T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 4 ©

N O V E M B E R 1 1 , 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2017-2018 TMSCA Mathematics Test Four

1. **Evaluate:** $0.08333... \div 0.75 + 1.125 - 0.8333... \times 3.2$.

   (A) $-\frac{11}{72}$ **(B)** $\frac{89}{72}$ **(C)** $\frac{7}{5}$ **(D)** $-\frac{103}{72}$ **(E)** $-\frac{283}{180}$

2. **Points A and B have coordinates** $(5,-5)$ **and** $(-9,17)$ **respectively. What is the *x*-coordinate of the *x*-intercept of the perpendicular bisector of the segment** $\overline{AB}$ **?**

   (A) $-\frac{80}{7}$ **(B)** $\frac{64}{7}$ **(C)** $-\frac{20}{7}$ **(D)** $-\frac{20}{11}$ **(E)** $\frac{84}{11}$

3. **66 yards per second is the same speed as _____miles per hour.**

   (A) 132 **(B)** 135 **(C)** 138 **(D)** 114 **(E)** 116

4. **If** $-11 = 3x - y$, $x - 4y = -22$ **and** $x + ay = 0$, **then** $a =$ **?**

   (A) 2.5 **(B)** –2.5 **(C)** 1.5 **(D)** 0.4 **(E)** 5

5. **Dallas is 415 miles from Corpus Christi. At 5:30 pm, Jennifer left Dallas and drove towards Corpus Christi at 68 mph. An hour later, Elisa left Corpus Christi and drove toward Dallas at 72 mph. What is the positive difference in their driving distances when they pass each other? (nearest tenth mile)**

   (A) 66.5 mi **(B)** 60.1 mi **(C)** 66.2 mi **(D)** 13.9 mi **(E)** 58.1 mi

6. **Which of the following is not a one-to-one function?**

   (A) $y = 8x^3$ **(B)** $y = \frac{2}{x^2 + 4}$ **(C)** $y = \sqrt[5]{5x}$ **(D)** $y = 2^{2x}$ **(E)** all are one to one

7. **Simplify:** $\frac{n!(n+3)!}{(n-2)!} \div \frac{(n+1)!(n+2)!}{(n-1)!}$.

   (A) $\frac{n^2 - 2n - 3}{n-1}$ **(B)** $\frac{n^2 + 2n - 3}{n+1}$ **(C)** $\frac{n^2 - n - 2}{n+1}$

   (D) $\frac{n^2 + 2n - 3}{n-1}$ **(E)** $\frac{n^2 - n - 2}{n-1}$

8. **Carl, Darren and Eli can each tile a floor in 8 hours, 8.4 hours and 9.5 hours respectively. How long would it take them to tile a floor that is twice as long and twice as wide together? (nearest minute)**

   (A) 11 hr 27 min **(B)** 11 hr **(C)** 13 hr 22 min **(D)** 11 hr 45 min **(E)** 5 hr 43 min

9. **If *p* and *q* are the zeros of the function** $f(x) = 18x^2 + 4x + 41$ **then** $pq^2 + p^2q =$

   (A) $\frac{41}{81}$ **(B)** $-\frac{41}{162}$ **(C)** $\frac{41}{162}$ **(D)** $-\frac{82}{81}$ **(E)** $-\frac{41}{81}$

10. **How many ordered pairs are solutions to the equation** $9x + 8y = 2017$, **where *x* and *y* are both non-negative integers?**

    (A) 27 **(B)** 28 **(C)** 30 **(D)** 29 **(E)** 31

11. The first term of an infinite geometric sequence is 48, while the third term is 12. What is the larger possible value for the sum of all the terms in the sequence?

(A) 48 (B) 96 (C) 72 (D) 32 (E) 36

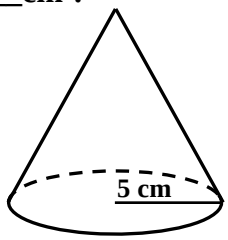
12. Let x vary inversely with y^2+2. If $x=8$ when $y=5$, find x when $y=4$.

(A) 12 (B) 18 (C) $\frac{2}{3}$ (D) $\frac{4}{9}$ (E) 24

13. How many distinct arrangements can be made with the letters in the word "ORIGAMI"?

(A) 720 (B) 5040 (C) 2520 (D) 1440 (E) 3240

14. The total surface area of the cone shown is 90π cm^2. The volume of the cone is _____cm^3.



(A) 90π (B) 108π (C) 96π (D) 144π (E) 100π

15. Let $f(x)=x^2+1$ and $g(x)=x^4$. Calculate $f(g(-2))$.

(A) 226 (B) 290 (C) 145 (D) 257 (E) 170

16. $6^3+5^3+\ldots+17^3=$

(A) 23,309 (B) 23,435 (C) 18,496 (D) 23,248 (E) 23,184

17. Given that the binomial $x+2$ is a factor of $Ax^3-x^2-Ax+22$, calculate the value of A.

(A) $\frac{13}{3}$ (B) 4 (C) $\frac{9}{5}$ (D) $-\frac{13}{3}$ (E) 3

18. The mathematical statement $(2x-3)(4x-7)=8x^2-14x-12x+21$ is an example of _____property.

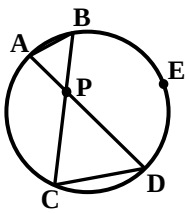
(A) Associative (B) Commutative (C) Identity

(D) Transitive (E) Distributive

19. Point $P(-3,2)$ lies on the x-y plane. Point P is reflected over the line $y=-x$ to point Q. Point Q is translated -6 units horizontally to point R. Point R is rotated 90° clockwise around the origin to point S. The coordinates of point S is (x,y). Find $x+y$.

(A) –1 (B) 11 (C) 1 (D) 4 (E) –3

20. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$ and $\overline{CD}$ are chords of the circle shown. Find mBED if $m\angle ADC=55^\circ$ and $m\angle APB=38^\circ$.



(A) 174° (B) 172° (C) 190° (D) 168° (E) 182°

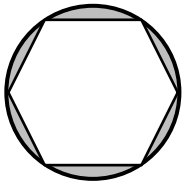
21. Which of the following are the side lengths of a scalene, obtuse triangle?

(A) 11, 11, 15 (B) 11, 13,16 (C) 9, 11, 14 (D) 11, 15, 18 (E) 11, 15, 21

22. A hexagonal dipyramid has 18 edges and the number of faces is four more than the number of vertices. How many vertices does it have?

A) 6 (B) 18 (C) 16 (D) 8 (E) 12

23. The regular hexagon in the illustration is inscribed in the circle. If a dart thrown strikes randomly inside the circle, what is the probability that it will land in the shaded region? (nearest hundredth)



(A) 0.54 (B) 0.17 (C) 0.18 (D) 0.46 (E) 0.19

24. If $\frac{A}{x+5}+\frac{B}{2x+3}=\frac{10x+8}{2x^2+13x+15}$ **, then** $A+B=?$

(A) −6 (B) 7 (C) −2 (D) 4 (E) −5

25. Using the following array, determine the value of the last number in the 25th row.

1					(row 1)
3	5				(row 2)
7	9	11			(row 3)
13	15	17	19		(row 4)
21	23	25	27	29	(row 5)
…					(…)

(A) 599 (B) 701 (C) 651 (D) 675 (E) 649

26. The bearing from town A to town B is 191°, and the bearing from town B to town C is 72°. It takes 1 hour and 40 minutes at 75 mph to go from town A to town B and 1 hour and 15 minutes to go from town B to town C travelling at 64 mph. How far is town A from town C? (nearest mile)

(A) 111 mi (B) 155 mi (C) 119 mi (D) 100 mi (E) 185 mi

27. If an integral factor of 252, not including 1 or 252 is chosen at random, what are the odds that it is a multiple of 7?

(A) 1:2 (B) 5:7 (C) 1:1 (D) 10:13 (E) 3:1

28. If $2+3i$ **is one of the zeros the polynomial** $f(x)=2x^3-13x^2+46x-65$ **, then another of its zeros is:**

(A) $\frac{2}{3}$ **(B)** $-\frac{3}{2}$ **(C)** $\frac{5}{2}$ **(D) −1 (E)** $\frac{2}{5}$

29. Find $\lim_{x\to-\infty}\frac{-4x^2+9x+7}{3x^3-9}$

(A) $-\frac{4}{3}$ **(B) −1 (C)** $\frac{3}{4}$ **(D) 0 (E) does not exist**

30. $330220_4-220330_4+2323_4=$ _____$_8$

(A) 2637 (B) 2647 (C) 2535 (D) 2651 (E) 2531

31. If $f''(x) = 54x - 10$ and $f(1) = -3$ and $f(-1) = -25$, then $f(2) =$ _____.

(A) 90 **(B)** 65 **(C)** 47 **(D)** 53 **(E)** 13

32. If $a_0 = -2$, $a_1 = 3$, $a_2 = 5$ and $a_n = (a_{n-3})(a_{n-1}) + a_{n-2}$ for $n \geq 3$, then $a_6 =$?

(A) −625 **(B)** 593 **(C)** 625 **(D)** −87 **(E)** −9575

33. If $\begin{bmatrix} 2 & -8 \\ a & -11 \end{bmatrix} - \begin{bmatrix} 3 & b \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ 10 & -16 \end{bmatrix}$ then $a + b =$?

(A) 24 **(B)** −4 **(C)** 36 **(D)** −7 **(E)** −16

34. Quentin is twice as old as Paul, and Quentin is five years younger than Ron. In four years the sum of their ages will be 57. How old is Quentin now?

(A) 8 **(B)** 6 **(C)** 16 **(D)** 12 **(E)** 20

35. Find the total area of the two regions enclosed by the curves $y = x^3 - 3x^2 - 9x + 27$ and $y = x + 3$.

(A) 127.50 **(B)** 101.75 **(C)** 85.75 **(D)** 92.75 **(E)** 106.75

36. A baseball diamond has the shape of a square with sides 90 ft. long. A player runs from first base to second base at a rate of 22 ft/sec. How fast is the player's distance from home plate changing when the player is 15 ft. from second base? (nearest tenth)

(A) 3.6 ft/sec **(B)** 13.9 ft/sec **(C)** 9.3 ft/sec **(D)** 14.1 ft/sec **(E)** 12.9 ft/sec

37. Let A and B be the roots of $f(x) = 2x^2 + 9x + 7$. Find the value of $A + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$.

(A) $\frac{6561}{16}$ **(B)** $\frac{2401}{10000}$ **(C)** $\frac{81}{256}$ **(D)** $\frac{625}{16}$ **(E)** $\frac{2401}{16}$

38. $\left(-2 - 3\sqrt{-10}\right)\left(6\sqrt{-8}\right)$

(A) $-72\sqrt{5} - 24\sqrt{2}\,i$ **(B)** $36\sqrt{10}$ **(C)** $72\sqrt{5} - 24\sqrt{2}\,i$

(D) $-24\sqrt{2} + 72\sqrt{5}\,i$ **(E)** $72\sqrt{10}$

39. Simplify: $\left(a^2 \div b^3\right)^{-4} \div a^7 \times b^5$.

(A) $\frac{b^{10}}{a^{15}}$ **(B)** $\frac{b^{11}}{a^8}$ **(C)** $\frac{b^{17}}{a^{15}}$ **(D)** $\frac{b^{14}}{a}$ **(E)** $\frac{b}{a^8}$

40. The function $f(x) = \frac{6x^2 + 5x - 25}{9x^2 + 6x - 35}$ has a vertical asymptote at $x = V$ and a horizontal asymptote at $y = H$ and a removable discontinuity when $x = R$. Find $V + H + R$.

(A) $-\frac{7}{3}$ **(B)** 0 **(C)** $\frac{2}{3}$ **(D)** −1 **(E)** $-\frac{1}{6}$

41. How many 3-digit numbers can be made with the digits 0, 0, 2, 4, and 6?

(A) 20 **(B)** 18 **(C)** 24 **(D)** 21 **(E)** 15

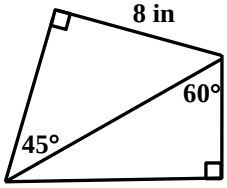
42. If $f(x)=\tan^3(x)$**, then** $\lim_{h\to 0}\dfrac{f\left(\frac{\pi}{4}+h\right)-f\left(\frac{\pi}{4}\right)}{h}$ **is**

(A) 6 **(B)** -1 **(C)** 1 **(D)** 3 **(E)** 0

43. If $f(x)=5^{2x}$, $g(x)=\log_5 x$ **and** $a\geq 3$**, then** $g\left(f(a-3)\right)=?$

(A) $2a$ **(B)** 5^{a-3} **(C)** $2a-6$ **(D)** $\log_3(a+1)$ **(E)** $a-3$

44. What is the perimeter of the quadrilateral shown? (nearest inch)



(A) 33 in **(B)** 31 in **(C)** 37 in **(D)** 33 in **(E)** 35 in

45. Which of the following functions expresses the perimeter, *P*, of an equilateral triangle in terms of the length of the height, *h*?

(A) $P=h^2\sqrt{3}$ **(B)** $P=3\sqrt{3}\,h$ **(C)** $P=\dfrac{3h}{2}$ **(D)** $P=\dfrac{h^2\sqrt{3}}{3}$ **(E)** $P=2\sqrt{3}\,h$

46. A set of five integers arranged least to greatest has a mean of 11, mode of 15, median of 13, and range of 10. Find the second number.

(A) 9 **(B)** 5 **(C)** 11 **(D)** 13 **(E)** 7

47. Find the area of a convex quadrilateral with vertices at $(-1,9)$, $(3,11)$, $(7,2)$ **and** $(2,-5)$ **.**

(A) 71.5 **(B)** 69.5 **(C)** 56.5 **(D)** 67.5 **(E)** 73

48. The repeating decimal 0.3111… in base 5 can be written as which of the following fractions in base 5 in simplified terms?

(A) $\dfrac{103}{330_5}$ **(B)** $\dfrac{24}{120_5}$ **(C)** $\dfrac{23}{40_5}$ **(D)** $\dfrac{2}{10_5}$ **(E)** $\dfrac{13}{20_5}$

49. If $\dfrac{x+7}{x-7}+\dfrac{x-7}{x+7}=2+\dfrac{A}{(x-7)(x+7)}$ **where** $A\in\mathbb{Z}^+$ **then** $A=?$

(A) 196 **(B)** 49 **(C)** 14 **(D)** 28 **(E)** 0

50. Find the range, or ranges of values *k* can take for $kx^2-8x+10-k=0$ **to have two distinct rational solutions.**

(A) $(2,8)$ **(B)** $(-\infty,-12)\cup(4,\infty)$ **(C)** $(-\infty,-8)\cup(-2,\infty)$

(D) $(-8,-2)$ **(E)** $(-\infty,2)\cup(8,\infty)$

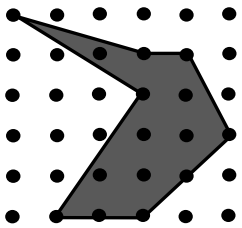
51. Find the *y*-intercept of the tangent line to $7x^2+2y^2=25$ **at the point** $(-1,-3)$**.**

(A) $\left(0,\dfrac{25}{6}\right)$ **(B)** $\left(0,-\dfrac{25}{6}\right)$ **(C)** $\left(0,\dfrac{25}{7}\right)$ **(D)** $\left(0,\dfrac{11}{6}\right)$ **(E)** $\left(-\dfrac{11}{7},0\right)$

52. A lightbulb company produces bulbs that are faulty on average 2.8% of the time. If 6 bulbs are packaged together, what is the probability that at least one of the bulbs is faulty? (nearest tenth)

(A) 16.9% (B) 16.3% (C) 15.2% (D) 15.7% (E) 18.2%

53. Find the area of the shaded region on the illustration shown if the dots on the grid are 6 cm. apart both horizontally and vertically.



(A) 57 cm^2 (B) 378 cm^2 (C) 360 cm^2 (D) 342 cm^2 (E) 63 cm^2

54. $\dfrac{1-\cos(2\theta)}{\sin(2\theta)} =$

(A) $\tan(2\theta)$ **(B)** $\tan\theta$ **(C)** $\cot(2\theta)$ **(D)** $1+\tan\theta$ **(E)** $\tan(2\theta)$

55. A game is played wherein two fair dice are rolled and the sum of the two top faces is calculated. If the sum is 6 or 8, the player wins \$3.50. If the sum is 7, then the player loses \$4.00. What is the mathematical expectation of a single roll? (nearest cent)

(A) \$0.58 loss (B) \$0.25 loss (C) \$0.31 gain (D) \$0.56 loss (E) \$0.25 gain

56. Find the digit in the millionths place of the sum of the series $0.7-\dfrac{(0.7)^3}{3!}+\dfrac{(0.7)^5}{5!}-\dfrac{(0.7)^7}{7!}+\dfrac{(0.7)^9}{9!}-\ldots$

(A) 4 (B) 6 (C) 9 (D) 5 (E) 7

57. If $PQR_5 + RPQ_4 - QPR_3$ **has a numeric value in base 10 of:**

(A) $26P-3Q+20R$ **(B)** $30P-3Q+14R$ **(C)** $26P-3Q+16R$

(D) $28P-3Q+16R$ **(E)** $30P+15Q+16R$

58. The lengths of the sides of triangle PQR are the roots of $f(x)=2x^3-29x^2+134x-198$**. The perimeter of triangle PQR is 14.5. Find the area of triangle PQR. (nearest tenth)**

(A) 9.1 (B) 6.4 (C) 7.3 (D) 6.2 (E) 7.8

59. $\det\begin{bmatrix}\sin A & \cos A\\ -\sin A & \cos A\end{bmatrix} =$

(A) $\cos 2A$ **(B)** $\cos^2 A-\sin^2 A$ **(C) 0 (D)** $\sin 2A$ **(E) 1**

60. Which of the following statements about $f(x)=7-|5x|$ **?**

I. $f'(x)$ **exists for all *x* in the domain of** $f(x)$

II. $f^{-1}(x)$ **is a function**

III. $\lim\limits_{x\to a} f(x)$ **exists for all *a* in the domain of** $f(x)$

(A) II only (B) I & II (C) III only (D) II & III (E) none of these

2017-2018 TMSCA Mathematics Test Four Answers

1. D	**21. E**	**41. D**
2. A	**22. D**	**42. A**
3. B	**23. B**	**43. C**
4. D	**24. D**	**44. B**
5. E	**25. E**	**45. E**
6. B	**26. A**	**46. E**
7. B	**27. C**	**47. D**
8. A	**28. C**	**48. C**
9. E	**29. D**	**49. A**
10. B	**30. B**	**50. E**
11. B	**31. C**	**51. B**
12. A	**32. B**	**52. D**
13. C	**33. E**	**53. D**
14. E	**34. C**	**54. B**
15. D	**35. B**	**55. C**
16. E	**36. D**	**56. E**
17. E	**37. A**	**57. C**
18. E	**38. C**	**58. B**
19. B	**39. C**	**59. D**
20. A	**40. B**	**60. C**

2017-2018 TMSCA Mathematics Test Four Select Solutions

2. $M=(-2,6)$ and the slope of the perpendicular is $m=\frac{7}{11}$, then to find the x-coordinate of the x-intercept: $\frac{7}{11}=\frac{0-6}{x+2}$ for $7x+14=-66$ and $x=-\frac{80}{7}$.

8. $t=\frac{2(2)}{\frac{1}{8}+\frac{1}{8.4}+\frac{1}{9.5}}\approx 11.45$ or 11 hr. and 27 min

9. This is the product of the sum of the roots and product of the roots, or $-\frac{4}{18}\left(\frac{41}{18}\right)=-\frac{41}{81}$

11. $r=\pm\frac{1}{2}$ for a larger sum of $\frac{48}{1-0.5}=96$

12. Only the 2-I's repeat, so $\frac{7!}{2!}=2520$

16. $\left(\frac{17\cdot 18}{2}\right)^2-(1+8+27+64+125)=23{,}184$

17. Solve $A(-2)^3-(-2)^2-A(-2)+22=0$ for $A=3$

19. $\mathrm{Q}(-2,3)$, $\mathrm{R}(-8,3)$ and $\mathrm{S}(3,8)$ for a sum of 11.

20. $\mathrm{mAC}=110^\circ$ and $\mathrm{mAB}+\mathrm{mCD}=76^\circ$, so $360^\circ-110^\circ-76^\circ=\mathrm{mBED}=174^\circ$

21. For an obtuse triangle, $a^2+b^2<c^2$, which is only true for $11^2+15^2<21^2$

22. $V+F-E=2$ for $V+V+4-18=2$ and $V=8$

24. $A(2x+3)+B(x+5)=10x+8$, so $\begin{matrix}2A+B=10\\3A+5B=8\end{matrix}$ for $A=6, B=-2$ and $A+B=4$

28. If $2+3i$ is a zero, then $2-3i$ is also a zero and the sum of the roots is $-\frac{b}{a}=6.5$ then $6.5-(2+3i)-(2-3i)=2.5=\frac{5}{2}$

Let x be the player's distance from 1st base and d be the player's distance from home plate. Then $\frac{dx}{dt}=22\ ft/s$ and $x^2+90^2=d^2$ for $x\frac{dx}{dt}=d\frac{dd}{dt}$ and at the given time, $75(22)=(117.15)\frac{dd}{dt}$ and $\frac{dd}{dt}\approx 14.1\ ft/s$

41. The 3-digit numbers with repeat digits are 200, 400 and 600 for the rest, there are 3 possible 1st digits, then 3 choices for the 2nd digit and two for the 3rd, for $3+3(3)(2)=21$

42. This is the definition of derivative of $f(x)=\tan^3 x$ when $x=\frac{\pi}{4}$

47. Find the area of any polygon using coordinates of vertices by $A=\left|\frac{(x_1y_2-y_1x_2)+(x_2y_3-y_2x_3)...+(x_ny_1-y_nx_1)}{2}\right|$ where the points are arranged in either clockwise or counter-clockwise order. So here $\left|\frac{(-1\cdot 11-9\cdot 3)+(3\cdot 2-11\cdot 7)+(-5\cdot 7-2\cdot 2)+(2\cdot 9-(-5)(-1))}{2}\right|=$ 67.5

50. $64-4k(10-k)>0$ for $(-\infty,2)\cup(8,\infty)$

51. $14x+4y\frac{dy}{dx}=0$ for $14(-1)+4(-3)\frac{dy}{dx}=0$ for $\frac{dy}{dx}=-\frac{7}{6}$, $-\frac{7}{6}=\frac{y+3}{0+1}$ and $y=-\frac{25}{6}$

52. $1-p(\text{no faulty bulbs})=1-(0.972)^6\approx 15.7\%$

54. $\frac{(\cos^2\theta+\sin^2\theta)-(\cos^2\theta-\sin^2\theta)}{2\sin\theta\cos\theta}=\frac{2\sin^2\theta}{2\sin\theta\cos\theta}=\tan\theta$

56. This is the McClaurin series for $f(x)=\sin x$ and the millionths place digit of $f(0.7)$ is 7.

58. The semi-perimeter of the triangle is 7.25 and the area of the triangle is $\sqrt{7.25\times\frac{f(7.25)}{2}}\approx 6.41$