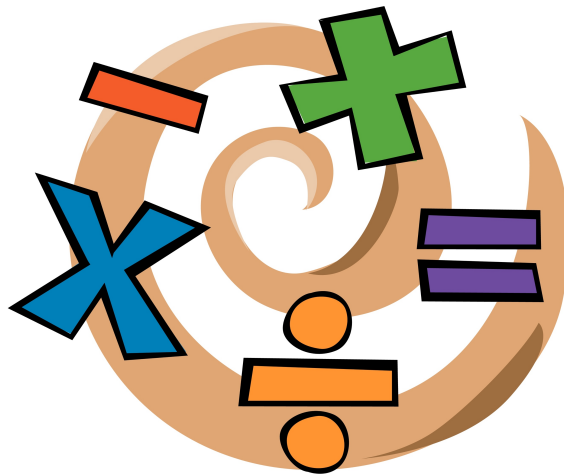# Mathematics

Invitational A • 2021

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1. Solve for k: $k - 20 \div (3 + 1) \times 2 + 0! = 22$

   (A) 25   (B) 26   (C) 28   (D) 30   (E) 31

2. Phil Detank travels 25 miles each way driving to and from work every day, Monday through Friday. His vehicle has a 15 gallon tank and averages 20 mpg. The tank is full when he heads to work on Monday. How much will it cost him to fill his tank when he gets home from work on Friday if gas costs $2.35 a gallon? (nearest cent)

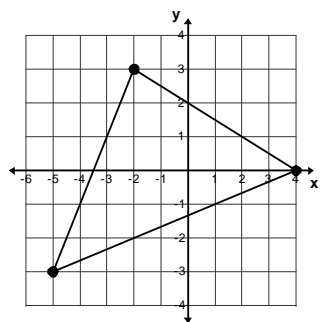   (A) $11.75   (B) $14.69   (C) $28.13   (D) $29.38   (E) $35.25

3. If P = {2,3,5,7,11}, T = {1,3,6,10,15}, F = {1,2,3,5,8}, and L = {1,2,3,4,7}, then $(P \cup T) \cap (F \cup L)$ contains how many elements?

   (A) 1   (B) 4   (C) 5   (D) 6   (E) 10

4. A line with a slope of $-\frac{5}{6}$ intersects the x-axis at x = 4 and intersects the y-axis at y = ?

   (A) $2\frac{1}{2}$ (B) $3\frac{1}{6}$ (C) $3\frac{1}{3}$ (D) $3\frac{2}{3}$ (E) $4\frac{5}{6}$

5. The triangle shown is considered to be which of the following types of triangles?

   (A)cute   (O)btuse   (R)ight   (E)quilateral   (S)calene   (I)sosceles

   (A) O & I   (B) O & S   (C) A & S   (D) R & S   (E) R & I

6. Papa Jawn is three times as old as Dom Knowles and Lil Seizer is 3 years younger than Dom. The sum of their ages 3 years ago was 63 years. How long from now will Papa's age be equal to the sum of the ages of Lil and Dom?

   (A) 12 yrs   (B) 15 yrs   (C) 18 yrs   (D) 27 yrs   (E) 45 yrs

7. Find the area of $\triangle ABC$. (nearest tenth)
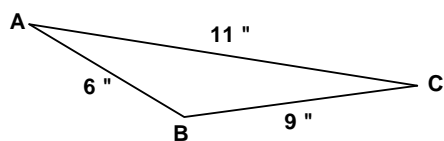
   (A) 26.0 in^2 (B) 27.0 in^2 (C) 24.8 in^2 (D) 22.4 in^2 (E) 21.2 in^2

8. If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is a(n) _____ triangle.

    (A) obtuse   (B) scalene   (C) acute   (D) isosceles   (E) right

9. If $-2, -1$, and 3 are the roots of $2x^3 + bx^2 + cx + d = 0$, then $b + c + d$ equals?

    (A) -26 (B) -13 (C) 12 (D) 13 (E) 14

10. Sir Cal Puhl is pouring a rectangular concrete patio to put his circular hot tub on. The diameter of the hot tub is 10 feet. The dimensions of the patio is 14 feet by 18 feet. What percent of the area of the patio is covered by the tub? (nearest whole percent)

    (A) 8%   (B) 12%   (C) 19%   (D) 21%   (E) 31%

11. If $\dfrac{2x-1}{x-2} + \dfrac{3x+2}{x+1} = \dfrac{ax^2 + bx + c}{dx^2 + ex + f}$, then $(a + b + c) \div (d + e + f)$ equals:

    (A) -2.5 (B) -0.5 (C) 1.5 (D) 0.75 (E) 2
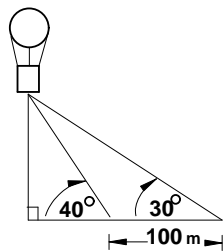
12. Let $a^4 \div b^3 \times a^{-5} \times b^6 \div (a^1)^7 \times b^2 = a^m \times b^n$. Find $m \times n$.

    (A) -1 (B) -9 (C) -40 (D) -60 (E) -88

13. $\triangle ABC$ is an isosceles triangle, where segment $\overline{BX}$ is the median to the base $\overline{AC}$. Find AB if AC = 8 in and BX = 4 in. (nearest tenth)

    (A) 8.9"   (B) 3.5"   (C) 6"   (D) 5.7"   (E) 4"

14. Three cables from an observation balloon are attached to the level ground below as show below. How high is the observation balloon? (nearest meter)

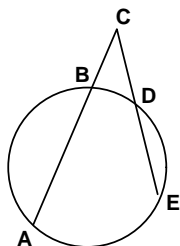   (A) 221 m   (B) 160 m   (C) 120 m   (D) 321 m   (E) 185 m

15. Find the focus of the graph of $y^2 + 6y + 4x + 25 = 0$

    (A) $(-1, -3)$ (B) $(-2, -3)$ (C) $(-3, -3)$ (D) $(-4, -3)$ (E) $(-5, -3)$

16. Which type of conic is the graph of the equation $x + 7y^2 + 3y + 5 = 0$?

    (A) circle   (B) ellipse   (C) hyperbola   (D) parabola   (E) not a conic

17. **Given: AB = 8 cm, ED = x cm, BC = 3 cm, and DC = x − 1 cm. Find CE. (nearest cm)**

(A) 9 cm   (B) 8 cm   (C) 7 cm   (D) 6 cm   (E) 5 cm

18. **The average rate of change of f(x) = x^2 + bx + 1 from x = 2 to x = 8 is 15. Find the value of b.**

(A) 3   (B) 3.75   (C) 5   (D) 6   (E) 7.5

19. **Which of the following statements is/are true for** $f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ -x^2 & \text{if } 0 < x \end{cases}$ **?**
   I. f is continuous at x = 0   II. $f'_-(0)$ exists   III. $f'_+(0)$ exists   IV. f is differentiable at x = 0

(A) I only   (B) I & IV   (C) I, II, & III   (D) All of them   (E) none of them

20. **I. C. Yew wants to find the probability that his class officers consisting of a president, treasurer, and secretary are randomly selected such that one of them has glasses, one has contacts, and one has no eyewear. How many members will be in the successful event's sample space?**

(A) 3   (B) 6   (C) 8   (D) 9   (E) 27

21. **Which of the following mathematicians developed formulas to calculate the sum of the first n integers, their squares and their cubes?**

(A) Diophantus   (B) Agnesi   (C) Aryabhata   (D) Erastosthenes   (E) Bigollo

22. **$(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ... + 20^3) \div (1 + 2 + 3 + 4 + 5 + ... + 20)$ = ?**

(A) 55   (B) 210   (C) 400   (D) 1,100   (E) 2,870

23. **The operation $\odot$ is defined as x $\odot$ y = $x^y - y^x$, where x, y are integers.**
    Find the value of (1 $\odot$ 2)$\odot$(2 $\odot$ 1).

(A) − 2   (B) − 1   (C) 0   (D) $\frac{1}{2}$ (E) 2

24. **Sluggy Snail crawled 2.5 feet in 45 seconds. How many inches will Sluggy crawl in 3 minutes?**

(A) 3,240 in   (B) 337.5 in   (C) 200 in   (D) 120 in   (E) 37.5 in

25. **Let $(ax^2 − 5x + 1)(x + d) = 4x^3 + bx^2 + cx − 3$. Find a + b − c + d.**

(A) − 32   (B) − 26   (C) − 8   (D) 0   (E) 34

26. Find the perimeter of the hexagon ABCDEF. (nearest tenth)
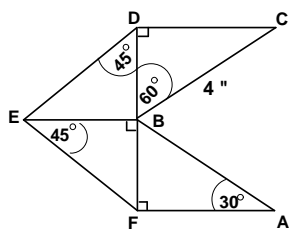
    (A) 17.7 in   (B) 24.7 in   (C) 18.9 in   (D) 26.6 in   (E) 20.6 in

27. If $a_1 = 2$, $a_2 = -1$, $a_3 = -3$ and $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$, where $n \geq 4$, then a_6 equals:

    (A) -29 (B) -19 (C) -13 (D) -5 (E) 8

28. Linda has a collection of green and red marbles. The ratio of greens to reds is 7 to 5 and the number of greens exceeds the number of reds by 48. How many red marbles does Linda have?

    (A) 120   (B) 132   (C) 144   (D) 156   (E) 168

29. Given: Points P, Q, R, and S lie on a circle with center O. Point T lies outside of the circle such that point R lies on segment TQ and point S lies on segment TP. Which of the following is true about $\angle RSP$?

        I. $m\angle RSP = m\angle QPT + m\angle QTP$ II. $\angle RSP$ and $\angle PQT$ are supplementary
        III. $\angle RSP$ is a right angle   IV. $m\angle RSP = m\angle RST + m\angle SRT$

    (A) I only   (B) II & IV   (C) I & III   (D) I & II   (E) III & IV

30. Find the domain of $f(x) = \dfrac{\sqrt{3x}}{|3x| - 2}$.

    (A) $[0, \infty)$ (B) $[0, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$ (C) $[0, 1) \cup (1, \infty)$
    (D) $(0, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$ (E) $(-\infty, \infty)$

31. Three ships left a tiny island at the same time. The *Friend* ship left on a bearing of 320° for 200 km then dropped anchor. The *Scholar* ship left on a bearing of 210° for 200 km and dropped anchor. The *Citizen* ship left on a bearing of 70° for 200 km and dropped anchor. The next day the *Citizen* sent out an SOS. How much closer to the *Citizen* was the closest ship than the other ship? (nearest km)

   (A) 45 km   (B) 48 km   (C) 58 km   (D) 65 km   (E) 93 km

32. The polar equation $r = 2\cos\theta$ written in rectangular coordinate form is:

    (A) $x^2 - y^2 = 2x$ (B) $y - x = 2$ (C) $x^2 + y^2 = 2y$ (D) $x^2 + y^2 = 2x$ (E) $x + y = 2$

33. How many four-letter arrangements of the letters CAPSULE having three consonants and one vowel can be formed if no letter is repeated?

    (A) 24   (B) 72   (C) 96   (D) 288   (E) 840

34 Three gears needed to run a particular machine need to be arranged and fit the specifications shown below. Find the measure of the cental angle of the smallest gear.
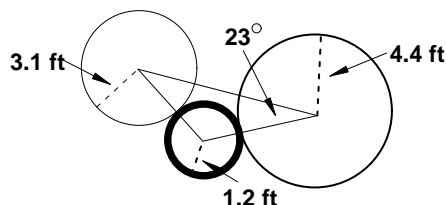

(A) 126°   (B) 112°   (C) 140°   (D) 123°   (E) 131°

35. Let X represent the number of people in a randomly selected household in Statville. Use the chart data to find the mean of the discrete random variable X.

Size of Households in Statville, USA

Number of People	1	2	3	4	5	6
Probability	.18	.36	.21	.15	.07	.03

(A) 2.62   (B) 2.64   (C) 2.66   (D) 2.68   (E) 2.70

36. Mei Telfone has a cell phone with a 7 digit number. The first and the last digits are 2's. The sum of any three consecutive digits is 9. What is the median digit?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

37. The dots below are one unit apart vertically and horizontally. Find the area of the shaded region. (square units)
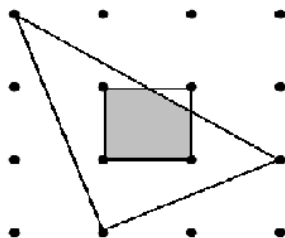
(A) 0.888...   (B) 0.900   (C) 0.91666...   (D) 0.9333...   (E) .9375

38. Jack went up a hill to fetch a 5-gallon pail of water. He drank a pint of water on his way back down. He used three gallons to water his bean stalk and two quarts to make some curds and whey. He gave Jill four cups of water to boil some eggs. How much water was left in the 5-gallon pail?

(A) 2 gals 1 qt 1 pt   (B) 1 gal 1 pt   (C) 1 qt 1 pt   (D) 1 gal   (E) 2 qts 1 pt

39. Let $f(x) = -3x^3 + 2x^2 - x + 4$. Which of the following is true about the end behavior of the graph?
  I. As x gets larger, f(x) gets smaller   II. As x gets larger, f(x) gets larger
  III. As x gets smaller, f(x) gets larger   IV. As x gets smaller, f(x) gets smaller

(A) I only   (B) II & IV   (C) I & III   (D) III only   (E) I & IV

40. **Moe Tell and Fran Tick are dishwashers at the *Hot Plate* diner. Moe washes 10 dishes per minute and Fran washes 14 dishes per minute. Moe starts washing the 800 dirty dishes twenty minutes before Fran starts. How many of the 800 dishes did Fran wash?**

   (A) 330   (B) 350   (C) 380   (D) 430   (E) 450

41. **Point A(2, 4) lies on the x-y plane. A is reflected across the y-axis to point B. Then B is reflected across the line y = — x to point C. Then C is translated horizontally 5 units to point D. Then D is translated vertically — 4 units to point E. Then E is rotated 90° clockwise about the origin to point F(x, y). Find x + y.**

   (A) — 6   (B) — 3   (C) — 1   (D) 0   (E) 2

42. **If Lotta Cache invested x dollars at a 4% simple interest rate and y dollars at a 3.5% simple interest rate, she would get $185.00 in interest in a year. However, if she invested x dollars at a 3.5% simple interest rate and y dollars at a 4% simple interest rate, she would get $199.00 in interest in a year. What is the total amount of her investment?**

   (A) $5,120.00   (B) $5,000.00   (C) $3,960.00   (D) $3,840.00   (E) $3,480.00

43. **Max Kutter has a piece of cardboard that is 2 feet long and 1 foot wide. He cuts a square out of each corner in order to create a cardboard box that has maximum volume. What is the area of the base of the cardboard box Max created? (nearest tenth)**

   (A) 131.1 sq. in   (B) 132.9 sq. in   (C) 134.5 sq. in   (D) 138.2 sq. in   (E) 150.2 sq. in

44. **Let (a , b) be a point on the graph of $y = 1 + 2\cos 3x$. Which of the following is a point on the graph of $y = -1 + 2\cos 3x$?**

   (A) (a , b — 2)   (B) (a , b + 2)   (C) (a — 2, b)   (D) (a + 2, b)   (E) (a + 2 , b — 2)

45. **When Doe gets to select one bill from a box containing bills as follows:**

Denomination of Bill	Number of Bills in the Box
1	16
5	10
10	8
20	6

   Based on the information, find the mathematically expected value.

   (A) $10.00   (B) $2.66   (C) $16.07   (D) $9.00   (E) $6.65

46. **Which of the following are considered to be *impolite* numbers?**
       I. 16   II. 48   III. 64   IV. 8,192

   (A) IV only   (B) II only   (C) I, III, & IV   (D) I, II, & III   (E) II & IV

47. A couple of years ago 80 young men and 40 young ladies representing Class AA took the state number sense test. Of the 120 students, 20% of the boys used pencils and 40% of the girls used pencils. If one of the students was chosen at random, what is the probability that the student used a pencil or is a young lady? (nearest whole percent)

(A) 9%   (B) 30%   (C) 45%   (D) 47%   (E) 60%

48. The graph of $h'(x)$ is shown below. Find h(10), when h(1) = 2.
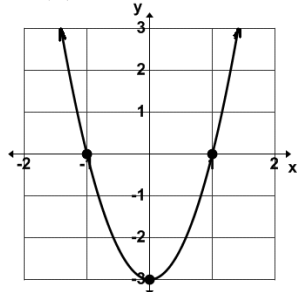
(A) 791   (B) 803   (C) 970   (D) 974   (E) 997

49. Let (x, y) be a solution to the equation $3(x - y) = xy + 1$, where x and y are positive digits. Find the sum of all such x and y digits.

(A) 12   (B) 9   (C) 8   (D) 7   (E) 5

50. $[(31F_{16}) + (A2B_{16})] \div 7_{16} =$ _____ $_{16}$.

(A) 19E   (B) 1E6   (C) 196   (D) D16   (E) D4A

51. The sequence $1 - \frac{5^2}{2!} + \frac{5^4}{4!} - \frac{5^6}{6!} + \frac{5^8}{8!} + ...$ simplifies to the decimal number 0.ABCDEFG..., where the letters represent digits. What digit does letter G represent?

(A) 0   (B) 1   (C) 2   (D) 5   (E) 6

52. The graph of the function $y = 5x + \sin(x) - 2$, where $-\pi \le x \le \pi$, lies in which quadrants?

(A) II & IV   (B) I & III   (C) I & IV   (D) II, III, & IV   (E) I, III, & IV

53. The graph of $f(x) \frac{8x - 3}{\sqrt{5x^2+1}}$ has horizontal asymptotes $y = \frac{a}{b}$ and $y = \frac{-a}{b}$.
Find the value of $\left|\frac{a}{b}\right|$ to four decimal places.

(A) 3.5333   (B) 3.5555   (C) 3.5777   (D) 3.5999   (E) 3.6111

54. If $53^{(45)} \cong x \pmod{12}$, then x =?

(A) 1   (B) 3   (C) 5   (D) 8   (E) 9

55. Which of the points lie on the line that is tangent to the curve $x^3 + y^2 = 2$ at point (1, 1)?

(A) $(-9, 11)$ (B) $(-20, -10.5)$ (C) $(-15, 20)$ (D) $(4, -3)$ (E) $(10, -12.5)$

56. Dee Grader has 5 different pens with black ink and 3 different pens with red ink. How many different sets of 5 pens can she make such that at least 3 of them are black ink pens?

(A) 46   (B) 75   (C) 56   (D) 13   (E) 30

57. Betty Duzett is taking this math test. If the odds of scoring 200 or higher on this test is $\frac{3}{5}$, what is the probability that Betty will score less than 200 on this test?

(A) 37.5%   (B) 40%   (C) 60%   (D) 62.5%   (E) 80%

58. Let $f_1 = 2$, $f_2 = 7$, $f_3 = 9$, $f_4 = 16$, ... be the terms of a Fibonacci characteristic sequence. If the ratio of f_n to f_{n-1} is the first ratio to equal the Golden Ratio rounded to the thousandth place, then $f_n - f_{n-1} = ?$

(A) 66   (B) 107   (C) 162   (D) 173   (E) 280

59. The positive even numbers are arranged as shown. What is the sum of the numbers in row 8?

$$\begin{array}{llll} & 2 & & \text{(row 1)} \\ & 4 \quad 6 & & \text{(row 2)} \\ & 8 \quad 10 \quad 12 & & \text{(row 3)} \\ & 14 \quad 16 \quad \cdots \quad \cdots & & \text{(row 4)} \end{array}$$

(A) 350   (B) 536   (C) 738   (D) 720   (E) 520

60. The base of a pyramid is a regular hexagon with each side equal to 24 cm. If the altitude of the pyramid is 42 cm, find the total surface area of the pyramid. (nearest cm^2)

(A) 4820 cm^2 (B) 4837 cm^2 (C) 4854 cm^2 (D) 4871 cm^2 (E) 4888 cm^2

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2021
Answer Key

1.	E	21.	C	41.	B
2.	D	22.	B	42.	A
3.	C	23.	A	43.	A
4.	C	24.	D	44.	A
5.	E	25.	A	45.	E
6.	C	26.	E	46.	C
7.	B	27.	B	47.	D
8.	C	28.	A	48.	D
9.	A	29.	D	49.	A
10.	E	30.	B	50.	B
11.	C	31.	B	51.	B
12.	C	32.	D	52.	E
13.	D	33.	D	53.	C
14.	E	34.	A	54.	C
15.	E	35.	C	55.	E
16.	D	36.	A	56.	A
17.	A	37.	C	57.	D
18.	C	38.	B	58.	B
19.	D	39.	C	59.	E
20.	B	40.	B	60.	D

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2021
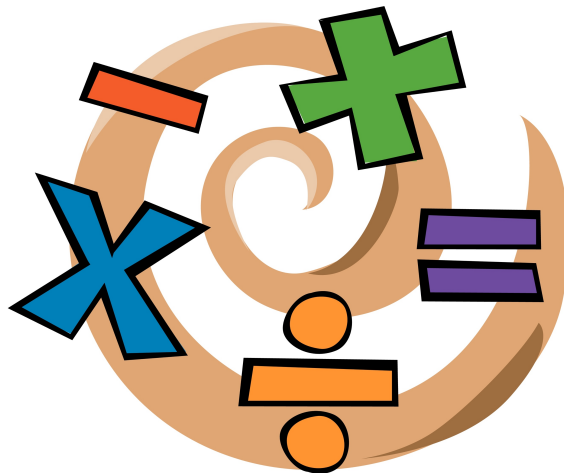
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1. **Evaluate:** $[(1+1)^2]! \div 3 - 5 \times 8 + 13$

   (A) -20 (B) -19 (C) 29 (D) 37 (E) 63

2. **Willie Prawfette bought four sets of used golf clubs at a garage sale for \$75.00 each. He sold one set at his garage sale for \$120.00, a second set for \$85.00, a third set for \$145.00, and the fourth set for \$55.00. What was his percent profit?**

   (A) 65%   (B) 60%   (C) 50%   (D) 45%   (E) 35%

3. **Three-fourths is to two-thirds as one-half is to what?**

   (A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{5}{6}$ (D) 1 (E) $1\frac{1}{2}$

4. **Forty percent of the Mustangs were added to twenty percent of the Plainsmen and the total was 33. Twice the number of Mustangs was fifteen less than three times the number of Plainsmen. How many were Plainsmen?**

   (A) 40   (B) 45   (C) 50   (D) 55   (E) 60

5. **150 students registered for Mr. White's problem writing camp. 85 students signed up for the math sessions. 70 signed up for the number sense sessions. 50 signed up for both the math and number sense sessions. How many students signed up for neither math nor number sense?**

   (A) 20   (B) 35   (C) 45   (D) 50   (E) 55

6. **Points A, B, and C are plotted below. Find point D such that $\overline{AB} \perp \overline{CD}$ and D is in quadrant III.**
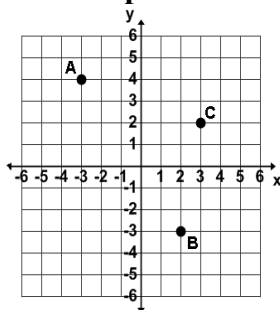
   (A) $(-4, -3)$ (B) $(-3, -2)$ (C) $(-2, -4)$ (D) $(-3, -4)$ (E) $(-2, -3)$

7. **Simplify:** $\dfrac{4x^2 - 25}{x + 1} \div \dfrac{6x^2 + 13x - 5}{3x^2 + 2x - 1}$

   (A) $2x - 5$ (B) $3x - 5$ (C) $\frac{2x - 5}{3x - 1}$ (D) $\frac{1}{2x + 5}$ (E) $2x + 5$

8. **Let $(4x - 1)(ax + b)(cx - 1) = 8x^3 + dx^2 + ex + 3$, where a,b,c,d, and e are integers and $a > c$. Find $a + b + c + d + e$.**

   (A) -16 (B) -13 (C) -5 (D) 0 (E) 1

9. Two trains, the *Clickety* and the *Clack*, left the home depot at the same time going in opposite directions. The *Clickety* reached the next depot in 1.5 hours traveling at an average speed of 50 mph. The *Clack* reached the next depot in 2 hours traveling at an average speed of 60 mph. How far apart were the two depots?

   (A) 385 miles   (B) 190 miles   (C) 295 miles   (D) 195 miles   (E) 115 miles

10. Line *m* and line *n* are skew. Point A is on line *m*. How many lines containing point A can be drawn perpendicular to line *n*?

   (A) none   (B) 1   (C) 2   (D) 3   (E) infinite

11. Given: AB = CD = 12, AD = 36 and $\angle ABC = 90°$. Find $\angle BCD$.
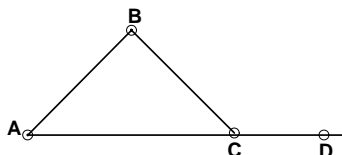
   (A) 120°   (B) 130°   (C) 140°   (D) 150°   (E) 160°

12. Paulie Gawn drew an isosceles trapezoid with base lengths of 10 feet and 5 feet and a height of 3 feet. If she increased the smaller base 1 foot and decreased the longer base 2 feet, how much would the height have to be for the area of the second trapezoid to be 80% of the area of the original trapezoid? (nearest half inch)

   (A) 2' 9"   (B) 2' 8"   (C) 2' 7.5"   (D) 2' 7"   (E) 2' 6.5"

13. Let $(a^3 \div b)^2 \times a^{-1} \times b^4 \div (a^{-1}) \times b^0 = a^m \times b^n$. Find $(m)^n$.

   (A) 8   (B) 9   (C) 12   (D) 36   (E) 64

14. Find the domain of $f(x) = \dfrac{\sqrt{12 - 2x}}{2x^2 + 13x + 21}$.

   (A) $x \in R \mid x \le 6$ (B) $x \in R \mid x \le 6, x \ne -3.5$ (C) $x \in R \mid x \le 6, x \ne -3$

   (D) $x \in R \mid x < 6, x \ne -3.5, -3$ (E) $x \in R \mid x \le 6, x \ne -3.5, -3$

15. Mr. White likes to take a daily walk at 3 miles per hour. Mrs. White picks him up in their golf cart at the end of his walk and returns home over the same route at 10 miles per hour. The time allotted for the entire trip is 2.5 hours. How far does Mr. White walk? (nearest tenth)

   (A) 2.5 miles   (B) 1.2 miles   (C) 5.8 miles   (D) 4.5 miles   (E) 3.6 miles

16. Chip Chawt has three white golf balls and two yellow ones. He wants to arrange them in random order and find the probability of having only two of the same colored balls next to each other given that a yellow ball is first or last. How many elements are in the total sample space?

   (A) 5   (B) 7   (C) 8   (D) 12   (E) 24

17. **Given the graph shown, what would the equation of this graph become if the amplitude is increased by 2, the phase shift is not changed, the period is doubled, and the displacement is cut in half. y = ?**
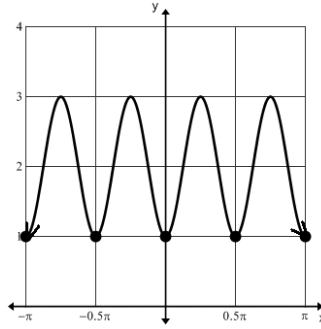

(A) $1 + 3\sin(2x - \frac{\pi}{4})$

(B) $4 - \frac{1}{2}\sin(8x - \frac{\pi}{2})$

(C) $1 + 2\sin(\pi x - \frac{\pi}{4})$

(D) $2 + \sin(2x - \frac{\pi}{2})$

(E) $1 + 3\sin(x - \frac{\pi}{4})$

18. **The remainder when the polynomial $2x^3 - 5x^2 + kx + 6$ is divided by $x - 4$ is 98. Find k.**

(A) 4   (B) 11   (C) 38   (D) 100   (E) 300

19. **$[(22A_{14}) - (BC_{14})] \times 7_{14} = $ _____ $_{14}$.**

(A) 1,064   (B) 757   (C) 1,848   (D) 132   (E) 960

20. **Ms. Daisey is driving to the market. She is waiting for the red light to turn green. When the light turns green she accelerates. Which graph best shows her speed versus time as she reaches the speed limit?**
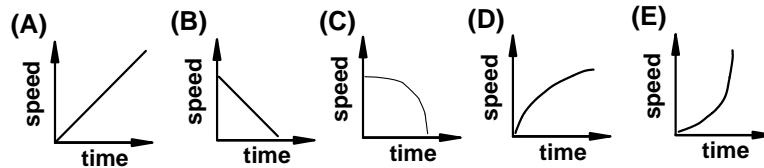

21. **Let $f(x) = x^2 - 4x + 3$ be continuous on [1, 3] and differentiable on (1, 3). Find the equation of the line tangent to the graph of $f(x)$ when the slope of the tangent line is 0.**

(A) $f(x) = 2$ (B) $f(x) = 0$ (C) $f(x) = -1$ (D) $f(x) = 2x$ (E) $f(x) = 2x - 4$

22. **If $f''(x) = 24x + 6$, $f'(-1) = 4$, and $f(1) = 6$, find $f(-2)$.**

(A) -15 (B) -16 (C) -23 (D) -37 (E) -42

23. **Which of the following mathematicians is known as the "father of geometry"?**

(A) Ptolemy (B) Archimedes (C) Rene Descartes (D) Leonard Euler (E) Euclid of Alexandria

24. **How many positive digits in base 10 are considered to be "unhappy odious" numbers?**

(A) 5   (B) 4   (C) 3   (D) 2   (E) 1

25. Willie When spins the wheel. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation of any one spin?
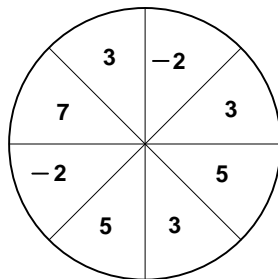
    (A) 3   (B) 2.2   (C) 2.25   (D) 2   (E) 2.75

26. Tai Bowe needs ribbons to wrap her presents. Each ribbon needs to be 5 feet 4 inches in length. How many ribbons of that length can she get from a 50-yard spool of ribbon?

    (A) 28   (B) 29   (C) 30   (D) 31   (E) 32

27. How many 4-digit even numbers between 2,500 and 5,000 can be created using the digits 2, 1, 3, 4, and 7? A digit cannot be used more than once in any of the 4-digit numbers.

    (A) 54   (B) 50   (C) 36   (D) 20   (E) 12

28. Which of the following equations has no real roots?

    (A) $x^2 + 4x - 5 = 0$ (B) $4x^2 + 12x + 9 = 0$ (C) $-2x^2 + 6x + 8 = 0$
    (D) $4x^2 + 6x + 1 = 0$ (E) $2x^2 + 8x + 15 = 0$

29. Given: m∠CBA = 30°, m∠ADE = 110°, ED ∥ AB, and BC = AC. Find m∠CAD.
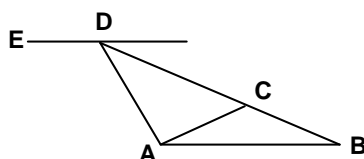
    (A) 30°   (B) 40°   (C) 60°   (D) 80°   (E) 110°

30. The roots of the equation $8x^2 - kx - 15 = 0$ are -1.25 and R. Find k.

    (A) -2 (B) -0.75 (C) 1.5   (D) 2   (E) 2.75

31. $\sin(\frac{\pi}{6}) + \sin(\frac{\pi}{3}) + \sin(\frac{\pi}{2}) + \sin(\frac{2\pi}{3}) + ... + \sin(\frac{7\pi}{3}) + \sin(\frac{5\pi}{2}) = ?$ (nearest hundredth)

    (A) 2.37   (B) 2.5   (C) 2.73   (D) 2.87   (E) 3

32. Given the function $f(x) = 5\sin(x) - 1$, find the slope of the secant line between $x = \frac{\pi}{2}$ and $x = \pi$.

    (A) $-\frac{10}{\pi}$ (B) $-\frac{5}{\pi}$ (C) $-\frac{1}{\pi}$ (D) zero slope   (E) no slope

33. Find the remainder when 31^{30} is divided by 29.

(A) 1   (B) 2   (C) 4   (D) 9   (E) 16

34. Let $f_1 = 3$, $f_2 = 8$, $f_3 = 11$, $f_4 = 19$, ... $f_{12} = 877$, ... be the terms of a Fibonacci characteristic sequence. Find f_{20}.

(A) 25,463   (B) 66,663   (C) 107,863   (D) 94,797   (E) 41,200

35. The *Bait Yor Hook* fishing pond has bass, crappie, catfish, perch, gar, and carp. The limit of fish you can keep is five and at least one of them has to be a perch. How many different stringers of fish could you keep?

(A) 252   (B) 120   (C) 210   (D) 126   (E) 600

36. The figures below are made up of little squares. The side length of each little square is 1 cm. If the pattern of the shapes continues what will the perimeter of figure 18 be?
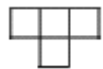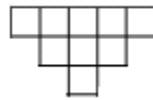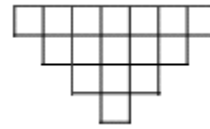
fig. 1   fig. 2   fig. 3   fig. 4

(A) 108   (B) 106   (C) 104   (D) 102   (E) 100

37. The top surface of a square board is painted red. A circle is inscribed on the board and its area is painted blue. What per cent of the square board remained red? (nearest tenth)

(A) 31.4%   (B) 78.5%   (C) 12.6%   (D) 21.5%   (E) 68.6%

38. A regular polygon of n sides exists such that its central angle is 24° and each side length is 4". Find the perimeter of the polygon. (nearest inch)

(A) 4' 0"   (B) 4' 4"   (C) 4' 6"   (D) 4' 8"   (E) 5' 0"

39. The *Wildflower* seed company's research data shows that the probability that a seed will germinate and grow into a plant is 85%. What are the odds that it won't germinate?

(A) $\frac{17}{20}$ (B) $\frac{5}{8}$ (C) $\frac{3}{17}$ (D) $\frac{3}{20}$ (E) $\frac{5}{13}$

40. Mark Ex and Drew Oh are playing Tic-tac-toe. Mark has won 5 games and Drew has won 3 games. If they continue to play, what is the least number of games they will have to play in order for Mark to have a winning probability of 75%?

(A) 7   (B) 6   (C) 5   (D) 4   (E) 3

41. If $a_1 = 2$, $a_2 = -3$, $a_3 = -1$ and $a_n = (a_{n-2})^{(a_{n-3})} - (a_{n-1})$, where $n \geq 4$, then a_6 equals:

(A) 12.1   (B) 11.1   (C) 1   (D) -1 (E) -11

42. Which type of conic is the graph of the equation $Ax^2 + By^2 + Cy + D = 0$, where A, B, C, and D are integers and A, C > 0 and B, D < 0?

    (A) circle   (B) ellipse   (C) hyperbola   (D) parabola   (E) not a conic

43. Find the units digit of $37^{(35)} - 33$.

    (A) 0   (B) 2   (C) 4   (D) 6   (E) 8

44. If $f(x) = 4x - 1$, $g(x) = 5x + 2$, $h(x) = 3 - 6x$, and $g\left(h\left(f(3x + 4)\right)\right) = ax + b$, then $a + b =$?

    (A) -840 (B) -793 (C) -773 (D) -465 (E) -384

45. Let R_1 and R_2 be the roots of $2x^2 - 3x + c = 0$, where the ratio of R_1 to R_2 is 5. Find c.

    (A) 0.15625   (B) 0.3125   (C) 0.625   (D) 1.25   (E) 2.5

46. Two circles, $(x - 4)^2 + (y + 5)^2 = 16$ and $x^2 + y^2 = 16$, intersect at two points. Find the slope of the line passing through the two points of intersection.

    (A) $\frac{4}{5}$ (B) $\frac{9}{16}$ (C) $\frac{1}{16}$ (D) $\frac{16}{25}$ (E) $1\frac{1}{4}$

47. If r, s, and t are real numbers such that $r + s + t = 20$, $t^2 = r^2 + s^2$, and $rs = 10$, find the value of t.

    (A) 8.5   (B) 9   (C) 9.5   (D) 10   (E) 10.5

48. Bill Durr is a surveyor working on a bridge that will span a small lake from point A to point B. The line of sight distances from him to point B and from him to point A are 456.2 ft. and 429.8 ft., respectively. He measures the angle between the two lines of sight to be 48.7 °. How long did he compute the length of the bridge to be? (nearest foot).

    (A) 496 ft   (B) 443 ft   (C) 391 ft   (D) 366 ft   (E) 313 ft

49. Given: $8\cos^2(4A) + 10\sin(4A) - 11 = 0$, where $0 \le A \le 4$ (rad).
    Find the largest value for A, in degrees. (nearest degree)

    (A) 188°   (B) 213°   (C) 218°   (D) 223°   (E) 230°

50. The vertical asymptote and the oblique asymptote of $f(x) = \frac{2x^2 + 3x + 5}{x - 1}$ intersect at point (x , y). Find the value of y.

    (A) 10   (B) 7   (C) 4   (D) 3   (E) 1

51. Find the sum of the first ten terms of an arithmetic sequence if the fourth term is 4 and the thirteenth term is 34.

    (A) $88\frac{2}{3}$ (B) $89\frac{1}{3}$ (C) 90   (D) $90\frac{2}{3}$ (E) $91\frac{1}{3}$

52. Which of the following is closest to the length of the segment bounded by the parametric equations: $x = 3t$, $y = 5t - 1$, $0 \le t \le 1$.

(A) 4.2   (B) 4.3   (C) 5.0   (D) 5.8   (E) 5.9

53. Find the area of the region bounded by the curve $8y = 5x^2 + 16$ and the lines $y = 0$, $x = 0$, $y = 12$, and $x = 6$.

(A) $45\frac{1}{3}$ (B) 46   (C) $46\frac{2}{3}$ (D) $47\frac{1}{3}$ (E) 48

54. The *Brite Lite* company statistics shows that only 3% of the light bulbs shipped out are defective. If a shipment of 24 bulbs is randomly selected for testing what is the probability that exactly 2 of them are defective? (nearest percent)

(A) 1%   (B) 4%   (C) 6%   (D) 7%   (E) 13%

55. The probability distribution of X, the number of hours Landon works on Thursdays is:

Hours (h)	P(X=h)
8	0.5
9	0.2
10	0.15
11	0.1
12	0.05

Find the variance of the probability distribution.

(A) 1.5   (B) 1.65   (C) 1.8   (D) 1.95   (E) 2.1

56. Point P($-$1, 6) lies on the x-y plane. P is reflected across the line $x = 1$ to point Q. Point Q is rotated $90°$ counterclockwise about the point (1, 0) to point R. Point R translated 4 units vertically and $-$2 units horizontally to point S (x, y). Find the distance from point S to point P. (nearest tenth)

(A) 4.5   (B) 6.0   (C) 7.0   (D) 7.5   (E) 9.2

57. Wendy Dae sailed her sloop from her pier traveling S50°W for 4 nautical miles. She turned and traveled N40°E for 4 nautical miles. Then the wind stopped and Wendy used the outboard motor to head straight back to the pier. What course did she need to take back to the pier?

(A) W40°N   (B) S45°E   (C) E50°S   (D) N45°W   (E) S35°E

58. $\triangle ABC$ is a right triangle with $\angle ABC$ being the right angle. Point D lies outside of $\triangle ABC$ such that $m\angle BCD = 90°$, $m\angle BAC = 59°$, and $\angle ACD$ is an obtuse angle. If AB = 6" and CD = 9", then BD = ? (nearest tenth)

(A) 9.8"   (B) 10.0"   (C) 10.8"   (D) 12.7"   (E) 13.4"

59. Points A and B lie on a circle with center C and a radius of 2.5". Point T lies outside of the circle such that segments AT and BT are tangent to the circle and m∠ACB = 130°. Find the area outside the circle and bounded by segments AT and BT.   (nearest tenth)

   (A) 3.6 sq. in   (B) 4.0 sq. in   (C) 6.3 sq. in   (D) 7.5 sq. in   (E) 8.0 sq. in

60. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 3rd term and the 9th term in row 11.

2								(row 1)
3	3							(row 2)
4	5	4						(row 3)
5	7	7	5					(row 4)
6	9	12	9	6				(row 5)
7	11	16	16	11	7			(row 6)
		...						(...)

   (A) 72   (B) 68   (C) 64   (D) 70   (E) 80

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2021
Answer Key

1.	B	21.	C	41.	B
2.	E	22.	A	42.	C
3.	B	23.	E	43.	A
4.	B	24.	C	44.	B
5.	C	25.	E	45.	C
6.	A	26.	A	46.	A
7.	A	27.	D	47.	C
8.	C	28.	E	48.	D
9.	D	29.	D	49.	C
10.	B	30.	D	50.	B
11.	D	31.	A	51.	C
12.	D	32.	A	52.	D
13.	D	33.	C	53.	A
14.	E	34.	E	54.	E
15.	C	35.	D	55.	A
16.	B	36.	B	56.	B
17.	A	37.	D	57.	B
18.	B	38.	E	58.	E
19.	E	39.	C	59.	C
20.	D	40.	D	60.	A

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2021
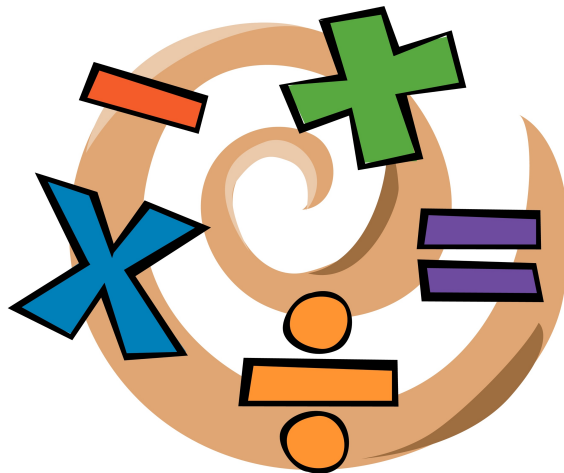
DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Solve for k: $(1+3)^2 \div 4! - 5 \times k + 7 = 8$

   (A) $-\frac{3}{5}$ (B) $-\frac{3}{15}$ (C) $-\frac{2}{15}$ (D) $-\frac{1}{15}$ (E) $-\frac{1}{30}$

2. Lotta Dough baked a batch of cookies. She put $\frac{1}{3}$ of them in a box for snacks at work, shared 75% of the remaining ones with her family, and took the remaining 5 cookies to her grandmother's house. How many cookies did she bake?

   (A) 18   (B) 24   (C) 30   (D) 36   (E) 42

3. Find the slope of the line perpendicular to the line shown and through the point $(-6, 7)$?
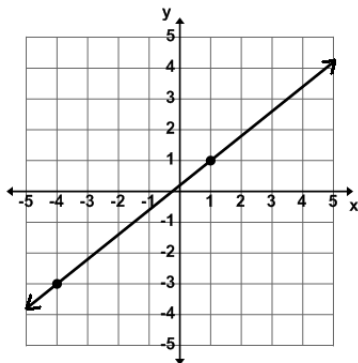
   (A) $-\frac{5}{4}$ (B) $-\frac{6}{7}$ (C) $-\frac{4}{5}$ (D) $-\frac{7}{6}$ (E) $-\frac{5}{6}$

4. Simplify: $\left(\frac{3x-3y}{x^2+2xy+y^2}\right) \times \left(\frac{x+y}{x-y}\right) \div \left(\frac{6}{x^2-y^2}\right)$

   (A) $\frac{x-y}{2}$ (B) $2x+2y$ (C) $2x-2y$ (D) $\frac{x+y}{2}$ (E) 2

5. Two complementary angles have measures of $2x-1$ degrees and $5x+3$ degrees. What would the measure of an angle be if it is supplementary to the smaller of the two complementary angles? (nearest whole degree)

   (A) $90°$ (B) $114°$ (C) $124°$ (D) $138°$ (E) $156°$

6. If $\frac{5+4x}{3x+2} - \frac{x-2}{2x-1} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, where A, B, C, P, Q, and R are integers. Find $A+B+C+P+Q+R$.

   (A) 3   (B) 9   (C) 15   (D) 19   (E) 25

7. Let $(a^5 \div b^2)^3 \times a \times b^{-4} \div (a^{-6}) \times b^0 = a^m \times b^n$. Find $m+n$.

   (A) -6 (B) 9   (C) 12   (D) 4.5   (E) 64

8. Determine the phase shift of $f(\theta) = 2 + 3\cos(\frac{4\pi}{5}\theta - 6)$. (nearest tenth)

   (A) 2.4   (B) 2.5   (C) 3   (D) 5.2   (E) 6

9. Which of the following is considered to be the first known female mathematician. Based on the works that have survived, it is thought that she worked on the Golden Mean and the Golden Rectangle.

(A) Agnesi   (B) Hypatia   (C) Lady Lovelace   (D) Freda Porter   (E) Theano

10. The four lines in the figure are coplanar with $m \| l$. Which of the following are true statements?
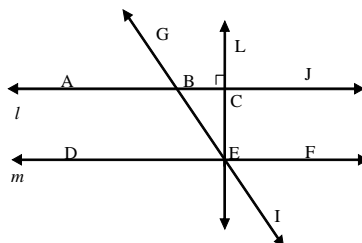
1. ∠ABE & ∠JBG are congruent   2. m∠DEI + m∠ABG = 180°
3. ∠JBI & ∠BEF are vertical angles   4. m∠CBE = 45°

(A) 3 only   (B) 1 & 2   (C) 1 & 4   (D) 3 & 4   (E) 4 only

11. Given: $\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find $(a + d) - (b + c)$.

(A) 1   (B) 0   (C) -1 (D) -2 (E) -3

12. If $f''(x) = 6x - 12$, $f'(1) = 0$, and $f(-2) = -49$, then $f(-1) = ?$

(A) 6   (B) 3   (C) -7 (D) -15 (E) -16

13. Wynn Zenn's science team consists of 5 seniors, 6 juniors, 3 sophomores, and 6 freshmen. In how many ways can he form a 6-member science team consisting of 2 seniors, 2 juniors, a sophomore, and a freshman?

(A) 2,700   (B) 59   (C) 38,760   (D) 10,800   (E) 34

14. Given that the set of natural numbers continues in the triangular pattern shown below, find the 3rd number in row 10.

				1					(row 1)
			2	3	4				(row 2)
		5	6	7	8	9			(row 3)
	10	11	12	13	14	15	16		(row 4)
				...					(...)

(A) 103   (B) 97   (C) 91   (D) 88   (E) 84

15. A string is 5 feet long. Three smaller strings with lengths of 1 foot 10 inches, 1 foot 8 inches, and 11 inches are cut from the original string. How long is the original string after the three cuts?

(A) 7 inches   (B) 8 inches   (C) 9 inches   (D) 11 inches   (E) 13 inches

16. Simplify: $\dfrac{(n-1)!}{(n)!} \times \dfrac{(n+1)!}{(n+2)!} \times n$

    (A) $n-1$ (B) $n+1$ (C) $\dfrac{1}{n}$ (D) $\dfrac{n}{n+1}$ (E) $\dfrac{1}{n+2}$
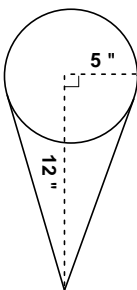
17. Let $(3x + A)(4x + B) = Cx^2 - 23x - 24$. Find $A - B + C$.

    (A) 23   (B) 17   (C) 12   (D) 7   (E) 1

18. Jack is twice as old as Jill. Seven years ago, the sum of their ages was 13. What will be the sum of their ages in five years?

    (A) 20 yrs   (B) 27 yrs   (C) 32 yrs   (D) 37 yrs   (E) 46 yrs

19. The least number of cups of water needed to fill the conic cup and spill over is:

    (A) 20 cups   (B) 21 cups   (C) 22 cups   (D) 23 cups   (E) 24 cups

20. A regular heptagonal prism has how many edges?

    (A) 12   (B) 14   (C) 18   (D) 21   (E) 24

21. If $\sqrt[4]{x^2\left(\sqrt[3]{x\left(\sqrt{x^6}\,\right)}\,\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $k = ?$

   (A) 12   (B) 10   (C) 6   (D) 5   (E) 3

22. Willett Staupp has a horse trough that holds 400 gallons of water with a tiny hole in it. The trough loses a cup of water every hour. What percent of the total gallons of water will still be in the trough 120 days after Willett fills it?

    (A) 55%   (B) 45%   (C) $33\frac{1}{3}$%   (D) 25%   (E) 10%

23. Allie Gater is a zoologist studying crocodiles. She spots two crocodiles on the bank. She estimates the distance from her boat is 30 meters to one croc and 40 meters to the other. She estimates the measure of the angle between her two lines of sight to be $28°$. How far apart are the two crocs? (nearest foot).

    (A) 20 ft   (B) 30 ft   (C) 35 ft   (D) 45 ft   (E) 50 ft

24. Find AM. (nearest tenth)
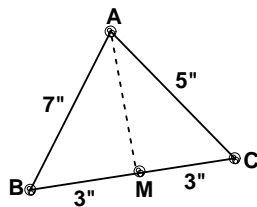
(A) 5.0"   (B) 5.1"   (C) 5.3"   (D) 5.5"   (E) 5.6"

25. Let $f(x) = 2x - 5$ and $g(x) = 3x + 1$. Find $f(g(x)) - g(f(-x))$.

(A) $12x + 11$ (B) -17 (C) $12x - 17$ (D) 11 (E) $12x - 11$

26. Which of the following is neither an even nor an odd function?
   I. $\sqrt{x^4 - x^2} + 4$ II. $\sqrt[3]{x}$ III. $x\sqrt{x^2 - 1}$

(A) I & III   (B) II only   (C) I, II, & III   (D) II & III   (E) none of them

27. Find the distance between the absolute maximum and the absolute minimum of
   $h(t) = 2t^3 + 3t^2 - 12t + 4$ on the interval $[0, 2]$. (nearest whole number)

(A) 7   (B) 9   (C) 11   (D) 12   (E) 15

28. Which of the points lie on the line that is tangent to the curve $x^2 + 2y^2 = 9$ at point $(1, 2)$.

(A) $(-40, 13)$ (B) $(-12, 5)$ (C) $(2, 2)$ (D) $(25, -4)$ (E) $(32, -6)$

29. Penni Lesse has two nickels and two dimes. She wants to arrange them in random order and find the probability of having two of the same coins next to each other given that a nickel is first or a dime is last. How many elements are in the successful sample space?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

30. The units digit of $13^{(2021)}$ is _____.

(A) 1   (B) 3   (C) 6   (D) 7   (E) 9

31. What value of k will make $x^2 + \frac{4}{5}x + k$ a trinomial square?

(A) .75   (B) 1   (C) .06   (D) .16   (E) .8

32. The vertex of the graph of the function $y = -x^2 - 3x + 5$ is (x, y). Find $x + y$.

(A) 5.25   (B) 5.75   (C) 7.25   (D) 8.75   (E) 11.75

33. $[(2A8_{12}) + (9B_{12})] \times 5_{12} = $ _____ $_{12}$.

(A) 1839   (B) 166B   (C) 2675   (D) 843   (E) 226B

34. Given:

x	-2	-1	1	a	3	5	0
g(x)	7	1	1	31	17	49	b

. Find $a + b$.

   (A) 11   (B) 7   (C) 5   (D) 3   (E) 2

35. How many positive digits k exists such that $k < 24$ and $24^k \div 7$ has a remainder of 1?

   (A) 3   (B) 4   (C) 6   (D) 7   (E) 9

36. How many positive digits in base 10 are considered to be "happy" and/or "evil" numbers?

   (A) 2   (B) 3   (C) 5   (D) 6   (E) 7

37. Given the Fibonacci characteristic sequence $f_0 = 2, f_1 = 5, f_2 = 7, f_3 = 12, ...$, find the sum of the f_5, f_7, and f_{11}.

   (A) 667   (B) 636   (C) 586   (D) 555   (E) 524

38. The digits 1, 2, 3, 4, and 5 are used once each to form the smallest possible five-digit odd number less than 40,000. What is the digit in the tens place?

   (A) 5   (B) 4   (C) 3   (D) 2   (E)

39. Thirty students at Venn U. took a survey about their favorite class. Twelve marked science, fourteen marked math, and seventeen marked English. Four students marked all three classes. Two marked math and science, but not English. Six marked math and English, but not science. Six marked only science. How many students did not mark any of these three classes?

   (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

40. Kay Ack paddled up a river for 3 hours 30 minutes. The return trip took 2 hours 45 minutes. If the speed of the current was 2 miles per hour, what was the speed of Kay's boat in still water?

   (A) $12\frac{1}{2}$ mph   (B) $15\frac{1}{3}$ mph   (C) $16\frac{2}{3}$ mph   (D) 17 mph   (E) $18\frac{1}{2}$ mph

41. Anne Teak took her history class on a bus trip to the Smithsonian museum. The expenses for the trip totaled $540.00 and were to be shared by the teacher and the students. When five students were unable to go due to bad grades, $1.50 was added to the cost per person going on the trip. How many people went on the trip?

   (A) 45   (B) 25   (C) 36   (D) 50   (E) 40

42. The letters from the word NUMBER are put into a bag. Two letters are selected at random without replacement. What is the probability that the second letter selected is a vowel?

   (A) 75%   (B) $66\frac{2}{3}$%   (C) 50%   (D) $33\frac{1}{3}$%   (E) 25%

43. **Chip Picker selects three chips without replacement from a bag containing four red chips and two white chips. What is the probability that he selects a red chip followed by a white chip followed by a red chip? (nearest whole percent)**

 (A) 24%   (B) 18%   (C) 15%   (D) 12%   (E) 20%

44. **Reid Allot has 5 math books, 3 science books, and 4 literature books to be arranged on a shelf. In how many ways can the books be arranged if they are to be grouped by topic?**

 (A) 17,280   (B) 51,840   (C) 103,680   (D) 311,040   (E) 479,001,600

45. **Find the area of △ABC. (nearest tenth)**
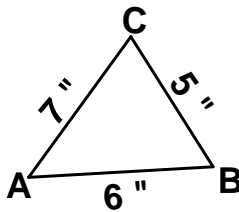
 (A) 12.3 in^2 (B) 14.5 in^2 (C) 14.7 in^2 (D) 17.0 in^2 (E) 17.4 in^2

46. **Let f(x) = 4x^3 — 3x^2 — x — 2. Which of the following is true about the end behavior of the graph?**
 I. As x gets larger, f(x) gets smaller   II. As x gets larger, f(x) gets larger
 III. As x gets smaller, f(x) gets larger   IV. As x gets smaller, f(x) gets smaller

 (A) II only   (B) II & IV   (C) II & III   (D) IV only   (E) I & IV

47. **Given: $\overline{BD} \parallel \overline{AE}$, BC = 30 cm, and AB = 20 cm. Find AC. (nearest tenth)**
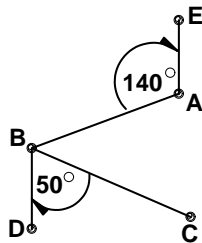
 (A) 33.0 cm   (B) 38.8 cm   (C) 34.2 cm   (D) 37.5 cm   (E) 36.1 cm

48. **Two circles, (x — 3)^2 + (y — 2)^2 = 25 and (x + 1)^2 + (y — 1)^2 = 9, intersect at two points. Find the slope of the line passing through the two points of intersection.**

 (A) — 4   (B) — $\frac{5}{8}$ (C) — $2\frac{1}{2}$ (D) — $\frac{1}{2}$ (E) — 2

49. **If you start at (— 1, 0) on a unit circle and travel clockwise 44 radians, where will you come to a stop on the unit circle?**
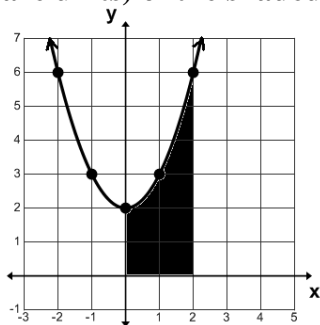
 (A) QI   (B) QII   (C) QIII   (D) QIV   (E) x-axis

50. **Professor Stats' probability class of 20 students took a pretest. The following chart shows the number of errors and the distribution. Find the standard deviation. (nearest hundredth)**

Number of errors	0	1	2	3	4	5
Frequency	1	5	4	5	2	3

(A) 0.92   (B) 0.98   (C) 1.08   (D) 1.47   (E) 2.15

51. **The area (in square units) of the shaded region below is:**

(A) $5\frac{2}{3}$ (B) $6\frac{1}{3}$ (C) $6\frac{2}{3}$ (D) $7\frac{1}{3}$ (E) $7\frac{2}{3}$

52. **Which of the equations in rectangular form describes the parametric equations $x = 5 - 3\cos(t)$ and $y = 4 + 2\sin(t)$, where $0 \le t \le 2\pi$?**

(A) $\frac{(x+5)^2}{9} + \frac{(y+4)^2}{4} = 1$ (B) $\frac{(y+4)^2}{2} - \frac{(x+5)^2}{3} = 1$ (C) $\frac{(y-4)^2}{2} - \frac{(x-5)^2}{3} = 1$

(D) $\frac{(x-5)^2}{3} + \frac{(y+4)^2}{2} = 1$ (E) $\frac{(x-5)^2}{9} + \frac{(y-4)^2}{4} = 1$

53. **Penni Lesse has two nickels and two dimes. She arranges them in random order. What are the odds of having two of the same coins next to each other, given that a nickel is first?**

(A) 2:5   (B) 2:1   (C) 2:3   (D) 3:5   (E) 3:2

54. **The coordinates of the vertices of a triangle are (0, 0), (0, 12), and (5, 0). The coordinates of the centroid of this triangle is (x, y). Find x + y.**

(A) $6\frac{2}{3}$ (B) $1\frac{4}{13}$ (C) $1\frac{1}{2}$ (D) $4\frac{8}{13}$ (E) $5\frac{2}{3}$

55. **Given the harmonic sequence $\frac{1}{12}, \frac{1}{19}, \frac{1}{26}, \frac{1}{33}, \ldots$, which of the following would be an element of this sequence?**
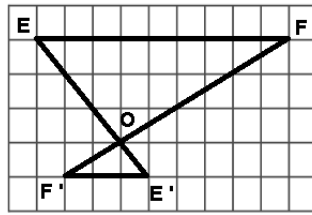
(A) $\frac{1}{336}$ (B) $\frac{1}{338}$ (C) $\frac{1}{340}$ (D) $\frac{1}{348}$ (E) $\frac{1}{352}$

56. **Use following table to calculate a midpoint Riemann sum on [0, 6], n = 3.**

x	0	1	2	3	4	5	6
f(x)	4	8	5	3	7	4	8

(A) 15   (B) 30   (C) 39   (D) 45   (E) 60

57. The dilation shown is?

(A) $D_O, -\frac{1}{2}$ (B) $D_O, -\frac{1}{3}$ (C) $D_O, \frac{2}{3}$ (D) $D_O, 2$ (E) $D_O, 3$

**58.** Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

$$f(x) = \begin{cases} -x^2 + 2x + 4, & \text{if } x \geq 1 \\ ax + b, & \text{if } -1 \leq x \leq 1 \\ -x, & \text{if } x \leq -1 \end{cases}$$

(A) -1.5 (B) -1 (C) 1 (D) 1.5 (E) 5

59. Find a positive number c whose existence is guaranteed by the Mean Value Theorem for the function $f(x) = x^3 + x$ on the interval $[-1, 1]$.

(A) $\frac{1}{3}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) 2

60. In rectangle ABCD, AB = 2 × BC. Equilateral triangle ABE overlaps rectangle ABCD. What percent of rectangle ABCD is covered by triangle ABE? (nearest whole percent)
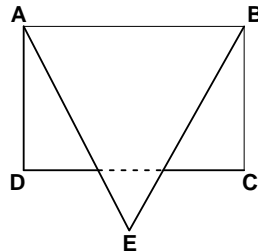
(A) 60%   (B) 65%   (C) 67%   (D) 71%   (E) 73%

University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2021
Answer Key

1. D	21. D	41. E
2. C	22. A	42. D
3. A	23. A	43. E
4. A	24. C	44. C
5. E	25. A	45. C
6. D	26. E	46. B
7. C	27. C	47. E
8. A	28. D	48. A
9. E	29. C	49. B
10. B	30. B	50. D
11. C	31. D	51. C
12. D	32. B	52. E
13. A	33. B	53. B
14. E	34. D	54. E
15. A	35. A	55. D
16. E	36. D	56. B
17. E	37. A	57. B
18. D	38. B	58. E
19. C	39. B	59. B
20. D	40. C	60. D

Mathematics

Region • 2021
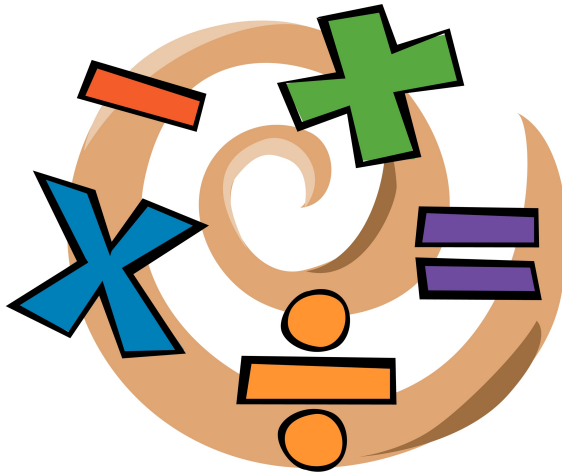
1. Solve for k: $(k - 4) \div 16 + 20 - 2 \times 1! = 17$

   (A) -12 (B) -13 (C) -15 (D) -17 (E) -20

2. *Cheap Flicks* streams movies online. There is a special price for any new members renting 5 movies. They can rent 2 movies for the regular price of 1, another one at 20% off the regular price, and two others at $\frac{2}{3}$ of the regular price. If they give them their email address, they will get $5.00 off of their total order. What would it cost a new member to rent 5 movies if the regular rental price is $7.50 per movie and they give them their email? (nearest cent)

   (A) $13.50   (B) $ 16.00   (C) $18.50   (D) $22.50   (E) $26.00
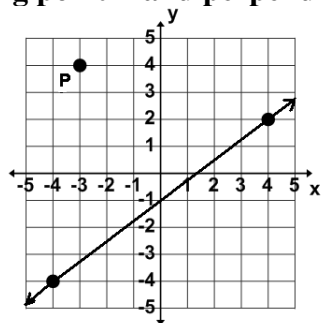
3. If $\frac{3}{4}$ of A is $\frac{4}{5}$ of B, then B is what percent of A?

   (A) $106\frac{2}{3}\%$ (B) 60%   (C) $166\frac{2}{3}\%$ (D) 93.75%   (E) 6.25%

4. Let $O = \{o, s, p, r, e, y\}$, $P = \{p, e, l, i, c, a, n\}$, $A = \{a, v, o, c, e, t\}$ and $B = \{b, a, r, n, o, w, l\}$. The number of elements in $(A \cup O) \cap (P \cup B)$ is:

   (A) 6   (B) 7   (C) 10   (D) 13   (E) 15

5. The line containing point P and perpendicular to the line shown contains the point (6, y). Find y.

   (A) $-6\frac{2}{3}$ (B) $-7\frac{1}{3}$ (C) -8 (D) $-8\frac{1}{3}$ (E) $-9\frac{1}{3}$

6. Which of the following equations has real roots?
   I. $4x^2 + 6x + 1 = 0$ II. $-\frac{1}{2}x^2 - 5x + 6 = 0$ III. $-x^2 + x - 1 = 0$

   (A) I only   (B) I & II   (C) II & III   (D) I & III   (E) I, II, & III

7. Let $(Ax + 3)(Bx + C) = 16x^2 + Dx + 12$, where A, B, C, and D are natural numbers and one of the roots of the equation $16x^2 + Dx + 12 = 0$ is -2. Find D.

   (A) 19   (B) 28   (C) 32   (D) 38   (E) 52

8. Which of the following mathematicians are thought of when working with prime numbers?
   I. Erastosthenes   II. Marin Mersenne   III. Sophie Germain

   (A) I only   (B) I & II   (C) I, II, & III   (D) I & III   (E) III only

9. Willit Gorown is trying to open the combination lock. He has to turn it clockwise to a prime number, then twice around counterclockwise to a Fibonacci number, then clockwise to a factor of 40. How many unique 3-number combinations of natural numbers fit the criteria?
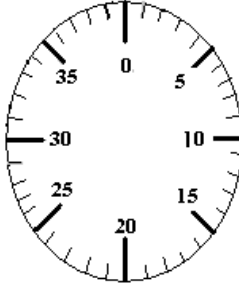
    (A) 28   (B) 144   (C) 672   (D) 768   (E) 864

10. Two supplementary angles have measures of $3x - 7$ degrees and $4x + 5$ degrees. What would the measure of an angle be if it is complementary to the smaller of these two angles? (nearest whole degree)

    (A) 19°   (B) 26°   (C) 38°   (D) 64°   (E) 71°

11. Rose Pedler has a square garden with a perimeter of 80 feet. Rose wants to increase the area of her garden by at least 30%. What is the least number of feet does she need to add to the length if she adds 2 feet to the width? (whole feet)

    (A) 1 foot   (B) 2 feet   (C) 3 feet   (D) 4 feet   (E) 6 feet

12. Find the total surface area of the cylinder shown, where both ends are closed. (nearest sq. inch)
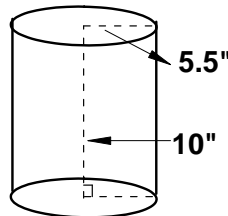
    (A) 346 sq. in   (B) 380 sq. in   (C) 536 sq. in   (D) 570 sq. in   (E) 691 sq. in

13. How many real roots less than zero exist given the equation $2x^4 + 9x^3 - 7x^2 - 54x - 40 = 0$?

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 5

14. Les Dough, Noah Dough, and Lotta Dough went to the local market to buy bread and milk. Les paid $3.47 for 2 loaves of bread and one quart of milk. Noah paid $3.07 for one loaf of bread and 2 quarts of milk. Based on these prices, how much should it cost Lotta for 4 loaves of bread and a gallon of milk?

    (A) $9.61   (B) $8.72   (C) $8.23   (D) $7.83   (E) $6.94

15. If $a_1 = -1$, $a_2 = 1$, $a_3 = 3$, and $a_n = (a_{n-2})(a_{n-3}) - (a_{n-1})$, where $n \geq 4$, then a_6 equals:

   (A) -19 (B) -9 (C) -4 (D) 7 (E) 10

16. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the numbers in row 9 minus the median number in row 9.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \text{(row 1)} \\
 & & & 3 & & 5 & & & \text{(row 2)} \\
 & & 7 & & 9 & & 11 & & \text{(row 3)} \\
 & 13 & & 15 & & 17 & & 19 & \text{(row 4)} \\
21 & & 23 & & 25 & & 27 & & 29 \quad \text{(row 5)} \\
 & & & & \cdots & & & & (\;\cdots\;)
\end{array}
$$

   (A) 810 (B) 793 (C) 747 (D) 729 (E) 648

17. Polly Ticks is randomly selecting the order of the candidates to be listed on the ballot for an upcoming election. There are 3 male candidates and 2 female candidates. How many elements are in the successful sample space when computing the probability that the 2 female names are listed back to back?

   (A) 60 (B) 48 (C) 24 (D) 8 (E) 4

18. The graph shown is the graph of which of the following equations.
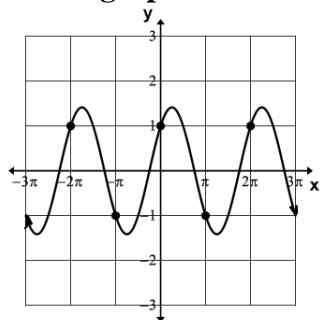
   (A) $y = \sin(x)\cos(x)$ (B) $y = \sin(x) - \cos(x)$ (C) $y = \cos^2(x) - \sin^2(x)$
   (D) $y = \sin^2(x) - \cos^2(x)$ (E) $y = \sin(x) + \cos(x)$

19. Simplify: $\dfrac{\cos\theta}{\sec\theta - \tan\theta}$

   (A) $\csc\theta - 1$ (B) $\dfrac{1 - \sin\theta}{\sin\theta}$ (C) $\cot\theta$ (D) $\cos\theta + 1$ (E) $\sin\theta + 1$

20. How many elements are in $\left\{ x \,\middle|\, \sin^2(x) + \sin(x) - 6 = 0, x \in [0, 2\pi) \right\}$?

   (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

21. If $244_P = 164$ and $355_Q = 187$, then $244_Q + 355_P = ?$

   (A) 339 (B) 351 (C) 367 (D) 452 (E) 599

22. The remainder when $f(x) = kx^3 + 3x^2 - 2x + 1$ is divided by $x - 2$ is 41. Find k.

    (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

23. Ann, Bea, Cay, Dee, Eva, and Felecia are to sit together in a row. In how many ways can they do this if Dee and Eva insist on sitting next to each other, and Felecia sits in the first chair?

    (A) 24   (B) 48   (C) 72   (D) 120   (E) 240

24. Which of the following statements is/are true for $f(x) = |x - 3|$?
    I. f is continuous at x = 3   II. $f'_-(3)$ exists   III. $f'_+(3)$ exists   IV. f is differentiable at x = 3

    (A) I only   (B) I & IV   (C) I, II, & III   (D) All of them   (E) none of them

25. Given the function $g(x) = x^3 - 3x^2 + 3$, at which of the following values of x is function g increasing?   I. -3 II. -1 III. 1   IV. 3

    (A) I & IV   (B) I & II   (C) I, II, & IV   (D) III only   (E) all of them

26. How many five-letter arrangements of the letters REGIONAL consisting of three consonants and two vowels can be formed if no letter is repeated?

    (A) 2,880   (B) 1,440   (C) 576   (D) 288   (E) 240

27. Given the Fibonacci characteristic sequence 2, p, q, 16, r, ... 107, 173, ..., find p + q + r.

    (A) 41   (B) 33   (C) 30   (D) 25   (E) 22

28. How many 4-digit odd numbers between 4,689 and 8,642 can be created using the digits 2, 3, 5, and 7? A digit cannot be used more than once in any of the 4-digit numbers.

    (A) 4   (B) 8   (C) 12   (D) 16   (E) 24

29. Which of the following points are collinear to $(-4, -4)$ and $(4, 2)$?
    I. $(-16, -13)$ II. $(-13, -16)$ III. $(11, 16)$ IV. $(12, 8)$

    (A) II & III   (B) I & IV   (C) II & IV   (D) I only   (E) I, II, & IV

30. Find the perimeter of the quadrilateral ABEF. (nearest tenth)
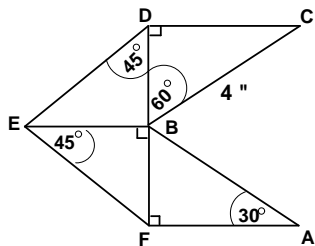
   (A) 10.3 in   (B) 12.6 in   (C) 14.3 in   (D) 10.5 in   (E) 12.3 in

31. If $x + y = -4$ and $xy = 3$ then $x^3 + y^3 = ?$

    (A) -343 (B) -37 (C) -28 (D) -21 (E) -1

32. Sumware is 70 miles due east of Noware. The bearing of Anyware is 15° from Noware and from Sumware is 290°. How much further is it from Anyware to Sumware than it is from Anyware to Noware? (nearest tenth)

    (A) 24.6 mi   (B) 26.2 mi   (C) 43.8 mi   (D) 45.9 mi   (E) 48.6 mi

33. Let $f(x) = \sin(x)$ and $g(x) = 2\cos(x)$. If (x, y) is the point where $f(x) = g(x)$ over the interval $0 \le x \le \pi$ radians, then $x + y = ?$ (nearest hundredth)

    (A) 1.98   (B) 2.00   (C) 2.01   (D) 2.03   (E) 2.05

34. The center of the graph of $2x^2 + y^2 + 8x - 8y - 48 = 0$ is (h, k). Find $h + k$.

    (A) -6 (B) -2 (C) 2   (D) 10   (E) 20

35. Let $f(x) = 7x - 1$ and $g(x) = 4x + 3$. Find $f(g(-x)) + g(f(x))$.

    (A) 21   (B) 20   (C) 19   (D) $56x + 19$ (E) $56x + 21$

36. The *Power Me Up* battery factory packages batteries in packs of 36. The actuaries' research data shows that the probability of at least one bad battery in a 36-pack is 12%. What are the odds that a randomly chosen 36-pack contains all good batteries?

    (A) $\frac{3}{25}$ (B) $\frac{3}{22}$ (C) $\frac{3}{1}$ (D) $\frac{1}{3}$ (E) $\frac{22}{3}$

37. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \ldots + \frac{1}{120} = ?$

    (A) $\frac{6}{7}$ (B) $\frac{7}{8}$ (C) $\frac{9}{11}$ (D) $\frac{13}{15}$ (E) $\frac{15}{17}$

38. How many positive numbers less than 50 are considered to be "tetrahedral" numbers?

    (A) 9   (B) 8   (C) 7   (D) 6   (E) 5

39. The area (in square units) of the shaded regions below is:
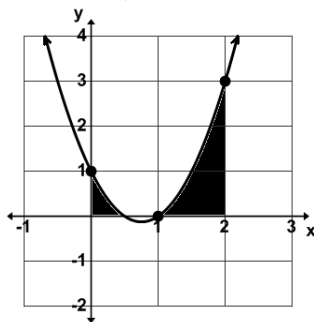
   (A) $1\frac{1}{4}$ (B) $1\frac{1}{6}$ (C) $1\frac{1}{3}$ (D) $1\frac{3}{8}$ (E) $1\frac{1}{2}$

40. **Willie Pass is calculating his 6-weeks average. His daily grade average is 92, quiz average is 70 and test average is 75. The 6-weeks average is calculated using 25% of the daily average, 30% of the quiz average, and 45% of the test average. What is Willie's 6-weeks average?**

   (A) 81.4　　　　(B) 80.25　　　　(C) 79　　　　(D) 77.85　　　　(E) 77.75

41. **The sum of two numbers is 86. If four times the larger number is decreased by the smaller number, the result is 144. Find the product of the two numbers?**

   (A) 1,849　　　　(B) 1,848　　　　(C) 1,845　　　　(D) 1,833　　　　(E) 1,840

42. **Penni Les has a coin purse containing dimes and quarters. The ratio of dimes to quarters is 8 to 3 and the number of dimes exceeds the number of quarters by 35. What is the monetary value of the coins in Penni's coin purse?**

   (A) $12.45　　　　(B) $9.75　　　　(C) $9.30　　　　(D) $11.75　　　　(E) $10.85

43. **The measure of a central angle of a regular pentadecagon is?**

   (A) 30　　　　(B) 24　　　　(C) 18　　　　(D) 15　　　　(E) 9

44. **Point P(3, — 2) lies on the x-y plane. P is rotated 90° clockwise about the origin to Point Q. Point Q is translated vertically + 4 units to point R. Point R is reflected across the line y = 2 — x to point S. Point S is _____ to point T(— 1, 4). (Fill in the blank.)**

   (A) translated vertically + 1 units　　　(B) rotated 90° counterclockwise about the origin
   (C) reflected across the x-axis　　　(D) translated horizontally — 2 units
   (E) translated vertically — 1 units

45. **Find the domain of $f(x) = \frac{\sqrt{8-4x}}{2x^2 - 7x + 5}$.**

   (A) $x \in R \mid x \le 2$　　　(B) $x \in R \mid x \le 2, x \ne 0.5, 5$　　(C) $x \in R \mid x \le 2, -5, -0.5$
   (D) $x \in R \mid x \le 2, x \ne 1$　　　(E) $x \in R \mid x \le 2, x \ne -2.5, -1$

46. **If $f(x) = \frac{2x-5}{3-4x}$, $f^{-1}(6) =$ _____. (nearest tenth)**

   (A) — .3　　　　(B) — .1　　　　(C) 0.2　　　　(D) 0.6　　　　(E) 0.9

47. **If the shadow of a building increases by 10 meters when the angle of elevation of the sun rays decreases from 70° to 60°, what is the height of the building? (nearest meter)**

   (A) 17 meters　　(B) 27 meters　　(C) 35 meters　　(D) 41 meters　　(E) 47 meters

48. **Which of the following is closest to the length of the segment bounded by the parametric equations: x = 2t — 3,   y = 3t + 2, 0 ≤ t ≤ 1.**

   (A) 8.1　　　　(B) 7.3　　　　(C) 5.0　　　　(D) 3.6　　　　(E) 2.2

49. Find the average value of the function f defined by $f(x) = x^3 + 1$ on the interval $[-2, 2]$.

   (A) 8   (B) 4   (C) 2   (D) 1   (E) 0

50. Given the function $f(x) = 2\cos(x) + 1$, find the slope of the secant line between $x = \frac{3\pi}{2}$ and $x = 2\pi$. (nearest tenth)

   (A) 0.4   (B) 0.8   (C) 1.3   (D) 2.5   (E) no slope

51. Polly Ticks is randomly selecting the order of the candidates to be listed on the ballot for an upcoming election. There are 3 male candidates and 2 female candidates. What is the probability that all three male candidates are listed back to back to back, given that the third candidate on the ballot is a male?

   (A) 20%   (B) 25%   (C) 30%   (D) 40%   (E) 50%

52. For the general population, IQ scores are normally distributed with a *mean* of 100 and a *standard deviation* of 15. Approximately, what percent of the population have IQ scores above 115?

   (A) 15%   (B) 16%   (C) 30%   (D) 32%   (E) 34%

53. Let g, $g + 2$, and $g + 8$ be the first three terms of a geometric sequence. What is the sum of the first four terms of this sequence?

   (A) 27   (B) 35   (C) 40   (D) 45   (E) 53

54. Find the slope of the tangent line to the curve whose equation is $r = 1 - \cos\theta$ at the point $(1 - \frac{\sqrt{2}}{2}, \frac{\pi}{4})$. (nearest tenth)

   (A) 1.4   (B) 1.7   (C) 2.4   (D) 2.9   (E) 3.4

55. Find distance, d, in order to balance the beam. (nearest cm)
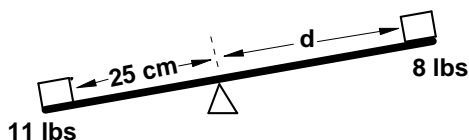
   (A) 35 cm   (B) 34 cm   (C) 28 cm   (D) 19 cm   (E) 18 cm

56. Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

$$f(x) = \begin{cases} x + a, & \text{if } x > 4 \\ 3 - |x - 1|, & \text{if } -1 \le x \le 4 \\ b, & \text{if } x < -1 \end{cases}$$

   (A) -3 (B) -1 (C) 1   (D) 4   (E) 5

57. Suppose $\triangle P'Q'R' = D_{O}, -\frac{2}{3}(\triangle PQR)$ and the coordinate of Q' is (x, y). Find $x + y$.
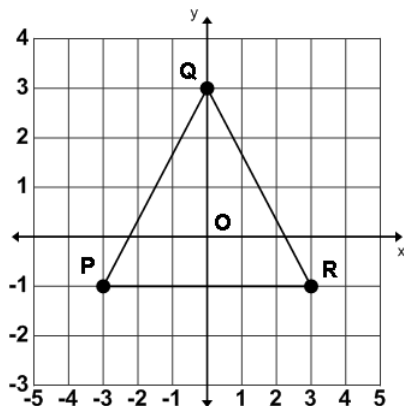
(A) -2 (B) $-1\frac{1}{3}$ (C) $-\frac{2}{3}$ (D) 1 (E) $1\frac{2}{3}$

58. Given the harmonic sequence $\frac{3}{11}, \frac{3}{17}, \frac{3}{23}, \frac{3}{29}, \ldots$., which of the following would be an element of this sequence?

(A) $\frac{3}{209}$ (B) $\frac{3}{205}$ (C) $\frac{3}{199}$ (D) $\frac{3}{193}$ (E) $\frac{3}{181}$

59. Given: $\triangle$ ABT is inscribed in the circle with center C; RS is tangent to the circle at point T; $m\angle ATB$ is $28°$; and $m\angle BAT$ is $71°$. Find $m\angle ATR$.
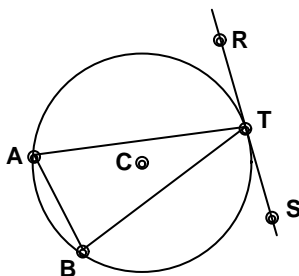
(A) $71°$ (B) $78°$ (C) $81°$ (D) $90°$ (E) $98°$

60. Two cardboard squares with side lengths 4" and 3" overlap with the corner of the smaller square at the center of the larger square. If the overlap portions are removed what is the difference between the remaining areas?
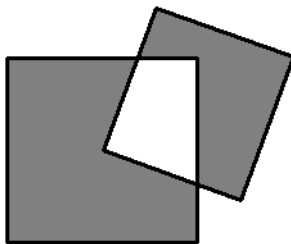
(A) 7 in^2 (B) 6 in^2 (C) 5 in^2 (D) 1 in^2 (E) can be determined

University Interscholastic League
MATHEMATICS CONTEST
HS ● Regional ● 2021
Answer Key

1. A	21. C	41. E
2. C	22. D	42. E
3. D	23. B	43. B
4. A	24. C	44. D
5. C	25. C	45. D
6. B	26. A	46. E
7. D	27. A	47. E
8. C	28. B	48. D
9. C	29. B	49. D
10. A	30. E	50. C
11. D	31. C	51. E
12. C	32. C	52. B
13. D	33. B	53. C
14. B	34. C	54. C
15. A	35. C	55. B
16. E	36. E	56. A
17. B	37. B	57. A
18. E	38. E	58. A
19. E	39. D	59. C
20. E	40. E	60. A

Mathematics

State • 2021
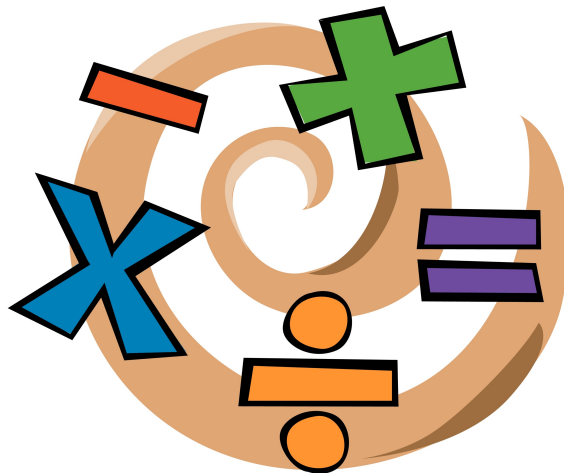
DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Solve for k: $3 \times 5 - 6 \times (1 + 2 \div k) = 6$

   (A) -6　　　　(B) $-1.666...$　　(C) 0.8　　　　(D) 4　　　　(E) 4.75

2. Trotter Turtle is loping at a speed of half a mile per hour. How long will it take Trotter to lope 20 feet? (nearest second)

   (A) 27 sec　　　(B) 2 sec　　　(C) 4 sec　　　(D) 30 sec　　　(E) 44 sec

3. Phil Hurrup has a pickup with a 25 gallon tank that averages 15 miles per gallon of gas. Phil spent \$50.00 for gas at \$2.32 per gallon to drive to his deer lease 600 miles away. How many miles was Phil able to drive before exhausting his \$50.00 worth of gas? (nearest mile)

   (A) 539 mi　　　(B) 216 mi　　　(C) 375 mi　　　(D) 277 mi　　　(E) 323 mi

4.

   |　　　　　A　　　　　B　　　　　C　　　　　D　　　　　E|

   `<---------------|---------------|---------------|---------------|---------------|--------------->`

   The distances between the hash marks (|) are equal. Find the midpoint of DE if
   $A = 3\frac{7}{10}$ and $E = 17\frac{3}{5}$.

   (A) $15\frac{9}{10}$　　　(B) $15\frac{69}{80}$　　　(C) $15\frac{1}{2}$　　　(D) $15\frac{2}{5}$　　　(E) $15\frac{9}{40}$

5. Given the statements, $3x - 2 = k + 4$ and $k + 4 = 5$, which of the following reasons justifies the conclusion $3x - 2 = 5$?

   (A) symmetric property　　　　(B) transitive property　　　　(C) distributive property
   (D) multiplicative property of -1　　　　　(E) additive identity property

6. Simplify: $\dfrac{3n^2 - 27}{6 - n - n^2} \times \dfrac{4 - 2n}{3 - n} \div \dfrac{6}{3n + n^2}$

   (A) -36　　　(B) -1　　　(C) $-n^2 - 3n$　　(D) $-\dfrac{3 + n}{n}$　　(E) $-(n + 3)$

7. Let $(4x - 3)(ax + b)(cx + 1) = 4x^3 + dx^2 + ex + 6$, where a,b,c,d, and e are non-distinct integers. Find $a + b + c + d + e$.

   (A) -12　　　(B) -8　　　(C) -2　　　(D) 7　　　(E) 16

8. The Electric Eel Energy Company set new rates for electricity for 2021.

Base cost	first 500 kwh	next 500 kwh	above 1000 kwh	Delivery rate
\$7.95	9.6¢ /kwh	9.0¢ /kwh	8.8¢ /kwh	3.2¢ /kwh

   If I. M. Shawked used 1400 kwh this month, what would be his total monthly bill, including the base cost, the usage cost, and the delivery cost? (tax not included)

   (A) \$168.95　　　(B) \$172.95　　　(C) \$175.95　　　(D) \$177.95　　　(E) \$180.95
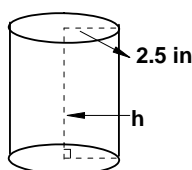
9. The set of points on the line below are collinear with which of these points?

   I. $(-17, 12)$ II. $(9, -10)$ III. $(7.5, -5.5)$
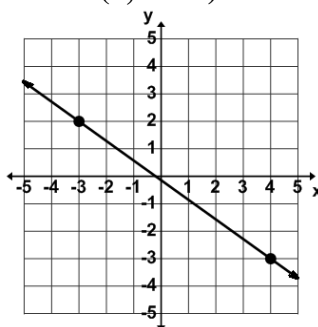
   (A) I only   (B) II only   (C) III only   (D) I & III   (E) I, II, & III

10. The cylinder below holds at least a gallon of water. What would the minimum height be? (nearest tenth)

   (A) 16.3 in   (B) 14.7 in   (C) 11.8 in   (D) 10.2 in   (E) 8.2 in

11. Point P(2, 5) lies on the x-y plane. P is translated -2 units vertically and -5 units horizontally to point Q. Point Q is reflected across the line y = 2 to point R. Point R is reflected across the line x = 5 to point S. Find the distance from point P to point S. (nearest tenth)

   (A) 10.0 units   (B) 6.4 units   (C) 10.2 units   (D) 7.8 units   (E) 11.7 units

12. Given the sides of three triangles, which of these triangles has its orthocenter outside of the triangle?   I. 3, 3, 4   II. 3, 4, 5   III. 3, 4, 6

   (A) I only   (B) II only   (C) III only   (D) I & II   (E) I & III

13. Let $f(x) = x^2 - 3$ and $g(x) = 4x + 5$. Find $f\big(g(-2)\big) + g\big(f(2)\big)$.

   (A) 15   (B) 12   (C) 9   (D) 6   (E) 3

14. $(2C2_{16}) \times (8_{16}) - (4E4_{16}) = $ _____ $_{16}$.

   (A) 4396   (B) 2,574   (C) 5648   (D) 1C56   (E) 112C
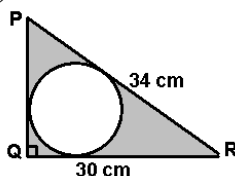
15. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - ...$? (nearest hundredth)

   (A) 1.05   (B) 0.63   (C) 0.79   (D) 0.39   (E) 1.57

16. Given the sequence: $-12, a, -3, b, c, \frac{3}{8}, d, ...$. Find the sum of these seven terms.

   (A) $-7\frac{5}{8}$ (B) $-7\frac{29}{64}$ (C) $-7\frac{7}{8}$ (D) $-8\frac{3}{64}$ (E) $-8\frac{1}{16}$

17. Given the circle inscribed in $\triangle$PQR, find the shaded area. (nearest cm^2)

   (A) 161 cm^2 (B) 139 cm^2 (C) 127 cm^2 (D) 117 cm^2 (E) 83 cm^2

18. The *Clickety Clack* and the *Choo Choo* leave the station on parallel tracks at the same time. *Clickety* travels 2 miles per hour faster than *Choo*. It takes *Choo* 15 minutes more time to reach the next station 300 miles away than *Clickety*. What was *Choo*'s rate of travel?

   (A) 40 mph   (B) 42 mph   (C) 45 mph   (D) 48 mph   (E) 50 mph

19. Which of the following is a false statement?

   (A) $\sin \theta = \cos (2\pi - \theta)$ (B) $\cos 3\theta = 4\cos^3\theta - 3\cos \theta$ (C) $\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

   (D) $\cos \theta = \sin (\frac{\pi}{2} - \theta)$ (E) $3\sin \theta = 4\sin^3\theta + \sin 3\theta$

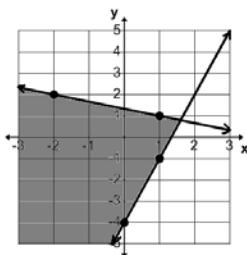20. If $\cos x \cos y = \frac{5}{8}$ and $\sin x \sin y = \frac{3}{8}$ then $\cos(x - y) = $?

   (A) $\cos \pi$ (B) $\cos \frac{5\pi}{12}$ (C) $\sin \frac{\pi}{12}$ (D) $\sin \frac{3\pi}{2}$ (E) $\sin \frac{5\pi}{2}$

21. Which type of conic is the graph of the equation $Ax^2 + By^2 + Cy + D = 1$, where A, B, C, and D are distinct integers and A, B, D > 0 and C < 0?

   (A) circle   (B) ellipse   (C) hyperbola   (D) parabola   (E) not a conic

22. The shaded area shown represents the solutions for which of the following pair of inequalities?

   (A) $x + 3y \leq 4$ (B) $x - 3y \leq 4$ (C) $x + 3y \geq 4$ (D) $x + 3y \leq 4$ (E) $x - 3y \leq 4$
       $3x - y \geq 4$ $3x + y \leq 4$ $3x - y \geq 4$ $3x - y \leq 4$ $3x + y \geq 4$

23. If $a_1 = 3, a_2 = 2, a_3 = -1$, and $a_n = [(a_{n-2}) - (a_{n-3})] \times (a_{n-1})$ for n ≥ 4, then a_6 equals:

   (A) -6 (B) -3 (C) 1   (D) 4   (E) 24

24. Chip Picker selects two chips at random from a box containing five chips numbered 2, 3, 5, 7, and 11. The probability that the sum of the two chips is a prime number is $\frac{c}{t}$. Find $t - c$.

   (A) 11   (B) 7   (C) 5   (D) 3   (E) 2

25. The graph of h(x) is shown below. Find h'(15.5) if h(1) = — 3.666... .
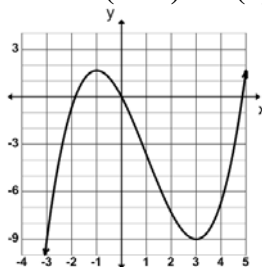

    (A) 205   (B) 205.75   (C) 206.25   (D) 207.5   (E) 212.25

26. The *Citizen* ship and the *Friend* ship are heading straight away from the same port along routes that make a 120° angle. The *Citizen* is traveling at 15 mph and the *Friend* is traveling at 25 mph. How fast are the ships moving apart when the *Citizen* is 5.8 miles from port and the *Friend* is 3.5 miles from port? (nearest whole mph)

    (A) 20 mph   (B) 21 mph   (C) 31 mph   (D) 34 mph   (E) 35 mph

27. Which of the following are considered to be happy numbers that are perfect or perfect numbers that are happy?   I. 6   II. 7   III. 28   IV. 496

    (A) I, III, & IV   (B) I & III   (C) II & III   (D) I & IV   (E) III & IV

28. Let U = {a, b, c, e, g, h, i, o, r, s, t, u} be a universal set. Let A = {a, c, u, t, e}, R = {r, i, g, h, t}, O = {o, b, t, u, s, e}, and C = (A ∪ R) ∩ (O). How many elements are in the complement of C?

    (A) 12   (B) 11   (C) 9   (D) 7   (E) 3

29. Mary D. Rapper bought a 5-yard spool of ribbon to wrap gifts. She used $\frac{1}{4}$ of the spool for one gift and 30% of the rest of the spool for a second gift. What was the length of the remaining ribbon on the spool?

    (A) 2 yds 1 ft 1.5 in   (B) 2 yds 2 ft 3 in   (C) 2 yds 1 ft 10.5 in   (D) 2 yds 9 in   (E) 1 yd 4.5 in

30. If y is inversely proportional to x + 2, and y = 13 when x = 2, find y when x = 1.25.

    (A) 16   (B) 20.8   (C) 8   (D) 41.6   (E) 10.5625

31. Let AB = 6". Find CF. (nearest tenth)
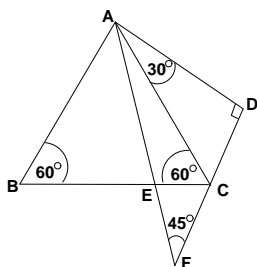
    (A) 2.4"   (B) 2.2"   (C) 1.4"   (D) 1.7"   (E) 3.7"

32. **STEM Tech Academy offers 8 advanced math courses, 6 advance science courses, and 3 robotics courses. In how many ways can a six course semester schedule be set up consisting of 3 math courses, 2 science courses and 1 robotics course?**

    (A) 30,240   (B) 2,520   (C) 369   (D) 12,376   (E) 74

33. **Which of the following mathematicians would most likely be linked to the sequence**
$\frac{1}{4} + \frac{3}{4} + 1 + 1\frac{3}{4} + 2\frac{3}{4} + ... + 7\frac{1}{4} + 11\frac{3}{4} + ...$?

    (A) Georg Cantor   (B) Alan Turing   (C) Mary Rudin   (D) Zeno of Elea   (E) Leonardo Bigollo

34. **The roots of $P(x) = x^3 + x^2 + kx + 8$ are integers. Find k.**

    (A) -10 (B) -4 (C) -2 (D) 10   (E) 14

35. **Solve for x:** $\log_3(x - 3) + \log_3(x - 1) = \log_3(3 - 3x)$

    (A) -3 (B) -1 (C) 1   (D) 2   (E) no solution

36. **Use following table to approximate a left hand Riemann sum of $\int f(x)$.**

x	0	2	3	4	9
f(x)	3	5	7	5	4

    (A) 16   (B) 19   (C) 43   (D) 47   (E) 96

37. **Rusty Yaht left his port and sailed his sloop 18 km on a bearing of 72° to Atoll. Then he sailed 24 km on a bearing of 336° to Eyland. He could have saved time by sailing straight to Eyland on a bearing of _____. (nearest degree)**

    (A) 9°   (B) 15°   (C) 39°   (D) 57°   (E) 345°

38. **Find the sum of all numbers c that satisfy the conclusions of the Mean Value Theorem for the function $f(x) = x^3 + x^2 - x$ on the interval $[-2, 1]$.**

    (A) $-\frac{1}{4}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{2}$ (D) $-\frac{2}{3}$ (E) -1

39. **Rollin Kubes rolls only one die. Assuming it is a fair die, what is the probability that the top face has 3 dots, 2 dots, or 1 dot showing, given that the number of dots showing is odd?**

    (A) $16\frac{2}{3}\%$ (B) 30%   (C) $33\frac{1}{3}\%$ (D) 60%   (E) $66\frac{2}{3}\%$

40. $333_8 + 222_4 + 111_2 =$ _____ $_{16}$.

    (A) 29A   (B) 2426   (C) 862C   (D) 10C   (E) 268

41. **Real numbers p, q, and r exist such that $p + q + r = 24$, $r^2 = p^2 + q^2$, and $pq = 12$, find the value of r.**

    (A) 12   (B) 11.5   (C) 10.5   (D) 8   (E) 5.75

42. The lengths of the sides of quadrilateral ABCD are given. If diagonal AC has integral length, find AC?
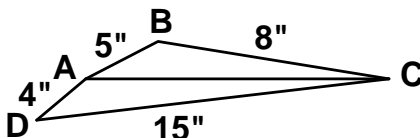
(A) 9" (B) 12" (C) 13" (D) 19" (E) 20"

43. A sine function, f(x), has a maximum value of 7, a minimum value of 3, a period of $\frac{2\pi}{3}$ and a horizontal phase shift of $\frac{\pi}{2}$. If f(0) = 3, then the value of $f(\frac{\pi}{5})$ is _____ . (nearest tenth)

(A) 5.6 (B) 5.4 (C) 5.3 (D) 5.2 (E) 5.0

44. Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

$$f(x) = \begin{cases} x - 3, & \text{if } x \geq 3 \\ \sqrt{a - x^2}, & \text{if } -3 < x < 3 \\ b - x, & \text{if } x \leq -3 \end{cases}$$

(A) -3 (B) 0 (C) 3 (D) 6 (E) 12

45. The sum of the y-values of all of the critical points of $f(x) = x^4 + 4x^3 - 2x^2 - 12x$ minus the sum of the y-values of all of the critical points of $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 4$ is ?
(A) -12 (B) -11 (C) -4 (D) 2 (E) 12

46. Find the coefficients of the x^4 term when $\left(x^{\frac{1}{2}} - x^{\frac{2}{3}} \right)^7$ is expanded.

(A) -21 (B) -35 (C) 7 (D) 21 (E) 35

47. Two squares have dimensions as indicated in the drawing. What is the area of the shaded region?

(A) 10.5 in^2 (B) 18.5 in^2 (C) 11.5 in^2 (D) 23.5 in^2 (E) 17.5 in^2

48. Poly Gawn has an irregular shaped quadrilateral garden. Two of the angles are equal. The third angle is equal to the sum of the two equal angles. The fourth angle is 60° less than twice the sum of the other three angles. How much bigger is the largest angle than the smallest angle?

(A) 220° (B) 167.5° (C) 150° (D) 115° (E) 185°

49. Points A, B, C, and D lie on a circle with center O, chord AC and chord BD are extended outside the circle intersecting at point P, BD = 5 cm, BP = 2 cm, and AP = 1.5 cm. Find AC.

(A) $8\frac{1}{2}$ cm (B) $7\frac{5}{6}$ cm (C) $7\frac{1}{3}$ (D) $6\frac{2}{3}$ (E) $4\frac{1}{2}$

50. Sir Vayer staked out a plot of land using the layout below. How far was it from stake A to stake B? (nearest hm)
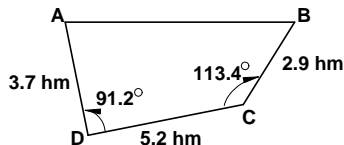
(A) 5.9 hm (B) 6.1 hm (C) 6.3 hm (D) 6.5 hm (E) 6.7 hm

51. Let $f(x) = \sin(\frac{\pi}{2} - x)$ and $g(x) = x$. If (x, y) is the point where $f(x) = g(x)$ over the interval $0 \le x \le 2\pi$ radians, then $x + y =$?   (nearest hundredth)

(A) 1.40 (B) 1.44 (C) 1.48 (D) 1.53 (E) 1.57

52. The sequence $1 - \frac{9}{2} + \frac{81}{24} - \frac{729}{720} + \frac{6561}{40320} - ...$ simplifies to the decimal number $-0.ABCDEFG...$, where the letters represent digits. What digit does letter G represent?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 9

53. A 5" by 7" photo is being enlarged. How long will it take for the area of the original photo to be at least 5 times as big if the length and width are increasing at the rate of 2" per second? (nearest hundredth)

(A) 3.65 sec (B) 3.63 sec (C) 3.41 sec (D) 3.36 sec (E) 2.50 sec

54. $(13121)^k \div 137$ has a remainder of 2 for which of these values of k?

(A) 138 (B) 134 (C) 133 (D) 131 (E) 130

55. Find the displacement of a particle traveling along the curve, $c(t) = 4^x$, from $t = 1$ to $t = 4.3$. (nearest tenth)

(A) 277.1 (B) 279.2 (C) 381.4 (D) 384.0 (E) 480.3

56. The circle shown is tangent to all sides of the square and has a radius shown. What are the odds that an arrow hits in the square but outside the circle? (nearest tenth)
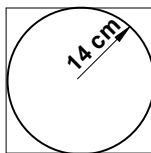
(A) 27.3% (B) 21.5% (C) 20.0% (D) 15.5% (E) 14.0%

57. A bag contains white balls and red balls. If two balls are randomly removed, the probability that they are both white is $\frac{1}{3}$. If three are randomly removed, the probability that they are all white is $\frac{1}{6}$. How many red balls are in the bag?

(A) 4   (B) 5   (C) 6   (D) 8   (E) 10

58. All 15 residents of Millersview were asked to rate something from 1 to 5 where 1 is the worst and 5 is the best. The responses are distributed as shown in the chart. Find the sum of the mean, the standard deviation, and the variance. (nearest tenth)

Response	1	2	3	4	5
Frequency	4	5	3	1	2

(A) 5.5   (B) 2.9   (C) 4.8   (D) 5.4   (E) 5.9

59. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. The sum of the numbers in column 16 row 10 and column 15 row 11 is?

1				1				2				3				5		
		2				3				5				8				
	3				5				8				13				21	
	5			8				13				21				34		
8			13			21			34						55			
		21				34				55				89				
	34			55			89				144				233		

(A) 1,595   (B) 1,974   (C) 2,584   (D) 2,843   (E) 3,571

60. Mr. White wrote 11,127 math problems for UIL and TMSCA over the last 19 years. Twenty-two of those problems were fill in the blank problems with each having only one correct answer. All of the other problems were multiple choice problems containing one correct answer and four distracters. The total number of correct answers and distracters Mr. White had to come up with over the 19 year period is?

(A) 55,635   (B) 55,613   (C) 55,569   (D) 55,566   (E) 55,547

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2021
Answer Key

1.	D	21.	B	41.	B
2.	A	22.	D	42.	B
3.	E	23.	A	43.	A
4.	B	24.	B	44.	D
5.	B	25.	C	45.	A
6.	C	26.	D	46.	B
7.	A	27.	E	47.	C
8.	E	28.	C	48.	E
9.	D	29.	C	49.	B
10.	C	30.	A	50.	D
11.	E	31.	B	51.	C
12.	C	32.	B	52.	C
13.	A	33.	E	53.	B
14.	E	34.	A	54.	A
15.	C	35.	E	55.	D
16.	E	36.	C	56.	A
17.	C	37.	B	57.	A
18.	D	38.	D	58.	A
19.	A	39.	E	59.	C
20.	E	40.	D	60.	E

University Interscholastic League
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

Final _____ _____

2nd _____ _____

1st _____ _____

Score   **Initials**

Contestant # _____

Conference _____

1. _____
2. _____
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60. _____

Larry White - Mathematics Contest Director - texasmath@centex.net

   **As you know, this will be the last update of my 'Mathematics Test Corrections and Comments'
   page for this 20-21 season and my last as the UIL Mathematics Contest Director.
   Mr. McCurdy will be taking over as the contest director for the 21-22 season. I will be his
   assistant and will work with him in helping to make this transition as smooth as possible. I
   believe he will be a great director and will bring a freshness of ideas mixed in with his unique
   style of problem writing. I thank all of you, past and present, for giving me the opportunity
   these past 19 years to be part of your academic growth through UIL competition. It has been a
   great joy and my pleasure to have served you in the best ways I could. I hope I have enhanced
   your love for mathematics and hope I have shown you the glorious paths in life you can choose
   to take. I have been deeply blessed.
   This has been a most unusual season. I encourage everyone to let the UIL staff know
   how thankful you are for all of the work it took them to put together a season amongst all of the
   surrounding difficulties. Finding ways to hold district, regional, and state competitions was not
   an easy task, to say the least. It would have been easier to cancel the season again as they had to
   last year, but that is not how UIL reacts to hardships. Kudos to all the UIL staff and all the
   coaches and hub workers for all their hard and tireless work to provide for our most valuable
   assets; our students and their competive spirit.
   I would like to congratulate all of the students for working through all of the issues facing us
   this year and continuing to grow academically through UIL competitions. The knowledge and
   skills gained through the UIL experience is everlasting and can never be taken away from you.
   I would like to congratulate all 125 of the math students who made to state and a special
   congratulations to those state championship individuals and those state championship teams. I
   missed not getting to see you all and I sure missed not getting to put your medals around your
   necks. Please don't forget to thank your parents, thank your coaches, thank your schools, and,
   most importantly, thank HIM. I hope you all continue to find a few minutes each day in your life
   to walk with, talk with, and give thanks to your creator. Without HIM the path is lonely and
   bleak. I am looking forward to seeing you all in the Fall.**

UIL Test Comments — 2020-21

*** NOTE: See **Off on a Tangent** below for information on workshops, Student Activity Conferences,
   and test discussions *******

SAC - - - > No errors, corrections, or comments reported at this time. (Release dates: 10/1/20)

A - - - >   # 53 needs an ' = ' sign after f(x)   (Release dates: 1/8/21 - 2/6/21)

B - - - >   # 35 needs a '5' between 'of ' and 'fish '   (Release dates: 2/12/21 - 3/13/21)

District - - - > #23 -- the units in the answer choices should have been meters not feet.
                #35 -- should have said 'integers' not 'digits'   (Release dates: 3/22/21 - 3/27/21)

Regional - - - > No errors, corrections, or comments reported at this time. (Release dates: 4/16/21 - 4/17/21)

State - - - > No errors, corrections, or comments reported at this time. (Release date: 4/29/21 - 5/1/21)

TMSCA Test Comments — 2020-21 (tests I write for TMSCA)

 # 6 - - - >   No errors, corrections, or comments reported at this time. (Release date: 12/07/19)

 # 13 - - - >   No errors, corrections, or comments reported at this time. (Release date: 3/07/20)

State - - - >   No errors, corrections, or comments reported at this time. (Release date: 3/21/20)

Off on a Tangent

Workshops and/or Presentations I will be doing:
1. UIL Capitol Conference, Austin - ~~June 23-24, 2020~~. Cancelled --- Virtual
   See the UIL Academic website for two recorded Zoom sessions and multiple downloads
   Keep an eye out on the UIL Academic Website for information of the upcoming 2021 virtual Capitol Conference.

2. *What's Your 11th Problem* - Math Camp at Texas Tech University, Lubbock on ~~July 13-18, 2020~~.
   Cancelled --- future camps have not been addressed at this time ---
   For more information contact Jack Barton at jack.barton@ttu.edu or 806-742-2350.
3. Student Activity Conferences: **The 2020 conferences will be virtual.**

   --- This Year in Number Sense and Mathematics: News -Updates- Hot Topics
   (prerecorded session to be posted on the UIL Academic website on Oct. 1)
   --- Number Sense Problem Solving
   (prerecorded session to be posted on the UIL Academic website on Oct. 29)
   --- Mathematics Problem Solving
   (prerecorded session to be posted on the UIL Academic website on Oct. 29)
   --- Number Sense and Math - Coaches Chat
   (live zoom session at 4:00 pm Wednesday, Nov. 4 --- register in advance)

Resources Update

The **mathematics contest is a curriculum based contest. The best resources for the contest are the courses and textbooks adopted by the state of Texas.**

Test Discussions

The district, regional, and state tests will be created based on the problems from these 6 tests:
2020SAC, TMSCA 6, UIL A, UIL B, TMSCA 13, & TMSCA STATE.
If a concept is addressed by a problem(s) from these 6 practice tests, then other types of problems in that concept area can appear on the district, regional, and state tests.

*** Not applicable for this year *** <u>STATE MEET PROOFERS AND GRADERS</u>