UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Invitational A • 2015

1. **Evaluate:** $1 \div (1+2)^{-1} \times 3 - \frac{5}{8} + 13 \times (21)^0$

   (A) -21.125 (B) -11.375 (C) 11.375 (D) 20.125 (E) 21.375

2. **Saul Wood had a two-by-four board that was 12 feet long. He cuts it into 3 pieces such that the ratio of the lengths of the pieces are 2:3:5 with a 8 inch board left over. How long was the longest piece?**

   (A) 2 yds   (B) 1 yd 2 ft 8 in   (C) 1 yd 2 ft 5 in   (D) 1 yd 2 ft 3.5 in   (E) 1 yd 2 ft 2 in

3. **Find the sum of the arithmetic mean, median, mode, and range of 2, 18, 4, 7, 1, 11, 29, & 3.**

   (A) $38\frac{3}{8}$ (B) $40\frac{3}{8}$ (C) $41\frac{5}{8}$ (D) $42\frac{7}{8}$ (E) $43\frac{7}{8}$

4. **Dee Orr rows his boat at 4 mph from his pier to a platform on the lake. A speed boat returns him to his pier at 45 mph. The complete trip took 25 minutes. How far is it from the pier to the platform? (nearest tenth)**

   (A) 0.8 miles   (B) 1.1 miles   (C) 1.3 miles   (D) 1.5 miles   (E) 1.8 miles

5. **Simplify:** $\left(\frac{x^3 - 21x - 20}{x-1} \right) \times \left(\frac{x+1}{x^2 - x - 20} \right) \div \left(\frac{1}{x^2 - 1} \right)$

   (A) 1 (B) $x+1$ (C) $x-1$ (D) $x^2 + 2x + 1$ (E) $x^3 + 3x^2 + 3x + 1$

6. **Given: $\angle P$ is supplementary to $\angle Q$; $m\angle R = 48°$; and $\angle Q$ is complementary to $\angle R$. Find $m\angle P$.**

   (A) $42°$ (B) $52°$ (C) $128°$ (D) $132°$ (E) $138°$

7. **Find the area of the shaded area. (nearest tenth)**
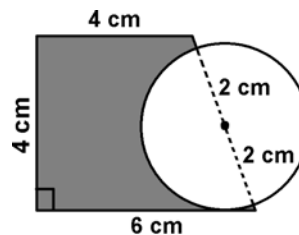
   (A) 7.4 cm^2 (B) 16.0 cm^2 (C) 17.7 cm^2 (D) 13.7 cm^2 (E) 22.3 cm^2

8. **The point of intersection of the 3 medians of a triangle is called a _____.**

   (A) center   (B) centroid   (C) circumcenter   (D) incenter   (E) orthocenter

9. **Lotta Cash, Les Sense, and Noah Dough have a total of $75.00. Noah has five dollars more than twice what Lotta has and Les has ten dollars less than Noah. How much more money does Les have then Lotta?**

   (A) $5.00   (B) $10.00   (C) $15.00   (D) $20.00   (E) $40.00

10. If $\frac{2x-3}{3x+2} - \frac{4x+1}{x-4} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

   (A) -2.8 (B) $-0.6181818...$ (C) $1.4666...$ (D) 1.8 (E) $2.1333...$

11. The fundamental period of the graph of $y = 1 - 2\sin^2(2x)$ is:

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π (E) π^2

12. $\sin(\frac{\pi}{2} - \theta)$ equals:

   (A) $-\cos(\frac{\pi}{2} - \theta)$ (B) $\sin(\theta + \frac{\pi}{2})$ (C) $\cos(\frac{\pi+\theta}{2})$ (D) $-\sin(\theta + \frac{\pi}{2})$ (E) $\sin(\frac{\pi-\theta}{2})$

13. Given the arithmetic sequence 15, a, b, 41.25, c, ..., find a + b + c.

   (A) 43.75   (B) 70.3125   (C) 97.5   (D) 106.25   (E) 123.75

14. Find m + n if $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

   (A) 2   (B) 4   (C) 8   (D) 10   (E) 18

15. Find the average rate of change over the interval [2,4] of $f(x) = x^2 + 4x$.

   (A) 32   (B) 21   (C) 12   (D) 10   (E) 8

16. How many distinguishable arrangements can be made from the letters "TOOTSIEPOPS"?

   (A) 720   (B) 831,600   (C) 15   (D) 55,440   (E) 1,663,200

17. Lou Cuss labels 8 blank cards with the numbers 2, 1, 3, 4, 7, 11, 18, and 29. One card is randomly drawn. What are the odds that the number is a Fibonacci number?

   (A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{2}{5}$ (D) $\frac{1}{4}$ (E) $\frac{3}{8}$

18. Which of the following female mathematicians is known for her work in differential calculus?

   (A) Agnesi   (B) Hypatia   (C) Germain   (D) Kovalevsky (E) Noether

19. Find the arithmetic mean of the first three harmonic numbers.

   (A) $2\frac{1}{6}$ (B) $1\frac{5}{6}$ (C) $1\frac{1}{2}$ (D) $1\frac{4}{9}$ (E) $\frac{11}{18}$

20. The number 678 in base 9 is equivalent to the number k in base 3. Find the sum of the digits in the number k.

   (A) 9   (B) 8   (C) 6   (D) 4   (E) 3

21. Using the partial ruler shown below, find the distance from A to B.
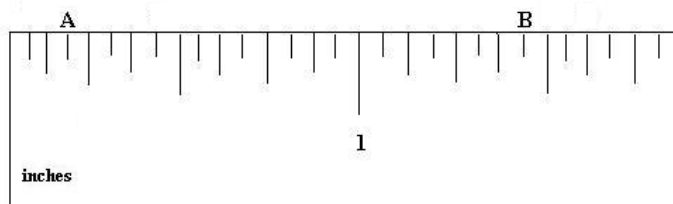
(A) $1\frac{3}{8}$" (B) $1\frac{3}{16}$" (C) $1\frac{5}{8}$" (D) $1\frac{1}{4}$" (E) $1\frac{7}{16}$"

22. The *Texas Wild Seed* farm mixes 3 pounds of Bluebonnet seeds with 1.5 pounds of Indian Blanket seeds to form a special mixture of wild flower seeds. Find the cost of a half pound of the mixture if Bluebonnet seeds cost $1.25 per pound and Indian Blanket seeds cost 80¢ per pound?

(A) $045 (B) $0.55 (C) $0.90 (D) $1.03 (E) $1.10

23. Which of the following sets are closed under addition and/or multiplication?
C = {composite numbers}   F = {Fibonacci numbers}   M = {multiples of 5}

(A) C & M (B) M only (C) F only (D) C & F (E) C, F, & M

24. Which of the following quadrant(s) does not contain a solution to $3x + 4y > 7$?

(A) QIV (B) QI & QII (C) QIII & QIV (D) QIII (E) Q1

25. Phil Whitwatter is filling up his empty circular water tank. The diameter of the tank is 12 feet and the height of the tank is 4 feet. What is the least number of whole gallons of water will he need to fill the tank half full?

(A) 1,129 gal (B) 1,693 gal (C) 1,765 gal (D) 1,975 gal (E) 2,257 gal

26. A triangle with side lengths of 11 dm, 8 dm, and 15 dm is a(n) _____ triangle.

(A) isosceles acute (B) scalene obtuse (C) isosceles obtuse (D) scalene acute (E) scalene right

27. Let $a_1 = 2$, $a_2 = 1$, $a_3 = 3$ and $a_n = (a_{n-3}) + \left[(a_{n-1}) - (a_{n-2})\right]$ for $n \geq 4$. Find a_6.

(A) -1 (B) 0 (C) 1 (D) 2 (E) 11

28. Simplify: $\log_3 x - 2\log_3 y + \log_3(0.5)$

(A) $-\log_3(xy^2)$ (B) $\frac{1}{2}\log_3(\frac{x}{y^2})$ (C) $\log_3(\frac{x}{2y^2})$ (D) $\log_3(\frac{x-y^2}{2})$ (E) $\log_3 x - y^2 + 0.5$

29. Which of the following equations in rectangular form can be written as $r - 12\cos\theta = 0$ in polar form?

(A) $x^2 - y^2 = 6$ (B) $x^2 + y^2 = 12$ (C) $x^2 + y^2 = 2\sqrt{3}$
(D) $y^2 - x^2 = 2\sqrt{3}$ (E) $(x - 6)^2 + y^2 = 36$

30. Find the area of △DEF to the nearest tenth.

P
4"
F
8"
E
8"
4"
R
8"   D   4"   Q
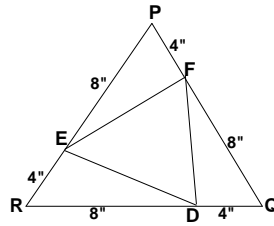
(A) 52.0 sq. in.   (B) 17.4 sq. in   (C) 62.4 sq. in   (D) 17.9 sq. in   (E) 20.8 sq. in

31. How many distinct solutions exist for $12\cos^2(x) - 5\cos(x) - 2 = 0$, where $-\frac{\pi}{2} < x < \frac{3\pi}{4}$?

(A) 8   (B) 5   (C) 3   (D) 4   (E) 7

32. Find the remainder when $f(x) = 4x^3 + 8x^2 - x - 2$ is divided by $x - 3$.

(A) -41 (B) -35 (C) 31   (D) 155   (E) 175

33. $\int (x^2 + 4x)\, dx =$ _____ $+ C$, where C is some arbitrary constant.

(A) $\frac{x^3}{3} + 2x^2$ (B) $2x + 4$ (C) $x^3 + 2x^2$ (D) $3x + 2x^2$ (E) $\frac{x^3}{3} + 2x$

34. Find the area bounded by $y = 2x^2 + 2x - 3$ and $y = 2x - 1$. (square units).

(A) 3.5   (B) 3.75   (C) 2.666...   (D) 4.25   (E) 4.5

35. Betty Kant has a stack of 8 cards consisting of J♠, J♥, J♦, J♣, Q♠, Q♥, Q♦, and Q♣. Betty shuffles the stack then deals out the top 3 cards. What is the probability that two of the cards dealt were Jacks and one was a Queen?

(A) $\frac{1}{56}$ (B) $\frac{1}{7}$ (C) $\frac{3}{28}$ (D) $\frac{3}{7}$ (E) $\frac{3}{8}$

36. Kandy Krunchur had a large bag of Tootsie Pops. She had chocolate ones, cherry ones, lime ones, strawberry ones, and raspberry ones. How many different small bags of 5 Pops can she package to sell?

(A) 126   (B) 25   (C) 120   (D) 24   (E) 1,512

37. Let P be a two-digit prime number less than 100 such that both digits are prime numbers. What is the sum of all such numbers, P?

(A) 348   (B) 253   (C) 221   (D) 186   (E) 113

38. Two of the roots of $f(x) = x^3 + bx^2 + cx + d$ are 3 and $2 + i$. Find $b + c + d$.

(A) -7 (B) -5 (C) 6   (D) 9   (E) 25

39. **How many proper fractions in lowest terms have a denominator of 24?**

    (A) 8   (B) 9   (C) 10   (D) 11   (E) 12

40. **Line m contains point $(-3, 4)$ and intersects the y-axis at $y = -5$. An equation for line m is:**

    (A) $3x - y = 5$ (B) $4x - 3y = -5$ (C) $3x + y = -5$ (D) $4x + 3y = 5$ (E) $-3x + 4y = 5$

41. **The point $(3, 4)$ lies on a circle whose center is $(0, 2)$. Where does the point $(-1, 5)$ lie in reference to the circle?**

    (A) on the circle   (B) outside the circle   (C) inside the circle

    (D) in quadrant III   (E) cannot be determined

42. **Consider the complex number $0 + i$, find the value of $i^{-1} + i^{-2} + i^{-3} + i^{-4}$.**

    (A) 0   (B) 1   (C) -1 (D) i (E) $-i$

43. **The Real value solution set for $2 + 3|5x - 7| < 11$ is?**

    (A) $\{x \mid \{0.2 < x < 2.6\}$ (B) $\{x \mid \{x > 2\} \cup \{x < 0.8\}\}$ (C) $\{x \mid -2 < x < -\frac{4}{5}\}$

    (D) $\{x \mid \{x > 2.6\} \cup \{x < -2\}$ (E) $\{x \mid 0.8 < x < 2\}$

44. **A baseball groundskeeper uses his line striping machine to create a triangle for a 3-bag baseball game. He marks a line 90 feet on a bearing of $100°$ from home base to first base. Then he marks a line 100 feet on a bearing of $80°$ from first base to second base. How long is the line he marked from second base to home base? (nearest foot)**

    (A) 109 ft   (B) 122 ft   (C) 136 ft   (D) 165 ft   (E) 187 ft

45. **The graph of the parametric equations $x = 3t$ and $y = 4t + 1$ is a(n) _____.**

    (A) circle   (B) ellipse   (C) hyperbola   (D) line   (E) parabola

46. **The harmonic mean of the real roots of $4x^3 + 8x^2 - x - 2 = 0$ is ?**

    (A) -1.5 (B) -2 (C) $-3.555...$ (D) -6 (E) -9

47. **Find the y-intercept of the line tangent to the $3x^2 + 4y^2 = 48$ at the point $(2,3)$.**

    (A) $(0, 4)$ (B) $(0, 8)$ (C) $(0, 9)$ (D) $(0, -2)$ (E) $(0, -3)$

48. **If $f''(x) = 24x + 16$ and $f'(0) = -1$ and $f(1) = 9$, then $f(-1) =$ _____.**

    (A) 6   (B) 3   (C) -4 (D) -5 (E) -9

49. **Willie Luze plays a dice game that costs 50¢ to play. He rolls two dice and sums up the top faces. He wins $1.00 if the sum is 7 or 11 and loses 25¢ if the sum is not 7 or 11. What is the mathematical expectation of a single roll? (nearest cent)**

  (A) 53¢ loss   (B) 47¢ loss   (C) 4¢ loss   (D) 3¢ gain   (E) 46¢ gain

50. **Let $f_0=0$, $f_1=1$, $f_2=1$, $f_3=2$, $f_4=3$, ... be the terms of the Fibonacci sequence. Find GCD(f_{15}, f_9).**

 (A) 8   (B) 6   (C) 3   (D) 2   (E) 1

51. **Ima Lost walks 1 foot north, then 2 feet west, then 3 feet south, then 4 feet east, then 5 feet north, then 6 feet west and so on, at 1 foot per second. What direction is Ima facing after walking 1 minute?**

  (A) north   (B) west   (C) south   (D) east   (E) northeast

52. **The sum of all of the real values of x such that $\sqrt{x-6} = x\sqrt{x-6}$ is:**

  (A) 13   (B) 7   (C) 6   (D) 1   (E) 0

53. **Given the circle O with perpendicular diameters and a chord, find BE if EF = 3" and DF = 7". (nearest tenth)**
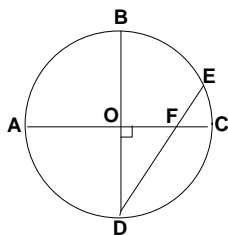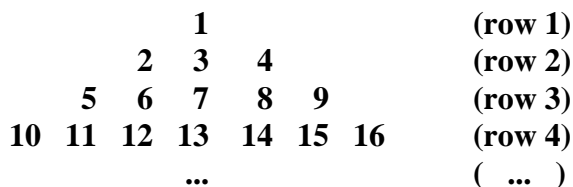

  (A) 5.2"   (B) 6.1"   (C) 6.3"   (D) 7.1"   (E) 7.6"

54. **Let $p^3 + q^3 = 4$ and pq = 0.666... . Find p + q.**

  (A) 2.666...   (B) 2   (C) 1   (D) 1.333...   (E) 0.1666...

55. **Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 12.**

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

  (A) 133   (B) 123   (C) 127   (D) 137   (E) 143

56. For which of the following values of θ is it true that $2^{\sin\theta} > 1$ and $3^{\cos\theta} < 1$?

    (A) 35°   (B) 70°   (C) 140°   (D) 280°   (E) 560°

57. Which of the following surfaces is generated by $9x^2 - 72y + 16z^2 = 0$?

   (A) cone   (B) cylinder   (C) ellipsoid   (D) hyperboloid   (E) paraboloid

58. Yu-Noh randomly selects a positive integer less than 60 that is a multiple of 7. Yu-Dont randomly selects a positive integer less than 60 that is a multiple of 9. What is the probability that they selected the same number? (nearest percent)

    (A) 78%   (B) 22%   (C) 15%   (D) 12%   (E) 0%

59. The square root of 1134 in base 5 is:

    (A) 13_5 (B) 113_5 (C) 23_5 (D) 114_5 (E) 33_5

60. Given the rectangular solid shown, find AE if AB = 3", BC = 5" and CE = 7". (nearest tenth)
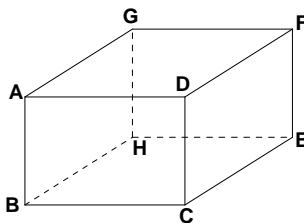
    (A) 15.0"   (B) 6.4"   (C) 7.5"   (D) 8.5"   (E) 9.1"

1.	E	21.	D	41.	C
2.	B	22.	B	42.	A
3.	D	23.	A	43.	E
4.	D	24.	D	44.	E
5.	E	25.	B	45.	D
6.	E	26.	B	46.	D
7.	D	27.	C	47.	A
8.	B	28.	C	48.	B
9.	B	29.	E	49.	B
10.	C	30.	E	50.	D
11.	C	31.	C	51.	C
12.	B	32.	E	52.	B
13.	D	33.	A	53.	C
14.	A	34.	C	54.	B
15.	D	35.	D	55.	A
16.	B	36.	A	56.	C
17.	A	37.	D	57.	E
18.	A	38.	B	58.	E
19.	D	39.	A	59.	C
20.	A	40.	C	60.	E

Mathematics

Invitational B • 2015

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $4! \div (16)^{\frac{1}{2}} - 4 \times (16)^{-1} + 4 \times 16^0$

   (A) $\frac{23}{8}$ (B) $\frac{31}{4}$ (C) $\frac{25}{4}$ (D) $\frac{31}{8}$ (E) $\frac{39}{4}$

2. **I. M. Broke borrowed $250.00 from his bank at a simple interest rate of 5%. He paid the loan off in 10 monthly payments. What was his monthly payments? (nearest cent)**

   (A) $26.04 (B) $26.15 (C) $26.25 (D) $28.06 (E) $28.82

3. **What is $8\frac{1}{3}\%$ of $(\frac{1}{16} \div (0.1666...)$?**

   (A) $\frac{1}{32}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$
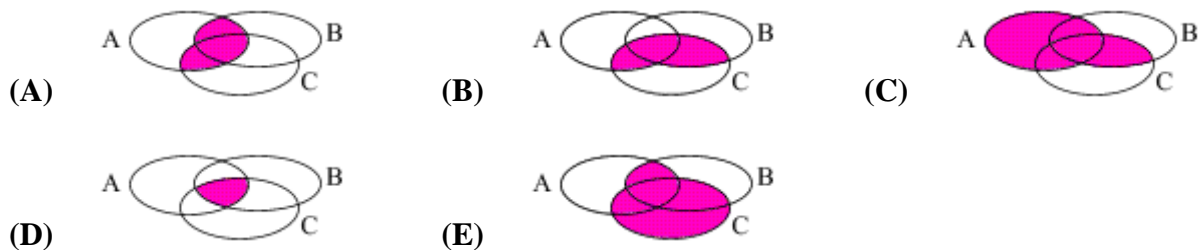
4. **Simplify:** $\left(\frac{16x^2 + 8x - 3}{16x^2 - 1}\right)\left(\frac{16x^2 + 8x + 1}{12x^2 + x - 6}\right)(3x - 2)$

   (A) $12x^2 - 5x - 2$ (B) $4x + 1$ (C) $\frac{4x - 1}{4x + 1}$ (D) $\frac{3x - 2}{4x + 1}$ (E) $12x^2 - 11x + 2$

5. **Justin Time is $\frac{4}{5}$ as old as Soh Yung. Fours years ago Justin was $\frac{3}{4}$ as old as Soh. What will the sum of their ages be in two years.**

   (A) 28 (B) 32 (C) 36 (D) 38 (E) 40

6. **In which of the following Venn diagrams does the shaded regions represent the set $A \cup (B \cap C)$?**

   (A) (B) (C)

   (D) (E)

7. **Three less than twice a number is the same as one more than twice the difference of four and the number. Find the number.**
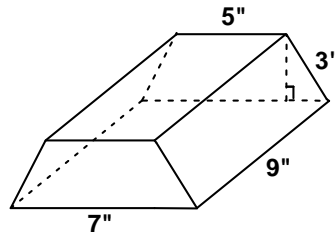
   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. **What is the probability that a factor of 160 is a multiple of 10?**

   (A) $71\frac{3}{7}\%$ (B) $33\frac{1}{3}\%$ (C) $45\frac{5}{11}\%$ (D) $41\frac{2}{3}\%$ (E) $6\frac{1}{4}\%$

9. **The measure of the interior angle of a regular n-gon is three times the measure of its exterior angle. How many sides does the regular n-gon have?**

   (A) 5 (B) 6 (C) 8 (D) 9 (E) 12

10. Find the volume of the isosceles trapezoid prism shown. (nearest cu. in). Drawing is not to scale.

(A) 153 cu. in   (B) 158 cu. in   (C) 162 cu. in   (D) 167 cu. in   (E) 171 cu. in

11. Which of the following points of concurrency are on the exterior of an obtuse triangle?
    (1) circumcenter   (2) centroid   (3) orthocenter   (4) incenter

(A) none of them   (B) 1 & 3   (C) 2 & 4   (D) 1 only   (E) all of them

12. The center of the circle, $x^2 + y^2 - 4x - 6y + 9 = 0$, is (h, k) and the radius is r.
Find $h + k + r$.

(A) 8   (B) 7   (C) 5   (D) 4   (E) -3

13. Simplify: Find $g(f(2a + 1))$ when $g(x) = x - 4a$ and $f(x) = 3x + a$.

(A) $3 - 5a$ (B) $3 - a$ (C) $3(a + 1)$ (D) $5a + 2$ (E) $3a - 5$

14. How many numbers k exist such that $10 \le k \le 99$ and the difference between k and the sum of the digits of k is 18.

(A) 10   (B) 9   (C) 8   (D) 6   (E) 5

15. The sum of the positive integral divisors of 120 is _____.

(A) 25   (B) 125   (C) 180   (D) 360   (E) 960

16. The fundamental period of the graph of $y = 2 - 3\cos^2(4x + 5)$ is:

(A) 2   (B) 3   (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{5}$ (E) 5

17. $\cos(x + \frac{5\pi}{2}) =$ _____.

(A) $-\cos x$ (B) $\sin x$ (C) $-\cos(\frac{\pi}{2}x)$ (D) $\sin 2x$ (E) $-\sin x$

18. Mack A. Roy drops a golf ball from a height of 5 feet. Each time it hits the ground it rebounds to a height of 80% of the distance it fell. Find the total distance the ball travels when it reaches the ground the third time. (nearest inch)

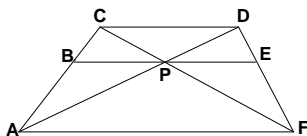(A) 22' 0''   (B) 19' 5''   (C) 18' 8''   (D) 16' 2''   (E) 14' 5''

19. Which of the following is a reference angle for $\frac{5\pi}{4}$?

   (A) $\frac{\pi}{4}$ (B) $\frac{4\pi}{5}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{8}$ (E) $\frac{\pi}{2}$

20. Evaluate: $\sum\limits_{k=1}^{5} (2)^{(2-k)} + k$

   (A) 13.625   (B) 16   (C) 18.875   (D) 25.375   (E) 32

21. Given the trapezoid shown where segments AF, BE, and CD are parallel to each other and the three interior segments are concurrent at point P, find the length of segment AF if CD = 20 cm and BE = 24 cm. (nearest tenth)

   (A) 26.2 cm   (B) 28 cm   (C) 28.8 cm   (D) 30 cm   (E) 32 cm

22. Let $f(x) = x^5 - 2x^4 + 2x^3 - 3x^2 + x - 3$. Find $f''(-1)$.

   (A) 20   (B) 12   (C) -1 (D) -15 (E) -62

23. Lotta Latts is building a rectangular parking lot for her *Lotta's Junk* store. The length of the lot will be bordered on one side by the store. She has 1200 feet of fence to enclose the lot. What is the maximum area of her parking lot?

   (A) 90,000 ft.^{2} (B) 160,000 ft.^{2} (C) 180,000 ft.^{2} (D) 360,000 ft.^{2} (E) 1,440,000 ft.^{2}

24. Let $f(x) = \dfrac{1}{\sqrt{x^2 + 3x - 10}}$. At which of these intervals is function f continuous?

   (A) $[-5, -2]$ (B) $(-5, 5]$ (C) $[-2, 2)$ (D) $(2, 5]$ (E) $(-2, 5)$

25. The number 15 is considered to be a "polite" number. The "politeness" of 15 is ____.

   (A) 1   (B) 2   (C) 3   (D) 5   (E) 15

26. N. A. Hurry enters a convenience store. The probability that she buys bread is 60%, the probability she buys milk is 50%, and the probability she buys both bread and milk is 30%. What is the probability that she will buy either bread or milk or both? (nearest percent)

   (A) 100%   (B) 80%   (C) 70%   (D) $53\frac{1}{3}\%$ (E) $46\frac{2}{3}\%$

27. Find the least positive integral sum of d and m if GCD(d , m) = 8 and LCM(d , m) = 320.

   (A) 48   (B) 104   (C) 112   (D) 176   (E) 328

28. Let k be a positive integer less than 100 such that k is a multiple of 4 and k is divisible by 3. Find the sum of all such numbers k.

   (A) 336   (B) 396   (C) 420   (D) 432   (E) 444

29. Les Cash, Lotta Dough, and Noah Moolah had piggy banks. The average of all 3 banks was $147.00. The average of Lotta's bank and Noah's bank was $141.00. How much money was in Les' bank?

   (A) $96.00   (B) $119.50   (C) $144.00   (D) $159.00   (E) $168.50

30. A line crosses the x-axis at $x = -2$ and goes through the point (3, 1). Another line crosses the y-axis at y = 2 and goes through the point $(-1, -3)$. The lines intersect at (x, y). Find $x + y$.

   (A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0   (D) $\frac{1}{2}$ (E) 1

31. Simplify this expression: $\left(\dfrac{x^2 y^{-3} z^3}{w^2 x^{-3} y^2} \right)^{-1} \times \left(\dfrac{x^2 z}{w y^3} \right)^2 \div \dfrac{1}{(xyz)^2}$

   (A) $(xz)^{-1}$ (B) xyz (C) $xy^2 z^3$ (D) $(xyz)^{-1}$ (E) $xy^3 z$

32. $\overline{AB}, \ \overline{AC}, \ \overline{BD},$ and $\overline{CD}$ are chords of circle O and point E lies on circle O. If $m\overset{\frown}{BC} = 46°$ and $m\angle APB = 78°$, then $m\angle ACD = ?$
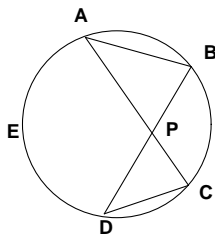
   (A) 101°   (B) 62°   (C) 75°   (D) 56°   (E) 79°

33. Betty Drawzette randomly selects a number from the set of all positive 2-digit numbers. What is the probability that the sum of the digits of the number selected is 11? (nearest percent)

   (A) 7%   (B) 8%   (C) 9%   (D) 10%   (E) 11%

34. Point P$(-3, 4)$ lies on the x-y plane. Point P is rotated 180° counter clockwise about the origin to point Q. Point Q is translated horizontally 5 units to the left to point R. Point R is reflected across the line $y = -x$ to point S. The coordinate of S is (x, y). Find $x + y$.

   (A) -7 (B) -2 (C) -1 (D) 5   (E) 6

35. How many integral values of n exist such that $n \leq 1$ and $\dfrac{(n+1)!}{(n-1)!} \leq 12$

   (A) none   (B) 2   (C) 4   (D) 6   (E) 7

36. The roots of the equation $x^3 - 13x + 12 = 0$ are 1, 3, and R. Find R.

   (A) 9   (B) 4   (C) -1 (D) -3 (E) -4

37. The graph of the polar equation $r = 5 + 2\cos(\theta)$ is a _____.

   (A) dimpled limacon   (B) convex limacon   (C) inner loop limacon

   (D) lemniscate   (E) cardioid

38. Find the smallest positive real number x such that $\sin x = \cos 2x$, where x is measured in radians.

   (A) $\frac{3\pi}{2}$ (B) $\frac{5\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{2\pi}{3}$ (E) $\frac{\pi}{12}$

39. The harmonic mean of the real roots of $2x^3 + 5x^2 - 4x - 3 = 0$ is _____.

   (A) $-0.666...$ (B) -0.75 (C) $-1.333...$ (D) -1.5 (E) -2.25

40. Simplify to the form $a + bi$: $(1 + 2i)(3 + 4i) \div (5i)$

   (A) $1 - 2i$ (B) $2 + i$ (C) $2 - 2i$ (D) $2 - i$ (E) $1 + 2i$

41. $221_3 + 102_3 + 121_3 = $ _____ $_9$

   (A) 63   (B) 57   (C) 48   (D) 36   (E) 12

42. Which of the following statements is a false statement for $f(x) = \begin{cases} \frac{1}{x-2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$?

   (A) f is continuous at $x = 1$ (B) $f(2)$ exists   (C) $\lim\limits_{x \to 2^-} f(x)$ exists

   (D) $\lim\limits_{x \to 2^+} f(x)$ exists   (E) f is continuous at 2

43. Let $f(x) = \frac{3x - 1}{2x + 5}$. Find $f'(4)$.

   (A) $\frac{17}{169}$ (B) $1\frac{1}{2}$ (C) $1\frac{2}{13}$ (D) $2\frac{6}{13}$ (E) $\frac{11}{13}$

44. If $\frac{1}{2} - \frac{3}{4x} = \frac{5y}{6}$, then x equals _____.

   (A) $\frac{3}{5} - \frac{9}{10y}$ (B) $-\frac{3}{10y}$ (C) $-\frac{5y}{3}$ (D) $-\frac{9}{10y - 6}$ (E) $-\frac{3}{5y - 2}$

45. Which of the following mathematicians is best known for their work in four dimensional geometry and introduced the term "polytope"?

   (A) Venn, John   (B) Hypatia   (C) Smith, Karen   (D) Zeno of Elea   (E) Stott, Alicia

46. Five girls were nominated for homecoming queen and four boys for homecoming king at Royal T high school. How many ways can a king, a queen, and their court be chosen if their court consists of one girl and one boy?

(A) 126   (B) 336   (C) 120   (D) 16   (E) 240

47. Which of the following is not a solution to $2 + |5x + 1| \leq 7$?

(A) $-1.333...$ (B) -0.7 (C) -0.15 (D) 0.4 (E) $0.666...$

48. Given the circle O with perpendicular diameters and a chord, find OF if EF = 4'' and DF = 8''. (nearest tenth)
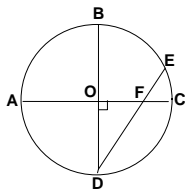
(A) 6.9''   (B) 5.7''   (C) 3.5''   (D) 4.0''   (E) 5.3''

49. The base of a tree is 20 feet from the base of a flagpole. Both the tree and the flagpole are on level ground. The tree is shorter than the 48-ft flagpole. At some time during the day the shadows of both the tree and the flagpole end at the same point 60 ft from the base of the flagpole. How tall is the tree? (nearest foot)

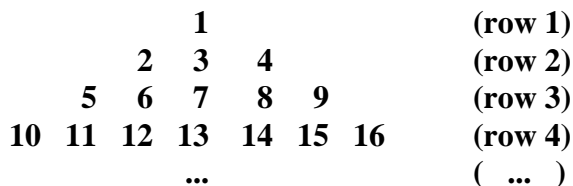(A) 25 ft   (B) 17 ft   (C) 38 ft   (D) 20 ft.   (E) 32 ft

50. Let x and y exist such that $4 < 8 < x < y$. If 4, 8, x form an arithmetic sequence and 8, x, y for a geometric sequence, then x + y = ?

(A) 32   (B) 30   (C) 26   (D) 24   (E) 18

51. Willie Byette borrowed $12,000.00 for a used car. Part of the loan was at the rate of 8% per year and the rest of the loan was at 18% per year. If the interest was $1000.00, how much of the loan was at 18%?

(A) $666.67   (B) $400.00   (C) $1,500.00   (D) $333.33   (E) $11,600.00

52. Given that the set of natural numbers continue in the triangular pattern shown below, how many numbers will be on row 47?

```
                        1                    (row 1)
                    2   3   4                (row 2)
                5   6   7   8   9            (row 3)
            10  11  12  13  14  15  16       (row 4)
                        ...                  ( ... )
```

(A) 91   (B) 92   (C) 93   (D) 94   (E) 95

53. The average monthly high temperature for Anchorage, Alaska in July is 65° F. The average monthly high temperature in January is 22° F. The average monthly high temperature of Anchorage varies sinusoidally with the month. What would be the predicted average high temperature for March? (nearest tenth)

   (A) 43.5 °   (B) 39.5 °   (C) 38.8 °   (D) 32.8 °   (E) 29.2 °

54. The graph of the parametric equations $x = \dfrac{2}{1+t^2}$ and $y = \dfrac{2t}{1+t^2}$ is a(n) _____.

   (A) circle   (B) ellipse   (C) hyperbola   (D) line   (E) parabola

55. Which of the following surfaces is generated by $x^2 = y^2 - z^2$?

   (A) elliptic cone   (B) cylinder   (C) ellipsoid
   (D) hyperbolic paraboloid   (E) elliptic paraboloid

56. Willie Score throws a dart at the February, 2015 calendar hanging on the wall. Assuming the dart hits one of the dates on the calendar, what are the odds that the date he hit was a Lucas number (2, 1, 3, 4, ...)? Each date has an equal chance of being hit.

   (A) $\dfrac{2}{7}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{7}$ (E) $\dfrac{7}{29}$

57. The *I Scream U Scream* Shoppe make great banana splits using three scoops of ice cream. The flavors of ice cream available are chocolate, vanilla, strawberry, mint swirl, pistachio, blueberry, and raspberry. How many different triple scoop banana splits can they create from the available flavors?

   (A) 84   (B) 72   (C) 36   (D) 35   (E) 21

58. How many 3-digit numbers exist such that the sum of their digits equals 4?

   (A) 8   (B) 9   (C) 10   (D) 11   (E) 12

59. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $(f_4)^2 + (f_5)^2$.

   (A) 18   (B) 20   (C) 34   (D) 40   (E) 55

60. Given the rectangular solid shown, find BF if AF = 6", FH = 4" and BG = 5". (nearest tenth)
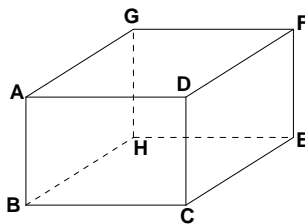
   (A) 8.8"   (B) 6.7"   (C) 6.5"   (D) 6.2"   (E) 5.1"

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2015
Answer Key

1. E	21. D	41. B
2. A	22. E	42. C, D, E
3. A	23. C	43. A
4. B	24. D	44. D
5. E	25. C	45. E
6. C	26. B	46. E
7. C	27. B	47. A
8. D	28. D	48. D
9. C	29. D	49. E
10. A	30. C	50. B
11. B	31. B	51. B
12. B	32. E	52. C
13. C	33. C	53. D
14. A	34. E	54. A
15. D	35. D	55. A
16. C	36. E	56. B
17. E	37. B	57. A
18. B	38. C	58. C
19. A	39. E	59. C
20. C	40. B	60. D

Mathematics
District 1 • 2015

1. $3 + 2 \times 3 - 3 \div 2 \times 8 + (20 - 15)$

    (A) 2   (B) 5.1875   (C) 5.375   (D) 13.8125   (E) 53

2. **Using the partial ruler shown below, find the difference in the lengths of AB and CD.**
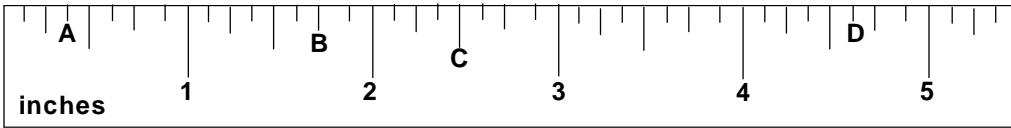
    (A) $\frac{3}{8}$"   (B) $\frac{3}{16}$"   (C) $\frac{1}{2}$"   (D) $\frac{3}{4}$"   (E) $\frac{1}{4}$"

3. **Three million two hundred eighty-two thousand fifteen plus five million one hundred two thousand three hundred twenty three is subtracted from one billion. What is the digit that appears the most number of times in the difference?**

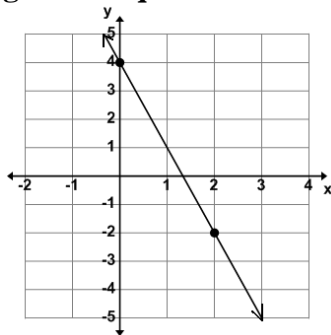   (A) 0   (B) 2   (C) 5   (D) 6   (E) 9

4. **Simplify:** $\left(a^{-3} \times b^{3}\right)^{-1} \div \left(a^{2} \times b^{-2}\right)^{2} \times \left(a \div b\right)^{3}$

    (A) $a^{3}b^{-3}$ (B) $a^{-2}b^{-4}$ (C) $a^{2}b^{-2}$ (D) ab (E) $a^{6}b^{6}$

5. **Bill Spender, Len Meekash, and Penni Les spent the day at the mall. Bill spent $3.00 more than Len spent. Len spent twice as much as Penni spent. When they left the mall Bill still had $5.00, Len had $2.00 and Penni had $.50. Together they spent $23.00. How much money did Len have when they went into the mall?**

    (A) $11.00   (B) $10.00   (C) $8.50   (D) $8.00   (E) $$4.50

6. **Which of the following linear equations is best represented by this graph?**

    (A) $3x + y = 4$ (B) $3x - 2y = 4$ (C) $2x + 3y = 4$ (D) $x - 3y = 4$ (E) $2x + 3y = 4$

7. **Simplify:** $\left(\frac{6x^{2} + x - 2}{4x^{3} - 16x^{2} - x + 4}\right) \div \left(\frac{9x^{2} + 12x + 4}{6x^{2} + 7x + 2}\right)$

    (A) $x^{2} - 8x + 16$ (B) $\frac{x-4}{x+4}$ (C) $x^{2} - 16$ (D) $\frac{1}{x-4}$ (E) $x + 4$

8. **Which of the following properties, is used to go from step 3 to step 4?**

Step

1	$5(k-2)$	$= 5$
2	$5k-10$	$= 5$
3	$5k-10+10$	$= 5+10$
4	$5k+0$	$= 5+10$
5	$5k$	$= 15$
6	$5k \times \frac{1}{5}$	$= 15 \times \frac{1}{5}$
7	k	$= 3$

(A) distributive (B) additive inverse (C) additive identity

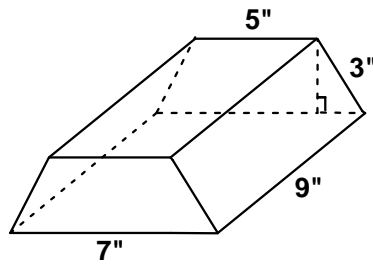(D) multiplicative identity (E) transitive

9. **Willis A. Nutt mixed some almonds worth $5.00 a pound with some cashews worth $6.50 a pound. How many pounds of cashews did he mix with the almonds to make 10 pounds of mixed nuts that sells for $6.00 a pound?**

(A) $4\frac{1}{3}$ lbs (B) $1\frac{1}{5}$ lbs (C) $6\frac{2}{3}$ lbs (D) 5 lbs (E) $1\frac{4}{5}$ lbs

10. **The set $\{-1, 0, 1\}$ is closed under which of the following operations:**
    + addition − subtraction × multiplication ÷ division

(A) $+$ & $\times$ (B) $+, -,$ & $\times$ (C) $\times$ only (D) none of the four (E) all four

11. **Find the lateral surface area of the isosceles trapezoid prism shown. (nearest sq. in). Drawing is not to scale.**

(A) 216 sq. in (B) 48 sq. in (C) 162 sq. in (D) 54 sq. in (E) 210 sq. in

12. **The ratio of the length to the width of a rectangle is 10:6. If 5 units are added to both the length and the width, then the ratio of the length to the width is now 3:2. What is the difference in the areas of the two rectangles? (square units)**

(A) 225 (B) 250 (C) 275 (D) 300 (E) 325

13. **The point of intersection of the 3 medians of a triangle is called a(n) _____.**

(A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

14. If $\dfrac{A}{3x-2} + \dfrac{B}{2x+1} = \dfrac{x-10}{6x^2-x-2}$, where A and B are constants, then $A + B$ equals:

    (A) -4 (B) -1 (C) 1   (D) 3   (E) 7

15. Let $A = \begin{bmatrix} -2 & -3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -6 \\ 3 & -1 \end{bmatrix}$. Find $\left| A^T + B^T \right|$.

    (A) 35   (B) -24 (C) 120   (D) 118   (E) -1

16. Find the value of $(4 + i^1) + (3 + i^2) + (2 + i^3) + (1 + i^4)$.

    (A) -10 (B) 0   (C) 2   (D) 9   (E) 10

17. Which of the following mathematicians is considered to be the "Father of Symbolic Logic"?

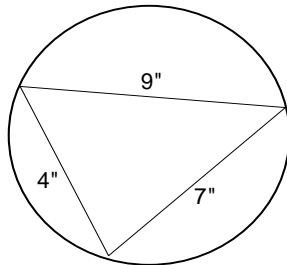   (A) Alicia Stott   (B) John Venn   (C) George Boole   (D) John Napier   (E) Georg Cantor

18. The *Wildflower* seed company's research data shows that the probability that a seed will germinate and grow into a plant is 70%. What are the odds that it won't germinate and grow into a plant?

    (A) $\dfrac{3}{10}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{3}{7}$ (E) $\dfrac{7}{10}$

19. How many 8-letter code words can be formed using the letters in the word COMMERCE?

    (A) 5,040   (B) 6,720   (C) 336   (D) 248   (E) 20,160

20. Find the diameter of the circle. Drawing is not to scale. (nearest tenth)

    (A) 9.4 "   (B) 10.0 "   (C) 11.9 "   (D) 12.6 "   (E) 13.3 "

21. The graph of the parametric equations $x = t^2 + t$ and $y = 2t - 1$ is a(n) _____.

    (A) semicircle   (B) ellipse   (C) line   (D) cycloid   (E) parabola

22. The frequency of the graph of $y = 1 + 2\sin^2(\frac{\pi}{6}x - 3)$ is:

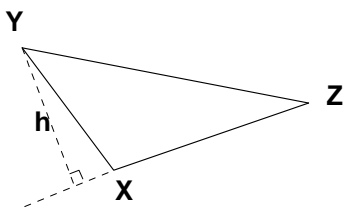   (A) .08333...   (B) 0.1666...   (C) 0.333...   (D) 0.314...   (E) 0.261666...

23. **Dawn Indyvalley sailed her scow 12 miles from Port A on a bearing of 125°. Then she changed her course and sailed 8 miles to buoy marker B on a bearing of 300°. How far is buoy marker B from Port A? (nearest tenth)**

   (A) 7.1 mi   (B) 6.7 mi   (C) 5.8 mi   (D) 5.0 mi   (E) 4.1 mi

24. **Use the Fibonacci characteristic sequence ..., p, q, 4, r, 9, ... to find p + q + r.**

   (A) 3   (B) 8   (C) 9   (D) 13   (E) 22

25. **The coefficient of the 3rd term in the expansion of $(4x - 5)^6$ is:**

   (A) 75,000   (B) 96,000   (C) 128,000   (D) 150,000   (E) 160,000

26. **Given that the set of natural numbers continue in the triangular pattern shown below, find the first number in row 11.**

$$
\begin{array}{cccccccc}
 & & & 1 & & & & \text{(row 1)} \\
 & & 2 & 3 & 4 & & & \text{(row 2)} \\
 & 5 & 6 & 7 & 8 & 9 & & \text{(row 3)} \\
10 & 11 & 12 & 13 & 14 & 15 & 16 & \text{(row 4)} \\
 & & & \ldots & & & & (\ \ldots\ )
\end{array}
$$

   (A) 101   (B) 104   (C) 107   (D) 110   (E) 111

27. **For what values of the domain is the function $f(x) = 15 - 2x - x^2$ decreasing?**

   (A) $(-\infty, 2)$ (B) $(-2, +\infty)$ (C) $(-1, +\infty)$ (D) $(-\infty, -1)$ (E) $(-\infty, +\infty)$

28. **Let $f(x) = \frac{4 - 3x}{5x}$. Find $f'(-2)$.**

   (A) -0.4 (B) -1 (C) -0.1 (D) -0.2 (E) -0.6

29. **The graph of $f(x) = \frac{x^3 - 64}{x^2 - 16}$ has how many asymptotes?**

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

30. **Wynn Zenn has a science club made up of six biology students, seven chemistry students and four physic students. How many ways can Wynn form a six member UIL science team if the team has to have one physic student, one chemistry student, and one biology student? The remaining three team members can be any one in the science club.**

   (A) 9,988   (B) 9,996   (C) 10,008   (D) 12,376   (E) 61,152

31. **How many positive integers less than or equal to 55 contain at least one 2 or at least one 3, but the sum of the digits do not equal 5?**

   (A) 30   (B) 28   (C) 26   (D) 24   (E) 23

32. **Will E. Pikett randomly selects an odd integer less than 100 that is a multiple of 3. Betty Wont randomly selects an odd integer less than 100 that is a multiple of 5. What is the probability that they selected the same number? (nearest tenth)**

    (A) 2.8%   (B) 3.0%   (C) 5.3%   (D) 3.7%   (E) 1.8%

33. **The sum of the first three *lucky prime numbers* is:**

    (A) 10   (B) 11   (C) 12   (D) 19   (E) 23

34. **$(323_5 + 201_5) \times 4_5 = $ _____ $_5$**

    (A) 4011   (B) 4101   (C) 4121   (D) 4201   (E) 4211

35. **The square root of 1161 in base 8 is:**

    (A) 31_8 (B) 34_8 (C) 27_8 (D) 41_8 (E) 37_8

36. **Cookie Baykur made a batch of cookies. She gave 60% of the cookies to her daughter for her class party. She kept $\frac{2}{3}$ of the cookies she had left for her evening tea party. She gave 4 of the remaining cookies to her son and his friend for an afternoon snack. There were 2 cookies left for her husband. How many cookies did she bake originally?**

    (A) 48   (B) 45   (C) 40   (D) 36   (E) 35

37. **Line *m* contains points (4, 1) and $(-2, -3)$. Which of the following equations of line *n* exists such that line n contains the point $(3, -2)$ and $m \perp n$.**

    (A) $2x + 3y = 5$ (B) $2x - 3y = -5$ (C) $5x + 3y = -2$ (D) $3x - 2y = -5$ (E) $3x + 2y = 5$

38. **If the roots of $2x^3 + bx^2 + cx + d = 0$ are -4, 2, and 6, then $b + c + d$ equals:**

    (A) 16   (B) -24 (C) 48   (D) -12 (E) 4

39. **The point $(3, -3)$ lies on a circle whose center is $(-3, -3)$. Where does the point (1, 2) lie in reference to the circle?**

    (A) on the circle   (B) outside the circle   (C) inside the circle
    (D) in quadrant II   (E) cannot be determined

40. **The range of the relation $(x + 3)^2 + (y + 3)^2 < 36$ is:**

    (A) $\left[-6, 6\right]$ (B) $\left(-8, 2\right)$ (C) $\left(-2, 5\right)$ (D) $\left(-4, 4\right)$ (E) $\left(-9, 3\right)$

41. **How many integral values of n exist such that $n \geq 0$ and $\frac{(n + 2)!}{n!} \leq 20$**

    (A) none   (B) 3   (C) 4   (D) 6   (E) 9

42. If $a_1 = -2$, $a_2 = 2$, $a_3 = 4$, $a_n = (a_{n-1}) \div (a_{n-3}) - (a_{n-2})$, where $n \geq 4$, then a_6 equals:

(A) 6   (B) 4.5   (C) 3   (D) 2.5   (E) 1.5

43. A triangle is drawn as shown. Find h if $m\angle XZY = 25°$, XY = 33", and YZ = 75". (nearest tenth)

(A) 36.9"   (B) 33.2"   (C) 31.4"   (D) 31.7 "   (E) 30.8"

44. $\triangle DEF$ exists such that $m\angle DEF = 90°$ and point M is the midpoint of segment DF. If EM = 12.5 cm, and DE = 24 cm, find $m\angle FME$. (nearest hundredth)

(A) 31.39°   (B) 32.52°   (C) 35.50°   (D) 36.87°   (E) 37.15°

45. The harmonic mean of the real roots of $2x^3 + 9x^2 + 3x - 4 = 0$ is ? (nearest tenth)

(A) 1.3   (B) 4.0   (C) 5.5   (D) -2.3 (E) -1.5

46. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $GCD(f_{16}, f_{12})$.

(A) 3   (B) 4   (C) 5   (D) 8   (E) 14

47. The probability that statement P is false is 3/10, and the probability that statement Q is true is 7/8. Determine the probability that $P \rightarrow Q$ is false.

(A) $\frac{21}{80}$ (B) $\frac{23}{40}$ (C) $\frac{3}{40}$ (D) $\frac{7}{80}$ (E) $\frac{3}{80}$

48. Find the area bounded by $y = 1 - x^2$, $y = x - 6$, $x = -1$, and $x = 1$. (square units).

(A) 12   (B) $12\frac{5}{6}$ (C) $13\frac{1}{3}$ (D) $13\frac{2}{3}$ (E) $14\frac{1}{6}$

49. The function $f(x) = x^4 - x^3 + 1$ has inflection points at $x = a$ and $x = b$. Find $a + b$.

(A) $\frac{1}{2}$ (B) 0   (C) 1   (D) -1 (E) $-\frac{1}{2}$

50. Which of the following equations in polar form can be written as $3x - 2y = 2$ in rectangular form?
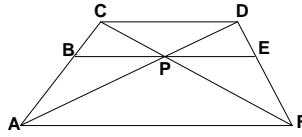
(A) $r(3\sin\theta - 2\cos\theta) = 2$ (B) $r(3\cos\theta + 2\sin\theta) = \sqrt{2}$ (C) $r(2\cos\theta - 3\sin\theta) = 2\sqrt{2}$

(D) $r(3\cos\theta - 2\sin\theta) = 2$ (E) $r(3\sin\theta + 2\cos\theta) = 4$

51. Let $f(x) = x^2 - bx + c$. If $f(x)$ is divided by $x - 2$ the remainder is 6 and if $f(x)$ is divided by $x + 3$ the remainder is 1. Find $b + c$.

   (A) -4 (B) -2 (C) 0   (D) 1   (E) 5

52. The expansion of $(3x - 1)(3x - 2)(3x - 3)$ is $ax^3 + bx^2 + cx + d$. Find $(ab) \div (cd)$.

   (A) 6.888...   (B) 7.363636...   (C) 1.222...   (D) 8.181818...   (E) 3.444...

53. Len Meecash borrowed \$500.00 to help pay for college books. Part of the loan was at the simple interest rate of 4% per year and the rest of the loan was at the simple interest rate of 6% per year. At the end of the year he paid off the loan and the interest totaling \$526.50. How much of the loan was at 4%?

   (A) \$375.00   (B) \$325.00   (C) \$250.00   (D) \$175.00   (E) \$125.00

54. Ranger Chris P. Kritter looks down from the Woodworth Tower and sees a forest fire coming toward the ranger station tower. The tower is 175 feet tall and the angle of depression from his eyes to the base of the fire is 10°. The speed of the fire is estimated to be moving at 3 feet per minute. How long does Chris have before the fire reaches the tower? (nearest minute).

   (A) 5 hrs 0 min   (B) 5 hrs 31 min   (C) 5 hrs 15 min   (D) 5 hrs 7 min   (E) 6 hrs 5 min

55. Seymore Wirk and Doug Upp can dig a trench for a water line in 8 hours when working at the same time. Seymore notices that Doug works twice as fast as he does. How long would it take Doug to dig the trench by himself?

   (A) 24 hrs   (B) 16 hrs   (C) 12 hrs   (D) 4 hrs   (E) 2.666... hrs

56. A right triangle has a hypotenuse of length 20". If one of the acute angles is decreasing at the rate of 6° per second, how fast is the area of the triangle decreasing when this acute angle is 21°? (nearest tenth)

   (A) 3.4 in²/sec   (B) 15.8 in²/sec   (C) 13.5 in²/sec   (D) 7.8 in²/sec   (E) 15.6 in²/sec

57. Willie Drawrite puts 5 country DVDs, 7 rock DVDs, and 3 blank DVDs in a bag. He randomly chooses 3 DVDs, without replacement. What is the probability that Willie chose one country, one rock, and one blank DVD? (nearest per cent)

   (A) 8%   (B) 12%   (C) 34%   (D) 58%   (E) 23%

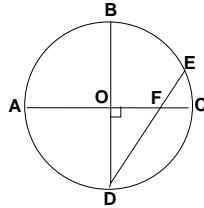58. Given the regular pentagon shown, find BD if AB = 8".   (nearest tenth)

   (A) 11.3"   (B) 12.9"   (C) 12.0"   (D) 13.9"   (E) 6.5"

59. Given the trapezoid shown where segments AF, BE, and CD are parallel to each other and the three interior segments are concurrent at point P, find the length of segment BP if CD = 18 cm and AF = 26 cm. (nearest hundredth)

   (A) 10.82 cm   (B) 9.91 cm   (C) 11.00 cm   (D) 10.35 cm   (E) 10.64 cm

60. Given the circle O with perpendicular diameters and a chord, find BE if DE = 11" and DF = 7". (nearest tenth)

   (A) 5.2"   (B) 6.3"   (C) 7.1"   (D) 6.1"   (E) 5.7"

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 1 ● 2015
Answer Key

1. A	21. E	41. C
2. D	22. B	42. D
3. D	23. E	43. D
4. C	24. C	44. B
5. B	25. B	45. B
6. A	26. A	46. A
7. D	27. C	47. D
8. B	28. D	48. C
9. C	29. C	49. A
10. C	30. B	50. D
11. C	31. C	51. A
12. A	32. E	52. B
13. B	33. E	53. D
14. B	34. E	54. B
15. C	35. A	55. C
16. E	36. B	56. E
17. C	37. E	57. E
18. D	38. C	58. B
19. A	39. B	59. E
20. A	40. E	60. E

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
District 2 • 2015

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1. Evaluate: $4! + 6 \times (4 - 11) \div 20 \times 15$

    (A) -157.5 (B) -13.5 (C) -7.5 (D) 81.75 (E) 111.75

2. Lacey Frills needs some ribbons for her dress. She has a ribbon that is 2 yards long. She cuts it into 4 pieces such that the ratio of the lengths of the pieces are 2:1:3:4 with a 7 inch piece left over. How long was the longest piece?

    (A) 1 ft 1.5 in   (B) 1 yd 2 ft 5 in   (C) 1 ft 7.5 in   (D) 1 yd 1 ft 6 in   (E) 2 ft 2 in

3.
$$-1\frac{3}{10} \qquad P \qquad Q \qquad R \qquad \frac{7}{10} \qquad S$$
<-------------|-------------|-------------|-------------|-------------|-------------|------------->
The distances between the hash marks (|) are equal. Find $P + Q + R + S$.

    (A) 0.2   (B) 0.3   (C) 0.5   (D) 1.0   (E) 2.3

4. What is $85\frac{5}{7}\%$ of ($\frac{5}{9} \div (0.909090...)$)?

    (A) $\frac{77}{108}$ (B) $\frac{55}{126}$ (C) $\frac{11}{21}$ (D) $\frac{5}{77}$ (E) $\frac{100}{231}$

5. Simplify: $\left(\frac{3n^2 - 27}{6 - n - n^2} \right) \times \left(\frac{4 - 2n}{3 - n} \right) \div \left(\frac{6}{3n + n^2} \right)$

    (A) $n^2 - 3n$ (B) $3 + n$ (C) $-n$ (D) $-n^2 + n$ (E) $-n^2 - 3n$

6. Which of the following properties is demonstrated from step 2 to step 3?

Step 1	$3x + 4$	$= 10$
2	$(3x + 4) + (-4)$	$= 10 + (-4)$
3	$3x + [4 + (-4)]$	$= 10 + (-4)$
4	$3x + 0$	$= 10 + (-4)$
5	$3x + 0$	$= 6$
6	$3x$	$= 6$
7	$\frac{1}{3}(3x)$	$= \frac{1}{3}(6)$
8	$(\frac{1}{3} \times 3)x$	$= \frac{1}{3}(6)$
9	$(1) \times x$	$= 2$
10	x	$= 2$

    (A) distributive   (B) additive inverse   (C) associative   (D) commutative   (E) transitive
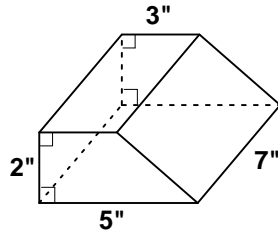
7. How many 12-letter code words can be formed using the letters DISTRICTMEET?

    (A) 19,958,400   (B) 495   (C) 40,320   (D) 95,040   (E) 967,680

8. The price of a diamond varies directly as the square of its weight. If a diamond weighing $\frac{3}{4}$ carat costs $621.00, find the cost of a diamond weighing 1.4 carats.

    (A) $2,163.84   (B) $2,090.82   (C) $1,622.88   (D) $1,545.60   (E) $1,159.20

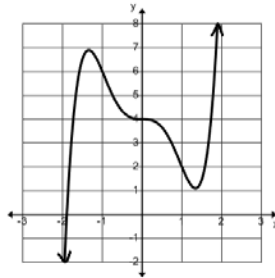9. **Find the total surface area of the prism shown. (nearest sq. in)**

   (A) 92 sq. in   (B) 96 sq. in   (C) 98 sq. in   (D) 100 sq. in   (E) 106 sq. in

10. **Point P(4, 3) lies on the x-y plane. Point P is rotated 90° clockwise about the origin to point Q. Point Q is reflected across the line y = — 1 to point R. Point R is translated vertically 5 units upward to point S. Point S is translated horizontally 5 units to the left to point T. The coordinates of point T are (x, y). Find x + y.**

   (A) — 2   (B) — 1   (C) 1   (D) 5   (E) 7

11. **△PQR is inscribed in circle C such that the measure of ∠PQR's intercepted arc is 76°. Find m∠PQR.**

   (A) $25\frac{1}{3}°$ (B) 76°   (C) 114°   (D) 38°   (E) 14°

12. **If $\frac{2x-7}{5x-3} - \frac{5x+3}{2x-7} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:**

   (A) $-\frac{9}{10}$ (B) $-\frac{1}{5}$ (C) $\frac{4}{5}$ (D) $2\frac{1}{10}$ (E) 5

13. **Simplify: $(2\log_5 X - 2\log_5 Y) + (\log_5 Y^3 - 2\log_5 X^3)$**

   (A) $\log_5(X^2 Y)$ (B) $\frac{Y}{X}$ (C) $4\log_5(\frac{Y}{X})$ (D) $X^4 Y$ (E) $\log_5 Y - 4\log_5 X$

14. **Lotta Sense has fifty coins consisting of nickels, dimes, and quarters. She has three times as many nickels as quarters and ten less quarters than dimes. How much money does she have?**

   (A) $5.30   (B) $6.50   (C) $5.00   (D) $6.25   (E) $5.70

15. **What are the odds that a factor of 120 is a multiple of 4?**

   (A) 1 to 1   (B) 1 to 2   (C) 4 to 1   (D) 2 to 1   (E) 1 to 4

16. **Determine the range of $f(x) = 3 - 5\cos(\frac{\pi}{4}x + \frac{\pi}{2})$.**

   (A) $[-8, 2]$ (B) $[-2, 2]$ (C) $[-\frac{3\pi}{4}, \frac{5\pi}{2}]$ (D) $[-\frac{\pi}{2}, \frac{\pi}{4}]$ (E) $[-2, 8]$

17. **$(1+i)^6$ equals:**

   (A) — 8i   (B) 6 — 8i   (C) 6i   (D) 8 — 8i   (E) 6 + 6i

18. **Given the geometric sequence 81, p, q, 7.111..., r, ..., find p + q + r.**

    (A) 52   (B) $55\frac{13}{81}$ (C) $59\frac{1}{9}$ (D) $62\frac{22}{81}$ (E) $66\frac{2}{9}$

19. **Which of the following is true about the relation graphed below?**

    (A) It is an odd function.   (B) It is an even function.   (C) It is not a function.

    (D) It is neither an even nor an odd function   (E) It is a one-to-one function.

20. **How many 3-digit numbers exist such that the sum of their digits equals 3?**

    (A) 10   (B) 8   (C) 6   (D) 5   (E) 3

21. **Find the digit in the millionths place of the sum of the series $1 + 3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + ...$**

    (A) 9   (B) 6   (C) 5   (D) 3   (E) 2

22. **Which of the following statements is a false statement for $f(x) = \begin{cases} 2x^2 - 2 & \text{if } x \leq 2 \\ 5x - 4 & \text{if } x > 2 \end{cases}$?**

    (A) $f(2)$ exists   (B) $\lim\limits_{x \to 2^-} f(x)$ exists   (C) $\lim\limits_{x \to 2^+} f(x)$ exists

    (D) f is continuous at 2   (E) none of them

23. **If $f''(x) = 6x + 6$ and $f'(1) = -4$ and $f(-1) = 0$, then $f(0) = $ _____.**

    (A) -15 (B) -3 (C) -1 (D) 9   (E) 12

24. **Roland Bones created a pair of special dice which have only three numbers on each die. The opposite side of each number is the same number. When the dice are rolled the die with the largest number on top wins. What is the probability that die A will win?**
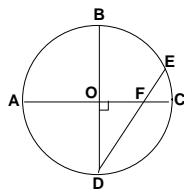

    (A) $55\frac{5}{9}\%$ (B) $44\frac{4}{9}\%$ (C) $66\frac{2}{3}\%$ (D) $33\frac{1}{3}\%$ (E) 20%

25. **The number 20 is considered to be a "polite" number. The "politeness" of 20 is _____.**

    (A) 10   (B) 5   (C) 4   (D) 2   (E) 1

26. There are 13 UIL Academic Contest directors, 4 females and 9 males. How many 5-member committees can Dr. Stevens appoint such that each committee must have at least 1 female director and at least 1 male director?

   (A) 178　　　　(B) 35,640　　　　(C) 5,940　　　　(D) 1,161　　　　(E) 1,512

27. If p is a prime number and $2p + 1$ is a prime number then $2p + 1$ is called a *safe prime* and p is called a _____ prime.

   (A) Hypatian　　(B) Euclidean　　(C) Boolean　　(D) Germain　　(E) Archimedian

28. How many proper fractions in lowest terms have a denominator of 36?

   (A) 21　　　　(B) 18　　　　(C) 15　　　　(D) 12　　　　(E) 9

29. $46_7 + 2015_6 + 411_5 =$ _____ $_{10}$

   (A) 139　　　　(B) 583　　　　(C) 672　　　　(D) 1,446　　　　(E) 2,472

30. Line *m* has a slope of -2 and goes through the point $(-4, 6)$. Line *n* goes through points $(1, -1)$ and $(2, 5)$. Line *m* intersect line *n* at (x, y). Find $x + y$.

   (A) 3.875　　(B) 2.875　　(C) 0.625　　(D) -2.625　　(E) -3.25

31. Which of the following is not a solution to $|4x + 11| - 20 \geq 15$?

   (A) -12.5　　(B) -11.875　　(C) -9.75　　(D) 6.555...　　(E) 11.5

32. M. T. Tank has a circular water tank with no water in it. The tank is 6 feet deep and has a diameter of 5 feet. How many gallons of water will M. T. have to put in the tank to be 75% full? (nearest gallon)

   (A) 661 gal　　(B) 793 gal　　(C) 565 gal　　(D) 881 gal　　(E) 656 gal

33. Latexo circle G19. Given the circle O with perpendicular diameters and a chord, find the area △DFO if EF = 5" and DE = 12" inches. (nearest tenth)
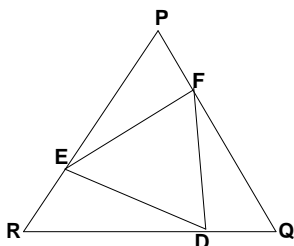

   (A) 8.6 sq. in　　(B) 8.9 sq. in　　(C) 9.1 sq. in　　(D) 9.3 sq. in　　(E) 9.8 sq. in

34. How many numbers k, where $10 \leq k \leq 50$, exist such that the number when the digits of k are reversed is subtracted from k, the differences greater than zero are divisible by 9?

   (A) 18　　　　(B) 15　　　　(C) 13　　　　(D) 11　　　　(E) 9

35. If $a_1 = -4$, $a_2 = 1$ and $a_n = [(a_{n-1}) + (a_{n-2})] \times (a_{n-1})$ for $n \geq 3$, then a_5 equals:

    (A) 9   (B) 12   (C) 18   (D) 36   (E) 54

36. Find the area of $\triangle PQR$ given that EF = DE = DF = 10.393 cm and FP = DQ = ER = 6 cm. (nearest tenth)

    (A) 93.5 sq. cm   (B) 108.6 sq. cm   (C) 121.9 sq. cm   (D) 140.3 sq. cm   (E) 187.1 sq. cm
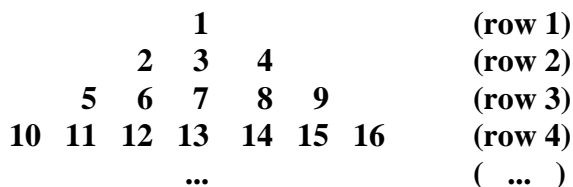
37. Frank Lynn is flying his kite. The distance from the level ground to Frank's eyes is 6 feet. The angle of elevation from his eyes to the kite is $23°$. What is the distance from level ground to the kite if all 100 feet of his string has been let out? (nearest foot)

    (A) 39 feet   (B) 45 feet   (C) 48 feet   (D) 52 feet   (E) 54 feet

38. The harmonic mean of the real roots of $x^3 + 3x^2 + kx - 15 = 0$ is $-3\frac{6}{13}$. Find k?

    (A) $5\frac{10}{13}$ (B) $\frac{7}{13}$ (C) -6 (D) -12 (E) -13

39. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 21.

```
                    1                        (row 1)
                 2  3  4                      (row 2)
              5  6  7  8  9                   (row 3)
          10 11 12 13  14 15 16               (row 4)
                   ...                        (  ...  )
```

    (A) 441   (B) 432   (C) 422   (D) 421   (E) 419

40. Let $f(x) = x^2 - 3x + 4$ and $g(x) = 2x - 5$. Find $g(f'(x + 1))$.

    (A) $4x - 15$ (B) $2x^2 - 2x + 5$ (C) $4x - 1$ (D) -1 (E) $4x - 7$

41. Let $f(x) = \frac{x^4}{4} + \frac{x^3}{2} - 3x^2 + 6$. The concavity of the curve is downward at which of the following values of x?

    (A) -2 (B) -1.75 (C) 1   (D) 1.5   (E) 2.25

42. **Wynn Lotts is playing the *10-digit Lotto* game. The game card has 10 covered spots, each one containing a different digit. He scratches off one spot to reveal the digit. If the digit is a Fibonacci number greater than zero he wins $1.00. If it is a zero, he wins $5.00. If it is not a Fibonacci number or a zero then he loses $1.00. What is the mathematical expectation of a single card? (nearest cent)**

 (A) 0¢ (B) 50¢ gain (C) 60¢ gain (D) $1.50 gain (E) $2.00 gain

43. **Cal Q. Later has a box of calculators. The probability that he will select one that works is 60%. If he puts 5 working calculators in the box, the probability of selecting a working calculator is 80%. How many non-working calculators are in the box?**

 (A) 10 (B) 8 (C) 5 (D) 3 (E) 2

44. **The repeating decimal 0.3222... in base 5 can be written as which of the following fractions in base 5 in simplified terms?**

 (A) $\frac{3}{10}_5$ (B) $\frac{11}{20}_5$ (C) $\frac{12}{20}_5$ (D) $\frac{7}{10}_5$ (E) $\frac{24}{40}_5$

45. **Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $(f_8)^2 + (f_9)^2$.**

 (A) 55 (B) 145 (C) 714 (D) 987 (E) 1,597

46. **If $x + y = 8$ and $4xy = -20$, then $x^2 + y^2 = $?**

 (A) 80 (B) 74 (C) 54 (D) 39 (E) 18*

47. **Larry is twice as old as Moe. Moe is three years older than Curly. Four years ago, Moe was twice Curly's age. How old will Larry be in five years?**

 (A) 12 (B) 15 (C) 20 (D) 25 (E) 37

48. **$\triangle ABC$ and $\triangle PQR$ exist such that $\angle BAC \cong \angle PRQ$, $\angle ACB \cong \angle RQP$, $AC = 28$, $CB = 24$, $PR = 16$, and $PQ = 18$. Find $AB + QR$.**

 (A) 24 (B) $25\frac{5}{7}$ (C) $30\frac{2}{3}$ (D) 33 (E) $42\frac{1}{3}$

49. **The point $(-1, -3)$ lies on a circle whose center is $(4, 2)$. Which of the following points lies inside the circle?**

 (A) $(2, 9)$ (B) $(-2, 5)$ (C) $(6, -5)$ (D) $(9, 7)$ (E) $(-3, 1)$

50. **If $x - \frac{1}{x} = 3$, then $x^3 - \frac{1}{x^3} = $?**

 (A) 11 (B) 49 (C) 33 (D) 7 (E) 36

51. Joy Ryder hops on a flatcar of a freight train that leaves the station heading due east at 8:00 p.m. Roland Along gets on a passenger train that leaves the same station heading due east at 11:00 p.m. The average speed of the freight train is 25 miles per hour, while the passenger train's average speed is 65 miles per hour. How far apart are the two trains after the passenger train has traveled for 2.5 hours?

   (A) 87.5 miles   (B) 40 miles   (C) 25 miles   (D) 100 miles   (E) 12.5 miles

52. Joy Ryder gets on a small Ferris wheel at the county fair. The radius of the Ferris wheel is 10 meters and it completes a revolution in 2 minutes. The bottom of the Ferris wheel where Joy gets in her seat to ride is 1 meter from the ground. How far from the ground will Joy be after riding 80 seconds? (nearest tenth)

   (A) 16.0 m   (B) 16.8 m   (C) 18.5 m   (D) 19.3 m   (E) 21.0 m

53. The graph of the parametric equations $5x = \cos(3t)$ and $2y = \sin(3t)$, where $0 \le t \le 2\pi$, is a(n):

   (A) circle   (B) ellipse   (C) hyperbola   (D) line   (E) parabola

54. The conic $4x^2 - 16x + 16 + y^2 + 8y = 0$ is a(n):

   (A) circle   (B) degenerate   (C) ellipse   (D) hyperbola   (E) parabola

55. Let $f(x) = \dfrac{x^2 - 3x + 9}{x + 3}$ and s(x) be the slant asymptote of f. Find the value of s($-$ 3).

   (A) 27   (B) 4.5   (C) $-$ 3   (D) $-$ 9   (E) undefined

56. Find the area bounded by the two parabolas. (square units).

   (A) 1.222...   (B) 1.333...   (C) 1.555...   (D) 1.666...   (E) 1.777...

57. Two of the roots of $f(x) = x^3 + bx^2 + cx + d$ are 1$-$ 2i and 3. Find b + c + d.

   (A) $-$ 21   (B) $-$ 9   (C) 1   (D) 11   (E) 31

58. Find the value of A $+$ 2B $+$ 3C, where A, B, and C are positive integers such that

$$\frac{57}{11} = A + \left(\cfrac{1}{B + \left(\frac{1}{C+1} \right)} \right).$$
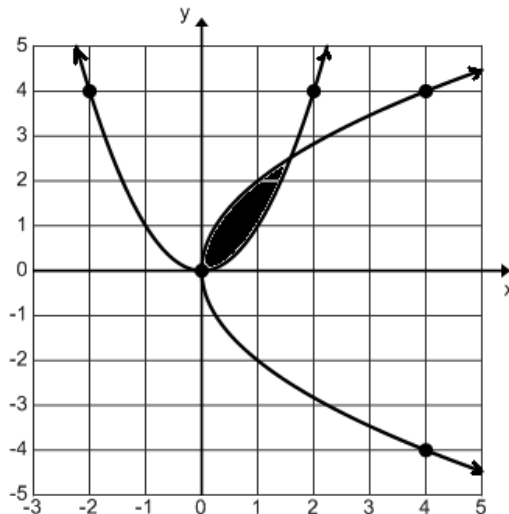
    (A) 11   (B) 12   (C) 15   (D) 18   (E) 20

59. N. D. Dark needs help solving this problem. Let $\oplus$ n $\oplus$ be the largest prime factor of n and let $\ominus$ n $\ominus$ be the smallest prime factor of n greater than 1.
What does $\oplus$ 20 $\oplus$ $-$ $\ominus$ 15 $\ominus$ $\times$ $\oplus$ 49 $\oplus$ $+$ $\ominus$ 57 $\ominus$ equal?

    (A) $-$13   (B) 17   (C) $-$14   (D) $-$6   (E) 13

60. Doug Upp, Doug Down, and Doug Aditch can shovel the snow from Mr. Paver's driveway in 4, 3, and 6 hours respectively. How long will it take them if all three Doug's worked together? (nearest minute)

    (A) 1 hr 20 min   (B) 2 hrs 10 min   (C) 1 hr 26 min   (D) 1 hr 30 min   (E) 40 min

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 2 ● 2015
Answer Key

1. C	21. B	41. B
2. E	22. E	42. C
3. B	23. A	43. E
4. C	24. B	44. C
5. E	25. E	45. E
6. C	26. D	46. B
7. A	27. D	47. D
8. A	28. D	48. E
9. E	29. B	49. B
10. D	30. D	50. E
11. D	31. C	51. C
12. A	32. A	52. A
13. E	33. A	53. B
14. C	34. D	54. C
15. A	35. C	55. D
16. E	36. D	56. B
17. A	37. B	57. B
18. B	38. E	58. D
19. D	39. D	59. A
20. C	40. E	60. A

Mathematics

Regional • 2015

1. $3! \div (6)^{-1} - 9 \div 3 \times 6 + (9)^{\frac{1}{2}}$

   (A) -17 (B) -15.5 (C) 18 (D) 21 (E) 57

2. Les Sense, Noah Moolah, and Ima Spender went to the movies. Ima bought popcorn and a soda. Les bought a candy bar and a soda. Noah ate some of Ima's popcorn, part of Les' candy bar, and bought his own soda. If movie tickets cost $6.50 each, popcorn cost $3.00, candy bars cost $1.75 and sodas are $3.50 each, determine the total amount they spent all together?

   (A) $44.25 (B) $38.50 (C) $34.75 (D) $32.25 (E) $21.75

3. Let R = {r, e, g, i, o, n, a, l}, M = {m, a, t, h}, and C = {c, h, a, m, p, i, o,n}. How many elements are in $(R \cap C) \cup (M \cap C)$?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

4. $32_6 - 32_8 + 201_5 =$ _____ $_7$

   (A) 63 (B) 47 (C) 54 (D) 102 (E) 65

5. Which of the following mathematicians showed that the set of rational numbers is countable and that the infinity of rational numbers is the same size as the infinity of natural numbers?

   (A) Christian Goldbach (B) Georg Cantor (C) George Boole
   (D) John Napier (E) Alicia Stott

6. Simplify: $\left(\frac{15 - 13x + 2x^2}{4x^2 - 9} \right) \left(\frac{2x + 1}{1 - 2x} \right) \div \left(\frac{5 - x}{2x - 1} \right)$

   (A) $2x^2 + 8x + 3$ (B) $\frac{2x - 1}{x + 1}$ (C) $2x^2 - 3$ (D) $\frac{x - 1}{2x + 3}$ (E) $\frac{2x + 1}{2x + 3}$

7. If $x - y = -3$ and $xy = -5$ then $x^3 - y^3 = $?

   (A) 98 (B) 18 (C) 8 (D) -50 (E) -72

8. Which of the following properties is demonstrated from step 5 to step 6?

Step 1	$3x + 4$	$= 10$
2	$(3x + 4) + (-4)$	$= 10 + (-4)$
3	$3x + [4 + (-4)]$	$= 10 + (-4)$
4	$3x + 0$	$= 10 + (-4)$
5	$3x + 0$	$= 6$
6	$3x$	$= 6$
7	$\frac{1}{3}(3x)$	$= \frac{1}{3}(6)$
8	$(\frac{1}{3} \times 3)x$	$= \frac{1}{3}(6)$
9	$(1) \times x$	$= 2$
10	x	$= 2$

   (A) reflexive (B) additive inverse (C) associative (D) commutative (E) additive identity

9. How many 8-letter code words can be formed using the letters in the word ABSCISSA?

   (A) 10,080   (B) 336   (C) 6,720   (D) 20,160   (E) 3,360

10. Find the lateral surface area of the regular pentagonal prism shown. (nearest sq. in).
    Drawing is not to scale.
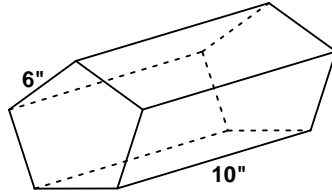
   (A) 124 sq. in   (B) 300 sq. in   (C) 308 sq. in   (D) 420 sq. in   (E) 424 sq. in

11. The ratio of the length to the width of a rectangle is 5:3. If 2 units are subtracted from the width
    and 2 units are added to the length, then the ratio of the width to the length is now 1:3. What is
    the difference in the areas of the two rectangles? (square units)

   (A) 12   (B) 28   (C) 4   (D) 30   (E) 26

12. △PQR is inscribed in circle C such that the measure of ∠PRQ's intercepted arc is 70° and
    m∠PQR = 50°. Find the measure of ∠QPR's intercepted arc.

   (A) 190°   (B) 170°   (C) 120°   (D) 100°   (E) 70°

13. Joy Ryder hops on a freight train that leaves the station at 9:00 a.m. The train enters a tunnel at
    9:45 a.m. at a constant speed of 40 mph. The end of the train exits the tunnel at 9:50 a.m. Find
    the length of the train if the length of the tunnel is 2 miles.

   (A) $\frac{2}{3}$ miles   (B) $\frac{3}{40}$ miles   (C) $1\frac{1}{4}$ miles   (D) $1\frac{1}{3}$ miles   (E) $1\frac{1}{2}$ miles

14. If $\frac{2x-5}{3x+4} - \frac{3x+4}{x-6} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

   (A) $1\frac{11}{41}$ (B) $1\frac{27}{35}$ (C) $1\frac{23}{41}$ (D) $1\frac{1}{34}$ (E) $\frac{34}{35}$
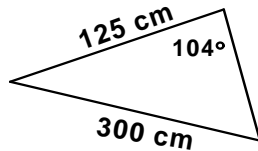
15. Let A $= \begin{bmatrix} 0 & 3 \\ -2 & 8 \end{bmatrix}$ and B $= \begin{bmatrix} 2 & 0 \\ 1 & -15 \end{bmatrix}$. Find $\left| A - B^T \right|$.

   (A) -42 (B) -13 (C) 4   (D) 80   (E) 81

16. Find m $-$ n if $\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 8 \\ 15 \end{bmatrix}$

   (A) $1\frac{2}{7}$ (B) $1\frac{3}{4}$ (C) $2\frac{7}{8}$ (D) $3\frac{1}{7}$ (E) $4\frac{1}{7}$

17. **Find the area of the triangle shown (nearest cm².).**

(A) 18,193 cm² (B) 17,149 cm² (C) 15,132 cm² (D) 14,806 cm² (E) 11,543 cm²

18. **Determine the range of $f(x) = -3\cos(2\pi x + 4\pi) - 1$.**

(A) $[-4, 2]$ (B) $[-2, 4]$ (C) $[-\frac{2\pi}{3}, \frac{4\pi}{3}]$ (D) $[-\frac{3\pi}{2}, \frac{3\pi}{4}]$ (E) $[-1, 1]$

19. **The directrix of the parabola $y = 1.25 - 1.5x - 0.25x^2$ is:**

(A) $y = 2$ (B) $y = 2.25$ (C) $y = 3.5$ (D) $y = 4.25$ (E) $y = 4.5$

20. **Which of the following is not an even function?**

(A) $y = \cos x$ (B) $y = x^2$ (C) $y = -5$ (D) $y = -1 + 2x^5$ (E) $y = -2x^4$

21. **If $f''(x) = 36x + 50$ and $f'(-1) = -30$ and $f(-1) = 9$, then $f(-2) =$ _____.**

(A) 144   (B) -26 (C) -21 (D) 25   (E) 40

22. **A right triangle has a hypotenuse of length 26". If one of the acute angles is decreasing at the rate of 10° per second, how fast is the area of the triangle decreasing when this acute angle is 13°? (nearest tenth)**

(A) 58.1 in²/sec   (B) 57.4 in²/sec   (C) 55.4 in²/sec   (D) 53.0 in²/sec   (E) 51.7 in²/sec

23. **The y-intercept of the line that is tangent to $y = 4x^2 - 4x + 1$ at $x = 1$ is (x, y). Find $x + y$.**

(A) 4   (B) 1   (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) -3

24. **Roland Bones created a pair of special dice which have only three numbers on each die. The opposite side of each number is the same number. When the dice are rolled the die with the largest number on top wins. What is the probability that die B will win?**

(A) $55\frac{5}{9}\%$ (B) $44\frac{4}{9}\%$ (C) $66\frac{2}{3}\%$ (D) $33\frac{1}{3}\%$ (E) 20%

25. Betty Luzes is playing the *Pic-A-Pie Digit* game. The game has 12 tiles, each tile containing one of the first 11 digits of pi and a decimal point. She puts the tiles in a bag, shakes them up and randomly selects one tile. If the tile contains a prime digit she gets 5 points. If it is a composite digit she loses 10 points. If it is a 1 or a decimal point she gets 25 points. What is the mathematical expectation of a single game? (nearest tenth)

(A) $3\frac{2}{11}$ points (B) $4\frac{3}{4}$ points (C) $6\frac{1}{4}$ points (D) $7\frac{1}{2}$ points (E) $8\frac{9}{11}$ points

26. How many 3-digit numbers exist such that the sum of their digits equals 12?

(A) 36 (B) 43 (C) 66 (D) 78 (E) 91

27. How many proper fractions in lowest terms have a denominator of 54?

(A) 27 (B) 24 (C) 18 (D) 15 (E) 9

28. Simplify: $\left(a^{-3} \times b^{-2}\right)^{-6} \div \left(a^2 \times b^8\right)^2 \times \left(a^0 \div b^1\right)^{-5}$

(A) $a^{14}b$ (B) $a^{-18}b^{-13}$ (C) $a^{-10}b^7$ (D) $a^{-10}b^{-13}$ (E) $a^{14}b^7$

29. Let k be a positive integer less than or equal to 120 such that k is a multiple of 3 and k is divisible by 4. Find the sum of all such numbers k.

(A) 792 (B) 660 (C) 540 (D) 600 (E) 936

30. The sum of the first 10 deficient numbers is a(n) _____ number.

(A) deficient (B) prime (C) abundant (D) harmonic (E) lucky

31. How many pounds of lima beans that cost 90¢ per pound must be mixed with 16 pounds of corn that cost 50¢ per pound to make a mixture of mixed vegetables that cost 65¢ a pound?

(A) 6.4 lbs (B) 2.6 lbs (C) 9.6 lbs (D) 18.4 lbs (E) 12.8 lbs

32. Tu Yung is two years younger than her sister, Soh Yung. Twelve years ago Soh was twice as old as Tu. What will the sum of their ages be in five years?

(A) 30 (B) 54 (C) 35 (D) 28 (E) 40

33. Which of the following polar equations has a graph of an inner-loop limacon?

(A) $r = 3 + 2\cos(\theta)$ (B) $r = 1 + \sin(\theta)$ (C) $r = 5 + 2\cos(\theta)$
(D) $r = 2 + \cos(\theta)$ (E) $r = 2 + 3\cos(\theta)$

34. If $a_1 = -4$, $a_2 = -1$, $a_3 = 2$, $a_n = (a_{n-2}) \times (a_{n-3}) - (a_{n-1})$, where $n \geq 4$, then a_6 equals:

(A) −16 (B) −7 (C) 5 (D) 8 (E) 11

35. **Mary Goround maps triangle 4 to triangle 5 by using which of the groups of three or four transformations in the order given?**
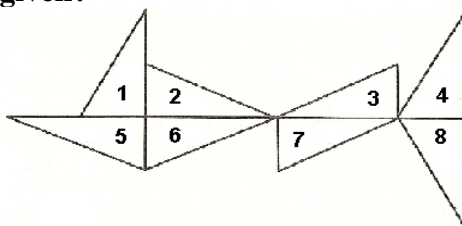
(w) half-turn   (x) reflection   (y) rotation (z) translation

(A) w, x, x, & z   (B) z, x, & x   (C) w, x, y, & z   (D) w, y, & z   (E) z, w, & x

36. **The ratio of the measure of an interior angle of a regular n-gon to the measure of its exterior angle is 3.5 to 1. How many sides does the regular n-gon have?**

(A) 5   (B) 7   (C) 9   (D) 11   (E) 14

37. **Which of the following is not a member of the solution set for $3\left|2x - 8\right| - 20 > 15$?**

(A) $-2\frac{1}{3}$ (B) $-1\frac{7}{8}$ (C) 2.666...   (D) $10\frac{2}{3}$ (E) $11\frac{1}{11}$

38. **The graph of the parametric equations x = 5cos(t) and y = 2sin(t), where $0 \le t \le 2\pi$ is a(n) ___.**

(A) semicircle   (B) ellipse   (C) line   (D) cycloid   (E) parabola

39. $(\sqrt{3}+i)^5$ **equals:**

(A) $16 - 16\sqrt{3}\,i$ (B) $5\sqrt{3}+5i$ (C) $-9\sqrt{3}+i$ (D) $-16\sqrt{3}+16i$ (E) $-32\sqrt{3}+32i$

40. **Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 6th number in row 7 and the 9th number in row 10.**

			1					(row 1)
		2	3	4				(row 2)
	5	6	7	8	9			(row 3)
10	11	12	13	14	15	16		(row 4)
			...					(...)

(A) 96   (B) 132   (C) 134   (D) 147   (E) 149

41. **Given the geometric sequence 5, p, q, $\frac{5}{8}$,..., find the sum of the first 8 terms. (nearest hundredth)**

(A) 9.96   (B) 9.92   (C) 9.87   (D) 9.625   (E) 9.375

42. **Let $f(x) = \frac{2x-1}{3x+4}$. Find $f'(-5)$.**

(A) $-\frac{5}{11}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{11}$ (D) $\frac{2}{3}$ (E) 1

43. Which of the following surfaces is generated by $x^2 - \frac{y^2}{4} - z^2 + 2z = 3$?

    (A) elliptic cone   (B) elliptic hyperboloids   (C) elliptic cylinder
    (D) hyperbolic paraboloid   (E) elliptic paraboloid

44. Willie Pikette randomly selects a factor of 144. Betty Wheel selects a factor of 88. What is the probability that they selected the same number? (nearest tenth)

    (A) 17.4%   (B) 3.3%   (C) 13.3%   (D) 6.1%   (E) 21.1%

45. Dee Deeler shuffles a standard 52-card deck. She deals out the top two cards. What are the odds that both cards are face cards?

    (A) $\frac{22}{321}$ (B) $\frac{11}{210}$ (C) $\frac{11}{130}$ (D) $\frac{11}{221}$ (E) $\frac{33}{400}$

46. The number of integers between 1 and 328 that are relatively prime to 328 is ?

    (A) 159   (B) 109   (C) 82   (D) 279   (E) 164

47. Find the sum of the lengths of the diagonals if the lengths of the sides of the regular pentagon shown is 6". (nearest tenth)
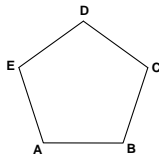
    (A) 30.0"   (B) 58.2"   (C) 29.1"   (D) 38.8"   (E) 48.5"

48. Line *m* goes through the point $(1, -4)$ and $(-1, -10)$. Line *n* goes through points $(-11, 0)$ and $(-4, 1)$. Line *m* intersect line *n* at (x, y). Find $x + y$. (nearest tenth)

    (A) 5   (B) 2   (C) 0   (D) -1 (E) -28

49. $\left\{(x, y)\,\middle|\, x,y \in \{\text{Integers}\}, -8 \le x \le 14, \text{ and } -14 \le y \le 8\right\}$ is the solution set of $5x + 3y = 28$. How many such ordered pairs exist?

    (A) 22   (B) 16   (C) 8   (D) 5   (E) 3

50. How many integral values of n exist such that $n > 3$ and $\frac{(n-1)!}{(n-3)!} \le 182$

    (A) 20   (B) 16   (C) 15   (D) 12   (E) 10

51. Lynn Kaln has some pennies. Penny Lesse has 4 times as many dimes as the number of pennies Lynn has. Nick Ohl has 16 less nickels than twice the number of dimes Penny has. How many nickels did Nick have if the total amount is they have $8.92?

    (A) 104   (B) 76   (C) 80   (D) 60   (E) 36

52. Find the shortest distance from the point (3, 2) and the line 5x + 12y = 13.

(A) 2   (B) $2\sqrt{3}$ (C) $3\sqrt{2}$ (D) 5   (E) $4\sqrt{2}$

53. Sir Vayer used his theodilite to find the angle measures of two points at the other end of a field. Point A was on a bearing of 290° and point B was on a bearing of 45° from where he stood. He was 150 yards from point A and 120 yards from point B. What was the distance from Point A to point B? (nearest yard)

(A) 240 yds   (B) 228 yds   (C) 222 yds   (D) 157 yds   (E) 151 yds

54. Phil Upp, Doug Upp, and Stan Upp are filling up a large hole. Phil can do the job by himself in 4 hours, Doug in 6 hours, and Stan in 8 hours. How long would it take them working together? (nearest minute)

(A) 2 hrs   (B) 1 hr 51 min   (C) 1 hr 40 min   (D) 1 hr 33 min   (E) 1 hr 18 min

55. Find C if the remainder of $x^3 - 3x^2 - 10x + C$ divided by $x - 4$ is 3.

(A) 75   (B) 21   (C) 27   (D) 53   (E) 34

56. Let $f(x) = \frac{1}{x^2 + 1}$. The concavity of the curve is upward at which of the following values of x?

I. $-\frac{2}{3}$ II. 0   III. $\frac{3}{4}$

(A) II only   (B) I, II, & III   (C) I & II   (D) I & III   (E) II & III

57. Cookie Baykur packages cookies 3 to a pack. The types of cookies she can choose from include chocolate chip, oatmeal, sugar-coated, sugar-free, peanut butter, and hazel-nut. How many different packs of 3 cookies can she package?

(A) 20   (B) 120   (C) 28   (D) 60   (E) 56

58. 0.5323232... in base 7 can be written as which of the following fractions in base 10?

(A) $\frac{131}{165}$ (B) $\frac{268}{343}$ (C) $\frac{263}{336}$ (D) $\frac{49}{76}$ (E) $\frac{524}{660}$

59. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $(f_6)^2 + (f_7)^2$.

(A) f_{26} (B) f_{14} (C) f_{13} (D) f_{12} (E) f_8

60. Given the circle O with perpendicular diameters and a chord, find the area of the circle if EF = 8" and DE = 20" inches. (nearest tenth)
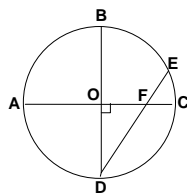
(A) 503 sq. in   (B) 377 sq. in   (C) 323 sq. in   (D) 176 sq. in   (E) 151 sq. in

University Interscholastic League
MATHEMATICS CONTEST
HS ● Regional ● 2015
Answer Key

1. D	21. E	41. A
2. C	22. D	42. C
3. C	23. E	43. B
4. A	24. A	44. B
5. B	25. C	45. B
6. E	26. C	46. ■
7. B	27. C	47. E
8. E	28. A	48. A
9. E	29. B	49. D
10. B	30. C	50. D
11. A	31. C	51. C
12. A	32. E	52. A
13. D	33. E	53. B
14. E	34. D	54. B
15. A	35. D	55. C
16. A	36. C	56. D
17. D	37. C	57. E
18. A	38. B	58. C
19. E	39. D	59. C
20. D	40. B	60. B

Mathematics

State • 2015

DO NOT TURN THIS PAGE UNTIL
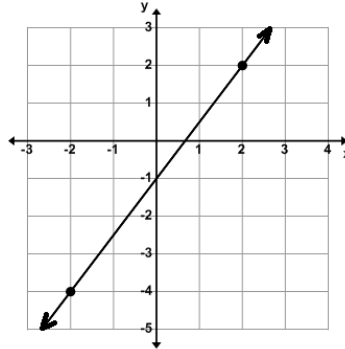YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $5 + 2 \times 6 - 5 \div 2 \times 6 + 5^2 - 6$

    (A) 130   (B) 55   (C) 51   (D) 21   (E) 17

2. The *Parr Fore* golf store sells a sleeve of 3 balls for $3.85, and a box of a dozen balls for $12.25. How much money would Ty Gerr save before sales tax if he bought 36 balls by the dozen instead of by the sleeve?

    (A) $8.40   (B) $9.45   (C) $9.55   (D) $10.55   (E) $10.95

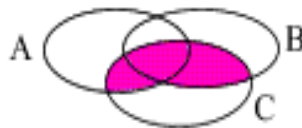3. Which of the following is an equation of the line shown?

    (A) $2x - 3y = -2$ (B) $2x + 3y = 2$ (C) $3x - 2y = -1$ (D) $3x + 2y = -1$ (E) $3x - 2y = 2$

4. Find the sum of the arithmetic mean, median, mode, and range of 5, 2, 6, 2, 8, 2, 0, 1, & 5.

    (A) $15\frac{4}{9}$ (B) $15\frac{7}{8}$ (C) $16\frac{4}{9}$ (D) $18\frac{7}{8}$ (E) $18\frac{4}{9}$

5. The shaded region of the Venn diagram shown represents which of the following sets:

    (A) $(A \cup B) \cap (C \cup B)$ (B) $(A \cap B) \cup C$ (C) $(A \cup B) \cap C$
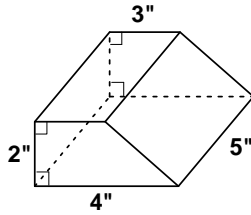   (D) $(A \cap C) \cup B$ (E) $(A \cap B) \cup (C \cap B)$

6. Simplify: $\left(\dfrac{4x^2 - 8x - 5}{x^3 + 2x^2 - 5x - 6} \right) \times \left(\dfrac{x^2 - x - 2}{5 - 2x} \right) \div \left(\dfrac{1 + 2x}{3 - x} \right)$

    (A) $\dfrac{3 - x}{x + 3}$ (B) $\dfrac{2x^2 - 11x + 15}{2x^2 + x - 15}$ (C) $\dfrac{x - 3}{x + 3}$ (D) $x^2 - 9$ (E) 1

7. Ten years ago Tu Yung's father was seven times as old as she. In five years, she will be half of her father's age. What is the sum of their ages now?

    (A) 62   (B) 56   (C) 48   (D) 44   (E) 40

8. Find the volume of the trapezoidal prism shown. (nearest cu. in).   Drawing is not to scale.


    (A) 40 cu. in   (B) 24 cu. in   (C) 39 cu. in   (D) 45 cu. in   (E) 35 cu. in

9. The ratio of the length to the width of a rectangle is 2.5:1. If 7 units are added to both the length and the width, then the ratio of the length to the width is now 4:3. What is the difference in the perimeters of the two rectangles? (units)

    (A) 56   (B) 49   (C) 28   (D) 21   (E) 14

10. The *Ice T* hockey team has 8 forwards, 9 defensemen, and 3 goaltenders. How many 6-member squads can be formed if each team needs 3 forwards, 2 defensemen, and 1 goaltender?

    (A) 7,056   (B) 6,048   (C) 95   (D) 2,016   (E) 38,760

11. Simplify: $\left(a^{-5} \times b^2\right)^{-6} \div \left(a^8 \times b^{-2}\right)^5 \times a^{20} \div b^{15}$

    (A) $a^{10}b^{-17}$ (B) $a^{-4}b^{-22}$ (C) a^{29} (D) $a^{10}b^{-9}$ (E) $a^{11}b^5$

12. The equation $4x^2 - 8x + k = 0$ always has two positive roots when which of the following is true?

    (A) $0 < k < 4$ (B) $8 > k > 4$ (C) $k > -2$ (D) $k > 4$ (E) $k < 0.5$

13. If $\frac{A}{5x+2} + \frac{B}{3x-1} = \frac{41x+1}{15x^2+x-2}$, where A and B are constants, then $A + B$ equals:

    (A) 4   (B) 6.9   (C) 7   (D) 11   (E) 13.8

14. Let $a_1 = 5$, $a_2 = -2$, $a_3 = 6$ and $a_n = (a_{n-2}) \times \left[(a_{n-3}) - (a_{n-1})\right]$ for $n \geq 4$. Find a_6.

    (A) 312   (B) 60   (C) 1,152   (D) 2   (E) 68

15. What are the odds of randomly selecting a number that is divisible by 3 from set of the triangular numbers less than 60?

    (A) 1.5:1   (B) 3:5   (C) 3:1   (D) 5:3   (E) 6:5

16. Let $A = \begin{bmatrix} 0 & 5 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ -1 & -5 \end{bmatrix}$. Find $\left|A^T + B\right|$.

    (A) -20 (B) -11 (C) -6 (D) -3 (E) 0

17. Given the circle with center O, perpendicular diameters and a chord, find the perimeter of △DFO if DE = 14 cm and DF = 10 cm. (nearest tenth)
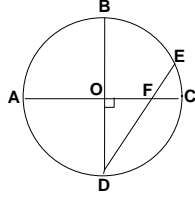
(A) 32.4 cm   (B) 29.5 cm   (C) 27.8 cm   (D) 23.8 cm   (E) 18.7 cm

18. Find m∠DCE, nearest degree, if AD = $\sqrt{48}$ inches.
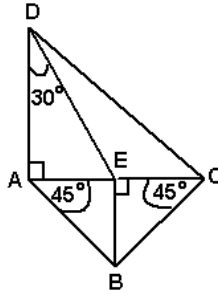
(A) 11°   (B) 30°   (C) 41°   (D) 49°   (E) 52°

19. Use the Fibonacci characteristic sequence ..., — 1, p, q, r, 4, 7, ... to find p + q + r.

(A) 5   (B) 6   (C) 9   (D) 10   (E) 16

20. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 2nd, 26th, and 50th numbers in row 26.

```
                      1                        (row 1)
                2   3   4                      (row 2)
            5   6   7   8   9                  (row 3)
       10  11  12  13   14  15  16             (row 4)
                     ...                       (  ...  )
```

(A) 1,875   (B) 1,914   (C) 1,953   (D) 1,991   (E) 2,028

21. A particle is moving along the straight line with a function of $f(t) = t^2 - t + 2$, where f(t) is the distance in meters per second. Find the instantaneous rate of change at a time of 2 seconds.

(A) 6 m   (B) 5 m   (C) 4 m   (D) 3 m   (E) 2 m

22. The directrix of the conic given by the equation $x^2 - 2y + 4x = -8$ is:

(A) y = — 1.5   (B) y = — 0.5   (C) y = 0.5   (D) y = 1.5   (E) y = 2.5

23. Let k be a positive integer less than or equal to 120 such that k is not a multiple of 2 and not a multiple of 3. How many such numbers exist?

(A) 100   (B) 80   (C) 60   (D) 40   (E) 20

24. Kandy Packer has gumballs, suckers, gumdrops, chocolate kisses, and bubble gum. She puts 6 pieces of candy in each pack to give to her students. How many different packs of candy can Kandy pack?

   (A) 2,310   (B) 462   (C) 210   (D) 120   (E) 30

25. The first term of an arithmetic sequence is 2 and the common difference is 6. How many terms are in the sequence if the sum of the terms is 420.

   (A) 12   (B) 18   (C) 20   (D) 24   (E) 35

26. If P, Q, and R represent digits then $RPQ_8 - QRP_4 - PQR_2$ has a numeric value in base 10 of:

   (A) $3P - 9Q + 11R$ (B) $8P - 13Q + 69R$ (C) $3P - 17Q + 59R$
   (D) $11P - 13Q + 61R$ (E) $5P - 10Q + 11R$

27. Lotta Dough had a bag of pennies. She gave her brother $\frac{1}{5}$ of her pennies. Then she gave her sister 20% of what she had left. Then she used her pennies to buy a 30¢ sucker. She put the remaining 50 pennies in her piggy bank. How many pennies did Lotta have in the bag, originally?

   (A) 120   (B) 125   (C) 130   (D) 134   (E) 156

28. Which of the following is not a solution to $5 + |2x - 6| \le 15$?

   (A) $-1.333...$ (B) -0.7 (C) $3\sqrt{8}$ (D) $2\sqrt{7}$ (E) $\frac{50}{7}$

29. If the roots of $2x^3 + bx^2 + cx + d = 0$ are $-3, 1$, and 2, then $b + c + d$ equals:

   (A) -2 (B) 26   (C) 0   (D) 4   (E) -13

30. Which of the following points of concurrency lies on the vertex of the right angle of a right triangle?   (1) circumcenter   (2) centroid   (3) orthocenter   (4) incenter

   (A) 1 & 2   (B) 1 only   (C) 3 only   (D) 2, 3, & 4   (E) none of these

31. Find the shortest distance from the point (2, 8) and the line $y = 1.25 - 0.75x$.

   (A) 8.6   (B) $5\frac{3}{14}$ (C) 7.4   (D) 4   (E) 6.6

32. How many integral values of n exist such that $n > 1$ and $\frac{(n + 1)!}{(n - 1)!} \le 26$?

   (A) 25   (B) 13   (C) 9   (D) 3   (E) 2

33. If (1, 1) and (2, -2) are members of the function $\{(x, y) \mid y = ax - 2b\}$, then $a + b = ?$

   (A) -5 (B) -4 (C) -3 (D) -2 (E) -1

34. The equation $y =$ _____ will produce this graph.
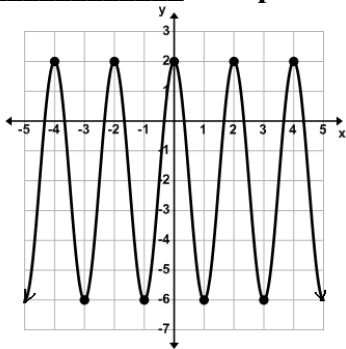
(A) $2 - 4\cos(\pi x - 4\pi)$ (B) $2 + 4\cos(\pi x - 3\pi)$ (C) $4\sin(\pi x - 3\pi) - 2$

(D) $4\cos(\pi x - 4\pi) - 2$ (E) $2 - 4\sin(\pi x - 2\pi)$

35. The graph of the parametric equations $x = 2\sin^2(t)$ and $y = \sin(t)$ is a(n) _____.

(A) circle (B) ellipse (C) hyperbola (D) cycloid (E) parabola

36. In the expansion of $(5x + 2)^6$, the sum of the coefficients of the 2nd, 3rd, 4th, 5th, and 6th terms is:

(A) 117,647 (B) 117,640 (C) 109,804 (D) 101,967 (E) 101,960

37. Simplify to the form $a + bi$: $(5 - 2i)(6 + i) \div (5i)$

(A) $-0.4 + 6i$ (B) $7 + 32i$ (C) $2.2 - 0.2i$ (D) $-5.6 - 3.4i$ (E) $-1.4 - 6.4i$

38. $F(x) = \dfrac{10}{x^2} + \dfrac{10}{x}$ has an inflection point at :

(A) $(-3\frac{1}{2}, -2\frac{1}{25})$ B) $(-3, -2\frac{2}{9})$ (C) $(-2, -2\frac{1}{2})$ (D) $(3, 4\frac{4}{9})$ (E) $(4, 3\frac{1}{8})$

39. Let $f(x) = 5x^2 - 2x - 6$ and $g(x) = 5x^2 + 2x - 8$. Find $f'(g'(1 + 5x))$.

(A) $500x + 82$ (B) $100x + 34$ (C) $100x + 20$ (D) $100x + 6$ (E) $500x + 118$

40. A star gazer is watching the night sky. The probability that he will see a satellite is 40%, the probability that he will see a shooting start is 25%, and the probability that he will see both is 15%. What is the probability that he will see either a satellite, a shooting star, or both? (nearest percent)

(A) 80% (B) 65% (C) 55% (D) 50% (E) 30%

41. Saul DeRod had 5 wooden rods with lengths of 5", 2", 6", 1" and 5". How many acute triangles can he form using only 3 rods at a time?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

42. **Given the pentagram shown, find the ratio of A to B. (nearest tenth)**
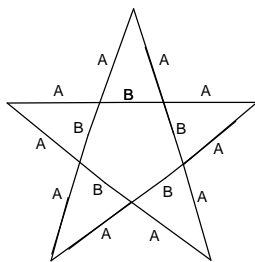
    (A) 0.5   (B) 0.6   (C) 1.6   (D) 1.9   (E) 2.0

43. **The harmonic mean, nearest tenth, of the real roots of $x^3 - 13x^2 + 52x - 60 = 0$ is:**

    (A) 4.7   (B) 4.3   (C) 3.9   (D) 3.5   (E) 3.1

44. **The graph of the polar equation $r = 7 + 4\cos(\theta)$ is a _____.**

    (A) **dimpled limacon**   (B) **convex limacon**   (C) **inner loop limacon**
    (D) **lemniscate**   (E) **cardioid**

45. **Let $f(x) = \dfrac{5x^3 - 8}{x^2 + 3x - 1}$ and s(x) be the slant asymptote of f. Find the value of s(3).**

    (A) 0   (B) $7\frac{8}{17}$ (C) 17   (D) 127   (E) **undefined**

46. **A standard deck of 52 cards is shuffled. The top 4 cards are dealt face up. What is the probability that they are all face cards (Jacks, Queens, and/or Kings)? (nearest hundredth)**

    (A) 0.03%   (B) 0.18%   (C) 0.23%   (D) 0.31%   (E) 0.83%

47. **The square root of 11661 in base 8 is what in base 10:**

    (A) 77   (B) 76   (C) 73   (D) 72   (E) 71

48. **Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, ...$ be the terms of the Fibonacci sequence. Find $GCD(f_m, f_n)$.**

    (A) $f_{(m+n)}$ (B) $f_m + f_n$ (C) $f_{(mn)}$ (D) $f_m \times f_n$ (E) $f_{GCD(m,n)}$

49. **The number 60 is considered to be a "polite" number. The "politeness" of 60 is ____.**

    (A) 5   (B) 3   (C) 2   (D) 1   (E) 0

50. **The *Slo-Poke* freight train leaves the station at 8:00 a.m. traveling at 35 mph. Later, the *Super-Speed* Amtrak left the same station traveling in the opposite direction at a speed of 75 mph. At 12:00 p.m. the two trains were 350 miles apart. At what time did the *Super-Speed* leave the station? (nearest minute)**

    (A) 8:45 a.m.   (B) 9:12 a.m.   (C) 9:45 a.m.   (D) 10:12 a.m.   (E) 10:48 a.m.

51. **Find the area of the shaded regions. (square units).**
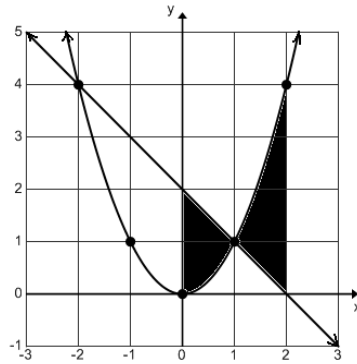
    (A) 4   (B) 3.5   (C) 3.333...   (D) 3   (E) 2.666...

52. **Bill Meelator borrowed $750.00 for his first semester books. Part of the loan was at the rate of 3% per year and the rest of the loan was at 8% per year. If the interest was $19.50 at the end of 6 months, how much of the loan was at 3%?**

    (A) $468.75   (B) $330.00   (C) $112.50   (D) $281.25   (E) $420.00

53. **Ranger Saul D. Smoke sees two fires from his ranger station. He uses a Triangulation Device to mark the point of each fire on his map. Then, using his protractor, he computes fire A to be 15 miles from his station on a bearing of 75° degrees and fire B to be 10 miles from his station on a bearing of 245°. How far apart are the two fires? (nearest mile)**

    (A) 20 mi   (B) 21 mi   (C) 23 mi   (D) 24 mi   (E) 25 mi

54. **The Ceehahks and the Paytritts play two games during the Foosball season. The Ceehahks are one and a half times as likely to win any game as is the Paytritts. What is the probability that the Ceehahks will win both games?**

    (A) $55\frac{5}{9}\%$ (B) 36%   (C) $44\frac{4}{9}\%$ (D) 16%   (E) 52%

55. **The point (— 2, 6) lies on a circle whose center is (1, 5). Which of the following points lie on the circle?**   P (4, 4)   Q (— 2, 4)   R (0, 2)

   (A) P only   (B) P & Q   (C) P & R   (D) Q & R   (E) P, Q, & R

56. **Let $f(x) = x^2 + bx + c$. If f(x) is divided by x — 3 the remainder is 2 and if f(x) is divided by x + 2 the remainder is 3. Find b + c.**

    (A) $-4\frac{3}{5}$ (B) $-3\frac{1}{2}$ (C) -1 (D) $1\frac{1}{2}$ (E) $2\frac{3}{5}$

57. **If the two-digit number 3Q is subtracted from the two-digit number P2 the difference is 27. Find the sum of the two-digit numbers PQ and QP, where P and Q are single digits.**

    (A) 130   (B) 121   (C) 112   (D) 111   (E) 97

58. Find the perimeter of the triangle shown (nearest cm).

75°

55°
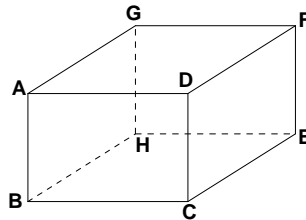
200 cm
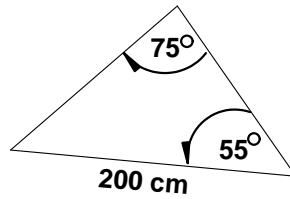
(A) 516 cm   (B) 528 cm   (C) 546 cm   (D) 560 cm   (E) 623 cm

59. Max Space needs two adjacent rectangular holding pens to separate his three cows from his bull. He has twenty 100' rolls of fencing. What is the maximum area that Max can fence in?

(A) 125,000 ft.^{2} (B) 133,333$\frac{1}{3}$ ft.^{2} (C) 166,666$\frac{2}{3}$ ft.^{2} (D) 175,000 ft.^{2} (E) 250,000 ft.^{2}

60. Given the rectangular solid shown, find AE if AF = y, BG = x and FH = z.

(A) $x^2 + y^2 + z^2$

(B) $\dfrac{x^2 + y^2 + z^2}{2}$

(C) $\sqrt{\dfrac{x^2 + y^2 + z^2}{2}}$

(D) $2(x^2 + y^2 + z^2)$

(E) $\dfrac{\sqrt{x + y + z}}{2}$

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2015
Answer Key

1. D	21. D	41. C
2. B	22. D	42. C
3. E	23. D	43. D
4. A	24. C	44. A
5. C	25. A	45. A
6. C	26. C	46. B
7. D	27. B	47. E
8. E	28. C	48. E
9. C	29. A	49. B
10. B	30. C	50. B
11. A	31. E	51. D
12. A	32. D	52. E
13. D	33. A	53. E
14. B	34. D	54. B
15. A	35. E	55. E
16. C	36. E	56. A
17. D	37. E	57. B
18. C	38. B	58. B
19. B	39. E	59. C
20. C	40. D	60. C