



TMSCA HIGH SCHOOL MATHEMATICS TEST # 12 © FEBRUARY 24, 2018

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

2017-2018 TMSCA Mathematics Test Twelve

1. Evaluate: $\sqrt[3]{729} \div (16)^{3/2} + 6 \times 3^{-1} - 4$.
 (A) $-\frac{119}{64}$ (B) $\frac{905}{64}$ (C) -2 (D) $\frac{1}{4}$ (E) $-\frac{29}{16}$
2. Find the sum of the multiples of 6 that are greater than 12 and less than 562.
 (A) 26,220 (B) 26,772 (C) 25,920 (D) 26,208 (E) 25,650
3. Lynn sold 28 school orchestra concert tickets for a total of \$225. Each adult ticket sold for \$10.50, and each student ticket sold for \$4.75. How many adult tickets did she sell?
 (A) 14 (B) 12 (C) 16 (D) 10 (E) 8
4. 23 miles per hour = _____ inches per second. (nearest inch)
 (A) 188 in. (B) 213 in. (C) 405 in. (D) 145 in. (E) 134 in.
5. Find the x -intercept of the line containing $(3, -8)$ that is perpendicular to $5x + 3y - 4 = 0$.
 (A) $\left(\frac{31}{3}, 0\right)$ (B) $\left(-\frac{49}{5}, 0\right)$ (C) $\left(\frac{39}{5}, 0\right)$ (D) $\left(\frac{9}{5}, 0\right)$ (E) $\left(\frac{49}{3}, 0\right)$
6. $(5x + 2)(2x - 1) = 10x^2 - x - 2$ and $2x(y + 9) = 2xy + 18x$ are examples of the _____ property of equality.
 (A) associative (B) distributive (C) symmetric (D) commutative (E) multiplication
7. Tom can paint a fence in 4 hours. His friend Huck can paint the same fence in 6 hours. How long will it take the two boys to paint the fence if they work together? (nearest minute)
 (A) 2 hr. 40 min. (B) 3 hr. 9 min. (C) 2 hr. 36 min. (D) 2 hr. 24 min. (E) 3 hr. 2 min.
8. The total volume of a cone is 653 cm^3 , and the vertex angle is 44° . The slant height is _____ cm. (nearest tenth)
 (A) 16.9 (B) 12.5 (C) 13.8 (D) 15.6 (E) 17.1
9. A box contains 2 black marbles, 8 red marbles and 6 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 red marbles?
 (A) 1:10 (B) 1:9 (C) 1:7 (D) 2:9 (E) 2:7
10. Simplify: $\left(18 \sqrt[3]{81x^7}\right) \div \left(12 \sqrt[3]{24x^4}\right)$
 (A) $\frac{9x^2}{4}$ (B) $\frac{9x}{2}$ (C) $\frac{3x^2}{4}$ (D) $\frac{3x}{4}$ (E) $\frac{9x}{4}$
11. A small motor boat travelling from point A to point B along a river can make the 36-mile trip in 5 hours. The return trip takes 20 hours. If the speed of the boat and current remain constant throughout both trips, what is the speed of the current?
 (A) 4.5 mph (B) 7.2 mph (C) 2.1 mph (D) 2.7 mph (E) 2.4 mph

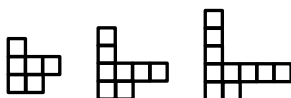
12. If $\int_3^k \frac{1}{x+5} dx = \ln 7$ find the value of k .

- (A) 61 (B) 56 (C) 10 (D) 15 (E) 51

13. Which of the following are the side lengths of a scalene, acute triangle?

- (A) 11, 15, 21 (B) 9, 10, 14 (C) 8, 8, 11 (D) 9, 12, 15 (E) 10, 24, 25

14. The three shapes below are made up of one-unit squares. If the pattern continues, the area of the figure with a perimeter of 110 units is _____square units.



- (A) 53 (B) 48 (C) 55 (D) 57 (E) 49

15. A particle's movement along the number line is defined by $f(t) = t^4 - 4t^3 - 26t^2 + 60t + 25$. At which of the following times is the particle moving to the left?

- (A) -2 (B) 0 (C) 2 (D) 6 (E) 8

16. Using the following pattern of numbers, determine the fifth term in the 19th row.

				1						(row 0)
			1		1					(row 1)
		1		2		1				(row 2)
	1		3		3		1			(row 3)
	1	4		6		4		1		(row 4)
1		5	10		10		5		1	(row 5)
			

- (A) 11,628 (B) 968 (C) 3,060 (D) 3,876 (E) 4,845

17. The letters "WEDNESDAY" are arranged in a line. How many distinct arrangements are possible if all of the vowels occur together?

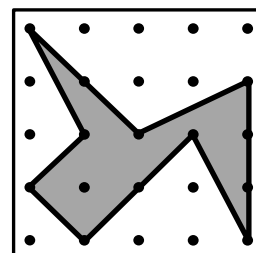
- (A) 4,320 (B) 30,240 (C) 17,280 (D) 8,640 (E) 15,120

18. Find a if $\begin{bmatrix} 3 & a \\ a & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} a \\ 33 \end{bmatrix}$.

- (A) -1 (B) -4 (C) -6 (D) 3 (E) 4

19. A rubber band was stretched on the geoboard to form this octagon. What is the area?

- (A) 6 units² (B) 7 units² (C) 7.5 units²
(D) 6.5 units² (E) 5 units²



20. If $\sin x + \cos x = \frac{2}{3}$, then $\tan x + \cot x = ?$

- (A) $-\frac{9}{4}$ (B) $-\frac{9}{5}$ (C) $-\frac{9}{2}$ (D) $-\frac{18}{5}$ (E) $\frac{9}{2}$

21. Let $f''(x) = 6x + 6$, $f'(2) = 19$ and $f(2) = 4$. Find $f(-1)$.

- (A) 1 (B) 2 (C) -1 (D) 3 (E) 0

22. Given that $(x - 3)$ is a factor of $6x^3 + kx^2 + 34x + 15$, find the value of k .

- (A) -27 (B) -156 (C) -26 (D) -31 (E) -14

23. The ellipse $9x^2 - 54x + 4y^2 + 8y = -49$ has foci at (x_1, y_1) and (x_2, y_2) . Find the value of $y_1 + y_2$.

- (A) $\sqrt{5}$ (B) $2\sqrt{5}$ (C) -2 (D) 6 (E) 2

24. Given the geometric sequence $-256, a, b, c, -81, \dots$ find the smallest possible value the sum of the sequence.

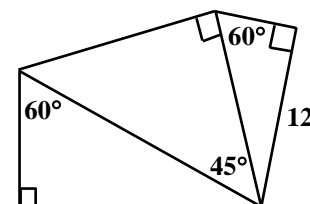
- (A) $-\frac{1024}{7}$ (B) -512 (C) $-\frac{512}{7}$ (D) $-\frac{1024}{3}$ (E) -1024

25. If $\log_8(5x - 1) - \log_8(x - 2) = 1$, then $x = ?$

- (A) 5 (B) 3 (C) 4 (D) 6 (E) No solution

26. The three triangles form a pentagon. What is the area of the pentagon?

- (A) $96 + 120\sqrt{3}$ (B) 240 (C) 180 (D) $96 + 72\sqrt{3}$ (E) $168\sqrt{3}$



27. If $(a - 3i)(b + 2i) = 13 + 11i$, where a and b are positive integers, find the value of $a + b$.

- (A) 3 (B) 1 (C) 8 (D) 7 (E) 2

28. Evaluate: $(0.777\dots)^{-1} \div (0.58333\dots)^{-1} \times (0.666\dots)^{-1}$.

- (A) $\frac{162}{49}$ (B) $\frac{18}{25}$ (C) $\frac{1}{2}$ (D) $\frac{9}{2}$ (E) $\frac{9}{8}$

29. The graph of $f(x) = (x^3 - 6x^2 - x + 30) \div (x^2 - 9)$ has a vertical asymptote(s) at:

- (A) $x = 3$ (B) $x = -3$ (C) $x = -3$ and $x = 3$ (D) $x = 0$ (E) no vertical asymptotes

30. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____ of the sides of the triangle.

- (A) Endpoints (B) Altitudes (C) Medians (D) Midpoints (E) Perpendicular Bisectors

31. Laura's Ice Cream shop specializes in double scoop sundaes. How many double scoop sundaes could be made using any of the 15 flavors on the menu?

- (A) 136 (B) 120 (C) 105 (D) 108 (E) 169

32. A cube is distorted so that the length is increased by 25%, the height is increased by 10% and the width is decreased by 8%. What is the percent change in the volume of the cube? (nearest 1%)

- (A) 31% (B) 27% (C) 25% (D) 26% (E) 29%

33. If $f(x) = ax^5 + bx^3 + cx - 9$ and $f(1) = -7$ then $f(-1) = \underline{\hspace{2cm}}$.

- (A) -11 (B) -7 (C) 1 (D) 11 (E) 5

34. If $5^x(25^{2y}) = 1$ and $3^{5x}(9^y) = \frac{1}{9}$ then $y = ?$

- (A) $-\frac{4}{9}$ (B) 1 (C) $\frac{1}{9}$ (D) $\frac{2}{3}$ (E) $-\frac{1}{3}$

35. Mr. Gardener's height is 6' 2". His is standing on a ladder with his feet 5' 7" from the ground trimming his oak tree. How long is Mr. Gardener's shadow, not including the ladder, when the angle of elevation of the sun from the ground is 66° ? (nearest inch)

- (A) 29' 10" (B) 32' 11" (C) 29' 8" (D) 32' 9" (E) 32' 3"

36. A set of positive integers has a mean of 20, a median of 22, a mode of 26 and a range of 19. If A, B, C, D and E are the integers arranged from least to greatest, what is the value of B?

- (A) 17 (B) 7 (C) 20 (D) 19 (E) 11

37. Evaluate $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{3} + h\right) - \tan\left(\frac{\pi}{3}\right)}{h}$.

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 4 (E) does not exist

38. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ be the terms of the Fibonacci sequence. Find $F_{22} - F_{20}$.

- (A) 17,711 (B) 4,181 (C) 21,892 (D) 6,765 (E) 10,946

39. If $\frac{2x-5}{x+3} + \frac{5x-2}{3x-1} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$ then $\frac{A+B+C}{P+Q+R} = ?$

- (A) 2 (B) $\frac{3}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{2}$ (E) $\frac{7}{8}$

40. Given that y is inversely proportional to the square of x and $y = 8$ when $x = 1.5$. Find the value of y when $x = 2\sqrt{3}$.

- (A) 2 (B) 1 (C) 2.5 (D) 1.5 (E) 3

41. The number 888 in base 9 is equivalent to the number k in base 3. Find the sum of the digits in the number k .

- (A) 12 (B) 7 (C) 4 (D) 8 (E) 5

42. What is the acute angle between the minute and hour hands on a circular clock at 3:07 pm?

- (A) 58.5° (B) 83° (C) 51.5° (D) 63.5° (E) 61.5°

43. Find the distance between the line $4x - 5y = 12$ and the point $(3, -7)$. (nearest tenth)

- (A) 9.2 (B) 4.6 (C) 7.1 (D) 8.9 (E) 5.5

44. Find the value of $A + B + C + D + E$ where A, B, C, D and E are integers greater than 0 and

$$\frac{972}{157} = A + \frac{1}{B + \frac{1}{C + \frac{1}{D + \frac{1}{E + 1}}}}$$

- (A) 19 (B) 21 (C) 22 (D) 20 (E) 23

45. Use the table of values to create a polynomial function and find K.

X	1	2	3	4	5	...	18
Y	-38	-33	-6	61	186		K

- (A) 15,279 (B) 12,762 (C) 18,106 (D) 5,307 (E) 5,981

46. Given that $12x + 9y = 48$ and $8x + 6y = k$. Find the value of k such that this system of equations has an infinite number of solutions.

- (A) 36 (B) 42 (C) 32 (D) 40 (E) 38

47. Find the Real number solution set of $3|4x + 2| > 18$.

- (A) $(-\infty, \infty)$ (B) $(-\infty, -1) \cup (2, \infty)$ (C) $(-2, 1)$ (D) $(-\infty, -2) \cup (1, \infty)$ (E) $(-1, 2)$

48. 487,32A,2B7 divided by 9 has a remainder of 8. Which of the following is a possible value of $A + B$.

- (A) 3 (B) 11 (C) 9 (D) 6 (E) 12

49. If $x - \frac{1}{x} = 7$, then $x^3 - \frac{1}{x^3} = ?$

- (A) 357 (B) 350 (C) 371 (D) 343 (E) 364

50. Find the slope of the graph of $4x^3 + 3y = 29$ at the point $(2, -1)$.

- (A) -12 (B) 8 (C) -16 (D) -8 (E) $-\frac{19}{3}$

51. $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \dots + \frac{1}{120} = \underline{\hspace{2cm}}$.

- (A) $\frac{28}{15}$ (B) $\frac{11}{6}$ (C) $\frac{13}{7}$ (D) $\frac{15}{8}$ (E) $\frac{20}{11}$

52. Let $\|V_1\| = 12$ and $\|V_2\| = 15$, where the direction angles of V_1 and V_2 are 45° and 60° respectively.

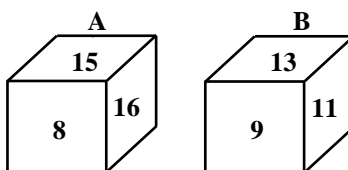
Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

- (A) 57° (B) 37° (C) 51° (D) 56° (E) 53°

53. How many 4-digit numbers can be created from the set of digits where the digits are not repeated?

- (A) 5,040 (B) 10,080 (C) 6,561 (D) 4,608 (E) 4,536

54. Leonard created a pair of special dice which have only three numbers each. The side opposite of each number is the same as the number. When the dice shown are rolled, the die with the smaller number on top wins. What is the probability that die B will win?



- (A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$ (E) $\frac{4}{9}$

55. The chart shows the losses and gains in an investment over the course of five years. What was the average annual growth rate over the course of the five years? (nearest hundredth of a percent)

Year	1	2	3	4	5
Percent Growth	+ 10%	-5 %	-3 %	+6%	+4%

- (A) +11.74% (B) +2.40% (C) +3.21% (D) +2.25% (E) +3.17%

56. Let $f(x) = \frac{2x^4 - 3x^3 + 5x^2}{x^3 - 8}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(3)$.

- (A) 2 (B) 3 (C) -3 (D) -2 (E) 6

57. The point $(7, -8)$ is rotated 1530° clockwise about the origin. The coordinates of the point after the rotations are _____.

- (A) $(-8, -7)$ (B) $(-8, 7)$ (C) $(8, 7)$ (D) $(-7, 8)$ (E) $(-7, -8)$

58. The lengths of the sides of triangle PQR are the roots of $f(x) = 2x^3 - 37x^2 + 206x - 315$. The perimeter of triangle PQR is 18.5. Find the area of triangle PQR. (nearest tenth)

- (A) 8.4 (B) 6.3 (C) 5.9 (D) 8.8 (E) 6.5

59. $f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} \dots$. Find the digit in the 10^{-8} place of $f(8)$.

- (A) 4 (B) 1 (C) 2 (D) 8 (E) 0

60. An insurance company sells 25,000 policies for \$1,050 each. They only have two levels of payouts. Their research shows that 1 in 10 policy holders will make a claim for \$8000 and 1 in 100 will make a claim for \$22,000. What is the company's expected profit on all the policies together?

- (A) \$1,000,000 (B) \$1,800,000 (C) \$100,000 (D) \$750,000 (E) \$1,500,000

2017 – 2018 TMSCA Mathematics Test Twelve Answers

1. A	21. A	41. A
2. D	22. D	42. C
3. C	23. C	43. E
4. C	24. E	44. A
5. E	25. A	45. A
6. B	26. D	46. C
7. D	27. C	47. D
8. A	28. E	48. B
9. B	29. B	49. E
10. E	30. E	50. C
11. D	31. B	51. D
12. E	32. B	52. E
13. E	33. A	53. E
14. C	34. C	54. D
15. C	35. B	55. D
16. D	36. D	56. B
17. E	37. D	57. A
18. A	38. E	58. C
19. A	39. B	59. B
20. D	40. D	60. D

2017-2018 TMSCA Mathematics Test Twelve Select Solutions

7. $\frac{1}{\frac{1}{4} + \frac{1}{6}} = 2.4$ hours for 2 hours 24 minutes

8. $r = l \sin 22$ and $h = l \cos 22$ then

$653 = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (l \sin 22)^2 (l \cos 22)$ for $l \approx 16.9$

12. $[\ln(x+5)]_3^k = \ln 7$ for $\ln(k+5) - \ln 8 = \ln 7$ then

$\frac{k+5}{8} = 7$ and $k = 51$

17. There are 6 consonants and treat the group of vowels as one block, for $7!$ arrangements, then within the vowels there will be $\frac{3!}{2!}$ arrangements because there are 3 vowels with a repeating E. The total number of arrangements will be the product: $7! \left(\frac{3!}{2!} \right) = 15,120$

19. Let P be the number of grid points that lie on the perimeter and I be the number of interior grid points, then

$A = \frac{P+2I}{2} - 1 = \frac{12+2}{2} - 1 = 6$

23. $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$ has the line $x = 3$ as the major axis and focal length of $\sqrt{9-4} = \sqrt{5}$ for foci $(3, -1-\sqrt{5})$ and $(3, -1+\sqrt{5})$, then $y_1 + y_2 = -2$

24. $-256r^4 = -81$ for $r = \pm \frac{3}{4}$ and sums $\frac{-256}{1-\frac{3}{4}} = -1024$ and

$\frac{-256}{1+\frac{3}{4}} = -\frac{1024}{7}$ with the first sum being the smallest.

25. $\frac{5x-1}{x-2} = 8$ for $x = 5$

31. ${}_{15+2-1}C_2 = 120$

33. Because the 3 terms with missing constants all have odd exponents on the variable, that portion of the function will yield opposite values for opposite values in the domain, so $-7 = C - 9$ and $C = 2$ then $f(-1) = -2 - 9 = -11$.

34. Solve the system $x + 4y = 0$ and $5x + 2y = -2$ for $y = \frac{1}{9}$

37. This is the definition of derivative of $f(x) = \tan x$ at

$x = \frac{\pi}{3}$.

41. In base 10, this number is $9^3 - 1 = 3^6 - 1$ which is 222222_3 for a sum of digits equal to 12.

43. $\frac{|3(4) + (-7)(-5) - 12|}{\sqrt{4^2 + (-5)^2}} \approx 5.5$

48. The sum of the known digits is 33, and $33 \div 9$ has a remainder of 6, which a division with a remainder of 6. If the sum of the digits was 35 or 44, then the remainder would be 8 making $A + B = 2$ or $A + B = 11$ with only 11 as an answer choice.

49. $x^3 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right) \left[\left(x - \frac{1}{x}\right)^2 + 3\right] = 7(49 + 3) = 364$

52. $\arctan\left(\frac{12 \sin 45^\circ + 15 \sin 60^\circ}{12 \cos 45^\circ + 15 \sin 60^\circ}\right) \approx 53^\circ$

53. The thousands place can't be 0, to the there are 9 choices for the thousands place and 9 choices for the hundreds place and a total $(9)(9)(8)(7) = 4536$

59. This is the McClaurin series expansion of $\sin 8 + \cos 8$ mode must be in radians.