UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Invitational A • 2011

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $10 \times 8 \div (6 + 4) - 2$

   (A) 6   (B) $15\frac{1}{3}$ (C) 10   (D) 24   (E) $9\frac{2}{3}$

2. Find the number of positive integral divisors of 354.

   (A) 16   (B) 12   (C) 10   (D) 8   (E) 6

3. The regular price for a Good Tread tire is $79.75. If Les Ruber buys a set of four of these tires he gets a 30% discount and he can buy a spare tire half off of the regular price. How much will Les have to pay before taxes if he buys all five tires?

   (A) $334.95   (B) $263.18   (C) $319.00   (D) $239.25   (E) $279.13

4. The statement, $[b + (-b)] + a = 0 + a$, is justified by which of the following properties of real numbers?

   (A) commutative   (B) inverse   (C) reflexive   (D) identity   (E) symmetric

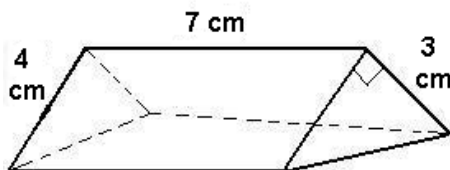5. Find the slope of a line through point P that is perpendicular to the line shown.
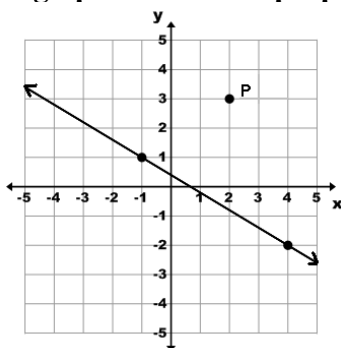
   (A) $-\frac{3}{5}$ (B) -2.5 (C) 0.6   (D) $\frac{2}{3}$ (E) $1\frac{2}{3}$

6. If the roots of $x^3 + bx^2 + cx + d = 0$ are $-1, 2,$ and 5, then $b + c + d$ equals:

   (A) -1 (B) 6   (C) 7   (D) 13   (E) 19

7. Find the lateral surface area of the polyhedron shown.

   (A) 12 cm^2 (B) 14 cm^2 (C) 84 cm^2 (D) 96 cm^2 (E) 98 cm^2

8. $\angle A$ and $\angle B$ are complementary angles. $\angle B$ and $\angle C$ are supplementary angles. Find m$\angle C$ if m $\angle A = 3x - 1$ and m $\angle B = 2x + 6$.

   (A) 163°   (B) 130°   (C) 136°   (D) 161°   (E) 140°

9. Hickory, Dickory, and Doc looked up at their circular clock. The time was precisely 12:34 pm. The smaller angle formed by the minute hand and the hour hand of the clock is:

    (A) 173 °   (B) 177°   (C) 180 °   (D) 183 °   (E) 187 °

10. The *Tortoise* leaves the Cross Country train station at 6 A.M. traveling 50 miles per hour. The *Hare* express leaves the same station on a parallel track at 9 A.M. traveling 75 miles per hour. How far from the train station will the trains be when the *Hare* overtakes the *Tortoise*?

    (A) 425 miles   (B) 450 miles   (C) 500 miles   (D) 625 miles   (E) 675 miles

11. If $8^{(k)} = 4^{(k + 2)}$, then $2^{(2k)} = ?$

    (A) 1024   (B) 128   (C) 64   (D) 256   (E) 512

12. The radius of the circle, $x^2 + 8x + y^2 - 4y = 16$ is:

    (A) 4   (B) $3\sqrt{2}$ (C) 6   (D) $2\sqrt{7}$ (E) 2

13. Which of the following is a triangular number?

    (A) 228   (B) 231   (C) 244   (D) 250   (E) 256

14. Simplify: $\cos\theta \tan\theta + \sin\theta$

    (A) $2\cos\theta$ (B) $\sin 2\theta$ (C) $\csc\theta$ (D) $\sec\theta$ (E) $2\sin\theta$

15. Ura Dizzie started at $(-1, 0)$ on a unit circle and traveled clockwise 98 radians along the circumference of the unit circle. Which quadrant or axis will Ura stop at?

    (A) x- axis   (B) QIV   (C) QIII   (D) QII   (E) y-axis

16. The equation $y =$ _____ will produce this graph.
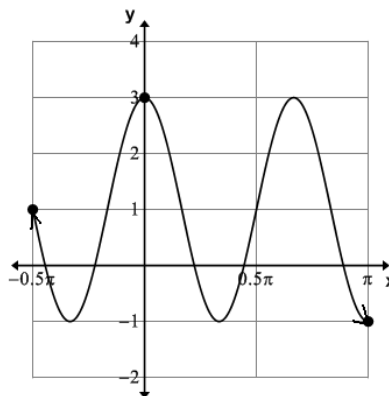
    (A) $3\sin(2x) - 1$ (B) $4\cos(2x) + 1$ (C) $2\sin(3x - 1)$

    (D) $2\cos(3x) + 1$ (E) $4\sin(3x) + 1$

17. In the binomial expansion of $(2x-5)^4$, the coefficient of the third term is:

    (A) 81   (B) -160 (C) 600   (D) -1000 (E) 486

18. Betty Goofsupp drops a ping pong ball from a height of 6 feet. Each time it hits the ground it rebounds to a height of 75% of the distance it fell. Find the total distance the ball travels when it reaches the ground the fourth time. (nearest tenth)

    (A) 26.8 ft   (B) 24.3 ft   (C) 20.8 ft   (D) 18.4 ft   (E) 16.4 ft

19. $\displaystyle\sum_{k=-1}^{2}(k+1)x-k=?$

    (A) $6x^3-7x^2+3x$ (B) $6x-2$ (C) $6x^2-7x+2x$ (D) $4x-3$ (E) $2x-1$

20. The directrix of the conic given by the equation $y=-2x^2+5x-3$ is:

    (A) $y=.125$ (B) $y=.25$ (C) $y=.4$ (D) $y=1.25$ (E) $y=1.5$

21. Find the sum of the series $1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}\ldots$. (nearest hundredth)

    (A) 3.00   (B) 2.88   (C) 2.72   (D) 2.64   (E) 2.58

22. If $y=x^3$, then $\frac{dy}{dx}=\frac{dx}{dy}$ when $x=?$

    (A) $\frac{1}{3}$ (B) $-\sqrt{3}$ (C) 3   (D) $\frac{\sqrt{3}}{3}$ (E) -3

23. Art has six different colored crayolas. How many ways can he draw a three-colored rainbow if each color is different?

    (A) 180   (B) 120   (C) 112   (D) 60   (E) 20

24. The ratio of the radii of the three concentric circles is 1:2:3. What is the probability that a dart that hits the target will hit the shaded section.?

    (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

25. Which of the following is not a POLITE number?

    (A) 26   (B) 28   (C) 30   (D) 32   (E) 34

26. Simplify: $\dfrac{1^2 + 2^2 + 3^2 + 4^2 + \ldots + 10^2 + 11^2}{1 + 3 + 6 + 10 + \ldots + 55 + 66}$

(A) $1\frac{10}{13}$　　　(B) $1\frac{1}{6}$　　　(C) $1\frac{5}{6}$　　　(D) $2\frac{11}{33}$　　　(E) $2\frac{3}{10}$

27. The mathematician Benoit Mandelbrot is known for his work with _____ .

(A) polyhedrons　　　(B) primes　　　(C) infinity　　　(D) integrals　　　(E) fractals

28. Let A = {0,2,1,3,4,7}, B = {2,3,5,7,11}, and C = {1,3,6,10,15}. How many elements are in the shaded areas of the Venn diagram shown?

(A) 5　　　(B) 16　　　(C) 4　　　(D) 3　　　(E) 11

29. Wynn Sumoor won 9 of the first 15 races he entered. How many of the remaining 12 races must he win to have at least a 70% winning percentage for the season?

(A) 5　　　(B) 6　　　(C) 7　　　(D) 9　　　(E) 10

30. Find the greatest common divisor of 324, 540, and 756.

(A) 6,480　　　(B) 2160　　　(C) 120　　　(D) 108　　　(E) 36

31. A set of 5 integers has a mean of 7, a mode of 6, and a median of 6. What is the smallest possible range of the set of integers?

(A) 3　　　(B) 4　　　(C) 5　　　(D) 6　　　(E) 7

32. $\triangle$PQR is similar to $\triangle$XYZ. $\angle$P $\cong$ $\angle$X, $\angle$R $\cong$ $\angle$Z, PQ = 25, XY = 20, QR = 30, and XZ = 16. Find PR $+$ YZ .

(A) 36.8　　　(B) 27　　　(C) 57.5　　　(D) 40　　　(E) 44

33. Find the diameter of the circle. (nearest tenth)

(A) 3.5 "　　　(B) 5.2 "　　　(C) 6.0 "　　　(D) 6.9 "　　　(E) 7.8 "

34. **Rob Burr looks out of a skyscraper window that is 250 feet above the ground. He sees a police car on the ground approaching the building. The line of sight from Rob's eyes to the police car has an angle of depression of 30 °. How far from the building is the police car? (nearest foot).**

   (A) 144 feet   (B) 216 feet   (C) 375 feet   (D) 403 feet   (E) 433 feet

35. **$\triangle PQR$ is a right triangle where $m\angle PQR = 90°$ and $m\angle RPQ = 30°$. Point M lies on segment PQ such that $m\angle RMQ = 60°$ and $MR = 10$ cm. Find the ratio of MQ to PQ.**

   (A) $\frac{3}{1}$ (B) $\frac{2}{1}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

36. **Three letters are randomly chosen from the word EIGHT. What are the odds that the set of three letters randomly chosen consist of at least 2 consonants?**

   (A) $\frac{7}{3}$ (B) $\frac{3}{10}$ (C) $\frac{7}{8}$ (D) $\frac{3}{7}$ (E) $\frac{7}{10}$

37. **The Yu Knee Cu tribe uses a different base other than base 10 to solve their math problems. Using their base they calculated that $34 + 43 = 121$ and $43 - 34 = 5$. What answer should they get for 33×4 ?**

   (A) 242   (B) 343   (C) 310   (D) 212   (E) 220

38. **A humming bird solution was made by mixing 8 pints of a 5% sugar solution with 6 pints of a 40% sugar solution. What was the percent sugar concentration of the humming bird solution?**

   (A) 14%   (B) 20%   (C) 22.5%   (D) 32%   (E) 45%

39. **Simplify:** $\left(\dfrac{2x - 2y}{x^2 + 2xy + y^2}\right) \div \left(\dfrac{x - y}{x + y}\right) \times \left(\dfrac{x^2 - y^2}{2}\right)$

   (A) $x + y$ (B) $2xy$ (C) $x - y$ (D) $x^2 - 2xy + y^2$ (E) 1

40. **Millie Tonne weighs 50 kilograms and is sitting on a seesaw 150 cm from the middle. Graham Meshur is sitting 100 cm from the center. How much does Graham weigh if the seesaw is balanced?**

   (A) 25 kg   (B) $33\frac{1}{3}$ kg   (C) 75 kg   (D) 125 kg   (E) 200 kg

41. **Which of the following points does not lie in the shaded portion of the graph of $2x - 5y \geq 3$?**

   (A) $(-1, -2)$ (B) $(5, 1)$ (C) $(1, -1)$ (D) $(3, -1)$ (E) $(4, 2)$

42. **Let $f(x) = x - 1$ and $g(x) = 2 - x$ and $h(x) = 2x + 1$. Find $h\left(g\left(f(-2x)\right)\right)$.**

   (A) $4x + 7$ (B) $2 + 4x$ (C) $5 - 4x$ (D) $4x$ (E) $2x - 4$

43. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\angle BPC$ if $m\angle ABD = 60°$ and $m\angle BDC = 20°$.
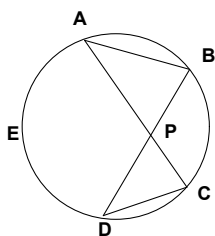
(A) 40°   (B) 80°   (C) 100°   (D) 120°   (E) 160°

44. The point of intersection of the three angle bisectors of a triangle is called the _____.

(A) median   (B) circumcenter   (C) Ceva   (D) incenter   (E) altitude

45. Find the minimum value for the function $f(x) = x^2 + 3x$

(A) -3 (B) -1.5 (C) 6   (D) 1.5   (E) -2.25

46. If $a_1 = 1$, $a_2 = 2$ and $a_n = a_{n-2} + a_{n-1}$, where n $\geq$ 3, then a_6 equals:

(A) 6   (B) 8   (C) 10   (D) 11   (E) 13

47. Determine the range of $f(x) = 4\cos(3x + 2) - 1$.

(A) $[-2, 0]$ (B) $[-2, 2]$ (C) $[-5, 3]$ (D) $[3, -5]$ (E) $[5, -5]$

48. Find the determinant of the 3 x 3 matrix.
$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & -2 & -3 \\ -2 & -1 & 0 \end{bmatrix}$$

(A) 2   (B) 0   (C) -1 (D) -3 (E) -4

49. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \ldots + \frac{1}{36} + \frac{1}{45} = ?$

(A) $\frac{3}{55}$ (B) $\frac{1}{5}$ (C) $\frac{9}{11}$ (D) $\frac{4}{5}$ (E) $\frac{1}{20}$

50. If $(3 + 4i)(2 - 3i) + (2 + 4i) = a + bi$ then $a + b = ?$

(A) -1 (B) 7   (C) 23   (D) -2 (E) 17

51. Which of the following is true about the relation $h(x) = x(2 - x)$

(A) odd function   (B) even function   (C) neither even nor odd function
(D) not a function   (E) none of these are true

52. **Find the area of the shaded region in square units.**
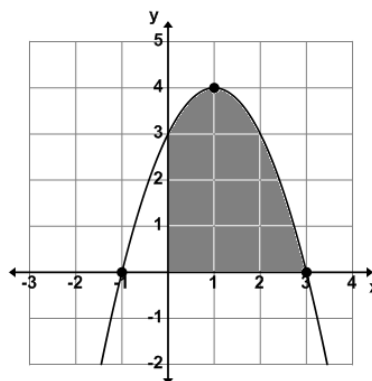
(A) $10\frac{2}{3}$ (B) $10\frac{1}{3}$ (C) $9\frac{2}{3}$ (D) 9 (E) $8\frac{1}{3}$

53. **If $f''(x) = 6x$ and $f'(-1) = 1$ and $f(-1) = 2$, then $f(1) = $ _____.**

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

54. **The Sweet Tooth candy store has four different types of gum balls, a sweet ball, a sour ball, a hot ball, and a red hot ball. They package them in sets of three. How many ways can the store package the gum balls?**

(A) 120 (B) 4 (C) 16 (D) 24 (E) 20

55. **Coach Wenzen recruits 5 physics students, 4 chemistry students, and 3 biology students for his science club. How many different 6-member science teams can he form if each team consists of 3 physics students, 2 chemistry students, and 1 biology student?**

(A) 19 (B) 66 (C) 75 (D) 180 (E) 240

56. **The figure shown was made from congruent cubes. What is the least number of congruent cubes needed to add to this figure to make a rectangular prism with no empty spaces? There are no hidden gaps.**

(A) 10 (B) 12 (C) 14 (D) 16 (E) 18

57. **Cher Debaker cooked four dozen cookies. She gave one-fourth of them to her grandparents, and two-thirds of what was left to her brothers and sisters. If she keeps one-half of what she has left, how many will be left to give to her teacher?**

(A) 3 (B) 6 (C) 8 (D) 9 (E) 12

58. Let $A = \begin{bmatrix} 7 & 5 \\ -2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -2 \\ 5 & -1 \end{bmatrix}$. Find $|AB|$.

(A) 3   (B) 9   (C) 18   (D) 37   (E) 51

59. Which of the following sequences is divergent?

(A) $\left\{ 4(0.2)^n \right\}$ (B) $\left\{ \frac{n-3}{n+3} \right\}$ (C) $\left\{ 6\left(\frac{1}{2}\right)^n \right\}$ (D) $\left\{ \frac{2n^2}{n^2+1} \right\}$ (E) $\left\{ \frac{10^n}{2} \right\}$

60. Ima Sportnutt has 5 baseball cards, 3 pitchers and 2 catchers. She has 6 football cards, 4 quarterbacks and 2 centers. Ima randomly selects two cards without replacement. What is the probability that both cards are quarterbacks? (nearest per cent)

(A) 15%   (B) 57%   (C) 11%   (D) 7%   (E) 40%

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Score   **Initials**

Contestant # _____

Conference _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2011
Answer Key

1. A	21. C	41. E
2. D	22. D	42. A
3. B	23. B	43. B
4. B	24. C	44. D
5. E	25. D	45. E
6. C	26. A	46. E
7. C	27. E	47. C
8. E	28. C	48. B
9. A	29. E	49. D
10. B	30. D	50. C
11. D	31. A	51. C
12. C	32. E	52. D
13. B	33. D	53. B
14. E	34. E	54. E
15. B	35. C	55. D
16. D	36. A	56. C
17. C	37. E	57. B
18. A	38. B	58. B
19. B	39. C	59. E
20. B	40. C	60. C

UIL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Invitational B • 2011

1. Evaluate: $5! \div 5 - 5 \times 5 + 5$

   (A) -6 (B) -5 (C) 4   (D) 5   (E) 100

2. Simplify: $a^3 \div b^5 \times a^{-2} \times b^2 \div a^5 \times b^{-3}$

   (A) $a^6 b^4$ (B) $a^{-4} b^4$ (C) $a^{-4} b^9$ (D) $a^6 b^{-6}$ (E) $a^{-4} b^{-6}$

3. If $\frac{4}{x} - \frac{3y}{2} = \frac{5}{6}$, then y equals _____.

   (A) $\frac{5x}{3}$ (B) $\frac{8x - 18}{3}$ (C) $\frac{9x}{5}$ (D) $\frac{16}{9x+5}$ (E) $\frac{24 - 5x}{9x}$

4. The ratio of Bob's age now to Ted's age now is 6:5. Fifteen years from now the ratio of Bob's age to Ted's age will be 9:8. How old will Ted be in ten years?

   (A) 25   (B) 30   (C) 35   (D) 40   (E) 45

5. Find the lateral surface area to the nearest square centimeter of the oblique cylinder shown.

   (A) 226 cm^2 (B) 396 cm^2 (C) 264 cm^2 (D) 113 cm^2 (E) 176 cm^2

6. A regular dodecahedron has _____ regular pentagonal faces.

   (A) 60   (B) 24   (C) 20   (D) 12   (E) 10

7. A road contractor hired 28 men to build a bridge in 90 days. He gets a bonus if he finishes the job in 84 days. How many more men will he need to hire to finish in 84 days and get the bonus?

   (A) 6   (B) 5   (C) 3   (D) 2   (E) 1

8. Find the product of the roots of $\frac{1 + x}{x - 2} = \frac{x + 3}{4 - x}$.

   (A) 4   (B) 2   (C) -1 (D) -3 (E) -5

9. Shirley Knott ties a 50' rope from a stake in the ground to a fence post in the north east corner of her yard. The she ties a 70' rope from the same stake to a fence post in the north west corner of her yard. The ropes and the stake form a 60 ° angle. How far apart are the two fence posts? (nearest inch)

   (A) 60' 0"   (B) 62' 1"   (C) 65' 4"   (D) 62' 5"   (E) 63' 7"

10. **Which of the following is a triangular number?**

   (A) 453   (B) 534   (C) 543   (D) 345   (E) 435

11. **A car is traveling 25 miles per hour in a school zone. At this rate, how many feet does the car travel in 1 second? (nearest foot)**

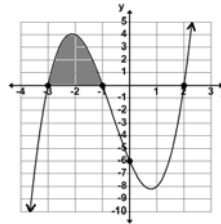   (A) 37 feet   (B) 35 feet   (C) 30 feet   (D) 25 feet   (E) 22 feet

12. **Find the remainder when $x^3 + 4$ is divided by $x - 2$.**

   (A) 0   (B) 2   (C) 4   (D) 6   (E) 12

13. **Find the instantaneous rate of change of $4x^3 + 2x - 3$ at $x = 2$.**

   (A) 24   (B) 26   (C) 31   (D) 48   (E) 50

14. **Find the area of the shaded region in square units.**

   (A) $4\frac{2}{3}$ (B) $4\frac{5}{6}$ (C) $5\frac{1}{3}$ (D) $6\frac{1}{6}$ (E) $7\frac{1}{3}$

15. **The probability of scoring above 180 on this test is $\frac{5}{9}$. If 45 students scored above 180 then how many students took this test?**

   (A) 70   (B) 75   (C) 81   (D) 90   (E) 135

16. **Roland Bawls putts a golf ball up a ramp and into one of 6 holes. The holes are numbered 2, 1, 3, 4, 7, and 11. If the ball goes into a hole numbered by a composite number, he loses 5 points. If it goes into a prime numbered hole, he wins 10 points. And if it goes into the hole numbered by a unit, he loses 15 points. What would the mathematical expectation be for any given putt?**

   (A) $6\frac{2}{3}$ pts   (B) $1\frac{1}{6}$ pts   (C) 5 pts   (D) 0 pts   (E) $3\frac{1}{3}$ pts

17. **Which of the following is a PRIMEVAL number?**

   (A) 2   (B) 3   (C) 5   (D) 7   (E) 11

18. **The mathematician best known for the invention of set theory is _____ .**

   (A) Georg Cantor   (B) Christian Goldbach   (C) Gottfried Leibniz

   (D) Alicia Stott   (E) Sophie Germain

19. **Willie Makette and Betty Wont were absent the day the other 15 Algebra students took their End of Class Exam. The class average on the exam was 72. What is the least score Willie can make on his test if Betty scores a 60 on her test and the overall class average has to be 70 or greater?**

   (A) 34   (B) 50   (C) 60   (D) 72   (E) 87

20. **Fourteen students compete in at least one of the events, number sense and math, at the TTU event. Eleven students took the math test. Eight students took the number sense test. How many students took both tests?**

   (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

21. **Find the least common multiple of 588, 252, and 126.**

   (A) 3,528   (B) 966   (C) 1,428   (D) 1,764   (E) 5,292

22. **The statement, $[b + (-c)] + a = b + [(-c) + a]$, is justified by which of the following properties of real numbers?**

   (A) commutative   (B) inverse   (C) subtraction   (D) identity   (E) associative

23. **How many ounces of water must be evaporated from $\frac{1}{4}$ gallon of a 4% salt solution to make a 6% salt solution?**

   (A) $1\frac{1}{2}$ oz   (B) $2\frac{2}{3}$ oz   (C) $6\frac{2}{3}$ oz   (D) $10\frac{2}{3}$ oz   (E) $13\frac{1}{4}$ oz

24. **Which of the following equations has a graph of a line with a negative slope and intersects the negative y-axis?**

   (A) $2x + y = 3$ (B) $3x + 2y = -1$ (C) $x - 3y = 2$ (D) $2x - 3y = -1$ (E) $x + y = 1$

25. **A circle has a circumference of 8π inches. If the radius of the circle is increased 50% then the area of the original circle is what part of the area of the new circle?**

   (A) $\frac{1}{2}$ (B) $\frac{5}{9}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$ (E) $\frac{4}{9}$

26. **The coordinates of the vertices of a square are $(-1, 1)$, $(-1, -5)$, (x, y) and $(2, -2)$. Find $x + y$.**

   (A) -8 (B) -6 (C) -4 (D) -2 (E) 0

27. **A parabola is the locus of points such that the distance to the focus is the same as the distance to its _____.**

   (A) asymptote   (B) directrix   (C) eccentricity   (D) axis of symmetry   (E) vertex

28. The three lines in the figure are coplanar with $m \parallel \ell$.
Which of the following are true statements?

1. $\angle ABG$ & $\angle EBC$ are vertical angles   2. $m\angle BED = \frac{1}{2}m\angle CBE$

3. $\angle BED \cong \angle GBA$ 4. $\angle DEG$ is $\angle BEH$'s complement

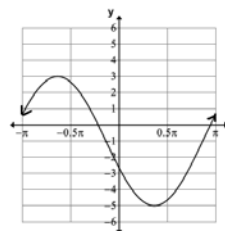(A) 1 & 2   (B) 1 & 3   (C) 1 only   (D) 2 & 4   (E) 3 & 4

29. Find the maximum value for the function $f(x) = 1 + 2x - 3x^2$

(A) 1   (B) $1\frac{1}{6}$ (C) $1\frac{1}{4}$ (D) $1\frac{1}{3}$ (E) $1\frac{1}{2}$

30. Let $A = \begin{bmatrix} 7 & 5 \\ -2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -2 \\ 5 & -1 \end{bmatrix}$. Find $|A + B|$.

(A) -19 (B) -37 (C) 3   (D) 6   (E) 19

31. The equation $y = $ _____ will produce this graph.

(A) $4\cos(x + 2) - 1$ (B) $4\sin(x - 1) + 2$ (C) $4\sin(x - 2)$

(D) $2\sin(x + 2) - 4$ (E) $2\cos(x - 4) + 1$

32. If $\sin\alpha = \frac{\sqrt{3}}{2}$, where $\frac{\pi}{2} < \alpha < \pi$ and $\cos\beta = -\frac{1}{2}$, where $\pi < \beta < \frac{3\pi}{2}$ then $\sin(\alpha - \beta) = $?

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3} - 1}{2}$ (E) $\frac{\sqrt{3} + 1}{2}$

33. $\frac{1 + 4 + 9 + 16 + ... + 49 + 64}{1 + 3 + 6 + 10 + ... + 28 + 36} = $?

(A) 2.25   (B) 2.0   (C) 1.7   (D) 1.4727272... (E) 1.4222...

34. The polar equation r = 7cos θ written in rectangular form is:

(A) $x^2 - 49 = 0$

(B) $(x - 7y)^2 = 0$

(C) $x^2 + y^2 + 7y = 0$

(D) $x^2 - 7x + y^2 = 0$

(E) $(x + 7)^2 = 0$

35. If the three numbers 246, 169, and 78 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 1   (B) 2   (C) 3   (D) 6   (E) 7

36. The infinite series $\frac{2}{5^0} - \frac{6}{5^1} + \frac{18}{5^2} - \frac{54}{5^3} + \frac{162}{5^4} - ...$ converges to:

(A) .75   (B) 1.25   (C) 1.5   (D) 1.75   (E) 2.25

37. If $y = 2x^2$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when x = ?

(A) $\frac{1}{8}$ (B) $-2\sqrt{2}$ (C) $\frac{1}{4}$ (D) $\frac{\sqrt{2}}{2}$ (E) $-\frac{\sqrt{2}}{8}$

38. The *Fishy Oter* Aquarium shoppe has 8 Guppies, 6 Tetras, and 4 Clownfish. How many ways can aquariums containing 2 Clownfish, 3 Tetras, and 4 Guppies be made?

(A) 48,620   (B) 94   (C) 2,240   (D) 8,400   (E) 5,600

39. Two black Jacks, a red Queen, a red King, and two black Aces are mixed up and placed face down on a table. Two cards are randomly selected and turned face up. What is the probability that one is a black card and one is a red card?

(A) 60%   (B) $46\frac{2}{3}$%   (C) 50%   (D) 40%   (E) $53\frac{1}{3}$%

40. Robbin D. Hood shoots 3 arrows at the square target containing 9 congruent smaller squares. Each arrow lands in a different small square. What are the odds that the 3 squares the arrows land in form a horizontal, vertical, or diagonal row?

(A) $\frac{1}{28}$ (B) $\frac{2}{21}$ (C) $\frac{4}{17}$ (D) $\frac{1}{27}$ (E) $\frac{2}{19}$

41. If $\frac{a+b}{a} = 6$ and $\frac{b+c}{c} = 7$, then $\frac{a}{c} = ?$

(A) $\frac{7}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{6}{5}$ (E) $\frac{5}{7}$

42. 210 is the _____ triangular number.

(A) 10th (B) 11th (C) 19th (D) 20th (E) 21st

43. The Red Abook store is having a new-used book sale. If you buy a new book and a used book you get 20% off of the regular price. If you buy a new book and two used books you get 50% off of the regular price. A new book costs $14.95 and a used book costs $10.50. How much would it cost for 2 new books and 3 used books? (tax not included)

   (A) $38.34   (B) $49.12   (C) $30.70   (D) $42.98   (E) $33.09

44. The number 3102 in base 4 is equivalent to the number xyz in base 8, where x,y, and z are digits. Find $x + y + z$.

   (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

45. A set of 5 integers has a mean of 46, a mode of 48, and a range of 9. What is the smallest possible integer in the set?

   (A) 39   (B) 40   (C) 41   (D) 42   (E) 43

46. Let p and q be the roots of $4x^2 + 8x - 5 = 0$. Find $p^3 + 3p^2q + 3pq^2 + q^3$.

   (A) -9 (B) -8 (C) $-4\frac{12}{125}$ (D) -1 (E) $-1\frac{61}{64}$

47. Sir Cusfood paid $4.75 for two ride tickets and three candy apples. His date, Lady Fare paid $4.50 for three ride tickets and one candy apple. What would it cost for the court jester to buy one ride ticket and two candy apples?

   (A) $2.00   (B) $2.25   (C) $2.75   (D) $3.25   (E) $3.00

48. The length of the inradius of a regular hexagon is 4 cm. Find the area of the regular hexagon. (nearest tenth)

   (A) 53.0 cm^2 (B) 53.9 cm^2 (C) 55.4 cm^2 (D) 58.1 cm^2 (E) 60.0 cm^2

49. △ABC is a scalene triangle. Point P lies on segment AB such that segment CP is the altitude of the triangle, m∠CBP = 40 °, AP = 10 cm, BP = 12 cm. Find m∠CAP. (nearest degree)

   (A) 38 °   (B) 41 °   (C) 45 °   (D) 50 °   (E) 60 °

50. The circle is tangent to all sides of the triangle. Find the diameter of the circle.
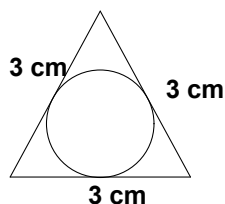
   (A) $2\sqrt{3}$ cm   (B) $\frac{\sqrt{3}}{2}$ cm   (C) $3\sqrt{3}$ cm   (D) $\frac{\sqrt{3}}{3}$ cm   (E) $\sqrt{3}$ cm

51. Simplify: $\log_b(2xy) + \log_b(\frac{x}{4y}) - \log_b(4x^2)$

    (A) $-\log_b(8)$ (B) $-3\log_b(x)$ (C) $2\log_b(4y)$ (D) $\log_b(32x^2y^2)$ (E) $\log_b(2x^3)$

52. The roots of the equation $8x^3 - 10x^2 + x + 1 = 0$ are 1, 0.5, and R. Find R.

    (A) -0.125 (B) -0.25 (C) -1.25 (D) -0.5 (E) -1.5

53. How many elements are in $\left\{x \mid 8\sin(x)\cos(x) = 2\sqrt{3}, x \in [0, 2\pi)\right\}$?

    (A) 1   (B) 2   (C) 4   (D) 6   (E) 8

54. Find an identity for $\tan\theta \sin\theta \cos\theta \cot\theta$.

    (A) 1   (B) $\csc\theta \sec\theta$ (C) $\frac{\sin\theta}{2}$ (D) $\frac{1}{2}\sin 2\theta$ (E) $\cos 2\theta$

55. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \dots + \frac{1}{105} + \frac{1}{120} = ?$

    (A) $\frac{7}{15}$ (B) $\frac{7}{8}$ (C) $\frac{8}{17}$ (D) $\frac{14}{15}$ (E) $\frac{15}{16}$

56. Seymore Shore can rake the private beach in 2.5 hours. If Sandy Soil helps Seymore they can rake it in 2 hours working together. How long would it take Sandy to rake it by herself?

    (A) 12.5 hours   (B) 10 hours   (C) 5 hours   (D) 4.5 hours   (E) 3.75 hours

57. Let $f(x) = \sqrt[3]{x^5}$. Find $f'(8)$.

    (A) $2\frac{2}{3}$ (B) $4\frac{1}{3}$ (C) $6\frac{2}{3}$ (D) $10\frac{2}{3}$ (E) $13\frac{1}{3}$

58. Find the slope of the line tangent to the curve $y = x^2 + 4x + 3$ at the point $(0, 3)$.

    (A) 4   (B) 3   (C) $1\frac{1}{3}$ (D) 1   (E) $\frac{3}{4}$

59. Snow White and her 7 dwarfs are sitting around King Arthur's octagonal table. How many ways can Snow White and her dwarfs be seated around the table?

    (A) 5,040   (B) 35,280   (C) 5760   (D) 720   (E) 40,320

60. The figure shown was made from congruent cubes. What is the least number of congruent cubes needed to add to this figure to make a rectangular prism with no empty spaces? There are no hidden gaps.

    (A) 16   (B) 17   (C) 23   (D) 28   (E) 29

University Interscholastic League
MATHEMATICS CONTEST

Final _____   _____

2nd _____   _____

1st _____   _____

Contestant # _____

Conference _____

Score   **Initials**

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

1. C	21. D	41. D
2. E	22. E	42. D
3. E	23. D	43. A
4. C	24. B	44. D
5. A	25. E	45. A
6. D	26. B	46. B
7. D	27. B	47. C
8. E	28. B	48. C
9. D	29. D	49. C
10. E	30. B	50. E
11. A	31. A	51. A
12. E	32. B	52. B
13. E	33. C	53. C
14. C	34. D	54. D
15. C	35. A	55. B
16. E	36. B	56. B
17. A	37. C	57. C
18. A	38. D	58. A
19. B	39. E	59. A
20. C	40. E	60. E

UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
District 1 • 2011

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $1 - 1 \times 2 + 3 \div (5 - 8) \times 13 + 21$

   (A) 7   (B) $\frac{1}{4}$ (C) 8   (D) 48   (E) $12\frac{1}{3}$

2. If x is 20% more than y and z is 40% less than y, then x is what percent of z?

   (A) 50%   (B) 60%   (C) 80%   (D) 180%   (E) 200%

3. The regular price for a single marker is 75¢ and a package of 5 markers cost $2.75. If Mark Etdown buys 3 packages at the regular price he will get a 40% discount on 2 more packages. How much money would Mark save if he bought 5 packages instead of 25 singles?

   (A) $7.50   (B) $10.04   (C) $7.20   (D) $11.55   (E) $8.30

4. Which of the following is an equation of the line shown?
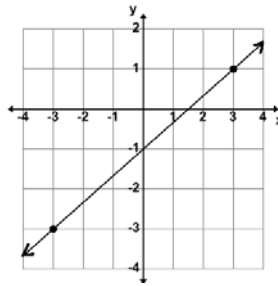
   (A) $2x - 3y = 3$ (B) $3x + 2y = 2$ (C) $2x - 3y = 1$ (D) $3x - 2y = 2$ (E) $2x + 3y = 3$

5. Simplify: $\left(\dfrac{x^3 - 9x^2 + 27x - 27}{x - 2}\right) \div \left(\dfrac{x - 3}{x^2 + x - 6}\right) \times \left(\dfrac{1}{x^2 - 9}\right)$

   (A) $x - 3$ (B) $x + 2$ (C) $x - 2$ (D) $x + 3$ (E) $2x - 3$

6. Donald D. Duck is sitting on one end of a seesaw. He weighs 170 lbs. His nephews, Huey, Dewey, and Louie, are sitting on the other end. Huey weights 40 lbs., Dewey 50 lbs. and Louie 60 lbs. Huey sits 3 feet, Dewey sits 4 feet, and Louie sits 6 feet from the fulcrum. How far must Donald sit from the fulcrum in order to balance his nephews?

   (A) 13 ft   (B) 7 ft   (C) 6 ft   (D) 4 ft   (E) 3 ft

7. Which of the following is true about a triangle with side lengths of 5", 5", and 8"?

   (1) acute   (2) obtuse   (3) right   (4) equilateral
   (5) isosceles   (6) scalene   (7) not a triangle

   (A) 1 & 5   (B) 2 & 4   (C) 3 & 6   (D) 2 & 5   (E) 7

8. The point of intersection of the three altitudes of a triangle is called the _____.

   (A) circumcenter   (B) orthocenter   (C) Ceva   (D) incenter   (E) centroid

9. Find the total surface area of the polyhedron shown.

13 cm 30 cm 5 cm
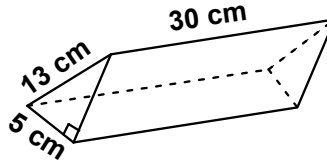
(A) 1,020 cm²   (B) 965 cm²   (C) 960 cm²   (D) 930 cm²   (E) 900 cm²

10. If $a_1 = -1$, $a_2 = 2$ and $a_n = a_{n-1} - a_{n-2}$, where $n \geq 3$, then a_6 equals:

(A) -3 (B) -2 (C) -1 (D) 1   (E) 3

11. The BRAINIAC computer can process the entire payroll for a large corporation in 9 hours. The SMARTIAC computer can do the same job in 6 hours. How fast could they do the job if they worked together?

(A) 7 hrs 30 min   (B) 4 hrs   (C) 3 hrs   (D) 3 hrs 36 min   (E) 1 hr 30 min

12. Let $A = \begin{bmatrix} -2 & 1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix}$. Find $|AB|$.

(A) 25   (B) 15   (C) 10   (D) 7   (E) 5

13. Which of the following is a triangular number?

(A) 505   (B) 511   (C) 516   (D) 528   (E) 531

14. Willie Landett looks down from the cockpit of his jet at an angle of depression of 52°20' and sees an aircraft carrier. The jet is 700 feet above level of the carrier's deck. How far away is the carrier? (nearest foot).

(A) 446 feet   (B) 516 feet   (C) 540 feet   (D) 547 feet   (E) 615 feet

15. Find the radius of the circle. (nearest tenth)

(A) 0.6 "   (B) 1.2 "   (C) 1.8 "   (D) 2.4 "   (E) 3.6 "

16. In the binomial expansion of $(3x + 4)^5$, the coefficient of the fourth term is:

(A) 2,160   (B) 2,880   (C) 3,840   (D) 4,320   (E) 5,760

17. If $(1 - 2i)(2 + 3i)(3 - 4i) = a + bi$ then $a + b = $?

(A) 55   (B) 20   (C) -15 (D) -18 (E) -35

18. A meat packing company that fails the Environmental Protection Agency (EPA) pollution standards is fined \$500.00 the first day and \$150.00 each day thereafter until they pass the standards. What would the total fine be if it takes the company eleven days to clean up and pass the standards?

(A) \$1,500.00   (B) \$1,650.00   (C) \$1,850.00   (D) \$2,000.00   (E) \$2,150.00

19. Find the area of the shaded region in square units.
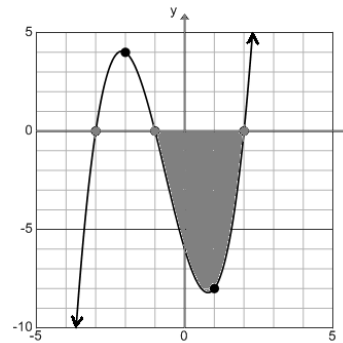
(A) $16\frac{1}{12}$ (B) $15\frac{3}{4}$ (C) $14\frac{1}{2}$ (D) $13\frac{1}{4}$ (E) $9\frac{7}{12}$

20. Let $f(x) = \frac{2x - 3}{5x}$. Find $f'(7)$.

(A) $\frac{3}{7}$ (B) $\frac{3}{49}$ (C) $\frac{3}{35}$ (D) $\frac{3}{245}$ (E) $\frac{3}{175}$

21. Simplify: $\dfrac{1^2 + 2^2 + 3^2 + 4^2 + \ldots + 16^2 + 17^2}{1 + 3 + 6 + 10 + \ldots + 136 + 153}$

(A) $1\frac{1}{9}$ (B) $1\frac{8}{19}$ (C) $1\frac{7}{9}$ (D) $1\frac{17}{21}$ (E) $1\frac{16}{19}$

22. Roland Bones tosses a pair of standard dice. What is the probability that the sum of the top faces of the dice is a prime number?

(A) $13\frac{8}{9}\%$ (B) $23\frac{17}{21}\%$ (C) $38\frac{8}{9}\%$ (D) $41\frac{2}{3}\%$ (E) $71\frac{3}{7}\%$

23. Kickem High School has 5 linemen, 4 receivers, 3 running backs, and 2 quarterbacks on their 6-man football team. How many different 6-man teams can the coach form if each team needs a quarterback, 2 running backs, 2 linemen and a receiver?

(A) 3,003   (B) 19   (C) 120   (D) 180   (E) 240

24. Find the greatest common divisor of 300, 360, and 2,250.

(A) 10   (B) 25   (C) 30   (D) 60   (E) 75

25. **"Every even integer greater than 2 can be expressed as a sum of two primes."**
    This statement is contributed to:

    (A) Georg Cantor   (B) Christian Goldbach   (C) Gottfried Leibniz

    (D) Benoit Mandelbrot   (E) George Boole

26. **Find the number of positive integral divisors of 882.**

    (A) 8   (B) 12   (C) 16   (D) 18   (E) 20

27. **On the map legend, $\frac{5}{8}$ of an inch represents 125 miles. The distance from El Paso to Tyler is approximately 750 miles. How far apart are they on the map?**

    (A) $3\frac{3}{4}$ in   (B) $2\frac{3}{8}$ in   (C) $7\frac{2}{5}$ cm   (D) $3\frac{1}{4}$ in   (E) $9\frac{3}{5}$ in

28. **The three lines in the figure are coplanar with $m \,//\, \ell$. Which of the following are true statements?**

    1. $\angle CBG$ & $\angle HEB$ are alternate exterior angles   2. $m\angle BED = \frac{1}{2}m\angle BEH$
    3. $\angle DEG$ is supplementary to $\angle BEH$ 4. $\angle DEF \cong \angle ABG$

    (A) 3 only   (B) 1 only   (C) 1 & 3   (D) 1, 2 & 4   (E) 2, 3 & 4

29. **Which of the following is NOT a solution to $5 - 4|\,3x + 2\,| \le 1$?**

    (A) -0.1 (B) -0.2 (C) -1 (D) -0.3 (E) -0.4

30. **Yul B. Speaden left Van Horn on I10 at 85 mph toward Ft. Stockton. Ida Slopoke left the same place as Yul going the opposite direction towards El Paso at 55 mph. How long would it take for Yul and Ida to be 245 miles apart?**

    (A) 1 hr 15 min   (B) 1 hr 30 min   (C) 1 hr 45 min   (D) 2 hrs   (E) 2 hrs 15 min

31. **A secant, $\overline{AP}$, and a tangent, $\overline{CP}$ intersect in the exterior of a circle. Point B lies on $\overline{AP}$ and on the circle such that CP = 16 and BP = 8. Find AB.**

    (A) 4   (B) 12   (C) 16   (D) 20   (E) 24

32. **If $27^{(k-1)} = 9^{(k)}$, then $3^{(k+1)} = $?**

    (A) 3   (B) 9   (C) 27   (D) 81   (E) 243

33. **Find the product of the roots of** $\frac{2x-1}{x+3} = \frac{3x-2}{x+5}$ **.**

    (A) -11 (B) -2 (C) -1 (D) 2   (E) 5
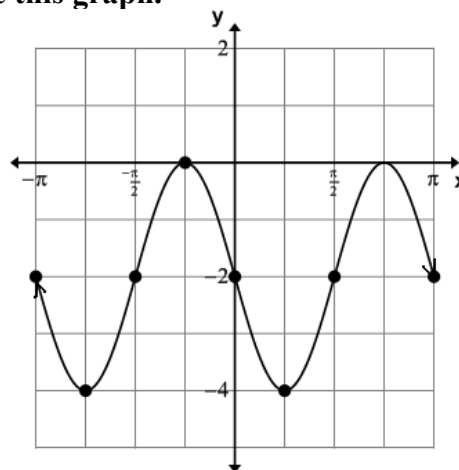
34. **How many of the ten digits used in the base 10 number system are POLITE numbers?**

    (A) 10   (B) 6   (C) 5   (D) 2   (E) 1

35. **Find** $\sin(\frac{7\pi}{6}) \times \cos(\frac{5\pi}{3}) \times \tan(\frac{3\pi}{4}) \times \cot(\frac{\pi}{4}) \times \sec(\frac{3\pi}{4}) \times \csc(\frac{5\pi}{4})$.

    (A) 1   (B) $\frac{1}{2}$ (C) 0   (D) $-\frac{1}{2}$ (E) -1

36. **The equation y = _____ will produce this graph.**

    (A) $2 + 2\sin[2(x + \frac{\pi}{2})]$ (B) $2\cos[2(x + \frac{\pi}{2})] - 2$ (C) $2\sin(2x + 2\pi) - 2$

    (D) $2 + 2\cos(2x + \frac{\pi}{2})$ (E) $2\sin[2(x + \frac{\pi}{2})] - 2$

37. **Find the fifth term of the geometric sequence: 3.6, a, b, 16.666..., c, ...**

    (A) 27.777...   (B) 24.333...   (C) 21.4   (D) 20.266...   (E) 18.222

38. **Given the function** $f(x) = 3x^5 - 2x^3 + x^2 - x + 4$ **there is a possibility of how many negative real zeroes?**

    (A) 1   (B) 3 or 1   (C) 0   (D) 2, 0   (E) 4, 2, or 0

39. **Find the sum of the series** $1 + 2 + 2 + 1\frac{1}{3} + \frac{2}{3} + \frac{4}{15} + ...$ **(nearest thousandth)**

    (A) 7.400   (B) 7.389   (C) 7.356   (D) 7.292   (E) 7.267

40. **If** $y = 2x^4$, **then** $\frac{dy}{dx} = \frac{dx}{dy}$ **when x = ?**

    (A) $\frac{12}{19}$ (B) $\frac{1}{8}$ (C) $\frac{7}{19}$ (D) $\frac{3}{8}$ (E) $\frac{1}{2}$

41. **Hope I. Whenn throws 3 darts at the square target containing 9 congruent smaller squares. Each dart lands in a different small square. What are the odds that the 3 squares the darts land in are the shaded squares?**

(A) $\frac{4}{5}$ (B) $\frac{1}{21}$ (C) $\frac{4}{9}$ (D) $\frac{1}{20}$ (E) $\frac{1}{27}$

42. **A tub of apples contains red apples and green apples. The probability of selecting a green apple is 25%. If 10 green apples are added to the tub, the probability of selecting a green apple becomes 40%. How many red apples are in the tub?**

(A) 30 (B) 25 (C) 20 (D) 15 (E) 10

43. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \frac{1}{36} + \frac{1}{45} + \frac{1}{55} + \frac{1}{66} + \frac{1}{78} + \frac{1}{91} = ?$

(A) $\frac{1}{13}$ (B) $\frac{6}{7}$ (C) $\frac{12}{13}$ (D) $\frac{12}{91}$ (E) $\frac{13}{15}$

44. **The number 10011110 in base 2 is equivalent to the number xyz in base 8, where x,y, and z are digits. Find $x + y + z$.**

(A) 16 (B) 11 (C) 10 (D) 8 (E) 6

45. **The *Sweet Tooth Shoppe* mixed 10 pounds of dark chocolate worth 75 cents a pound with 6 pounds of white chocolate to create a special chocolate mixture worth $1.20 a pound. How much per pound was the white chocolate worth?**

(A) $2.10 (B) $1.95 (C) $1.70 (D) $1.50 (E) $1.25

46. **If the roots of $x^3 + bx^2 + cx + d = 0$ are 3, 2, and -1, then $b + c + d$ equals:**

(A) 11 (B) 9 (C) 4 (D) 3 (E) -1

47. **The coordinates of the vertices of a rectangle are $(-5, 2), (-2, 5), (x, y)$ and $(2, 1)$. Find $x + y$.**

(A) 3 (B) 1 (C) 0 (D) -1 (E) -3

48. **The ellipse $x^2 - 6x + 4y^2 - 16y = 11$ has a major axis with a length of:**

(A) 14 (B) 12 (C) 9 (D) 8 (E) 6

49. **How many elements are in $\left\{ x \mid 2\cos^2(x) - 2\sin^2(x) = \sqrt{2}, x \in [-\pi, 2\pi) \right\}$?**

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

50. The type of graph of the polar equation $r = b + a\cos(\theta)$, where a and b are arbitrary real numbers is called a:

    (A) Archimedian spiral   (B) cardioid   (C) lemniscate   (D) limacon   (E) rose

51. Let $f(x) = x^2 - 4$, $g(x) = x - 3$, and $S(x)$ be the slant (oblique) asymptote of $\frac{f(x)}{g(x)}$. Find $S(-2)$.

    (A) -5 (B) -1 (C) $-\frac{1}{5}$ (D) 0 (E) 1

52. Find the instantaneous rate of change of $x^3 + 2x^2 - 5x - 6$ at $x = -1$.

    (A) -6 (B) -4 (C) 0 (D) 2 (E) 12

53. The function $f(x) = x^3 + 2x^2 - 5x - 6$ has an inflection point at (x,y). Find x.

    (A) -1 B) $-\frac{2}{3}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$ (E) $-\frac{1}{6}$

54. Snow White and 6 of her dwarfs are going to sit in the same row at the *Bad Apple* concert. How many ways can Snow White seat her 6 dwarfs in the row if she is going to sit in the middle?

    (A) 36   (B) 1,440   (C) 72   (D) 720   (E) 5,040

55. A booth at the local carnival has a circular board filled with balloons. The booth operator spins the wheel, the player throws a dart and pops a balloon. If one of the 12 red balloons is popped the player loses 80¢. If one of the 8 white balloons is popped the player gets 20¢. If one of the 5 blue balloons is popped the player gets 60¢. What is the mathematical expectation of a single throw?

    (A) loses 20¢   (B) loses 15¢   (C) breaks even   (D) gains 5¢   (E) gains 9¢

56. Simplify: $\dfrac{1 + 8 + 27 + 64 + ... + 729 + 1000}{1 + 3 + 6 + 10 + ... + 45 + 55}$

    (A) $10\frac{1}{4}$ (B) $11\frac{9}{11}$ (C) $12\frac{5}{6}$ (D) $13\frac{3}{4}$ (E) $18\frac{2}{11}$

57. In $\triangle ABC$, $m\angle BAC = 100°$ and $m\angle ABC = 50°$. Point H lies on segment BC such that segment AH is an altitude of $\triangle ABC$. Point M lies on segment AC such that segment BM is a median of $\triangle ABC$. Find $m\angle MHC$.

    (A) 15°   (B) 22.5°   (C) 30°   (D) 40°   (E) 45°

58. Let $f(x) = 2x + 5$ and $g(x) = 3x - 2$. If $h(x)$ is the inverse function of $\frac{f(x)}{g(x)}$, find the value of $h(-4)$.

    (A) $-1\frac{3}{10}$ (B) -4 (C) $4\frac{2}{3}$ (D) $\frac{10}{13}$ (E) $\frac{3}{14}$

59. A set of 5 positive integers has a mode of 15, a median of 15 and a range of 15. What is the smallest possible mean of the set of integers?

   (A) 15.0   (B) 13.0   (C) 10.1   (D) 9.8   (E) 9.6

60. The figure shown was made from congruent cubes. Each cube has 6 faces. How many faces are not visible? There are no hidden gaps.

   (A) 248   (B) 164   (C) 40   (D) 84   (E) 204

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Score   **Initials**

Contestant # _____   **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 1 ● 2011
Answer Key

1.	A	21.	E	41.	D
2.	E	22.	D	42.	A
3.	C	23.	E	43.	B
4.	A	24.	C	44.	B
5.	A	25.	B	45.	B
6.	D	26.	D	46.	D
7.	D	27.	A	47.	E
8.	B	28.	A	48.	B
9.	C	29.	E	49.	D
10.	A	30.	C	50.	D
11.	D	31.	E	51.	E
12.	A	32.	D	52.	A
13.	D	33.	C	53.	B
14.	C	34.	C	54.	D
15.	B	35.	B	55.	A
16.	E	36.	E	56.	D
17.	C	37.	A	57.	C
18.	D	38.	A	58.	E
19.	B	39.	B	59.	D
20.	D	40.	E	60.	B

UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
District 2 • 2011

1. Evaluate: $2 - 1 \times (3 + 4) \div (7 + 11) \times 18 - 29$

   (A) $-18\frac{2}{3}$ (B) -20 (C) -22 (D) $-28\frac{1}{3}$ (E) -34

2. Let A = {0,1,2,3,4,5}, B = {2,3,4,5}, and C = {0,1,4,6}. The number of distinct elements in
   $(A \cap C) \cup B$ is _____ .

   (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

3. Mark Etupp pays $50 each for 5 used printers at the Cheap Shack. After testing them he sells
   1 of them for $60 and 3 of them for $80 each. What is the least amount he can sell the last one
   for to make a 70% profit on the 5 printers. (tax not included)

   (A) $135.00   (B) $125.00   (C) $120.00   (D) $85.00   (E) $75.00

4. Simplify: $\left(\dfrac{3x + 9y}{x - 3y}\right) \div \left(\dfrac{3x^2 + 8xy - 3y^2}{9x^2 - 6xy + y^2}\right) \times \left(\dfrac{x - 3y}{3}\right)$

   (A) $3x - y$ (B) $9x^2 - y^2$ (C) $x + 3y$ (D) $x^2 + 6xy + 9y^2$ (E) 1

5. The statement, $a + b \times \frac{1}{b} = a + 1$, is justified by which of the following properties of real
   numbers?

   (A) commutative   (B) inverse   (C) associative   (D) identity   (E) distributive

6. Rose Upp weighs 140 lbs and is sitting on a seesaw 7 feet from the center. Will Godown is sitting
   8 feet from the center. How much does Will weigh if the seesaw is balanced?

   (A) 150 lbs   (B) 131.25 lbs   (C) 75 lbs   (D) 122.5 lbs   (E) 160 lbs

7. Find the total surface area of the polyhedron shown. (nearest cm^2)

   (A) 245 cm^2 (B) 217 cm^2 (C) 207 cm^2 (D) 195 cm^2 (E) 179 cm^2

8. In a plane, the locus of points equidistant from the sides of an angle is the:

   (A) midpoint   (B) angle bisector   (C) centroid   (D) incenter   (E) perpendicular bisector

9. A rectangular prism tank without a top has a length of 6 feet, a width of 3 feet, and a height of
   2 feet. What is the greatest number of gallons of water that could be poured into the prism
   before flowing over?

   (A) 193 gals   (B) 693 gals   (C) 216 gals   (D) 538 gals   (E) 269 gals

10. The radius of the circle $x^2 - 6x + y^2 + 8y = 2$ is:

(A) 2   (B) $3\sqrt{3}$ (C) $\sqrt{23}$ (D) $3\sqrt{2}$ (E) 3

11. Which of the following equations will produce the graph shown here?
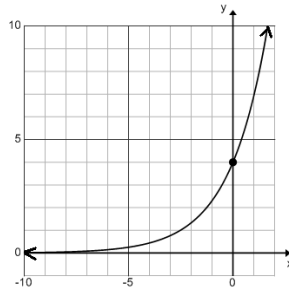
(A) $y = e^{3x} + 4$ (B) $y = 4\ln(3^x)$ (C) $y = \ln(3^x) - 4$ (D) $y = 4\sqrt{3^x}$ (E) $y = \sqrt{x^3} + 4$

12. The *I. M. AHED* freight train leaves the station at 7:00 A.M. traveling 60 miles per hour. The *U. R. BEEHEIN* passenger train leaves the same station at 9:30 A.M. traveling 70 miles per hour in the same direction as the freight train. How far apart are the two trains at 11:00 A.M. ?

(A) 135 miles   (B) 130 miles   (C) 105 miles   (D) 75 miles   (E) 10 miles

13. 528 is the _____ triangular number?

(A) 23rd (B) 30th (C) 35th (D) 28th (E) 32nd

14. If θ is in QIV then $\dfrac{\cos^2(\theta) + \cos(\theta) + \sin^2(\theta)}{\sin(\theta)} = \cot(k\theta)$ and k equals:

(A) -2 (B) -1 (C) 2   (D) 1   (E) $\frac{1}{2}$

15. Bea Safe installs a security sensor light over her back door. The sensor is 8 feet above the driveway. What will the angle of depression need to be in order for the security light to cover all 40 feet of the driveway? (nearest minute)

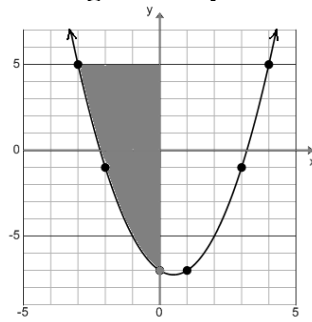(A) $11° 19'$ (B) $20° 41'$ (C) $3° 28'$ (D) $11° 32'$ (E) $20° 18'$

16. In the binomial expansion of $(2x + 3y)^4$, the sum of the coefficients of the x^3y term and the xy^3 term is:

(A) 432   (B) 312   (C) 216   (D) 192   (E) 160

17. $\displaystyle\sum_{k=-1}^{2} (k-1)x + (k)(x-1) = ?$

(A) -2 (B) 0   (C) $8x - 2$ (D) $6x - 1$ (E) $-2x - 2$

18. **Find the area of the shaded region in square units.**

(A) $20\frac{1}{4}$ (B) $21\frac{3}{4}$ (C) $22\frac{1}{2}$ (D) $23\frac{1}{4}$ (E) $24\frac{1}{2}$

19. **If** $y = \sqrt{x - 1}$, **then** $\frac{dy}{dx} = \frac{dx}{dy}$ **when** $x = $?

(A) 2   (B) 1.25   (C) 0.25   (D) -0.75 (E) -2

20. **Circles A and C are tangent to the large circle and to circle B. Circles A, B, and C are congruent. Cy Klopps shoots an arrow and hits the circular target. What are the odds that the arrow hit the shaded section.?**

(A) 1:4   (B) 1:3   (C) 1:2   (D) 2:3   (E) 3:4

21. **The geometric mean of 213 and 112 is what % of the arithmetic mean of 213 and 112? (nearest whole percent)**

(A) 90 %   (B) 95%   (C) 99%   (D) 101%   (E) 105%

22. **The *politeness* of the polite number 26 is:**

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

23. **Gottfried Leibniz invented at least three things that are essential for the modern world. One of these was:**

(A) fractals (B) logs (C) prime number generator (D) binary arithmetic (E) geo-graphers

24. **A science lab beaker contains 500 ml of a 12% acid solution. How much water needs to be evaporated to increase the acid concentration to 15%?**

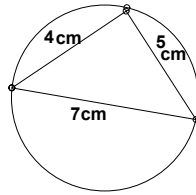(A) 30 ml   (B) 50 ml   (C) 100 ml   (D) 125 ml   (E) 166 ml

25. **Which of the following points does not lie in the shaded portion of the graph of** $2y \leq |3x + 1|$ **?**

(A) $(-2, 2)$ (B) $(4, 7)$ (C) $(0, 0)$ (D) $(-4, 5)$ (E) $(1, 2)$

26. The value of $(0.222...) \times (0.41666...)^{-1} \div (0.1875)^{-1}$ is:

(A) $\frac{1}{10}$ (B) $2\frac{1}{2}$ (C) $11\frac{1}{4}$ (D) $\frac{5}{288}$ (E) $2\frac{38}{45}$

27. Find the area of the circle. (nearest tenth)

(A) 22.4 cm²   (B) 64.1 cm²   (C) 39.5 cm²   (D) 50.3 cm²   (E) 40.1 cm²

28. Cher Sense has \$2.00 in pennies. She gives one-fourth of them to her brother, one-third of what was left to her sister and keeps one-tenth of what is left for herself. She puts what the remaining pennies in her piggy bank. How much did she put in her bank?

(A) 10¢   (B) 50¢   (C) 60¢   (D) 90¢   (E) \$1.10

29. Simplify: $\log_b(x^3y) - \log_b(2xy^2) + \log_b(3x^{-1}y^3)$

(A) $\log_b(\frac{2x^5}{3})$ (B) $\log_b(\frac{6y^4}{x^3})$ (C) $\log_b(\frac{3xy^2}{2})$ (D) $\log_b(\frac{2xy^2}{3})$ (E) $\log_b(\frac{3x^2y}{2})$
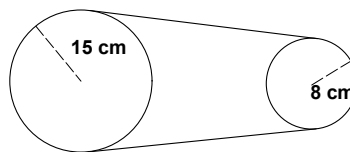
30. Let $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$. Find $\left|AB\right|$.

(A) 2   (B) 4   (C) 5   (D) 8   (E) 25

31. Find the least common multiple of 84, 180, and 350.

(A) 210   (B) 44,100   (C) 614   (D) 15,120   (E) 6,300

32. A belt joins the two pulleys shown. If the larger pulley is rotating at 15 rpm, then the smaller pulley is rotating at _____ rpm .

(A) $28\frac{1}{8}$ (B) $26\frac{1}{4}$ (C) $21\frac{1}{8}$ (D) 16   (E) 8

33. △ABC is a right triangle where m∠ABC = 90 ° and m∠ACB = 45 °. Point D lies on segment AC such that m∠ADB = 90 ° and BC = 5 cm. Find the ratio of CD to AB.

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{5}$ (C) $\frac{5\sqrt{2}}{2}$ (D) $\frac{2\sqrt{2}}{5}$ (E) $\frac{\sqrt{2}}{2}$

34. A yellow ribbon is 6 yards long. It is going to be cut into three pieces such that the longest piece is twice as long as the shortest piece and the other piece is 18 inches longer than the shortest piece. How long is the longest piece?

(A) 1 yd 1 ft 1.5 in

(B) 3 yds 9 in

(C) 2 yds 2 ft 3 in

(D) 1 yd 2 ft 7.5 in

(E) 2 yds 1 ft 9 in

35. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \dots + \frac{1}{136} + \frac{1}{153} = ?$

(A) $\frac{8}{9}$ (B) $\frac{5}{6}$ (C) $\frac{16}{17}$ (D) $\frac{15}{17}$ (E) $\frac{17}{19}$

36. Find $a + b + c + d$ given the arithmetic sequence: $-7, a, b, c, 5, d, \dots$

(A) 2   (B) 3   (C) 5   (D) 8   (E) 13

37. $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = ?$

(A) 1,296   (B) 1,000   (C) 784   (D) 512   (E) 441

38. The directrix of the conic given by the equation $y = 4x^2 - 4x + 1$ is:

(A) $y = -\frac{1}{4}$ (B) $y = -\frac{1}{16}$ (C) $y = -\frac{1}{8}$ (D) $y = \frac{1}{16}$ (E) $y = \frac{1}{4}$

39. The *I Scream 4 Ice Cream* Shoppe has a special sale for triple scoop cones. The flavors available are vanilla, chocolate, strawberry, banana-nut, pecan praline, and peppermint. How many different triple scoop cones can they create from the available flavors?

(A) 504   (B) 168   (C) 120   (D) 56   (E) 20

40. Which of the following polar equations will generate the following graph?

(A) $r = 1 - 2\cos(\theta + 1)$

(B) $r = 1 + 2\sin(\theta + 1)$

(C) $r = 1 - 2\cos(\theta - 1)$

(D) $r = -1 + 2\sin(\theta - 1)$

(E) $r = 1 + 2\cos(\theta + 1)$

41. A set of 5 positive integers has a mean of 22, a median of 23, and a mode of 27. What is the largest possible range of the set of integers?

(A) 11   (B) 15   (C) 16   (D) 23   (E) 32

42. If $\frac{5}{6x} + \frac{3y}{4} = \frac{1}{2}$, then x equals _____.

   (A) $\frac{8}{7y}$
   (B) $\frac{10}{6-9y}$
   (C) $7y$
   (D) $\frac{7y}{8-9y}$
   (E) $\frac{6y-10}{9}$

43. $\{(x,y)| \; x,y \in \{\text{Integers}\}, -8 \le x \le 5, \text{ and } -3 \le y \le 6\}$ is the solution set of $2x + 3y = 5$. How many such ordered pairs exist?

   (A) 6
   (B) 5
   (C) 4
   (D) 3
   (E) 2

44. Hickory, Dickory, and Doc looked up at the circular clock. The time shown was 1:30 a.m. Find the measure of the smaller angle between the big hand and the little hand.

   (A) 135°
   (B) 145°
   (C) 138°
   (D) 142°
   (E) 150°

45. $\overline{AB}, \; \overline{AC}, \; \overline{BD}, \text{ and } \overline{CD}$ are chords of circle O and point E lies on circle O. If $m\overarc{BC} = 50°$ and $m\angle APB = 80°$, then $m\angle ACD = ?$
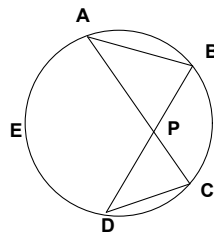
   (A) 25°
   (B) 40°
   (C) 50°
   (D) 75°
   (E) 80°

46. How many of the Platonic solids have faces that are equilateral triangles?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

47. If $a_3 = 1$, $a_4 = 8\frac{1}{4}$, and $a_5 = 15\frac{1}{2}$ then $a_1 =$ _____.

   (A) $-20\frac{3}{4}$
   (B) $-13\frac{1}{2}$
   (C) $-9\frac{3}{4}$
   (D) $-7\frac{3}{4}$
   (E) $-6\frac{1}{4}$

48. A special deck of cards consists of the A♥, A♣, 2♦, 3♠, 5♠, 8♦, J♣, Q♥, and K♣. The special deck is shuffled. The top three cards are dealt face up. What is the probability that the first card is a spade (♠), the second card is an ace (A), and the third card is a face card (J, Q, K)? (nearest tenth %)

   (A) 2.4%
   (B) 2.6%
   (C) 0.7%
   (D) 0.8%
   (E) 1.1%

49. The graph of $2x^2 + 4xy - y^2 - 2x + 3y = 6$ is a(n):

   (A) circle
   (B) ellipse
   (C) hyperbola
   (D) line
   (E) parabola

50. $\sin(\frac{3\pi}{2} - \theta)$ equals:

    (A) $\sin(\theta + 2\pi)$ (B) $\cos(\theta + \frac{2\pi}{3})$ (C) $\sin(\theta - \frac{\pi}{3})$ (D) $\cos(\theta - \pi)$ (E) $\cos(\theta + \frac{3\pi}{2})$

51. Simplify: $\cot\theta \sec\theta \sin\theta$

    (A) -1 (B) $\tan\theta$ (C) 1   (D) $\cos\theta$ (E) $\csc\theta$

52. If the three numbers 135, 234, and 366 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

    (A) 9   (B) 7   (C) 6   (D) 3   (E) 1

53. Tye Gehr drops a golf ball from a platform 12 feet above the ground. Each time it hits the ground it rebounds two-thirds of the distance it fell. Find the total distance the golf ball travels when it hits the ground the sixth time. (nearest foot)

    (A) 50 ft   (B) 51 ft   (C) 52 ft   (D) 53 ft   (E) 54 ft

54. The complex numbers $w = 3 + i$ and $z = i - 4$ are graphed as vectors. What is the measure of the angle formed between them? (nearest tenth)

    (A) 4.4 °   (B) 85.6 °   (C) 94.4 °   (D) 122.5 °   (E) 147.5 °

55. $\int (-\sin x \cos x)\ dx = $ _____ $+\ C$, where C is some arbitrary constant.

    (A) $\frac{1}{2}\cos^2(x)$ (B) $\frac{1}{2}\sin x \cos x$ (C) $-\frac{1}{2}\cos(2x)$ (D) $\frac{1}{2}\sin^2(x)$ (E) $2\cos^2(2x)$

56. Let $f(x) = \sqrt{x^3}$. Find $f'(4)$.

    (A) $\sqrt{2}$ (B) $2\sqrt{2}$ (C) $2\frac{2}{3}$ (D) 3   (E) 4

57. The probability of a student in class 1A scoring above 146 on this test is 48%. How many 1A students took this test if 240 students scored above 146?

    (A) 482   (B) 485   (C) 500   (D) 520   (E) 544

58. Max Profitt has 50 used CDs that he can package in sets of 4 or sets of 5. He gets $15.00 for a set of 4 and $18.00 for a set of 5. The maximum amount of money he can get for selling all 50 CDs is:

    (A) $174.00   (B) $177.00   (C) $180.00   (D) $183.00   (E) $186.00

59. $\displaystyle\sum_{1}^{17} (n^2) \div \sum_{1}^{17} \left(\frac{n(n+1)}{2}\right) = ?$

    (A) $1\frac{17}{21}$ (B) $1\frac{16}{19}$ (C) $1\frac{8}{19}$ (D) $1\frac{7}{9}$ (E) $1\frac{1}{9}$

60. The figure shown was made by gluing 16 congruent cubes together. How many of the cubes faces in the figure shown are not visible? Do not count the faces that are glued together.

    (A) 29   (B) 28   (C) 27   (D) 26   (E) 25

University Interscholastic League
MATHEMATICS CONTEST

Final _____   _____

2nd _____   _____

1st _____   _____

Score   **Initials**

Contestant # _____   **Conference** _____

1. _____   21. _____   41. _____

2. _____   22. _____   42. _____

3. _____   23. _____   43. _____

4. _____   24. _____   44. _____

5. _____   25. _____   45. _____

6. _____   26. _____   46. _____

7. _____   27. _____   47. _____

8. _____   28. _____   48. _____

9. _____   29. _____   49. _____

10. _____   30. _____   50. _____

11. _____   31. _____   51. _____

12. _____   32. _____   52. _____

13. _____   33. _____   53. _____

14. _____   34. _____   54. _____

15. _____   35. _____   55. _____

16. _____   36. _____   56. _____

17. _____   37. _____   57. _____

18. _____   38. _____   58. _____

19. _____   39. _____   59. _____

20. _____   40. _____   60. _____

1. E	21. B	41. C
2. C	22. A	42. B
3. B	23. D	43. C
4. A	24. C	44. A
5. B	25. B	45. D
6. D	26. A	46. C
7. A	27. E	47. B
8. B	28. D	48. A
9. E	29. C	49. C
10. B	30. B	50. D
11. D	31. E	51. C
12. A	32. A	52. D
13. E	33. E	53. E
14. E	34. C	54. E
15. A	35. A	55. A
16. B	36. C	56. D
17. A	37. C	57. C
18. C	38. B	58. E
19. B	39. D	59. B
20. C	40. E	60. B

UIL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Regional • 2011

1. Evaluate: $28 \div 21 \times 15 - 10 + 6 \div 3 - 1$

   (A) $3\frac{2}{9}$ (B) 4   (C) $4\frac{1}{3}$ (D) 9   (E) 11

2. I. M. Hungree can buy three 10" pizzas for $10.00 and two 2-liter sodas for 95¢ each. He has a 50¢ off coupon. If he makes his purchase before 5:00 p.m. he gets an extra 20% off of his total bill after the coupon is applied. At 4:30 p.m. he orders three 10" pizzas and two 2-liter sodas. How much change should he get back if he pays with a twenty dollar bill?

   (A) $9.02   (B) $9.12   (C) $10.88   (D) $10.98   (E) $11.28

3. The value of $(0.08333...)^{-1} \times (0.111...) \div (0.1666...) \times (0.333...)$ is:

   (A) $2\frac{8}{9}$ (B) $2\frac{2}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{15}$ (E) $\frac{1}{6}$

4. Dee Hunter found 30 eggs at the annual Easter egg hunt. She gave 20% of them to her brother. Then she gave one-fourth of what she had left to her little sister. If she kept $33\frac{1}{3}\%$ of what remained, how many did she have left to give to her parents?

   (A) 5   (B) 8   (C) 10   (D) 12   (E) 15

5. Find the y-intercept of the line through point P that is perpendicular to the line shown.
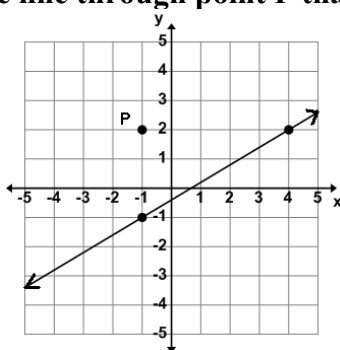
   (A) $(0, \frac{2}{5})$ (B) $(0, \frac{1}{4})$ (C) $(0, \frac{1}{3})$ (D) $(0, \frac{2}{3})$ (E) $(0, \frac{7}{5})$

6. The set of Prime numbers is closed under which one of the following four basic operations?
        addition ($+$)   subtraction ($-$)   multiplication ($\times$)   division ($\div$)

   (A) $+, -, \times, \div$ (B) $+, -$ (C) $\times, \div$ (D) $+$ (E) not closed under any of these

7. The ratio of Snap's age to Crackle's age now is 3:2 and the ratio of Crackle's age to Pop's age now is 1:6. In 9 years the ratio of Crackle's age to Pop's age will be 1:3. What is the sum of the their ages now?

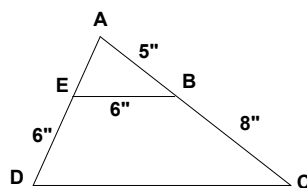   (A) 81   (B) 63   (C) 60   (D) 51   (E) 34

8. Which of the Platonic solids have faces that are regular pentagons?

   (A) tetrahedron   (B) octahedron   (C) icosahedron   (D) dodecahedron   (E) cube

9. The area of a rectangle is 108 sq. cm. The ratio of its length to its width is 4:3. Find the perimeter of the rectangle.

   (A) 14 cm   (B) 24 cm   (C) 36 cm   (D) 42 cm   (E) 48 cm

10. Find the perimeter of △ACD if $\overline{BE} \parallel \overline{CD}$.

   (A) 41.8"   (B) 39.333..."   (C) 38.35"   (D) 32.35"   (E) 29.5"

11. Les Dough has $100.00 in his savings account. He has to withdraw 10% of his monthly balance each month. How much will he have left after 6 monthly withdrawals? (nearest cent)

   (A) $40.00   (B) $47.84   (C) $53.14   (D) $55.00   (E) $59.05

12. Let $f(x) = 2 - x$, $g(x) = 3 + 2x$, and $h(x) = 4x - 3$. Find $h(g(f(1 - x)))$.

   (A) $8x - 3$ (B) $8x - 7$ (C) $8x + 7$ (D) $8x + 17$ (E) $8x + 19$

13. Let $A = \begin{bmatrix} 4 & 9 \\ -1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -9 \\ 1 & 4 \end{bmatrix}$. Find $|A| \times |B|$.

   (A) 121   (B) 22   (C) 0   (D) -22 (E) -121

14. An eagle looks down from his nest on a cliff and spots a rodent for dinner. The rodent is 120 feet away from the base of the cliff and the angle of depression from the eagle to the rodent is 14°45'. How far is the rodent from the eagle? (nearest inch).

   (A) 41 yds 1' 1"   (B) 41 yds 2' 4"   (C) 49 yds 1' 3"   (D) 49 yds 1"   (E) 52 yds 1' 1"

15. $\cos(\theta + \frac{\pi}{2})$ equals:

   (A) $\sin(\pi - \theta)$ (B) $\sin(\theta + 2\pi)$ (C) $-\sin(\theta - \frac{\pi}{2})$ (D) $-\sin(\theta + \frac{\pi}{2})$ (E) $\sin(\theta - \pi)$

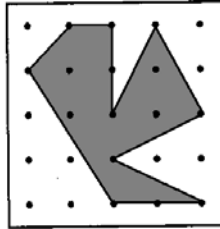16. Which of the following cannot be a triangular number?

   (A) 6,105   (B) 5,048   (C) 4,560   (D) 3,403   (E) 2,926

17.

| P | $-\frac{1}{3}$ | Q | R | S | $\frac{2}{9}$ |

   <------------|------------|------------|------------|------------|------------|------------>

   The distances between the hash marks (|) are equal. Find the midpoint of segment QR.

   (A) $-\frac{1}{8}$ (B) $\frac{10}{27}$ (C) $-\frac{1}{18}$ (D) $\frac{1}{4}$ (E) $\frac{5}{36}$

18. The horizontal and vertical distances between each pair of dots is 1 cm. Find the area of the shaded figure.

(A) 7 sq. cm   (B) 7.5 sq. cm   (C) 8 sq. cm   (D) 8.5 sq. cm   (E) 9.5 sq. cmx

19. Find the remainder when $4x^3 - 4x^2 + x - 5$ is divided by $x - 2$.

(A) -56 (B) -55 (C) 13   (D) 33   (E) 65

20. $\sum\limits_{1}^{10} (n^3) \div \sum\limits_{1}^{10} \left(\frac{n(n+1)}{2}\right) = ?$

(A) 12.25   (B) 13.5   (C) 13.75   (D) 14.75   (E) 16.5

21. Let $f(x) = \frac{3-x}{2x^2+x-1}$. Find $f'(1)$.

(A) $1\frac{3}{4}$ (B) 1   (C) -2 (D) -3 (E) $-4\frac{1}{4}$

22. The eccentricity of the conic given by the equation $8y^2 - 2x^2 = 16$ is:

(A) $\sqrt{34}$ (B) $2\sqrt{17}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{\sqrt{10}}{4}$ (E) $\sqrt{5}$

23. If $y = 3\sqrt{x^2+1}$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$

(A) $\frac{\sqrt{10}}{10}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{\sqrt{6}}{6}$ (D) $\frac{\sqrt{2}}{4}$ (E) $\frac{\sqrt{3}}{4}$

24. Circles X and Z are tangent to the large circle and to circle Y. Circles X, Y, and Z are congruent. Willie Tale shoots an arrow and hits the big circular target. What is the probability that the arrow hit the shaded section?

(A) 25%   (B) 50%   (C) $66\frac{2}{3}\%$ (D) 75%   (E) $33\frac{1}{3}\%$

25. What are the odds that a factor of the triangular number 276 is a triangular number?

(A) 1:2   (B) 1:3   (C) 1:4   (D) 3:1   (E) 2:1

26. Which of the following mathematicians is known as the "father of algebra" and was one of the first to write about polygonal numbers?

(A) Diophantus   (B) Goldbach   (C) Leibniz   (D) Pythagoras   (E) Theano

27. The *politeness* of the *polite* number 15 is:

(A) 5   (B) 4   (C) 3   (D) 2   (E) 1

28. A set of 5 positive integers {c, d, e, f, g) is listed in numeric order from least to greatest. The set has a mean of 30, a median of 30, a range of 26, and a mode of 41. Find d.

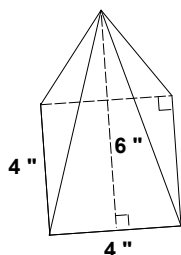(A) 18   (B) 23   (C) 25   (D) 28   (E) 30

29. Which of the following is NOT a solution to $|2x-4| \geq 3x+1$?

(A) $\frac{3}{4}$ (B) $\frac{2}{5}$ (C) 0   (D) $-\frac{1}{3}$ (E) $-\frac{3}{7}$

30. Let p and q be the roots of $x^2 + 2x - 15 = 0$. Find $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.

(A) -30 (B) -15 (C) 4   (D) 16   (E) 40

31. The pyramid shown has a square base. Find the total surface area.

(A) 96 in^2 (B) 64 in^2 (C) 48 in^2 (D) 32 in^2 (E) 24 in^2

32. Point P (3 , 2) is translated horizontally 6 units to the left to point Q. Point Q is rotated 90° counterclockwise about the origin to point R. Point R is reflected across the origin to point S. Point S is translated vertically down 5 units to point T (x, y). Find x + y.

(A) -3 (B) -1 (C) 0   (D) 1   (E) 4

33. Let $f(x) = 4-x$ and $g(x) = 2+3x$. If h(x) is the inverse function of $\frac{f(x)}{g(x)}$, find the value of h(-5).

(A) $-1\frac{4}{9}$ (B) -1 (C) $-\frac{1}{4}$ (D) $\frac{3}{7}$ (E) 1

34. If $a_1 = 2$, $a_2 = 3$, $a_3 = 4$ and $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, where $n \geq 4$, then a_6 equals:

(A) -3 (B) -1 (C) 2   (D) 3   (E) 4

35. A belt joins the two pulleys shown. If the angular velocity of the larger pulley is 60 rpm and the angular velocity of the smaller pulley is 150 rpm then the radius of the smaller pulley is:

(A) 90 cm   (B) 50 cm   (C) $41\frac{2}{3}$ cm   (D) 40 cm   (E) 25 cm

36. In △ABC, m∠BAC = 70° and m∠ABC = 50°. Point H lies on segment AB such that segment CH is an angle bisector of ∠ACB. Point M lies on segment BC such that BM = BH. Find m∠CHM.

(A) 35°   (B) 40°   (C) 45°   (D) 50°   (E) 60°
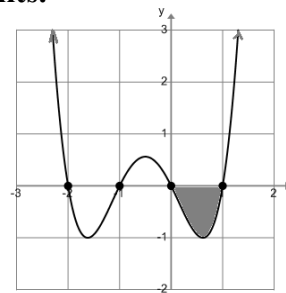
37. $\displaystyle\sum_{k=-2}^{1} (k+1)x + (k-1) = ?$

(A) $2x + 5$ (B) $4x + 6$ (C) $3x - 7$ (D) $4x - 5$ (E) $2x - 6$

38. In the binomial expansion of $(2x - 3)^6$, the sum of the coefficients of the 3rd and 5th terms is:

(A) 9,720   (B) 7,020   (C) 4,320   (D) $-2,160$ (E) $-2,020$

39. Find the area of the shaded region in square units.

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{19}{30}$ (D) $\frac{7}{10}$ (E) $\frac{3}{4}$

40. Which of the following is an inflection point of the function $f(x) = x^4 - 3x^3 + 3x^2 + 1$?

(A) $(\frac{1}{2}, 0)$ (B) $(1, 2)$ (C) $(\frac{1}{2}, 1)$ (D) $(1, 0)$ (E) $(1, \frac{1}{2})$

41. Zane E. Fella lost his marbles. He had 8 green ones, 10 blue ones, 5 red ones, and 2 clear ones. He randomly picked 3 of them from the jar, dropped them, and they rolled away. What is the probability that all 3 of them were the same color? (nearest percent)

(A) 3%   (B) 8%   (C) 11%   (D) 13%   (E) 29%

42. Gene Yus has ten math dominoes turned face down. The face of each domino has a different digit, 0 - 9, written on it. Gene shuffles the dominoes then flips one of them over. If the digit is a factor of 9 then he gets 9 points. If the digit is a positive multiple of 2 he gets 2 points. If the digit is neither of these he loses 5 points. What is the mathematical expectation of a single draw?

(A) 3.5 pts (B) 3 pts (C) 2.9 pts (D) 2.7 pts (E) 2 pts

43. Simplify: $\dfrac{1^2 + 2^2 + 3^2 + 4^2 + ... + 23^2 + 24^2}{1 + 3 + 6 + 10 + ... + 276 + 300}$

(A) $1\frac{25}{28}$ (B) $1\frac{23}{25}$ (C) $1\frac{8}{9}$ (D) $1\frac{23}{26}$ (E) $1\frac{22}{25}$

44. Find the midpoint of the domain of the real valued function $f(x) = \sqrt{5 - \sqrt{3x - 2}}$.

(A) $4\frac{1}{6}$ (B) $4\frac{1}{4}$ (C) $4\frac{1}{2}$ (D) $4\frac{3}{4}$ (E) $4\frac{5}{6}$

45. If the GCD(x, 63) = 9 and the LCM (x, 63) = 756, then x = ?

(A) 7 (B) 18 (C) 108 (D) 567 (E) 972

46. Simplify: $a^5 \div (ab^3) \times a^{-2} \times a^{-3}b^2 \div a^{-1} \times b$

(A) b^{-2} (B) a^{-4} (C) $a^{-4}b^{-2}$ (D) $a^{-2}b^{-4}$ (E) 1

47. The *Mixed Nutt Shoppe* mixes 9 pounds of cashews, 12 pounds of pecans, and 15 pounds of walnuts to create their special of the day. How much per pound should their special sell for if cashews sell for $7.00 a pound, pecans for $5.00 a pound, and walnuts for $2.00 a pound?

(A) $2.58 (B) $3.88 (C) $4.00 (D) $4.25 (E) $4.67

48. $\left\{(x , y) \mid x,y \in \{\text{Integers}\}, -3 \le x \le 6, \text{ and } -3 \le y \le 6\right\}$ is the solution set of x + 3y = 6. How many such ordered pairs exist?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

49. The incenter, circumcenter, centroid, and orthocenter are always inside the triangle for which of these types of triangles: 1. Equilateral   2. Right   3. Acute   4. Obtuse

(A) 1 & 3 (B) 2 & 4 (C) 1, 3, & 4 (D) 1 only (E) 1, 2, 3, & 4

50. Find the circumference of the circle. (nearest tenth)

(A) 45.1 cm (B) 20.6 cm (C) 25 cm (D) 35.5 cm (E) 23.8 cm

51. Three workers can wax four cars in five hours. How long would it take five workers to wax six cars if they work at the same rate as the three workers?

(A) 5 hrs   (B) 4 hrs 40 min   (C) 4 hrs 45 min   (D) 4 hrs 30 min   (E) 4 hrs

52. The radius of the circle, $x^2 + y^2 + 2x - 4y - 7 = 0$ is:

(A) $\sqrt{2}$ (B) 2   (C) 3   (D) $2\sqrt{3}$ (E) 6

53. If $\sin \alpha = -\frac{1}{2}$, where $\pi < \alpha < \frac{3\pi}{2}$, and $\cos \beta = \frac{\sqrt{3}}{2}$, where $\frac{3\pi}{2} < \beta < 2\pi$, then $\cos(\alpha - \beta)$=?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{\sqrt{3}}{2}$ (E) -1

54. Mark Despot is staking out a triangular garden. After placing his first stake he steps off 50 feet on a bearing of 75° and places a second stake. Then he walks 75 feet on a bearing of 100° and places the third stake. What is the distance from the third stake to the first stake? (nearest feet)

(A) 125 ft   (B) 94 ft   (C) 90 ft   (D) 115 ft   (E) 122 ft

55. The complex numbers $w = 2 + 5i$ and $z = 3 - 4i$ are graphed as vectors. What is the measure of the angle formed between them? (nearest tenth)

(A) 58.7 °   (B) 74.9 °   (C) 105.1 °   (D) 121.3 °   (E) 123.9 °

56. Find the determinant of the 3 x 3 matrix $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$.

(A) -4 (B) -2 (C) 0   (D) 1   (E) 2

57. $\int \left(\frac{3x}{3x^2 - 1} \right) dx = $ _____ $+ C$, where $x > \frac{\sqrt{3}}{3}$ and C is some arbitrary constant.

(A) $\frac{\ln(3x^2 - 1)}{2}$ (B) $3\ln(3x^2 - 1)$ (C) $\frac{1}{\ln(3x^2 - 1)}$ (D) $\ln(3x^2 - 1)$ (E) $\frac{\ln(3x^2 - 1)}{3}$

58. Each of the 20 mobile homes at Rainbow Park have 1 car or 2 cars and 1 dog or 2 dogs. Thirty percent of the homes have 2 cars in their driveway and 2 dogs in their backyard. Forty-five percent of them have 1 car in their driveway and 2 dogs in their backyard. What is the probability that a home chosen randomly has at least 1 car in their driveway, but only 1 dog in their backyard?

(A) 55%   (B) $37\frac{1}{2}$%   (C) $33\frac{1}{3}$%   (D) 25%   (E) 15%

59. How many positive three-digit integers exist such that the digit in the hundreds place is an evil number, the tens digit is a primeval number, and the ones digit is a polite number?

(A) 32   (B) 40   (C) 64   (D) 80   (E) 100

60. The figure shown was made from congruent cubes. How many faces are not visible? There are no hidden gaps.

(A) 99   (B) 96   (C) 72   (D) 52   (E) 48

University Interscholastic League
MATHEMATICS CONTEST
HS ● Regional ● 2011
Answer Key

1. E	21. D	41. B
2. C	22. E	42. E
3. B	23. D	43. D
4. D	24. C	44. E
5. C	25. A	45. C
6. E	26. A	46. E
7. D	27. C	47. D
8. D	28. B	48. B
9. D	29. A	49. A
10. C	30. D	50. E
11. C	31. B	51. D
12. D	32. C	52. D
13. A	33. B	53. C
14. A	34. D	54. E
15. E	35. D	55. D
16. B	36. A	56. A
17. A	37. E	57. A
18. D	38. B	58. D
19. C	39. C	59. B
20. C	40. B	60. B

UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
State • 2011

1. Evaluate: $9 \times 8 \times 7 \div 6! - (5+4)^{\frac{1}{2}} + 3 \div 2 - 1$

(A) -1.8 (B) -1.4 (C) -0.65 (D) 0 (E) 10

2. Will I. Byette purchased five videos. The regular price of each video was $9.00. The store had a 20% off sale and he received a special "10% off the sale price" discount for buying five videos. How much change should Will have received if he paid for his purchase with two $20-bills?

(A) $8.50   (B) $8.35   (C) $8.20   (D) $7.95   (E) $7.60

3. Phil Upp's compact car has a gas tank with a capacity of 15 gallons. The gauge shows that it is $\frac{1}{4}$ full. How much would it cost to fill it up at $2.75 per gallon?

(A) $41.25   (B) $35.63   (C) $33.65   (D) $31.69   (E) $30.94

4. Simplify: $a^0 b \times (ab^2)^3 \times a^{-5}b^{-3} \div a^{-2} \div (a^{-1}b^{-1})$

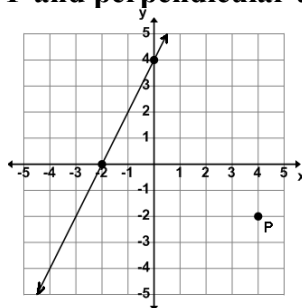(A) $a^{-1} b^3$ (B) $a^4 b$ (C) ab^5 (D) $a^{-3} b^4$ (E) b^5

5. The statement, $a(bc) = a(cb)$, is justified by which of the following properties of real numbers?

(A) commutative   (B) inverse   (C) associative   (D) identity   (E) distributive

6. Simplify: $\dfrac{9x^2 - 16}{12x^2 - 25x + 12} \div \dfrac{9x^2 + 24x + 16}{8x - 6}$
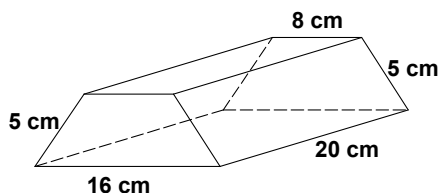
(A) $\dfrac{2}{4x-3}$ (B) $8x+6$ (C) 2 (D) $\dfrac{3x+4}{4x-3}$ (E) $\dfrac{2}{3x+4}$

7. The line containing point P and perpendicular to the line shown intersects the x-axis at:

(A) $x = -\frac{1}{2}$ (B) $x = -\frac{1}{4}$ (C) $x = -\frac{1}{8}$ (D) $x = 0$ (E) $x = \frac{1}{4}$

8. Find the total surface area of the trapezoidal prism.

(A) 652 cm^2 (B) 752 cm^2 (C) 760 cm^2 (D) 776 cm^2 (E) 840 cm^2

9. The *C-Shore* salt water store has a 5 gallon salt water tank that is 5% salt. How much water would they have to evaporate so that it is 8% salt?

   (A) 3 gals 2 qts   (B) 1 gal 2 qts 1 pts   (C) 3 qts 1 pt   (D) 1 gal 3 qts 1 pt   (E) 3 gals 1 pt

10. In a plane, given angle $\angle ABC$, the locus of points equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is a(n):

   (A) arch   (B) semi circle   (C) angle bisector   (D) incenter   (E) centroid

11. A secant, $\overline{AP}$, and a tangent, $\overline{CP}$ intersect in the exterior of a circle. Point B lies on $\overline{AP}$ and on the circle such that AB = 15" and CP = 10". Find BP.

   (A) 25"   (B) 20"   (C) 12.5"   (D) 7.5"   (E) 5"

12. Point A $(0, -2)$ is reflected across the line $y = -x$ to point B. Then point B is rotated 90° counterclockwise around the origin to point C. Then point C is translated horizontally 2 units to the left to point D. Then point D is reflected across the origin to point E (x, y). Find y.

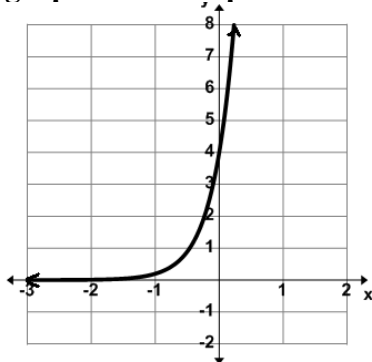   (A) -2 (B) -1 (C) 0   (D) 1   (E) 2

13. Let $A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ -5 & -2 \end{bmatrix}$. Find $|A - B|$.

   (A) 0   (B) 1   (C) 17   (D) 21   (E) 29

14. I. M. Slow leaves Kerrville on Interstate 10 driving at a constant speed of 65 mph. Will I. Ketchum leaves Kerrville 15 minutes later. It takes Will an hour and thirty minutes driving at a constant speed to catch up to I. M. Slow. What was Will's constant speed? (nearest mph).

   (A) 70 mph   (B) 72 mph   (C) 76 mph   (D) 80 mph   (E) 83 mph

15. Which of the following equations will produce the graph shown here?

   (A) $y = e^{3x} + 4$ (B) $y = 4e^{3x}$ (C) $y = \sqrt{e^{3x} - 4}$ (D) $y = 4\sqrt{e^{-3x}}$ (E) $y = \sqrt{e^{-3x}} + 4$

16. If $a_1 = -2$, $a_2 = 0$, $a_3 = 1$ and $a_n = (a_{n-3})(a_{n-1}) - a_{n-2}$, where $n \geq 4$, then a_7 equals:

   (A) 2   (B) 1   (C) 0   (D) -1 (E) -2

17. 325 is the _____ triangular number?

    (A) 21st (B) 22nd (C) 23rd (D) 24th (E) 25th

18. $\triangle ABC$ exists such that AB = AC. P lies on $\overline{AC}$ and Q lies on $\overline{BC}$ such that AP = AQ. Find m∠PQC if m∠BAQ = 30 °

    (A) 60 °   (B) 22.5 °   (C) 15 °   (D) 12.5 °   (E) 45 °

19. If $\sin \alpha = \frac{\sqrt{3}}{2}$, where $\frac{\pi}{2} < \alpha < \pi$, and $\cos \beta = - \frac{\sqrt{3}}{2}$, where $\pi < \beta < \frac{3\pi}{2}$, then $\sin (\alpha + \beta)$=?

    (A) -1 (B) $-\frac{1}{2}$ (C) 0   (D) $\frac{3}{4}$ (E) $1\frac{1}{2}$

20. Sir Vayer drove four stakes in the ground as shown. He used his surveying equipment to find the approximate measures as shown. From this information he was able to approximate the perimeter of the triangle to the nearest inch. What did he calculate the perimeter to be?
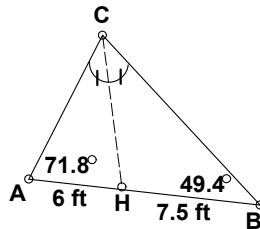
    (A) 40' 6"   (B) 41'4"   (C) 38'6"   (D) 39'7"   (E) 41'5"

21. $\sum_{n=1}^{k} (n^2) \div \sum_{n=1}^{k} \left(\frac{n(n+1)}{2}\right) = ?$

    (A) $\frac{3k}{k+2}$ (B) $\frac{(k)(k+1)}{2}$ (C) $\frac{2k+1}{k+2}$ (D) $\frac{2k}{k+3}$ (E) $\frac{(k)(k+1)(k+2)}{6}$

22. Mr. White and his dog, Lady, try to speed walk 2 miles each day when the weather permits. The following table shows Mr. White's speed for each half mile walked. Find Mr. White's average walking speed. (nearest hundredth)

Distance	walking speed
First 1/2 mile	4.3 mph
Second 1/2 mile	3.0 mph
Third 1/2 mile	3.3 mph
Fourth 1/2 mile	2.7 mph

    (A) 3.27 mph   (B) 3.38 mph   (C) 3.30 mph   (D) 3.33 mph   (E) 3.23 mph

23. Find the remainder when $x^3 - 5x^2 + 8x - 4$ is divided by $x + 3$.

    (A) -100 (B) -28 (C) 2   (D) 5   (E) 65

24. If $(1 - i)(2 + 3i)(5 - 8i) = a + bi$ then $a + b = ?$

   (A) -3 (B) -2 (C) 1   (D) 13   (E) 16

25. Find the equation of the line tangent to the curve $y = \sqrt[3]{x}$ at the point (8, 2).

   (A) $12x + y = 98$ (B) $x - 12y = 16$ (C) $4x + 3y = 38$ (D) $x - 12y = -16$ (E) $x + 12y = 32$

26. If $2x^3 - 5y = 7$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when x = ?

   (A) $\frac{5}{6}$ (B) $\frac{\sqrt{6}}{5}$ (C) $\frac{\sqrt{5}}{6}$ (D) $\frac{\sqrt{30}}{5}$ (E) $\frac{\sqrt{30}}{6}$

27. $\int (3\sqrt{6x + 9})\ dx = $ _____ $+ C,$ where C is some arbitrary constant.

   (A) $\frac{\sqrt{(6x+9)^3}}{3}$ (B) $2(\sqrt{6x + 9})^3$ (C) $\frac{(\sqrt{6x+9})^3}{2}$ (D) $\sqrt{\frac{(6x+9)^3}{3}}$ (E) $3(6x + 9)^{\frac{2}{3}}$

28. Betty Won't throws 3 darts at the square target containing 9 congruent smaller squares. Each dart lands in a different small square. She wins if the sum of the numbers in the 3 squares the darts land in is a Fibonacci number. What is the probability that she wins?

   (A) 30 %   (B) $28\frac{4}{7}$ %   (C) $22\frac{2}{9}$ %   (D) $21\frac{3}{7}$ %   (E) 20 %

29. Roland Cubes tosses a fair die three times. What is the probability that the face with six dots comes up at least two times? (nearest tenth)

   (A) 8.3 %   (B) 7.4 %   (C) 6.9 %   (D) 8.0 %   (E) 7.1 %

30. Which of the following are two dimensional perspectives of the figure shown?

   (1)   (2)   (3)   (4)   (5)

   (A) 1 & 5   (B) 2 & 4   (C) 1, 3, & 5   (D) 1, 2, & 5   (E) all of them

31. $246_{16} + 357_{16} + 981_{16} = $ _____ $_{16}$

    (A) F7E   (B) 1584   (C) F74   (D) EHE   (E) F1E

32. Lotta Sense had 50 quarters. She spent 20% of them playing a pinball machine. Then she spent 40% of what she had left at a candy shop. When she got home she put $33\frac{1}{3}\%$ of what she had left in her piggy bank and the rest in her pocket. How much did she put in her pocket?

    (A) $4.00   (B) $4.50   (C) $5.00   (D) $5.25   (E) $6.00

33. Let A = {m,e,a,n}, B = {m,e,d,i,a,n}, C = {m,o,d,e}, and D = {r,a,n,g,e} . The number of distinct elements in $(A \cap D) \cup (B \cap C)$ is _____ .

    (A) 4   (B) 5   (C) 6   (D) 8   (E) 9

34. Graham Waite places a weight 120 cm from the fulcrum of a beam. The weight weighs 75 kg. How far on the other side of the fulcrum will he have to put a 90 kg weight in order to balance the beam?

    (A) 144 cm   (B) 105 cm   (C) 100 cm   (D) 82.5 cm   (E) 60 cm

35. If $\frac{1}{x} - \frac{2y}{3} = \frac{5}{8}$, then x equals:

    (A) $\frac{16y + 15}{24}$ (B) $\frac{24}{15 - 16y}$ (C) $\frac{24}{15 + 16y}$ (D) $\frac{24}{15y - 16}$ (E) $\frac{15y - 24}{16}$

36. A triangle has integral side lengths of 4", 7", and k". How many values of k exist such that the triangle is an acute, scalene triangle?

    (A) 1   (B) 2   (C) 3   (D) 5   (E) 7

37. Six computer techs can assemble nine identical computers in eight hours. If two techs don't show up to work one day, how many of the identical computers could the others assemble in eight hours working at the same rate as the six workers?

    (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

38. How many elements are in $\left\{ x \mid 1 + \cos(x + \pi) = \sin(x - \pi), x \in [-2\pi, 2\pi\,] \right\}$?

    (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

39. $\displaystyle\sum_{k=-2}^{2} (kx - 1)(kx + 1) = ?$

    (A) $10x^2 - 5$ (B) $10x^2 - 4$ (C) $10x^2 - 7$ (D) $5x^2 - 4$ (E) $5x^2 - 5$

40. **Which of the following polar equations will generate the following graph?**

(A) $r = 2\cos(5\theta)$ (B) $r = 2\sin(5\theta)$ (C) $r = 2\cos(\theta + 5)$ (D) $r = 5\sin(2\theta)$ (E) $r = 2\sin(\theta + 5)$

41. **Les Sense has jars containing pennies, nickels, dimes, quarters, half-dollars, and silver dollars. He wants to give each of his nieces and nephews four coins. How many different sets of four coins can he choose from to give his nieces and nephews?**

(A) 126   (B) 21   (C) 360   (D) 252   (E) 90

42. **Simplify:** $\dfrac{1 + 8 + 27 + 64 + \ldots + 1728 + 2197}{1 + 3 + 6 + 10 + \ldots + 78 + 91}$

(A) $24\frac{1}{7}$ (B) $20\frac{6}{7}$ (C) $18\frac{1}{5}$ (D) 13   (E) $1\frac{4}{5}$

43. **How many prime numbers can be obtained from the primeval number 113?**

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

44. **Which of these mathematicians proved that the number of elements in the set of real numbers is more numerous than the number of elements in the set of natural numbers?**

(A) Georg Cantor   (B) Christian Goldbach   (C) Gottfried Leibniz

(D) Benoit Mandelbrot   (E) John Venn

45. **Find the sum of the mean, median, mode, and range of this set of numbers, {1, 3, 6, 10, 15, 21}.**

(A) $93\frac{1}{3}$ (B) $14\frac{1}{3}$ (C) $41\frac{1}{3}$ (D) $37\frac{1}{3}$ (E) $17\frac{1}{3}$

46. **Coach Bear has 6 boys and 4 girls on his math team. One of the boys, Yogi, and one of the girls, Prissy, cannot be on the same 4-member number sense team. How many different 4-member number sense teams made up of two boys and two girls can Coach Bear form from his math team?**
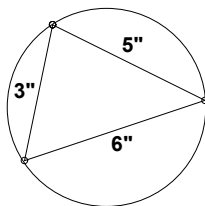
(A) 45   (B) 60   (C) 75   (D) 90   (E) 105

47. **The number 235 in base 8 is equivalent to the number wxyz in base 4, where w, x, y, and z are digits. Find $w + x + y + z$.**

(A) 10   (B) 8   (C) 7   (D) 5   (E) 4

48. If the roots of $x^3 + bx^2 + cx + d = 0$ are 1, 3, and -6, then $b + c + d$ equals:

    (A) -2 (B) -1 (C) 1   (D) 5   (E) 41

49. Find the area of the circle. (nearest tenth)

    (A) 20.2 in^2 (B) 24.8 in^2 (C) 26.8 in^2 (D) 28.4 in^2 (E) 35.9 in^2

50. The circumcenter and the orthocenter are on the triangle for which of these types of triangles:
        1. Equilateral   2. Right   3. Acute   4. Obtuse

    (A) 2 only   (B) 1 & 2   (C) 3 & 4   (D) 1 only   (E) 1, 2, 3, & 4

51. Section 301 at the NASCAR race track has 10 seats in row 1, 11 seats in row 2, 12 seats in row 3, 13 seats in row 4, etc. There are 20 rows in section 301. How many seats are in section 301?

    (A) 400   (B) 390   (C) 375   (D) 350   (E) 325

52. Find the eccentricity of the ellipse $4x^2 + 6y^2 = 36$. (nearest hundredth)

    (A) 0.58   (B) 0.60   (C) 0.64   (D) 0.74   (E) 0.75

53. Captains Chip Ahoy and Rusty Nale leave port in their respective ships at 2 p.m. Chip's ship sails on a bearing of 40° traveling at a speed of 10 mph. Rusty's ship sails on a bearing of 140° traveling at a speed of 15 mph. How far apart will the ships be at 5 p.m.? (nearest mile)

    (A) 19 mi   (B) 52 mi   (C) 54 mi   (D) 58 mi   (E) 71 mi

54. The equation $\sec^2(\theta) + \cos^2(\theta) = 2$ can be simplified to the equation:

    (A) $\cos(\theta) = \sqrt{2}$ (B) $\sin^2(\theta) = -1$ (C) $\cos^2(\theta) = \frac{1}{2}$ (D) $\sin(\theta) = \sqrt{2}$ (E) $\cos^2(\theta) = 1$

55. In the binomial expansion of $(3x + 4)^5$, the coefficient of the 4th term minus the coefficient of the 2nd term is:

    (A) 1,260   (B) 2,220   (C) 2,520   (D) 4,140   (E) 4,320

56. Which of the following is true about the relation $h(x) = \sin(x)\cos(x)$?

    (A) even function   (B) odd function   (C) neither even nor odd function

    (D) not a function   (E) none of these are true

57. **Saul D. Would precut 40 identical poles to make 3-legged easels and 4-legged easels for art class. He earns $50 for each 3-legged easel and $60 for each 4-legged easel. The maximum amount of money he can earn using all 40 poles is:**

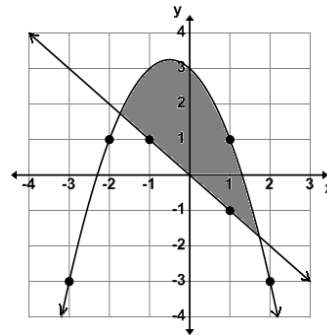   (A) $640.00   (B) $620.00   (C) $660.00   (D) $680.00   (E) $600.00

58. **What are the odds that a factor of the triangular number 300 is a triangular number?**

   (A) 5:13   (B) 5:11   (C) 3:5   (D) 1:2   (E) 1:3

59. **How many three digit numbers that are not divisible by five can be formed from the digits 0, 1, 2, 3, and 5 without repetition ?**

   (A) 27   (B) 25   (C) 32   (D) 36   (E) 50

60. **Find the area to the nearest hundredth of the shaded region in square units.**

   (A) 5.90   (B) 5.97   (C) 6.25   (D) 6.93   (E) 7.25

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Score   **Initials**

Contestant # _____

Conference _____

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

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9. _____

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58. _____

59. _____

60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2011
Answer Key

1. A	21. C	41. A
2. E	22. E	42. C
3. E	23. A	43. D
4. C	24. B	44. A
5. A	25. D	45. D
6. E	26. E	46. C
7. D	27. A	47. C
8. B	28. D	48. B
9. D	29. B	49. D
10. C	30. C	50. A
11. E	31. E	51. B
12. A	32. A	52. A
13. D	33. B	53. D
14. C	34. C	54. E
15. B	35. C	55. D
16. D	36. B	56. B
17. E	37. E	57. C
18. C	38. D	58. D
19. B	39. A	59. A
20. A	40. B	60. D