T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 8 ©

J A N U A R Y 2 0 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2017-2018 TMSCA Mathematics Test Eight

1. **Evaluate:** $0.41666... \div 0.75 + 1.75 - 0.8333... \times 1.4$.

   (A) $\frac{203}{360}$ **(B)** $1\frac{22}{45}$ **(C)** $1\frac{49}{240}$ **(D)** $1\frac{5}{36}$ **(E)** $\frac{43}{48}$

2. **Carol started her weekend with \$80. She spent \$22 on ingredients for dinner Friday night and \$11.50 on a movie Saturday. On Sunday, she paid \$4.50 each for 3 games at the bowling alley and \$6.50 on snacks. How much money did Carol have left at the end of the weekend?**

   (A) \$7.96 **(B)** \$12.25 **(C)** \$8.46 **(D)** \$8.49 **(E)** \$26.50

3. **Lynn sold 83 school orchestra concert tickets for a total of \$673.75. Each adult ticket sold for \$9.50, and each student ticket sold for \$5.25. How many adult tickets did she sell?**

   (A) 56 **(B)** 27 **(C)** 43 **(D)** 40 **(E)** 51

4. **Which of the following relations describes a 1-1 function?**

   (A) $\{(2,4),(-2,-4),(3,5),(-3,-5)\}$ **(B)** $\{(2,4),(-2,4),(3,5),(-3,5)\}$ **(C)** $\{(2,4),(2,-4),(3,5),(3,-5)\}$

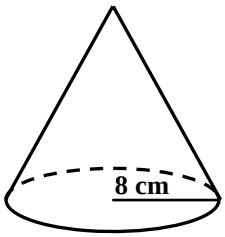
   (D) $\{(-2,4),(-2,-4),(-3,5),(-3,-5)\}$ **(E)** none of these

5. **Larry, Mark and Norman can each tile a floor in 12 hours, 9 hours and 10 hours respectively. How long would it take them to tile a floor that is twice as long and twice as wide together? (nearest minute)**

   (A) 13 h 58 min **(B)** 6 h 48 min **(C)** 6 h 59 min **(D)** 13 h 35 min **(E)** 11 h 8 min

6. **The total surface area of the cone shown is 653 cm^2. The slant height is _____cm. (nearest tenth)**

   (A) 25.9 **(B)** 18.0 **(C)** 9.0 **(D)** 23.9 **(E)** 21.7



7. **Simplify:** $\frac{n!(n+3)!}{(n-2)!} \div \frac{(n+1)!(n+2)!}{(n-1)!}$.

   (A) $\frac{n^2-2n-3}{n-1}$ **(B)** $\frac{n^2-n-2}{n-1}$ **(C)** $\frac{n^2-n-2}{n+1}$

   (D) $\frac{n^2+2n-3}{n-1}$ **(E)** $\frac{n^2+2n-3}{n+1}$

8. **If** $21x^2-2x-8=(ax+b)(cx+d)$ **where *a, b, c* and *d* are all integers then** $ab+cd=$ **?**

   (A) 4 **(B)** 13 **(C)** 11 **(D)** 22 **(E)** 3

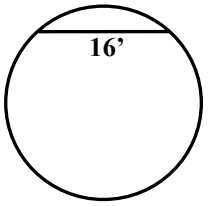
9. **Find the greatest common divisor of** $2^3\times7^2$, $3^2\times2^7$ **and** $2^3\times7^3$.

   (A) 2 **(B)** 21 **(C)** 42 **(D)** 14 **(E)** 8

10. If $\frac{x-4}{x+5}+\frac{x+5}{x-4}$ is written as the mixed number $A\frac{B}{C}$ then $B=$?

(A) 9 (B) 18 (C) 81 (D) 400 (E) 20

11. The circle C shown has a radius of 12 feet. How far is the chord shown from the center of the circle?



(A) $4\sqrt{7}$ ft (B) $4\sqrt{5}$ ft (C) $2\sqrt{22}$ ft (D) $2\sqrt{14}$ ft (E) $2\sqrt{10}$ ft

12. A highway exit ramp must cover a horizontal distance of 483 ft. and a vertical distance of -27 ft. What is the angle of depression for the ramp? (nearest minute)

(A) 3° 18' (B) 3° 6' (C) 3° 12' (D) 3° 26' (E) 3° 20'

13. Which of the following are the side lengths of a isosceles, obtuse triangle?

(A) 11, 15, 21 (B) 13, 13,16 (C) 9, 9, 12 (D) 11, 11, 18 (E) 10, 24, 26

14. Sherry has 10 non-fiction books to shelve. She wants to keep her 4 math books together, but otherwise any order is fine. In how many different ways can Lily organize her shelf?

(A) 20160 (B) 3,628,800 (C) 151,200 (D) 720 (E) 120,960

15. A particle's movement along the number line is defined by $f(t)=t^4-4t^3-26t^2+60t+25$. At which of the following times is the particle moving to the left?

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

16. Simplify: $\frac{\left(b^{-2}\right)^3\left(\sqrt[3]{b}\right)}{b^{(-1/2)}}\times b^3$

(A) $\left(\sqrt[6]{b}\right)^{-19}$ (B) $\left(\sqrt[6]{b}\right)^{-13}$ (C) $\left(\sqrt[6]{b}\right)^{-7}$ (D) $\left(\sqrt[6]{b}\right)^{29}$ (E) $\left(\sqrt[6]{b}\right)^{-15}$

17. Let $f(x)=x^2+1$ and $g(x)=x^4$. Calculate $f(g(-2))$.

(A) 257 (B) 65 (C) 625 (D) 125 (E) 81

18. Find $m+n$ if $\begin{bmatrix}4 & 3\\ 2 & 1\end{bmatrix}\bullet\begin{bmatrix}m\\ n\end{bmatrix}=\begin{bmatrix}5\\ -3\end{bmatrix}$.

(A) –11 (B) –4 (C) 18 (D) –15 (E) 4

19. If the pattern of the sequence 9046, 9042, 9035, 9022, 9000, 8966, …continues, find the smallest positive term.

(A) 890 (B) 529 (C) 192 (D) 2 (E) 167

20. On triangle ABC, AB = 40 cm, BC = 36 cm and $m\angle A = 60^\circ$. What is the sum of the two possible values of AC?

(A) 20 cm (B) 25 cm (C) 40 cm (D) 22 cm (E) 30 cm

21. Using the following array, determine the value of the median of the 23rd row.

2					(row 1)
4	6				(row 2)
8	10	12			(row 3)
14	16	18	20		(row 4)
22	24	26	28	30	(row 5)
…					(…)

(A) 530 **(B)** 485 **(C)** 577 **(D)** 531 **(E)** 483

22. Which of the following is the reference angle for 600°?

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{6}$ **(E)** $\frac{\pi}{7}$

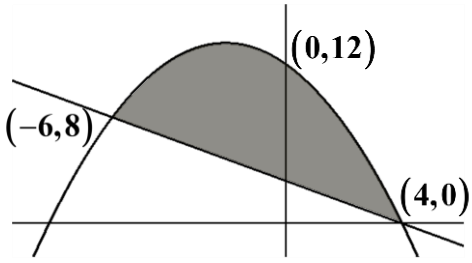
23. The ellipse $9x^2 + y^2 - 54x + 45 = 0$ has foci at (x_1, y_1) and (x_2, y_2). Find the value of $x_1 + x_2$.

(A) $4\sqrt{2}$ **(B)** 0 **(C)** 3 **(D)** 6 **(E)** $-3 + 4\sqrt{2}$

24. What is the constant term in the binomial expansion of $\left(3x - \frac{2}{x^4}\right)^5$?

(A) −32 **(B)** −1458 **(C)** −30 **(D)** 240 **(E)** −810

25. Find volume of the solid generated when the shaded region bounded by the parabola and the line in the illustration is rotated 360° around the *x*-axis. (nearest cubic unit)



(A) 937 **(B)** 2944 **(C)** 1408 **(D)** 2176 **(E)** 996

26. $(212121_4 + 121212_4) \times 11_4 =$ _____$_4$.

(A) 1,333,323 **(B)** 10,333,323 **(C)** 10,323,333 **(D)** 10,333,233 **(E)** 1,333,233

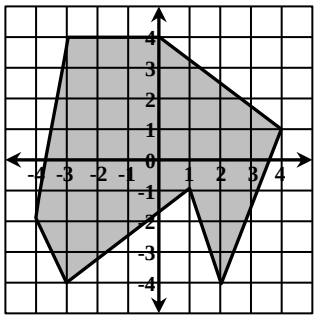
27. What is the probability that a factor of 441 is a multiple of 7?

(A) $\frac{3}{7}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{2}{7}$

28. Evaluate: $(0.555...)^{-1} \div (0.8333...)^{-1} \times (0.333...)^{-1}$

(A) $\frac{162}{25}$ **(B)** $\frac{18}{25}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{2}$ **(E)** $\frac{25}{18}$

29. The coordinates of the vertices of the heptagon shown are all integers. What is the area of the figure?



(A) 44.5 **(B) 23** **(C) 48** **(D) 41.5** **(E) 28**

30. The number 456 in base 9 is equivalent to the number *k* in base 3. Find the sum of the digits in the number *k*.

(A) 7 **(B) 8** **(C) 6** **(D) 5** **(E) 4**

31. The United States Supreme Court consists of 4 women and 8 men. How many distinct 3-person sub-committees could be formed from the justices if the committee must contain at least one woman?

(A) 112 **(B) 220** **(C) 160** **(D) 56** **(E) 164**

32. If $f''(x) = 12x - 6$ and $f(1) = -7$ and $f(2) = -1$, then $f(-1) =$ _____.

(A) -37 **(B) -13** **(C) 1** **(D) 11** **(E) 5**

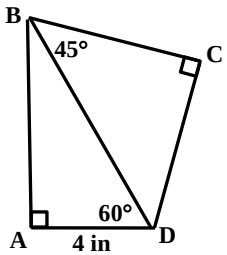
33. If $3 + 2i$ is one of the zeros the polynomial $f(x) = 2x^3 - 11x^2 + 20x + 13$, then another of its zeros is:

(A) -2 **(B) $-\frac{3}{2}$** **(C) $\frac{5}{2}$** **(D) $-\frac{1}{2}$** **(E) $\frac{3}{2}$**

34. Two unbiased standard dice are thrown. Find the probability that the product of the numbers on the two dice is a multiple of 4.

(A) $\frac{4}{9}$ **(B) $\frac{5}{12}$** **(C) $\frac{5}{9}$** **(D) $\frac{1}{2}$** **(E) $\frac{11}{18}$**

35. What is the perimeter of the quadrilateral ABCD? (nearest inch)



(A) 27 in **(B) 23 in** **(C) 22 in** **(D) 26 in** **(E) 29 in**

36. How many elements are in $\left\{ x \middle| \sin^2(2x) = \cos x, x \in (0, 2\pi] \right\}$?

(A) 0 **(B) 4** **(C) 6** **(D) 7** **(E) 9**

37. Kayla is three times as old as Lynn, and Lynn is twelve years younger than Mary. In 4 years, the sum of their ages will be 64? How old is Lynn?

(A) 8 **(B) 6** **(C) 16** **(D) 12** **(E) 20**

38. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ be the terms of the Fibonacci sequence. Find $f_{12} + f_{14}$.

(A) 377 **(B)** 449 **(C)** 570 **(D)** 521 **(E)** 510

39. Let x vary inversely with $y^2 + 2$. If $x = 8$ when $y = 5$, find x when $y = 4$.

(A) 24 **(B)** 18 **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{9}$ **(E)** 12

40. What is the angle between the minute and hour hands on a circular clock at 2:36 pm?

(A) 130° **(B)** 138° **(C)** 156° **(D)** 131° **(E)** 134°

41. Which of the following is closed under addition?

I. natural numbers **II.** irrational numbers **III.** negative numbers **IV.** odd integers

(A) II **(B)** I, III & IV **(C)** I, II & III **(D)** I & III **(E)** none of these

42. If the height of a right conical container is cut in half and the radius of the base is doubled, then what is the ratio of the volume of the original container to the volume of the new container?

(A) 1:2 **(B)** 1:1 **(C)** 4:1 **(D)** 1:4 **(E)** 2:1

43. The function $f(x) = \frac{6x^2 + 5x - 25}{9x^2 - 25}$ has a vertical asymptote at $x = V$ and a horizontal asymptote at $y = H$. Find $V + H$

(A) 2 **(B)** 1 **(C)** 3 **(D)** –1 **(E)** $\frac{9}{10}$

44. How many 3-digit numbers exist such that the sum of their digits equals 12?

(A) 68 **(B)** 66 **(C)** 71 **(D)** 67 **(E)** 69

45. If $f(x) = \tan x$, then $\lim_{h \to 0} \frac{f\left(\frac{\pi}{3} + h\right) - f\left(\frac{\pi}{3}\right)}{h}$ is

(A) 4 **(B)** –1 **(C)** 2 **(D)** $\frac{1}{2}$ **(E)** undefined

46. The inequality $x + 2 > \frac{12}{x - 2}$ is true for what percentage of the real number values on the interval $[-10, 10]$?

(A) 30% **(B)** 70% **(C)** 80% **(D)** 60% **(E)** 40%

47. The repeating decimal 0.3111… in base 5 can be written as which of the following fractions in base 5 in simplified terms?

(A) $\frac{103}{330_5}$ **(B)** $\frac{24}{120_5}$ **(C)** $\frac{13}{20_5}$ **(D)** $\frac{2}{10_5}$ **(E)** $\frac{23}{40_5}$

48. If $x-\frac{1}{x}=6$, then $x^3-\frac{1}{x^3}=?$

(A) 228 (B) 210 (C) 222 (D) 218 (E) 234

49. How many integral values of *n* exist such that $n\geq 1$ and $\frac{(n+4)!}{(n+2)!}<120$?

(A) 4 (B) 6 (C) 5 (D) 7 (E) 8

50. Lauren has bins of apple, cherry, grape and strawberry lollipops. In how many ways can she package them in bags of eight for a fundraiser?

(A) 495 (B) 165 (C) 1287 (D) 792 (E) 330

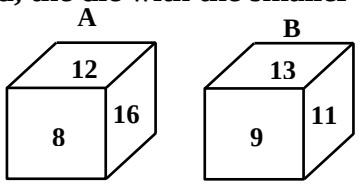
51. Which of the following mathematicians is an American Indian who received a Ph.D. in mathematics?

(A) Marsenne (B) Noether (C) Porter (D) Germain (E) Williams

52. Find the distance between the line $7x+3y=42$ and the point $(-8,3)$. (nearest tenth)

(A) 9.7 (B) 10.5 (C) 9.1 (D) 11.7 (E) 10.1

53. Leonard created a pair of special dice which have only three numbers each. The side opposite of each number is the same as the number. When the dice shown are rolled, the die with the smaller number on top wins. What is the probability that die B will win?



(A) $\frac{1}{9}$ (B) $\frac{5}{9}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$ (E) $\frac{4}{9}$

54. Myrtle left her dock on the lake shore and swam 0.9 km on a bearing of 307°, then she changed directions and swam another 0.7 km on a bearing of 33°. If there are no obstructions, how far will Myrtle need to swim to go directly back to the dock? (nearest meter)

(A) 510 m (B) 984 m (C) 1178 m (D) 775 m (E) 1099 m

55. Find the *y*-intercept of the tangent line to $5x^2+3y^2=17$ at the point $(-1,2)$.

(A) $\left(0,\frac{17}{6}\right)$ (B) $(0,-17)$ (C) $\left(0,\frac{17}{4}\right)$ (D) $\left(0,\frac{1}{6}\right)$ (E) $\left(\frac{13}{6},0\right)$

56. $f(x)=1+x-\frac{x^2}{2}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}-\frac{x^6}{6!}\ldots$. Find the digit in the 10^{-8} place of $f(3)$.

(A) 4 (B) 5 (C) 8 (D) 2 (E) 0

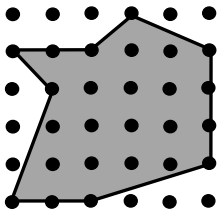
57. The lengths of the sides of triangle PQR are the roots of $f(x)=2x^3-39x^2+238x-441$. The perimeter of triangle PQR is 19.5. Find the area of triangle PQR. (nearest tenth)

(A) 15.9 (B) 12.3 (C) 13.1 (D) 11.2 (E) 10.7

58. Solve $2^{2x+1} - 2^{x+1} + 1 = 2^x$ for x.

(A) $-1, \frac{1}{2}$ **(B)** $\frac{1}{2}, 1$ **(C)** $-1, 0$ **(D)** $0, 1$ **(E)** $0, \frac{1}{2}$

59. Find the area of the shaded region on the illustration shown if the dots on the grid are 2 cm. apart both horizontally and vertically.



(A) 36 cm^2 **(B)** 76 cm^2 **(C)** 38 cm^2 **(D)** 72 cm^2 **(E)** 74 cm^2

60. $B478C_{13} + 5A937_{13} =$ _____$_{13}$.

(A) 1323B6 **(B)** C23C6 **(C)** 1123B6 **(D)** 1423B5 **(E)** 1423C6

2017-2018 TMSCA Mathematics Test Eight Answers

1. D	**21. A**	**41. D**
2. E	**22. A**	**42. A**
3. A	**23. D**	**43. D**
4. A	**24. E**	**44. B**
5. D	**25. B**	**45. A**
6. B	**26. B**	**46. D**
7. E	**27. C**	**47. E**
8. D	**28. D**	**48. E**
9. E	**29. D**	**49. D**
10. C	**30. A**	**50. B**
11. B	**31. E**	**51. C**
12. C	**32. B**	**52. D**
13. D	**33. D**	**53. B**
14. E	**34. B**	**54. C**
15. E	**35. C**	**55. A**
16. B	**36. C**	**56. C**
17. A	**37. A**	**57. D**
18. E	**38. D**	**58. C**
19. E	**39. E**	**59. D**
20. C	**40. B**	**60. E**

2017-2018 TMSCA Mathematics Test Eight Selected Solutions

14. The possible arrangements of the math books is 4!, then treat them as a group along with the other books for 7! for $(4!)(7!) = 120{,}960$

19. Use the method of finite differences to identify this as a cubic function. Find a cubic regression function, then use table to find the smallest positive value, 167.

20. Use law of cosines to set up the quadratic relationship: $36^2 = 40^2 + x^2 - 2x(40)\cos 60$**. Either graph both sides and find the sum of the two solutions, or simplify and use** $-\frac{b}{2a}$ **to find the sum of the roots or 40.**

24. $5(3)^4\left(-\frac{2}{x^4}\right)^1 = -810$

31. $4\left({}_8C_2\right) + 8\left({}_4C_2\right) + \left({}_4C_3\right) = 164$

39. $k = x\left(y^2 + 2\right)$ **solve** $k = 8(27) = 216$ **then** $216 = 18x$ **for** $x = 12$

45. This is the definition of the derivative of $f(x) = \tan x$ **when** $x = \frac{\pi}{3}$ **for 4.**

47. In base 5,
$$\begin{array}{rcl} 10n & = & 3.111... \\ n & = & 0.311... \\ \hline 4n & = & 2.3 \end{array}$$
then $n = \frac{2.3}{4} = \frac{23}{40}$.

48. $\left(x - \frac{1}{x}\right)^3 = x^3 - 3x + \frac{3}{x} - \frac{1}{x^3} = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x^3}\right)$ **for** $\left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$ **and** $6^3 + 3(6) = 234$

50. ${}_{4+8-1}C_8 = 165$

52. The distance between the line $7x + 3y - 42 = 0$ **and** $(-8, 3)$ **is** $\frac{|7(-8) + 3(3) - 42|}{\sqrt{49 + 9}} \approx 11.7$

56. This is the McClaurin series expansion of $f(x) = \sin x + \cos x$**. Make sure your mode is set to radians then evaluate** $\sin 3 + \cos 3$ **for a** 10^{-8} **digit 8.**

59. Let I **= dots interior to the polygon and** P **= dots on the perimeter.** $A = \frac{2I + P - 2}{2} = \frac{36 + 12 - 2}{2} = 18$ **, but each square on the grid represents 4 square units, so the actual area is 72 u^2.**