1^{st} Annual Harvard-MIT November Tournament

Saturday 8 November 2008

Individual Round

1. [**2**] Find the minimum of $x^2 - 2x$ over all real numbers x.

2. [**3**] What is the units digit of 7^{2009}?

3. [**3**] How many diagonals does a regular undecagon (11-sided polygon) have?

4. [**4**] How many numbers between 1 and $1,000,000$ are perfect squares but not perfect cubes?

5. [**5**] Joe has a triangle with area $\sqrt{3}$. What's the smallest perimeter it could have?

6. [**5**] We say "s grows to r" if there exists some integer $n > 0$ such that $s^n = r$. Call a real number r "sparse" if there are only finitely many real numbers s that grow to r. Find all real numbers that are sparse.

7. [**6**] Find all ordered pairs (x, y) such that
$$(x - 2y)^2 + (y - 1)^2 = 0.$$

8. [**7**] How many integers between 2 and 100 inclusive *cannot* be written as $m \cdot n$, where m and n have no common factors and neither m nor n is equal to 1? Note that there are 25 primes less than 100.

9. [**7**] Find the product of all real x for which
$$2^{3x+1} - 17 \cdot 2^{2x} + 2^{x+3} = 0.$$

10. [**8**] Find the largest positive integer n such that $n^3 + 4n^2 - 15n - 18$ is the cube of an integer.