

# 1<sup>st</sup> Annual Harvard-MIT November Tournament

Saturday 8 November 2008

## Theme Round

### Triangular Areas [25]

Triangles have many properties. Calculate some things about their area.

1. [3] A triangle has sides of length 9, 40, and 41. What is its area?
2. [4] Let  $ABC$  be a triangle, and let  $M$  be the midpoint of side  $AB$ . If  $AB$  is 17 units long and  $CM$  is 8 units long, find the maximum possible value of the area of  $ABC$ .
3. [5] Let  $DEF$  be a triangle and  $H$  the foot of the altitude from  $D$  to  $EF$ . If  $DE = 60$ ,  $DF = 35$ , and  $DH = 21$ , what is the difference between the minimum and the maximum possible values for the area of  $DEF$ ?
4. [6] Right triangle  $XYZ$ , with hypotenuse  $YZ$ , has an incircle of radius  $\frac{3}{8}$  and one leg of length 3. Find the area of the triangle.
5. [7] A triangle has altitudes of length 15, 21, and 35. Find its area.

### Chessboards [25]

Joe B. is playing with some chess pieces on a  $6 \times 6$  chessboard. Help him find out some things.

1. [3] Joe B. first places the black king in one corner of the board. In how many of the 35 remaining squares can he place a white bishop so that it does not check the black king?
2. [4] Joe B. then places a white king in the opposite corner of the board. How many total ways can he place one black bishop and one white bishop so that neither checks the king of the opposite color?
3. [5] Joe B. now clears the board. How many ways can he place 3 white rooks and 3 black rooks on the board so that no two rooks of **opposite** color can attack each other?
4. [6] Joe B. is frustrated with chess. He breaks the board, leaving a  $4 \times 4$  board, and throws 3 black knights and 3 white kings at the board. Miraculously, they all land in distinct squares! What is the expected number of checks in the resulting position? (Note that a knight can administer multiple checks and a king can be checked by multiple knights.)
5. [7] Suppose that at some point Joe B. has placed 2 black knights on the original board, but gets bored of chess. He now decides to cover the 34 remaining squares with 17 dominos so that no two overlap and the dominos cover the entire rest of the board. For how many initial arrangements of the two pieces is this possible?

**Note:** Chess is a game played with pieces of two colors, black and white, that players can move between squares on a rectangular grid. Some of the pieces move in the following ways:

- **Bishop:** This piece can move any number of squares diagonally if there are no other pieces along its path.
- **Rook:** This piece can move any number of squares either vertically or horizontally if there are no other pieces along its path.
- **Knight:** This piece can move either two squares along a row and one square along a column or two squares along a column and one square along a row.
- **King:** This piece can move to any open adjacent square (including diagonally).

If a piece can move to a square occupied by a king of the opposite color, we say that it is *checking* the king.

If a piece moves to a square occupied by another piece, this is called *attacking*.