2nd Annual Harvard-MIT November Tournament

Saturday 7 November 2009

Theme Round

Shortest Paths

1. [**3**] Paul starts with the number 19. In one step, he can add 1 to his number, divide his number by 2, or divide his number by 3. What is the minimum number of steps Paul needs to get to 1?

2. [**4**] You start with a number. Every second, you can add or subtract any number of the form $n!$ to your current number to get a new number. In how many ways can you get from 0 to 100 in 4 seconds? ($n!$ is defined as $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$, so $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, etc.)

3. [**5**] Let C be the circle of radius 12 centered at $(0, 0)$. What is the length of the shortest path in the plane between $(8\sqrt{3}, 0)$ and $(0, 12\sqrt{2})$ that does not pass through the interior of C?

4. [**6**] You are given a 5×6 checkerboard with squares alternately shaded black and white. The bottom-left square is white. Each square has side length 1 unit. You can normally travel on this board at a speed of 2 units per second, but while you travel through the interior (not the boundary) of a black square, you are slowed down to 1 unit per second. What is the shortest time it takes to travel from the bottom-left corner to the top-right corner of the board?

5. [**7**] The following grid represents a mountain range; the number in each cell represents the height of the mountain located there. Moving from a mountain of height a to a mountain of height b takes $(b-a)^2$ time. Suppose that you start on the mountain of height 1 and that you can move up, down, left, or right to get from one mountain to the next. What is the minimum amount of time you need to get to the mountain of height 49?

1	3	6	10	15	21	28
2	5	9	14	20	27	34
4	8	13	19	26	33	39
7	12	18	25	32	38	43
11	17	24	31	37	42	46
16	23	30	36	41	45	48
22	29	35	40	44	47	49

Five Guys

6. [**3**] There are five guys named Alan, Bob, Casey, Dan, and Eric. Each one either always tells the truth or always lies. You overhear the following discussion between them:

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Alan: "All of us are truth-tellers."
Bob: "No, only Alan and I are truth-tellers."
Casey: "You are both liars."
Dan: "If Casey is a truth-teller, then Eric is too."
Eric: "An odd number of us are liars."
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   Who are the liars?

7. [**4**] Five guys are eating hamburgers. Each one puts a top half and a bottom half of a hamburger bun on the grill. When the buns are toasted, each guy randomly takes two pieces of bread off of the grill. What is the probability that each guy gets a top half and a bottom half?

8. [**5**] A single burger is not enough to satisfy a guy's hunger. The five guys go to Five Guys' Restaurant, which has 20 different meals on the menu. Each meal costs a different integer dollar amount between \$1 and \$20. The five guys have \$20 to split between them, and they want to use all the money to order five different meals. How many sets of five meals can the guys choose?

9. [**6**] Five guys each have a positive integer (the integers are not necessarily distinct). The greatest common divisor of any two guys' numbers is always more than 1, but the greatest common divisor of all the numbers is 1. What is the minimum possible value of the product of the numbers?

10. [**7**] Five guys join five girls for a night of bridge. Bridge games are always played by a team of two guys against a team of two girls. The guys and girls want to make sure that every guy and girl play against each other an equal number of times. Given that at least one game is played, what is the least number of games necessary to accomplish this?