



**TMSCA HIGH SCHOOL
MATHEMATICS
TEST #3 ©
NOVEMBER 2, 2019**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

1. Evaluate: $12 \times 8 + 25 \div \left(\frac{1}{5}\right)^{-1} - (7^2 + 2)$

- (A) 48 (B) 50 (C) 52 (D) 54 (E) 56

2. Rachel gave her sweetie \$600 for his birthday. He plans to camp out in Colorado next week, so he went to Academy Sports and purchased some new hiking boots for \$145.00, a new backpack for \$225.00, 3 pairs of socks for \$7.85 each, and 4 shirts for \$22.50 each. If the sales tax rate is 8.25%, how much of his birthday money does he still have?

- (A) \$72.36 (B) \$74.46 (C) \$76.56 (D) \$78.66 (E) \$80.76

3. If 40% of A is 60% of B, then A is what percent of B?

- (A) 50% (B) $66\frac{2}{3}\%$ (C) 75% (D) $133\frac{1}{3}\%$ (E) 150%

4. Which of the following points is not a solution to $5x + 3y > 6$?

- (A) $(-4, 9)$ (B) $(-2, 6)$ (C) $(0, 3)$ (D) $(2, -1)$ (E) $(4, -5)$

5. If $6x^2 + 11x + c = (3x - 5)(ax + b)$, then $a + b + c =$ _____.

- (A) -26 (B) -18 (C) -9 (D) 6 (E) 14

6. Find the sum of the mean, median, range and mode of these number sense test scores. 166, 202, 158, 246, 141, 202, 288 and 133.

- (A) 721 (B) 725 (C) 729 (D) 733 (E) 737

7. $\angle A$ is complementary to $\angle B$ and $\angle B$ is supplementary to $\angle C$. If $m\angle A = 3x + 10$ and $m\angle B = 2x + 5$, then $m\angle C =$ _____.

- (A) 125° (B) 130° (C) 135° (D) 140° (E) 145°

8. Line L_1 has a y-intercept at $(0, 8)$ and an x-intercept at $(5, 0)$. Line L_2 contains the point $(4, 6)$ and it is perpendicular to line L_1 . Find the y-intercept of line L_2 .

- (A) $(0, 2)$ (B) $\left(0, \frac{5}{2}\right)$ (C) $(0, 3)$ (D) $\left(0, \frac{7}{2}\right)$ (E) $(0, 4)$

9. Simplify: $\left(\frac{2x^2 - 5x - 3}{3x^2 + 10x + 8}\right)\left(\frac{3x^2 - 2x - 8}{2x^2 - 5x - 3}\right)$

- (A) $\frac{x-2}{x+2}$ (B) $\frac{x+2}{x-2}$ (C) $\frac{x+3}{x-3}$ (D) $\frac{2x-1}{2x+1}$ (E) $\frac{3x+4}{3x-4}$

10. $f(x) = 3x^2 + 6$ and $g(x) = \sqrt[4]{x}$. If $g(f(a)) = 3$ and $a > 0$, then $a =$ _____.

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

11. $\frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \dots + \frac{1}{120} + \frac{1}{136} =$ _____.

- (A) $\frac{8}{17}$ (B) $\frac{28}{51}$ (C) $\frac{11}{17}$ (D) $\frac{33}{51}$ (E) $\frac{15}{17}$

12. Find the sum of all the prime numbers between 100 and 130.

- (A) 551 (B) 595 (C) 660 (D) 724 (E) 789

13. Which of the following is equivalent to $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$?

- (A) 0 (B) $\sin(2x)$ (C) $\cos(2x)$ (D) $\csc x + \sec x$ (E) 2

14. Yogi's team has 10 infielders, 8 outfielders, 3 catchers, and 9 pitchers. How many ways can he select the starters for tonight's game if he must choose 4 infielders, 3 outfielders, 1 catcher and 1 pitcher?

- (A) 2160 (B) 105600 (C) 244000 (D) 317520 (E) 410240

15. Richard's pool has developed a leak. A completely full pool will empty in 24 hours. His hose would completely fill the pool when empty in 10 hours before the pool started to leak. He did not use his pool for a few days and the pool is now empty. If he does not fix the leak, how long will it take him to fill the pool? (nearest minute)

- (A) 17 hr 9 min (B) 17 hr 22 min (C) 17 hr 35 min (D) 17 hr 48 min (E) 18 hr 1 min

16. Cindy has a collection of 84 marbles that are red, green, blue or black. She has 9 more black ones than blue ones. She has 3 more blue ones than green ones. She has twice as many black ones as red ones. How many green marbles does Cindy have?

- (A) 16 (B) 18 (C) 20 (D) 22 (E) 24

17. Consider the graph of the ellipse with equation $9x^2 + 25y^2 + 36x - 50y = 164$. One of the vertices is

- (A) $(-6, 1)$ (B) $(-5, 1)$ (C) $(-2, 1)$ (D) $(3, 1)$ (E) $(4, 1)$

18. $\ln(x-4) + \ln(x-3) = \ln(3x-12)$. $x =$ _____.

- (A) no solution (B) 4 only (C) 6 only (D) 4 and 6 (E) 6 and 8

19. Consider the curve with equation $y = 6 - 2\sin 3\left(x - \frac{\pi}{6}\right)$. A maximum occurs when $x =$ _____.
- (A) $\frac{7\pi}{6}$ (B) $\frac{4\pi}{3}$ (C) $\frac{11\pi}{6}$ (D) $\frac{7\pi}{3}$ (E) $\frac{5\pi}{2}$
20. Twenty ping pong balls are numbered from 1 to 20 and placed in a box. One of the balls is randomly drawn out of the box. What are the odds that this ball does not have a Fibonacci number on it?
- (A) $\frac{11}{9}$ (B) $\frac{3}{2}$ (C) $\frac{13}{7}$ (D) $\frac{7}{3}$ (E) $\frac{17}{3}$
21. The difference between the 7th hexagonal number and the 7th triangular number is _____.
- (A) 51 (B) 54 (C) 57 (D) 60 (E) 63
22. The function $y = ax^4 + bx^2 + 6$ has a point of inflection at $\left(\frac{\sqrt{2}}{2}, \frac{7}{2}\right)$. $a - b =$ _____.
- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12
23. The sum of the coefficients of the 1st term and the 7th term of the expansion of $(2x - 3y)^6$ is _____.
- (A) 769 (B) 781 (C) 793 (D) 805 (E) 817
24. The graph of the polar equation $r = 3\cos(4\theta)$ is a/an _____.
- (A) 3-petaled rose (B) cardioid (C) 4-petaled rose (D) lemniscate (E) 8-petaled rose
25. This mathematician was the daughter of George Boole and she is remembered for her work in 4-dimensional geometry.
- (A) Alicia Stott (B) Freda Porter (C) Lady Lovelace (D) Emmy Noether (E) Olga Olenik
26. The 7th term of an arithmetic sequence is 9 and the 10th term is 18. Find the sum of the first 15 terms.
- (A) 177 (B) 180 (C) 183 (D) 186 (E) 189
27. The lateral area of a cone is 204.2 cm^2 and the ratio of the diameter to the height is 5:6. Find the volume of the cone. (nearest hundredth of a cm^3)
- (A) 308.22 cm^3 (B) 310.20 cm^3 (C) 312.18 cm^3 (D) 314.16 cm^3 (E) 316.14 cm^3
28. If $x - y = 5$ and $xy = 84$, then $x^3 - y^3 =$ _____.
- (A) 1385 (B) 1557 (C) 1728 (D) 1900 (E) 2071

29. Let $f(x) = -2x^3 + 6x^2 - 4$. Find the x-intercept of the line tangent to the graph of $f(x)$ when $x = 2.1$ (nearest tenth)

- (A) 4.8 (B) 5.0 (C) 5.2 (D) 5.4 (E) 5.6

30. Find the sum of all zeros of $f(x) = 2\sin^2(x) - \sqrt{3}\sin(x)$ in the interval $0 \leq x \leq \pi$.

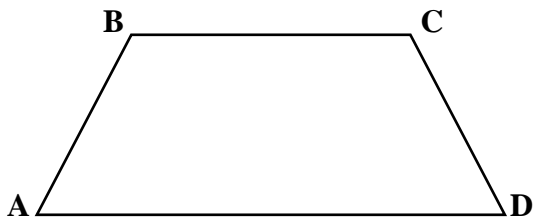
- (A) $\frac{2\pi}{3}$ (B) π (C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$ (E) 2π

31. If $f(x) = x^2 + 6x$ and $g(x) = 3x - 4$, then $f(g'(2)) =$ _____.

- (A) 0 (B) 15 (C) 27 (D) 36 (E) 42

32. $2044_b - 156_b + 303_b = 2161_b$. $3456_b =$ _____ base 10.

- (A) 1156 (B) 1202 (C) 1234 (D) 1266 (E) 1304



Isosceles Trapezoid ABCD with area = 450 in^2

33. If $BC = 20 \text{ in}$ and $AD = 30 \text{ in}$, then $AB =$ _____ (nearest tenth of an inch)

- (A) 17.5 in (B) 17.8 in (C) 18.1 in (D) 18.4 in (E) 18.7 in

34. Change $0.3404040\dots$ base 5 to a base 6 decimal.

- (A) 0.3555... (B) 0.4202020... (C) 0.4333... (D) 0.4505050... (E) 0.5111...

35. The number 84 is classified as which of the following types of numbers?

- I. abundant II. happy III. lucky IV. odious

- (A) III, IV only (B) II, III only (C) I, IV only (D) I, II, III only (E) I, II, III and IV

36. The sum of the positive integral divisors of 1800 = _____.

- (A) 5875 (B) 5925 (C) 5960 (D) 6025 (E) 6045

37. Find the area of the region bounded by the curves $y_1 = 2x^2 - 8$ and $y_2 = -\frac{1}{2}x^2 + 8$. (nearest tenth)

- (A) 54.0 (B) 55.3 (C) 56.6 (D) 57.9 (E) 59.2

38. When $f(x) = 2x^3 + bx^2 + 4x - 5$ is divided by $(x - 2)$, the remainder is 7. $b =$ _____.

- (A) -1 (B) 2 (C) -3 (D) 4 (E) -5

39. Consider circle O with a diameter of 22 cm. Points A and C lie on the circle. If $m\angle AOC = 120^\circ$, then the area of sector AOC is _____. (nearest cm^2)

- (A) 122.5 cm^2 (B) 124.6 cm^2 (C) 126.7 cm^2 (D) 128.8 cm^2 (E) 130.9 cm^2

40. What is the fifth harmonic number?

- (A) 2.0333... (B) 2.1666... (C) 2.222... (D) 2.28333... (E) 2.3444...

41. If $y^3 = -46 - 9i$ and $y^4 = -119 + 120i$ where $y = a + bi$, then $a + b =$ _____.

- (A) -1 (B) 1 (C) 3 (D) 5 (E) 7

42. Bob is 5 years older than Judy. In 20 years, Judy will be $\frac{7}{8}$ of Bob's age. How old is Judy?

- (A) 10 (B) 12 (C) 15 (D) 18 (E) 20

43. $\begin{bmatrix} 2 & 1 & 3 \\ 6 & 2 & 4 \\ -1 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ k \\ 7 \end{bmatrix} = \begin{bmatrix} 28 \\ 52 \\ -20 \end{bmatrix}$ $k =$ _____.

- (A) -5 (B) -3 (C) -1 (D) 1 (E) 3

44. Eliminate the parameter and find the x-intercept of the resulting rectangular equation.

$$x = -5t$$

$$y = 4 - 2t$$

- (A) -16 (B) -14 (C) -12 (D) -10 (E) -8

45. Find the sum of the integers that are in the solution set for $|4x + 3| - 10 \leq 13$ is _____.

- (A) -6 (B) 0 (C) 5 (D) 6 (E) 13

46. Vivian is a lifetime 86% free throw shooter. In a game against Nebraska, she attempted 18 free throws. Find the probability that she made at least 15 of them. (nearest hundredth)

- (A) 0.70 (B) 0.72 (C) 0.74 (D) 0.76 (E) 0.78

47. What is the sum of the Fibonacci numbers between 500 and 1000?

- (A) 1466 (B) 1513 (C) 1597 (D) 1644 (E) 1721

48. The function $f(x) = \frac{x^2 + 2x - 35}{x^2 - 25}$ has a removable discontinuity at $x =$ _____.

- (A) -7 (B) -5 (C) 1 (D) 5 (E) 7

49. Consider the function, $f(x)$, that fits the data in the table. $f(3) =$ _____.

X	-2	-1	0	1	2
f(x)	-24	-1	4	3	8

- (A) 18 (B) 21 (C) 24 (D) 27 (E) 31

50. The circles $(x - 2)^2 + (y + 3)^2 = 36$ and $(x + 4)^2 + (y - 5)^2 = 49$ intersect in two points. Find the slope of the line that contains these two points.

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

51. Consider the Fibonacci characteristic sequence $-2, a, b, 6, c, d, 22, \dots$ $a + b + c + d =$ _____.

- (A) 24 (B) 26 (C) 28 (D) 30 (E) 32

52. Kermit Donuts offers glazed, chocolate, jelly, maple and Boston Crème donuts. Kevin purchased 12 donuts to take on the trip to region. How many different ways could he select 12 donuts?

- (A) 60 (B) 360 (C) 840 (D) 1820 (E) 6188

53. A right triangle has legs with lengths 9 and 40. Find the area of the inscribed circle. (nearest tenth)

- (A) 49.1 (B) 50.3 (C) 51.5 (D) 52.7 (E) 53.9

54. In how many distinct ways can 4 of the letters of ODESSA be arranged to form a 4 letter code?

- (A) 120 (B) 144 (C) 192 (D) 216 (E) 244

55. Ray inherited \$120,000 from his grandmother and he wisely decided to place the money in 3 savings accounts. He placed \$40,000 into an account that earns 4.5% annual interest compounded quarterly and he placed \$50,000 into an account that earns 6.25% annual interest compounded monthly. He placed the rest in an account that earns 3.6% annual interest compounded quarterly. At the end of 7 years, how much interest will he have earned?

- (A) \$50,077.44 (B) \$50,101.64 (C) \$50,541.75 (D) \$50,588.22 (E) \$50,622.10

56. Let $g(x) = \frac{4x - 7}{9 - 2x}$. $g'(4) =$ _____.

- (A) 10 (B) 13 (C) 16 (D) 19 (E) 22

57. Evaluate: $\int_{-a}^a (4x^5 - 3x^3 + 5x) dx$

- (A) 0 (B) $6a^6 - 6a^4 + 6a^2$ (C) $4a^4 - 3a^2$ (D) $2a^2$ (E) $22a^4 - 18a^2$

58. How many non-negative proper fractions in lowest terms have a denominator of 72?

- (A) 20 (B) 24 (C) 28 (D) 32 (E) 36

59. Let $f(x) = \frac{x^2 - 6x + 4}{x - 2}$ and let $s(x)$ be the slant asymptote of the graph of $f(x)$. $s(9) = \underline{\hspace{2cm}}$.

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

60. A hoot owl is perched on the edge of the roof of the tallest building in Tenaha. He spots a mouse on the ground 280 feet from the base of the building. The angle of depression from the owl to the mouse is 22.5° . How tall is the tallest building in Tenaha? (nearest foot)

- (A) 104 ft (B) 108 ft (C) 112 ft (D) 116 ft (E) 120 ft

2019 – 2020 TMSCA High School Mathematics Test # 3
Answer Key

1. B	21. E	41. A
2. C	22. C	42. C
3. E	23. C	43. B
4. E	24. E	44. D
5. A	25. A	45. A
6. D	26. B	46. D
7. E	27. D	47. C
8. D	28. A	48. D
9. A	29. C	49. E
10. C	30. E	50. E
11. B	31. C	51. C
12. C	32. D	52. D
13. E	33. E	53. B
14. D	34. C	54. C
15. A	35. C	55. E
16. B	36. E	56. E
17. D	37. A	57. A
18. C	38. C	58. B
19. B	39. C	59. C
20. D	40. D	60. D

19-20 TMSCA HSMA Test # 3 Selected Solutions

$$2. 600 - (\pi 45 + 225 + 3(7.85) + 4(22.50))(1.0825) = 76.56$$

$$4. y > -\frac{5}{3}x + 2$$

$$m = -\frac{8}{5}, m_p = \frac{5}{8}$$

$$5. (3x-5)(2x+7) = 6x^2 + 11x - 35$$

$$a + b + c = -26$$

$$6. \bar{x} = 192, M = 184, R = 155, m = 202$$

$$733$$

$$8. y - 6 = \frac{5}{8}(x - 4)$$

$$y = \frac{5}{8}x + \frac{7}{2} \rightarrow \left(0, \frac{7}{2}\right)$$

$$10. \sqrt[4]{3a^2 + 6} = 3$$

$$a = 5$$

$$11. \frac{15}{17} - \frac{1}{3} = \frac{28}{51}$$

$$12. 101 + 103 + 107 + 109 + 113 + 127 = 660$$

$$13. \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) +$$

$$\sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x)$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$2(1) = 2$$

$$14. \frac{{}^{10}C_4 \cdot {}^8C_3 \cdot 3 \cdot 9}{317,520}$$

$$\frac{t}{10} - \frac{t}{24} = 1$$

$$15. t = 17\frac{1}{7}$$

$$17 \text{ hr } 9 \text{ min}$$

$$R + G + B + X = 84$$

$$X = B + 9$$

$$16. B = G + 3$$

$$X = 2R$$

$$G = 18$$

$$9x^2 + 36x + 25y^2 - 50y = 164$$

$$9(x+2)^2 + 25(y-1)^2 = 225$$

$$17. \frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$$

$$-2 + 5 = 3 \rightarrow (3, 1)$$

$$1, 2, 3, 5, 8, 13$$

$$20. \frac{14}{6} = \frac{7}{3}$$

$$21. 91 - 28 = 63$$

$$y'' = 12ax^2 + 2b = 0$$

$$12a\left(\frac{1}{2}\right) + 2b = 0$$

$$22. \frac{7}{2} = a\left(\frac{1}{4}\right) + b\left(\frac{1}{2}\right) + 6$$

$$a = 2, b = -6$$

$$2 - -6 = 8$$

$$9 + 3d = 18, d = 3$$

$$9 = a_1 + 6(3), a_1 = -9$$

$$26. a_{15} = -9 + 14(3) = 33$$

$$S = \left(\frac{15}{2}\right)(-9 + 33) = 180$$

$$204.2 = \pi RL$$

$$\frac{R}{H} = \frac{5}{12}$$

$$27. R^2 + H^2 = L^2$$

$$V = \left(\frac{1}{3}\right)\pi(5^2)(12)$$

$$V = 314.16$$

$$y = -1.26x + 6.584$$

$$29. 0 = -1.26x + 6.584$$

$$x = 5.2$$

$$30. 0 + \frac{\pi}{3} + \frac{2\pi}{3} + \pi$$

$$2\pi$$

$$g'(2) = 3$$

$$31. f(3) = 3^2 + 6(3)$$

$$27$$

$$32. \frac{b}{1266} = 7$$

$$450 = \left(\frac{1}{2}\right)h(20 + 30)$$

$$33. h = 18$$

$$5^2 + 18^2 = L^2$$

$$L = 18.7$$

$$34. \frac{35}{50} = .4333... \quad 36. \frac{2^3 \cdot 3^2 \cdot 5^2}{6045} (15)(13)(31) \quad 37. \int_{-2.5298221281}^{2.5298221281} (y_2 - y_1) dx = 54.0 \quad 39. \left(\frac{120}{360} \right) \cdot \pi \cdot 11^2 = 126.7$$

$$40. 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 2.28333... \quad 41. \frac{-119 + 120i}{-46 - 9i} = 2 - 3i \quad 42. J + 20 = \frac{7}{8}(B + 20) \quad 43. \frac{10 + k + 21}{k} = 28$$

$$B = J + 5 \quad B = 20, J = 15$$

$$44. t = -\frac{x}{5} \quad 45. -\frac{13}{2} \leq x \leq 5 \quad 46. \text{binomial Cdf}(18, .86, 15, 18) = .76 \quad 47. 610 + 987 = 1597$$

$$y = 4 - 2\left(-\frac{x}{5}\right) \quad 0 = 4 + \frac{2}{5}x \quad x = -10$$

$$48. \frac{(x+7)(x-5)}{(x+5)(x-5)} \quad 49. y = 2x^3 - 3x^2 + 4 \quad 50. -4x + 6y - 23 = 8x - 10y - 8 \quad 51. -2 + a = b$$

$$y(3) = 31 \quad 16y = 12x + 15 \quad a + b = 6$$

$$y = \frac{3}{4}x + \frac{15}{16} \quad 51. a = 4, b = 2, c = 8, d = 14$$

$$52. {}_{16}C_{12} = 1820 \quad 53. c = 41 \quad 54. 5 \cdot 4 \cdot 3 \cdot 2 = 120 \quad 58. {}^{72}\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)$$

$$r = \frac{(9)(40)}{9 + 40 + 41} \quad 4C_2 \cdot 4 \cdot 3 = 72 \quad 120 + 72 = 192 \quad 24$$

$$r = 4 \quad A = \pi \cdot 4^2 = 50.3$$

$$59. s(x) = x - 4 \quad 60. \tan(22.5^\circ) = \frac{h}{280}$$

$$s(9) = 5 \quad h = 116$$