T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 3 ©

N O V E M B E R 3 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Three

1. **Evaluate:** $\left(6!+4^2\right)\div 46\times 2^{-2}+3$.

   (A) 4 **(B)** 3 **(C)** 7 **(D)** 67 **(E)** 15

2. **Kayla has a job at a retail store where she receives a discount on any purchases she makes. At Christmastime, the store ran a sale with 30% off all ornaments and 40% off all lights. Kayla purchased 4 ornaments and 6 strings of lights with full prices of $6.99 each and $8.95 each respectively. Kayla's employee discount gives her an additional 20% off of her purchase, and the local sales tax is 8.35%. How much change will Kayla receive if she pays with a $100 bill?**

   (A) \$58.57 **(B)** \$61.58 **(C)** \$65.50 **(D)** \$55.11 **(E)** \$64.54

3. **If** $m\angle A+m\angle B+m\angle C=180°$ **and** $m\angle C+m\angle D=180°$ **then** $m\angle A+m\angle B+m\angle C=m\angle C+m\angle D$ **is an example of the _____ property.**

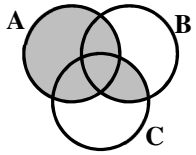
   (A) Transitive **(B)** Reflexive **(C)** Symmetric **(D)** Distributive **(E)** Addition

4. **Find an equation of the line that is perpendicular to** $2x+7y=9$ **and has a** x**-intercept of** $(-2,0)$.

   (A) $7x-2y=-14$ **(B)** $2x+7y=-4$ **(C)** $7x-2y=-2$

   (D) $2x+7y=6$ **(E)** $7x+2y=-14$

5. **Which of the following is a symbolic representation for the Venn diagram shown?**



   (A) $A\cap(B\cap C)$ **(B)** $(A\cap B)\cup(B\cap C)$ **(C)** $A\cup(B\cap C)$

   (D) $(A\cup C)\cap B$ **(E)** $(A\cup C)\cap(B\cap C)$

6. **Andrew's Organic Farm consists of 13 "labors" of land. How many total acres of land does this include?**

   (A) 1872 **(B)** 2301 **(C)** 832 **(D)** 8320 **(E)** 2210

7. **Simplify:** $\left(\dfrac{4x^2-24x+35}{4x^2-25}\right)\left(\dfrac{2x^2+21x+40}{7-2x}\right)$

   (A) $2x-5$ **(B)** $8-x$ **(C)** $2x+5$ **(D)** $-x-8$ **(E)** $5-2x$

8. **Quadrilateral ABCD is inscribed in circle O,** $\angle A$ **and** $\angle C$ **are not consecutive angles, and the measure of** $\angle A$ **is eight times the measure of** $\angle C$**. Find** $m\angle A$.

   (A) 160° **(B)** 120° **(C)** 20° **(D)** 80° **(E)** 100°

9. **If** $\dfrac{x-6}{x+8}+\dfrac{x+8}{x-6}$ **is written as the mixed number** $A\dfrac{B}{C}$**, then** $B=$ **?**

   (A) 28 **(B)** 98 **(C)** 196 **(D)** 4 **(E)** 48

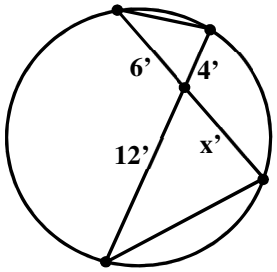
10. **Simplify:** $\left(\sqrt[3]{3a^3b^2}\right)\left(\sqrt[6]{243a^3b^2}\right)$

    (A) $3ab\sqrt[3]{3b^3}$ **(B)** $3ab\sqrt[3]{3a^3}$ **(C)** $3ab\sqrt[6]{3a^3}$ **(D)** $3ab\sqrt[6]{3b^3}$ **(E)** $3ab\sqrt[6]{3a^3b^3}$

11. Events *A* and *B* are independent events such that $p(B)=3p(A)$ and $p(A\cup B)=\frac{213}{400}$. Find $p(A)$.

(A) 0.15 (B) 0.22 (C) 0.45 (D) 0.66 (E) 0.05

12. The line segments shown are all chords of the circle. Find the value of *x*.



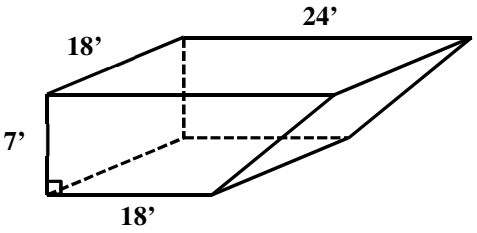
(A) 9' (B) 7.2' (C) 8'

(D) 8.4' (E) 9.3'

13. Two standard dice are rolled. What are the odds that the numbers on each of the dice are the different?

(A) 6:1 (B) 5:2 (C) 5:1 (D) 3:1 (E) 4:1

14. Wade's pool is shaped like a trapezoidal prism as shown. How many gallons (nearest gallon) will fill the pool completely?



(A) 14,244 gal (B) 16,966 gal (C) 22,621 gal

(D) 18,437 gal (E) 19,793 gal

15. The point (x,y) is a point of inflection on $f(x)=\frac{\cos x}{1+\sin x}$, where $0\le x\le 2\pi$. Find the value of x.

(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ (E) 2π

16. Using the following pattern of numbers, which of the following numbers will not be in row 15?

						1									row 0
					1		1								row 1
				1		2		1							row 2
			1		3		3		1						row 3
		1		4		6		4		1					row 4
	1		5		10		10		5		1				row 5
1		6		15		20		15		6		1			row 6
						…									…

(A) 15 (B) 455 (C) 1365 (D) 2002 (E) 5005

17. The measure of an interior angle of a regular octagon is _____. (nearest degree)

(A) 144° (B) 135° (C) 120° (D) 150° (E) 132°

18. Find the range of the function $y=3+2|1-x|$ if the domain is restricted to $\{x\,|\,x\in\mathbb{R},-4\le x\le 4\}$.

(A) $[3,13]$ (B) $[3,9]$ (C) $[2,11]$ (D) $[2,17]$ (E) $[0,17]$

19. $456_7 + 567_8 + 678_9 =$ _____$_{10}$

(A) 1701 **(B)** 1435 **(C)** 1571 **(D)** 1091 **(E)** 1169

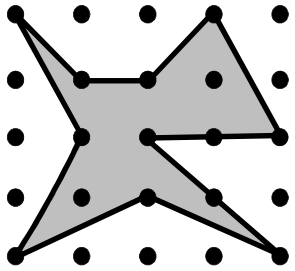
20. Which of the following functions is neither even nor odd? $f(x) =$ _____.

(A) $(4x+3)^2$ **(B)** $x^3 - x$ **(C)** $|3x|$ **(D)** $\tan(\pi x)$ **(E)** $\cos(2x)$

21. Determine the frequency of $f(x) = 7 + 3\cos\left[2\left(x - \frac{\pi}{2}\right)\right]$.

(A) π **(B)** $\frac{1}{2\pi}$ **(C)** $\frac{2}{\pi}$ **(D)** $\frac{1}{\pi}$ **(E)** 2

22. If the dots on the grid shown below are 5 cm apart both vertically and horizontally, then the area of the shaded region is _____cm².



(A) 175 **(B)** 35 **(C)** 7

(D) 200 **(E)** 225

23. Let $f(x) = x^3 + 5$ **and** $g(x) = x + 1$. **Find** $f(g(x))$.

(A) $x^3 + 6$ **(B)** $x^3 + 3x^2 + 3x + 1$ **(C)** $x^3 + 3x^2 + 3x + 6$

(D) $x^3 + 3x^2 + 3x + 5$ **(E)** $x^3 + 3x^2 + 3x + 8$

24. Find $m + n$ **if** $\begin{bmatrix} 3 & 7 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 42 \\ 17 \end{bmatrix}$.

(A) 9 **(B)** –7 **(C)** 3 **(D)** 2 **(E)** 16

25. How many distinct 4-letter arrangements can be made from the letters in the word "CALCULATOR"?

(A) 1380 **(B)** 1038 **(C)** 1398 **(D)** 1218 **(E)** 1200

26. Let P, Q, and R be positive integers. If $P + \dfrac{1}{Q + \dfrac{1}{R+1}} = \dfrac{229}{25}$ **, then** $P + Q + R =$

(A) 19 **(B)** 20 **(C)** 16 **(D)** 18 **(E)** 17

27. Let $f(x) = \dfrac{9x^3 + 8x^2 - 9}{3x^2 + 2}$ **and** $s(x)$ **be the slant asymptote of** $f(x)$. **Find the value of** $s(3)$.

(A) $\frac{19}{3}$ **(B)** 11 **(C)** 8 **(D)** $-\frac{97}{3}$ **(E)** $\frac{35}{3}$

28. An icosahedron has 20 faces, *V* vertices and 30 edges. Find the value of *V*.

(A) 12 **(B)** 10 **(C)** 14 **(D)** 16 **(E)** 18

29. The odds of drawing a pink raffle ticket at random from a bucket containing 300 tickets is 5:7. How many pink tickets would have to be removed from the bucket to reduce the odds to 12:25?

(A) 62 (B) 38 (C) 41 (D) 49 (E) 72

30. The roots of $x^3-4x^2-11x+30$ are p, q and r. Find $(p+q)(q+r)(p+r)$.

(A) −14 (B) −20 (C) −30 (D) −12 (E) −28

31. The harmonic mean of the roots of $0=x^4-16x^3+91x^2-216x+180$ is:

(A) $\frac{30}{91}$ (B) $\frac{6}{5}$ (C) $\frac{30}{31}$ (D) $\frac{10}{3}$ (E) 3

32. Given the sequence 10, 8, 2, 10, 50, 140, 298, … , find the 20th term.

(A) 14,050 (B) 19,730 (C) 16,730 (D) 3,503 (E) 3917

33. If $\log_2(x)+\log_2(x-7)=\log_2(4)+\log_2(15)$, then x equals____.

(A) 3 (B) 5 (C) 10 (D) 6 (E) 12

34. Which of the following is closed under multiplication?

I. irrational numbers II. whole numbers III. negative numbers IV. even integers

(A) II & IV (B) II, III & IV (C) I, II & III (D) II (E) none of these

35. The area under the curve $f(x)=\sin(2x)$ on the interval $\left[k,\frac{4\pi}{3}\right]$ is 0.5. Find k to the nearest tenth, where $\pi \le k \le \frac{4\pi}{3}$.

(A) 3.2 (B) 3.7 (C) 3.5 (D) 3.8 (E) 4.0

36. Carl would like to know the height of his peach tree before he buys a ladder. The angle of elevation to the top of the tree at a point 30 feet from the base of the tree is 40°. How tall is the tree? (nearest foot)

(A) 22 ft (B) 23 ft (C) 24 ft (D) 25 ft (E) 26 ft

37. Given that $x+\frac{1}{x}=32$, find $x^3+\frac{1}{x^3}$.

(A) 32,704 (B) 32,736 (C) 32,864 (D) 32,832 (E) 32,672

38. An operation "⊙" is defined by: $a \odot b = a-b^2$. What is the value of $(-1\odot 2)(2\odot -1)$?

(A) 6 (B) −3 (C) −4 (D) −6 (E) −18

39. Meredith set out to row on a lake. She rowed 600 m on a bearing of 75°, then 300 m on a bearing of 25°, then 350 m on a bearing of 52°. How far is she from her original starting point? (nearest meter)

(A) 1090 m (B) 1174 m (C) 1201 m (D) 1152 m (E) 1075 m

40. Given $f(x)=2x+5$ and $g(x)=x^2-3$ find $f(g'(2))$.

(A) 13 (B) 7 (C) 12 (D) 6 (E) 15

41. Given $f(x)=ax^4+bx^2+8x$ and $f(5)=25$, calculate $f(-5)$.

(A) –25 (B) 25 (C) –55 (D) 55 (E) –15

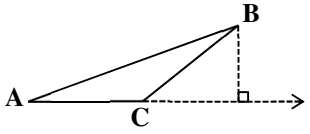
42. Avery has a large stack of $5.00 bills, $10.00 bills and $20.00 bills. She wants to put 4 bills into an envelope to give to charity. In how many ways can Avery stuff the envelope?

(A) 15 (B) 35 (C) 5 (D) 30 (E) 45

43. Point M$(-2,5)$ is the midpoint of the line segment with endpoints P$(-4,y)$ and Q$(x,12)$. Find PQ. (nearest tenth)

(A) 11.7 (B) 12.0 (C) 17.1 (D) 14.6 (E) 13.8

44. On the triangle ABC shown, m∠BAC = $\frac{\pi}{6}$ radians, AB = 24 and AC = 18. Find the area of triangle ABC.



(A) $54\sqrt{3}$ (B) 144 (C) 108

(D) $144\sqrt{3}$ (E) 54

45. Find the sum of the coefficients of the 2nd and 3rd terms in the polynomial expansion of $(2x-3)^9$.

(A) 40,704 (B) 34,560 (C) 44,928 (D) –44,928 (E) –6400

46. How many four-digit numbers exist such that all of the digits are prime?

(A) 24 (B) 625 (C) 120 (D) 500 (E) 256

47. The probability that it will rain on any single May day in Bowie, TX is 0.15. If Sarah plans a seven-day trip in May, what is the probability that it will rain at least one day while she is there? (nearest thousandth)

(A) 0.679 (B) 0.321 (C) 0.623 (D) 0.377 (E) 0.396

48. Simplify $\sqrt[3]{2}\left(\sqrt[3]{4}-2\sqrt[3]{32}\right)$.

(A) –4 (B) –2 (C) 4 (D) –6 (E) –8

49. The repeating decimal 0.4222… in base 5 can be written as which of the following fractions in base 5 in simplified terms?

(A) $\frac{33}{44_5}$ (B) $\frac{12}{20_5}$ (C) $\frac{14}{20_5}$ (D) $\frac{4}{10_5}$ (E) $\frac{23}{40_5}$

50. Given $y=\ln|5x-11|$, find the value(s) of x for which $\frac{dy}{dx}=\frac{dx}{dy}$.

(A) 1.2 (B) 0.6 (C) 0.6 and 1.6 (D) 3.2 (E) 1.2 and 3.2

51. Let $-4,a,1,b,c,...$ be the terms of a Fibonacci-type sequence. Find $a+b+c$.

(A) 15 (B) 18 (C) 16 (D) 11 (E) 9

52. The graph of $h(x)=\dfrac{2x^2-19x+24}{x-8}$ suggests that the *discontinuity* at $x=8$ is *removable* by defining $h(8)$ to be_____.

(A) 0 (B) 16 (C) 13 (D) 19 (E) 14

53. What is the 10^{-8} digit in the expansion of $1+(x-2)+\dfrac{(x-2)^2}{2!}+\dfrac{(x-2)^3}{3!}+\ldots$, when $x=1$?

(A) 0 (B) 4 (C) 5 (D) 8 (E) 9

54. Which of the following is a reference angle for $\dfrac{29\pi}{6}$?

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3}$ (C) $\dfrac{\pi}{4}$ (D) 0 (E) $\dfrac{2\pi}{3}$

55. Andrew is 6 feet tall. He is walking at a rate of 5 feet per second toward a street light that is 25 feet tall. What is the rate of change of the length of Andrew's shadow? (nearest tenth ft/s)

(A) –2.1 ft/s (B) –1.4 ft/s (C) –1.8 ft/s (D) –1.6 ft/s (E) –2.0 ft/s

56. If the three numbers 227, 292 and 370 are each divided by the number D, each of their quotients has the same remainder R. Find R.

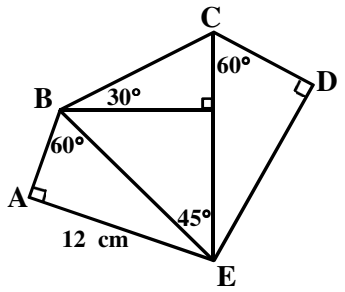
(A) 5 (B) 6 (C) 0 (D) 16 (E) 8

57. Which of the following mathematicians are associated with working with prime numbers?

I. Erastosthenes II. Marin Mersenne III. Sophie Germain

(A) I only (B) I & II (C) II & III (D) I & III (E) I, II & III

58. Find the perimeter of the pentagon ABCDE to the nearest centimeter.



(A) 53 cm (B) 57 cm (C) 51 cm

(D) 61 cm (E) 58 cm

59. The sum of the first *n* terms of a sequence is given by $S_n=\dfrac{3}{2}\left(n^2+3n\right)$, where $n\in\mathbb{Z}^+$. Find the 8th term of the sequence.

(A) 27 (B) 132 (C) 30 (D) 24 (E) 105

60. $\begin{vmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{vmatrix}=$

(A) $\sin(2\alpha)$ (B) $\sec^2(\alpha)$ (C) $\cos(2\alpha)$ (D) $\csc^2(\alpha)$ (E) 1

2018-2019 TMSCA Mathematics Test Three Answers

1. C	21. D	41. C
2. D	22. A	42. A
3. A	23. C	43. D
4. A	24. D	44. C
5. C	25. C	45. B
6. B	26. D	46. E
7. D	27. E	47. A
8. A	28. A	48. D
9. C	29. C	49. C
10. C	30. A	50. E
11. A	31. D	51. B
12. C	32. C	52. C
13. C	33. E	53. B
14. E	34. A	54. A
15. C	35. B	55. D
16. D	36. D	56. B
17. B	37. E	57. E
18. A	38. D	58. C
19. E	39. B	59. A
20. A	40. A	60. C

2018-2019 TMSCA Mathematics Test Three Solutions

6. One "labor" of land is about 177 acres, so 13 labors is 2301 acres.

8. $m\angle C + 8m\angle C = 180°$ **for** $m\angle C = 20°$ **and** $m\angle A = 160°$

11.

$$p(A \cup B) = p(A) + p(B) - p(A) \times p(B)$$

$$\frac{213}{400} = p(A) + 3p(A) - 3\left(p(A)\right)^2$$

for $p(A) = 0.15$.

16. The numbers in the 15th row will be ${}_{15}C_x$**, where** x **is an integer from 0 to 15. It works well to enter this as a function and look at the table to see that 2002 is not a function value.**

22. $\frac{2I + 2P - 2}{2} = \frac{4 + 12 - 2}{2} = 7$ **square units where each square unit represents 25 cm² and a total of 175 cm².**

25. CALCULATOR has 2-C's, 2-A's and 2-L's and a total of 7 distinct letters.

No repeats: ${}_7C_4 \times 4! = 840$

One letter repeating: $3 \times {}_6C_2 \times \frac{4!}{2!} = 540$

Two letters repeating: $3 \times \frac{4!}{(2!)(2!)} = 18$

For a sum of 1398.

31. The roots are 2, 3, 5 and 6, so the harmonic mean is

$$\frac{4}{\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{6}} = \frac{10}{3}$$

37. $x^3 - \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 - 3\right] = 32\left(32^2 - 3\right) =$

32,672.

42. ${}_{3+4-1}C_4 = {}_6C_4 = 15$

45. $9(2)^8(-3)^1 + {}_9C_2(2)^7(-3)^2 = 34{,}560$

47. $1 - p(no\ rain) = 1 - (0.85)^7 \approx 0.68$

52. $h(x) = \frac{(2x-3)(x-8)}{x-8}$ **is equal to** $g(x) = 2x - 3$ **at all points except when** $x = 8$**, so the value that removes the discontinuity is** $g(8) = 16 - 3 = 13$

53. This is the MacClaurin series representation of the function $f(x) = e^{x-2}$**, so** e^{1-2} **and the** 10^{-8} **place digit 4.**

55. Set up the similar triangles letting x **be the distance from the pole and** s **be the length of the shadow for** $\frac{6}{s} = \frac{25}{x+s}$ **then simplify to** $6x = 19s$ **and** $6\frac{dx}{dt} = 19\frac{ds}{dt}$ **then substitute in the rate of change for** x **which is -5 ft/s and solve for** $\frac{ds}{dt} = -\frac{30}{19} \approx -1.6$.

60. $\cos^2\alpha - \sin^2\alpha = \cos(2\alpha)$