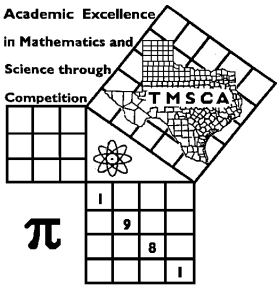

TMSCA HIGH SCHOOL MATHEMATICS TEST #8 ©

JANUARY 25, 2020

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2019-2020 TMSCA High School Math Test 8

1. $(17 \times 3! \times 2^{-1} + 9 - 13 \times 3) \div 7 =$

(A) $\frac{8}{7}$ (B) 3 (C) $\frac{25}{7}$ (D) 6 (E) $\frac{48}{7}$

2. Vidit went to Academy Sports and purchased some Nike racing shoes for \$125.65, three Under Armour running shirts for \$24.95 each, and some New Balance running socks for \$14.95. If the tax rate is 8.25%, how much change would he get if he paid with three one-hundred dollar bills?

(A) \$60.45 (B) \$62.56 (C) \$64.67 (D) \$66.78 (E) \$68.89

3. $(3x-5)^3 = ax^3 + bx^2 + cx + d$. $a + b + c + d =$

(A) -8 (B) -6 (C) -4 (D) 4 (E) 8

4. A = {o, r, g, a, n, i, c}. B = {c, h, e, m, i, s, t, r, y}. C = {i, s, a, b, e, l}. Find the number of elements in $(A \cap B) \cup C$?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

5. Kunal goes to Winco and purchases a total of 90 bags of fresh berries. His cart is filled with blueberries, blackberries, and raspberries. He has 12 more bags of blackberries than bags of blueberries. He has 12 more bags of raspberries than bags blackberries. How many bags of blueberries are in his cart?

(A) 16 (B) 18 (C) 20 (D) 22 (E) 24

6. Line L_1 contains the points $(-8, 6)$ and $(2, -4)$. Line L_2 contains the point $(0, 3)$ and is perpendicular to L_1. L_1 intersects L_2 at the point (a, b). $a + b =$ _____.

(A) -3 (B) -2.5 (C) -2 (D) -1.5 (E) -1

7. If Q is 75% of P and R is 50% of Q, then P is what percent of R?

(A) $37\frac{1}{2}\%$ (B) 75% (C) $112\frac{1}{2}\%$ (D) 175% (E) $266\frac{2}{3}\%$

8. Given the system: $.35x + .75y = 6.2$ and $\frac{3}{5}x + \frac{3}{8}y = \frac{243}{40}$. If the solution to the system is (a, b), then $a + b =$ _____.

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

9. If $\sqrt{5x-9} = x - 3$, then $x =$ _____

(A) 2 (B) 4 (C) 7 (D) 9 (E) 12

10. An eagle is perched on the top of a tree overlooking the Snake River. The eagle spots a rabbit on the bank of the river on the other side. If the angle of depression from the eagle to the rabbit is 12.2° and the tree is 38 feet tall, how far is it across the Snake River? (nearest foot)

(A) 176 ft (B) 179 ft (C) 182 ft (D) 185 ft (E) 188 ft

11. Given: $m\angle A = 5x + 2$ and $m\angle B = 4x + 7$. $\angle A$ is complementary to $\angle B$ and $\angle C$ is supplementary to $\angle A$. Find $m\angle C$.

(A) 121° (B) 125° (C) 129° (D) 133° (E) 137°

12. Consider triangle ABC with coordinates $A(0, 0)$, $B(3, 8)$, and $C(9, -2)$. If point $D(a, b)$ is the centroid of the triangle, then a + b = _____.

(A) $5.\overline{3}$ (B) $5.\overline{6}$ (C) 6 (D) $6.\overline{3}$ (E) $6.\overline{6}$

13. After winning the region tournament, Russell took the team to the Idalou Pizza Hut and ordered 8 extra-large supreme pizzas. He noticed that each pizza was divided into eight equal slices, the diameter of each pizza was 14 inches, and the thickness of each pizza was 0.25 inches. If Becci only ate one slice, what volume of pizza did she consume? (nearest hundredth)

(A) 4.69 in^3 (B) 4.81 in^3 (C) 4.93 in^3 (D) 5.05 in^3 (E) 5.17 in^3

14. Consider isosceles trapezoid ABCD with perimeter = 69. $\overline{BC}$ is parallel to $\overline{AD}$, AB = 12, BC = 18, and $\angle DAB = 70°$. Find the area of the trapezoid. (nearest whole number)

(A) 254 (B) 258 (C) 262 (D) 266 (E) 270

15. The point at which the three bisectors of the interior angles of a triangle intersect is the _____.

(A) incenter (B) circumcenter (C) orthocenter (D) Euler point (E) centroid

16. Consider regular pentagon ABCDE with AC = 12. Find the perimeter of the pentagon. (nearest tenth)

(A) 36.2 (B) 36.5 (C) 36.8 (D) 37.1 (E) 37.4

17. A quadrilateral has a perimeter of 93 in and one side is 21 in long. The lengths of the other three sides have a 3:4:5 ratio. Find the length of the longest side of the quadrilateral. (nearest tenth)

(A) 26.4 in (B) 27.6 in (C) 28.8 in (D) 30.0 in (E) 31.2 in

18. A solid metal cone with a radius of 12 and a lateral area of 754 is melted down and recast as a solid ball. Find the surface area of the solid ball. (nearest tenth)

(A) 854.8 (B) 858.6 (C) 862.4 (D) 866.2 (E) 870.0

19. If $a_1 = 7,\ a_2 = 5,\ a_n = (a_{n-2} - a_{n-1})(a_{n-1} - 2)$, then $a_5 =$ _____.

(A) -60 (B) -55 (C) -50 (D) -45 (E) -45

20. Consider the geometric sequence 30, a, b, c, 3.888. If a > 0, then a + b + c = _____.

(A) 33.60 (B) 34.44 (C) 35.28 (D) 36.12 (E) 36.96

21. Consider an arithmetic sequence in which the first term is 6, the common difference is 6, and the sum of the sequence is 396. Find the last term in the sequence.

(A) 60 (B) 62 (C) 64 (D) 66 (E) 68

22. What is the value of c if $(2x^3 + 4x^2 + cx - 8) \div (2x + 1)$ has a remainder of -4.25?

(A) -6 (B) -3 (C) 0 (D) 3 (E) 6

23. Mark has a ranch north of Roanoke. He has a cylindrical water trough that holds 2538 gallons of water when it is filled to 60% of capacity. If the diameter of the trough is 12 feet, find the height of the trough. (nearest inch)

(A) 54 (B) 56 (C) 58 (D) 60 (E) 62

24. If $\begin{bmatrix} 2 & b & 1 \end{bmatrix} \times \begin{bmatrix} -7 \\ -3 \\ 4 \end{bmatrix} = 5$, then b = _____.

(A) -5 (B) -3 (C) -1 (D) 1 (E) 3

25. Find the area of an ellipse with foci at $(0, 0)$ and $(4, 0)$, and with a major axis of length 6. (nearest tenth)

(A) 20.7 (B) 21.1 (C) 21.5 (D) 21.9 (E) 22.3

26. If $u = 5i + 6j$ and $v = 8i - 5j$, then the dot product of the vectors equals _____.

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

27. The line $y = -2x + 10$ intersects the circle $x^2 + y^2 = 25$ at points P and Q. Find the length of $\overline{PQ}$. (nearest hundredth)

(A) 3.99 (B) 4.11 (C) 4.23 (D) 4.35 (E) 4.47

28. $\sin(135°) + \cos(225°) + \cot(315°) =$ _____.

(A) $-2\sqrt{2} - 1$ (B) $-2\sqrt{2} + 1$ (C) -1 (D) $2\sqrt{2} - 1$ (E) 1

29. $A + B = 23$ and $A \times B = 112$. If $A > B$, then $A - B =$ _____.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

30. Consider quadrilateral ABCD with $BC = CD = 20$, $AD = 24$, $m\angle BCD = 132°$ and $m\angle ADC = 108°$. Find the area of the quadrilateral. (nearest whole number)

(A) 569 (B) 573 (C) 577 (D) 581 (E) 585

31. The graph of $r = \cos(4\theta)$ is a/an _____ .

(A) 2-leaved rose (B) 4-leaved rose (C) 8-leaved rose (D) circle (E) spiral

32. Find the sum of the Fibonacci characteristic sequence $2, 5, 7, 12, ..., 343, 555$

(A) 1440 (B) 1444 (C) 1448 (D) 1452 (E) 1456

33. Find the angle between the vectors $u = \langle 2, -3, 4 \rangle$ and $v = \langle 5, -1, 6 \rangle$. (nearest tenth)

(A) 29.2° (B) 31.4° (C) 33.6° (D) 35.8° (E) 38.0°

34. Consider the hyperbola $9y^2 - x^2 + 54y + 10x + 55 = 0$. The vertices are (a, b) and (a, c). $b + c =$ ___

(A) -6 (B) $-\frac{16}{3}$ (C) $-\frac{14}{3}$ (D) -4 (E) $-\frac{10}{3}$

35. Convert the polar equation $r = \frac{4}{2 - 3\cos\theta}$ to a rectangular equation.

(A) $4x^2 - 5y^2 - 24y - 16 = 0$ (B) $4x^2 - 9y^2 + 4x + 18y = 0$ (C) $4y^2 - 5x^2 - 12y - 18 = 0$
(D) $4x^2 - 5y^2 - 12x - 18 = 0$ (E) $4y^2 - 5x^2 - 24x - 16 = 0$

36. $\frac{13x - 1}{x^2 + x - 6} = \frac{A}{x + 3} + \frac{B}{x - 2}$ $A - B =$ _____.

(A) 1 (B) 1.5 (C) 2 (D) 2.5 (E) 3

37. Given: $f''(x) = 8$, $f(2) = 15$, $f(-2) = 23$. $f(4) =$ _____.

(A) 47 (B) 50 (C) 53 (D) 56 (E) 59

38. Consider the tangent line to the function $f(x) = (x - 2)^2 + 3$ at x = 1. The x-intercept of the tangent line is $(a, 0)$. a = _____.

(A) $2.\overline{3}$ (B) $2.\overline{6}$ (C) 3 (D) $3.\overline{3}$ (E) $3.\overline{6}$

39. Find the area bounded by the graphs of $y = .5 \cdot e^{(x-2)}$, $y = 3 \cdot \sqrt{x+2}$, and $x = 3$. (bounded region to the right of x = 3) (nearest hundredth)

(A) 5.25 (B) 5.50 (C) 5.75 (D) 6.00 (E) 6.25

40. Find $\frac{dy}{dx}$ for $3x^2 + 2y^2 - 8x + 4y = 32$ at the point $(4, 2)$.

(A) $-\frac{4}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

41. Consider the graph of $f(x) = -x^4 - 2x^3 + 3x^2 - 4x + 5$. A point of inflection occurs at (a, b) where $a > 0$. Find the value of a + b. (nearest tenth)

(A) 3.4 (B) 3.6 (C) 3.8 (D) 4.0 (E) 4.2

42. If $\frac{d}{dx}\left(\frac{6x+4}{3x-7}\right) = \frac{a(bx+c) - b(ax+d)}{(bx+c)^2}$, then a + b + c + d = _____.

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

43. Use the first three terms of the Maclaurin series for $f(x) = \cos x$ to approximate the value of $\cos\left(\frac{\pi}{3}\right)$. What digit is in the ten-thousandths place?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

44. $\int \frac{2}{x} dx =$ _____ + C.

(A) e^{2x} (B) ln(x) (C) $\ln\left(x^2\right)$ (D) $-\frac{2}{x^2}$ (E) $\ln|2x|$

45. The graph of $f(x) = \frac{5x+1}{\sqrt{3x^2+2}}$ has two horizontal asymptotes. One of them is the line _____.

(A) $y = -\frac{5}{\sqrt{3}}$ (B) $y = 3$ (C) $y = -\frac{28}{9}$ (D) $y = \frac{32}{11}$ (E) $y = -\frac{9}{\pi}$

46. Ryan took six number sense tests for practice on Saturday. His scores were 341, 345, 310, 346, 355, and 301. What was Ryan's mean score on Saturday? (nearest whole number)

(A) 330 (B) 331 (C) 332 (D) 333 (E) 334

47. Sonali played six rounds of golf last week at the Robson Ranch course. Her scores were 76, 79, 82, 80, 77, and 79. Find the sum of the mode, median and range of the scores? (nearest whole number)

(A) 163 **(B) 164** **(C) 165** **(D) 166** **(E) 167**

48. The probability of a senior at Oklahoma Baptist University being accepted into the Princeton Center for Theoretical and Applied Physics is 0.006. If all 244 seniors apply, what is the probability that at least one senior will be accepted? (nearest thousandth)

(A) 0.714 **(B) 0.728** **(C) 0.742** **(D) 0.756** **(E) 0.770**

49. Eli has 12 red marbles, 15 blue marbles and 18 green marbles. He places all of the marbles in a jar and shakes the jar for 30 seconds. He puts on a blindfold and randomly selects a marble from the jar and keeps it. He does this two more times and then looks at the three marbles he selected. Find the probability that all three marbles he selected are red. (nearest ten-thousandth)

(A) 0.0155 **(B) 0.0166** **(C) 0.0177** **(D) 0.0188** **(E) 0.0199**

50. Haley did some research on the Titanic and she found that of the 319 first class passengers, 197 survived. Out of the 261 second class passengers, 94 survived and out of the 627 third class passengers, 151 survived. If a person selected did not survive, what is the probability that the person selected was a first class passenger? (nearest hundredth)

(A) 0.13 **(B) 0.16** **(C) 0.19** **(D) 0.22** **(E) 0.25**

51. Jasmin rolled a pair of fair dice and noted the sum of the top faces. What is the probability that the sum was less than 6 or greater than 9?

(A) $\frac{7}{18}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{17}{36}$ **(E)** $\frac{1}{2}$

52. For least squares regression lines, the difference between an observed value of the response variable and the value predicted by the regression line is called a/an _____.

(A) correlation **(B) balance** **(C) residual** **(D) outlier** **(E) IQR**

53. Assume that the mean height of high school senior females in Texas is 66 inches with a standard deviation of 3 inches. What is the probability that a randomly chosen Texas senior female has a height between 70 and 75 inches? (nearest hundredth of a percent)

(A) 4.55% **(B) 5.66%** **(C) 6.77%** **(D) 7.88%** **(E) 8.99%**

54. At the 1976 U.S. Open tennis tournament, Jimmy Connors beat Bjorn Borg in the men's final. He won the first set, lost the second set, then won sets three and four. In men's tennis, the first player to win 3 sets wins the match. How many different ways can a player win a match?

(A) 8 **(B) 9** **(C) 10** **(D) 11** **(E) 12**

55. 1000 in base 10 equals _____ in base 15.

(A) 450 (B) 45B (C) 45D (D) 46A (E) 46C

56. $\frac{10}{11}$ base 2 + $\frac{11}{13}$ base 6 = _____ base 10 mixed number.

(A) $1\frac{1}{4}$ (B) $1\frac{1}{3}$ (C) $1\frac{2}{5}$ (D) $1\frac{4}{9}$ (E) $1\frac{1}{2}$

57. If $623_b + 552_b = 1375_b$, then 222_b = _____ base 10.

(A) 146 (B) 155 (C) 164 (D) 173 (E) 182

58. Find the sum of the product of the roots taken four at a time for $x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12 = 0$.

(A) −3 (B) −5 (C) 15 (D) 4 (E) −12

59. Thirteen cards are dealt to James from a well-shuffled standard deck of 52 cards. What is the probability that he got 7 hearts, 3 diamonds, and 3 spades? (3 SD)

(A) .000155 (B) .000221 (C) .000287 (D) .000353 (E) .000419

60. Lauren's favorite topic in BC Calculus is finding the area of a polar region. Here is one of her homework problems. "Find the area of one petal of the rose curve $r = 3\sin(3\theta)$." The correct answer is _____.

(A) $\frac{7\pi}{12}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$ (E) $\frac{11\pi}{12}$

2019 – 2020 TMSCA High School Mathematics Test 8
Answer Key

1. B	**21. D**	**41. E**
2. D	**22. A**	**42. B**
3. A	**23. D**	**43. D**
4. D	**24. A**	**44. C**
5. B	**25. B**	**45. A**
6. C	**26. C**	**46. D**
7. E	**27. E**	**47. B**
8. C	**28. C**	**48. E**
9. D	**29. B**	**49. A**
10. A	**30. E**	**50. B**
11. D	**31. C**	**51. C**
12. C	**32. C**	**52. C**
13. B	**33. A**	**53. E**
14. A	**34. A**	**54. C**
15. A	**35. E**	**55. D**
16. D	**36. E**	**56. D**
17. D	**37. E**	**57. A**
18. E	**38. C**	**58. D**
19. A	**39. E**	**59. B**
20. C	**40. A**	**60. C**

19-20 TMSCA HSMA Test 8 Selected Solutions

2. $300-\left(125.65+3(24.95)+14.95\right)\left(1.0825\right)=66.78$

3. $27x^3-135x^2+225x-125$

$27-135+225-125=-8$

4. $\frac{i,s,a,b,e,l,r,c}{8}$

5.
$x+y+z=90$
$y=12+x$
$z=12+y$
$x=18$

6.
$y+4=-(x-2)$
$y=-x-2$
$y-3=x$
$y=x+3$
$(-2.5,0.5)$
$-2.5+.5=-2$

7. Let $P=100$

$Q=75,\ R=37.5$

$\frac{100}{37.5}\cdot 100\%=266\frac{2}{3}\%$

8. Solve

$x=7,\ y=5$

$7+5=12$

9. Solve

$x=9$

10.
$\tan(12.2°)=\frac{38}{d}$
$d=176\text{ ft}$

11.
$9x+9=90$
$x=9$
$A=47$
$C=180-47=133°$

12.
$a=\frac{0+3+9}{3}=4$
$b=\frac{0+8-2}{3}=2$
$4+2=6$

13. $\frac{1}{8}\cdot\pi\cdot 7^2\cdot\frac{1}{4}=4.81\text{ in}^3$

14.
$\sin(70°)=\frac{h}{12}$
$h=11.2763...$
$A=\frac{1}{2}(h)(18+27)=254$

16.
$AB=7.4164...$
$5(AB)=37.1$

17.
$3x+4x+5x=93-21$
$x=6$
$5x=30$

18.
$754=\pi(12)L$
$12^2+h^2=L^2$
$V=\frac{\pi}{3}(12)^2(h)$
$V=\frac{4}{3}(\pi)R^3$
$A=4\pi R^2=870.0$

19.
$a_3=(2)(3)=6$
$a_4=(-1)(4)=-4$
$a_5=(10)(-6)=-60$

20.
$3.888=30\cdot r^4$
$r=.6$
$18+10.8+6.48=35.28$

21.
$396=\frac{n}{2}\left(6+(6+(n-1)6)\right)$
$n=11$
$a_{11}=6+(11-1)(6)=66$

23.
$\frac{2538}{.6}=4230$
$\frac{\pi(6\cdot 12)^2 h}{231}=4230$
$h=60$

24.
$-14-3b+4=5$
$b=-5$

25.
$9=4+b^2$
$\frac{(x-2)^2}{9}+\frac{(y-0)^2}{5}=1$
$A=\pi(3)\left(\sqrt{5}\right)=21.1$

26. $(5)(8)+(6)(-5)=10$

27. $y_1 = -2x + 10$
$y_2 = \sqrt{25 - x^2}$
$y_3 = -y_2$
intersections
$(3,4)$ and $(5,0)$
$d = \sqrt{4^2 + 2^2} = 4.47$

28. $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 1 = -1$

29. $A = 16,\ B = 7$
$16 - 7 = 9$

30. $DC = 36.5418....$
$AB = 41.5687...$
Heron $\rightarrow 148.628...$
Heron $\rightarrow 436.099...$
Total $= 585$

31. $(4)(2) = 8$

32. $(2)(555) + 343 - 5 = 1448$

33. $\cos\theta = \frac{10 + 3 + 24}{\sqrt{29} \cdot \sqrt{62}}$
$\theta = 29.2°$

34. $9y^2 + 54y - x^2 + 10x = -55$
$\frac{(y+3)^2}{\frac{1}{9}} - \frac{(x-5)^2}{1} = 1$
$\left(5, -3 - \frac{1}{3}\right)\ \left(5, -3 + \frac{1}{3}\right)$
$-\frac{10}{3} - \frac{8}{3} = -6$

35. $2r - 3r\cos\theta = 4$
$2\sqrt{x^2 + y^2} - 3x = 4$
$4x^2 + 4y^2 = (3x + 4)^2$
$4y^2 - 5x^2 - 24x - 16 = 0$

36. $A = 8,\ B = 5$
$8 - 5 = 3$

37. $f' = 8x + c$
$f = 4x^2 + cx + d$
$15 = 16 + 2c + d$
$23 = 16 - 2c + d$
$f(x) = 4x^2 - 2x + 3$
$f(4) = 59$

38. $y = -2x + 6$
$0 = -2x + 6$
$x = 3$

39. $\int_3^{4.746253} (y_2 - y_1)\,dx = 6.25$

40. $\frac{dy}{dx} = \frac{8 - 6x}{4y + 4}$
$\frac{8 - 24}{8 + 4} = -\frac{4}{3}$

41. $(.366025, 3.8217968)$
$sum = 4.2$

42. $\frac{6(3x - 7) - 3(6x + 4)}{(3x - 7)^2}$
$6 + 3 - 7 + 4 = 6$

43. $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$
$x = \frac{\pi}{3} \rightarrow 0.5017962$

44. $2\int\left(\frac{1}{x}\right)dx = 2\ln|x| + C$
$\ln\left(x^2\right) + C$

45. $\lim_{x \to -\infty} \left(f(x)\right) = -\frac{5}{\sqrt{3}}$

46. $\frac{341 + 345 + 310 + 346 + 355 + 301}{6} = 333$

47. $79 + 79 + 6 = 164$

48. $1 - (.944)^{244} = 0.770$

49. $\frac{12}{45}\cdot\frac{11}{44}\cdot\frac{10}{43}=0.0155$

50. $319-197=122$
$261-94=167$
$627-151=476$
$122+167+476=765$
$\frac{122}{765}=0.159477$

51. $\frac{1+2+3+4+3+2+1}{36}=\frac{4}{9}$

52. residual

53. $\text{normalCdf}(70,75,66,3)=0.08986...$
8.99%

54. WWW
WWLW
WWLLW
WLWW
WLWLW
WLLWW
LWWW
LWLWW
LWWLW
LLWWW

55. 46A
$4(15)^2+6(15)+10=1000$

56. $\frac{2}{3}+\frac{7}{9}=1\frac{4}{9}$

57. $222_8=146_{10}$

59. $\frac{{}_{13}C_7\cdot{}_{13}C_3\cdot{}_{13}C_3}{{}_{52}C_{13}}=0.000221$

60. $A=\frac{1}{2}\int_0^{\frac{\pi}{3}}(3\sin(3\theta))^2\,d\theta=\frac{3\pi}{4}$