Mathematics

Invitational A • 2022

1. In a recent game, Kevin made 4 out of 11 three-point shots, 6 out of 13 two-point shots, and 7 out of 8 free throws (one point each). How many points did he score?

   (A) 29   (B) 30   (C) 31   (D) 32   (E) 33

2. Tom selected his favorite number and doubled it, then he added 12, next he divided by 4, then he multiplied by 3, and finally he subtracted 11. If the result was 22, what is Tom's favorite number?

   (A) 16   (B) 20   (C) 24   (D) 28   (E) 32

3. Karen's Kennel will keep your dog when you are out of town. They charge a basic fee of $9.95 per day, plus $1.95 per meal for premium dog food, and $29.95 per hour to exercise your dog. Find the cost to leave a dog with Karen for two weeks if you want your dog to receive three meals each day and you want your dog to be exercised 45 minutes each day.

   (A) $534.57   (B) $535.68   (C) $536.79   (D) $537.90   (E) $539.01

4. Consider a line, $y = f(x)$, that contains the point $P(-3, 2)$ and is also perpendicular to the line shown. Which point lies on the line $y = f(x)$?

   (A) $(-4, 3)$
   (B) $(-2, 1)$
   (C) $(2, -5)$
   (D) $(4, -7)$
   (E) $(6, -10)$

5. Joe paid $5,000 down on a new RAV 4, which had a total cost of $25,700. The dealership is offering to finance the remainder of the cost, interest free, over 48 months. What will be the amount of each monthly payment?

   (A) $431.25   (B) $433.50   (C) $435.75   (D) $438.00   (E) $440.25

6. Lisa accidentally spilled a quart of water into her swimming pool. The pool is rectangular in shape and measures 12 ft by 20 ft. By how much did the water level in the pool increase?
   (nearest hundred thousandth)

   (A) 0.00167 in   (B) 0.00334 in   (C) 0.00668 in   (D) 0.0134 in   (E) 0.0267 in

7. During the 1970 season, Gary completed 62.5% of his passes. During the first quarter of a game against Monterey, he attempted 4 passes. Find the probability that he completed 3 of these passes.
   (nearest tenth of a percent)

   (A) 32.2%   (B) 34.4%   (C) 36.6%   (D) 38.8%   (E) 41.0%

8. If $h(x) = -x^2 + bx + c$, and has zeros at $x = 3$ and $x = 7$, then the maximum value of $h(x)$ is _____.

   (A) 3   (B) 3.25   (C) 3.5   (D) 3.75   (E) 4

9. The graph of the function $f(x) = \dfrac{6x+4}{2x+5}$ has asymptotes $x = d$ and $y = b$ and a zero at $x = c$.

   $d+b+c =$ _____.

   (A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{3}$ (C) $-\dfrac{1}{6}$ (D) $\dfrac{1}{6}$ (E) $\dfrac{1}{3}$

10. Mary has ten pictures of her grandma. She plans to choose six of these, frame them, and hang them on her wall. If she hangs these six in one row, how many different arrangements are possible?

    (A) 210   (B) 37,958   (C) 75,705   (D) 113,453   (E) 151,200

11. A sheet of graph paper consists of squares, each with an area of 1.00 cm². A small bug starts at point $A(5,7)$ and walks to point $B(-6,-1)$. From there the bug walks to point $C(3,-8)$. Find the total distance traveled by the bug. (nearest tenth)

    (A) 24.1 cm   (B) 24.4 cm   (C) 24.7 cm   (D) 25.0 cm   (E) 25.3 cm

12. Consider a line segment with endpoints $P(-5,7)$ and $Q(4,-9)$. Which of the following points lies on the perpendicular bisector of $\overline{PQ}$?

    (A) $\left(-6, -\dfrac{129}{32}\right)$ (B) $\left(-3, -\dfrac{75}{32}\right)$ (C) $\left(3, \dfrac{31}{32}\right)$ (D) $\left(6, \dfrac{87}{32}\right)$ (E) $\left(9, \dfrac{141}{32}\right)$

13. The circumference of the Earth at sea level along the equator is 4.0074×10^7 m. Cindy is at rest in a city at sea level and located at 40° north latitude. Find her linear speed due to the rotation of the Earth. (nearest whole number)

    (A) 335 m/s   (B) 340 m/s   (C) 345 m/s   (D) 350 m/s   (E) 355 m/s

14. Find the $m\angle CBD$ if $m\angle ECF = 65°$ and $m\angle CAB = 59°$.

    (A) 120°   (B) 122°   (C) 124°
    (D) 126°   (D) 128°

15. Consider $\triangle DEF$ with vertices $D(-3,8)$, $E(-1,2)$ and $F(10,6)$. If the coordinates of the centroid of $\triangle DEF$ are (a,b), then $a+b =$ _____.

    (A) $7\dfrac{1}{3}$ (B) $7\dfrac{2}{3}$ (C) 8   (D) $8\dfrac{1}{3}$ (E) $8\dfrac{2}{3}$

16. Consider rhombus ABCD with AB = 16 cm and $m\angle ABC = 60°$. If circle O has the same area as rhombus ABCD, find the circumference of circle O. (nearest whole number)

    (A) 49 cm   (B) 51 cm   (C) 53 cm   (D) 55 cm   (E) 57 cm

17. **Point $A(3,-4)$ is rotated 90° counterclockwise about the origin to point B. Point B is reflected across the y-axis to point C. Point C is translated vertically upward 4 units to point $D(a,b)$. $a+b=$ _____.**

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

18. **Consider $\triangle STR$ with point V on side $\overline{ST}$ and point U on side $\overline{SR}$. Line $\overleftrightarrow{VU}$ is parallel to line $\overleftrightarrow{TR}$. If $SV=6$, $VT=12$, and $VU=4$, then $TR=$ _____.**

(A) 6   (B) 8   (C) 9   (D) 12   (E) 15

19. **Consider $\triangle ABC$ with point D on $\overline{AC}$ such that $\overline{BD}\perp\overline{AC}$. If $AB=10$, $BC=17$, and $AD=8$, then the area of $\triangle ABC=$ _____. (nearest whole number)**

(A) 68   (B) 72   (C) 76   (D) 80   (E) 84

20. **Points A, B, C and D lie on circle O such that chord $\overline{AC}$ intersects chord $\overline{BD}$ at point E. If $AE=24$, $BE=16$, and $DE=36$, then $CE=$ _____. (nearest tenth)**

(A) 22.8   (B) 23.2   (C) 23.6   (D) 24.0   (E) 24.4

21. **Find the area of a triangle with vertices $A(2,8)$, $B(-6,-4)$, and $C(6,-2)$.**

(A) 56   (B) 58   (C) 60   (D) 62   (E) 64

22.

x	-3	-2	-1	1	2	3
f(x)	2	8	6	2	12	38

Use the table to find the value of $f(-4)$.

(A) −20   (B) −18   (C) −16   (D) −14   (E) −12

23. **The relative intensity of a sound, β in dB, is calculated using the formula $\beta=10\log\left(\dfrac{I}{10^{-12}}\right)$ where $I=$ the intensity of a sound in W/m². If your eardrums will burst when the relative intensity of a sound reaches 160 dB, what is the intensity of such a sound?**

(A) 0.0001 W/m²   (B) 0.01 W/m²   (C) 1 W/m²   (D) 100 W/m²   (E) 10,000 W/m²

24. **If $f(x)=2x^3+6x^2+kx-10$ and $f(-3)=-22$, then $f(1)=$ _____.**

(A) 2   (B) 4   (C) 6   (D) 8   (E) 10

25. **If $s(x)$ is the slant asymptote of the function $f(x)=\dfrac{x^2+x+6}{x-1}$, then $s(6)=$ _____.**

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

26. Consider the orbit of Mars around the Sun. The distance from Mars to the Sun at perihelion is 206.617×10^6 km and the distance at aphelion is 249.229×10^6 km. Find the eccentricity of the orbit of Mars. (nearest hundred-thousandth)

(A) 0.0935   (B) 0.1843   (C) 0.2751   (D) 0.3659   (E) 0.4567

27. A superball is dropped from a height of 12 feet. Each time it hits the floor, it bounces $\frac{11}{12}$ of its previous height. Find the total distance traveled by the ball.

(A) 242 ft   (B) 254 ft   (C) 264 ft   (D) 276 ft   (E) 288 ft

28. Simplify the expression. $\dfrac{1 + \csc(x)}{\cos(x) + \cot(x)}$

(A) $\sec(x)$ (B) $\csc(x)$ (C) $\sin(x)$ (D) $\tan(x)$ (E) $1 - \csc(x)$

29. Consider the formula $y = y_0 + v_0 t + \frac{1}{2}at^2$ for the height of a projectile at time t, t in seconds. If $a = -32.174$ ft/s^2 and the projectile was launched straight up from the ground at 200 ft/s, how high will it go? (nearest whole number)

(A) 618 ft   (B) 622 ft   (C) 626 ft   (D) 630 ft   (E) 634 ft

30. Cobalt 57 has a half life of 271.8 days. If Karen has 12 g today, how many days from now will it be until she has only 2 g? (nearest tenth)

(A) 702.6   (B) 704.7   (C) 706.8   (D) 708.9   (E) 711.0

31. An airplane is flying horizontally and it is at an altitude of 5.46 miles when it passes over an observer on the ground. At the instant that the angle of elevation from the observer to the plane is $33°\,33'\,33''$, what is the distance from the observer to the plane? (nearest foot)

(A) 52,144 ft   (B) 52,151 ft   (C) 52,158 ft   (D) 52,165 ft   (E) 52,172 ft

32. Which of the following is one of the four fourth roots of the complex number $81i$? (answers are in standard form and have been rounded to the nearest hundredth)

(A) $-2.61 - 0.99i$ (B) $-2.65 - 1.03i$ (C) $-2.69 - 1.07i$ (D) $-2.73 - 1.11i$ (E) $-2.77 - 1.15i$

33. At the NCTC mathematics contest, the top 10 students received cash prizes. First place received $350, second place received $315, third place received $280, fourth placed received $245, and so on. What was the total amount of prize money awarded?

(A) $1910   (B) $1915   (C) $1920   (D) $1925   (E) $1930

34. If the focus of the parabola $y^2 + 6y + 4x + 21 = 0$ is (a, b), then $a + b =$ _____.

   (A) -7 (B) -6 (C) -5 (D) -4 (E) -3

35. Find the rectangular coordinates of the point given in polar coordinates. $\left(6, \dfrac{13\pi}{9}\right)$

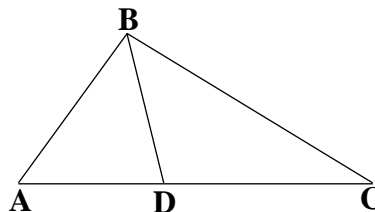 (all answers are rounded to the nearest hundredth)

   (A) $(-0.96, -5.83)$ (B) $(-1.00, -5.87)$ (C) $(-1.04, -5.91)$ (D) $(-1.08, -5.95)$ (E) $(-1.12, -5.99)$

36. Find the distance from the point $(1, 2, 3)$ to the plane $x + y - 2z = 6$. (nearest hundredth)

   (A) 3.45   (B) 3.67   (C) 3.89   (D) 4.11   (E) 4.33

37. $m\angle ABD = m\angle CBD,\ AB = 10,\ BC = 16,\ DC = 10$

    Find AD.

   (A) 6
   (B) 6.25
   (C) 6.5
   (D) 6.75
   (E) 7

38. Find the number that is $\dfrac{3}{8}$ of the way from $-6\dfrac{4}{7}$ to $12\dfrac{2}{7}$.

   (A) $\dfrac{1}{2}$ (B) $\dfrac{4}{7}$ (C) $\dfrac{5}{7}$ (D) $\dfrac{3}{4}$ (E) $\dfrac{6}{7}$

39. Evaluate: $\displaystyle\lim_{x \to \left(\frac{\pi}{2}\right)^+} (\tan x)$

   (A) -1 (B) 1 (C) $-\infty$ (D) $+\infty$ (E) 0

40. Consider the function $f(x) = \dfrac{12x}{x^2 + 4}$. Find the x-intercept of the line tangent to $f(x)$ when $x = 1$.

   (A) -1 (B) $-\dfrac{3}{4}$ (C) $-\dfrac{2}{3}$ (D) $-\dfrac{1}{2}$ (E) $-\dfrac{1}{3}$

41. Consider the function $f(x) = -x^4 + 2x^2$. Which of the values in the interval $(-3, 2)$ satisfy the mean value theorem for $f(x)$ on $[-3, 2]$? (nearest thousandth)

   (A) -1.979 (B) -1.865 (C) -1.751 (D) -1.637 (E) -1.523

42. Given: $\displaystyle\int_3^8 f(x)dx = 12$ and $\displaystyle\int_3^8 g(x)dx = -4$. Evaluate $\displaystyle\int_3^8 \left[3f(x) - 2g(x)\right]dx$.

   (A) 16   (B) 23   (C) 30   (D) 37   (E) 44

43. Consider the graph of $f(x) = e^{2x}\ln(x)$. Find the y-intercept of the line tangent to $f(x)$ at the point (a, b) if $b = 2$. (nearest tenth)

   (A) −14.7   (B) −14.4   (C) −14.1   (D) −13.8   (E) −13.5

44. The rate of change of the number of bunnies $B(t)$ in a population of bunnies is directly proportion to $1000 - B(t)$, where t is the time in years. At t = 0, the population is 100. When t = 3, the population has increased to 300. Find the population when t = 4.

   (A) 356   (B) 367   (C) 378   (D) 389   (E) 400

45. Find the volume of the solid generated when the region bounded by the graphs of $y = \sqrt{3x}$ and $y = .6x^2$ is revolved around the line $x = -3$. (nearest whole number)

   (A) 29   (B) 32   (C) 35   (D) 38   (E) 41

46. Find the third degree Maclaurin polynomial for $f(x) = \sin(x)$. Then calculate the magnitude of the exact error when evaluating $f(0.8)$ using this polynomial. (nearest hundred-thousandth)

   (A) 0.00203   (B) 0.00236   (C) 0.00269   (D) 0.00302   (E) 0.00335

47. When analyzing home prices in Denton, these are considered resistant to extreme values.
           I. mean   II. median   III. standard deviation   IV. IQR

   (A) I, II   (B) I, III   (C) III, IV   (D) II, IV   (E) I, II, III, IV

48. A large sample of students at Newberg High School took an IQ test and it was first reported that the mean score was 100 and the standard deviation was 15. However, an error was found in the scoring and the corrected IQ score is found by taking each individual score and adding 2 and then multiplying by 1.12. Find the mean and standard deviation of the corrected IQ scores.

   (A) 102, 17   (B) 114.24, 15   (C) 114.24, 16.8   (D) 114.24, 17   (E) 114.24, 19.04

Cookies	0	2	6	12	20	30
Score	284	276	262	232	208	164

49. The number sense team at Sabine High School performed an experiment. Each person ate a specified number of chocolate chip cookies and then they took a practice test. When examining the data, it appeared that the test scores decreased linearly. The table above shows the results of one of the students. Use the data in the table to create an appropriate model and predict this student's score if she eats 27 cookies.

   (A) 173   (B) 175   (C) 177   (D) 179   (E) 181

50. The basic principle/principles of designing an experiment include which of the following?
 I. Control   II. Random Assignment   III. Replication

(A) I only   (B) II only   (C) I, II only   (D) II, III only   (E) I, II, III

Event	Probability
Runs over 40 miles per week	0.36
Runs over 40 miles per week and gets injured	0.12
Does not run over 40 miles per week and does not get injured	0.57

51. An adult male is randomly selected from a large group of runners. Use the information above to find the probability that the individual selected gets injured given that he runs over 40 miles per week.

(A) 0.09   (B) 0.21   (C) $0.\overline{3}$ (D) 0.43   (E) 0.48

52. Each child of a particular pair of parents has probability 0.36 of being gifted in statistics. If the parents have 6 children, find the probability that exactly 4 of their children are gifted in statistics. (nearest thousandth)

(A) 0.103   (B) 0.127   (C) 0.151   (D) 0.175   (E) 0.199

53. If you reject the null hypothesis when the null hypothesis is true, you have committed a Type _____ error.

(A) I   (B) II   (C) III   (D) IV   (E) V

54. $CBA2_{15} - ABC8_{15} = $ _____ $_{15}$

(A) 1EC9   (B) 1ECD   (C) 1ED3   (D) 1EDA   (E) 1EE1

55. Find the sum of the sequence.   9, 8, 17, 25, 42, 67, …, 461, 746

(A) 1935   (B) 1945   (C) 1955   (D) 1965   (E) 1975

56. Jerry's Ice Cream Shoppe in Highland Park has chocolate, vanilla, strawberry, butter pecan, peppermint, licorice, and chocolate mint flavors of ice cream. How many different ways can Mr. Speir purchase a six-scoop bowl of ice cream?

(A) 384   (B) 924   (C) 3003   (D) 84,400   (E) 117,649

57. According to Wikipedia, this English mathematician "is widely considered to be the father of theoretical computer science and artificial intelligence…. For a time, he led Hut 8, the section that was responsible for German naval cryptanalysis."

(A) Charles Babbage   (B) Jon von Neumann   (C) Tommy Flowers
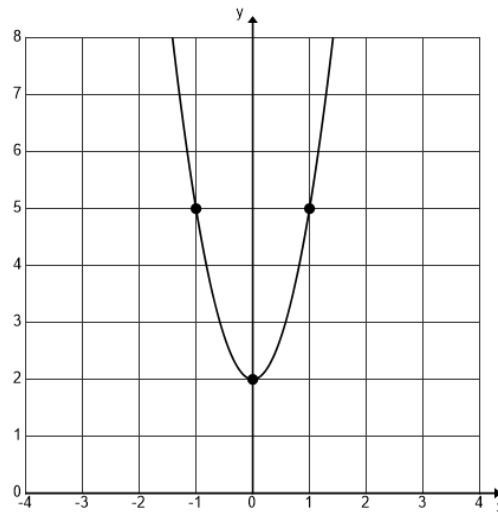   (D) Alan Turing   (E) Christian Goldbach

58. The number 44 is considered to be which of the following types of numbers?
  I. Happy   II. Polite   III. Odious   IV. Extravagant

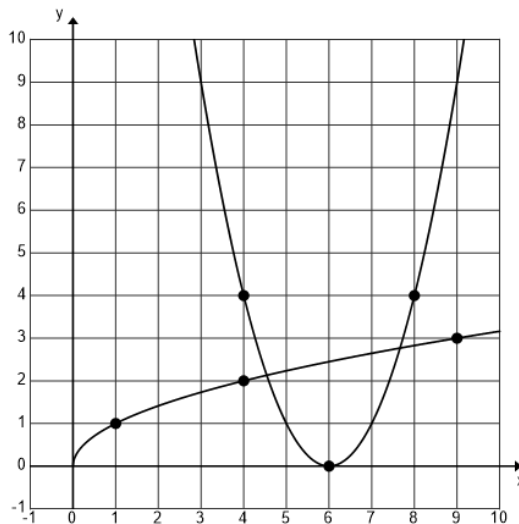 (A) I, II only   (B) I, III only   (C) I, II, IV only   (D) I, III, IV only   (E) I, II, III, IV

59. The graph of $g'(x)$ is shown
  on the right. If $g(-1) = -2$,
  then $g(2) = $ _____.

  (A) 12
  (B) 13
  (C) 14
  (D) 15
  (E) 16

60. Find the area bounded by
  the two curves shown on
  the right. (nearest tenth)

  (A) 4.3
  (B) 4.5
  (C) 4.7
  (D) 4.9
  (E) 5.1

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2022
Answer Key

1. C	21. E	41. D
2. A	22. B	42. E
3. B	23. E	43. D
4. E	24. A	44. A
5. A	25. E	45. E
6. A	26. A	46. C
7. C	27. D	47. D
8. E	28. A	48. C
9. C	29. B	49. C
10. E	30. A	50. E
11. D	31. B	51. C
12. C	32. E	52. A
13. E	33. D	53. A
14. C	34. A	54. A
15. A	35. C	55. B
16. C	36. B	56. B
17. C	37. B	57. D
18. D	38. A	58. E
19. B	39. C	59. B
20. D	40. C	60. E

Mathematics

Invitational B • 2022

1. Solve for w if $\sqrt{3w+3} - 6 = 0$.

   (A) 9   (B) 10   (C) 11   (D) 12   (E) 13

2. Rickie's Roller Palace charges $6.00 per session. They rent skates for $4.00 and knee pads for $5.00. Ted took Martha skating Friday night. They both needed skates and Ted also rented knee pads for himself, as he is a bit clumsy. If the tax rate is 8.25%, what was the total cost?

   (A) $27.06   (B) $27.20   (C) $27.34   (D) $27.48   (E) $27.62

3. Line L_1 contains the points $(-5, 6)$ and $(3, -2)$. Line L_2 is parallel to L_1 and contains the point $(-2, -4)$. Which of the following points lies on L_2 ?

   (A) $(-8, 3)$ (B) $(-4, -1)$ (C) $(2, -6)$ (D) $(4, -9)$ (E) $(8, -14)$

4. Danny the deer has a top speed of 30 mph and Charlie the cougar has a top speed of 45 mph. Charlie was 150 feet west of Danny when Danny spotted Charlie sneaking up. They both took off at top speed heading east. How long will it take Charlie to catch Danny? (nearest tenth)

   (A) 6.6 sec   (B) 6.8 sec   (C) 7.0 sec   (D) 7.2 sec   (E) 7.4 sec

5. The Tigard City Park has a jogging path along its perimeter. The rectangular shaped park measures 0.45 mile by 0.75 mile when measuring inside the jogging path. If the path is two yards wide, even at the corners, find the total area of the jogging path. (nearest square foot)

   (A) 76,010 ft^2
   (B) 76,055 ft^2
   (C) 76,100 ft^2
   (D) 76,145 ft^2
   (E) 76,190 ft^2
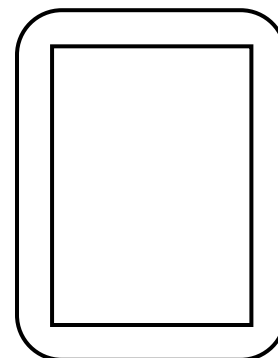
   Problem 5

6. The number of students attending LWU near Eden was 1014 in 1998. Since then, enrollment has increased by 4% each year. The projected enrollment in 2024 is _____ students.

   (A) 2802   (B) 2811   (C) 2820   (D) 2829   (E) 2838

7. Sarah's Gym in Seymour charges members $10.00 per month, plus $1.50 every time a member comes in to exercise. They also charge $1.00 if you need a towel. In a typical month, Katie exercises 12 times and she needs a towel six times. What is Katie's annual cost to exercise at Sarah's gym?

   (A) $408.00   (B) $412.00   (C) $416.00   (D) $420.00   (E) $424.00

8. If $\dfrac{x+4}{x^2-9x+20} - \dfrac{x-5}{x^2-16} = \dfrac{ax^2+bx+c}{dx^4+ex^3+fx^2+gx+h}$, then $\dfrac{a+b+c}{d+e+f+g+h} =$ _____.

   (A) 0.09   (B) 0.12   (C) 0.15   (D) 0.18   (E) 0.21

9. Jake starts at the Square in Jackson and walks 4 blocks north. Then he turns and heads 6 blocks east. Next, he turns and walks 12 blocks south and stops for lunch at The Mighty Moose Pizzeria. If each block in Jackson is one-eighth of a mile long, how far is The Mighty Moose from the Square?

(A) 6400 ft   (B) 6450 ft   (C) 6500 ft   (D) 6550 ft   (E) 6600 ft

10. Find the area of the triangle on the right.

(A) 58
(B) 60
(C) 62
(D) 64
(E) 66
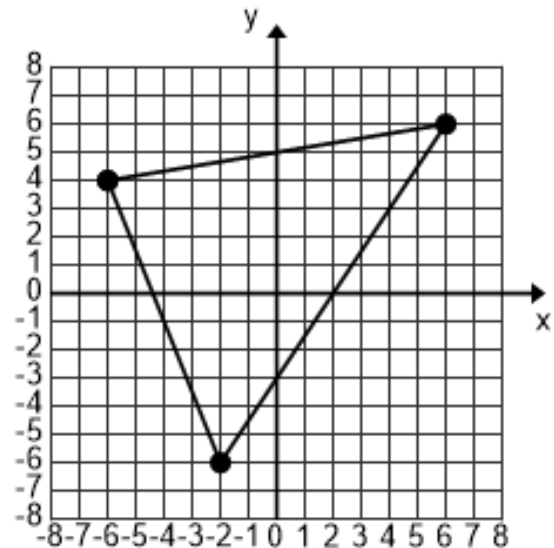
11. Find the perimeter of the triangle on the right.
(nearest tenth)

(A) 37.1
(B) 37.4
(C) 37.7
(D) 38.0
(E) 38.3

Problems 10, 11

12. Consider the conditional statement "If Vidit has a good day, then he breaks 300." Also, consider the statement "If Vidit does not break 300, then he does not have a good day." This is called the _____ of the conditional statement.

(A) Inverse   (B) Contrapositive   (C) Converse   (D) Transpose   (E) Antithesis

13. Consider triangle ABC with $m\angle A = 13x$, $m\angle B = 5x$, and $m\angle C = 4x + 4$. Find the value of $m\angle A - m\angle C$.
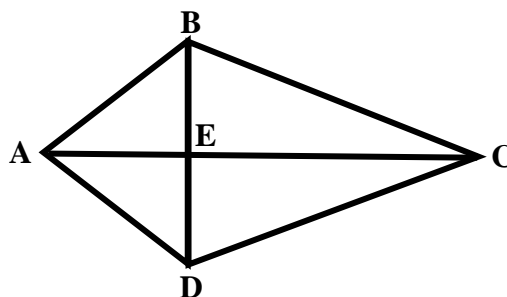
(A) 64°   (B) 66°   (C) 68°   (D) 70°   (E) 72°

14. Consider acute triangle DEF with point G on $\overline{DE}$ such that G is the midpoint of $\overline{DE}$. If point P is the centroid of the triangle and PG = 12, then FG = _____.

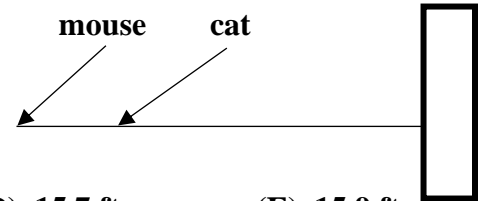(A) 18   (B) 24   (C) 30   (D) 36   (E) 42

15. Kite ABCD has an area of 120 cm².
If BE = 6 cm, then AC = _____ cm.

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26

Problem 15

16. The angle of depression from the roof to the cat is 20°. The angle of depression from the roof to the mouse is 18°. If the distance from the cat to the building is 132 ft, find the distance from the mouse to the cat. (nearest tenth)

mouse   cat

(A) 15.1 ft   (B) 15.3 ft   (C) 15.5 ft   (D) 15.7 ft   (E) 15.9 ft

17. By road, it is 670 miles from Bonner's Ferry to Preston. Lura leaves Preston at 8:00 AM and begins driving at an average speed of 64 mph towards Bonner's Ferry. Phil leaves Bonner's Ferry at 10:00 AM and begins driving towards Preston at an average speed of 60 mph. How far from Preston are they when they meet? (nearest mile)

(A) 400 mi   (B) 404 mi   (C) 408 mi   (D) 412 mi   (E) 416 mi

18. Which point lies on the curve shown on the right?

(A) $(11, 16)$
(B) $(14, 20)$
(C) $(17, 25)$
(D) $(20, 33)$
(E) $(23, 41)$
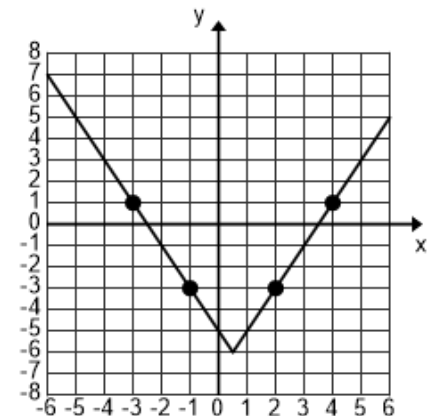
Problem 18

19. The diagonal of a television screen measures 51 inches. The width of the rectangularly shaped television screen is 21 inches greater than the height. Find the area of the television screen.

(A) 1072 in^2 (B) 1080 in^2 (C) 1088 in^2 (D) 1096 in^2 (E) 1104 in^2

20. Consider the parent function $f(x) = x^2$. If the graph of $y = f(x)$ is reflected across the x-axis, shifted horizontally 6 units left, and then shifted vertically 4 units down, the point $(-1, b)$ lies on the graph. Find the value of b.

(A) −29   (B) −28   (C) −27   (D) −26   (E) −24

21. Margarita deposits $10,000 into an account that earns 6.25% annual interest compounded quarterly. Juan deposits $12,000 into an account that earns 3.75% annual interest compounded monthly. How many years are required for the balance in Margarita's account to equal the balance in Juan's account? (nearest tenth)

(A) 6.6   (B) 6.8   (C) 7.0   (D) 7.2   (E) 7.4

22. The graph of $f(x) = \dfrac{4x^2}{x^2 - 16}$ has _____ asymptotes.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

UIL Mathematics

23. The graph of the circle $x^2 + y^2 = 25$ and the graph of the line $y = .8x + 2$ intersect at points A and B. Find the length of $\overline{AB}$. (nearest tenth)

    (A) 8.7   (B) 8.9   (C) 9.1   (D) 9.3   (E) 9.5

24. Let $g(x)$ be the inverse function of $f(x) = 5x\sqrt{x}$. Find the smallest positive integer value of x such that $g(x) > 2$.

    (A) 13   (B) 14   (C) 15   (D) 16   (E) 17

25. Let w and m be the complex solutions to the equation $3x^2 - 2x + 15 = 0$. If $w^2 - m^2 = \dfrac{a\sqrt{b}}{c}i$ where a, b and c are positive integers then the value of a + b + c = _____.

    (A) 24   (B) 26   (C) 28   (D) 30   (E) 32

26. Find x if $4\left(3^{2x-1}\right) - 5 = 6$.

    (A) $\dfrac{1}{2} + \dfrac{2\ln\left(\dfrac{11}{4}\right)}{\ln(9)}$ (B) $\dfrac{\ln\left(\dfrac{33}{4}\right)}{\ln(3)}$ (C) $\dfrac{1}{2} + \dfrac{\ln\left(\dfrac{11}{4}\right)}{\ln(3)}$ (D) $\dfrac{\ln\left(\dfrac{33}{4}\right)}{\ln(6)}$ (E) $\dfrac{1}{2} + \dfrac{\ln\left(\dfrac{11}{4}\right)}{\ln(9)}$

27. The graph of $y = \dfrac{1}{4}\tan(3x)$ has a vertical asymptote at x = _____.

    (A) $\dfrac{3\pi}{4}$ (B) $\dfrac{4\pi}{3}$ (C) 2π (D) $\dfrac{13\pi}{6}$ (E) $\dfrac{29\pi}{12}$

28. The vector $u = \langle 6, -4 \rangle$ is orthogonal to the vector $v = $ _____.

    (A) $\langle 2, 3 \rangle$ (B) $\langle -2, 3 \rangle$ (C) $\langle 2, -3 \rangle$ (D) $\langle 3, -2 \rangle$ (E) $\langle -4, 6 \rangle$

29. Consider the system of inequalities $\begin{array}{c} x^2 - y \le 4 \\ y - x \le 6 \end{array}$ Which of the following points does not satisfy the system?

    (A) $(-2.6, 3.3)$ (B) $(-1.9, -0.3)$ (C) $(1.1, -2.7)$ (D) $(2.2, 0.8)$ (E) $(3.6, 9.6)$

30. Phillip Physics was finding the current (in amps) in different parts of a circuit. He came up with the following system: $6 - 12I_1 - 24I_2 = 0$, $6 - 12I_1 - 6I_3 - 3 = 0$, $I_1 = I_2 + I_3$ Find I_3. (nearest hundredth)

    (A) 0.07 A   (B) 0.11 A   (C) 0.14 A   (D) 0.18 A   (E) 0.21 A

UIL Mathematics

31. **Assume the license plates in a small country must consist of 4 letters followed by 3 digits. How many distinct license plates can be formed in this country?**

    (A) 11,232,000   (B) 122,668,000   (C) 234,104,000   (D) 345,540,000   (E) 456,976,000

32. **Consider the graph of $x^2 - 4y^2 - 4x - 24y - 33 = 0$. The vertices of the graph are (a, b) and (c, b). If $a < c$, then a = _____.**

    (A) 1   (B) 1.25   (C) 1.5   (D) 1.75   (E) 2

33. **Find the angle between the vectors $u = \langle 2, -3, 4 \rangle$ and $v = \langle -1, -2, 5 \rangle$. (nearest tenth)**

    (A) 35.2°   (B) 35.5°   (C) 35.8°   (D) 36.1°   (E) 36.4°

34. **Juanita is flying a kite. The kite string has a length of 333 ft and the angle of elevation from Juanita to the kite is 58.6°. If the end of the string Juanita is holding is 4 ft above the ground, how high above the ground is the kite? (nearest foot)**

    (A) 282 ft   (B) 284 ft   (C) 286 ft   (D) 288 ft   (E) 300 ft

35. **Simplify:** $\dfrac{a}{b + \dfrac{c}{d + \dfrac{e}{f}}}$

    (A) $\dfrac{af + ae}{bdf + be + cf}$ (B) $\dfrac{adf + ae}{bd + be + cf}$ (C) $\dfrac{ad + aef}{bdf + be + cf}$ (D) $\dfrac{ad + ae}{bf + be + cf}$ (E) $\dfrac{adf + ae}{bdf + be + cf}$

36. **Five physics books and six chemistry books are on a shelf. How many ways can they be arranged if the physics books are kept together and the chemistry books are kept together?**

    (A) 86,400   (B) 129,600   (C) 172,800   (D) 216,000   (E) 259,200

37. **Simplify: $39^{\log_{39}(7)}$**

    (A) 0   (B) 1   (C) 7   (D) 39   (E) 49

38. **Twelve men can do 20 jobs in five days. How many days would it take forty men to do 180 jobs?**

    (A) 12   (B) 13.5   (C) 15   (D) 16.5   (E) 18

39. **Find the slope of the line tangent to the graph of $f(x) = x + \dfrac{6}{x}$ at the point $(6, 7)$.**

    (A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{5}{6}$ (D) 1   (E) $\dfrac{7}{6}$

40. Consider the graph of $f(x) = 5x^5 - 3x^3$. Find the sum of the y-values of the points of inflection. (nearest hundredth)

    (A) −0.32　　　(B) −0.16　　　(C) 0.00　　　(D) 0.16　　　(E) 0.32

41. Use four rectangles of equal widths to find an approximation of the area of the first quadrant region bounded by the curves $y_1 = 4 - x^2$, $y_2 = 0$, and $x = 0$. Use left endpoints of each interval to find an upper sum.

    (A) 5.5　　　(B) 5.75　　　(C) 6　　　(D) 6.25　　　(E) 6.5

42. A group of 100 mice was accidentally released on a small island west of the state of Washington on March 1, 2018. A University of Washington professor estimates that the island can support no more than 5,000 mice. On March 1, 2020, the mice population had reached 1,200. The professor used a logistic model to predict the population on March 1, 2025. He predicted _____ mice.

    (A) 4979　　　(B) 4981　　　(C) 4983　　　(D) 4985　　　(E) 4987

43. When evaluating $\int \left(x^2 \ln(x) \right) dx$ using the method of integration by parts, the best choice for dv is _____.

    (A) $x\,dx$　　　(B) x^2　　　(C) $x^2\,dx$　　　(D) $\ln(x)$　　　(E) $x^3\,dx$

44. Let $\sum a_n$ be a series with nonzero terms. The Ratio Test for determining the convergence or divergence of the series fails when $\lim\limits_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| =$ _____.

    (A) 0　　　(B) 0.5　　　(C) 1　　　(D) 1.5　　　(E) 2

45. Find the area of the region bounded by the curves $y_1 = 8 - 2x$ and $y_2 = \dfrac{1}{x}$. (nearest tenth)

    (A) 10.8　　　(B) 11.0　　　(C) 11.2　　　(D) 11.4　　　(E) 11.6

46. A particle was traveling along the x-axis. It was accelerating to the right at a constant 2.44 m/s². At t = 0, the position of the particle was at x = 3.00 m and the velocity of the particle was −5.66 m/s. The position of the particle at t = 4.22 s is at x = _____ m. (nearest hundredth)

    (A) 0.62　　　(B) 0.84　　　(C) 1.06　　　(D) 1.28　　　(E) 1.30

47. A rectangular sheet of cardboard is to be used to make an open top box by cutting out equal sized squares from each corner. If the dimensions of the cardboard sheet are 36 in by 48 in, then the maximum volume possible of the open top box is _____ in³. (nearest whole number)

    (A) 5240　　　(B) 5244　　　(C) 5248　　　(D) 5252　　　(E) 5256

48. A woman is 5 ft 6 in tall and walks at a rate of 6 ft/s away from a streetlight that is 18 ft above the ground. When she is 20 ft from the base of the light, at what rate is the length of her shadow increasing? (nearest hundredth)

    18 ft

    5 ft 6 in

    Problem 48

    (A) 2.64 ft/s
    (B) 2.75 ft/s
    (C) 2.86 ft/s
    (D) 2.97 ft/s
    (E) 3.03 ft/s
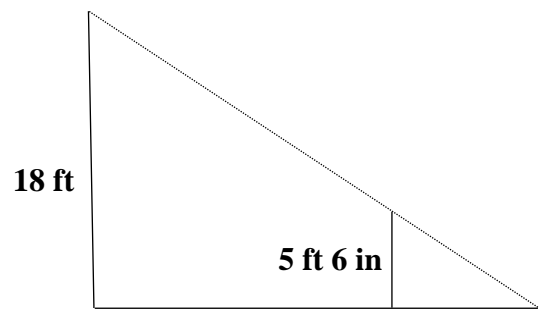
49. Northern Arizona won the NCAA DI men's cross-country race held on March 15, 2021. The 10-K times of their top five runners were 29:58, 30:02, 30:05, 30:10 and 30:50. Coach Smith analyzed the times and found that the difference between the mean and median was _____. (nearest tenth)

    (A) 7.1 sec   (B) 7.4 sec   (C) 7.7 sec   (D) 8.0 sec   (E) 8.3 sec

50. In a normal distribution with mean μ and standard deviation σ, _____% of the observations fall within 2σ of μ. (nearest whole number)

    (A) 91   (B) 93   (C) 95   (D) 97   (E) 99

51. Maria received some data for 100 girls attending Chavez Middle School. She entered the data into a computer and the statistical software on the computer generated a Least Squares Regression Line with a correlation of r = 0.825. The measurements of the girls' heights and weights were made in centimeters and pounds. Her teacher told her to convert the heights to inches and recompute the LSRL. What effect did this have on the value of the correlation, r?

    (A) r decreased by a factor of $\dfrac{2.54}{100}$ (B) r decreased by a factor of $\dfrac{2.54}{\sqrt{100}}$ (C) no effect

    (D) r increased by a factor of $\dfrac{2.54}{100}$ (E) r increased by a factor of $\dfrac{2.54}{\sqrt{100}}$

AP courses	0	1	2	3	4
Probability	.02	.14	.28	.32	.24

52. Students at the Texline Young Men's Academy are encouraged to take AP courses their senior year. They are not allowed to take more than four AP courses. Use the table above to find the expected number of AP courses a student at TYMA will take his senior year.

    (A) 2.62   (B) 2.64   (C) 2.66   (D) 2.68   (E) 2.70

53. Mr. Stat Teacher brought a huge jar containing 5000 marbles to class. Each marble was either red or blue. Michael took an SRS of 200 marbles from the jar. He separated the marbles into two bowls and ended up with 112 blue marbles in one bowl and 88 red marbles in the other bowl. Construct a 95% confidence interval for p, the proportion of blue marbles in the jar. (nearest ten-thousandth)

    (A) (.4835, .6365)   (B) (.4912, .6288)   (C) (.5066, .6134)   (D) (.5176, .6024)   (E) (.5312, .5888)

54. Coach uses one of three players to bat lead-off in games. Joe is selected to lead off 20% of the time and he leads off with a hit 30% of the time. Bob is selected to lead off 30% of the time and he leads off with a hit 20% of the time. Mike is selected to lead off 50% of the time and leads off with a hit 40% of the time. Find the probability that the lead-off hitter for this team leads off with a hit. (nearest hundredth)

   (A) 0.28   (B) 0.30   (C) 0.32   (D) 0.34   (E) 0.36

55. Suppose the distribution of the length of drives by Golfer George in his long career is approximately normal with a mean of 315 yards and a standard deviation of 15 yards. What percent of his drives are between 330 yards and 345 yards? (nearest tenth)

   (A) 12.5%   (B) 13.6%   (C) 14.7%   (D) 15.8%   (E) 16.9%

56. This mathematician is credited for developing differential and integral calculus independently of Isaac Newton.

   (A) George Boole   (B) John Napier   (C) John Venn   (D) Gottfried Leibniz   (E) Alicia Stott
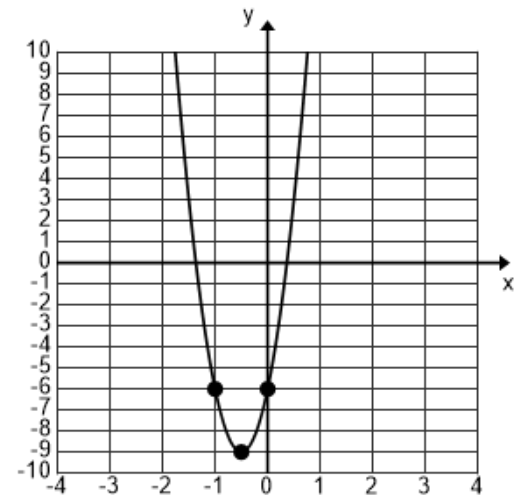
57. Which of the following numbers are classified as "perfect" numbers?
   I. 6   II. 28   III. 496   IV. 8,132

   (A) I, II only   (B) I, III only   (C) II, III only   (D) I, II, II only   (E) I, II, III, IV

58. Find the focus of the parabola shown on the right.

   (A) $\left(-\dfrac{1}{2},-8\right)$

   (B) $\left(-\dfrac{1}{2},-8\dfrac{3}{4}\right)$

   (C) $\left(-\dfrac{1}{2},-8\dfrac{11}{12}\right)$

   (D) $\left(-\dfrac{1}{2},-8\dfrac{23}{24}\right)$

   (E) $\left(-\dfrac{1}{2},-8\dfrac{47}{48}\right)$

Problems 58, 59

59. Suppose the graph on the right is the graph of $f''(x)$. If $f'(1) = 4$ and $f(-1) = 0$, then $f(-2) = $ _____.

   (A) −14   (B) −12   (C) −10   (D) −8   (E) −6

60. $ABC6_{14} + ABC6_{15} = $ _____ $_{16}$

   (A) 106C5   (B) 105B5   (C) 104A5   (D) 10395   (E) 10285

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2022
Answer Key

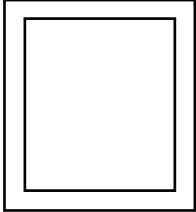|		
1. C	21. E	41. D
2. A	22. D	42. C
3. E	23. E	43. C
4. B	24. C	44. C
5. D	25. C	45. E
6. B	26. E	46. B
7. A	27. D	47. A
8. C	28. A	48. A
9. E	29. D	49. D
10. D	30. A	50. C
11. B	31. E	51. C
12. B	32. A	52. A
13. C	33. B	53. B
14. D	34. D	54. C
15. B	35. E	55. B
16. E	36. C	56. D
17. C	37. C	57. D
18. D	38. B	58. E
19. B	39. C	59. D
20. A	40. C	60. E

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2022

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Solve for k if $12|8k - 27| - 20 = 16$ and $k > 3$.

   (A) 3.25   (B) 3.5   (C) 3.75   (D) 4   (E) 4.25

2. Teresa received a $125 Barnes and Noble gift card for her birthday. She used it to purchase two paperback novels for $11.95 each, one hardback travel book for $39.75, and two AP Calculus prep books for $23.65 each. If the tax rate was 8.25%, how much remained on her gift card? (nearest cent)

   (A) $4.90   (B) $5.02   (C) $5.14   (D) $5.26   (E) $5.38

3. Lisa made a 256 on Test A, 264 on Test B, 242 on Test C, 272 on Test D, and 248 on Test E. What score will she need to make on Test F to obtain an overall average of exactly 258?

   (A) 260   (B) 262   (C) 264   (D) 266   (E) 268

4. Jim earns $12.75 per hour for the first forty hours each week at his job at Oscar's Oil Change and Lube. He earns $19.15 for any hours more than forty, but not exceeding fifty. He earns $25.50 for any hours more than fifty. If Jim worked 62 hours last week, how much did he earn?

   (A) $1002.50   (B) $1003.75   (C) $1005.00   (D) $1006.25   (E) $1007.50

5. The Mendoza Construction Company is repaving a 16-mile stretch of highway between American Falls and Aberdeen. On a previous job, 22 workers repaved an 8 mile stretch of highway in 28 days. If 18 workers are used, how long will it take to repave the 16-mile stretch of highway? (nearest tenth)

   (A) 67.5 days   (B) 67.8 days   (C) 68.1 days   (D) 68.4 days   (E) 68.7 days

6. Farmer Jones has a small farm with hogs, dogs and cows. The number of hogs exceeds four times the number of dogs by four. Seven times the number of dogs is twenty-eight less than four times the number of cows. Four times the sum of the number of dogs and cows is twelve greater than three times the number of hogs. How many animals are on Farmer Jones farm?

   (A) 34   (B) 36   (C) 38   (D) 40   (E) 42

7. Which ordered pair is a solution of the system? $y > -x - 3$ and $y \leq 2x + 4$

   (A) $(8, -11)$ (B) $(4, -8)$ (C) $(-2, -1)$ (D) $(4, 13)$ (E) $(8, 20)$

8. The picture frame on the right is for an 8 in by 10 in picture. The width of the frame is the same on every side. If the total area of the frame, not including the picture, is 101.25 square inches, what is the width of the frame? (nearest hundredth)

   (A) 2.05 in   (B) 2.15 in   (C) 2.25 in   (D) 2.35 in   (E) 2.45 in

9. Bob's Burger Shack offers single and double meat burgers with the following optional condiments: mustard, ketchup, pickles, tomatoes, onions. How many different ways can I order a burger?

   (A) 32   (B) 48   (C) 64   (D) 72   (E) 128

UIL Mathematics

10. Simplify: $\dfrac{\dfrac{a^2}{b^2}+\dfrac{9c}{d}}{\dfrac{e^2}{bd}-\dfrac{5}{b^2}}$

(A) $\dfrac{a^2d+9bc}{be^2-5d}$ (B) $\dfrac{ad+9b^2c}{be^2-5d}$ (C) $\dfrac{a^2d+9b^2c}{be^2-5d}$ (D) $\dfrac{a^2d+9b^2c}{be-5d}$ (E) $\dfrac{a^2d+9b^2c}{be^2-5d^2}$

11. The midpoint of line segment $\overline{AB}$ is the point C with coordinates $(1,-1)$. The coordinates of point A are $(-3,-6)$. Find the length of line segment $\overline{AB}$. (nearest tenth)

(A) 12.2   (B) 12.4   (C) 12.6   (D) 12.8   (E) 13.0

12. Three of the vertices of a parallelogram have coordinates $(2,-2)$, $(6,1)$ and $(-3,4)$. When finding the coordinates of the fourth vertex, you find that there are _____ distinct correct answers.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

13. Points A, B, C and D lie on a circle with center O. Chords $\overline{CD}$ and $\overline{AB}$ intersect at point P. If $AP=6x$, $BP=6x+9$, $DP=6x-3$ and $CD=18x+6$, then $AB=$ _____. (nearest whole number)

(A) 26   (B) 27   (C) 28   (D) 29   (E) 30

14. Given: Triangle ABC is similar to triangle DEF, $BC=19$, $m\angle ABC=48°$, and $m\angle BCA=73°$. If $DF=15$, then $AB-EF=$ _____. (nearest tenth)

(A) 3.9   (B) 4.1   (C) 4.3   (D) 4.5   (E) 4.7

15. Consider the conditional statement "If a triangle is equilateral, then it is equiangular." Which of the following are true?
I. Converse   II. Inverse   III. Contrapositive

(A) I only   (B) III only   (C) I, II only   (D) I, III only   (E) I, II, III

16. Square ABCD has a perimeter of 60 and the center of a circle with a diameter of 30 is the point C. Find the area of the union of the two geometric figures. (nearest whole number)

(A) 743   (B) 747   (C) 751   (D) 755   (E) 759

17. Find the domain of the function. $f(x)=\dfrac{\sqrt{4x-12}}{8x^2-8x+2}$

(A) $(-\infty,\infty)$ (B) $x\geq 2$ (C) $x\geq 3$ (D) $x>3$ (E) $x\geq 3,\ x\neq 4$

18. Suppose you could wrap a cable around the Earth along a path of constant latitude at 45° North. Assume the Earth is a sphere with a radius of 3960 miles. Find the length of the cable. (nearest whole number)

   (A) 17,582 mi   (B) 17,588 mi   (C) 17,594 mi   (D) 17,600 mi   (E) 17,606 mi

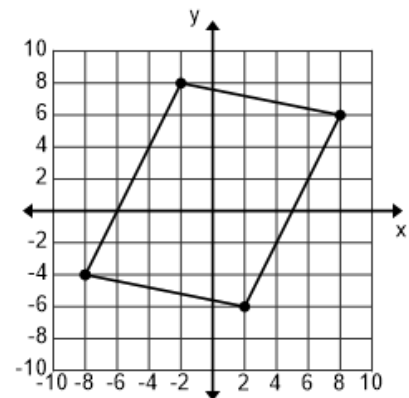19. If the area of the circle $x^2 + y^2 + 6x + 14y + k = 0$ is 201.062, what is the value of k? (nearest tenth)

   (A) −6.3   (B) −6.0   (C) −5.7   (D) −5.4   (E) −5.1

20. Find the perimeter of the quadrilateral shown on the right. (nearest tenth)

   (A) 46.6   (B) 46.9   (C) 47.2   (D) 47.5   (E) 47.8

21. Find the area of the quadrilateral shown on the right. (nearest tenth)

   (A) 130.8   (B) 131.1   (C) 131.4   (D) 131.7   (E) 132.0


Problems 20, 21

22. Consider acute triangle △ABC with AB = 12 and BC = 15. If the area of the triangle equals 84.572, what is the measure of ∠BCA? (nearest whole number)

   (A) 46°   (B) 52°   (C) 58°   (D) 64°   (E) 70°

23. Farmer Jones has a water trough that is shaped like a prism. The ends of the trough are isosceles trapezoids, and the other sides are rectangles six feet long. How many gallons of water will the trough hold when it is full? (nearest gallon)

3 feet

18 inches

30 inches
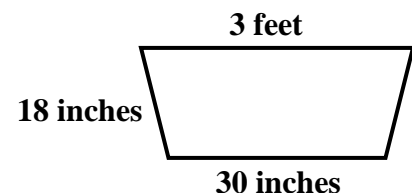
   (A) 175   (B) 177   (C) 179   (D) 181   (E) 183

24. Consider a regular octagon with an area of 204. Find the area of a circle inscribed in the octagon? (nearest whole number)

   (A) 190   (B) 193   (C) 196   (D) 199   (E) 202

25. Consider the geometric sequence $6, a, b, c, 30\frac{3}{8}, d, \dots$ If $a > 0$, then $a + b + c =$ _____.

   (A) $42\frac{1}{4}$ (B) $42\frac{3}{8}$ (C) $42\frac{1}{2}$ (D) $42\frac{5}{8}$ (E) $42\frac{3}{4}$

26. Find the sum of the series $1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + 13 + 14 + 16 + 17 + \dots + 304 + 305$

   (A) 31,194   (B) 31,203   (C) 31,212   (D) 31,221   (E) 31,230

UIL Mathematics

27. The amount of medicine in Norv's circulatory system when he takes a dose gradually dilutes with time. The half-life of his current medication is 36 hours. If he took an 8.00 mL dose at 8:00 AM on Monday, how much of the medicine is present in his circulatory system at 11:30 PM on Thursday of the same week? (nearest hundredth)

   (A) 1.39 mL   (B) 1.42 mL   (C) 1.45 mL   (D) 1.48 mL   (E) 1.51 mL

28. Find the eccentricity of an ellipse with its center at $(-12, -5)$ and given that the ellipse is tangent to both axes. (nearest hundredth)

   (A) 0.83   (B) 0.85   (C) 0.87   (D) 0.89   (E) 0.91

29. Consider the graph of the hyperbola $16x^2 - 9y^2 - 128x + 18y + 103 = 0$. The asymptotes of the graph have y-intercepts with coordinates $(0, a)$ and $(0, b)$. $a + b = $ _____.

   (A) $\dfrac{2}{3}$ (B) 1   (C) $1\dfrac{1}{3}$ (D) $1\dfrac{2}{3}$ (E) 2

30. Consider the function $f(x) = \text{Sin}^{-1}(x)$. The domain of f(x) is _____.

   (A) $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ (B) $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$ (C) $-1 < x < 1$ (D) $-1 \le x \le 1$ (E) $-\pi \le x \le \pi$

31. Tina is standing on a platform at the water's edge as she watches a Nimitz class carrier sail directly away from where she is watching. The high point of the carrier is 57 feet above the waterline and Tina's eye level is 32 feet above the waterline. How far is the high point of the carrier from Tina when the high point of the carrier disappears from her sight? The radius of the earth is 3960 miles. (nearest tenth)

   (A) 15.9 mi   (B) 16.2 mi   (C) 16.5 mi   (D) 16.8 mi   (E) 17.1 mi

32. The period of $h(x) = \dfrac{1}{k}\sin\left(3\pi kx\right)$ is 4. Find the amplitude of $h(x)$.

   (A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{2}$ (C) 2π (D) 3   (E) 6

33. Consider the sequence $3, 4, 7, 10, 16, 21, 30, 40, a, b, 120, \dots$ $a + b = $ _____.

   (A) 138   (B) 139   (C) 140   (D) 141   (E) 142

34. Consider the graph of the polar equation $r = \dfrac{3}{1 + 2\sin\theta}$. The coordinates of the foci are (a, b) and (a, c). $a + b + c = $ _____.

   (A) 2   (B) $1 + 2\sqrt{2}$ (C) 4   (D) $2 + 2\sqrt{3}$ (E) 6

35. **Find the equation of a line such that every point on the line is the same distance from point A as it is from point B.**

    (A) $18x - 10y + 15 = 0$
    (B) $9x - 5y + 6 = 0$
    (C) $9x - 5y + 7 = 0$
    (D) $9x - 5y + 8 = 0$
    (E) $18x - 12y + 13 = 0$
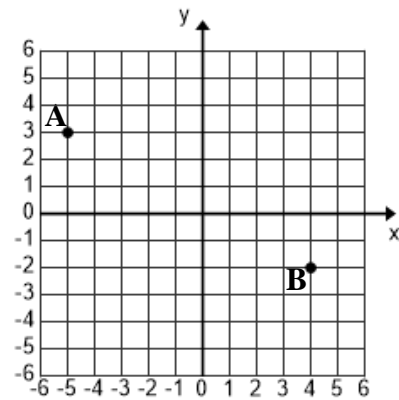
36. **Find the distance between the plane $3x - 5y + 2z = 6$ and the point $(2, 3, 4)$. (nearest tenth)**

    (A) 0.9   (B) 1.1   (C) 1.3   (D) 1.5   (E) 1.7

37. **A shipment of 20 fuses contains 3 defective fuses. In how many ways can a person purchase 5 fuses and get at least 4 good fuses?**

    (A) 11,684   (B) 12,432   (C) 13,328   (D) 14,512   (E) 15,772

38. **If $[x]$ = the greatest integer less than or equal to x, then**

    $$\left[\sqrt{2}\right] + \left[\sqrt{3}\right] + \left[\sqrt{4}\right] + \left[\sqrt{5}\right] + \left[\sqrt{6}\right] + ... + \left[\sqrt{23}\right] + \left[\sqrt{24}\right] = \underline{\hspace{2cm}}.$$

    (A) 65   (B) 66   (C) 67   (D) 68   (E) 69

39. **Identify the conic and calculate the angle of rotation for $4x^2 - 10xy + 4y^2 + 10x + 2y + 1 = 0$.**

    I. Parabola   II. Ellipse   III. Hyperbola   IV. 30°   V. 45°   VI. 60°

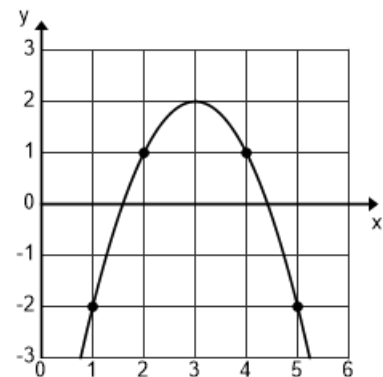   (A) I, IV   (B) II, V   (C) III, VI   (D) II, VI   (E) III, V

40. **The coordinates of the focus of the parabola shown on the right are (a, b). $a + b = $ _____.**

    (A) 4   (B) $4\frac{1}{4}$ (C) $4\frac{1}{2}$ (D) $4\frac{3}{4}$ (E) $4\frac{7}{8}$

41. **The graph of $f'(x)$ is shown on the right. If $f(3) = 3$, then $f(1) = $ _____.**

   (A) 1   (B) $1\frac{1}{3}$ (C) $1\frac{1}{2}$ (D) $1\frac{2}{3}$ (E) 2

Problems 40, 41

42. **Find the volume of the solid generated by revolving the region bounded by $y = -x^2 + 10x - 17$ and $y = x^2 - 8x + 19$ about the line $x = -6$. (nearest whole number)**

    (A) 582   (B) 585   (C) 588   (D) 591   (E) 594

UIL Mathematics

43. Consider two vertical posts that are 60 feet apart. Post A is 16 feet tall and post B is 24 feet tall. They are to be stayed by two wires attached to a single stake. A wire runs from ground level to the top of each post. Find the least amount of wire needed. (nearest inch)
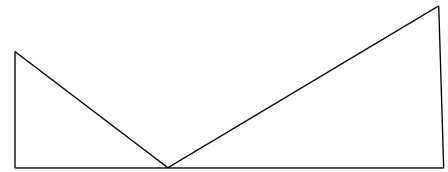
    (A) 71 ft 1 in   (B) 71 ft 5 in   (C) 71 ft 9 in   (D) 72 ft 1 in   (E) 72 ft 5 in

44. Consider the region bounded by the graph of the function $f(x) = 16 - x^2$, the x-axis, and the lines $x = 1$ and $x = 3$. Find an approximation of the area of this region by dividing the interval $[1, 3]$ into four subintervals of equal width and then constructing rectangles using the right endpoints of each subinterval. One of the rectangles is shown on the right. Find the value of this approximation.
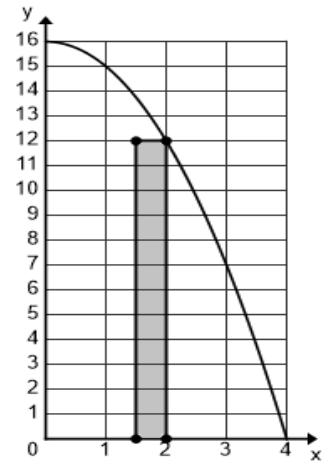
    (A) 20.25   (B) 20.50   (C) 20.75   (D) 21.00   (E) 21.25

45. Find the exact value of the area of the region described in problem 44.

    (A) $23\frac{1}{4}$ (B) $23\frac{1}{3}$ (C) $23\frac{1}{2}$ (D) $23\frac{2}{3}$ (E) $23\frac{3}{4}$

    Problems 44, 45

46. Consider the function $g(x)$, which is continuous on $[-4, 0]$, and with $g(-1) = 6$ and $g(-2) = 6$. If $g''(x)$ is continuous and negative on $[-4, 0]$, then which of the following must be true?

    (A) $g'(-1.5) = 0$ (B) $g'(-1.5) > 0$ (C) $g'(-1.5) < 0$ (D) $g'(-3) > 0$ (E) $g'(-3) < 0$

47. A particle is moving along the x-axis. At t = 0, the particle is located at x = 0 and the acceleration of the particle is $a(t) = 12t$, $t \geq 0$. If the maximum displacement of the particle in the negative direction is -32, find the velocity of the particle at t = 0. (nearest whole number)

    (A) -24 (B) -22 (C) -20 (D) -18 (E) -16

48. A conservation group releases 25 prairie dogs into a large park northeast of Lubbock at 1:00 AM on April 1st, 2022. The carrying capacity of the park is 500 prairie dogs. On April 1st, 2025, there were 100 prairie dogs in the park. Using a logistic model, the fastest rate of growth of the prairie dog population should occur on or about _____.

    (A) 06 / 01 / 2027   (B) 09 / 01 / 2027   (C) 12 / 01 / 2027   (D) 03 / 01 / 2028   (E) 06 / 01 / 2028

49. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a famous series in mathematics. Which of the following statements are true?

    I. The series is known as the harmonic series.
    II. The terms of the series tend to zero as n increases.
    III. The series converges.

    (A) II only   (B) I, II only   (C) II, III only   (D) I, III only   (E) I, II, III

50. Find the interval of convergence of $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$.

(A) $(-1, 1)$ (B) $[-1, 1]$ (C) $(0, 2)$ (D) $[0, 2]$ (E) $(-\infty, \infty)$

51. The scatterplot of some data indicated a moderately strong linear relationship between two variables. A least-squares regression line was computed and the correlation coefficient was 0.80. However, the explanatory and response variables had been switched. A new regression line was computed using the correct explanatory and response variables. The new correlation coefficient was _____.

(A) -0.80 (B) 0.80   (C) -1.25 (D) 1.25   (E) 0.40

52. There are 250 seniors at the High Plains Stem Academy in Borger. Twenty-eight students are taking Calculus, Physics and Statistics. Thirty-six students are taking Physics and Statistics, but not Calculus. Twenty-four students are taking Calculus and Statistics, but not Physics. Forty-four are taking Calculus and Physics, but not Statistics. If 126 students are taking Physics, 110 are taking Statistics, and 144 are taking Calculus, how many seniors are not taking any of these courses?

(A) 28   (B) 30   (C) 32   (D) 34   (E) 36

53. Tim is 54 inches tall. This places him at the 95th percentile of all boys his age. The heights for boys this age form an approximately normal distribution with a mean of 51 inches and a standard deviation of _____ inches. (nearest hundredth)

(A) 1.70   (B) 1.76   (C) 1.82   (D) 1.88   (E) 1.94

54. In the Deschutes County lottery, the cost of a lottery ticket is $20. Six positive integers, from 1 to 30, are randomly chosen without repetition. If you correctly pick all 6 numbers, you win $1,000,000. If you correctly pick exactly 5 of the 6 numbers, you win $10,000. If you correctly pick exactly 4 of the 6 numbers, you win $1,000. When the cost of the ticket is considered, what is the expected value of a lottery ticket?

(A) $-$8.92   (B) $-$8.88   (C) $-$8.84   (D) $-$8.80   (E) $-$8.76

55. A professor at Baylor took a random sample of students at Baylor to find the proportion of students who love math. She found that 67 out of the 150 students questioned love math. A professor at Rice took a similar random sample of students at Rice and he found that 53 out of 120 students questioned love math. Calculate a 95% confidence interval for the difference between the proportions of students at the two universities who love math. (nearest thousandth)

(A) $(-.114, .124)$ (B) $(-.124, .134)$ (C) $(-.134, .144)$ (D) $(-.144, .154)$ (E) $(-.154, .164)$

56. A linear regression was performed using the following data points.
$A(1, 17)$, $B(3, 56)$, $C(5, 80)$, $D(7, 126)$, $E(9, 142)$
The absolute value of the residual for point C is _____. (nearest tenth)

(A) 3.4   (B) 3.6   (C) 3.8   (D) 4.0   (E) 4.2

	Under $40,000	$40,000 - $60,000	Over $60,000	Total
Mathematics	12	40	108	160
English	77	29	14	120
Political Science	102	38	10	150
Total	191	107	132	430

57. A professor at Texas Tech believes that graduates with math degrees are more likely to have good starting salaries than graduates with English degrees or Political Science degrees. The table above shows data collected from a random sample of Tech graduates. What is the expected cell count for the cell denoting English and $40,000 - $60,000 if there is no difference between the starting salaries of the three majors? (nearest hundredth)

   (A) 28.44   (B) 29.86   (C) 31.28   (D) 32.70   (E) 34.12

58. Dr. Bixler determined that there is a strong, positive, linear relationship between student IQ scores and their scores on the physics section on the regional science test. He randomly selected 90 students who competed at region and found the IQ mean was 120 with a standard deviation of 9. The mean physics score was 50 with a standard deviation of 18. His analysis also found that $r^2 = 0.64$. What is the predicted physics score for a student with an IQ of 140? (nearest whole number)

   (A) 70   (B) 74   (C) 78   (D) 82   (E) 86

59. A company produces barbells that have a mean weight of 45 pounds with a standard deviation of 0.25 pounds. The production process is such that the weight of each barbell produced is independent of the weights of other barbells. The weights are approximately normally distributed. If three barbells are randomly selected, what is the probability that all three will weigh more than 45.25 pounds? (nearest ten-thousandth)

   (A) 0.0034   (B) 0.0040   (C) 0.0046   (D) 0.0052   (E) 0.0058

60. There are more than 30,000 children in Idaho needing a volunteer to listen to them read after school. The I.E.A. is attempting to estimate the proportion of all retired adults in Idaho who are willing to listen to children read after school. They plan to poll a random sample of retired adults to help them estimate this proportion. Of the following, which is the smallest sample size that will ensure a margin of error of no more than 4 percent for a 96% confidence interval estimate of the proportion of retired adults in Idaho who are willing to listen to children read after school?

   (A) 435   (B) 670   (C) 905   (D) 1130   (E) 1355

University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2022
Answer Key

1. C	21. E	41. D
2. A	22. A	42. E
3. D	23. E	43. D
4. E	24. B	44. E
5. D	25. E	45. B
6. C	26. C	46. D
7. E	27. D	47. A
8. C	28. E	48. C
9. C	29. E	49. B
10. C	30. D	50. E
11. D	31. B	51. B
12. D	32. E	52. B
13. B	33. D	53. C
14. A	34. C	54. A
15. E	35. C	55. A
16. D	36. B	56. E
17. C	37. C	57. B
18. C	38. E	58. D
19. B	39. E	59. B
20. C	40. D	60. B

University Interscholastic League
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

Final _____ _____

2nd _____ _____

1st _____ _____

Contestant # _____

Conference _____

Score **Initials**

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

Mathematics

Region • 2022

UNIVERSITY INTERSCHOLASTIC LEAGUE

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. If $6 \times \sqrt{k-16} - 20 = 4,$ then $k^2 =$ _____.

   (A) 4   (B) 16   (C) 32   (D) 256   (E) 1024

2. Helen Hoopster received a $250 gift card for her birthday. She purchased some shoes for $125.95, some shorts for $22.50, two t-shirts for $19.90 each, and some socks for $9.85. If the tax rate was 8.25%, how much remained on her gift card? (nearest cent)

   (A) $35.56   (B) $35.69   (C) $35.82   (D) $35.95   (E) $36.08

3. Biker Bob cycled due north for 45 minutes at 25 mph. Then he cycled due east for 30 minutes at 24 mph. Next, he cycled due south for 75 minutes at 28 mph. How far was Bob from where he started? (nearest tenth)

   (A) 19.6 mi   (B) 19.8 mi   (C) 20.0 mi   (D) 20.2 mi   (E) 20.4 mi

4. The public swimming pool in Crosbyton has two pipes that fill the pool. One of them can fill the pool by itself in 48 hours. The second one can fill the pool by itself in 36 hours. The drain can empty the pool in 60 hours if both fill pipes are shut off. If the pool is empty, how long will it take to fill the pool when both fill pipes are open and the drain is also functioning? (nearest tenth)

   (A) 31.3 hr   (B) 31.5 hr   (C) 31.7 hr   (D) 31.9 hr   (E) 32.1 hr

5. Carrie baked a batch of chocolate cookies. She gave one-sixth of them to Landon. Next, she gave one-fourth of what remained to Rose. Then she gave half of what remained to Caleb. Next, she gave 18 cookies to Elise. If she had twelve cookies left for herself, how many cookies did she give to Caleb?

   (A) 26   (B) 28   (C) 30   (D) 32   (E) 34

6. Find the number that is $\dfrac{7}{13}$ of the way from $-4\dfrac{7}{12}$ to $10\dfrac{3}{8}$.

   (A) $3\dfrac{6}{13}$ (B) $3\dfrac{49}{104}$ (C) $3\dfrac{25}{52}$ (D) $3\dfrac{51}{104}$ (E) $3\dfrac{1}{2}$

7. Turbo is an ultramarathoner. In a recent 100-mile race, he ran the first 28 miles at an average speed of 9.20 mph. He ran the next 38 miles at an average speed of 8.70 mph. If his goal was to average to average 9.00 mph for the entire race, at what average speed must he run for the remainder of the race to meet his goal pace? (nearest hundredth)

   (A) 9.05 mph   (B) 9.12 mph   (C) 9.19 mph   (D) 9.26 mph   (E) 9.33 mph

8. Cindy's Sporting Goods purchased 75 Tour Edition bowling balls for $60 each. The price of a bowling ball in June was $119.95 and they sold 15. In July, they reduced the price by 20% and they sold 20. In August, they reduced the price by 25% of the July price and they sold all of the remaining balls. What was the average profit on a single bowling ball? (nearest cent)

   (A) $27.96   (B) $28.02   (C) $28.08   (D) $28.14   (E) $28.20

UIL Mathematics

9. Three times Juanita's age exceeds twice Carmen's age by 5. In fifteen years, four times Carmen's age will equal five times Juanita's age. How old is Carmen?

    (A) 20 yr   (B) 25 yr   (C) 30 yr   (D) 35 yr   (E) 40 yr

10. Consider positive integers a, b, and c such that ab = 96, ac = 144 and bc = 216. Find the value of $a+b+c$.

   (A) 36   (B) 38   (C) 40   (D) 42   (E) 44

11. Traffic can vary on my morning drive to work. On Thursday, I drove at an average speed of 45 mph and arrived 16 minutes late. On Saturday, I drove at an average speed of 90 mph and arrived 16 minutes early. At what average speed do I need to drive to arrive on time?

    (A) 57.5 mph   (B) 60 mph   (C) 62.5 mph   (D) 65 mph   (E) 67.5 mph

12. Find the y-intercept of the perpendicular bisector of line segment on the right.

    (A) $\left(0, \dfrac{1}{6}\right)$ (B) $\left(0, \dfrac{1}{4}\right)$ (C) $\left(0, \dfrac{1}{3}\right)$

    (D) $\left(0, \dfrac{1}{2}\right)$ (E) $\left(0, \dfrac{2}{3}\right)$
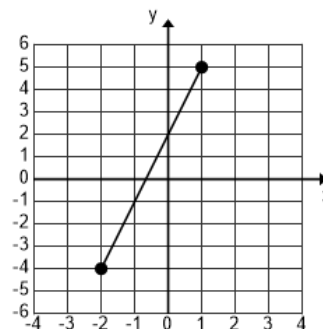
13. Consider triangle △ABC with m∠A = 66°, m∠B = 56°, and AC = 12 in.
    Point D is the midpoint of line segment $\overline{AB}$. Point F is the centroid. DF = _____. (nearest tenth)

    (A) 3.1 in   (B) 3.3 in   (C) 3.5 in   (D) 3.7 in   (E) 3.9 in

14. Consider equilateral triangle △ABC with BC = 9 in. Point D lies on ray $\overrightarrow{BC}$ with BD = 18 in. Find the area of triangle △ABD. (nearest tenth)

    (A) 69.5 in²   (B) 69.8 in²   (C) 70.1 in²   (D) 70.4 in²   (E) 70.7 in²

15. Consider triangle △ABC with AB = 16, AC = 24, and m∠BAC = 60°. Point D lies on line segment $\overline{AC}$ and $\overline{AC} \perp \overline{DB}$. m∠DBC = _____. (nearest tenth)

   (A) 48.2°   (B) 48.5°   (C) 48.8°   (D) 49.1°   (E) 49.4°

16. Consider a circle with center O and chord $\overline{AB}$. Point C is the midpoint of $\overline{AB}$. If AB = 10 cm and CO = 12 cm, find the area of the circle. (nearest whole number)

    (A) 524 cm²   (B) 531 cm²   (C) 538 cm²   (D) 545 cm²   (E) 552 cm²

17. A right circular cone has a radius of 12 cm and a slant height of 18 cm. Find the volume of the cone. (nearest whole number)

    (A) 2023 cm³   (B) 2031 cm³   (C) 2039 cm³   (D) 2047 cm³   (E) 2055 cm³

UIL Mathematics

18. $\overline{PT}$ is tangent to the circle on the right.
    If PT = 24 and PR = 18, then PQ = _____.
    (nearest tenth)
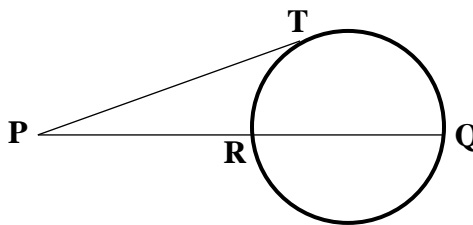
   (A) 31.6   (B) 31.8   (C) 32.0
                (D) 32.2   (E) 32.4

19. Consider $\triangle ABC$ with AB = 18 m, BC = 14 m, and AC = 16 m. If point D is the midpoint of $\overline{AB}$, E is
    the midpoint of $\overline{AC}$ and F is the midpoint of $\overline{BC}$, then the area of $\triangle DEF$ = _____. (nearest tenth)

   (A) 26.2 m²   (B) 26.4 m²   (C) 26.6 m²   (D) 26.8 m²   (E) 27.0 m²

20. One edge of a cube was increased by 2 in, one edge was decreased by 2 in, and the third edge was
    unchanged. If the volume of the rectangular solid is 32 in³ less than the cube, what is the volume of
    the cube?

   (A) 64 in³   (B) 216 in³   (C) 512 in³   (D) 1000 in³   (E) 1728 in³

21. An athletic field is rectangular in shape, has an area of 360 sq yd and a diagonal of 41 yd. What is
    the perimeter of the field? (nearest whole number)

   (A) 294 ft   (B) 297 ft   (C) 300 ft   (D) 303 ft   (E) 306 ft

22. Find the shortest distance between the lines $9x - 40y = 49$ and $18x - 80y = 16$. (nearest tenth)

   (A) 1.0   (B) 1.2   (C) 1.4   (D) 1.6   (E) 1.8

23. If $f(x) = \dfrac{1}{8}x - 3$ and $g(x) = 2x - 5$, then $\left(g^{-1} \circ f^{-1}\right)(2) =$ _____

   (A) 18   (B) 19.5   (C) 21   (D) 22.5   (E) 24

24. Monty has 5 red marbles, 7 green marbles, and 10 blue marbles. They are identical except for the
    color. How many distinguishable ways can Monty arrange them in a row?

   (A) 512,143,600   (B) 512,143,608   (C) 512,143,616   (D) 512,143,624   (E) 512,143,632

25. A company has received 16 applications from qualified applicants for six entry level analyst
    positions. Ten are from Montana and six are from Wyoming. In how many ways can the positions
    be filled if exactly two of the positions must be filled with applicants from Wyoming?

   (A) 720   (B) 3,150   (C) 12,000   (D) 84,600   (E) 151,200

26. A regular dodecagon is inscribed in a circle. If the area of the circle is 804 in², what is the area of the
    dodecagon? (nearest whole number)

   (A) 752 in²   (B) 756 in²   (C) 760 in²   (D) 764 in²   (E) 768 in²

27. Which are true for the function $g(x) = \dfrac{4}{x-5} + 6$?

   I. The domain is $(-\infty, 5) \cup (5, \infty)$.   II. The range is $(-\infty, 6) \cup (6, \infty)$.   III. The function is odd.

   (A) I only   (B) II only   (C) I, II only   (D) I, III only   (E) I, II, III

28. Which are true for the graph of $h(x) = \dfrac{x^2 - 16}{x^2 - 9x + 20}$?

   I. $(-4, 0)$ is an x-intercept.   II. $(4, 0)$ is an x-intercept.   III. $x = -5$ is a vertical asymptote.
   IV. $x = 4$ is a vertical asymptote.   V. $y = 1$ is a horizontal asymptote.

   (A) I, V only   (B) I, IV, V only   (C) I, III, IV only   (D) I, III, IV, V only   (E) III, IV, V only

29. $\dfrac{30(\cos 95° + i \sin 95°)}{5(\cos 125° + i \sin 125°)} = $ _____ .

   (A) $6 - 6\sqrt{3}\,i$ (B) $-3\sqrt{3} - 3i$ (C) $6 + 6\sqrt{3}\,i$ (D) $3\sqrt{3} - 3i$ (E) $6\sqrt{3} - 6i$

30. If $\sin \theta = \dfrac{12}{13}$ and θ is in quadrant II, and if $\tan \alpha = -\dfrac{5}{12}$ and α is in quadrant IV, then what is
   the value of $\cos(\theta - \alpha)$?

   (A) $-\dfrac{120}{169}$ (B) $-\dfrac{90}{169}$ (C) $-\dfrac{60}{169}$ (D) $\dfrac{60}{169}$ (E) $\dfrac{90}{169}$

31. The sound level in decibels, β, is given by $\beta = 10 \log\left(\dfrac{I}{I_0}\right)$, where I is the intensity in watts per

   square meter and I_0 is the threshold of hearing, which equals 10^{-12} watts per square meter. If Kim
   is playing a violin that is producing sound at 87.0 dB, what is the sound level if 11 other musicians
   join Kim and they all play their violins at 87.0 dB? (nearest tenth)

   (A) 97.5 dB   (B) 97.8 dB   (C) 98.1 dB   (D) 98.4 dB   (E) 98.7 dB

32. The equations of the asymptotes of a hyperbola are $y = \dfrac{5}{9}x$ and $y = -\dfrac{5}{9}x$. There is a vertex

   at $(0, 5)$. The coordinates of the foci are (a, b) and (a, c). $|b - c| = $ _____ . (nearest tenth)

   (A) 19.8   (B) 20.0   (C) 20.2   (D) 20.4   (E) 20.6

33. The graph of a curve represented by the parametric equations $x = 4\sec(\theta)$ and $y = 3\tan(\theta)$ has
   foci at (a, b) and (c, b). $|a - c| = $ _____ . (nearest tenth)

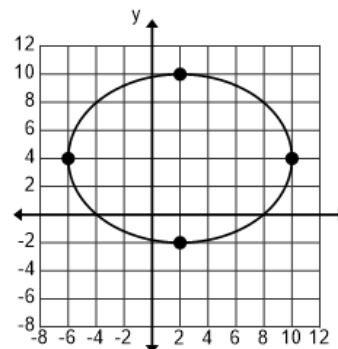   (A) 5.3   (B) 6.5   (C) 7.6   (D) 8.8   (E) 10.0

34. Find the angle between the vectors $\mathbf{u} = \langle 2, -3, 4 \rangle$ and $\mathbf{v} = \langle -5, 1, -2 \rangle$. (nearest tenth)

    (A) 131.2°   (B) 133.3°   (C) 135.4°   (D) 137.5°   (E) 139.6°

35. The coordinates of the foci of the ellipse shown on the right are (a, b) and (c, b). $a + c = $ _____. (nearest tenth)

    (A) 3.8
    (B) 3.9
    (C) 4.0
    (D) 4.1
    (E) 4.2

36. Consider the sequence $2, 4, 8, 13, 20, 29, 41, 57, 79, k, 155, \dots$ $k = $ _____.

    (A) 106   (B) 108   (C) 110   (D) 112   (E) 114

37. A new card game consists of 88 cards that are identical, except for color. There are 20 blue, 20 red, 20 green, and 20 yellow, and 8 purple cards. If you are dealt eight cards, what is the probability of being dealt 4 blue cards, 2 red cards, 1 green card and 1 purple card? (nearest hundred-thousandth)

    (A) 0.00118   (B) 0.00229   (C) 0.00340   (D) 0.00451   (E) 0.00562

38. Some professionals are playing poker. Poker Pete has a full house. This is when a player has a five-card combination with three cards of one type and two of another, such as three 5s and two 9s. Given that Pete has a full house, what is the probability that he has two kings and three queens?

    (A) $\dfrac{1}{624}$ (B) $\dfrac{1}{312}$ (C) $\dfrac{1}{156}$ (D) $\dfrac{1}{78}$ (E) $\dfrac{1}{39}$

39. Convert the rectangular equation $\left(x^2 + y^2\right)^3 = 64x^2y^2$ to a polar equation.
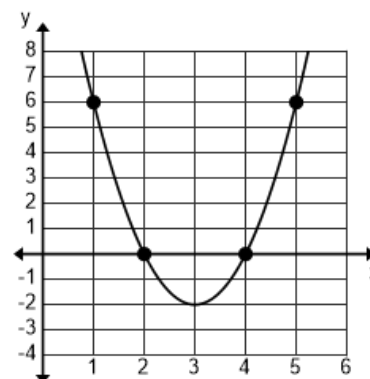
    (A) $r = 4\sin 2\theta$ (B) $r = 4\cos 2\theta$ (C) $r = 8\sin 2\theta$ (D) $r = 8\cos 2\theta$ (E) $r = 16\sin 2\theta$

40. If the focus of the parabola on the right is (a, b), then $a + b = $ _____.

    (A) $\dfrac{7}{8}$ (B) $\dfrac{15}{16}$ (C) 1   (D) $\dfrac{17}{16}$ (E) $\dfrac{9}{8}$

41. The graph on the right is the graph of $f'(x)$. If $f(1) = 1$, then $f(4) = $ _____.

    (A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) 1   (D) $1\dfrac{1}{3}$ (E) $1\dfrac{2}{3}$ **Problems 40, 41**

UIL Mathematics

42. Consider the differential equation $\dfrac{dy}{dx} = 2x^2 - 3y$. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(1) = 2$. Use Euler's method with two steps of equal size, starting at $x = 1$ to approximate $f(2)$. (nearest hundredth)

(A) 2.25   (B) 2.30   (C) 2.35   (D) 2.40   (E) 2.45

43. Suppose g is a differentiable function and suppose f is a function defined by $f(x) = g\left(x^3 - 5\right)$. $f'(2) =$ _____.

(A) $3g'(2)$ (B) $6g'(2)$ (C) $6g'(3)$ (D) $12g'(2)$ (E) $12g'(3)$

44. Consider the graph of the parabola $y = x^2 + x$ in the x-y plane. A particle is moving at a constant speed of $3\sqrt{26}$ along the graph of the parabola. Given that $\dfrac{dx}{dt} > 0$ when the particle is at $(2, 6)$, find the value of $\dfrac{dy}{dt}$ when the particle is at $(2, 6)$. (nearest tenth)
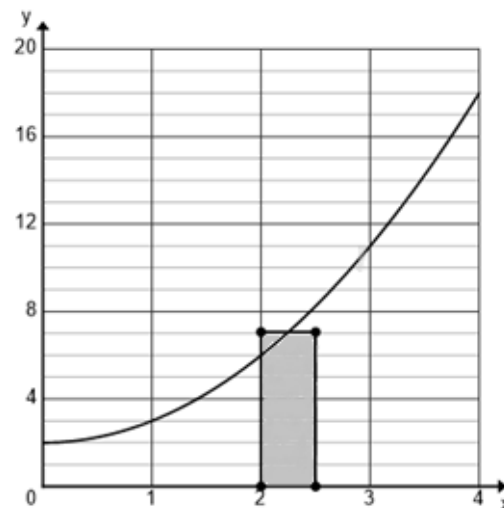
(A) 12.6   (B) 13.2   (C) 13.8   (D) 14.4   (E) 15.0

45. Approximate the area bounded by the curves $y = x^2 + 2$, $y = 0$, $x = 1$, and $x = 3$. Divide the region into 4 subintervals of equal width. Then find the areas of four rectangles formed using the midpoints of each subinterval to find the heights. One of these rectangles is shown on the right. (nearest thousandth)

(A) 12.600   (B) 12.625   (C) 12.650   (D) 12.675   (E) 12.700

46. Find the exact area of the region described in problem 45.

(A) $12\dfrac{1}{3}$ (B) $12\dfrac{1}{2}$ (C) $12\dfrac{2}{3}$ (D) $12\dfrac{5}{6}$ (E) 13   **Problems 45, 46**


x	2	4	6	10	16
f(x)	3	8	4	-12	2

47. Use the table above to evaluate $\displaystyle\int_{2}^{16}\left(4 - 3f'(x)\right)dx$.

(A) 51   (B) 53   (C) 55   (D) 57   (E) 59

48. Lauren is riding her bicycle from her house on Dove Ave to Amy's house, which is also on Dove Ave. On this day, it took her 10 minutes to go from her house to Amy's house. A graph of her velocity as a function of time is shown on the right. How far is it from Lauren's house to Amy's house? (nearest hundredth)

    (A) 1.10 mi   (B) 1.15 mi   (C) 1.20 mi
              (D) 1.25 mi   (E) 1.30 mi
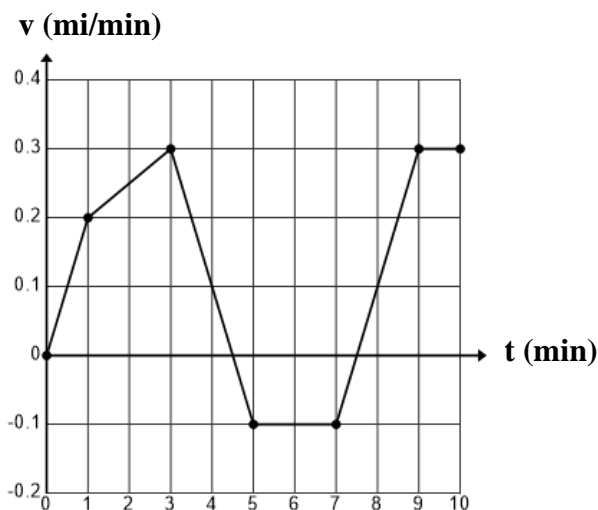
49. Use the graph on the right to find Lauren's acceleration at t = 8 min. (nearest thousandth)

    (A) 0.293 ft/s²   (B) 0.313 ft/s²   (C) 0.333 ft/s²
              (D) 0.353 ft/s²   (E) 0.373 ft/s²

v (mi/min)

Problems 48, 49

50-51 A particle is traveling along the x-axis. The position of the particle is given by
$x(t) = t^4 - 5t^3 + 5t^2 + 5t - 6$, $t \geq 0$, $x(t)$ in feet and t in seconds.

50. Find the total distance traveled by the particle from t = 0 to t = 4. (nearest tenth)

    (A) 36.0 ft   (B) 37.2 ft   (C) 38.3 ft   (D) 39.5 ft   (E) 40.6 ft

51. During the interval when the particle is traveling to the left, what is the maximum speed of the particle? (nearest tenth)

    (A) 2.5 ft/s   (B) 2.7 ft/s   (C) 2.9 ft/s   (D) 3.1 ft/s   (E) 3.3 ft/s

52. Suppose 85% of athletes who use steroids have a positive result when tested. Also, suppose 85% of athletes who do not use steroids have a negative result when tested. If 10% of athletes use steroids, what percent of athletes would test positive when tested? (nearest tenth)

    (A) 20.8%   (B) 21.2%   (C) 21.6%   (D) 22.0%   (E) 22.4%

    The following statistics represent a summary of the distribution in weights for a simple random sample of 1000 high school football players. All measures are in pounds.
    Mean: 210   Median: 190   Standard Deviation: 30   1st Quartile: 165   3rd Quartile: 220

53. Using the above information, which of the following values are considered outliers?
    I. 84 lb   II. 88 lb   III. 300 lb   IV. 306 lb

    (A) IV only   (B) I, IV only   (C) III, IV only   (D) I, III, IV only (E) I, II, III, IV

54. Wildlife employees recently stocked Springfield Lake with 1,000 bass of various weights. Previously, the lake had no bass in it. The distribution of weights of the bass is approximately normal with a mean weight of 3.00 pounds and a standard deviation of 0.75 pounds. A researcher collected a random sample of 25 bass immediately after the lake was stocked. Find the probability that the mean weight of the fish he collected is greater than 3.25 pounds. (nearest thousandth)

    (A) 0.048   (B) 0.128   (C) 0.209   (D) 0.289   (E) 0.369

UIL Mathematics

X	1	2	3	4	5
P(X)	0.02	0.06	0.24	0.32	0.36

55. **Find the standard deviation of X from the probability distribution table above. (nearest hundredth)**

(A) 0.98   (B) 1.01   (C) 1.04   (D) 1.07   (E) 1.10

56. **Assume that the mean number of miles run each week by world class distance runners is 106. If the weekly mileages of world class distance runners are approximately normally distributed and a weekly mileage of 125 represents the 94th percentile, what is the approximate standard deviation of the weekly mileages of world class distance runners? (nearest tenth)**

(A) 11.0 mi   (B) 11.3 mi   (C) 11.6 mi   (D) 11.9 mi   (E) 12.2 mi

57-58 **Farmer Fred thinks the new sprinkler system he installed this year has increased the mean weight of his pears. Last year, the mean weight of his pears was 6.8 ounces. He took a random sample of 20 of this year's pears and found the mean weight was 7.2 ounces with a standard deviation of 0.9 ounces. Assume the weights of his pears are approximately normally distributed. Fred performed an appropriate statistical test on this data.**

57. **The proper null hypothesis is $H_0 : \mu = 6.8$ and the proper alternative hypothesis is _____.**

(A) $H_a : \mu = 7.2$ (B) $H_a : \mu > 7.2$ (C) $H_a : \mu \neq 6.8$ (D) $H_a : \mu > 6.8$ (E) $H_a : \mu < 6.8$

58. **At the $\alpha = .05$ level, he rejected H_0 because the P-value was _____. (nearest ten-thousandth)**

(A) 0.0125   (B) 0.0216   (C) 0.0307   (D) 0.0398   (E) 0.0489

Use the table below for problems 59 and 60. AU = astronomical unit

Planet	Distance From the Sun (AU)	Period (Earth Years)
Mercury	0.387	0.241
Venus	0.722	0.615
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.20	11.9
Saturn	9.58	29.5
Uranus	19.2	84.0
Neptune	30.1	165

59. **Consider the distance from the Sun as L_1 and the period as L_2. From these choices, the best way to linearize the data is to choose ____ as the independent variable and _____ as the dependent variable.**

(A) $\ln(L_1)$, L_2 (B) L_1, $\log(L_2)$ (C) L_1, $(L_2)^2$ (D) L_1, $\ln(L_2)$ (E) $\log(L_1)$, $\log(L_2)$

60. **Use an appropriate model to predict the period of Pluto if it is 39.5 AU from the Sun. (nearest whole number)**

(A) 244 yr   (B) 248 yr   (C) 252 yr   (D) 256 yr   (E) 260 yr

UIL Mathematics

University Interscholastic League
MATHEMATICS CONTEST
HS ● Region ● 2022
Answer Key

1. E	21. A	41. C
2. A	22. A	42. A
3. D	23. D	43. E
4. A	24. E	44. E
5. C	25. B	45. B
6. B	26. E	46. C
7. C	27. C	47. E
8. A	28. A	48. A
9. D	29. D	49. A
10. B	30. A	50. E
11. B	31. B	51. D
12. C	32. E	52. D
13. D	33. E	53. A
14. C	34. C	54. A
15. D	35. C	55. B
16. B	36. C	56. E
17. A	37. B	57. D
18. C	38. C	58. C
19. D	39. A	59. E
20. C	40. E	60. B

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Contestant # _____

Conference _____

Score   **Initials**

|		
1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2022

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. The equation for a thin lens is $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}$. Find the value of d_i if $f = 18$ cm and $d_o = 12$ cm.

   (A) −36 cm   (B) −30 cm   (C) −24 cm   (D) −18 cm   (E) −12 cm

2. Six times the complement of $\angle A$ is 4 greater than twice the supplement of $\angle A$. $m\angle A =$ _____.

   (A) 40°   (B) 42°   (C) 44°   (D) 46°   (E) 48°

3. Consider three consecutive odd numbers such that five times the third is 21 less than three times the sum of the first two. What is the square of the largest number?

   (A) 625   (B) 729   (C) 961   (D) 1225   (E) 1521
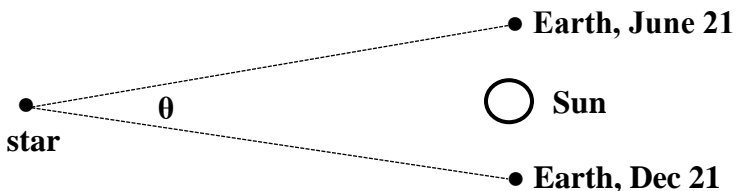
4. Find the value of θ. Ray L₁ is tangent to the circle.

   (A) 46.0°
   (B) 46.5°
   (C) 47.0°
   (D) 47.5°
   (E) 48.0°
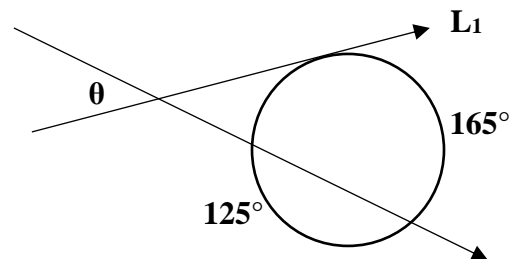
   Problem 4

5. Lib is taking a three-day trip to see her parents in Wyoming. On day one, she traveled 480 miles at an average speed of 65 mph. On day 2, she traveled at an average speed of 72 mph. On day 3 she traveled 420 miles at an average speed 68 mph. If she averaged 68.14 mph for the entire trip, how far did she travel on day 2? (nearest mile)

   (A) 446 mi   (B) 449 mi   (C) 452 mi   (D) 455 mi   (E) 458 mi

6. Twelve men can paint three Nash-Phillips homes in 60 hours. Ten men are sent to paint four Nash-Phillips homes. If these men work for forty-five hours and then half of them quit, how long will it take the remaining men to finish the job? (nearest whole number)

   (A) 90 hr   (B) 93 hr   (C) 96 hr   (D) 99 hr   (E) 102 hr

7. The time on my clock shows that it is exactly 8:18. Find the obtuse angle formed by the hour hand and the minute hand. (nearest whole number)

   (A) 137°   (B) 139°   (C) 141°   (D) 143°   (E) 145°

8. Last fall, Professor C attended the Dallas Cowboys game against Denver with his wife and two sons. Parking cost $40. For this game, they got a "Great Deal" on Club Main, row 1 tickets. They only cost $2887 each. Each of them had a Green Chile Kobe Burger, which cost $13.50 each. Each of them got a coke for $6 each and fries for $5 each. If the cost of gas to drive from Salado to Dallas and back was $27.77, what was the total cost of attending the game?

   (A) $11,705.77   (B) $11,709.77   (C) $11,713.77   (D) $11,718.77   (E) $11,723.77

UIL Mathematics

9. Rocket told his computer science students that their semester grades will be weighted by counting the average of their six test grades as 81% of their numerical grade and their semester exam grade as the other 19%. Joe has the following test scores: 86, 91, 85, 95, 91 and 88. Of the following, which is the lowest score Joe can make on the semester exam in order to make an "A" for the semester if Rocket will give you an "A" if your weighted average is 89.5 or higher?

   (A) 89   (B) 90   (C) 91   (D) 92   (E) 93

10. At the Tokyo Olympics held last summer, Grant Fisher of the USA placed sixth in the 10,000 meters. The Stanford alum's time of 27:46.39 was only three seconds slower than the time of Ethiopia's Selemon Barega, who won the gold medal. What was Grant's average pace?
   (minute, nearest second / mile)

   (A) (4 min 24 sec) / mi   (B) (4 min 26 sec) / mi   (C) (4 min 28 sec) / mi
        (D) (4 min 30 sec) / mi   (E) (4 min 32 sec) / mi

11. Consider a circle with center O. Points A and B lie on the circle such that the length of chord $\overline{AB}$ is 48 and the length of segment $\overline{OB}$ is 26. Find the length of minor arc AB. (nearest tenth)

   (A) 60.0   (B) 60.3   (C) 60.6   (D) 60.9   (E) 61.2

12. A total of 60 people were on the bus transporting The Brock Eagles to their game with Lorena. The average age of the 8 cheerleaders was exactly 17 years, the average age of the 12 coaches was exactly 38 years and the overall average age of the 60 people on the bus was exactly 21 years. What was the average age of the 40 football players? (nearest tenth)

   (A) 16.5 yr   (B) 16.7 yr   (C) 16.9 yr   (D) 17.1 yr   (E) 17.3 yr

13. At the Hunter's Swap Meet, 7 quail were worth 2 ducks and 1 duck was worth 5 doves. How many quail would you get for 30 doves?

   (A) 21   (B) 24   (C) 28   (D) 32   (E) 35

14. Consider a triangle with side lengths 16, 30, 34. What is the length of the shortest altitude of the triangle? (nearest tenth)

   (A) 13.3   (B) 13.5   (C) 13.7   (D) 13.9   (E) 14.1

15. Consider an equilateral triangle inscribed in a circle. If the perimeter of the triangle is equal to the area of the circle, what is the area of the circle minus the area of the triangle? (nearest tenth)

   (A) 5.0   (B) 5.2   (C) 5.4   (D) 5.6   (E) 5.8

16. A card was drawn from a full deck of 52 cards. What are the odds that the card was a queen or a heart or the queen of hearts?

   (A) $\dfrac{4}{9}$ (B) $\dfrac{9}{20}$ (C) $\dfrac{23}{50}$ (D) $\dfrac{17}{35}$ (E) $\dfrac{1}{2}$

17. Coach Griffin has 18 players on his baseball team. How many different nine-person batting orders are possible?

    (A) 48,620   (B) 128,720   (C) 1,682, 400   (D) 886,723,500   (E) 17,643,225,600

18. Consider a regular hexagon with an inscribed circle and a circumscribed circle. If the area of the circumscribed circle is 452 cm², what is the area of the inscribed circle? (nearest square centimeter)

    (A) 327 cm²   (B) 331 cm²   (C) 335 cm²   (D) 339 cm²   (E) 343 cm²

19. There is a row of 5 empty chairs in a room. Three boys and two girls enter the room and are told to sit in the chairs. How many different seating arrangemnets are possible if the girls are not allowed to sit next to each other?

    (A) 60   (B) 72   (C) 84   (D) 96   (E) 120

20. The distance from Earth's center to the Sun's center is 1.496×10^8 km. If $\theta = 7.5 \times 10^{-6}$ rad, the distance from the Sun to the star is _____ km. (two significant digits)

    (A) 3.1×10^{13} (B) 3.4×10^{13} (C) 3.7×10^{13} (D) 4.0×10^{13} (E) 4.3×10^{13}

21. If $f(x) = \dfrac{1}{32}x - 5$ and $h(x) = x^5$, then $\left(f^{-1} \circ h^{-1}\right)(243) =$ _____ .

    (A) 96   (B) 176   (C) 256   (D) 336   (E) 416

22. If 0.5 is a zero of $6x^3 - 9x^2 + kx + 6 = 0$, then $k =$ _____ .

    (A) −11   (B) −9   (C) −7   (D) −5   (E) −3

23. Becci placed \$123,456.78 into an account that earns 6.45% annual interest compounded monthly. How long will it take for the balance in her account to triple? (nearest whole number)

    (A) 14 yr   (B) 15 yr   (C) 16 yr   (D) 17 yr   (E) 18 yr

24. Find the period of the function $f(x) = -3 + 2\sec\left(3x - \dfrac{\pi}{4}\right)$.

    (A) $\dfrac{2\pi}{3}$ (B) π (C) $\dfrac{4\pi}{3}$ (D) $\dfrac{5\pi}{3}$ (E) 2π

25. Given: $\cos\theta = \dfrac{12}{13}$, $\sin\alpha = -\dfrac{15}{17}$. If both θ and α are in quadrant IV, then $\sec(\theta - \alpha) =$ _____ . (nearest hundredth)

    (A) 1.17   (B) 1.20   (C) 1.23   (D) 1.26   (E) 1.29

26. $m\angle BAC =$ _____ . (nearest tenth)

    (A) 37.1°   (B) 37.4°   (C) 37.7°   (D) 38.0°   (E) 38.3°

27. Point D (not shown) lies on $\overline{AC}$ and $\overline{BD} \perp \overline{AC}$.
Find AD. (nearest tenth)

    (A) 12.7   (B) 13.0   (C) 13.3   (D) 13.6   (E) 13.9
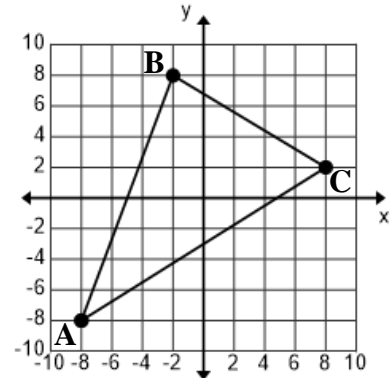
Problems 26, 27

28. A boat is traveling due south, parallel to the shoreline. It is
traveling at a constant speed of 15 mph. At $t = 0$, the bearing from the
boat to a tall motel at the shoreline was W 72° S. At $t = 20$ minutes, the bearing was W 36 ° S. Find
the shortest distance from the shoreline to the boat. (nearest tenth)

    (A) 1.3 mi   (B) 1.5 mi   (C) 1.7 mi   (D) 1.9 mi   (E) 2.1 mi

29. Which of the following are the three cube roots of $1 + i$ in polar form ?

    I. $\sqrt[6]{2}\text{cis}(15°)$ II. $\sqrt[6]{2}\text{cis}(45°)$ III. $\sqrt[6]{2}\text{cis}(135°)$ IV. $\sqrt[6]{2}\text{cis}(225°)$ V. $\sqrt[6]{2}\text{cis}(255°)$

    (A) I, II, III   (B) II, III, IV   (C) I, III, V   (D) II, III, IV   (E) III, IV, V

30. Find the acute angle formed by the two intersecting
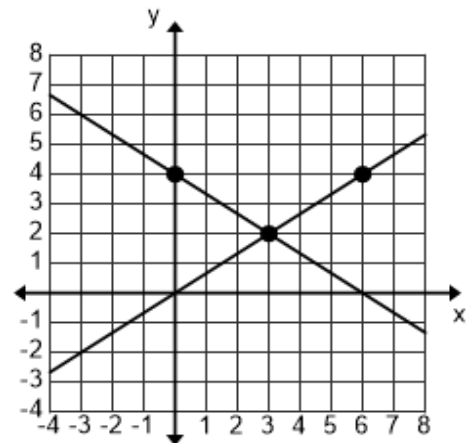lines shown on the right. (nearest tenth)

    (A) 67.0°   (B) 67.2°   (C) 67.4°   (D) 67.6°   (E) 67.8°

31. The asymptotes of a hyperbola are graphed on the right.
If the equation of the hyperbola centered at (h, k) is
$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$ and one of the foci is the point
$(h, k+c)$. $a+b+c =$ _____ . (nearest tenth)

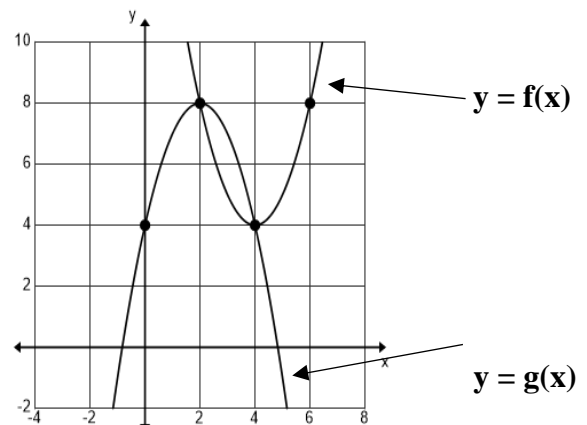   (A) 8.0   (B) 8.2   (C) 8.4   (D) 8.6   (E) 8.8   Problems 30, 31

32. The equation of the directrix of the graph of
$y = f(x)$ is $y =$ _____ .

    (A) $\dfrac{63}{16}$ (B) $\dfrac{31}{8}$ (C) $\dfrac{15}{4}$ (D) $\dfrac{7}{2}$ (E) 3

33. The distance from the focus of the graph of $y = f(x)$ to
the focus of the graph of $y = g(x)$ is _____ .
(nearest tenth)

    (A) 3.6   (B) 3.8   (C) 4.0   (D) 4.2   (E) 4.4

Problems 32, 33

34. Convert the rectangular equation $xy = 8$ to a polar equation.

(A) $r^2 = 8\sec(\theta)$ (B) $r^2 = 16\csc(2\theta)$ (C) $r^2 = 8\csc(\theta)$ (D) $r^2 = 16\sec(2\theta)$ (E) $r^2 = 8\cot(\theta)$

35. Just as an airplane leaves the runway, the plane's angle of ascent is 16° and its speed is 200 mph. If it continues on this path at this speed, how long will it take the plane to reach an altitude of 8,000 ft? (nearest second)

(A) 87 s   (B) 90 s   (C) 93 s   (D) 96 s   (E) 99 s

36. The partial fraction decomposition of $\dfrac{6x^2 + 5x + 12}{x^3 + 3x}$ is $\dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 3}$. $A + B + C = $ _____.

(A) 11   (B) 12   (C) 13   (D) 14   (E) 15

37. Consider the graph produced by the parametric equations $x = 4\sec\theta$ and $y = 2\tan\theta$. Which of the following points is a focus of the graph?

(A) $\left(-2\sqrt{6}, 0\right)$ (B) $\left(-2\sqrt{5}, 0\right)$ (C) $\left(-2\sqrt{3}, 0\right)$ (D) $\left(-2\sqrt{2}, 0\right)$ (E) $(-2, 0)$

38. $\sqrt{5} + \sqrt{10} + \sqrt{15} + \sqrt{20} + \sqrt{25} + ... + \sqrt{220} + \sqrt{225} = $ _____. (nearest whole number)

(A) 453   (B) 455   (C) 457   (D) 459   (E) 461

39. The circle $(x + 6)^2 + (y + 3)^2 = 5$ is tangent to the circle $x^2 + y^2 = 20$. The common internal tangent is a line with x-intercept $(a, 0)$ and y-intercept $(0, b)$. $a + b = $ _____. (nearest tenth)

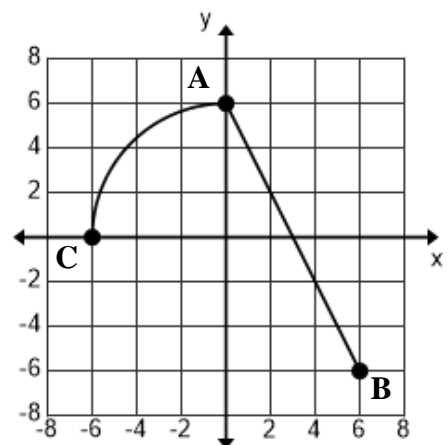(A) −17.0   (B) −16.5   (C) −16.0   (D) −15.5   (E) −15.0

For problems 40 and 41, consider the graph of the piecewise function, $y = f(x)$, which is shown on the right. The domain of $f(x)$ is $-6 \le x \le 6$.

40. The y-intercept of the perpendicular bisector of line segment $\overline{AB}$ is the point $(0, b)$. $b = $ _____.

(A) $-\dfrac{5}{4}$ (B) $-\dfrac{4}{3}$ (C) $-\dfrac{3}{2}$ (D) $-\dfrac{5}{3}$ (E) $-\dfrac{7}{4}$

41. $\displaystyle\int_{-6}^{6} f(x)dx = $ _____. (nearest tenth)

(A) 28.3   (B) 29.4   (C) 30.6   (D) 31.7   (E) 32.8

42. Given: $\displaystyle\int_{-6}^{6} f(x)dx = 24$ and $\displaystyle\int_{6}^{10} f(x)dx = 8$. If $f(x)$ is an even function, then $\displaystyle\int_{0}^{10} f(x)dx =$ _____.

   (A) 8   (B) 12   (C) 16   (D) 20   (E) 24

43. If $\displaystyle\int_{1}^{x} f(t)dt = \frac{33x}{\sqrt{9x^2 - 12}} + 9$, then $\displaystyle\int_{1}^{\infty} f(t)dt =$ _____.

   (A) 9   (B) 12   (C) 20   (D) 24   (E) 42

44. The area in the first quadrant bounded by the curve $y = 2\cos\left(\dfrac{x}{4}\right)$, the line $x = \dfrac{3\pi}{2}$, x-axis, and the

   y-axis is divided into two equal parts by the line $x = k$. Find k. (nearest hundredth)

   (A) 1.92   (B) 1.94   (C) 1.96   (D) 1.98   (E) 2.00

45. Find the area of the inner loop of the graph of the limacon $r = 1 + 2\sin\theta$. (nearest hundredth)

   (A) 0.54   (B) 0.57   (C) 0.60   (D) 0.63   (E) 0.66

46. The position of a particle moving in the x-y plane is given by the parametric functions $x(t)$ and

   $y(t)$, where $0 \le t \le 2\pi$. If $x'(t) = t\cos t$ and $y'(t) = 3e^{-5t} + 6$, what is the slope of the line tangent to
   the graph of the particle's path when $t = 3$? (nearest hundredth)

   (A) −3.68   (B) −2.02   (C) 0.87   (D) 2.33   (E) 3.72

47. The function f is defined by $f(x) = 6x + e^{2x}$. If $g(x) = f^{-1}(x)$ for all x, what is the value of $g'(1)$?
   (nearest thousandth)

   (A) 0.125   (B) 0.250   (C) 0.500   (D) 0.750   (E) 1.000

**Use this information for problems 48 and 49.**
The position of a particle traveling along the x-axis is given by $x(t) = \sin(t)$, for $0 \le t \le 2\pi$.

48. Find the position of the particle, along the x-axis, when the velocity of the particle is $\dfrac{\sqrt{2}}{2}$ to the left,

   and the acceleration of the particle is $\dfrac{\sqrt{2}}{2}$ to the right.

   (A) −1   (B) $-\dfrac{\sqrt{2}}{2}$ (C) 0   (D) $\dfrac{\sqrt{2}}{2}$ (E) 1

49. Find the total distance traveled by the particle from $t = 0$ to $t = 2\pi$.

   (A) $2\sqrt{2}$ (B) π (C) 4   (D) $4\sqrt{2}$ (E) 2π

UIL Mathematics

50. Consider the volume of a solid formed using cross sections. The base is the region in the fourth quadrant bounded by the graph of the ellipse $25x^2 + 16y^2 = 400$. Each cross-section is perpendicular to the x-axis and is an isosceles right triangle with a leg as the base. Find the volume of the solid. (nearest tenth)

    (A) 29.1   (B) 29.4   (C) 29.7   (D) 33.0   (E) 33.3

51. Lance is cycling south on FM 2464 at 30 mph at the same time a truck is traveling west on FM 3131. At the moment Lance is 300 feet north of FM 3131, the truck is 400 feet west of FM 2464 and the distance between Lance and the truck is increasing at 26.4 ft/s. What is the speed of the truck? (nearest tenth)
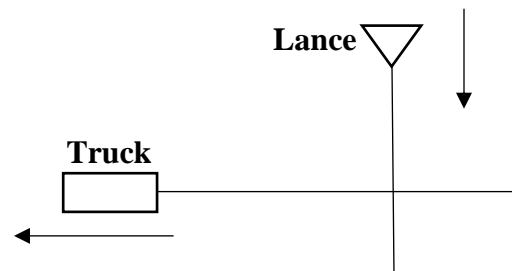
    (A) 42.6 mph   (B) 45.0 mph   (C) 47.4 mph
        (D) 49.8 mph   (E) 52.2 mph

    Problem 51

52. The illumination from a light source varies inversely with the square of the distance from the light source and directly with intensity of the light source. Two light sources are placed 7.00 m apart. The intensity of light source A is 54 cd and the intensity of light source B is 128 cd. Find the distance from light source A to the darkest point along a line connecting the two light sources. (nearest tenth)

    (A) 2.4 m   (B) 2.6 m   (C) 2.8 m   (D) 3.0 m   (E) 3.2 m

53. In a test of $H_0 : p = 0.37$ against $H_a : p \neq 0.37$, a random sample of size 200 yields a test statistic of $z = 2.25$. The P-value of this test is approximately equal to _____. (nearest thousandth)

    (A) 0.012   (B) 0.024   (C) 0.036   (D) 0.048   (E) 0.060

54. Suppose the mean SAT math score for students in Idaho is 523 with a standard deviation of 117 and the mean ACT math score is 20.4 with a standard deviation of 7.1. Jim scored 690 on the math section of the SAT test. Of the following, which is the lowest score he can make on the math section of the ACT test in order have done at least as well on the ACT test as he did on the SAT test?

    (A) 29   (B) 30   (C) 31   (D) 32   (E) 33

55. Mr. King retired and opened a candy store in a small town in New Mexico. He kept careful records and discovered that 72% of all customers purchase chocolate truffles, 48% purchase licorice and 36% purchase both. If a customer is randomly selected, what is the probability that she orders either chocolate truffles or licorice, but not both?

    (A) 40%   (B) 42%   (C) 44%   (D) 46%   (E) 48%

56. Dr. Good specializes in research on Australian wombats. This year, as part of her research, she needs to construct a 98% confidence interval for the average weight of adult wombats. Based on previous research, she estimates that the standard deviation is 2.20 kg. Dr. Good plans to select a random sample of wombats and record their weights. Of the following, which is the smallest sample size that will result in a margin of error of 0.125 kg or less for the confidence interval?

    (A) 1240   (B) 1350   (C) 1460   (D) 1570   (E) 1680

57. Professor Taylor drinks a bottle of V-2 vegetable juice for breakfast every day. V-2 is a blend of carrot juice and beet juice. The amount of carrot juice in each bottle can be modeled using a normal distribution with a mean of 4.0 ounces and a standard deviation of 0.08 ounces. The amount of beet juice can also be modeled using a normal distribution with a mean of 3.8 ounces and a standard deviation of 0.07 ounces. The Walmart Distribution Center in Sabine received a shipment of over 900,000 bottles. If six bottles are randomly selected, what is the probability that at least two of them will have more beet juice than carrot juice? (nearest ten-thousandth)

    (A) 0.0088   (B) 0.0106   (C) 0.0124   (D) 0.0142   (E) 0.0160

Use the table below for problems 58 and 59.

	Sports Station	Movie Station	Nature Station	Total
Beef	65	55	30	150
Turkey	50	60	80	190
Veggie	40	65	75	180
Total	155	180	185	520

58. Market researchers believe that the station on the TVs in a restaurant may affect the choices customers make when they order one of the three plates available at Maria's in Abilene. A study was done in which three TV stations were randomly assigned as treatments and the resulting number of each type of lunch plate ordered was recorded. The table above summarizes the data collected. A chi-square test was used to analyze the results. What was the contribution of the Veggie/Nature cell to the chi-square statistic? (nearest hundredth)

    (A) 1.88   (B) 2.00   (C) 2.12   (D) 2.24   (E) 2.36

59. Further analysis produced a chi-square statistic of 28.645416 and a p-value of _____. (3 significant digits)

    (A) 0.00000923   (B) 0.0000817   (C) 0.000711   (D) 0.00605   (E) 0.0499

60. Does spending six weeks of the summer attending the Newberry Number Sense camp improve the number sense scores of high school students? To find out, a pre-camp test was administered at the beginning of camp and a post-camp test was administered at the conclusion of camp. The two tests were written by Mr. White and determined to be of equal difficulty by Mr. Barton. The results are shown for the ten students who attended the camp last summer.

Student	1	2	3	4	5	6	7	8	9	10
Pre	122	144	116	148	166	152	202	104	216	188
Post	168	178	124	198	172	156	244	156	262	202

   Which of the following procedures is the most appropriate to determine if there is a significant difference in the Pre and Post number sense scores?

    (A) Two-proportion z test
    (B) Two sample t test for means
    (C) Chi-square test for independence
    (D) t-test for the slope of a regression line
    (E) Matched pairs t test for means

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2022
Answer Key

1. A	21. C	41. A
2. C	22. B	42. D
3. E	23. D	43. C
4. D	24. A	44. A
5. B	25. E	45. A
6. E	26. B	46. B
7. C	27. D	47. A
8. C	28. E	48. B
9. C	29. C	49. C
10. C	30. C	50. E
11. E	31. D	51. B
12. B	32. C	52. D
13. A	33. C	53. B
14. E	34. B	54. C
15. A	35. E	55. E
16. A	36. A	56. E
17. E	37. B	57. C
18. D	38. C	58. A
19. B	39. E	59. A
20. D	40. C	60. E

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Score **Initials**

Contestant # _____

Conference _____

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59. _____
60. _____