T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 2 ©

O C T O B E R 2 8, 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2017-2018 TMSCA Mathematics Test Two

1. **Evaluate:** $24\times15-4!-27\div(18-6)\times4+28$.

   A. 363.4 **B.** 375 **C.** 356.6 **D.** 403 **E.** 355

2. **Caroline had a rope that was 30 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:12 with 20 inches of string left over. How long was the shortest piece?**

   A. 4 ft. 10 in. **B.** 3 ft. **C.** 2 ft. 6 in. **D.** 3 ft. 8 in. **E.** 3 ft. 4 in.

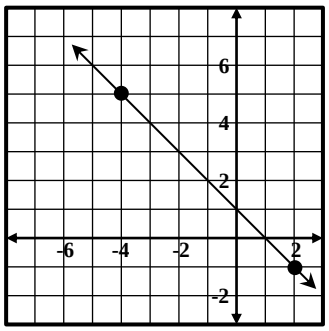
3. **The data set** $9,a,b,c,24,35$ **is shown least to greatest and has a mean of 20, mode of 24 and median of 20. Calculate the value of** $a+c$.

   A. 39 **B.** 48 **C.** 40 **D.** 36 **E.** 35

4. **What is the sum of the first four abundant numbers?**

   A. 56 **B.** 28 **C.** 84 **D.** 74 **E.** 86

5. **Which of the following is the standard form of the equation of the perpendicular bisector of the line shown?**



   A. $x+y=3$ **B.** $x+y=-3$ **C.** $x+y=1$

   D. $x-y=-1$ **E.** $x-y=-3$

6. **Evaluate:** $\dfrac{(x+2)!}{(x-3)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^6-5x^4+4x^2$ **B.** x^4+5 **C.** x^4-5x^2+4 **D.** x^6+4x^2 **E.** x^4-5x^2+5

7. **If** $\theta=5\lambda$ **and** $\alpha+\theta=\varphi$, **then** $\alpha+5\lambda=\varphi$. **This is an example of the _____ property.**

   A. Substitution **B. Transitive** **C. Commutative** **D. Associative** **E. Reflexive**

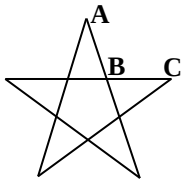
8. **Two consecutive angles in a pentagon are supplementary. The other three angles are congruent. What is the measure of one of the three congruent angles?**

   A. 60° **B.** 120° **C.** 150° **D.** 90° **E.** 135°

9. **What is the area of the region entirely bounded by the two functions** $f(x)=x^2-4x+3$ **and** $g(x)=5x-11$**?**

   A. $\dfrac{125}{6}$ **B.** 21 **C.** $\dfrac{32}{3}$ **D.** $\dfrac{125}{3}$ **E.** $\dfrac{32}{6}$

10. **The angles at each point on the star shown are congruent. What is** $m\angle ABC$**?**



   A. 120° **B.** 108° **C.** 96° **D.** 72° **E.** 144°

11. If $x+y=-3$, and $xy=-8$, then $x^3+y^3=$

A. 45 **B. −75** **C. −51** **D. −99** **E. −45**

12. The four brothers Lester, Morris, Nigel and Porter wanted to go on a road trip, but Lester had no money. Morris, Nigel and Porter each gave Lester one-fourth, one-fifth and one-sixth of his money respectively. If each gave Lester the same amount, what fraction of the money did Lester possess after the exchange?

A. $\frac{1}{6}$ **B.** $\frac{2}{5}$ **C.** $\frac{1}{5}$ **D.** $\frac{37}{60}$ **E.** $\frac{1}{3}$

13. Find the value of the arithmetic mean for terms *a*, *b* and *c* in the geometric sequence: $6561, 4374, a, b, c \ldots$.

A. 1944 **B. 0** **C. 2052** **D. 2916** **E. 1458**

14. $\tan\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)\div\cot\left(\frac{5\pi}{3}\right)\csc\left(\frac{\pi}{6}\right)\div\cos\left(\frac{5\pi}{3}\right)\csc\left(\frac{5\pi}{3}\right)=$

A. 8 **B. 4** **C. 2** **D.** $\frac{2\sqrt{3}}{3}$ **E.** $\frac{1}{2}$

15. The intersection of the medians of a triangle is called the _____.

A. Centroid **B. Incenter** **C. Median** **D. Circumcenter** **E. Orthocenter**

16. How many integral values of *n* exist such that $n>3$, and $\frac{n!}{(n-3)!}\le 1000$?

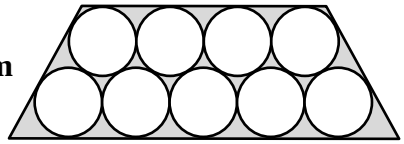
A. 0 **B. 5** **C. 9** **D. 8** **E. 10**

17. There are two values of *k* for which $\det\begin{bmatrix} k+1 & 3 \\ 5 & 2k \end{bmatrix}=129$**. The sum of those two values is**

A. 1 **B. 17** **C. 3** **D. −2** **E. −1**

18. The radius of each circle is 3.5 cm. Find the perimeter of the trapezoid. (nearest tenth centimeter)

A. 90.4 cm **B. 64.6 cm** **C. 123.3 cm** **D. 95.3 cm** **E. 82.4 cm**



19. The number 458 in base 9 is equivalent to the number *k* in base 3. Find the sum of the digits in *k*.

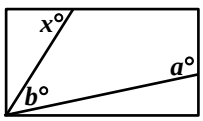
A. 8 **B. 19** **C. 9** **D. 7** **E. 10**

20. Find the mean value of $f(x)=4x^3-6x^2+2x-1$ for $[-1,4]$.

A. 27 **B. 19** **C. 9** **D. 33.75** **E. 7**

21. In the rectangle shown right, what is *x* in terms of *a* and *b*?

A. $90-a-b$ **B.** $90-a+b$ **C.** $a+b$ **D.** $a+b-90$ **E.** $180+a-b$



22. How many distinct arrangements can be formed using all of the letters in the word "TRIGONOMETRY"?

A. 79,833,600 **B.** 479,001,600 **C.** 4,838,400 **D.** 59,875,200 **E.** 29,937,600

23. If $g(x)=x-2$ and $f(x)=x^4$, find $g(f(x+1))$.

A. $x^4+3x^3+3x^2+x$ **B.** x^4-1 **C.** $x^4+4x^3+6x^2+4x-1$

D. $x^4+4x^3+6x^2+4x$ **E.** x^4-2

24. A chemistry student needs to mix a 100-fluid ounce solution containing 54% glucose. The pharmacist has 30% and 90% solutions on hand. How much of the 30% solution should she use?

A. 60 ounces **B.** 54 ounces **C.** 40 ounces **D.** 46 ounces **E.** 50 ounces

25. Which of the following quadrants does not contain a solution to $4x+3y\geq 11$?

A. QIII **B.** QI & QII **C.** QIV **D.** QI & QIV **E.** QI

26. A triangle with side lengths 12 cm, 12 cm and 22 cm is a(n)_____triangle.

A. isosceles acute **B.** scalene acute **C.** isosceles obtuse **D.** scalene obtuse **E.** scalene right

27. Which of the following is an equation of the tangent line of $f(x)=2x^2-x+\frac{32}{x}$ for $x=4$?

A. $y=13x+88$ **B.** $y=13x-52$ **C.** $y=13x-36$ **D.** $y=13x-16$ **E.** $y=13x+36$

28. If $\log 4=P$, and $\log 5=Q$, then $\log(0.64)=?$

A. P^2-Q^2 **B.** $2P+2Q$ **C.** $\frac{P-Q}{2}$ **D.** $2P-2Q$ **E.** $\frac{P+Q}{2}$

29. If $U=\{a,b,c,d,e,f,g,h\}$, $A=\{a,c,e,g\}$, and $B=\{b,c,d,e\}$, find $A'\cap B$.

A. $\{b,d\}$ **B.** $\{a,b,c,d,e,g\}$ **C.** $\{b,d,f,h\}$ **D.** $\{b,c,d,e,f,h\}$ **E.** $\{f,h\}$

30. If *P*, *Q* and *R* are real numbers such that $P+Q+R=10$, $R^2=P^2+Q^2$ and $PQ=8$, then $R=?$

A. 2.1 **B.** –4.2 **C.** –2.5 **D.** 4.2 **E.** 3.6

31. There are 7 girls and 9 boys in Ms. Angle's homeroom class. She must select a group of 2 girls and 2 boys to represent her class in a Veterans Day ceremony. How many distinct groups does she have to choose from?

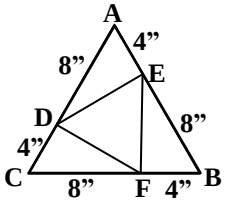
A. 1512 **B.** 1820 **C.** 910 **D.** 240 **E.** 756

32. Which of the following equations in rectangular form can be written as $r-8\sin\theta=0$ in polar form?

A. $x^2+y^2=16$ **B.** $x^2+y^2=4$ **C.** $x^2+y^2=2\sqrt{3}$

D. $x^2-8x+y^2=0$ **E.** $x^2+y^2-8y=0$

33. Find the area of $\triangle DEF$. (nearest tenth)



A. 5.2 in^2 **B.** 20.8 in^2 **C.** 6.9 in^2 **D.** 15.6 in^2 **E.** 11.7 in^2

34. Find the remainder when $f(x) = 6x^3 - x^2 - 7x + 5$ is divided by $x - 5$.

A. 715 B. 695 C. −705 D. −735 E. 710

35. A sales clerk is packaging blue, red and black pens for a back-to-school sale. How many distinct packages of 5 pens can he make?

A. 56 B. 126 C. 28 D. 56 E. 21

36. Two roots of $f(x) = x^3 + bx^2 + cx + d$ are 4 and $3 + i$. Find $b + c + d$.

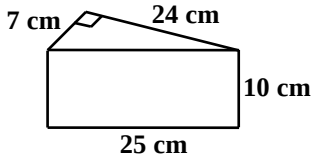
A. 54 B. −26 C. −16 D. 64 E. −4

37. If $g(x) \le f(x) \le h(x)$ for all *x*, *k* in $[a,b]$, where $x \neq k$, and $\lim_{x \to k} g(x) = L$ and $\lim_{x \to k} h(x) = L$ then $\lim_{x \to k} f(x) = L$. This theorem is known as:

A. Sandwich Theorem B. Rolle's Theorem C. Fundamental Theorem of Calculus

D. Intermediate Value Theorem E. Fundamental Theorem of Algebra

38. Calculate the total surface area of the triangular prism shown.



A. 728 cm² B. 644 cm² C. 840 cm² D. 924 cm² E. 560 cm²

39. Find the sum of all the three-digit numbers whose digits have a sum of ten and whose digits can all be used to form a perfect cube.

A. 343 B. 1998 C. 1386 D. 1110 E. 776

40. The ratio of length to width of a rectangle is 13:3 and the perimeter is 3072 in. What is the area of the rectangle?

A. 359,424 ft² B. 4,992 ft² C. 718,848 ft² D. 2,496 ft² E. 9,984 ft²

41. The function $f(x) = \dfrac{2x^3}{x^2 - 3}$ is increasing at which of the following values of *x*?

A. −3 B. 0 C. −1 D. −4 E. 2

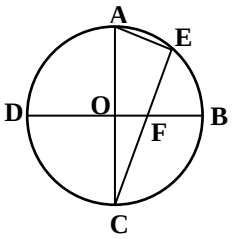
42. How many distinct solutions exist for $2\sin^2 x = 2\cos x$, where $0 \le x < \pi$?

A. 0 B. 1 C. 2 D. 3 E. 4

43. Meredith set out to row on a lake. She rowed 600 m on a bearing of 75°, then 200 m on a bearing of 25°, then 350 m on a bearing of 52°. How far is she from her original starting point? (nearest meter)

A. 1090 m B. 615 m C. 775 m D. 526 m E. 994 m

44. Circle O has perpendicular diameters and a chord, find AE if CF = 9 inches and EF = 7 inches. (nearest tenth)



A. 4.5 in B. 3.7 in C. 6.2 in D. 5.7 in E. 4.7 in

45. What is the harmonic mean of the roots of the function $f(x)=10x^2-11x-6$?

A. $\frac{11}{12}$ **B.** $\frac{12}{11}$ **C.** $\frac{11}{20}$ **D.** $-\frac{12}{11}$ **E.** $-\frac{11}{20}$

46. Find $f(x)$ given that $f''(x)=18x-14$ and that $f(1)=-2$ and $f(-1)=-14$.

A. $3x^3-7x^2+3x-1$ **B.** $3x^3-7x^2+5x-1$ **C.** $3x^3-7x^2+4x-1$

D. $3x^3-7x^2-4x-1$ **E.** $3x^3-7x^2$

47. What is the constant term in the binomial expansion of $\left(3x^2-\frac{2}{x}\right)^6$?

A. 4320 **B.** 960 **C.** 2160 **D.** –4320 **E.** 729

48. A contestant on a game show rolls a single, fair, standard die. The player loses \$200 if an odd number is rolled. If he rolls an even prime, he gets a \$500 payout. If he rolls a perfect number, he gets \$1000 payout. Otherwise, nothing happens. What are his expected winnings?

A. \$200 **B.** \$250 **C.** \$240 **D.** \$150 **E.** \$275

49. The point $(6,-2)$ lies on a circle whose center is $(1,10)$. Where does the point $(8,13)$ lie in reference to the circle?

A. Inside **B.** Outside **C.** On the Circle **D.** Q II **E.** Unknowable

50. How many solutions are there for the equation $2x+5y=175$, where both *x* and *y* are non-negative integers?

A. 17 **B.** 16 **C.** 14 **D.** 12 **E.** 18

51. Forty-eight percent of homes in a town have pets. If four homes are chosen at random for a survey, find the probability that at least three have pets. (nearest percent)

A. 26% **B.** 11% **C.** 28% **D.** 5% **E.** 23%

52. If $\frac{10x+38}{x^2+4x-5}=\frac{A}{x+5}+\frac{B}{x-1}$, then $AB=$

A. –8 **B.** 6 **C.** –10 **D.** 16 **E.** 10

53. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 12.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			…				(…)

A. 111 **B.** 157 **C.** 145 **D.** 134 **E.** 133

54. The square root of 1013_6 is:

A. 111_6 **B.** 23_6 **C.** 35_6 **D.** 25_6 **E.** 151_6

55. If $y^2 = -16 - 30i$ and $y^3 = -198 - 10i$ where $y = a + bi$ then $a + b =$

A. 8 **B. –5** **C. 5** **D. –2** **E. 3**

56. $4^3 + 5^3 + 6^3 + ... + 12^3 + 13^3 + 14^3 =$

A. 11,025 **B. 11017** **C. 10,989** **D. 11025** **E. 2744**

57. What is the area of a regular hexagon in terms of the length, *a*, of the apothem?

A. $\frac{3a^2\sqrt{3}}{4}$ **B.** $\frac{4a^2\sqrt{3}}{3}$ **C.** $2a^2\sqrt{3}$ **D.** $\frac{3a^2\sqrt{3}}{2}$ **E.** $3a^2\sqrt{3}$

58. Find the units digit of 17^{2017}.

A. 3 **B. 1** **C. 0** **D. 7** **E. 9**

59. Simplify to the nearest ten-thousandth place: $1 + (1.3) + \frac{(1.3)^2}{2!} + \frac{(1.3)^3}{3!} + \frac{(1.3)^4}{4!} + ...$.

A. 0.2624 **B. 0.2675** **C. 3.6693** **D. 3.6302** **E. 0.9636**

60. The function f is such that $\int_{-1}^{8} f(x)dx = 12$. What is the value of $\int_{-1}^{8} (2f(x) + 3)dx$?

A. 51 **B. 39** **C. 27** **D. 32** **E. 57**

2017-2018 TMSCA Mathematics Test Two Answers

1. E	**21. D**	**41. D**
2. E	**22. D**	**42. B**
3. D	**23. C**	**43. A**
4. D	**24. A**	**44. D**
5. E	**25. A**	**45. D**
6. C	**26. C**	**46. A**
7. A	**27. D**	**47. C**
8. B	**28. D**	**48. D**
9. A	**29. A**	**49. A**
10. B	**30. D**	**50. E**
11. D	**31. E**	**51. C**
12. C	**32. E**	**52. D**
13. C	**33. B**	**53. E**
14. B	**34. B**	**54. B**
15. A	**35. E**	**55. D**
16. D	**36. C**	**56. C**
17. E	**37. A**	**57. C**
18. D	**38. A**	**58. D**
19. C	**39. D**	**59. C**
20. A	**40. A**	**60. A**

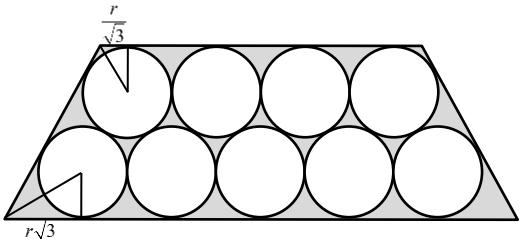
2017 – 2018 TMSCA Mathematics Test Two Select Solutions

10. Think of the same pentagon inscribed in a circle. The desired angle is formed by two intersecting chords with intercepted arcs of 144° and 72°, so $m\angle ABC = \frac{144+72}{2} = 108$

11. $x^3+y^3 = (x+y)\left(x^2-xy+y^2\right) = (x+y)\left[(x+y)^2-3xy\right] =$ $-3\left[(-3)^2-3(-8)\right] = -99$

17. $(k+1)(2k)-15 = 129$, so $2k^2+2k-144 = 0$ and the sum of the roots will be $-\frac{b}{a} = -\frac{2}{2} = -1$

18.



$P = 18r + 4\left(\frac{r}{\sqrt{3}}\right) + 4\left(r\sqrt{3}\right) \approx 95.3$

22. $\frac{12!}{(2!)(2!)(2!)} = 59{,}875{,}200$

24. Solve: $0.3x + 0.9(100-x) = 0.54(100)$ for 60 oz.

28. $\log(0.64) = \log\left(\frac{4^2}{5^2}\right) = 2\log 4 - 2\log 5 = 2P - 2Q$

30. Arrange to $(P+Q)^2 = (10-R)^2$ for $P^2+2PQ+Q^2 = 100-20R+R^2$ then $2(8) = 100-20R$ and $R = 4.2$

31. $({}_7C_2)({}_9C_2) = 756$

33. Each side of triangle DEF is also a long leg of a 30-60-90 triangle, so they all have lengths of $4\sqrt{3}$ for an area of $\frac{s^2\sqrt{3}}{4} \approx 20.8$

35. There are 3 colors of pens, so ${}_{(3+5-1)}C_5 = 21$

36. Multiply: $(x-4)\left[(x-3)^2+1\right] = x^3-10x^2+34x-40$, so $b+c+d = -16$

39. The only perfect cube with a sum of digits equal to 10 is 343, so $343+334+433 = 1110$

43. Let $x = 600\cos 75 + 200\cos 25 + 350\cos 52$ and $y = 600\sin 75 + 200\sin 25 + 350\sin 52$, then the distance from the start will be $\sqrt{x^2+y^2} \approx 1090$

44. $r^2+(OF)^2 = 81$ and $r^2-(OF)^2 = 63$ solve for $r = 6\sqrt{2}$, then $AE = \sqrt{\left(12\sqrt{2}\right)^2-16^2} \approx 5.7$

46. $f(x) = 3x^3-7x^2+Ax+B$ then use the two function values to find $A = 3$ and $B = -1$

51. $p(at\ least\ 3) = p(3)+p(4) = 4(0.48)^3(0.52)+(0.48)^4 \approx$ 28%

52. $A(x-1)+B(x+5) = 10x+38$ so that $\begin{matrix} A+B &= 10 \\ -A+5B &= 38 \end{matrix}$ for $B = 8$, $A = 2$ and $AB = 16$

59. This is the McClaurin series for $f(x) = e^x$, so $f(1.3) \approx 3.6693$

60. $2(12)+3(8+1) = 51$