



UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

# Mathematics

## Invitational A • 2016



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1. Evaluate:  $1 - 8 \times 1 + 6 \div (1 \times 30) + 1 - 6$

- (A)  $-35$       (B)  $-11.8$       (C)  $-11.2$       (D)  $-8$       (E)  $-5.0333\ldots$

2. Max Spender had \$40.00 to buy school supplies. He bought six notebooks at \$2.75 each, three reams of paper at \$1.20 each, two 4-packs of highlighter pens at \$3.10 per pack, twelve pencils at 8¢ each, and six 3-color pens at 79¢ each. How much money did he have left?

- (A) \$4.17      (B) \$5.00      (C) \$6.57      (D) \$7.12      (E) \$8.00

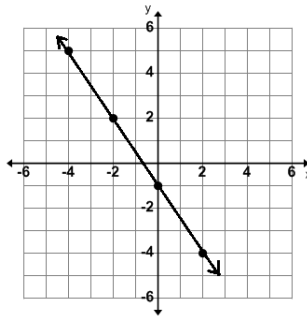
3. 45 miles per hour is the same speed as \_\_\_\_\_ inches per second.

- (A) 792      (B) 3,240      (C) 47,520      (D) 880      (E) 66

4. If  $P = \{p, l, a, t, o\}$ ,  $O = \{p, t, o, l, e, m, y\}$ , and  $E = \{e, u, c, l, i, d\}$  then  $(P \cup O) \cap E = ?$

- (A)  $\{\emptyset\}$       (B)  $\{p, o, e\}$       (C)  $\{e, l\}$       (D)  $\{p, o, e, m\}$       (E)  $\{p, l, o, t\}$

5. An equation for the line shown is:



- (A)  $3x - 2y = 1$       (B)  $2x - 3y = 2$       (C)  $x + y = -1$       (D)  $3x + 2y = -2$       (E)  $x - y = 1.5$

6. Which of the following relations describes a function?

- (A)  $\{(2,3), (3,3) (4,3) (5,3)\}$       (B)  $\{(-2,0), (0, -2) (0,2) (2,0)\}$       (C)  $\{(0,0), (2, -2) (2,2) (3,3)\}$   
(D)  $\{(3,3), (3,4) (3,5) (3,6)\}$       (E) none of these are a function

7. Multiply:  $(x - 4)(2x - 1)(3x + 2)$

- (A)  $6x^3 - 23x^2 - 6x + 8$       (B)  $6x^3 + 4x^2 + 12x + 8$       (C)  $6x^3 - 27x^2 - 18x + 8$   
(D)  $6x^3 + 6x^2 - 10x + 8$       (E)  $6x^3 - 31x^2 - 30x + 8$

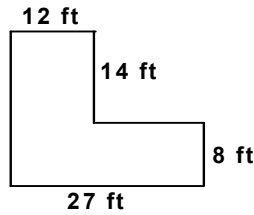
8. Betty Falls off weighs 100 pounds and is sitting on a seesaw 6.5 feet from the middle. Lotta Wait weighs 150 pounds. How far from the middle must Lotta sit in order to balance the seesaw?

- (A) 5 ft 6 in      (B) 4 ft 8 in      (C) 4 ft 4 in      (D) 4 ft 0 in      (E) 3 ft 3 in

9. Given:  $\angle P$  is complementary to  $\angle Q$ ;  $m\angle R = 135^\circ$ ; and  $\angle Q$  is supplementary to  $\angle R$ . Find  $m\angle P$ .

- (A)  $22.5^\circ$       (B)  $45^\circ$       (C)  $55^\circ$       (D)  $67.5^\circ$       (E)  $95^\circ$

10. Poly Gawn wants to carpet her L-shaped hexagonal room shown below. If she buys the least amount of whole yards of carpet to cover the room, what will the carpet cost if the cost is \$25.00 per square yard?



- (A) \$1,050.00    (B) \$3,200.00    (C) \$1,075.00    (D) \$1,066.67    (E) \$2,450.00
11. Sandy Shore is ordering single topping pizzas for her beach party. The menu shows:  
size: 8", 12", or 16"  
crust: thin, thick, hand tossed, or cheese filled  
toppings: cheese, pepperoni, mushrooms, onions, or Jalapenos  
How many different pizzas could she order?
- (A) 12    (B) 15    (C) 20    (D) 45    (E) 60
12. Jay R. Lington has 10 students in his advanced science club. He wants to arrange them into 6-member teams for UIL competition. How many 6-member teams can he arrange?
- (A) 210    (B) 144    (C) 60    (D) 5,040    (E) 24
13. Which of the following prime numbers are considered to be Mersenne primes?  
I. 3    II. 7    III. 13    IV. 17
- (A) I only    (B) II only    (C) I & II    (D) I & III    (E) II & IV
14. Simplify:  $(a)^{\left(\frac{(a^{-3})^3(\sqrt{a})}{a^{-3}}\right)}$
- (A)  $(\sqrt{a})^{-11}$     (B)  $(\sqrt{a})^{-9}$     (C)  $(\sqrt{a})^{-8}$     (D)  $(\sqrt{a})^3$     (E)  $(\sqrt{a})^4$
15. Let  $2a - 5b = 8$  and  $3b = 4a + 6$ . Find  $a - b$ .
- (A)  $-7$     (B)  $-3\frac{6}{7}$     (C)  $-\frac{5}{7}$     (D) 3    (E)  $3\frac{1}{7}$
16. Alexis, Ayleen, and Angel can paint Ed Inberg's house in 2 days working together. When working alone Alexis can paint the house in 6 days. Ayleen can paint the house in 4 days when working alone. How long would it take Angel to paint the house working alone?
- (A) 12 days    (B) 10 days    (C) 8 days    (D) 6 days    (E) 4 days
17. Determine the frequency of  $f(\theta) = 5 + 4\cos\left(\frac{3\pi}{2}\theta - 1\right)$ .
- (A)  $\frac{2}{3}$     (B)  $\frac{3}{4}$     (C) 3    (D)  $1\frac{1}{3}$     (E)  $1\frac{1}{2}$

18. Otis got on the elevator on the first floor with four other people. Two people got off on the third floor and four got on. One got off on the fifth floor and 3 got on. Otis and five others got off on the seventh floor and one got on. The remaining people got off on the ninth floor. How many got off on the ninth floor?

(A) 1                      (B) 4                      (C) 5                      (D) 7                      (E) 9

19. Which of the following is a reference angle for  $150^\circ$  ?

(A)  $\frac{\pi}{2}$                       (B)  $\frac{\pi}{3}$                       (C)  $\frac{\pi}{4}$                       (D)  $\frac{\pi}{5}$                       (E)  $\frac{\pi}{6}$

20. Given that the set of natural numbers continue in the triangular pattern shown below, find the 4<sup>th</sup> term in row 6.

			2				(row 1)
		4	6	8			(row 2)
	10	12	14	16	18		(row 3)
20	22	24	26	28	30	32	(row 4)
			...				( ... )

(A) 54                      (B) 58                      (C) 62                      (D) 64                      (E) 68

21. Find the first term of the arithmetic sequence: a, b, 33, c, d, 45, ...

(A) 13                      (B) 17                      (C) 20                      (D) 25                      (E) 27

22. The sum of the coefficients of the 2<sup>nd</sup> term and the 4<sup>th</sup> term in the expansion of  $(x + 2)^5$  is:

(A) 10                      (B) 20                      (C) 40                      (D) 80                      (E) 90

23. Let  $f(x) = x^2 - 2x - 8$  and  $g(x) = 3x + 1$ . Find  $(f \circ g)(-1)$ .

(A) 0                      (B) 14                      (C) -5                      (D) 3                      (E) -14

24. If  $y = x^3 - 3x^2 - 9x + 7$  has an inflection point at (x,y) then y equals \_\_\_\_\_.

(A) -6                      (B) -4                      (C) -3                      (D) 1                      (E) 5

25. Let  $f(x) = x^3 - 2x^2 - x + 2$ . Find  $f'(3)$ .

(A) 6                      (B) 8                      (C) 14                      (D) -3                      (E) -40

26. A candy bag contains red, orange, green, and blue sour balls. There are 12 red, 6 orange, 21 green and 18 blue sour balls in the bag. What are the odds of reaching in the bag and randomly pulling out a sour ball that is neither red or green?

(A)  $\frac{11}{19}$                       (B)  $\frac{8}{13}$                       (C)  $\frac{8}{11}$                       (D)  $\frac{11}{13}$                       (E)  $\frac{8}{19}$

27. If the pattern of the sequence 1, 3, 7, 13, 21, 31, ... continues, find the 21<sup>st</sup> term.

- (A) 413                      (B) 417                      (C) 419                      (D) 421                      (E) 441

28.  $[(1 + 2 + 3 + 4 + \dots + 38 + 39) \times 40] \div [(41 + 42 + 43 + 44 + \dots + 78 + 79) \times 80] = ?$

- (A)  $\frac{1}{1600}$                       (B)  $\frac{1}{40}$                       (C)  $\frac{1}{6}$                       (D)  $\frac{1}{4}$                       (E)  $\frac{1}{2}$

29. How many proper fractions in lowest terms have a denominator of 18?

- (A) 9                      (B) 8                      (C) 7                      (D) 6                      (E) 5

30.  $101110_2 + 10111_2 + 1101_2 = \underline{\hspace{2cm}}_8$

- (A) 103                      (B) 112                      (C) 122                      (D) 376                      (E) 443

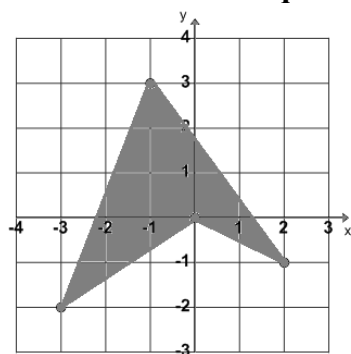
31. Set  $F = \{\dots, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, \dots\}$  is a recursive sequence. Set F is closed under which of the following operation? (A)ddition (S)ubtraction (M)ultiplication (D)ivision

- (A) A                      (B) M                      (C) A & M                      (D) all of them                      (E) none of them

32. Find the greatest common divisor of  $2^3 \times 3^2 \times 5$ ,  $2^2 \times 3 \times 5^3$ , and  $2^3 \times 3^3 \times 5$ .

- (A) 10                      (B) 15                      (C) 30                      (D) 45                      (E) 60

33. Monsieur De Kartz drew this quadrilateral on the coordinate plane below. The coordinates of the vertices are integers. What is the area of his quadrilateral?



- (A) 7.5 units<sup>2</sup>                      (B) 7.75 units<sup>2</sup>                      (C) 8 units<sup>2</sup>                      (D) 8.25 units<sup>2</sup>                      (E) 8.5 units<sup>2</sup>

34. Which of the following are the side lengths of a scalene obtuse triangle?

- (A) 7, 10, 13                      (B) 7, 7, 12                      (C) 7, 8, 9                      (D) 23, 24, 25                      (E) 7, 24, 25

35. A square has sides that measure 15 cm. A rectangle has a length of 18 cm. The perimeter of the square is equal to the perimeter of the rectangle. What is the area of the rectangle?

- (A) 108 cm<sup>2</sup>                      (B) 112 cm<sup>2</sup>                      (C) 135 cm<sup>2</sup>                      (D) 162 cm<sup>2</sup>                      (E) 216 cm<sup>2</sup>

36. Find  $g(h(-1))$  when  $g(x) = 2x - 5$  and  $h(x) = 4x - 3$ .
- (A)  $-1$       (B)  $-3$       (C)  $-14$       (D)  $-19$       (E)  $-31$
37. If  $a_1 = 1$ ,  $a_2 = -1$  and  $a_n = (a_{n-1})(a_{n-2}) - (a_{n-2})$  for  $n \geq 3$ , then  $a_5$  equals:
- (A)  $-8$       (B)  $-4$       (C)  $-2$       (D)  $-1$       (E)  $0$
38. The circle,  $x^2 + y^2 - 6x + 8y - 11 = 0$  has radius  $r$  and center  $(h, k)$ . Find  $r + h + k$ .
- (A)  $35$       (B)  $24$       (C)  $13$       (D)  $6$       (E)  $5$
39. If  $\sin x \cos x = -\frac{\sqrt{3}}{4}$ , then  $\cos 2x =$  \_\_\_\_\_.
- (A)  $-\sqrt{2}$       (B)  $-\frac{1}{2}$       (C)  $\frac{\sqrt{3}}{2}$       (D)  $\frac{1}{2}$       (E)  $\frac{\sqrt{2}}{2}$
40. Annie Ware sails 120 kilometers from Sumplace to Sumwear on a bearing of  $240^\circ$ . Then she sails 80 kilometers from Sumwear to Knoplace on a bearing of  $100^\circ$ . How far is Knoplace from Sumplace? (nearest kilometer)
- (A) 65 km      (B) 72 km      (C) 78 km      (D) 92 km      (E) 119 km
41. If  $P = \begin{bmatrix} -2 & -3 \\ 5 & 7 \end{bmatrix}$  then  $P^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Find  $(a + b) - (c + d)$ .
- (A)  $-85$       (B)  $-61$       (C)  $-58$       (D)  $-33$       (E)  $-29$
42. Let  $x^5 + x^4 - px^3 + qx^2 - x + 1 = 0$ , where  $p, q > 0$ . According to Descartes' Rule of Signs, how many possible positive roots are there?
- (A) 3, 1, or 0      (B) 1 or 0      (C) 4, 2, or 0      (D) 2 or 0      (E) 0
43. Let  $f(x) = 2x^2 - 3x - 5$ ,  $g(x) = x + 1$ , and  $S(x)$  be the slant (oblique) asymptote of  $\frac{f(x)}{g(x)}$ . Find  $S(4)$ .
- (A) 20      (B) 15      (C) 5      (D) 3      (E) 0
44. The graph of  $y = x^2 - 10x$  has a vertex at  $(5, -25)$ . Find the area of the region bounded by the  $x$ -axis and the graph of  $y$ .
- (A)  $55\frac{1}{2}$       (B)  $93\frac{3}{4}$       (C) 125      (D)  $166\frac{2}{3}$       (E) 200
45. The function  $f(x) = \frac{x^2}{4x + 4}$  is decreasing over which of the following intervals?
- (A)  $[0, 4]$       (B)  $[-4, -2]$       (C)  $[-3, -1]$       (D)  $[-4, -1]$       (E)  $(-1, 0]$

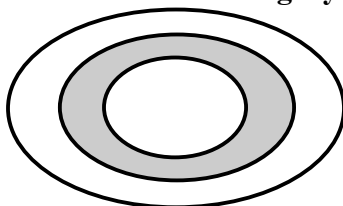
46. Juan N. Itself randomly selected two different numbers from the set  $\{2, 3, 5, 7, 11, 13\}$ . What is the probability that the sum of the two numbers selected is a composite number?

- (A) 0%                      (B) 20%                      (C) 25%                      (D) 40%                      (E) 80%

47. Nicole Moore had some pennies, nickels, dimes, and quarters. She put them in stacks of 4 coins. What is the maximum number of different stacks could she make?

- (A) 16                      (B) 28                      (C) 32                      (D) 35                      (E) 70

48. The diameter of the smallest circle is 9 cm. The diameter of the middle circle is 15 cm. The diameter of the largest circle is 21 cm. Robin D. Hood throws a dart that sticks in the target. What is the probability that the dart sticks in the gray area of the target? (nearest percent)



- (A) 33%                      (B) 30%                      (C) 25%                      (D) 16%                      (E) 13%

49. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number, and  $P_n$  be the  $n$ th pentagonal number. Then  $P_3 + S_3$  has the same value as:

- (A)  $P_9$                       (B)  $T_3$                       (C)  $P_6$                       (D)  $T_6$                       (E)  $S_9$

50. How many positive 3-digit numbers exist such that the sum of their digits equals 11?

- (A) 54                      (B) 61                      (C) 64                      (D) 65                      (E) 66

51. Let  $f_0 = 0$ ,  $f_1 = 1$ ,  $f_2 = 1$ ,  $f_3 = 2$ ,  $f_4 = 3$ , ... be the terms of the Fibonacci sequence. Find  $f_{20}$ .

- (A) 6,765                      (B) 9,349                      (C) 5,168                      (D) 4,181                      (E) 10,946

52.  $\{(x, y) \mid x, y \in \{\text{Integers}\}, -3 \leq x \leq 3, \text{ and } -4 \leq y \leq 4\}$  is the solution set of  $3x + 4y = 5$ . How many such ordered pairs exist?

- (A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4

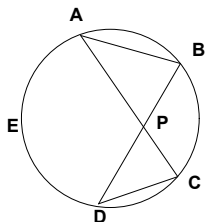
53. The point  $(5, 4)$  lies on a circle whose center is  $(0, 3)$ . Where does the point  $(0, -3)$  lie in reference to the circle?

- (A) on the circle                      (B) outside the circle                      (C) inside the circle  
(D) in quadrant III                      (E) cannot be determined

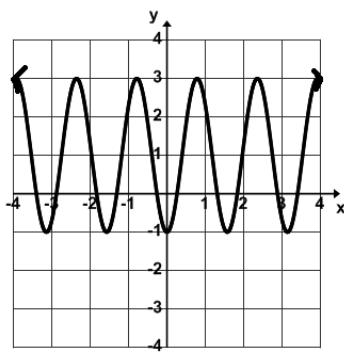
54. Solve:  $2\log_2(4) + \log_2(x) = 5$ .

- (A)  $\sqrt{2}$                       (B) 2                      (C) 2.5                      (D) 5.2                      (E)  $\sqrt{5}$

55.  $\overline{AB}$ ,  $\overline{AC}$ ,  $\overline{BD}$ , and  $\overline{CD}$  are chords of circle O. Find  $m\widehat{CD}$  if  $m\widehat{AB} = 60^\circ$  and  $m\angle APB = 40^\circ$ .



- (A)  $50^\circ$       (B)  $40^\circ$       (C)  $30^\circ$       (D)  $20^\circ$       (E)  $10^\circ$
56. The equation  $y = \underline{\hspace{2cm}}$  will produce this graph.



- (A)  $\cos(4x + \pi) - 2$       (B)  $2\cos(4x - \pi) + 1$       (C)  $\cos(2x - \pi) + 4$   
 (D)  $2\sin(4x - \pi) + 1$       (E)  $\sin(4x + \pi) - 2$
57. Which of the following statements about  $f(x) = x^2 - x$  is/are true?  
 I.  $f(x)$  is a function.  
 II.  $f(x)$  is a one to one function on its domain.  
 III.  $f(x)$  has an inverse function on its domain.
- (A) I & II      (B) I only      (C) I & III      (D) all of them      (E) none of them

58. Which of the following statements about  $f(x) = \sqrt{x - 1}$  is/are true?

- I.  $f(x)$  is defined at 1      II.  $\lim_{x \rightarrow 1} f(x)$  exists  
 III.  $f(x)$  is continuous at 1      IV.  $f(x)$  is differentiable at 1

- (A) I, II, & III      (B) I & III      (C) II & III      (D) I only      (E) none of them
59. The repeating decimal  $0.3414141\dots$  in base 5 can be written as which of the following fractions in base 5 in simplified form?

- (A)  $\frac{131}{220}_5$       (B)  $\frac{151}{220}_5$       (C)  $\frac{111}{130}_5$       (D)  $\frac{101}{110}_5$       (E)  $\frac{302}{440}_5$

60. The length of the sides of  $\triangle PQR$  are the roots of  $f(x) = x^3 - 12x^2 + 47x - 60$ . The perimeter of  $\triangle PQR$  is 12. Find the area  $\triangle PQR$ . (nearest tenth unit)

- (A) 3.0      (B) 7.5      (C) 10.0      (D) 3.5      (E) 6.0



University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final \_\_\_\_\_  
2nd \_\_\_\_\_  
1st \_\_\_\_\_  
Score \_\_\_\_\_  
Initials \_\_\_\_\_

Contestant # \_\_\_\_\_ Conference \_\_\_\_\_

- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitation A • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. B  | 21. D | 41. A |
| 2. E  | 22. E | 42. C |
| 3. A  | 23. A | 43. D |
| 4. C  | 24. B | 44. D |
| 5. D  | 25. C | 45. E |
| 6. A  | 26. C | 46. E |
| 7. A  | 27. D | 47. D |
| 8. C  | 28. C | 48. A |
| 9. B  | 29. D | 49. D |
| 10. C | 30. C | 50. B |
| 11. E | 31. E | 51. A |
| 12. A | 32. E | 52. C |
| 13. C | 33. C | 53. B |
| 14. A | 34. A | 54. B |
| 15. C | 35. E | 55. D |
| 16. A | 36. D | 56. B |
| 17. B | 37. B | 57. B |
| 18. B | 38. E | 58. B |
| 19. E | 39. D | 59. C |
| 20. B | 40. C | 60. E |



UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

# Mathematics

## Invitational B • 2016



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1. Evaluate:  $(18 + 11) \times 7 \div 4 + 3 \div (1 - 2)$

- (A)  $-53.7$       (B)  $-37.25$       (C)  $25.75$       (D)  $47.75$       (E)  $51.7$

2. Willie Reedem bought 4 novels at the discount book store. He got one at the regular price, the second one at 20% off, the third one was  $\frac{1}{3}$  off, and \$2.50 was taken off the regular price for the fourth one. What would the total cost of the 4 novels be before taxes if the regular price of the books were \$6.00? (to the nearest cent)

- (A) \$18.30      (B) \$ 16.20      (C) \$ 13.50      (D) \$12.30      (E) \$11.70

3. 150 revolutions per hour = \_\_\_\_\_ degrees per second.

- (A) 900      (B) 90      (C) 15      (D) 10      (E) 1.5

4. In which of the following Venn diagrams does the shaded regions represent the set  $(B \cap C) \cup (A \cap C)$ ?



5. Use the table of values to create a function to find K.

X	1	2	3	4	...	15	...
Y	2	5	8	11	...	K	...

- (A) 46      (B) 44      (C) 35      (D) 28      (E) 22

6. If  $y = 3x - 1$ ,  $2x + y = 4$  and  $ax + y = 5$ , then a equals:

- (A) 7      (B) 6      (C) 5      (D) 4      (E) 3

7. Nicole Saver has some dimes and some nickels in a jar. The number of dimes she has is three less than twice the number of nickels she has. The monetary value of her nickels and dimes is \$7.20. What is the monetary value of her dimes?

- (A) \$3.00      (B) \$6.30      (C) \$5.70      (D) \$4.20      (E) \$1.50

8. Which of the following mathematicians is one of the few American Indians who received a Ph.D. in mathematics?

- (A) Sophie Germain      (B) Marin Mersenne      (C) Emmy Noether  
(D) Grace Williams      (E) Freda Porter

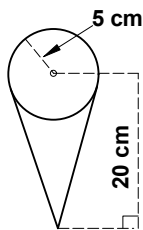
9. Solve for x in terms of y:  $\frac{4y}{5} - \frac{2}{3x} = \frac{y}{6}$

- (A)  $\frac{20}{19y}$  (B)  $3.8y$  (C)  $\frac{19}{20y}$  (D)  $1.05y$  (E)  $\frac{5}{19y}$

10.  $(21216g + 31216g) \times 4g = \underline{\hspace{2cm}} 8$

- (A) 210156 (B) 211730 (C) 211736 (D) 250150 (E) 252160

11. Find the total surface area, nearest square cm, of the cone with the top on.



- (A)  $247 \text{ cm}^2$  (B)  $283 \text{ cm}^2$  (C)  $314 \text{ cm}^2$  (D)  $325 \text{ cm}^2$  (E)  $402 \text{ cm}^2$

12. Herr Oiler's polyhedron has 5 faces and 9 edges. How many vertices does it have?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

13.  $\angle A$  and  $\angle B$  are complementary angles.  $\angle A$  and  $\angle C$  are supplementary angles. Find  $m\angle B$  if  $m\angle C = 2x + 8$  and  $m\angle A = x - 3$ .

- (A)  $58\frac{1}{3}^\circ$  (B)  $55\frac{1}{3}^\circ$  (C)  $37\frac{2}{3}^\circ$  (D)  $34\frac{2}{3}^\circ$  (E)  $31\frac{2}{3}^\circ$

14. Trey Scidid cut a flat, narrow board into four pieces. The lengths of the pieces were 11", 8", 7", and 5". How many triangles could Trey make using only three pieces at a time?

- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

15. If  $\frac{x-5}{x+3} - \frac{4x}{x+2} = \frac{ax^2+bx+c}{px^2+qx+r}$ , then  $(a + b + c) - (p + q + r)$  equals:

- (A)  $-40$  (B)  $-28$  (C)  $-18$  (D)  $-14$  (E)  $-12$

16. Points  $(2, -3)$  and  $(-3, 1)$  are members of the function  $\{(x, y) \mid y = ax - b\}$ . Find  $a + b$ .

- (A) 2.2 (B) 1.4 (C) 0.8 (D) 0.6 (E)  $-0.2$

17. Les Speed leaves the Van Horn truck stop on I10 heading east at 8:00 a.m. at an average speed of 60 miles per hour. Willie Ketchum leaves the same truck stop on I10 heading east at 9:30 a.m. at an average speed of 75 miles per hour. How far from the truck stop will they be when Willie catches Les?

- (A) 750 miles (B) 450 miles (C) 225 miles (D) 135 miles (E) 112.5 miles

18. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots, f_k = 6,765, \dots$  be the terms of the Fibonacci sequence. Find  $f_{k+1}$ .

(A) 8,383      (B) 8,856      (C) 10,946      (D) 11,912      (E) 13,237

19.  $\cos(\frac{3\pi}{2} + \theta)$  equals:

(A)  $\cos(\theta - \frac{\pi}{2})$     (B)  $\sin(\theta - \frac{3\pi}{2})$     (C)  $-\cos(\theta + \frac{\pi}{2})$     (D)  $-\sin(\theta + \frac{\pi}{2})$     (E)  $\cos(\frac{\pi}{2} + \theta)$

20. Let  $4\sin^2(x) + 2\cos(x) = 4$ . How many solutions are there if  $-\frac{5\pi}{4} < x < \frac{7\pi}{4}$ ?

(A) 2      (B) 3      (C) 4      (D) 5      (E) 6

21. Which of the following statements about  $f(x) = \sqrt{x+3} - 1$  is/are true?

- I.  $f^{-1}(x)$  is a function.  
 II. the domain of  $f(x)$  is  $\{x|x \geq -3\}$ .  
 III. the domain of  $f^{-1}(x)$  is  $\{x|x \geq -1\}$ .

(A) I & II      (B) I only      (C) I & III      (D) all of them      (E) none of them

22. Given that the set of natural numbers continue in the triangular pattern shown below, find the 9th number in row 10.

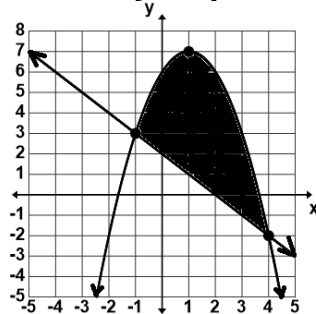
			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				( ... )

(A) 98      (B) 96      (C) 94      (D) 92      (E) 90

23. Let  $A = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$ . Find  $|(A+B)^T|$ .

(A) 2      (B) 1      (C) 0      (D) -1      (E) -2

24. Find the area of the region bounded by the parabola and the line. (square units).



(A)  $21\frac{1}{3}$       (B)  $20\frac{5}{6}$       (C) 20      (D)  $18\frac{2}{3}$       (E)  $16\frac{1}{2}$

25. Let  $f(x) = 5x^3 - 3x^2 + 2x + 20$ . Find  $f'(-1)$ .

- (A) 10                      (B) 11                      (C)  $-7$                       (D) 23                      (E)  $-19$

26. Find  $\lim_{x \rightarrow 2} \frac{2x^2 - 4x}{x^2 - 5x + 6}$ .

- (A)  $-4$                       (B)  $-2$                       (C) 0                      (D) 2                      (E) does not exist

27. The Two-Squared 4-H club has 5 senior students, 3 juniors, 2 sophomores and 6 freshmen. A group of 6 students will attend the state convention. Each group has to have 2 seniors, 1 junior, 1 sophomore, and 2 freshmen. How many different groups of 6 students can be formed?

- (A) 30                      (B) 150                      (C) 900                      (D) 3,600                      (E) 8,008

28. What are the odds of randomly selecting a letter M or a letter T from the letters in the word "MATHEMATICS"?

- (A)  $\frac{2}{9}$                       (B)  $\frac{4}{7}$                       (C)  $\frac{1}{2}$                       (D)  $\frac{4}{11}$                       (E)  $\frac{2}{7}$

29. Evaluate:  $(0.777\ldots)^{-1} \div (0.08333\ldots)^{-1} \times (0.666\ldots)^{-1}$

- (A)  $6\frac{2}{9}$                       (B)  $1\frac{41}{42}$                       (C)  $\frac{1}{162}$                       (D)  $\frac{42}{83}$                       (E)  $\frac{9}{56}$

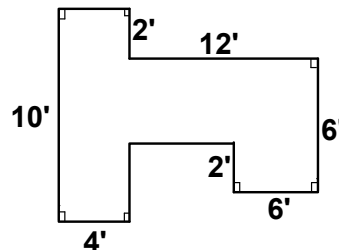
30. Find the number of positive integral divisors of 420.

- (A) 42                      (B) 36                      (C) 30                      (D) 24                      (E) 12

31. If  $x - y = 2$  and  $xy = 4$  then  $x^3 - y^3 = ?$

- (A) 512                      (B) 32                      (C) 76                      (D)  $-56$                       (E)  $-16$

32. Polly Scidid wants to paint this decagon red. A pint of paint will cover 16 square feet. How many pints will she need to paint the decagon?

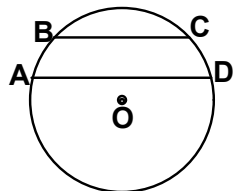


- (A) 3 pints                      (B) 5 pints                      (C) 6 pints                      (D) 7 pints                      (E) 9 pints

33. If  $\log_4(x - 4) + \log_4(x) = \log_4(12)$ , then  $x$  equals \_\_\_\_\_.

- (A) 2                      (B) 4                      (C) 6                      (D) 8                      (E) 10

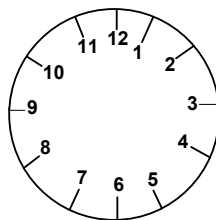
34. If  $a_1 = -4$ ,  $a_2 = -1$ ,  $a_3 = 2$ , and  $a_n = [(a_{n-3})(a_{n-2})] - (a_{n-1})$ , where  $n \geq 4$ , then  $a_8 = ?$
- (A) 4                      (B) 8                      (C)  $-8$                       (D)  $-16$                       (E)  $-32$
35. The circle,  $x^2 + y^2 - 4x - 8y - 5 = 0$  has radius  $r$  and center  $(h, k)$ . Find  $r + h + k$ .
- (A)  $6 + \sqrt{15}$                       (B) 9                      (C)  $2 + \sqrt{21}$                       (D)  $6 + \sqrt{5}$                       (E) 11
36. In the binomial expansion of  $(2x + 5)^6$ , the coefficient of the fourth term is:
- (A) 1,000                      (B) 343                      (C) 20,000                      (D) 4,096                      (E) 200
37. Find  $a + b + c + d$  given the Fibonacci characteristic sequence:  $a, b, -2, c, 3, d, \dots$
- (A) 19                      (B) 16                      (C) 15                      (D) 12                      (E) 11
38. Let 4,  $a$ , 18 be the first three members of an arithmetic sequence and 18,  $b$ , 32 be the first three members of a geometric sequence. Find  $a + b$ .
- (A) 13                      (B) 24                      (C) 35                      (D) 37                      (E) 46
39. The directrix of the parabola  $y = 2x^2 + 4x - 3$  is:
- (A)  $y = -5.125$                       (B)  $y = -4.875$                       (C)  $y = -4.125$                       (D)  $y = -0.875$                       (E)  $y = -1.125$
40. Which of the following are considered to be *transcendental functions*?
- I.  $y = \ln(x)$                       II.  $y = e^x$                       III.  $y = \csc(x)$
- (A) I only                      (B) II only                      (C) I & II                      (D) II & III                      (E) I, II, & III
41. How many positive 3-digit numbers exist such that the sum of their digits equals 10?
- (A) 54                      (B) 55                      (C) 57                      (D) 58                      (E) 61
42. Given:  $\odot O$ ;  $\overline{AD} \parallel \overline{BC}$ ; , measure of minor arc  $AB = 33^\circ$ ; and measure of minor arc  $AD = 150^\circ$ . Find the measure of minor arc  $BC$ .



- (A)  $117^\circ$                       (B)  $84^\circ$                       (C)  $75^\circ$                       (D)  $66^\circ$                       (E)  $51^\circ$



43. A recipe for butternut ice cream requires  $2\frac{1}{4}$  cups of cream and serves 5 people. How many cups of cream would be needed to make enough for 8 people?
- (A) 3.75      (B) 3.6      (C) 3      (D) 2.85      (E) 2.5
44. I.C. Delite is in his boat in the bay. He wants to find the distance from the dock to the lighthouse. The measure of the angle formed from the dock to the boat to the lighthouse is  $30^\circ$ . The measure of the angle formed from the boat to the dock to the lighthouse is  $100^\circ$ . What is the distance from the dock to the lighthouse if the boat is a mile from the dock? (nearest foot).
- (A) 1,834 ft      (B) 2,681 ft      (C) 3,446 ft      (D) 2,599 ft      (E) 3,772 ft
45. A heavy duty fire hose can fill a fire truck tank in 3 hours. A smaller hose takes 5 hours to fill the tank. The tank can be emptied using a spray hose in 8 hours. If the tank is empty and all 3 hoses are running, how long will it take to fill up the tank? (nearest minute)
- (A) 3 hrs 23 min      (B) 2 hrs 27 min      (C) 2 hrs 30 min      (D) 2 hrs 6 min      (E) 1 hr 31 min
46. Miss Dee Odds is demonstrating a probability experiment by flipping two coins then rolling one die. She writes the sample space on the board. How many outcomes are there in the sample space?
- (A) 64      (B) 48      (C) 32      (D) 24      (E) 12
47. A line crosses the x-axis at  $x = -1$  and goes through the point  $(-3, -3)$ . Another line crosses the y-axis at  $y = 6$  and goes through the point  $(3, -3)$ . The lines intersect at  $(x, y)$ . Find  $x + y$ .
- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5
48. Will Tern can't remember the combination to the padlock shown. He knows that the first number is a Fibonacci number, the second number is a triangular number, and the third is a prime number. How many combinations can he try to open the lock?



- (A) 125      (B) 100      (C) 80      (D) 75      (E) 14
49.  $\{(x, y) \mid x, y \in \{\text{Integers}\}, -2 \leq x \leq 5, \text{ and } -3 \leq y \leq 5\}$  is the solution set of  $2x + 3y = 5$ . How many such ordered pairs exist?
- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

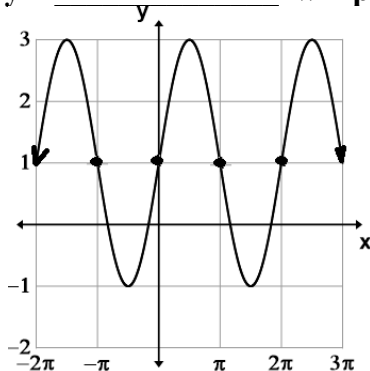
50. Point P(1, 3) lies on the x-y plane. Point P is rotated  $90^\circ$  counter clockwise about the origin to point Q. Point Q is translated horizontally 2 units to the left to point R. Point R is reflected across the line  $y = x$  to point S. The coordinate of S is (x, y). Find  $x + y$ .

(A)  $-6$       (B)  $-5$       (C)  $-4$       (D)  $2$       (E)  $4$

51. A bag of jelly beans had 10 licorice ones, 4 lemon ones, 8 lime ones, and 5 mint ones. Three beans are randomly taken from the bag without replacement. What is the probability that the first two are licorice and the third one is not licorice? (nearest percent)

(A) 46%      (B) 37%      (C) 260%      (D) 9%      (E) 8%

52. The equation  $y = \underline{\hspace{2cm}}$  will produce this graph.



(A)  $2\sin(x) + 1$     (B)  $2\cos(x) - 1$     (C)  $1 - 2\sin(x)$     (D)  $2\cos(x) + 1$     (E)  $2\sin(x) - 1$

53. Using the equation  $y = \frac{2}{3}\cos(6x + \frac{\pi}{2}) + \frac{3}{4}$  which of the following has the largest numeric value?

(A) amplitude    (B) displacement    (C) frequency    (D) period    (E) phase shift

54. The graph of  $h(x) = \frac{x^2 + x - 6}{x - 2}$  suggests that the *discontinuity* at  $x = 2$  is *removable* by defining  $h(2)$  to be \_\_\_\_\_.

(A)  $-2$       (B)  $-3$       (C)  $2$       (D)  $3$       (E)  $5$

55. The geometric mean of 2016 and 6102 is \_\_\_\_\_ % of the arithmetic mean of 2016 and 6102. (nearest whole %)

(A) 86      (B) 33      (C) 140      (D) 75      (E) 116

56. Which of the following statements about  $f(x) = \frac{2x-3}{x-4}$  is/are true?

I.  $f(x)$  is defined at 4      II.  $\lim_{x \rightarrow 4} f(x)$  exists  
 III.  $f(x)$  is continuous at 4      IV.  $f(x)$  is differentiable at 4

(A) I, II, & III    (B) I & III    (C) II & III    (D) IV only    (E) none of them

57. The repeating decimal  $0.4131313\dots$  in base 6 can be written as which of the following fractions in base 10 in simplified form?

- (A)  $\frac{51}{110}$       (B)  $\frac{23}{35}$       (C)  $\frac{149}{210}$       (D)  $\frac{31}{42}$       (E)  $\frac{415}{550}$

58. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number, and  $P_n$  be the  $n$ th pentagonal number. Then  $T_9 + S_5$  has the same value as:

- (A)  $P_4$       (B)  $T_{14}$       (C)  $S_7$       (D)  $T_7$       (E)  $P_7$

59. The length of the sides of  $\triangle PQR$  are the roots of  $f(x) = x^3 - 8x^2 + 21x - 18$ . Find the area of  $\triangle PQR$ . (nearest tenth unit<sup>2</sup>)

- (A) 2.8      (B) 3.0      (C) 3.3      (D) 4.2      (E) 4.5

60. Find the sample standard deviation of this set of numbers  $\{29, 48, 73\}$ . (nearest tenth)

- (A) 22.1      (B) 23.3      (C) 24.0      (D) 26.7      (E) 28.2

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
--------------------	------------------

- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitation B • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. D  | 21. D | 41. A |
| 2. A  | 22. E | 42. B |
| 3. C  | 23. A | 43. B |
| 4. B  | 24. B | 44. C |
| 5. B  | 25. D | 45. B |
| 6. E  | 26. A | 46. D |
| 7. C  | 27. C | 47. D |
| 8. E  | 28. B | 48. B |
| 9. A  | 29. E | 49. C |
| 10. E | 30. D | 50. C |
| 11. E | 31. B | 51. D |
| 12. C | 32. D | 52. A |
| 13. D | 33. C | 53. E |
| 14. A | 34. D | 54. E |
| 15. A | 35. E | 55. A |
| 16. D | 36. C | 56. E |
| 17. B | 37. E | 57. C |
| 18. C | 38. C | 58. E |
| 19. A | 39. A | 59. A |
| 20. E | 40. E | 60. A |



UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

# Mathematics

## District 1 • 2016



DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $3 + 2 \times (1 - 3) \div (2 - 6) \times 20 + 16$

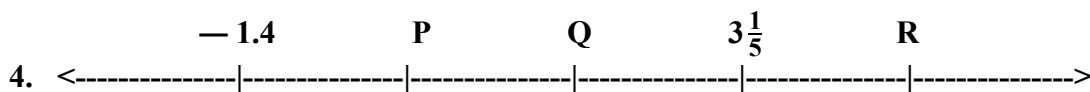
- (A)  $-84$       (B)  $-1$       (C)  $24$       (D)  $39$       (E)  $66$

2. Lotta Dough had \$150.00 for her Spring Break trip. She spent 35% of her money on shorts and tank tops,  $\frac{1}{10}$  of her money on sandals, and \$7.50 on a hat. She needed \$30.00 for gas. How much spending money did she have left?

- (A) \$40.75      (B) \$42.50      (C) \$45.00      (D) \$47.75      (E) \$50.25

3. If  $B = \{b, y, r, o, n\}$ ,  $C = \{c, a, n, t, o, r\}$ , and  $N = \{n, a, p, i, e, r\}$ , then  $(C \cap N) \cup B$  contains how many distinct elements?

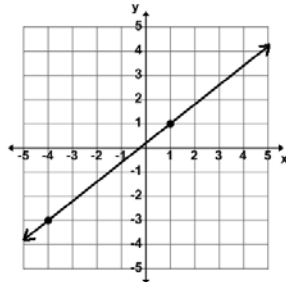
- (A) 6      (B) 5      (C) 4      (D) 2      (E) 1



The distances between the hash marks ( | ) are equal. Find  $P + Q + R$ .

- (A)  $6\frac{1}{30}$       (B) 6.1      (C)  $6\frac{2}{15}$       (D) 6.3      (E)  $6\frac{8}{15}$

5. Find an equation of the line perpendicular to the line shown and through the point (1, 1)?



- (A)  $4x + 5y = 9$     (B)  $5x - 4y = -9$     (C)  $5x + 4y = -9$     (D)  $4x + 4y = -9$     (E)  $5x + 4y = 9$

6. Simplify:  $\left(\frac{x^2 - 3x - 4}{x^2 + 2x - 3}\right) \div \left(\frac{x^2 - 5x + 4}{x^2 + 4x + 3}\right)$

- (A) 1      (B)  $x^2 - 2x - 1$       (C)  $x^2 + 2x + 1$       (D)  $\frac{x^2 + 2x + 1}{x^2 - 2x + 1}$       (E)  $\frac{x + 1}{x - 1}$

7. The Jolly Trolley was empty when Fran Sisco and six others got on at the first stop. Three got on at the second stop and two got off. Four got on at the third stop and five got off. Two got on at the fourth stop. Fran got off with three others at the fifth stop. How many rode to the next stop?

- (A) 3      (B) 4      (C) 5      (D) 6      (E) 7

8.  $[(1 + 2 + 3 + 4 + \dots + 16 + 17) \times 18] \div [(19 + 20 + 21 + 22 + \dots + 34 + 35) \times 36] = ?$

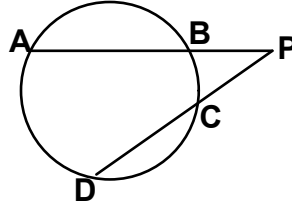
- (A)  $\frac{1}{54}$       (B)  $\frac{1}{6}$       (C)  $\frac{1}{18}$       (D)  $\frac{1}{648}$       (E)  $\frac{1}{2}$

9. Which of the following relations describes a function?

- I.  $\{ (2,1), (3,2) (4,3) (5,4) \}$
- II.  $\{ (-1,1), (2, -2) (1, -1) (0,0) \}$
- III.  $\{ (1,1) (1,2) (1,3) (1,4) \}$

(A) I only      (B) III only      (C) I & II      (D) II & III      (E) I & III

10. Points A, B, C, and D lie on the circle shown. Find  $x$  if  $AB = x$ ,  $BP = 6''$ ,  $DC = x + 4$ , and  $CP = 5''$ . (drawing is not to scale)



(A) 20"      (B) 11"      (C) 9"      (D) 3.5"      (E) 3"

11. M. T. Tank's cylindrical water tank has a 5 feet diameter and is 8 feet tall. How many gallons of water will M. T. need to fill the tank? (nearest gallon)

(A) 940 gal      (B) 979 gal      (C) 1,097 gal      (D) 1,175 gal      (E) 1,253 gal

12. An enneahedron is a polyhedron with 9 faces and 9 vertices. How many edges does it have?

(A) 16      (B) 10      (C) 18      (D) 9      (E) 20

13. If  $\frac{2}{(x+1)(x-3)} - \frac{4}{(x-3)(x+k)} = \frac{k}{(x+1)(x-3)(x+k)}$ , then  $k$  equals:

(A)  $2x + 4$       (B)  $\frac{6x+4}{3}$       (C)  $6x + 4$       (D)  $x + 2$       (E)  $\frac{2x+4}{3}$

14. If  $3 - 2i$  is one of the zeros of the polynomial  $x^3 - 4x^2 + x + 26$ , then another of its zeros is:

(A)  $-4$       (B)  $-2$       (C)  $-1$       (D)  $1$       (E)  $4$

15. Find the sum of the coefficients of the quotient:  $(x^4 + 2x^3 + 3x + 6) \div (x + 2)$

(A) 6      (B) 5      (C) 4      (D) 3      (E) 2

16. Which of the following prime numbers are considered to be Germain primes?

- I. 5      II. 11      III. 23      IV. 41

(A) I only      (B) I & II      (C) I, II, & IV      (D) all of them      (E) none of them

17. The graph of  $y = 4 + 3\cos(5x - 70^\circ)$  reaches a maximum value at:

(A)  $(-56^\circ, 5)$       (B)  $(-70^\circ, 7)$       (C)  $(14^\circ, 5)$       (D)  $(14^\circ, 7)$       (E)  $(84^\circ, 5)$



18. Simplify:  $\sin(\theta) + \cos(\theta)\cot(\theta) - \csc(\theta)$

- (A) 0                      (B)  $2\sec(\theta)$                       (C) 1                      (D)  $\tan(\theta)$                       (E)  $2\tan(\theta)$

19. In the expansion of  $(x - 3y)^5$ , the sum of the coefficients of the 2<sup>nd</sup> and the 5<sup>th</sup> terms is:

- (A)  $-320$                       (B)  $-255$                       (C) 390                      (D) 420                      (E) 480

20. Use the Fibonacci characteristic sequence ..., p,  $-5$ , q, r, 8, ... to Find  $p + q + r$ .

- (A)  $-3.5$                       (B)  $-.5$                       (C) 16.5                      (D) 19.5                      (E) 22.5

21. Using the following array, determine the value of the last term of row 24.

1					(row 1)
2	3				(row 2)
4	5	6			(row 3)
7	8	9	10		(row 4)
11	12	13	14	15	(row 5)
...					( ... )

- (A) 293                      (B) 296                      (C) 300                      (D) 305                      (E) 311

22. Let  $f(x) = \frac{x+2}{x-4}$ , where  $x \neq 4$ . Find  $f^{-1}(x)$ .

- (A)  $\frac{x+1}{2x-4}$                       (B)  $\frac{4x+2}{x-1}$                       (C)  $\frac{4x-2}{x+1}$                       (D)  $\frac{2x-4}{x+2}$                       (E)  $\frac{x-4}{x+2}$

23. Find  $\lim_{x \rightarrow -1} \frac{x^2 - 5x + 6}{x^2 - x - 2}$

- (A)  $-2$                       (B)  $-1$                       (C) 0                      (D) 1                      (E) does not exist

24. There are 7 schools in the Big 12 football conference. How many inconference football games are played during the season if the schools all play each other exactly once?

- (A) 28                      (B) 21                      (C) 14                      (D) 12                      (E) 7

25. Daisy Field picked a bunch of wild flowers. She picked 6 Bluebonnets, 4 Larkspurs, 2 Baby Blue Eyes, and 5 Cosmos. If Daisy randomly chooses a wild flower from her bunch what is the probability of choosing a Larkspur or a Cosmos? (nearest %)

- (A) 69%                      (B) 61%                      (C) 58%                      (D) 55%                      (E) 53%

26. Let  $f_0 = 0$ ,  $f_1 = 1$ ,  $f_2 = 1$ ,  $f_3 = 2$ ,  $f_4 = 3$ , ... be the terms of the Fibonacci sequence. Find  $f_{30}$ .

- (A) 830,422                      (B) 831,516                      (C) 831,872                      (D) 832,040                      (E) 833,658

27. Find the number of positive integral divisors of 540.

- (A) 26                      (B) 24                      (C) 22                      (D) 20                      (E) 18

28.  $(356_8 + 27_8) \times 4_8 = \underline{\hspace{2cm}}_8$
- (A) 2142      (B) 2024      (C) 1532      (D) 1204      (E) 1044
29. The slope of the line going through the points  $(-6, y)$ ,  $(-1, -5)$  and  $(x, -2)$  is  $\frac{3}{5}$ . Find  $x + y$ .
- (A)  $-6$       (B)  $-5$       (C)  $-4$       (D)  $-3$       (E)  $-$
30. If  $3x - y = 2$ ,  $x + 2y = 4$  and  $ax + y = 1$ , then  $a$  equals:
- (A)  $-\frac{3}{8}$       (B)  $-\frac{3}{7}$       (C)  $-1\frac{1}{8}$       (D)  $1\frac{3}{7}$       (E)  $2\frac{4}{7}$
31. The circle,  $x^2 + y^2 - 4x + 10y + 20 = 0$  has radius  $r$  and center  $(h, k)$ . Find  $r + h + k$ .
- (A) 12      (B) 9      (C) 6      (D) 3      (E) 0
32. If  $a_1 = 2$ ,  $a_2 = -3$ ,  $a_3 = 1$  and  $a_n = (a_{n-3}) - (a_{n-1})(a_{n-2})$  for  $n \geq 4$ , then  $a_7$  equals:
- (A) 39      (B) 41      (C) 161      (D) 323      (E) 333
33. Find the sample standard deviation of this set of numbers  $\{83, 83, 91, 97\}$ . (nearest tenth)
- (A) 3.4      (B) 5.2      (C) 4.0      (D) 10.4      (E) 6.8
34. Let  $A = \begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ . Find  $|(AB)^T|$ .
- (A) 145      (B) 132      (C)  $-74$       (D)  $-110$       (E)  $-125$
35. Find the first term of the arithmetic sequence:  $a, 8\frac{1}{4}, b, c, d, 24\frac{3}{4}, \dots$
- (A)  $2\frac{3}{4}$       (B)  $3\frac{3}{8}$       (C)  $3\frac{1}{2}$       (D)  $3\frac{7}{8}$       (E)  $4\frac{1}{8}$
36. What is the probability that a factor of 240 is a multiple of 4 greater than zero?
- (A) 60%      (B) 50%      (C) 40%      (D) 25%      (E) 24%
37. Find the sum of all the  $x$ -values of the critical points of  $f(x) = 2x^3 + x^2 - 20x + 4$ .
- (A) 6      (B)  $\frac{1}{12}$       (C)  $-\frac{1}{3}$       (D)  $-2\frac{1}{3}$       (E)  $-3\frac{2}{3}$
38. The function  $f(x) = x^4 - 2x^2$  is increasing over which of the following intervals?
- (A)  $[0, 4]$       (B)  $[-2, -1]$       (C)  $[-1, 1]$       (D)  $[2, 4]$       (E)  $[1, -4]$

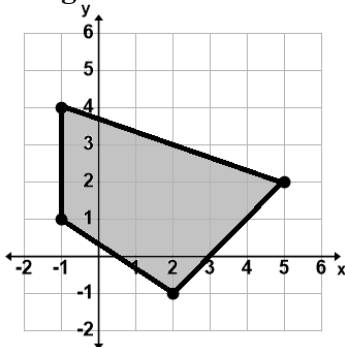
39. If the pattern of the sequence 1, 5, 11, 19, 29, 41, ... continues, find the average of the 24<sup>th</sup> and 26<sup>th</sup> term.

- (A) 602      (B) 626      (C) 650      (D) 671      (E) 675

40. How many positive 3-digit numbers exist such that the sum of their digits equals 14?

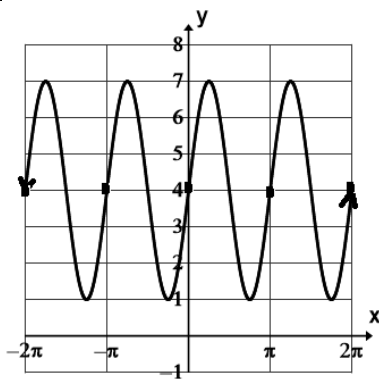
- (A) 100      (B) 63      (C) 83      (D) 54      (E) 70

41. Dee Cortez drew this figure on the coordinate plane below. The coordinates of the vertices are integers. What is the area of the figure?



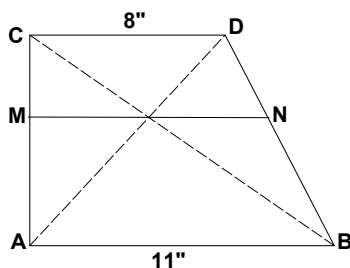
- (A) 16.75 units<sup>2</sup>   (B) 16.5 units<sup>2</sup>   (C) 16 units<sup>2</sup>   (D) 15 units<sup>2</sup>   (E) 15.5 units<sup>2</sup>

42. The equation  $y =$  \_\_\_\_\_ will produce this graph.



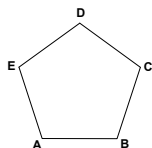
- (A)  $3 + 4\cos(2x - \frac{\pi}{2})$       (B)  $4 - 3\cos(2x + \frac{\pi}{2})$       (C)  $3 - 4\cos(2x + \frac{\pi}{2})$   
 (D)  $4 + 3\cos(2x + \frac{\pi}{2})$       (E)  $4 - 3\cos(2x - \frac{\pi}{2})$

43. Given the trapezoid shown where  $AB \parallel MN \parallel CD$ , find MN. (nearest hundredth)



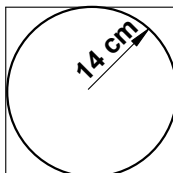
- (A) 9.26"      (B) 9.38"      (C) 9.5"      (D) 9.62"      (E) 9.74"

44. Find the sum of the lengths of all of the diagonals of the regular pentagon if the lengths of each side is 8". (nearest tenth)



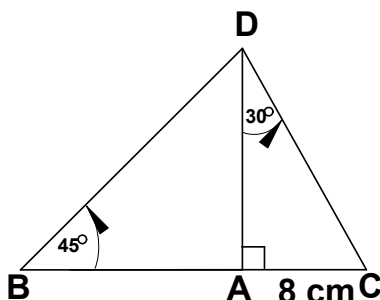
- (A) 64.7"      (B) 64.0"      (C) 61.8"      (D) 49.4"      (E) 24.7"

45. The circle shown is tangent to all sides of the square and has a radius shown. What is the probability that an arrow that hits the target hits in the circle? (nearest tenth)



- (A) 75.0%      (B) 78.5%      (C) 80.0%      (D) 84.5%      (E) 86.0%

46. Find the perimeter of  $\triangle BCD$ . (nearest tenth).



- (A) 55.3 cm      (B) 46.6 cm      (C) 63.2 cm      (D) 60.5 cm      (E) 57.5 cm

47.  $\{(x, y) \mid x, y \in \{\text{Integers}\}, -2 \leq x \leq 5, \text{ and } -4 \leq y \leq 10\}$  is the solution set of  $5x - 3y = 2$ . How many such ordered pairs exist?

- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

48.  $(-6, -6)$  and  $(7, 0)$  are opposite vertices of a parallelogram. If  $(2, -10)$  is the third vertex, then the fourth vertex is:

- (A)  $(1, 3)$       (B)  $(-1, 4)$       (C)  $(-3, 5)$       (D)  $(3, 8)$       (E)  $(-2, 3)$

49. If  $\cos \alpha = -\frac{\sqrt{3}}{2}$ , where  $\frac{\pi}{2} < \alpha < \pi$ , and  $\cos \beta = -\frac{1}{2}$ , where  $\pi < \beta < \frac{3\pi}{2}$ , then  $\sin(\alpha + \beta) = ?$

- (A) 0      (B)  $\frac{1}{2}$       (C)  $-\frac{1}{2}$       (D)  $-\frac{\sqrt{3}}{2}$       (E)  $\frac{\sqrt{2}}{2}$

50. Which of the following statements about  $f(x) = \sqrt{x-2}$  is/are true?
- $f(x)$  is a one to one function on its domain.
  - $f(x)$  has an inverse function on its domain.
  - $f(x)$  has a one to one inverse function on its domain.
- (A) I & II      (B) I only      (C) I & III      (D) all of them      (E) none of them
51. The graph of  $f(x) = x^4 - 6x^2 + 8x + 10$  is concave down for which of the following values of  $x$ .
- (A) 1.75      (B) 1.5      (C) 1.25      (D)  $-0.75$       (E)  $-1.75$
52. Which of the following statements about
- $$f(x) = \begin{cases} -1 & \text{if } x \leq 2 \\ x^2 + x & \text{if } x > 2 \end{cases} \quad \text{is/are true?}$$
- $f(x)$  is defined at 2
  - $\lim_{x \rightarrow 2} f(x)$  exists
  - $f(x)$  is continuous at 2
  - $f(x)$  is differentiable at 2
- (A) I, II, & IV      (B) I & II      (C) I only      (D) all of them      (E) none of them
53. The repeating decimal  $0.4222\dots$  in base 5 can be written as which of the following fractions in base 5 in simplified form?
- (A)  $\frac{21}{40}_5$       (B)  $\frac{34}{40}_5$       (C)  $\frac{4}{10}_5$       (D)  $\frac{12}{20}_5$       (E)  $\frac{14}{20}_5$
54. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number, and  $P_n$  be the  $n$ th pentagonal number. Then  $T_8 + S_9$  has the same value as:
- (A)  $P_9$       (B)  $P_{10}$       (C)  $S_{10}$       (D)  $T_9$       (E)  $T_{10}$
55. The length of the sides of  $\triangle PQR$  are the roots of  $f(x) = x^3 - 20x^2 + 127x - 252$ . Find the area of  $\triangle PQR$ . (nearest tenth unit)
- (A) 12.1      (B) 13.0      (C) 13.4      (D) 13.7      (E) 14.0
56. Tye Purr can proof a stack of books in 10 hours. It takes 14 hours for Ed Itter to proof the same stack of book. How long would it take if Tye and Ed worked together to proof the stack of books? (nearest minute)
- (A) 5 hrs 50 min      (B) 6 hrs 10 min      (C) 6 hrs      (D) 5 hrs 7 min      (E) 7 hrs
57. The perimeter of a square is 40 inches. The length of a rectangle is twice its width. The square and the rectangle have the same perimeter. What is the difference in the two areas?
- (A)  $55\frac{5}{9} \text{ in}^2$       (B)  $48\frac{1}{9} \text{ in}^2$       (C)  $20 \text{ in}^2$       (D)  $11\frac{1}{9} \text{ in}^2$       (E)  $10 \text{ in}^2$

58. Saul T. Cee has a container with 1,000 milliliters of a 2% salt solution. How many grams of salt must he add to get a 4% salt solution?
- (A)  $30\frac{1}{6}$  grams    (B) 30 grams    (C)  $25\frac{1}{2}$  grams    (D)  $20\frac{5}{6}$  grams    (E) 20 grams
59. Sir Vayer places a blue stake in the ground. He places a red stake 200 feet from the blue stake on a bearing of  $300^\circ$  and a white stake 140 feet from the blue stake on a bearing of  $40^\circ$ . He ties a rope from the blue stake to the red stake to the white stake and back to the blue stake. How long does the rope need to be if he needs 8 feet extra to tie it to the stakes? (nearest foot)
- (A) 611 ft    (B) 619 ft    (C) 625 ft    (D) 629 ft    (E) 631 ft
60. Betty Luzes is considering buying a \$2.00 raffle ticket at the local store. The weekly raffle has a grand prize of \$5000.00 and two second prizes of \$500.00 each. What would her expected winnings be on one ticket if 800 tickets are sold each week and she plays each week for several years? (plus or minus one cent)
- (A) \$5.50    (B) \$22.50    (C) \$16.50    (D) — \$2.00    (E) — \$2.66

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • District 1 • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. D  | 21. C | 41. B |
| 2. C  | 22. B | 42. B |
| 3. A  | 23. E | 43. A |
| 4. E  | 24. B | 44. A |
| 5. E  | 25. E | 45. B |
| 6. D  | 26. D | 46. E |
| 7. C  | 27. B | 47. B |
| 8. B  | 28. B | 48. B |
| 9. C  | 29. C | 49. B |
| 10. C | 30. A | 50. D |
| 11. D | 31. E | 51. D |
| 12. A | 32. E | 52. C |
| 13. A | 33. E | 53. E |
| 14. B | 34. D | 54. A |
| 15. C | 35. E | 55. C |
| 16. D | 36. A | 56. A |
| 17. D | 37. C | 57. D |
| 18. A | 38. D | 58. E |
| 19. C | 39. C | 59. A |
| 20. D | 40. E | 60. A |

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
--------------------	------------------

- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |





UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

# Mathematics

## District 2 • 2016



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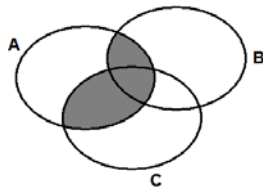
1. Evaluate:  $(2 + 1 \times 3! + 4) \div 7^0 - 11 + 18$

- (A) 7                      (B) 19                      (C) 24                      (D) 29                      (E) 37

2. The absolute value difference between one billion two hundred three million forty-five thousand six hundred seven and eight million nine hundred ten thousand seven hundred twelve is  $x$ . What is the sum of the digits in the difference  $x$  when written out using digits instead of words?

- (A) 56                      (B) 54                      (C) 48                      (D) 45                      (E) 43

3. Which of the following is a symbolic representation for the Venn diagram shown?



- (A)  $A \cap B \cap C$                       (B)  $(A \cup B) \cap (A \cup C)$                       (C)  $(A \cap C) \cup (C \cap B)$   
(D)  $A \cup (B \cap C)$                       (E)  $(A \cap C) \cup (A \cap B)$

4.  $3(4 + 5) = 12 + 15$  and  $(15 - 12) \div 3 = 5 - 4$  are examples of the \_\_\_\_\_ property of equality.

- (A) associative    (B) commutative    (C) distributive    (D) identity    (E) transitive

5. Otto Driver took a 750 mile trip from Heer to Thare. His car gets 22 miles per gallon. If the cost per gallon of gas was \$2.14, how much did it cost Otto for gas to make the trip?

- (A) \$70.50                      (B) \$72.76                      (C) \$72.95                      (D) \$76.72                      (E) \$77.10

6. Simplify:  $\left( \frac{6x^2 + 15x - 9}{6x^2 + x - 2} \right) \left( \frac{9x^2 - 4}{9x - 6} \right)$

- (A)  $x + 3$                       (B)  $3x + 1$                       (C)  $\frac{x+3}{2x-1}$                       (D)  $\frac{x+3}{3}$                       (E)  $\frac{3x+9}{3x+2}$

7. Which of the following relations describes a one to one function?

- I.  $\{ (2,5), (3,4) (4,3) (5,2) \}$   
II.  $\{ (-1,1), (2, -2) (1, 1) (0,0) \}$   
III.  $\{ (1,1) (2,2) (3,3) (4,4) \}$

- (A) I only                      (B) III only                      (C) I & II                      (D) I & III                      (E) II & III

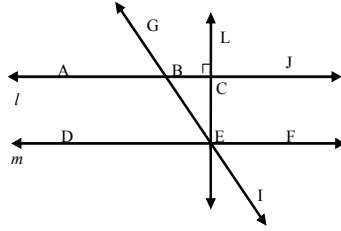
8. Find the greatest common divisor of  $2^4 \times 3^3 \times 5^2$ ,  $2^3 \times 3^2 \times 5^4$ , and  $2^2 \times 3^4 \times 5^3$ .

- (A) 30                      (B) 67,500                      (C) 380                      (D) 10,800                      (E) 900

9. Trey Engals had five pieces of uncooked spaghetti. The lengths of the pieces were 12", 8", 7", 5", and 4". How many obtuse triangles could Trey make using only three pieces at a time?

- (A) 4                      (B) 5                      (C) 6                      (D) 7                      (E) 8

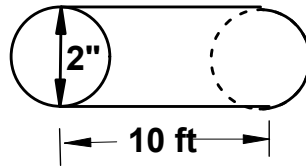
10. The four lines in the figure are coplanar with  $m \parallel l$ . How many other angles have the same measure as  $\angle ABG$ ?



- (A) 5                      (B) 4                      (C) 3                      (D) 2                      (E) 1
11. If  $\frac{4x-1}{2x+5} - \frac{3}{x-6} = \frac{ax^2+bx+c}{px^2+qx+r}$ , then  $(a+b+c) - (p+q+r)$  equals:
- (A)  $-71$                       (B)  $-41$                       (C)  $-39$                       (D)  $-21$                       (E)  $-1$
12. Let  $3P - 4Q = -1$  and  $2P + 3Q = 5$ . Find  $P + Q$ .
- (A)  $\frac{43}{51}$                       (B)  $\frac{1}{17}$                       (C) 13                      (D) 2                      (E)  $1\frac{37}{51}$
13. If  $16^{(k+1)} = 4^{(k)}$ , then  $2^{(k-1)} = ?$
- (A) 0.125                      (B) .25                      (C) 2                      (D) 0.5                      (E) 0.0625
14. Let  $\sin^2(x) = \frac{1}{2}$ , where  $\pi \leq x \leq \frac{3\pi}{2}$  and  $\cos^2(y) = \frac{3}{4}$ , where  $\frac{\pi}{2} \leq y \leq \pi$ .  
Find  $\sin(x)\cos(y) - \sin(y)\cos(x)$ . (nearest hundredth)
- (A) 0.20                      (B) 1.25                      (C) 0.97                      (D) 0.44                      (E) 0.26
15. Let  $f(x) = 3 - x$  and  $g(x) = 2x + 1$ . Find  $f(g(x)) + g(-f(x))$ .
- (A)  $-3$                       (B)  $-4x - 3$                       (C)  $4x + 7$                       (D)  $4x - 3$                       (E) 7
16. Let  $x^5 - 4x^4 - 7x^3 + 14x^2 - 44x + 120 = 0$ . How many possible negative real roots are there?
- (A) 0                      (B) 1                      (C) 4, 2, or 0                      (D) 3 or 1                      (E) 5, 3, or 1
17. Find C if the remainder when  $x^3 + 2x^2 + 5x + C$  is divided by  $x - 2$  is 3.
- (A) 29                      (B) 26                      (C) 13                      (D)  $-7$                       (E)  $-23$
18. The *Cheapway Deli* lets the customer build a sandwich their way. The menu shows:
- |        |  |
|--------|--|
| bread: | white, wheat, rye, or pita                       |
| sauce: | mayonnaise, mustard, or ketchup                  |
| meats: | ham, bologna, turkey, or chicken                 |
| sides: | tomatoes, lettuce, onions, pickles, or jalepenos |
- How many different sandwiches could be built using one item from each section?

- (A) 240                      (B) 16                      (C) 1,820                      (D) 64                      (E) 120

19. The diameter and length of the joint of PVC pipe is shown. What is the maximum number of fluid ounces of water the joint of pipe could hold if both ends were capped?



- (A) 210 fl. oz.    (B) 208 fl. oz.    (C) 205 fl. oz.    (D) 203 fl. oz.    (E) 200 fl. oz
20. If the pattern of the sequence 3, 7, 13, 21, 31, 43, ... continues, find the 30<sup>th</sup> term.
- (A) 921            (B) 923            (C) 927            (D) 931            (E) 933
21. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number, and  $H_n$  be the  $n$ th hexagonal number. Then  $2(T_4) + S_5$  has the same value as:
- (A)  $S_9$             (B)  $H_6$             (C)  $T_5$             (D)  $H_5$             (E)  $T_7$
22. How many proper fractions in lowest terms have a denominator of 30?
- (A) 8            (B) 9            (C) 10            (D) 12            (E) 15
23.  $110110_2 - 10101_2 + 1111_2 = \underline{\hspace{2cm}}_4$
- (A) 122            (B) 200            (C) 300            (D) 1100            (E) 1122
24. If  $4(2x - 3) = 3(2 + 4x)$  then  $3x - 4$  equals:
- (A)  $-17.5$             (B)  $-13.5$             (C)  $-9.9$             (D)  $-4.9$             (E)  $-0.5$
25. If  $a_1 = -1$ ,  $a_2 = -2$ , and  $a_n = -3(a_{n-1}) + 4(a_{n-2})$ , then  $a_5$  equals:
- (A) 50            (B)  $-29$             (C) 1            (D)  $-214$             (E)  $-233$
26. The sample standard deviation of the set of numbers {4, 5, 5, 4, 4, 2, 2, 6} is:
- (A)  $\sqrt{2}$             (B)  $\frac{\sqrt{7}}{2}$             (C)  $7\sqrt{2}$             (D)  $2\sqrt{7}$             (E)  $\frac{\sqrt{2}}{7}$
27. The directrix of the parabola  $x^2 - 8x - y + 2 = 0$  is:
- (A)  $y = -16.25$     (B)  $y = -16.75$     (C)  $y = -15.125$     (D)  $y = -14.25$     (E)  $y = -13.75$
28. If  $[(5 + 2i)(3 + 4i)] \div (1 - i) = a + bi$ , then  $a + b = ?$
- (A) 26            (B) 22.5            (C) 14            (D) 2.5            (E) 7

29. If  $f'(x) = 12x^2 - 6x - 25$  and  $f(1) = 2$ , find  $f(-2)$ .

- (A)  $-19$       (B)  $-13$       (C)  $2$       (D)  $32$       (E)  $35$

30. How many positive 3-digit numbers exist such that the sum of their digits equals 18?

- (A)  $54$       (B)  $62$       (C)  $70$       (D)  $76$       (E)  $82$

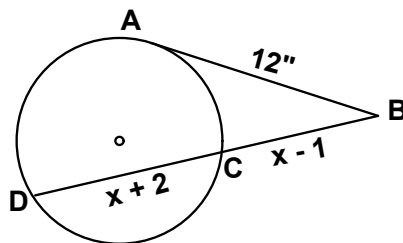
31. Find the slope of the line tangent to the curve  $y = 12x^2 - 6x - 25$  at the point  $(2, 11)$ .

- (A)  $24$       (B)  $13$       (C)  $42$       (D)  $9$       (E)  $18$

32. The slope of the line going through the points  $(5, y)$ ,  $(-1, -2)$  and  $(x, 4)$  is  $\frac{2}{3}$ . Find  $x + y$ .

- (A)  $16$       (B)  $10$       (C)  $6$       (D)  $3$       (E)  $2$

33. Points A, C, and D lie on the circle shown. Find  $x$ . (nearest tenth)



- (A)  $12.6''$       (B)  $11.6''$       (C)  $9.0''$       (D)  $8.8''$       (E)  $8.3''$

34. Point  $P(3, -2)$  and point  $Q(-1, 4)$  lie on the  $x$ - $y$  plane.  $P$  is rotated  $90^\circ$  clockwise about the origin to  $P_1$ .  $Q$  is reflected across the  $y$ -axis to  $Q_1$ . What is the distance between  $P_1$  and  $Q_1$ ? (nearest tenth of a unit)

- (A)  $6.8$       (B)  $7.1$       (C)  $7.2$       (D)  $7.6$       (E)  $7.8$

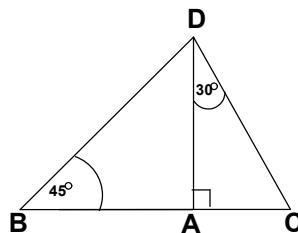
35. The *Nuttin Honey Shoppe* mixed some peanuts costing  $\$1.80$  a pound with some chocolate costing  $\$3.20$  a pound to make 50 pounds of a sweet and salty mixture that costs  $\$2.92$  pound. How many more pounds of chocolate than peanuts did they use?

- (A)  $15$  lbs      (B)  $20$  lbs      (C)  $30$  lbs      (D)  $35$  lbs      (E)  $40$  lbs

36. Quadrilateral  $ABCD$  has two angles with the same measure. A third angle has a measure equal to the sum of the two equal angles. The fourth angle is  $60^\circ$  less than twice the sum of the other three angles. What is the measure of the obtuse angle?

- (A)  $140^\circ$       (B)  $105^\circ$       (C)  $220^\circ$       (D)  $160^\circ$       (E)  $200^\circ$

37. Mr. White's height is 6' 1". He is standing on a ladder 5'3" from the ground trimming his pecan trees. How long of a shadow is cast when the angle of elevation of the sun from the ground is  $60^\circ$ ? (nearest inch)
- (A) 5' 8"      (B) 6' 4"      (C) 6' 7"      (D) 9' 6"      (E) 9' 8"
38. Lotta Proffitt sold books and magazines to raise money for her favorite charity. She sold four times as many books as she did magazines. The profit she earned for each book sold was \$3.25 and the profit for each magazine was \$2.00. She made a total profit of \$135.00. How many books did she sell?
- (A) 20      (B) 24      (C) 28      (D) 32      (E) 36
39. Hi Stepper starts at the midpoint of the goal line of a football field. He runs 20 yards on a bearing of  $70^\circ$  then turns and runs 30 yards on a bearing of  $320^\circ$  where he is tackled. How far from his starting point would he have run if he would have gone in a straight line to the point where he was tackled? (nearest inch)
- (A) 36 yds 3"      (B) 16 yds 6"      (C) 26 yds 1' 3"      (D) 29 yds 2' 6"      (E) 23 yds 0"
40. The Cow-Belles and Tex-Ants will play each other twice during the season. The Tex-Ants are twice as likely to win any game against the Cow-Belles. What is the probability that the Tex-Ants will win one game and the Cow-Belles will win the other?
- (A)  $\frac{1}{2}$       (B)  $\frac{1}{3}$       (C)  $\frac{2}{3}$       (D)  $\frac{4}{9}$       (E)  $\frac{5}{9}$
41. Let  $AD = 6$  inches. Find the area of  $\triangle BCD$ . (nearest tenth).



- (A)  $27.0 \text{ in}^2$       (B)  $22.4 \text{ in}^2$       (C)  $30.7 \text{ in}^2$       (D)  $36.0 \text{ in}^2$       (E)  $28.4 \text{ in}^2$
42. In one year Tu Old will be six times as old as Soh Yung. Evan Yung is three years younger than Soh Yung. In ten years Tu will be fourteen years older than the sum of the ages of Soh and Evan. What is the sum of all of their ages?
- (A) 36      (B) 34      (C) 33      (D) 30      (E) 29
43. Les Cash has a bag of \$1.00 bills, a bag of \$5.00 bills, a bag of \$10.00 bills, and a bag of \$20.00 bills. He wants to put five bills in an envelope to give to his various charities. How many different ways can Les stuff the envelopes?
- (A) 120      (B) 70      (C) 56      (D) 36      (E) 24

44. Given that the set of natural numbers continue in the triangular pattern shown below, find the 2<sup>nd</sup> number in row 12.

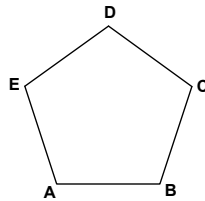
			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				( ... )

- (A) 141                      (B) 131                      (C) 123                      (D) 121                      (E) 145
45. The point (1, 4) lies on a circle whose center is  $(-1, 0)$ . Where does the point  $(-3, -4)$  lie in reference to the circle?
- (A) on the circle                      (B) outside the circle                      (C) inside the circle  
(D) in quadrant II                      (E) cannot be determined

46. Find the sum of the values of  $\theta$  of  $\left\{ \theta \mid 2\sin^2(\theta) - \sin(\theta) - 1 = 0, \theta \in [-\pi, \pi] \right\}$ .  
(nearest hundredth)

- (A) 4.71                      (B) 3.14                      (C)  $-7.85$                       (D)  $-3.14$                       (E)  $-1.57$

47. Find the perimeter of the regular pentagon if  $EB = 5''$ . (nearest tenth)



- (A) 8.1"                      (B) 9.7                      (C) 13.1"                      (D) 15.0"                      (E) 15.5"
48. The solutions for the system  $r = 2\sin(\theta)$  and  $r = 1 + \cos(\theta)$  are the pole and the point  $(r, \theta)$ . Find  $r + \theta$ . (nearest hundredth)

- (A) 1.07                      (B) 2.53                      (C) 2.20                      (D) 2.13                      (E) 0.67

49. Which of the following statements about  $f(x) = x^3 + 3$  is/are true?

- I.  $f(x)$  is a one to one function on its domain.  
II.  $f(x)$  has an inverse function on its domain.  
III.  $f(x)$  has a one to one inverse function on its domain.

- (A) I & II                      (B) I only                      (C) I & III                      (D) all of them                      (E) none of them

50. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots, f_k = 610, \dots$  be the terms of the Fibonacci sequence. Find  $f_{k-1}$ .

- (A) 987                      (B) 358                      (C) 583                      (D) 377                      (E) 317

51. Let  $f(x) = 6 - \frac{2+x}{x-1}$ , where  $x \neq 1$ . Find  $f^{-1}(x)$ .

- (A)  $\frac{x+1}{x-2} + 6$  (B)  $\frac{x-7}{x+4}$  (C)  $\frac{x-5}{x-7}$  (D)  $\frac{x+7}{x+8}$  (E)  $\frac{x-8}{x-5}$

52. Which of the following prime numbers are considered to be Mersenne primes?

- I. 13 II. 8,191 III. 524287

- (A) I only (B) II only (C) III only (D) I & II (E) II & III

53.  $(A59B_{13} + 32C6_{13}) = \underline{\hspace{2cm}}_{13}$

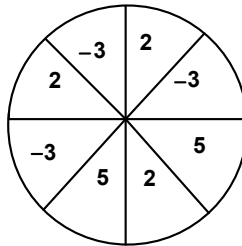
- (A) C8C4 (B) A0927 (C) 10894 (D) ABC4 (E) 13927

54. Which of the following statements about  $f(x) = (x-3)^{(-2)}$  is/are true?

- I.  $f(x)$  is defined at 3 II.  $\lim_{x \rightarrow 3} f(x)$  exists  
III.  $f(x)$  is continuous at 3 IV.  $f(x)$  is differentiable at 3

- (A) I only (B) I & II (C) I, II, & III (D) all of them (E) none of them

55. Roland Around spins the wheel. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation on any one spin?



- (A) 2.333... (B) 0.5 (C) 0.875 (D) 1.375 (E) 3

56. The graph of  $h(x) = \frac{x^3 + x^2 - 3x - 3}{x + 1}$  suggests that the *discontinuity* at  $x = -1$  is *removable* by defining  $h(-1)$  to be \_\_\_\_.

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 3

57. Dee V. Dee is organizing some movies on his iPad. He has 8 movies to choose from, but only wants to put 5 of them on his iPad. How many ways can he organize a set of 5 movies?

- (A) 120 (B) 6,720 (C) 40,320 (D) 280 (E) 720

58. The base 10 fraction  $\frac{5}{21}$  can be written as which of the following repeating decimals in base 6?

- (A) 0.1222...<sub>6</sub> (B) 0.1232323...<sub>6</sub> (C) 0.2131313...<sub>6</sub> (D) 0.151515...<sub>6</sub> (E) 0.1252525...<sub>6</sub>



59. The length of the sides of  $\triangle PQR$  are the roots of  $f(x) = x^3 - 14x^2 + 63x - 90$ . Find the area of  $\triangle PQR$ . (nearest tenth unit)

(A) 7.0

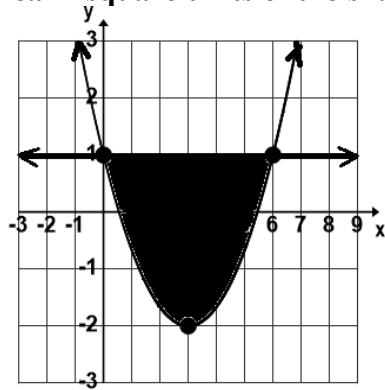
(B) 7.5

(C) 7.8

(D) 8.0

(E) 8.2

60. Find the area in square units of the shaded region.



(A) 10

(B) 12

(C)  $13\frac{1}{3}$

(D)  $11\frac{2}{3}$

(E) 14.5

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • District 2 • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. B  | 21. D | 41. E |
| 2. D  | 22. A | 42. B |
| 3. E  | 23. C | 43. C |
| 4. C  | 24. A | 44. C |
| 5. C  | 25. A | 45. A |
| 6. A  | 26. A | 46. E |
| 7. D  | 27. D | 47. E |
| 8. E  | 28. E | 48. B |
| 9. A  | 29. D | 49. D |
| 10. C | 30. A | 50. D |
| 11. E | 31. C | 51. E |
| 12. D | 32. B | 52. E |
| 13. A | 33. D | 53. C |
| 14. C | 34. D | 54. E |
| 15. A | 35. C | 55. C |
| 16. B | 36. C | 56. A |
| 17. E | 37. C | 57. B |
| 18. A | 38. E | 58. B |
| 19. B | 39. D | 59. B |
| 20. D | 40. D | 60. B |

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
--------------------	------------------

- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

# Mathematics

## Regional • 2016



DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $0.25 \div 0.08333... + 0.75 - 0.1666... \times 0.375 \div 0.0625$

- (A)  $\frac{37}{48}$  (B)  $1\frac{11}{32}$  (C)  $2\frac{3}{4}$  (D)  $20\frac{5}{24}$  (E)  $21\frac{1}{2}$

2. Harry Brush went to the paint store to get supplies to paint his shed. He bought 4 gallons of paint at \$19.95 per gallon, 3 brushes at \$7.50 each, a roller for \$2.69, a plastic tarp for \$8.25, and 2 pints of cleaning fluid at \$3.95 per pint. How much change would he get back if he gave the cashier two \$100.00 bills and the tax rate was 8%?

- (A) \$21.14 (B) \$30.83 (C) \$57.66 (D) \$69.17 (E) \$78.86

3. 70 miles per hour is the same speed as \_\_\_\_\_ inches per second.

- (A)  $102\frac{2}{3}$  (B)  $410\frac{2}{3}$  (C) 905 (D) 1,023 (E) 1,232

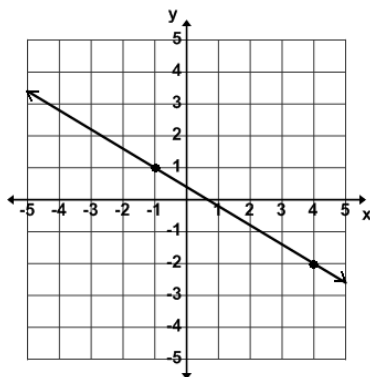
4. A recipe for 1 pint of vanilla ice cream requires 4 egg yolks,  $\frac{2}{3}$  cup of sugar,  $1\frac{1}{2}$  cups of milk, 1 vanilla bean, and  $1\frac{1}{2}$  cups of cream. How much sugar would be needed to make  $2\frac{1}{2}$  quarts of vanilla ice cream?

- (A)  $3\frac{1}{3}$  cups (B)  $1\frac{11}{15}$  cups (C)  $5\frac{2}{3}$  cups (D)  $3\frac{7}{15}$  cups (E)  $1\frac{2}{3}$  cups

5. If  $5 = 3x - y$ ,  $x + 4y = 6$  and  $ax + y = 11$ , then a equals:

- (A) 7 (B) 6 (C) 5 (D) 4 (E) 3

6. The x-intercept of a line that is perpendicular to the line shown at point (4, -2) is (x,y). Find x + y.



- (A) 6 (B) 5.2 (C) 2.8 (D) 5 (E) 7.333...

7. Which of the following is not a one-to-one function?

- (A)  $y = 2^x$  (B)  $y = \sqrt{2x}$  (C)  $y = \ln(2x)$  (D)  $y = |2x|$  (E) all are one to one

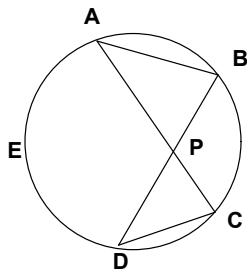
8. The Fibonacci sequence,  $F = \{0, 1, 1, 2, 3, 5, 8, \dots\}$  is closed under which of the following operation(s)? (A) addition (S)ubtraction (M)ultiplication (D)ivision

- (A) A (B) M (C) A & M (D) all of them (E) none of them

9. Point P( $-1, -2$ ) lies on the x-y plane. Point P is translated  $-3$  units vertically to point Q. Point Q is translated  $-4$  units horizontally to point R. Point R is reflected across the line  $y = -x$  to point S. The coordinates of point S is  $(x, y)$ . Find  $x + y$ .

(A) 12                      (B) 10                      (C) 6                      (D) 5                      (E) 0

10.  $\overline{AB}$ ,  $\overline{AC}$ ,  $\overline{BD}$ , and  $\overline{CD}$  are chords of circle O. Find  $m\widehat{AED}$  if  $m\angle BDC = 55^\circ$  and  $m\angle APB = 50^\circ$ .



(A)  $37.5^\circ$                       (B)  $100^\circ$                       (C)  $105^\circ$                       (D)  $110^\circ$                       (E)  $150^\circ$

11. Tri Phen chooses an element T at random from  $\{1, 3, 6, 10, 15\}$  and an element P at random from  $\{1, 5, 12, 22, 35\}$ . What are the odds that the product of P and T is odd?

(A) 64%                      (B) 43.75%                      (C) 36%                      (D) 24%                      (E) 56.25%

12. Which of the following are the side lengths of an acute, non-right triangle?

(A) 7, 10, 13                      (B) 7, 7, 12                      (C) 7, 8, 9                      (D) 6, 20, 21                      (E) 7, 24, 25

13. A triangular dipyrmaid is an isohedron with 6 faces and 9 edges. How many vertices does it have?

(A) 5                      (B) 6                      (C) 9                      (D) 12                      (E) 13

14. If  $\frac{4}{x-4} + \frac{2}{x+2} - \frac{3}{x+5} = \frac{Ax^2 + Bx + C}{Px^3 + Qx^2 + Rx + S}$ , then  $\frac{A+B+C}{P+Q+R+S}$  equals:

(A)  $\frac{63}{62}$                       (B) 1                      (C)  $-\frac{9}{11}$                       (D)  $-\frac{7}{6}$                       (E)  $-\frac{1}{6}$

15. If  $a_1 = 2$ ,  $a_2 = -3$ ,  $a_3 = 5$ , and  $a_n = (a_{n-2}) + (a_{n-3})(a_{n-1})$ , where  $n \geq 4$ , then  $a_7 = ?$

(A)  $-527$                       (B)  $-357$                       (C) 34                      (D) 265                      (E) 353

16. If the pattern of the sequence 3, 8, 15, 24, 35, 48, ... continues, find the 24<sup>th</sup> term.

(A) 630                      (B) 624                      (C) 618                      (D) 605                      (E) 588

17. Find the sum of the coefficients of the quotient:  $(x^4 + 4x^3 + 3x^2 + 4) \div (x + 2)$

(A)  $-4$                       (B)  $-1$                       (C) 0                      (D) 2                      (E) 4

18. The bearing from Sumware to Noware is  $153^\circ$ , and the bearing from Noware to Overthare is  $63^\circ$ . It takes Sumwon 1 hour and 15 minutes at 60 mph to go from Sumware to Noware and 1 hour and 45 minutes at 60 mph to go from Noware to Overthare. How far is Sumware from Overthare? (nearest mile)

(A) 73 mi      (B) 90 mi      (C) 114 mi      (D) 129 mi      (E) 144 mi

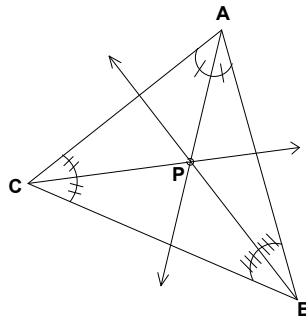
19. Find the sum of the solutions of  $\sec(x) \csc(x) = 2\csc(x)$ , where  $0 \leq x < 2\pi$ ?

(A)  $2\pi$       (B)  $\frac{5\pi}{3}$       (C)  $3\pi$       (D)  $\frac{8\pi}{3}$       (E)  $\frac{7\pi}{3}$

20. The graph of  $y = 3\cos(2x + \frac{3\pi}{2}) - 3$  reaches a maximum value at:

(A)  $(-\frac{\pi}{4}, 0)$       (B)  $(-\frac{\pi}{3}, -6)$       (C)  $(\frac{3\pi}{2}, 0)$       (D)  $(\frac{\pi}{3}, 2)$       (E)  $(\frac{\pi}{4}, 0)$

21. The point of intersection, P, of the triangle is called the \_\_\_\_\_.



(A) center      (B) centroid      (C) circumcenter      (D) incenter      (E) orthocenter

22. Let  $x^5 + x^4 - 2x^3 + x^2 - 1 = 0$ . According to Descartes' Rule of Signs, how many possible negative roots are there?

(A) 5, 3, 1, or 0      (B) 1 or 0      (C) 3, 1, or 0      (D) 2 or 0      (E) 0

23. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the first term and the last term in row 10.

				1	(row 1)
			3	5	(row 2)
		7	9	11	(row 3)
	13	15	17	19	(row 4)
21	23	25	27	29	(row 5)
				...	( ... )

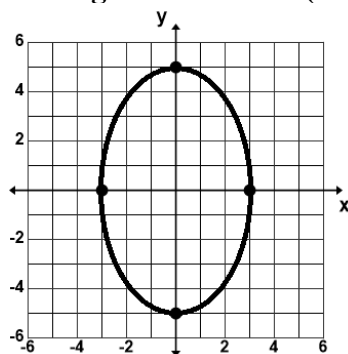
(A) 110      (B) 151      (C) 200      (D) 209      (E) 91

24. If  $a_1 = 0$ ,  $a_2 = -1$ ,  $a_3 = 2$ ,  $a_4 = -3$ , and  $a_n = (a_{n-4})(a_{n-1}) + (a_{n-2})(a_{n-3})$  for  $n \geq 5$ , then  $a_7$  equals:

(A)  $-4$       (B)  $-2$       (C)  $0$       (D)  $4$       (E)  $6$

25. Phil Itup turns on two different size water pipes to fill up his pool. With both pipes open, it takes 16 hours to fill the pool. The smaller pipe takes 10 hours more than the larger pipe to fill the pool. How long would it take the larger pipe to fill the pool running by itself? (nearest minute)
- (A) 27 hrs 46 min (B) 24 hrs 31 min (C) 21 hrs 32 min (D) 2 hrs 6 min (E) 11 hrs 32 min
26. Let  $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$  and  $g(x) = 4x^2 - 4x + 1$ . Find  $g'(f''(-2))$ .
- (A) 8 (B) 12 (C) 84 (D) 92 (E) 132
27. Find  $\lim_{x \rightarrow -\infty} \frac{-2x^3 - 2x + 3}{3x^3 + 3x^2 - 5x}$
- (A)  $-1\frac{1}{2}$  (B)  $-\frac{2}{3}$  (C) 1 (D)  $\frac{2}{3}$  (E) does not exist
28. If  $f''(x) = 6x - 22$  and  $f'(1) = 20$  and  $f(-1) = -96$ , then  $f(1) = \underline{\hspace{2cm}}$ .
- (A)  $-8$  (B)  $-16$  (C)  $-22$  (D)  $-28$  (E)  $-38$
29.  $102221_3 - 12012_3 + 2121_3 = \underline{\hspace{2cm}}_9$
- (A) 310 (B) 53 (C) 630 (D) 475 (E) 145
30. The *Fifty-Fifty* rental car company rents cars for \$50.00 a day and 50¢ a mile for any miles driven over 100 miles. The *Cheep* rental car company charges \$40.00 a day and 60¢ for each mile driven. How much would be saved by renting a car for three days traveling 500 miles from *Fifty-Fifty* instead of *Cheep*?
- (A) \$20.00 (B) \$30.00 (C) \$70.00 (D) \$80.00 (E) \$210.00

31. The eccentricity of the figure shown is: (nearest hundredth)

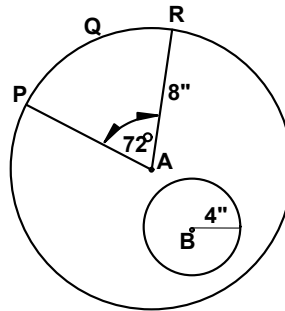


- (A) 0.64 (B) 0.60 (C) 0.80 (D) 0.333... (E) 0.75
32. Ima Yung is 7 years younger than Heath Yung and Soh Yung is four years older than Ima. In four years the sum of all of their ages will be 50. How old is Heath now?
- (A) 9 (B) 11 (C) 13 (D) 16 (E) 20



33. Auna Diette bought 3 containers of yogurt and 5 grapefruits for \$2.79 at the health store. Les Wait bought 4 containers of yogurt and 7 grapefruits for \$3.81 at the same store. What would it cost Lotta Pounds to buy 1 container of yogurt and 1 grapefruit at the same health store?
- (A) \$0.72      (B) \$0.75      (C) \$0.96      (D) \$1.02      (E) \$1.35

34. A point is selected inside the large circle with center A. What is the probability that the point is neither in the  $72^\circ$  sector nor the smaller circle with center B? (nearest percent)



- (A) 55%      (B) 32.5%      (C) 70%      (D) 45%      (E) 67.5%
35. Mike Rowe's computer company produces computer chips. On average 2% of the chips produced are defective. If 4 chips are chosen at random for testing, what is the probability that exactly one chip is defective? (nearest tenth)
- (A) 2.5%      (B) 7.5%      (C) 7.8%      (D) 8.0 %      (E) 9.4%
36. How many prime numbers,  $P$ , where  $P \leq 20$ , exist that are both a Germain prime and a Mersenne prime?
- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4
37. A set of five positive integers  $\{p, q, r, s, t\}$  is listed in numeric order from least to greatest. The set has a mean of 82, a median of 82, a range of 15, and a mode of 75. Find  $s$  if  $t = 90$ .
- (A) 84      (B) 85      (C) 86      (D) 87      (E) 88
38. Let  $p$  and  $q$  be the roots of  $2x^2 + 7x - 4 = 0$ . Find  $p^3 + 3p^2q + 3pq^2 + q^3$ .
- (A)  $-8$       (B)  $-21.75$       (C)  $-42.875$       (D)  $-10.5$       (E)  $-49.125$
39. Simplify:  $\left(\frac{2x^2 - 7x - 4}{x^2 - 6x + 9}\right) \times \left(\frac{x^3 - 9x^2 + 27x - 27}{4x^2 - 1}\right) \div (x - 4)$
- (A)  $\frac{x-3}{2x-1}$       (B)  $\frac{x^2+6x+9}{2x^2-5x+1}$       (C)  $\frac{x-3}{2x+1}$       (D)  $\frac{x^2-7x+12}{x^2+6x+9}$       (E)  $\frac{x-1}{x+1}$
40. If the side of a square based pyramid is doubled and the height is cut in half, then the ratio of the volume of the new pyramid to the volume of the original pyramid is:
- (A) 1:1      (B) 2:1      (C) 1:4      (D) 4:1      (E) 1:2

41. Simplify:  $(a^4)\left(\frac{(a^{-2})^3}{\sqrt{a}}\right)^{-5}$
- (A)  $(\sqrt{a})^{-7}$  (B)  $(\sqrt{a})^{-3}$  (C)  $(\sqrt{a})^{15}$  (D)  $(\sqrt{a})^{24}$  (E)  $(\sqrt{a})^{73}$
42. How many integral values of  $n$  exist such that  $n \geq 1$  and  $\frac{(n+4)!}{(n+2)!} \leq 51$
- (A) 2 (B) 3 (C) 5 (D) 7 (E) 11
43. Find the equation of the line tangent to the curve  $y = 1 + 2x - x^3$  at the point  $(1, 2)$ .
- (A)  $y = 3 - x$  (B)  $y = -6x + 3$  (C)  $y = 2x$  (D)  $y = 1 - x$  (E)  $y = -3x + 1$
44. Which of the following polar equations is the graph of a looped limaçon?
- (A)  $r = 2 + \cos \theta$  (B)  $r = 1.3 + \cos \theta$  (C)  $r = 1 + \cos \theta$  (D)  $r = 0.5 + \cos \theta$  (E)  $r = \cos \theta$
45. How many positive 3-digit numbers exist such that the sum of their digits equals 23?
- (A) 21 (B) 10 (C) 16 (D) 28 (E) 15
46. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$  be the terms of the Fibonacci sequence. Find  $k$  if  $f_k = 28,657$ .
- (A) 31 (B) 23 (C) 25 (D) 28 (E) 20
47. Rose Thorn's flower shop has red roses, white roses, yellow roses, pink roses, and 49er roses. How many sets of three roses can she put in a vase?
- (A) 10 (B) 21 (C) 35 (D) 56 (E) 60
48. The diameter of a penny is 0.75 inch and the diameter of a quarter is 0.95 inch. The entire penny is placed face down on the surface of the quarter. What is area of the part of the quarter not covered by the penny? (nearest hundredth)
- (A)  $0.03 \text{ in}^2$  (B)  $0.10 \text{ in}^2$  (C)  $0.13 \text{ in}^2$  (D)  $0.27 \text{ in}^2$  (E)  $0.39 \text{ in}^2$
49. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number,  $P_n$  be the  $n$ th pentagonal number, and  $H_n$  be the  $n$ th hexagonal number. Then  $T_{20}$  has the same value as:
- (A)  $2H_8$  (B)  $H_{10}$  (C)  $3P_7$  (D)  $5T_9$  (E)  $8S_4$
50. Which of the following statements about  $f(x) = |2x - 1| + 3$  is/are true?
- I.  $f^{-1}(x)$  is a function.  
 II. the domain of  $f(x)$  is  $\{x | x \in \{\text{Reals}\}\}$ .  
 III. the range of  $f(x)$  is  $\{y | y \geq 3\}$ .
- (A) I & II (B) II only (C) II & III (D) all of them (E) none of them

51. Which of the following statement(s) is/are true about  $f(x)$ ?

$$f(x) = \begin{cases} \sqrt{1-x} & \text{if } x < 1 \\ (1-x)^2 & \text{if } 1 \leq x \end{cases}$$

- I.  $f(x)$  is defined at 1                      II.  $\lim_{x \rightarrow 1} f(x)$  exists  
 III.  $f(x)$  is continuous at 1                  IV.  $f(x)$  is differentiable at 1

(A) I only              (B) I & II              (C) I, II, & III              (D) I, II, III, & IV              (E) none of them

52. The chart below shows the estimates of the number of people, P, in a car driving through downtown Millersview in a day and the probability distribution. Based on the information what is the expected value of P number of people in a car driving through Millersview on a given day?

P (number of people)	1	2	3	4	5	6
probability	38%	27%	14%	12%	6%	3%

(A) 2.1              (B) 2.3              (C) 2.8              (D) 3.0              (E) 3.5

53. A West Texas pump jack goes up and down pumping oil out of the ground. The pump jack is horizontal to the ground when not running. Once the pump is started it will reach a maximum height of 18 feet above the ground in 3 seconds and a minimum height of 2 feet above the ground in 6 seconds. What height above the ground will the pump be after pumping for 20 seconds? (nearest inch)

(A) 14'              (B) 14' 6"              (C) 16'              (D) 16' 3"              (E) 17' 4"

54. Using the equation  $y = 3\sin(4x - \frac{2\pi}{5}) - 1$  which of the following has the least numeric value?

(A) amplitude    (B) displacement    (C) frequency    (D) period    (E) phase shift

55. Let P, Q, and R be the real roots of  $x^3 - 7x^2 + 14x - 8 = 0$ . Find the harmonic mean of P, Q, and R.

(A)  $1\frac{1}{14}$               (B)  $2\frac{1}{3}$               (C)  $1\frac{1}{2}$               (D) 2              (E)  $1\frac{5}{7}$

56. Find the sum of all of the vertical, horizontal, and oblique asymptotes of the relation

$$y = \frac{4x^2 - 3}{2x^2 - 3x + 1}.$$

(A) 0              (B) 1              (C) 1.5              (D) 2              (E) 3.5

57. Find the area of the region bounded by  $x = y^2 - 2$  and  $x = -y^2 + 6$ .

(A)  $20\frac{2}{3}$               (B)  $21\frac{1}{3}$               (C) 16              (D)  $23\frac{1}{3}$               (E) 32

58. Willis Rong is going to take a multiple choice quiz that has 5 questions. Each question has 4 answer choice, 1 correct and 3 incorrect. If Willis is going to guess the answers at random, what is the probability he will get exactly 1 question correct? (nearest percent)

(A) 76%              (B) 40%              (C) 24%              (D) 20%              (E) 0.01%

59. The base 9 fraction  $\frac{5}{26}$  can be written as which of the following repeating decimals in base 9?

- (A)  $0.1777..._9$     (B)  $0.1555..._9$     (C)  $0.2111..._9$     (D)  $0.444..._9$     (E)  $0.256256256..._9$

60.  $A6B9C_{15} - 7D5E3_{15} = \underline{\hspace{2cm}}_{15}$

- (A)  $27BA_9$     (B)  $286B_9$     (C)  $285A_9$     (D)  $2765_9$     (E)  $2860_9$

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Regional • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. C  | 21. D | 41. E |
| 2. D  | 22. D | 42. B |
| 3. E  | 23. C | 43. A |
| 4. A  | 24. B | 44. D |
| 5. C  | 25. A | 45. E |
| 6. B  | 26. D | 46. B |
| 7. D  | 27. B | 47. C |
| 8. E  | 28. B | 48. D |
| 9. B  | 29. A | 49. C |
| 10. E | 30. C | 50. C |
| 11. E | 31. C | 51. C |
| 12. C | 32. D | 52. B |
| 13. A | 33. B | 53. A |
| 14. D | 34. A | 54. B |
| 15. A | 35. B | 55. E |
| 16. B | 36. B | 56. E |
| 17. E | 37. E | 57. B |
| 18. D | 38. C | 58. B |
| 19. A | 39. A | 59. A |
| 20. E | 40. B | 60. C |

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
--------------------	------------------

- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE  
Making a World of Difference

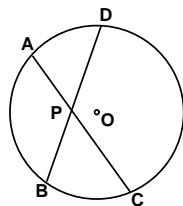
# Mathematics

## State • 2016



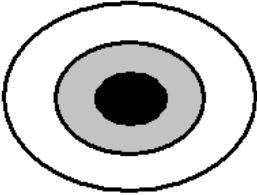
DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $5^2 + 3 \times (5 - 2^4) \div (5 - 2) \times 5 + (20 - 16)!$
- (A) 46.8                      (B) 9                      (C) 4                      (D)  $-6$                       (E)  $-9$
2. The *No Tread* tire store is having a sale on used tires. Noah Rubber needs to buy 4 tires for his old sedan. Which of the following is the least expensive deal?
- (A) \$17.50 each                      (B) 3 tires at \$22.25 each and get 1 free  
(C) buy 4 for \$72.00 get 10% off                      (D) buy 2 at \$25.00 each and get 2 for half price  
(E) buy 2 at \$21.00 each, 1 for \$15.00 and 1 for \$12.00
3. Simplify:  $\left( \frac{3x-1}{3x^2} \right) \div \left( \frac{3x+1}{3x} \right) \left( \frac{9x^2+6x+1}{9x^2-1} \right)$
- (A)  $\frac{3x+1}{x}$                       (B)  $\frac{3x+1}{3x-1}$                       (C)  $\frac{1}{x}$                       (D)  $\frac{3x-1}{3x+1}$                       (E)  $\frac{3x-1}{3}$
4. Let  $D = \{2, 1, 3, 4\}$  be the domain of relation  $g$  and  $R = \{k, 1, 2, 3\}$  be the range of relation  $g$ . Which of the following values of  $k$  would make relation  $g$  a one to one function?
- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) none of these
5. Given:  $\angle A$  is complementary to  $\angle C$ ;  $m\angle B = 123^\circ$ ; and  $\angle C$  is supplementary to  $\angle B$ . Find  $m\angle A$ .
- (A)  $67^\circ$                       (B)  $57^\circ$                       (C)  $53^\circ$                       (D)  $43^\circ$                       (E)  $33^\circ$
6. Given the circle with center O shown,  $AP = 3''$ ,  $DP = 5''$ , and  $AC = 10''$ . Find  $CP + BP$ .



- (A) 8.666... "                      (B) 10.333... "                      (C) 11.2"                      (D) 12"                      (E) 16"
7.  $\begin{array}{ccccccc} & P & & Q & & -1\frac{1}{8} & & R & & 2.75 \\ & | & & | & & | & & | & & | \end{array}$   
 The distances between the hash marks ( | ) are equal. Find  $P + Q + R$ .
- (A)  $-7.25$                       (B)  $-7.5625$                       (C)  $-6.375$                       (D)  $-5.625$                       (E)  $-8.25$
8. Evaluate:  $(0.272727\ldots)^{-1} + (0.41666\ldots)^{-1} + (0.4375)^{-1}$
- (A)  $1\frac{131}{397}$                       (B)  $1\frac{317}{2500}$                       (C)  $\frac{315}{877}$                       (D)  $8\frac{1}{3}$                       (E)  $8\frac{37}{105}$
9. If the roots of  $x^3 + bx^2 + cx + d = 0$  are  $-5$ ,  $-2$ , and  $4$ , then  $b + c + d$  equals:
- (A)  $-3$                       (B)  $-25$                       (C)  $-19$                       (D)  $-55$                       (E)  $-61$

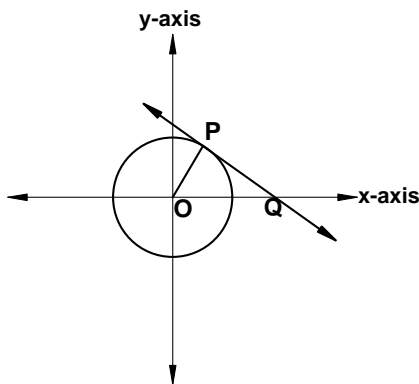


10. Line  $l$  contains the points  $(3, -2)$  and  $(-4, 1)$ . Which of the following equations of a line contain the point  $(0, 0)$  and is perpendicular to line  $l$ ?
- (A)  $7x - 3y = 0$  (B)  $3x + 7y = -5$  (C)  $7x - 3y = 5$  (D)  $3x - 7y = 0$  (E)  $3x + 7y = 0$
11. Point  $P(3, -5)$  lies on the  $x$ - $y$  plane. Point  $P$  is reflected across the line  $y = x$  to point  $Q$ . Point  $Q$  is translated  $+4$  units horizontally to point  $R$ . Point  $R$  is reflected across  $x$ -axis to point  $S$ . Point  $S$  is translated  $-4$  vertically to point  $T(x, y)$ . Find  $x + y$ .
- (A)  $-1$  (B)  $-4$  (C)  $-6$  (D)  $-7$  (E)  $-8$
12. The height of a right cylindrical cone is decreased by one-third and the radius is tripled. What is the ratio of the volume of the original cone to the volume of the new cone?
- (A) 1:3 (B) 1:6 (C) 1:9 (D) 3:1 (E) 9:1
13. The target shown consists of three concentric circles. The radius of the largest circle is 8". The radius of the smallest circle is 3". The radius of the middle circle is 5". Bill Tale shot an arrow that sticks in the target. What is the probability that the arrow does not stick in the smallest circle? (nearest percent)
- 
- (A) 14% (B) 36% (C) 69% (D) 86% (E) 91%
14. Sonya is creating a list of Mersenne primes using natural numbers,  $n$ . Which of the following values of  $n$  will not result in creating a Mersenne prime for her list?
- (A) 17 (B) 13 (C) 11 (D) 7 (E) 3
15. How many proper fractions in lowest terms have a denominator of 54?
- (A) 27 (B) 24 (C) 21 (D) 18 (E) 15
16.  $12301_4 - 2031_4 + 312_4 = \underline{\hspace{2cm}}_{16}$
- (A) 160 (B) 34 (C) 1B6 (D) 166 (E) 15A
17. Let  $g(x) = 5x - 2$  and  $h(x) = 5x + 2$ . Find  $g(h(4)) - h(g(5))$ .
- (A)  $-11$  (B)  $-9$  (C)  $-8$  (D)  $-5$  (E)  $-1$
18. Points  $(-2, -4)$  and  $(2, 5)$  are members of the function  $\{(x, y) \mid y = ax + b\}$ . Find  $a - b$ .
- (A) 1.75 (B) 2 (C) 2.5 (D) 2.5 (E) 3

19. Find  $a + b + c + d$  given the Fibonacci characteristic sequence:  $-4, a, b, 4, c, d, \dots$ .

- (A) 16                      (B) 4                      (C) 20                      (D) 12                      (E) 8

20. Find the perimeter of  $\triangle OPQ$  given that  $PQ$  is tangent to the circle at  $P$  and intersects the  $x$ -axis at  $Q$ ,  $m\angle PQO = 35^\circ$ , and the radius of the circle is  $5''$ . (nearest tenth)



- (A) 20.9"                      (B) 19.8"                      (C) 19.3"                      (D) 17.8"                      (E) 16.6"

21. Given that the set of even numbers continue in the triangular pattern shown below, find the middle term in row 21.

				2	(row 1)
			4	6	(row 2)
		8	10	12	(row 3)
	14	16	18	20	(row 4)
22	24	26	28	30	(row 5)
				...	( ... )

- (A) 422                      (B) 440                      (C) 442                      (D) 462                      (E) 466

22. Given the sequence  $2, 0, 5, 8, 26, 63, k, 440, 1157, \dots$  find  $k$ .

- (A) 156                      (B) 163                      (C) 168                      (D) 170                      (E) 195

23. If  $a_1 = -2$ ,  $a_2 = 0$ ,  $a_3 = 1$  and  $a_n = (a_{n-2}) - (a_{n-3}) + (a_{n-1})$  for  $n \geq 4$ , then  $a_6$  equals:

- (A) 3                      (B) 4                      (C) 6                      (D) 7                      (E) 10

24. If the pattern of the sequence  $1, 3, 7, 13, 21, 31, \dots$  continues, which term has a value of 1,407?

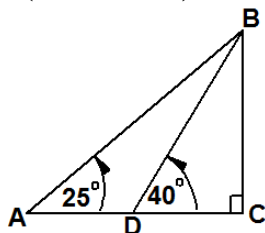
- (A) 40<sup>th</sup>                      (B) 38<sup>th</sup>                      (C) 37<sup>th</sup>                      (D) 35<sup>th</sup>                      (E) 30<sup>th</sup>

25. Let  $f_0 = 0$ ,  $f_1 = 1$ ,  $f_2 = 1$ ,  $f_3 = 2$ ,  $f_4 = 3$ , ...,  $f_k = 75,025$ , ... be the terms of the Fibonacci sequence. Find  $f_{k+3}$ .

- (A) 317,811                      (B) 196,418                      (C) 317,791                      (D) 307,302                      (E) 514,229

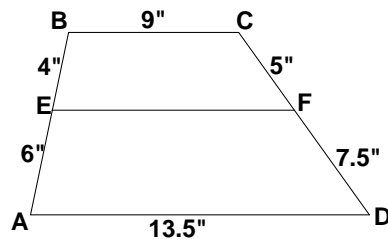
26. Mr White interviewed 100 state meet competitors. 52 of them liked number sense, 63 liked math, and 25 liked calculator. 24 liked number sense and math. 9 liked number sense and calculator. 11 liked math and calculator. All of them liked at least one contest. If 4 liked all three contests, how many liked just one contest?
- (A) 68                      (B) 64                      (C) 60                      (D) 56                      (E) 52
27. Mie Yung is twice as old as Soh Yung. Soh Yung is eight years older than Tu Yung. In five years Mie will be three times as old as Tu. How old is Soh now?
- (A) 4                      (B) 6                      (C) 10                      (D) 12                      (E) 14
28. Lotta Dough, Les Dough, and Noah Dough had a total of \$150.00. At the Cheep Mall, Lotta spent half as much as Noah spent and Les spent \$10.00 less than Lotta spent. How much money did they have when they left the mall?
- (A) \$0.00                      (B) \$10.00                      (C) \$12.50                      (D) \$18.50                      (E) \$20.00
29. Dee Putter digs three holes in his back yard to practice his putting. Hole #2 is 20 feet from hole #1 on a bearing of  $160^\circ$ . Hole #3 is 30 feet from hole #2 on a bearing of  $330^\circ$ . What is the bearing from hole #3 to hole #1? (nearest degree)
- (A)  $113^\circ$                       (B)  $119^\circ$                       (C)  $131^\circ$                       (D)  $169^\circ$                       (E)  $209^\circ$
30. Trudy Frudy mixed 9 gallons of *Green Apple Ade* with 8 gallons of *Red Apple Ade* containing 45% apple cider to make a fruit drink containing 30% apple cider. What percent of apple cider did *Green Apple Ade* contain?
- (A) 15%                      (B)  $16\frac{2}{3}\%$                       (C) 18%                      (D)  $18\frac{3}{4}\%$                       (E) 21%
31. Buck Doenater has a bag containing seven \$1 bills, two \$5 bills, one \$10 bill and four \$20 bills. He randomly selects one bill from the bag and gives it to the homeless shelter. The odds in favor of Buck selecting a \$20 bill is:
- (A)  $\frac{2}{7}$                       (B)  $\frac{1}{3}$                       (C)  $\frac{4}{5}$                       (D)  $\frac{1}{4}$                       (E)  $\frac{2}{5}$
32. Let  $T_n$  be the  $n$ th triangular number,  $S_n$  be the  $n$ th square number,  $P_n$  be the  $n$ th pentagonal number, and  $H_n$  be the  $n$ th hexagonal number. Then  $T_6 + S_5 + P_4 + H_3 = ?$
- (A) 74                      (B) 76                      (C) 79                      (D) 83                      (E) 88

33. Find BD if  $AB = 35$  cm. (nearest cm)



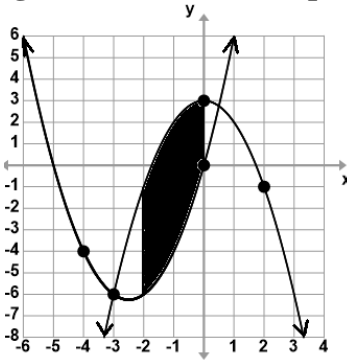
- (A) 17 cm                      (B) 19 cm                      (C) 21 cm                      (D) 23 cm                      (E) 25 cm

34. Given the trapezoid shown where  $AD \parallel EF \parallel BC$ , find EF. (nearest tenth)



- (A) 10.8 "      (B) 11.5 "      (C) 11.8 "      (D) 11.3 "      (E) 11.0 "

35. Find the area of the shaded region between the two parabolas. (square units).



- (A)  $9\frac{1}{6}$       (B)  $9\frac{5}{6}$       (C) 10      (D)  $10\frac{1}{3}$       (E)  $10\frac{2}{3}$

36. Point P(− 5, 3) and point Q(2, − 4) lie on the x-y plane. P is reflected across the x-axis to  $P_1$ . Q is reflected across the y-axis to  $Q_1$ . What is the distance between  $P_1$  and  $Q_1$ ? (nearest tenth)

- (A) 1.6      (B) 3.2      (C) 5.1      (D) 7.1      (E) 9.9

37. How many unique rectangles exist such that the lengths of the sides are integers, the perimeter is less than or equal to 144 units, and the area is 144 square units?

- (A) 4      (B) 6      (C) 8      (D) 10      (E) 12

38. The base 5 fraction  $\frac{412}{1300}$  can be written as which of the following repeating decimals in base 5?

- (A) 0.2141414...<sub>5</sub>      (B) 0.12434343...<sub>5</sub>      (C) 0.24313131...<sub>5</sub>  
 (D) 0.23141414...<sub>5</sub>      (E) 0.10242424...<sub>5</sub>

39. Miss Dee Shought shoots 5 free throws. She averages making 70% of her free throws. What is the probability that she makes exactly 3 of the 5 shots? (nearest percent)

- (A) 14%      (B) 20%      (C) 31%      (D) 42%      (E) 60%

40. If  $2\log_5(3) \times \log_9(k) = \log_5(3) - 1$ , then k equals \_\_\_\_\_.

- (A)  $1\frac{2}{3}$       (B)  $\frac{2}{3}$       (C) 1.6      (D) 5.4      (E) 0.6

41. Point P has polar coordinates of  $(-4, \frac{2\pi}{3})$  and rectangular coordinates of  $(x, y)$ . Where does point P lie on the Cartesian coordinate plane?
- (A) QI                      (B) QII                      (C) QIII                      (D) QIV                      (E) on the x-y axis
42. Which of the following statements about  $f(x) = e^{(-x)} + x$  is/are true?
- I.  $f^{-1}(x)$  is a function.  
 II. the domain of  $f(x)$  is  $\{x | x \in \{\text{Reals}\}\}$ .  
 III. the range of  $f(x)$  is  $\{y | y \geq 0\}$ .
- (A) I & II                      (B) II only                      (C) II & III                      (D) all of them                      (E) none of them
43. The function  $f(x) = x^4 - x^2 + 1$  is increasing over which of the following intervals?
- I.  $[-1.5, -0.5]$                       II.  $[0, 0.5]$                       III.  $[0.5, 1.5]$
- (A) none of these                      (B) I only                      (C) II & III                      (D) I & III                      (E) all of these
44.  $\{(x, y) | x, y \in \{\text{Integers}\}, -8 \leq x \leq 8, \text{ and } -4 \leq y \leq 4\}$  is the solution set of  $2x - 5y = 4$ . How many such ordered pairs exist?
- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6
45. Point B(3, 5) and point C(5, 3) are opposite vertices of a rhombus. Point A(1, 1) and point D(x, y) are the other vertices of the rhombus. Find the distance from point A to point D.
- (A)  $6\sqrt{2}$                       (B)  $5\sqrt{2}$                       (C)  $3\sqrt{5}$                       (D)  $2\sqrt{5}$                       (E)  $2\sqrt{2}$
46. If you connect the centers of adjacent faces of a regular tetrahedron with segments these segments will form the edges of a(n):
- (A) triangle                      (B) icosahedron                      (C) tetrahedron                      (D) decahedron                      (E) octahedron
47. Find the sum of the values of  $\theta$  of  $\left\{ \theta \mid 4\sin^3(\theta) + 2\sin^2(\theta) - 2\sin(\theta) - 1 = 0, \theta \in \left[-\frac{3\pi}{2}, \pi\right] \right\}$ ? (nearest hundredth)
- (A)  $-9.16$                       (B)  $-7.07$                       (C)  $-6.81$                       (D)  $-4.44$                       (E)  $-3.14$
48. If  $A = \begin{bmatrix} 0 & 5 \\ 2 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 0 \\ 1 & 6 \end{bmatrix}$ , then  $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Find  $(a + d) - (b + c)$ .
- (A)  $-12$                       (B) 2                      (C)  $-10$                       (D) 22                      (E)  $-9$
49. Find the sum of the y-values of the inflection points of  $y = x^4 - x^2 - 1$ .
- (A)  $-4\frac{1}{6}$                       (B)  $-2\frac{5}{18}$                       (C)  $-2$                       (D) 0                      (E)  $1\frac{5}{6}$

50. Let  $f(x) = \frac{1}{\sqrt{1+x^2}}$ , where  $x \in [0, \infty)$ . Find  $f^{-1}(x)$ .

- (A)  $\frac{\sqrt{x^2-1}}{x}$  (B)  $-\frac{\sqrt{1+x^2}}{x}$  (C)  $\frac{x\sqrt{1-x^2}}{1-x}$  (D)  $\frac{\sqrt{1-x^2}}{x}$  (E)  $\frac{x\sqrt{1+x^2}}{1+x}$

51. Lotta Sense is demonstrating a probability experiment by flipping a penny, a dime, and a quarter. Each coin has a head and a tail. She writes the sample space on the board listing them in the order they were flipped. How many possible elements are there in the sample space?

- (A) 64 (B) 48 (C) 32 (D) 24 (E) 8

52.  $B75A_{16} - 6C84_{16} = \underline{\hspace{2cm}}_4$

- (A) 11010312 (B) 10223112 (C) 11110312 (D) 10101312 (E) 10113212

53. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$  be the terms of the Fibonacci sequence. Find  $k$  if  $f_k = 5,702,887$ .

- (A) 31 (B) 23 (C) 34 (D) 28 (E) 20

54. Let  $[(5 - 2i)(4 - 5i)] \div (2 - 5i) = a + bi$ , then  $a + b = ?$

- (A)  $7\frac{22}{29}$  (B)  $8\frac{1}{21}$  (C) 10 (D)  $5\frac{24}{29}$  (E)  $9\frac{4}{7}$

55. The function  $f(x) = \sqrt{\frac{9-x^2}{4-x}}$  is continuous on which of these intervals?

I.  $(-\infty, -3)$  II.  $[-3, 3)$  III.  $[3, 4]$  IV.  $(4, +\infty)$

- (A) II only (B) I & III (C) II & IV (D) II, III, & IV (E) all of them

56. The graph of the polar equation  $r^2 = 4 \sin(2\theta)$  is called a(n) \_\_\_\_\_.

- (A) looped limaçon (B) lemniscate (C) convex limaçon (D) circle (E) cardioid

57. Roland Bones has a special cube. Each face is numbered using the digits 1, 1, 2, 3, 5, and 8. Roland rolls the cube. What is the probability that the top face is greater than 1 or that it is a prime number? (nearest %)

- (A)  $33\frac{1}{3}\%$  (B)  $58\frac{2}{3}\%$  (C)  $83\frac{1}{3}\%$  (D) 50% (E)  $66\frac{2}{3}\%$

58. The length of the sides of  $\triangle PQR$  are the roots of  $f(x) = x^3 - 9x^2 + 26x - 24$ . The area of  $\triangle PQR$  is  $k$ . Find  $k^2$ .

- (A)  $8\frac{1}{8}$  (B)  $8\frac{3}{16}$  (C)  $8\frac{7}{16}$  (D)  $8\frac{5}{8}$  (E) 9

59. The Oranges and the Maroons are playing three games of Texball. The Maroons are twice as likely to win any game as is the Oranges. What is the probability that the Maroons will win two or more games?
- (A) 78%      (B) 74%      (C)  $66\frac{2}{3}\%$       (D) 64%      (E) 50%
60. Farmer Fudd grew 5 carrots for Buggy Bunnie. The lengths of the carrots were 3", 4.5", 6", 3.25", and 8". What is the sample standard deviation of the lengths of these carrots? (nearest hundredth)
- (A) 1.86      (B) 2.08      (C) 2.22      (D) 2.47      (E) 4.33

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • State • 2016  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. D  | 21. C | 41. D |
| 2. C  | 22. D | 42. A |
| 3. C  | 23. C | 43. A |
| 4. D  | 24. B | 44. C |
| 5. E  | 25. A | 45. A |
| 6. C  | 26. B | 46. C |
| 7. A  | 27. E | 47. B |
| 8. E  | 28. A | 48. E |
| 9. D  | 29. C | 49. B |
| 10. A | 30. B | 50. D |
| 11. E | 31. E | 51. B |
| 12. B | 32. D | 52. B |
| 13. D | 33. D | 53. C |
| 14. C | 34. E | 54. D |
| 15. D | 35. E | 55. C |
| 16. E | 36. B | 56. B |
| 17. B | 37. B | 57. E |
| 18. A | 38. D | 58. C |
| 19. A | 39. C | 59. B |
| 20. A | 40. E | 60. B |