate the value of** $\int_5^{12}[3f(x)+5]dx$.

(A) 93.5 **(B)** 63.5 **(C)** 58.5 **(D)** 54.5 **(E)** 70.5

52. How many solutions are there to $7x+8y=221$ such that $x, y \in \mathbb{Z}^+$.

(A) 4 (B) 24 (C) 7 (D) 18 (E) 8

53. Boyd has pennies, nickels and dimes. He puts them in stacks of 5 coins. What is the maximum number of different stacks he could make?

(A) 35 (B) 56 (C) 70 (D) 21 (E) 28

54. If $y^2 = 5-12i$ and $y^3 = -9-46i$ where $y = a+bi$ then $a+b=$?

(A) −38 (B) −62 (C) 1 (D) 5 (E) 6

55. Find the sum of all 3-digit numbers whose digits have a sum of eight and whose digits can all be used to form a perfect cube.

(A) 1925 (B) 861 (C) 915 (D) 1420 (E) 1776

56. $f(x) = ax^5 + bx^3 + cx + 8$. If $f(-5) = 52$ then $f(5) =$?

(A) 44 (B) −36 (C) −44 (D) −52 (E) −60

57. The chart shows the losses and gains in an investment over the course of five years. What was the average annual growth rate over the course of the five years? (nearest hundredth of a percent)

Year	1	2	3	4	5
Percent Growth	+ 10%	–5 %	–3 %	+9%	+6%

(A) +3.40% (B) +2.87% (C) +3.21% (D) +2.91% (E) +3.17%

58. Which of the following are the side lengths of a scalene obtuse triangle?

(A) 5, 9, 9 (B) 8, 10, 12 (C) 5, 24, 25 (D) 5, 12, 13 (E) 9, 9, 14

59. $f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} \ldots$. Find the digit in the 10^{-8} place of $f(5)$.

(A) 4 (B) 5 (C) 2 (D) 8 (E) 0

60. The repeating decimal 0.545454… in base 7 can be written as which of the following fractions in base 7?

(A) $\frac{36}{101_7}$ (B) $\frac{3}{11_7}$ (C) $\frac{3}{22_7}$ (D) $\frac{16}{22_7}$ (E) $\frac{6}{11_7}$

2017 – 2018 TMSCA Mathematics Test Nine Answers

1. D	**21. B**	**41. A**
2. C	**22. A**	**42. C**
3. E	**23. E**	**43. D**
4. A	**24. C**	**44. D**
5. A	**25. C**	**45. D**
6. C	**26. C**	**46. C**
7. E	**27. D**	**47. B**
8. C	**28. D**	**48. D**
9. A	**29. B**	**49. E**
10. B	**30. B**	**50. C**
11. A	**31. A**	**51. A**
12. E	**32. E**	**52. A**
13. C	**33. D**	**53. D**
14. B	**34. E**	**54. C**
15. D	**35. B**	**55. E**
16. A	**36. B**	**56. B**
17. D	**37. C**	**57. C**
18. B	**38. D**	**58. C**
19. C	**39. C**	**59. D**
20. D	**40. B**	**60. D**

2017-2018 TMSCA Mathematics Test Nine Select Solutions

3. Solve the system $97=\frac{4}{2}(a+a+3d)$ **and** $234=\frac{8}{2}(a+a+7d)$ **for** $a=20.5$ **and** $d=2.5$

8. $0.05x+2=0.5(x+2)$ **to get** $2.2\overline{}$ **pounds which rounds to 36 oz.**

15. $-3|2x-5|>-12$ **then** $|2x-5|<4$ **for** $-4<2x-5<4$ **and** $\frac{1}{2}<x<\frac{9}{2}$

19. Let *p* be the number of grid points on the perimeter, and *i* be the number of grid points on the interior, then $\frac{p+2i-2}{2}=\frac{13+4-2}{2}=7.5$

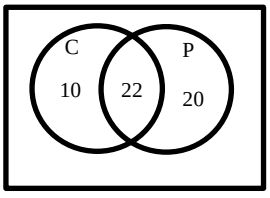
21. This is the polynomial expansion of the cube of the sum of the roots or $\left(-\frac{b}{a}\right)^3=\left(\frac{5}{2}\right)^3=\frac{125}{8}$

23. $-3=-48r^4$ **for** $r=\pm\frac{1}{2}$ **and possible sums of** -96 **and** -32 **with the** -96 **as the smaller sum.**

25. There are 5 distinct letters, M, A, T, H and E, and 2 letters that each repeat once M and T, so the options are"

No repeats: ${}_5P_3=60$
One letter repeated (3 possibilities and 4 options for the remaining letter each time): $3\times4\times\frac{3!}{2}=36$
For 96 possibilities.

30. $82-10-22-20=30$



32. 7^x **forms a pattern in the last two digits, 07, 49, 43, 01, 07, 49, 43, 01 that repeats every 4. So** $102\div2$ **has a remainder 2, and the second term is 49 for a sum of 13.**

37. The numbers in each row of Pascal's triangle are the expansion of $(1+1)^n$ **, so the 5th term in the 21st row will be** ${}_{21}C_4(1)^{17}(1)^4$ **or just 5985.**

38. $({}_6C_1)({}_9C_3)+({}_6C_2)({}_9C_2)+({}_6C_3)({}_9C_1)+({}_6C_4)({}_9C_0)=1239$

42. This is a play on Heron's theorem for the area of a triangle. The semi-perimeter is half the sum of the roots or 9.5, then $a=\sqrt{9.5f(9.5)}\approx9.3$

43. Use $\lim_{x\to0}\frac{\sin x}{x}=1$ **for** $\lim_{x\to0}\frac{3\sin(2x)}{x}=\lim_{x\to0}\left[6\times\frac{\sin(2x)}{2x}\right]=6$

51. $3(19.5)+5(12-5)=93.5$

56. Let the sum of all the odd powered terms be C when $x=-5$ **for** $52=C+8$ **and** $C=44$**. Then** $f(5)=-44+8=-36$

59. This is the McClaurin series expansion of $f(5)=\sin5+\cos5$**. Make sure your calculator is set in radians and evaluate for a** 10^{-8} **place 8.**