



ALGEBRA

This test consists of 10 short-answer problems to be solved individually in 50 minutes. Problems will be weighted with point values after the contest based on how many competitors solve each problem. There is no penalty for guessing.

No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted. Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.

Answers should be simplified as much as is reasonably possible and must be exact unless otherwise specified. Rational numbers should be written in lowest terms, although denominators of irrationals need not be rationalized. An n th root should be simplified so that the radicand is not divisible by the n th power of any prime.

Correct mathematical notation must be used. No partial credit will be given unless otherwise specified.

If you believe the test contains an error, please submit your protest in writing to the Science Center Lobby during lunchtime.

Enjoy!

HMMT 2014
Saturday 22 February 2014

Algebra

1. Given that x and y are nonzero real numbers such that $x + \frac{1}{y} = 10$ and $y + \frac{1}{x} = \frac{5}{12}$, find all possible values of x .

2. Find the integer closest to

$$\frac{1}{\sqrt[4]{5^4 + 1} - \sqrt[4]{5^4 - 1}}.$$

3. Let

$$A = \frac{1}{6} \left((\log_2(3))^3 - (\log_2(6))^3 - (\log_2(12))^3 + (\log_2(24))^3 \right).$$

Compute 2^A .

4. Let b and c be real numbers, and define the polynomial $P(x) = x^2 + bx + c$. Suppose that $P(P(1)) = P(P(2)) = 0$, and that $P(1) \neq P(2)$. Find $P(0)$.

5. Find the sum of all real numbers x such that $5x^4 - 10x^3 + 10x^2 - 5x - 11 = 0$.

6. Given that w and z are complex numbers such that $|w + z| = 1$ and $|w^2 + z^2| = 14$, find the smallest possible value of $|w^3 + z^3|$. Here, $|\cdot|$ denotes the absolute value of a complex number, given by $|a + bi| = \sqrt{a^2 + b^2}$ whenever a and b are real numbers.

7. Find the largest real number c such that

$$\sum_{i=1}^{101} x_i^2 \geq cM^2$$

whenever x_1, \dots, x_{101} are real numbers such that $x_1 + \dots + x_{101} = 0$ and M is the median of x_1, \dots, x_{101} .

8. Find all real numbers k such that $r^4 + kr^3 + r^2 + 4kr + 16 = 0$ is true for exactly one real number r .

9. Given that a , b , and c are complex numbers satisfying

$$a^2 + ab + b^2 = 1 + i$$

$$b^2 + bc + c^2 = -2$$

$$c^2 + ca + a^2 = 1,$$

compute $(ab + bc + ca)^2$. (Here, $i = \sqrt{-1}$.)

10. For an integer n , let $f_9(n)$ denote the number of positive integers $d \leq 9$ dividing n . Suppose that m is a positive integer and b_1, b_2, \dots, b_m are real numbers such that $f_9(n) = \sum_{j=1}^m b_j f_9(n-j)$ for all $n > m$. Find the smallest possible value of m .

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Algebra

PUT LABEL HERE

Name _____ Team ID# _____

Organization _____ Team _____

1. _____
2. _____
3. _____
4. _____
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Score: _____