GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $6! - 6^3 + 6 \times 6 + (36)^{3/2} \div (6)^{-1}$

    (A) 576   (B) 864   (C) 549   (D) 1836   (E) 1812

2. Shelly bought 4 shirts each marked $32.99 and 2 pairs of shoes each marked $47.95. The shirts were on sale for 20% off, and the shoes were on sale for 40% off. When she got to the counter to pay, the salesperson informed her that her whole purchase was an additional 10% off. If the local tax rate was 8.25%, how much did Lindsay pay?

    (A) $151.90   (B) $168.78   (C) $176.56   (D) $167.91   (E) $158.91

3. Find the sum of the multiples of six that are greater than 0 and less than 384.

    (A) 12,288   (B) 12,096   (C) 12,084   (D) 12,090   (E) 12,102

4. $(x+3)(x-7) = x^2 - 7x + 3x - 21$ and $9(2x+7) = 18x + 63$ **are examples of the _____ property of equality.**
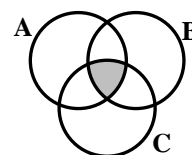
    (A) associative   (B) commutative (C) addition   (D) distributive (E) multiplication

5. If P is 24% less than Q and Q is 28% more than R, then R is what percent of P? (nearest one percent)

    (A) 98%   (B) 92%   (C) 101%   (D) 104%   (E) 103%

6. Which of the following is a symbolic representation for the Venn diagram shown?

    (A) $(A \cap C) \cap (B \cap C)$ (B) $(A \cup C) \cap (B \cup C)$ (C) $(A \cap C) \cup (B \cap C)$

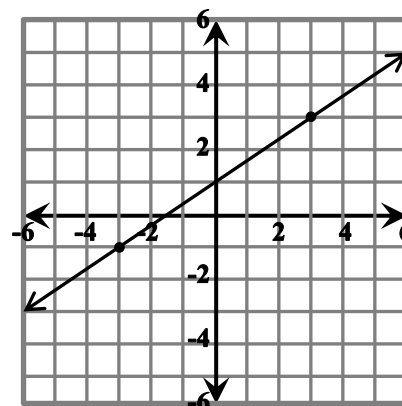   (D) $(A \cup C) \cup (B \cup C)$ (E) $(A \cup C) \cap (B \cap C)$

7. If $5x - 2y = -20$, $2x - 5y = -29$ **and** $ax + y = 13$, **find the value of** a.

    (A) 7   (B) −5   (C) 2   (D) 5   (E) −4

8. An equation of the line shown is:

    (A) $3x + 2y = 1$ (B) $2x - 3y = -3$ (C) $3x - 2y = 7$
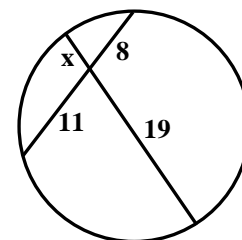
    (D) $3x + 2y = -7$ (E) $2x - 3y = -1$

9. Given that quadrilateral ABCD is inscribed in circle O, $\angle A$ **and** $\angle C$ **are not consecutive angles,** $m\angle A = (x^2 - 4x)°$ **and** $m\angle C = (5x - 2)$, **find** $m\angle A$.

    (A) 117°   (B) 156°   (C) 24°   (D) 63°   (E) 92°

10. Simplify: $16 \sqrt[3]{16x^8} \div \left(24 \sqrt[3]{54x^5}\right)$

(A) $\dfrac{4x^2}{9}$ (B) $\dfrac{2}{9}$ (C) $\dfrac{4x}{9}$ (D) $\dfrac{2x}{9}$ (E) $\dfrac{2x^2}{9}$

11. The two segments in the illustration shown are chords of the circle. Find the value of x. (nearest tenth)

A)   4.2   (B)   3.7   (C)   5.8   (D)   4.6   (E)   5.1

12. Which of the following are the side lengths of an obtuse, scalene triangle?

(A)   7, 7, 13   (B)   7, 10, 13   (C)   7, 8, 10   (D)   7, 20, 21   (E)   7, 24, 25

13. Let $f(x)=3x-3$, $g(x)=2x-4$, $h(x)=2-5x$, and $g\big(h(f(a))\big)=10$. Find $6a-7$

(A)   −1   (B)   −3   (C)   7   (D)   2   (E)   1

14. The repeating decimal 0.52525252… in base 6 can be written as which of the following fractions in simplest form?

A) $\dfrac{32}{55}_6$ (B) $\dfrac{10}{11}_6$ (C) $\dfrac{5}{11}_6$ (D) $\dfrac{52}{55}_6$ (E) $\dfrac{32}{35}_6$

15. A particle's movement along the number line is defined by $f(t)=t^4-4t^3-26t^2+60t+25$. At which of the following times is the particle moving to the left?

(A)   1   (B)   0   (C)   −1   (D)   −2   (E)   3

16. Which of the following set(s) of numbers is/are closed under both addition and multiplication?

  I. Real Numbers   II. Even Numbers   III. Odd Numbers

(A)   I only   (B)   III only   (C)   I & III   (D)   I & II   (E)   None

17. The measure of an interior angle of a regular dodecagon is _____. (nearest degree)

(A)   144°   (B)   150°   (C)   120°   (D)   130°   (E)   132°

18. If $z=a+bi$, $z^4=28+96i$ and $z^5=-316-12i$, then $a+b=$?

(A)   −3   (B)   2   (C)   3   (D)   −1   (E)   1

19. Determine the range of $f(x)=4-3\cos(2x+1)$.

(A) $[-2,10]$ (B) $[-3,3]$ (C) $[1,7]$ (D) $[-1,7]$ (E) $[-2,4]$

20. If $\sin x+\cos x=\dfrac{1}{3}$, then $\tan x+\cot x=$? (nearest tenth)

(A)   −2.3   (B)   2.7   (C)   −2.8   (D)   −3.6   (E)   3.9

21. Find k if the remainder when $x^3 + 3x^2 + kx + 12$ is divided by $x - 4$ is 104.

   (A) 5   (B) −3   (C) 2   (D) −5   (E) 3

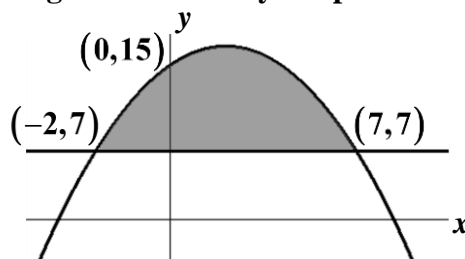**22.** Find $a + b + c + d$ given the Fibonacci characteristic sequence: $4, a, b, 2, 5, c, d, \ldots$.

   (A) 19   (B) 23   (C) 21   (D) 25   (E) 27

23. If $\log_6(x+8) - \log_6(x+3) = 1$, then $x = ?$

   (A) 5.2   (B) −2   (C) 1   (D) 2   (E) No solution

24. Find the area of the shaded region bounded by the parabola and the line in the illustration. (nearest square unit)

   (A) 132   (B) 78   (C) 113   (D) 69   (E) 83

25. $A = \begin{bmatrix} 3 & 5 \\ 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$. Calculate $\det(B - 2A)$.

   (A) −119   (B) −18   (C) 9   (D) 133   (E) −194

26. Karen left her car and hiked 3 miles on a course heading 30° east of south. She then turned and walked 1.8 miles on a course heading 15° east of north. How far will she have to hike directly back to her car? (nearest tenth)

   (A) 1.8 mi   (B) 2.1 mi   (C) 2.3 mi   (D) 2.7 mi   (E) 1.2 mi

27. Calculate $9^3 + 10^3 + 11^3 + \ldots + 21^3 = ?$

   (A) 51,336   (B) 52,577   (C) 63,225   (D) 52,065   (E) 58,293

28. How many distinct 4-letter words can be made from the letters in the word "STATE MEET"?

   (A) 270   (B) 296   (C) 230   (D) 308   (E) 302

29. Let $a_1 = 2$, $a_2 = -1$, $a_3 = 3$ and $a_n = (a_{n-2})(a_{n-3}) - (a_{n-1})$ where $n > 3$ then $a_7 = ?$

   (A) 2   (B) 13   (C) −13   (D) −17   (E) 7

30. $\displaystyle\int \frac{\sin(x)}{\cos^2(x)} dx = \underline{\hspace{1cm}} + C$, where C is some arbitrary constant.

   (A) $\tan x$ (B) $\dfrac{1}{\sin^2 x}$ (C) $-\dfrac{1}{\sin x}$ (D) $-\dfrac{1}{\cos x}$ (E) $\dfrac{1}{\cos x}$

31. Using the following pattern of numbers, which of the following numbers will not be in row 13?

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                    1                           row 0
               1         1                      row 1
          1         2         1                 row 2
      1        3         3         1            row 3
   1       4        5        4        1         row 4
 1      5       7        7       5       1      row 5
1     6      9      10      9      6      1     row 6
                   ...                         ...
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(A) 792   (B) 78   (C) 1287   (D) 715   (E) 286

32. The hypotenuse of the inscribed triangle is a diameter of the circle shown. What is the probability that a dart landing inside the circle at random will land inside the shaded area? (nearest whole percent)

A) 28%   (B) 14%   (C) 32%   (D) 16%   (E) 24%

33. The slope of the line going through the points $(-5, y)$, $(4, -3)$ and $(x, 7)$ is $-\dfrac{2}{3}$. Find $x + y$.

(A) 11   (B) −5   (C) 14   (D) −8   (E) 8

34. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____of the sides of the triangle.

(A) Endpoints (B) Altitudes (C) Medians (D) Midpoints (E) Perpendicular Bisectors

35. Two unbiased standard dice are thrown. Find the probability that the product of the numbers on the two dice is a multiple of 4.

(A) $\dfrac{4}{9}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{5}{9}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{11}{18}$

36. Mr. Gardener's height is 5' 11". His is standing on a ladder with his feet 8' 7" from the ground trimming his oak tree. How long is Mr. Gardener's shadow, not including the ladder, when the angle of elevation of the sun from the ground is 66°? (nearest inch)

(A) 3' 10''   (B) 2' 10''   (C) 2' 8''   (D) 5' 7''   (E) 6' 4''

37. Which of the following mathematicians is one of a small group of American Indians who have earned a Ph.D. in mathematics and is doing work applying mathematical models to the study of groundwater contamination?

(A) Hypatia (B) Ada Bryon   (C) Alicia Scott   (D) Freda Porter   (E) Emmy Noether

38. If y varies directly with x and $y = 28$ when $x = 200$, calculate y when $x = 167$

(A) 24.57   (B) 23.38   (C) 26.91   (D) 18.72   (E) 14.04

39. What is the constant term in the binomial expansion of $\left(x + \dfrac{3}{x^3} \right)^8$ **?**

   (A) 9   (B) 70   (C) 252   (D) 28   (E) 630

40. The coordinates of the vertices of the heptagon shown are all integers. The area of the figure is ____square units?
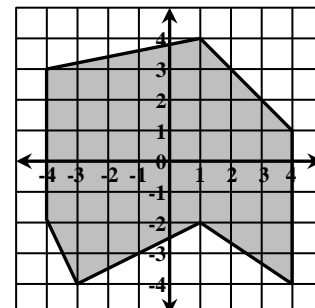
   (A) 47   (B) 47.5   (C) 49.5

   (D) 49   (E) 48.5

41. Find $\lim\limits_{x \to 2} \dfrac{3x^2 - 2x - 8}{2x^2 - 9x + 10}$ **.**

   (A) $\dfrac{3}{2}$ (B) 10   (C) −10   (D) 1   (E) Does not exist

42. $111000_2 + 1001_4 + 10_8 = $ _____$_{16}$ **.**

   (A) 90   (B) D6   (C) 8C   (D) 81   (E) C8

43. Given $f(x) = ax^5 + bx^3 + cx - 12$ **and** $f(5) = 16$ **, calculate** $f(-5)$ **.**

   (A) 12   (B) −40   (C) −52   (D) 28   (E) −28

44. A recipe for mac & cheese calls for $4\dfrac{1}{3}$ **cups of milk and serves 12 people. How many cups of milk would be needed to make enough for 50 people? (nearest quarter cup)**

   (A) 17.75 c.   (B) 17.5 c.   (C) 18 c.   (D) 17 c.   (E) 18.25 c

45. Simplify: $\dfrac{4x^3 - 4x^2 - 9x + 9}{3x^2 - 8x + 5} \div \dfrac{2x^2 + 5x + 3}{9x^2 - 30x + 25} =$

   (A) $\dfrac{3x - 5}{x + 1}$ (B) $\dfrac{6x^2 - 19x + 15}{x + 1}$ (C) $\dfrac{2x - 3}{x - 1}$ (D) $\dfrac{6x^2 - 9x - 15}{x + 1}$ (E) $\dfrac{3x - 5}{x - 1}$

46. On the triangle ABC shown, m∠BAC = $\dfrac{\pi}{6}$ **radians, AB = 24 and AC = 18. Find the area of triangle ABC.**
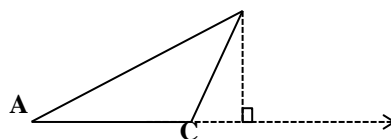
   (A) 144   (B) 216   (C) $144\sqrt{3}$ (D) $108\sqrt{3}$ (E) 108

47. Quentin is twice as old as Paul, and Quentin is five years younger than Ron. In four years, the sum of their ages will be 62. How old is Quentin now?

   (A) 13   (B) 18   (C) 22   (D) 9   (E) 23

48. $\{(x,y)|x,y \in \mathbb{Z}, -5 \le x \le 7, and -2 \le y \le 9\}$ is the solution set of $3x+4y=6$. How many such ordered pairs exist?

    (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

49. The real solution set to $2|3x-7|-7 \le 11$ is:

    (A) $\left\{ x \middle| -\dfrac{16}{3} \le x \le \dfrac{2}{3} \right\}$ (B) $\left\{ x \middle| \dfrac{2}{3} \le x \le \dfrac{16}{3} \right\}$ (C) $\left\{ x \middle| \left\{ x \le -\dfrac{16}{3} \right\} \cup \left\{ x \ge \dfrac{2}{3} \right\} \right\}$

    (D) $\left\{ x \middle| \left\{ x \le -\dfrac{2}{3} \right\} \cup \left\{ x \ge \dfrac{16}{3} \right\} \right\}$ (E) $\left\{ x \middle| -\dfrac{2}{3} \le x \le \dfrac{16}{3} \right\}$

50. Given that the set of integers in ascending order $\{a,b,c,d,e\}$ has a median of 27, a mean of 24.8, a mode of 19 and a range of 12. Find the value of d.

    (A) 12   (B) 19   (C) 25   (D) 28   (E) 27

51. Find the distance between the line $7x+3y=42$ and the point $(-6,-2)$. (nearest tenth)

    (A) 13.4   (B) 11.8   (C) 9.1   (D) 11.2   (E) 10.1

52. The graph of $h(x)=\dfrac{2x^2-7x-15}{2x+3}$ suggests that the *discontinuity* at $x=-\dfrac{3}{2}$ is *removable* by defining $h\left(-\dfrac{3}{2}\right)$ to be_____.

    (A) −1.5   (B) −14   (C) −7   (D) −6.5   (E) −8.5

53. P and Q are the roots of $x^3+3x^2-9x-27$. Evaluate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

    (A) 1296   (B) 81   (C) 729   (D) 9   (E) 36

54. Buffi has bins containing 3 different colors of markers, 3 different colors of pens and 8 different colors of crayons. In how many ways can Buffi assemble bags containing 2 markers, 2 pens and 2 crayons?

    (A) 864   (B) 576   (C) 252   (D) 504   (E) 1296

55. Let $f(x)=\dfrac{3x^3-4x^2}{x^2-1}$ and $s(x)$ be the slant asymptote of f. Find the value of $s(3)$.

    (A) 2   (B) 5   (C) −3   (D) −2   (E) 6

56. Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.

    (A) Sandwich Theorem   (B) Rolle's Theorem   (C) Fundamental Theorem of Calculus

    (D) Intermediate Value Theorem   (E) Fundamental Theorem of Algebra

57. How many positive 3-digit numbers exist such that the sum of their digits equals 12?

    (A) 54   (B) 60   (C) 62   (D) 65   (E) 66

58. Find the sum of the arithmetic sequence: $-5, -8, -11, -14, ..., -44$.

    (A) -352 (B) -319 (C) -294 (D) -343 (E) -367

59. If $\dfrac{-3x-17}{x^2+2x-3} = \dfrac{A}{x+3} + \dfrac{B}{x-1}$, then $AB = $?

    (A) -10 (B) -6 (C) 10   (D) -3 (E) 6

60. An insurance company sells 10,000 policies for $ 850 each. They only have two levels of payouts. Their research shows that 1 in 10 policy holders will make a claim for $5000 and 1 in 100 will make a claim for $25,000. What is the company's expected profit on all the policies together?

    (A) $1,000,000   (B) $1,800,000   (C) $100,000   (D) $3,000,000   (E) $1,500,000

2017 – 2018 TMSCA Mathematics Test Ten Answers

1. D	21.D	41.C
2. E	22.C	42.D
3. B	23.B	43.B
4. D	24.D	44.C
5. E	25.E	45.B
6. A	26.B	46.E
7. E	27.D	47.B
8. B	28.E	48.B
9. A	29.E	49.E
10.C	30.E	50.D
11.D	31.A	51.B
12.B	32.A	52.D
13.B	33.D	53.A
14.D	34.E	54.E
15.E	35.B	55.B
16.D	36.C	56.B
17.B	37.D	57.E
18.B	38.B	58.D
19.C	39.C	59.A
20.A	40.D	60.A

2017-2018 TMSCA Mathematics Test Ten Select Solutions

3. This will be an arithmetic sequence with 63 terms for a sum of $\dfrac{63}{2}(6+378)=12{,}096$.

9. In an inscribed quadrilateral, the inscribed angles are supplementary, so solve $x^2-4x+5x-2=180$ **for** $x=13$ **and** $m\angle A=117°$.

11. Solve $19x=88$ **for** $x\approx 4.6$.

18. $\dfrac{-316-12i}{28+96i}=-1+3i$

21. Solve $f(4)=64+48+4k+12=104$ **for** $k=-5$

22. $a+b=2$, $c=7$ **and** $d=12$ **for** $a+b+c+d=21$

23. Solve $\dfrac{x+8}{x+3}=6$ **for** $x=-2$

27. $\displaystyle\sum_{k=1}^{n}k^3=\left[\dfrac{n(n+1)}{2}\right]^2$, **so the sum is**

$\left(\dfrac{21\times 22}{2}\right)^2-\left(\dfrac{8\times 9}{2}\right)^2=52{,}065$

28. There are 5 distinct letters, STAEM, 3-E's and 3-T's the possible configurations are:

No repeats: ${}_5P_4=120$

1 letter repeating once: $2\times({}_4C_2)\times\dfrac{4!}{2!}=144$

2 letters each repeating: $\dfrac{4!}{2!2!}=6$

1 triple: $2\times{}_4C_1\times\dfrac{4!}{3!}=32$ **for a total of 302.**

31. The numbers in the 13th row are the numbers in the un-simplified expansion of $(1+1)^{13}$ **for 1, 13, 78, 286, 715, 1287 and 1716, so 792 is the one that is not in the row.**

32. Let the radius be 1, for a circle area of π. **Then the sides of the triangle are 1 and** $\sqrt{3}$, **and the area of the triangle are is** $\dfrac{\sqrt{3}}{2}$ **for a probability of about 28%.**

39. ${}_8C_2\left(x^6\right)\left(\dfrac{3}{x^3}\right)^2=252$

43. $16=X-12$ **for** $X=28$ **then** $f(-5)=-28-12=-40$

51. $\dfrac{|7(-6)+3(-2)-42|}{\sqrt{49+9}}\approx 11.8$

52. $f(x)=\dfrac{(2x+3)(x-5)}{2x+3}$ **is the same as** $g(x)=x-5$ **at every point except** $\left(-\dfrac{3}{2},-\dfrac{13}{2}\right)$ **which could "remove" the discontinuity.**

53. The roots of the function are 3 and -3 , **and the expression is the** $(-3-3)^4=(3-(-3))^4=1296$

54. $\left({}_{3+2-1}C_2\right)\left({}_{3+2-1}C_2\right)\left({}_{8+2-1}C_2\right)=1296$.

59. $A(x-1)+B(x+3)=-3x-17$ **for the system of equations** $A+B=-3$ **and** $-A+3B=-17$ **for** $A=2$ **and** $B=-5$ **with** $AB=-10$.