TMSCA HIGH SCHOOL MATHEMATICS TEST #11 ©

FEBRUARY 9, 2019

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Eleven

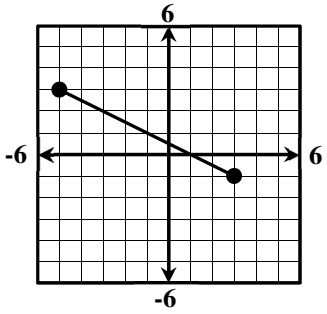
1. **Evaluate** $\dfrac{19(7!)\div 8}{7+2^3}$.

   (A) 1718 **(B)** 214 **(C)** 798 **(D)** 6384 **(E)** 13,680

2. **Caroline had a rope that was 18 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:9 with 20 inches of string left over. How long was the longest piece?**

   (A) 9' 8" **(B)** 9' 11" **(C)** 3' 6" **(D)** 10' 6" **(E)** 10'

3. **What is the x – intercept of the perpendicular bisector of the line segment shown?**

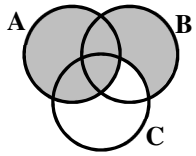


   (A) $\left(0,\frac{1}{2}\right)$ **(B)** $(0,3)$ **(C)** $(1,0)$ **(D)** $\left(-\frac{3}{2},0\right)$ **(E)** $(3,0)$

4. **Libby made a cake that called for 2 cups of flour, 1.5 cups of sugar, 1 cup of butter, 1 tablespoon of baking powder, 1 cup of milk and 1 teaspoon of vanilla. Calculate the price of the cake if flour costs \$2.89 for 20 cups; sugar costs \$1.79 for 12 cups; butter costs \$3.99 for 2 cups; baking powder costs \$1.42 for 30 tablespoons; milk costs \$2.69 per gallon and vanilla costs \$13.99 for 1.5 cups. All of the ingredients in Libby's cake are tax exempt groceries.**

   (A) \$2.38 **(B)** \$3.27 **(C)** \$4.32 **(D)** \$3.14 **(E)** \$2.92

5. **Which of the following is a symbolic representation for the Venn diagram shown?**



   (A) $B\cap(A\cap C)$ **(B)** $(A\cup B)\cap C$ **(C)** $A\cup(B\cap C)$

   (D) $A\cup C'$ **(E)** $B\cup A$

6. **Events A and B are independent events such that $P(B)=5P(A)$ and $P(A\cup B)=0.648$. Find $P(A)$.**

   (A) 0.15 **(B)** 0.20 **(C)** 0.48 **(D)** 0.16 **(E)** 0.12

7. **If A is $66\frac{2}{3}\%$ of B, and C is 120% of B, then A = _____% of C?**

   (A) $71\frac{3}{7}$ **(B)** $28\frac{4}{7}$ **(C)** $77\frac{7}{9}$ **(D)** $55\frac{5}{9}$ **(E)** $57\frac{1}{7}$

8. **The graph of $25x^2+9y^2-150x-18y=-9$ is an ellipse. Find the area of the ellipse.**

   (A) 9π **(B)** 25π **(C)** 15π **(D)** 8π **(E)** 30π

9. **Approximately one million black wildebeest migrate every year between the Serengeti in Tanzania and Maasai Mara in Kenya. The average wildebeest is 6 feet long. If all of the wildebeest lined up end-to-end, the line would be _____kilometers long. (nearest kilometer)**

   (A) 1288 **(B)** 305 **(C)** 1890 **(D)** 1829 **(E)** 1578

10. Simplify: $\sqrt[3]{a}\left(\sqrt[3]{2a^2}+\sqrt[3]{16a^2}\right)$

(A) $3a\sqrt[3]{2}$ **(B)** $3a\sqrt[3]{2a}$ **(C)** $2a\sqrt[3]{2}$ **(D)** $2a\sqrt[3]{2a}$ **(E)** $4a\sqrt[3]{2}$

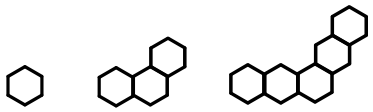
11. $\angle A$ **and** $\angle B$ **are supplementary. If** $m\angle A=(4x)°$ **and** $m\angle B=(5x+18)°$**, find** $m\angle B$**.**

(A) 90° **(B)** 108° **(C)** 36° **(D)** 72° **(E)** 18°

12. The total surface area of a right cone with a radius at the base of 19 ft and a vertex angle of 22° is ____ft^2. (nearest square foot)

(A) 36,952 **(B)** 7,078 **(C)** 4,162 **(D)** 6,969 **(E)** 37,644

13. The three shapes below are made up of regular hexagons with side lengths of 1 unit. If the pattern continues, the perimeter of the shape with 56 hexagons will is _____units.



(A) 224 **(B)** 230 **(C)** 242 **(D)** 220 **(E)** 226

14. The measure of an interior angle of a regular nonagon is _____. (nearest degree)

(A) 144° **(B)** 135° **(C)** 120° **(D)** 140° **(E)** 132°

15. $\left(8+\sqrt{-25}\right)\left(6+\sqrt{-36}\right)=$

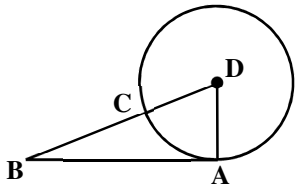
(A) $78-18i$ **(B)** $78+18i$ **(C)** $18+78i$ **(D)** $18-78i$ **(E)** undefined

16. The circumcenter of a circle can be found by constructing the _____ of the triangle and finding the point of concurrency.

(A) medians **(B)** altitudes **(C)** perpendicular bisectors of the sides

(D) angle bisectors **(E)** sides

17. On the illustration, $\overline{AB}$ **is tangent to circle D, AB = 12 and BC = 8. Find the area of circle D.**



(A) 25π **(B)** 24π **(C)** 10π **(D)** 16π **(E)** 64π

18. If $(2x-1)^2+8=ax^2+bx+c$ **then** $a+b+c=$ _____.

(A) 14 **(B)** 5 **(C)** 8 **(D)** 9 **(E)** 4

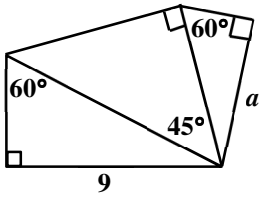
19. How many distinct 4-letter arrangements can be made from the letters in the word "SKEWED"?

(A) 360 **(B)** 264 **(C)** 120 **(D)** 192 **(E)** 72

20. A small motor boat traveling from point A to point B along the river can make the 14-mile trip in 6 hours. The return trip takes 8 hours and 24 minutes. If the speed of the boat and current remain constant throughout both trips, what is the speed of the current?

(A) 8,800 ft/hr **(B)** 10,560 ft/hr **(C)** 12,320 ft/hr **(D)** 4,928 ft/hr **(E)** 1,760 ft/hr

21. Find the value of a on the picture shown.



(A) $\frac{9\sqrt{2}}{2}$ **(B)** $3\sqrt{6}$ **(C)** $6\sqrt{3}$ **(D)** $\frac{27\sqrt{2}}{2}$ **(E)** $6\sqrt{6}$

22. $\int \ln(x)dx =$ _____$+C$ **, where C is some arbitrary constant.**

(A) $-x + x\ln x$ **(B)** $-x\ln x + x$ **(C)** $-x\ln x$ **(D)** $x\ln x$ **(E)** $x\ln x + x$

23. Two numbers are in a ratio of 5:13. If the first is decreased by 5 and the second is increased by 5, the resulting numbers will be in a ratio of 2:7. Find the sum of the two numbers.

(A) 63 **(B)** 87 **(C)** 112 **(D)** 74 **(E)** 90

24. A fair tetrahedral die with sides labeled 4, 6, 6, 12 is thrown. What is the expected value of a single roll?

(A) 5 **(B)** $\frac{11}{3}$ **(C)** 6 **(D)** $\frac{16}{3}$ **(E)** 7

25. Sammy bought seven boxes of chocolate bars from a school fundraiser. He gave half of his order to his niece, then two-thirds of the remaining bars to his nephew. He ate six bars, then put the remaining box away to use for gifts. How may bars came in each box?

(A) 48 **(B)** 72 **(C)** 36 **(D)** 12 **(E)** 24

26. What is the equation of a line tangent to $y^2 - x = 191$ at $(5,14)$.

(A) $x - 28y = 397$ **(B)** $28x + y = 126$ **(C)** $28x + y = 397$

(D) $x - 28y = -387$ **(E)** $28x + y = 154$

27. The area of a sector of a circle with a central angle $\frac{5\pi}{8}$ in a circle with a diameter of 44 cm is _____cm^2. (nearest square centimeter)

(A) 1425 **(B)** 356 **(C)** 2851 **(D)** 713 **(E)** 475

28. The third term of an arithmetic sequence is -2, and the sum of the first fourteen terms is 98. Find the first term.

(A) –4 **(B)** –6 **(C)** 2 **(D)** –2 **(E)** 4

29. Determine the period of $f(x)=\frac{3}{2}\sin\left(\frac{1}{3}x-7\right)+8$

(A) $\frac{7\pi}{3}$ **(B)** 6π **(C)** $\frac{3\pi}{2}$ **(D)** 2π **(E)** $\frac{2\pi}{3}$

30. If $g(x)=x-1$ **and** $g(x)=x^4$**, find** $f(g(x+1))$

(A) x^4-2 **(B)** x^4 **(C)** $x^4+4x^3+6x^2+4x-2$

(D) $x^4+3x^3+3x^2+x$ **(E)** $x^4+4x^3+6x^2+4x$

31. When $3x^3+2x^2-7x+k$ **is divided by** $(x-2)$ **the remainder is** m**. Find the value of** m **in terms of** k**.**

(A) $m=k-30$ **(B)** $m=k+30$ **(C)** $m=k-2$ **(D)** $m=k+46$ **(E)** $m=k+18$

32. Myrtle has bins containing 9 flavors of lollipops. In how many ways can she package 5 to sell if she can repeat flavors?

(A) 2002 (B) 715 (C) 3003 (D) 1287 (E) 1001

33. Two workers can paint a fence in three hours. How long would it take five workers to paint a fence twice as long and twice as high if they each paint at the same rate?

(A) 4 hr 8 min (B) 2 hr 24 min (C) 1 hr 12 min (D) 6 hr (E) 4 hr 48 min

34. On triangle ABC, AB = 24 cm, BC = 21 cm, and $m\angle A=60°$**. The smallest possible value of AC is _____cm.**

(A) 9 (B) 3.6 (C) 15 (D) 6.9 (E) 38.1

35. If $f(x)=\sin(2x)$**, then** $\lim_{h\to 0}\frac{f(\pi+h)-f(\pi)}{h}=$

(A) 0 (B) 1 (C) –1 (D) 2 (E) undefined

36. Solve $\log_2(x)+\log_2(x-6)=4$**.**

(A) 4 (B) –3 (C) 6 (D) 8 (E) 9

37. If $\tan\theta<0$ **and** $\csc\theta>0$**, where will** θ **terminate?**

(A) QI (B) QII (C) QIII (D) QIV (E) x**-axis**

38. $\sum_{k=1}^{7}\left(3k^2+2k-1\right)=$

(A) 385 (B) 476 (C) 357 (D) 441 (E) 469

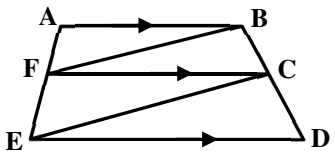
39. According to Descartes' Rule of Signs, how many negative real zeros will $f(x)=x^4+x^3-3x^2-x+2$ **have?**

(A) 3 or 1 (B) 4, 2 or 0 (C) 1 (D) 0 (E) 2 or 0

40. A particular model of car has an advertised gas mileage of 32 mpg for in-town driving. Upon further investigation, a consumer discovers that the gas mileage is normally distributed with a standard deviation of 3 mpg. What is the probability that the driver will get over 33 mpg for in-town driving? (nearest percent)

(A) 27% **(B)** 37% **(C)** 84% **(D)** 63% **(E)** 16%

41. On the diagram, FC is the geometric mean of AB and ED, AB = 29.6, FC = 37 and EC = 42.8. Find FB. (nearest tenth)



(A) 31.9 **(B)** 30.5 **(C)** 34.6 **(D)** 34.2 **(E)** 36.4

42. Evaluate: $\prod_{k=2}^{6}(2k+4)$

(A) 21,504 **(B)** 215,040 **(C)** 15,360 **(D)** 26,880 **(E)** 276,480

43. At which of these x-values is the graph of $f(x)=x^4-12x^3+48x^2-64x$ concave down?

(A) 5 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

44. Find the values of k for which $kx^2-4x+5-k$ has two distinct real roots.

(A) $-6<k<2$ **(B)** $k<\frac{4}{5}$ **(C)** $k<-6$ or $k>2$ **(D)** $k<1$ or $k>4$ **(E)** $1<k<4$

45. The operation ξ is defined as $A\xi B=A^3+2A+B^3$. Compute $3\xi(1\xi 2)$.

(A) 36 **(B)** 42,876 **(C)** 1,364 **(D)** 1,764 **(E)** 756

46. If $f(x)=ax^4+bx^2+x$ and $f(2)=30$ then $f(-2)$.

(A) 26 **(B)** 32 **(C)** 30 **(D)** 28 **(E)** −30

47. If A represents a digit 0 – 9 in the equation $6A5_8+3A2_6=1000101011_2$, find the value of A.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

48. If $g(x)\le f(x)\le h(x)$ for all x, k in $[a,b]$, where $x\neq k$, $\lim_{x\to k} g(x)=L$ and $\lim_{x\to k} h(x)=L$ then $\lim_{x\to k} f(x)=L$. This is known as _____.

(A) Rolle's Theorem **(B)** Sandwich Theorem **(C)** Intermediate Value Theorem

(D) Fundamental Theorem of Algebra **(E)** Fundamental Theorem of Calculus

49. Given $y=-2\cos x$, find the value of x for which $\frac{dy}{dx}=\frac{dx}{dy}$, where $0\le x\le\frac{\pi}{2}$

(A) π **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$ **(E)** 0

50. Given $GCF(36,k)=12$ and $LCM(36,k)=252$, find the value of k.

(A) 84 (B) 63 (C) 42 (D) 48 (E) 126

51. Find the sum of all 3-digit numbers whose digits have a sum of eight and whose digits can all be used to form a perfect cube.

(A) 1420 (B) 915 (C) 861 (D) 1925 (E) 1776

52. P, Q and R are real numbers such that $P+Q+R=8$, $R^2=P^2+Q^2$ and $PQ=8$. Find the value of R.

(A) 1 (B) 6 (C) 3 (D) 8 (E) 4

53. If a hiker travels 6 miles on a bearing of 12°, then another 5 miles on a bearing of 334°, what is the shortest distance back to her starting point? (nearest tenth of a mile)

(A) 2.3 mi (B) 10.1 mi (C) 13.1 mi (D) 10.0 mi (E) 10.4 mi

54. If $\frac{x-20}{x+20}+\frac{x+20}{x-20}=A\frac{B}{C}$ then $B=$

(A) 2400 (B) 1000 (C) 3200 (D) 1600 (E) 400

55. Given $y=x-15$ and $xy=18$. Calculate x^3-y^3.

(A) 3915 (B) 3645 (C) 2565 (D) 2835 (E) 4185

56. Find the sum of the solutions of the equation $|9x+8|=|x-17|$.

(A) $-2\frac{9}{40}$ (B) $4\frac{1}{40}$ (C) $-1\frac{3}{8}$ (D) $-4\frac{1}{40}$ (E) $-1\frac{3}{5}$

57. Find the area of the region defined by the inequalities $x\geq 0$, $y\geq 0$ and $y\leq 2\cos\left(\frac{x}{4}\right)$.

(A) 1 (B) 2 (C) 4 (D) 8 (E) 16

58. A fair coin is tossed seven times. What is the probability of at least 5 heads in a row?

(A) $\frac{1}{32}$ (B) $\frac{5}{128}$ (C) $\frac{1}{128}$ (D) $\frac{1}{16}$ (E) $\frac{3}{64}$

59. If $\log 9=P$ and $\log 5=Q$ then $\log 0.6=$

(A) $\frac{\sqrt{P}}{Q}$ (B) $\frac{P-2Q}{2}$ (C) $\log\left(\frac{\sqrt{P}}{Q}\right)$ (D) $2PQ$ (E) $\frac{PQ}{2}$

60. $\int_{-2}^{4}\left(kx^2+5x+2\right)dx=114$, what is the value of k?

(A) 1 (B) −3 (C) 3 (D) 2 (E) −1

2018-2019 TMSCA Mathematics Test Ten Answers

1. C	**21. A**	**41. D**
2. D	**22. A**	**42. B**
3. D	**23. E**	**43. C**
4. E	**24. E**	**44. D**
5. D	**25. C**	**45. C**
6. E	**26. D**	**46. A**
7. D	**27. E**	**47. D**
8. C	**28. B**	**48. B**
9. D	**29. B**	**49. B**
10. A	**30. E**	**50. A**
11. B	**31. E**	**51. E**
12. B	**32. D**	**52. C**
13. E	**33. E**	**53. E**
14. D	**34. A**	**54. D**
15. C	**35. D**	**55. E**
16. C	**36. D**	**56. A**
17. A	**37. B**	**57. D**
18. D	**38. E**	**58. D**
19. D	**39. E**	**59. B**
20. E	**40. B**	**60. C**

2018-2019 TMSCA Mathematics Test Eleven Solutions

6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **, so** $0.648 = 5A + A - 5A^2$**, solve quadratic equation for** $P(A) = 0.12$

12. $l = \dfrac{19}{\sin 11}$ **then** $SA = \pi(19)^2 + \pi(19)l \approx 7078$

17. Set up the Pythagorean theorem $12^2 + r^2 = (8+r)^2$ **for** $r = 5$ **and the area is** 25π**.**

19. If there are no repeating letters, there are ${}_5P_4 = 120$ **arrangements. If the E's repeat, then there are** $({}_4C_2)\left(\dfrac{4!}{2!}\right) = 72$ **arrangements for a total of 192.**

24. $E(x) = \dfrac{1}{4}(4+6+6+12) = 7$

28. $-2 = a + 2d$ **then** $98 = \dfrac{14}{2}(a + a + 13d)$ **which simplifies to** $14 = 2a + 13d$**. Solve the system for** $a = -6$**.**

32. ${}_{9+5-1}C_5 = 1287$

33. Each worker could paint a fence alone in 1.5 hours. Then $\dfrac{4}{\frac{1}{1.5}+\frac{1}{1.5}+\frac{1}{1.5}+\frac{1}{1.5}+\frac{1}{1.5}} = 1.2$ **hours or 1 hour and 12 minutes. The "4" in the numerator comes from the fact that a fence that is twice as high and twice as long will have 4 times the area.**

34. Use law of cosines for $21^2 = 24^2 + (AC)^2 - 2(AC)(24)\cos 60°$ **which gives values of 9 and 15 for** AC**.**

35. This is the definition of derivative of $f(x) = \sin(2x)$ **at** π**, which is 2. Make sure to set the mode to radian if using the calculator.**

38. Use formulas for sum of first n numbers and squares for $3\left[\dfrac{7(7+1)(14+1)}{6}\right] + 2\left[\dfrac{7(8)}{2}\right] - 1(7) = 469$

39. Use the cumulative normal distribution function using lower limit 33, upper limit $32 + 5(3) = 47$ **which leaves negligible value at the upper end and the probability is 0.369.**

41. The geometric mean of the bases of a trapezoid cuts the trapezoid into two similar pieces, so $\dfrac{29.6}{37} = \dfrac{x}{42.8}$ **for** $x \approx 34.2$**.**

42. This is the product of each term, so $(8)(10)(12)(14)(16) = 215{,}040$

51. The two 3-digit cubes whose digits add up to 8 are 125 and 512, both have the same digits, the sum of all the numbers with those three digits will be $125 + 152 + 251 + 215 + 512 + 521 = 1776$

52. $P + Q = 8 - R$ **and** $P^2 + 2PQ + Q^2 = 64 - 16R + R^2$**, then** $2PQ = 64 - 16R$ **since** $P^2 + Q^2 = R^2$**. Finally, solve** $2(8) = 64 - 16R$ **for** $R = 3$**.**

58. Let H be a group of 5 heads in a row, h represent a single head and t represent a single tail. The possibilities with at least 5 heads in a row are:
Htt, tHt, ttH, htH, Hth, Hht, tHh and all heads for $8\left(\dfrac{1}{2}\right)^7 = \dfrac{1}{16}$