T M S C A H I G H S C H O O L

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T E S T # 1 0 ©

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GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2018 – 2019 TMSCA Mathematics Test Ten

1. **Evaluate:** $(9-3)!\times 5^{-2}-3\div(-1)\times\left(3^2+1\right)$.

   (A) 18,030 **(B)** $58\frac{4}{5}$ **(C)** $15{,}544\frac{24}{25}$ **(D)** $29\frac{1}{10}$ **(E)** $483\frac{21}{25}$

2. **Find the arithmetic mean of the median, mode and range of the quiz grades: 86, 75, 38, 96, 86, 88 & 64. (nearest whole number)**

   (A) 73 **(B)** 71 **(C)** 86 **(D)** 77 **(E)** 75

3. **If 45% of A is $8\frac{2}{5}$ of B, then B is what percent of A?**

   (A) $5\frac{5}{14}\%$ **(B)** $4\frac{3}{7}\%$ **(C)** $10\frac{5}{7}\%$ **(D)** $12\frac{1}{2}\%$ **(E)** $4\frac{1}{6}\%$

4. $\overline{PQ}$ **has endpoints P(3, 11) and Q(−2, 6). Which of the following is an equation of the perpendicular bisector of** $\overline{PQ}$ **?**

   (A) $x-y=-8$ **(B)** $x-y=-9$ **(C)** $x+y=9$

   (D) $x+y=4$ **(E)** $x+y=3$

5. **Let** $(3x+2)^2(5x-2)=ax^3+bx^2+cx+d$. **Find** $a+b+c+d$.

   (A) 91 **(B)** 99 **(C)** 83 **(D)** 88 **(E)** 75

6. **Simplify:** $\dfrac{6x^2-11x-10}{2x^2-3x-5}\div\dfrac{9x^2+12x+4}{x^2+8x+7}$.

   (A) $\dfrac{x+7}{3x+2}$ **(B)** $\dfrac{x+7}{3x-2}$ **(C)** $\dfrac{x}{3x+2}$ **(D)** $x+7$ **(E)** $\dfrac{x}{3x-2}$

7. **A polyhedral die has 60 edges and 30 vertices. How many faces does it have?**

   (A) 28 **(B)** 30 **(C)** 32 **(D)** 34 **(E)** 36

8. **Let** $O=\{1,3,5,7,...29\}$, $S=\{1,4,9,16,27,...,100\}$ **and** $F=\{1,2,3,5,8,13,...89\}$. **How many elements are in** $(O\cup S)\cap F$ **?**

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. **If** $m\angle A+m\angle B=180°$, **then** $180°=m\angle A+m\angle B$ **is an example of the _____ property.**

   (A) Associative **(B)** Commutative **(C)** Distributive **(D)** Symmetric **(E)** Transitive

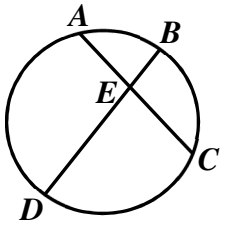
10. **Martha likes to mix her regular blend coffee beans with special dark roast coffee beans in a ratio of 4:1. The regular blend sells for $9.00 per pound, and the special dark roast sells for $13.50 per pound. How much should Martha plan to spend making one pound of her mix? (nearest cent)**

    (A) $10.13 **(B)** $9.90 **(C)** $12.60 **(D)** $11.10 **(E)** $11.40

11. **Let** $x-3y=10$, $2y+z=1$ **and** $z-x=1$. **Find** $x+yz$.

    (A) 6 **(B)** 18 **(C)** 7 **(D)** −6 **(E)** −14

12. $\overline{AC}$ **and** $\overline{BD}$ **are both chords of a circle and intersect at point** E**. If** $m\overset{\frown}{AB} = 32°$ **and** $m\angle DEC = 82°$ **find** $m\overset{\frown}{DC}$**.**



(A) $138°$ **(B)** $132°$ **(C)** $142°$

(D) $140°$ **(E)** $134°$

13. Find the leading digit of the integer 7^{888}**.**

(A) 2 **(B)** 3 **(C)** 3 **(D)** 7 **(E)** 9

14. An ounce of canned spinach occupies 1.8 in^3 of space. A manufacturer plans to make 14.5-ounce cans that are 4.5 inches tall. Calculate the diameter of the cans? (nearest tenth of an inch)

(A) 1.4 inches **(B)** 1.8 inches **(C)** 3.8 inches **(D)** 2.7 inches **(E)** 3.1 inches

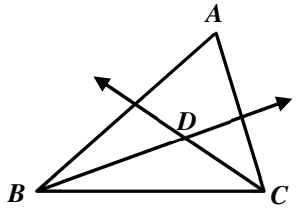
15. Which of the following mathematicians is known as the "Father of Modern Algebra" and is associated with the formulas for finding the sum of the roots and the product of the roots, $-\frac{b}{a}$ **and** $\frac{c}{a}$**?**

(A) Goldbach **(B)** Vieta **(C)** Napier **(D)** Turing **(E)** Mersenne

16. How many non-negative proper fractions in lowest terms have a denominator of 48?

(A) 12 **(B)** 18 **(C)** 14 **(D)** 16 **(E)** 10

17. In triangle ABC**, the bisectors of** $\angle B$ **and** $\angle C$ **meet at** D**. Find** $m\angle BDC$ **if** $m\angle A = 58°$**.**



(A) $122°$ **(B)** $121°$ **(C)** $120°$

(D) $119°$ **(E)** $118°$

18. Find $g\left(f\left(a+1\right)\right)$ **when** $f\left(x\right) = 2x - 4$ **and** $g\left(x\right) = 2 - x$**.**

(A) $2a$ **(B)** $2a + 4$ **(C)** $4 - 2a$ **(D)** $2 - 2a$ **(E)** $-2a$

19. Given $f\left(x\right) = -3\sin\left[2\pi\left(x-5\right)\right]+4$**, find the sum of the numeric values of the frequency and vertical displacement.**

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

20. According to Descartes' Rule of Signs, how many positive real zeros will $f\left(x\right) = 5x^4 - 3x^3 + 7x^2 - 12x + 4$ **have?**

(A) 3 or 1 **(B)** 4, 2 or 0 **(C)** 1 **(D)** 0 **(E)** 2 or 0

21. Solve $\left|4x+2\right|-1 \geq 5$

(A) $\left(-\infty,-1\right]\cup\left[2,\infty\right)$ **(B)** $\left(-\infty,1\right]\cup\left[2,\infty\right)$ **(C)** $\left[-2,1\right]$

(D) $\left(-\infty,-2\right]\cup\left[1,\infty\right)$ **(E)** $\left[-1,2\right]$

22. Nine students including 3 girls sat at a circular table. If the seating arrangement was random, what is the probability that all of the girls sat together?

(A) $\frac{1}{84}$ **(B)** $\frac{1}{72}$ **(C)** $\frac{1}{24}$ **(D)** $\frac{3}{56}$ **(E)** $\frac{3}{28}$

23. The vertex of a parabola is located at $(-3,1)$ and the focus is located at $(-3,0)$. Find the directrix of the parabola.

(A) $y=0$ **(B)** $y=2$ **(C)** $y=-2$ **(D)** $y=-1$ **(E)** $y=1$

24. Let $f(x)=\frac{2x-5}{4x+2}$. Find $f'(4)$.

(A) $-\frac{13}{14}$ **(B)** $\frac{1}{6}$ **(C)** $-\frac{4}{81}$ **(D)** $\frac{2}{27}$ **(E)** $-\frac{3}{14}$

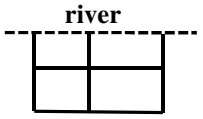
25. Which of the following is an identity for $2\sin(2x)\cos x-\sin x$?

(A) $\cos^2 x$ **(B)** $\sin(3x)$ **(C)** $\sin x\cos x$ **(D)** $\sin x+\cos x$ **(E)** $\cos(2x)$

26. Let $f(x)=\frac{2x-5}{4x+2}$. Find $f^{-1}(4)$.

(A) $-\frac{1}{2}$ **(B)** $\frac{2}{27}$ **(C)** $-\frac{13}{14}$ **(D)** $-\frac{7}{18}$ **(E)** $-\frac{13}{18}$

27. Carl has 120 meters of fencing. He wants to create a rectangular livestock enclosure divided into four separate sections. He has a river that will serve as one side of the enclosure. The maximum area he can fence is _____m^2.



(A) 200 **(B)** 300 **(C)** 150 **(D)** 600 **(E)** 720

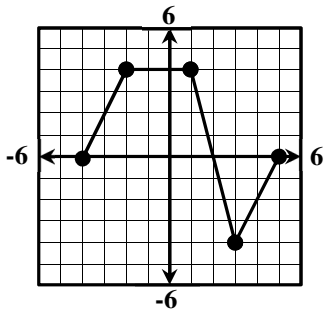
28. Let K be an integer such that K has exactly eight factors, including 1 and itself. The numbers 91 and 133 are two of the factors. What is the sum of the digits of K?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

29. Carl has a bag containing 6 red stones, 4 green stones and 5 black stones. What is the probability of Carl randomly selecting two balls (without replacement) that are two different colors? (nearest %)

(A) 66% **(B)** 67% **(C)** 69% **(D)** 70% **(E)** 72%

30. Find the value of $\int_{-4}^{5} f(x)dx$ for the piecewise-linear function f, $-4\le x\le 5$, shown.



(A) 18 **(B)** 24 **(C)** 12

(D) 16 **(E)** 20

31. Let $878_b + 788_b = 1111_b$. Find 272_b in base 10.

(A) 492 (B) 557 (C) 626 (D) 699 (E) 776

32. The sum of the coefficients of the 3rd term in the expansion of $(x+2)^3$, the 4th term of $(x+2)^4$ and the 5th term of $(x+2)^5$ is:

(A) 124 (B) 65 (C) 12 (D) 62 (E) 28

33. $\{(x,y)|x,y\in\mathbb{Z},-15\le x\le 15,-15\le y\le 15\}$ is the solution set of $3x+2y=12$. How many such ordered pairs exist?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

34. If $5^{2x+y}=125$ and $4^{x+2y}=16$ then xy equals_____.

(A) $\frac{5}{9}$ (B) $1\frac{4}{5}$ (C) $\frac{4}{9}$ (D) $3\frac{1}{8}$ (E) $3\frac{1}{5}$

35. The cube root of 10404_5 is $\_\_\_\_\_{}_5$.

(A) 14 (B) 20 (C) 13 (D) 21 (E) 12

36. Find A if the remainder of $7x^5+Ax^4+3x^3-3x^2+x$ divided by $(x+1)$ is -18.

(A) 32 (B) -4 (C) 14 (D) -28 (E) 36

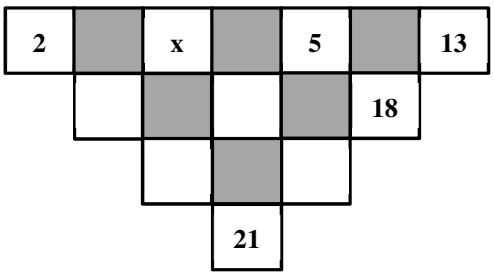
37. Let $\begin{bmatrix}-3 & 7\\ -1 & 5\end{bmatrix}^{-1}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$. Find d.

(A) $-\frac{5}{8}$ (B) $\frac{7}{8}$ (C) $-\frac{1}{8}$ (D) $\frac{3}{8}$ (E) $\frac{3}{22}$

38. Meredith can peel a bushel of ripe peaches in 1 hour and 15 minutes by herself. Nancy takes 1 hour and 45 minutes to do the same job. How long would it take the two together to peel six bushels? (nearest minute)

(A) 8 h. 45 m. (B) 6 h. 11 m. (C) 4 h. 23 m. (D) 4 h. 37 m. (E) 5 h. 17 m.

39. The number in each unshaded box is found by adding the numbers connected to it from the row above. (ex. 18 is found by adding 5 and 13). What is the value of x?



(A) 5 (B) 2 (C) -3 (D) -2 (E) 1

40. Given $y=x-12$ and $xy=18$. Calculate x^3-y^3.

(A) 2376 (B) 2160 (C) 1944 (D) 1080 (E) 1296

41. A group agrees to share equally in the cost of a \$48,000 piece of machinery. If they can find two more group members, each member's share will decrease by \$4000. How many are presently in the group?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

42. Find the area of the quadrilateral with the vertices $(-11,12)$, $(9,16)$, $(15,-2)$ and $(-2,-12)$ respectively?

(A) 432 (B) 439 (C) 435 (D) 434 (E) 441

43. $\prod_{k=0}^{2}(-1)^{k+1}(y-kx) =$

(A) $y^3-3xy^2+2x^2y$ **(B)** $y^3-2xy^2+2x^2y$ **(C)** $-y^3+3xy^2-2x^2y$

(D) $-y^3+2xy^2-2x^2y$ **(E)** $-y^3-3xy^2-2x^2y$

44. Find $a+b+c+d$ given the Fibonacci characteristic sequence: $-4,a,b,6,c,13,d$.

(A) 13 (B) 32 (C) 30 (D) 27 (E) 33

45. The apothem of a regular hexagon has a length of $14\sqrt{3}$ cm. The area of the hexagon is _____cm².

(A) 1176 (B) 588 (C) $588\sqrt{3}$ (D) 2037 (E) $1176\sqrt{3}$

46. Given that $(a-10i)(b+5i)=58$, where $a,b\in\mathbb{Z}$, which of the following is a possible value of $a+b$.

(A) 6 (B) -7 (C) -4 (D) -2 (E) 5

47. Triangle *PQR* is such that $m\angle R=60°$, $PR=16$ and $PQ=14$. There are two possible values of QR, find the sum of the two values.

(A) 24 (B) 16 (C) 10 (D) 14 (E) 30

48. How many distinct 4-letter words can be formed using the letters in the word BUBBLE?

(A) 12 (B) 120 (C) 60 (D) 72 (E) 48

49. The fraction $\frac{2}{10}$ in base 7 can be written as which of the following decimals in base 7?

(A) 0.2 (B) 0.252525… (C) 0.1666… (D) 0.125 (E) 0.333…

50. $\frac{27}{10}\pi$ **radians = _____°**

(A) 648 (B) 486 (C) 972 (D) 243 (E) 612

51. Let $f(x)=\frac{x^3-7x^2+2x-14}{2x^2-9x-35}$. **A *removable discontinuity* exists at $x=$?**

A) 5 (B) 6 (C) 7 (D) 8 (E) 9

52. If $f(x)=3^x$, $g(x)=\log_3 x$ and $a\geq 2$ then $g(f(a+1))$.

(A) 3^{a+1} **(B)** $3a+3$ **(C)** $\log_3(a+1)$ **(D)** $a+1$ **(E)** $3a+1$

53. The point $(2,7)$ is reflected over the x-axis, reflected over the line $y = x$, rotated 180° clockwise around the origin, then shifted down three units to the point (a,b). Find $a+b$.

(A) 4 (B) −3 (C) 1 (D) 6 (E) 2

54. Let K harmonic mean of the real roots of $f(x) = 8x^3 - 55x^2 + 73x - 15$. What digit is in the 10^{-5} place of K?

(A) 0 (B) 6 (C) 2 (D) 3 (E) 8

55. Calculate $12 - \frac{12^3}{6} + \frac{12^5}{120} - \frac{12^7}{5040} + \frac{12^9}{362880}$.... to the nearest ten-thousandth

(A) 0.8439 (B) −0.6359 (C) 1.0792 (D) −0.5366 (E) 2.4849

56. There are two values of k for which $\begin{vmatrix} -k & 5 \\ -4 & k+2 \end{vmatrix} = 5$. Find the larger value of k.

(A) 5 (B) 15 (C) 3 (D) −5 (E) −3

57. Evaluate: $\sum_{n=1}^{\infty}\left[\frac{1}{4}\left(\frac{2}{5}\right)^n\right]$

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

58. If P, Q and R represent digits, then $PRQ_7 + RPQ_8 - QRP_9$ has a numeric value in base 10 of:

(A) $58P + 83Q + 80R$ (B) $56Q - 79Q + 80R$ (C) $58P - 83Q + 64R$

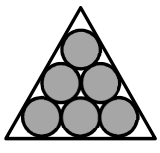
(D) $56P - 79Q + 62R$ (E) $58P - 83Q + 64R$

59. Levi and CW are riding a seesaw. Levi weighs 85 pounds and is sitting 5 feet from the center of the seesaw. CW weighs 91 pounds and is sittign on the other end of the seesaw. If the seesaw is balanced, how far is CW from the center? (nearest inch)

(A) 4' 6" (B) 4' 8" (C) 4'10" (D) 5' 0" (E) 4' 9"

60. Six circles are tangent to each other and an equilateral triangle as shown. What is the probability that a dart landing randomly within the triangle will not land inside one of the circles? (nearest percent)

(A) 40% (B) 42% (C) 22% (D) 32% (E) 78%



2018-2019 TMSCA Mathematics Test Ten Answers

1. B	**21. D**	**41. C**
2. D	**22. E**	**42. E**
3. A	**23. B**	**43. A**
4. C	**24. D**	**44. E**
5. E	**25. B**	**45. E**
6. A	**26. C**	**46. A**
7. C	**27. D**	**47. B**
8. B	**28. D**	**48. D**
9. D	**29. D**	**49. A**
10. B	**30. C**	**50. B**
11. D	**31. B**	**51. C**
12. B	**32. A**	**52. D**
13. A	**33. E**	**53. E**
14. D	**34. C**	**54. D**
15. B	**35. A**	**55. D**
16. D	**36. B**	**56. C**
17. D	**37. D**	**57. A**
18. C	**38. C**	**58. D**
19. E	**39. C**	**59. B**
20. B	**40. A**	**60. C**

2018-2019 TMSCA Mathematics Test Ten Solutions

7. Use Euler's formula for the relationship $V + F - E = 2$ for 32 faces.

12. $\frac{32° + x°}{2} = 82°$ **for 132°**

13. $\log x = 888 \log 7 = 750.447$, **so** $x = 10^{0.447} \times 10^{750}$ **or** 2.79×10^{750} **and a leading digit 2.**

22. Treat the group of girls as one item, and the 6 boys as individuals. The number of circular arrangements of 7 items is $\frac{7!}{7} = 720$, **but the number of arrangements within the group of 3 girls is 3! or 6, so the total arrangements with the girls together is 4320. To find the denominator, the number of circular arrangements of all 9 students is** $\frac{9!}{9} = 40320$ **and a probability of** $\frac{4320}{40320} = \frac{3}{28}$.

27. Let x be the width of the enclosure, so maximize $A = x\left(\frac{120 - 3x}{2}\right)$ **for an area of 600 m².**

28. The two numbers given are comprised of 3 prime numbers, 7, 13 and 19. The 8 factors will be 1, 7, 13, 19, 91, 133, 247 and 1729, for the sum of the digits in 1729 equal to 19.

29. To get two different colors, it could be rr', gg' or bb' for

$$\frac{6}{15} \times \frac{9}{14} + \frac{4}{15} \times \frac{11}{14} + \frac{5}{15} \times \frac{10}{14} \approx 70\%$$

30. The area of the trapezoid above the x-axis minus the area of the triangle below the x-axis for

$$\left(\frac{6+3}{2}\right)(4) - \frac{1}{2}(3)(4) = 12$$

34. Solve the system $2x + y = 3$ **and** $x + 2y = 2$ **for** $x = \frac{4}{3}$ **and** $y = \frac{1}{3}$ **and a product of** $\frac{4}{9}$.

38. $\frac{6}{\frac{1}{75} + \frac{1}{105}} \approx 263$ **minutes or 4 hours 23 minutes.**

40. $x - y = 12$ **and**

$$x^3 - y^3 = (x - y)\left[(x - y)^2 + 3xy\right] = 12(144 + 54) = 2376$$

43. This is the product of

$$\left[(-1)(y)\right]\left[(1)(y - x)\right]\left[(-1)(y - 2x)\right] =$$

$$(y)(y - x)(y - 2x) = y\left(y^2 - 3xy + 2x^2\right) = y^3 - 3xy^2 + 2x^2 y$$

47. Use law of cosines $14^2 = 16^2 + x^2 - 2(16)(x)\cos 60°$ **which simplifies to** $0 = x^2 - 16x + 60$ **and a sum of the values** $-\frac{b}{a} = 8$.

48. There are 3 B's and 4 distinct letters.

No repeats: 4! = 24

2 B's: ${}_3C_2 \times \frac{4!}{2!} = 36$

3 B's: ${}_3C_1 \times \frac{4!}{3!} = 12$ **for a total of 72 4-letter arrangements.**

51. When the numerator and denominator are both factored, they both have a single factor of $(x - 7)$ **which leaves a removable discontinuity of** $x = 7$.

57. This is an infinite geometric series with a common ratio of $\frac{2}{5}$ **and an initial term** $\frac{1}{10}$ **and a sum** $\frac{1/10}{1 - 2/5} = \frac{1}{6}$.