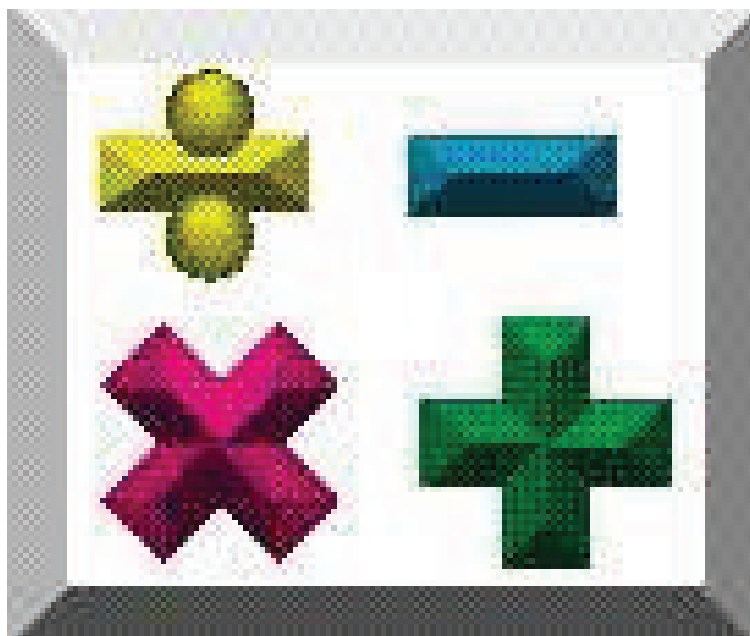




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2010



**WRITE ALL ANSWERS WITH
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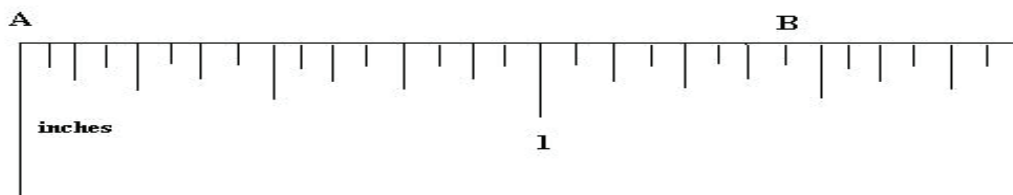
1. Evaluate: $30 - 24 \div 18 \times 12 + 6$

- (A) 6 (B) 10 (C) 20 (D) 35.888... (E) 350

2. Reid Moore went to the Ye Olde Book store to buy 3 copies of the same book for gifts. The regular price of the book is \$19.95. Because he is buying 3 copies he gets 25% off of the regular price of the second copy and 40% off the regular price of the third copy. What would the total cost of the 3 books be before taxes? (to the nearest cent)

- (A) \$ 26.93 (B) \$ 32.92 (C) \$ 38.91 (D) \$ 40.40 (E) \$ 46.88

3. Using the partial ruler shown below, find the distance from A to B.



- (A) $1\frac{1}{8}$ " (B) $1\frac{1}{4}$ " (C) $1\frac{3}{8}$ " (D) $1\frac{7}{16}$ " (E) $1\frac{1}{2}$ "

4. Which of the following is not a solution to $|8x - 6| - 4 \geq 2$?

- (A) $-2\frac{1}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $1\frac{4}{5}$ (E) 2

5. The function $f(x) = x^2 - x - 12$ crosses the x-axis at two points. Find the distance between the two points.

- (A) 8 (B) 7 (C) 6 (D) 4 (E) 1

6. A male zebra fish has 8 stripes. A female zebra fish has 7 stripes. What is the ratio of male fish to female fish, if the total number of stripes on all of the zebra fish in an aquarium totals 87?

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$ (E) $\frac{3}{1}$

7. A box contains four rods whose lengths are 2", 3", 5", and 7". How many different triangles can be made using only three rods at a time.

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. A right cylinder water tank is 6 feet high and has an inside radius of 3 feet. The amount of water in the tank is 75% of its maximum capacity. How much water is in the tank? (nearest gallon)

- (A) 1270 gal (B) 635 gal (C) 734 gal (D) 317 gal (E) 952 gal

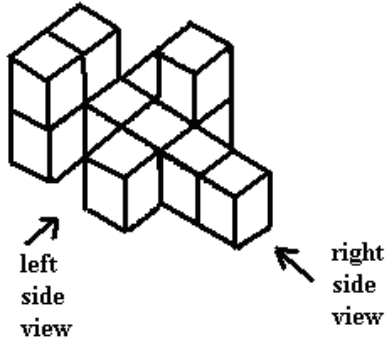
9. The region bounded by two radii of a circle and their intercepted arc is called a:

- (A) slice of pi (B) semicircle (C) secant (D) sector (E) segment

10. Noah Sense has 28 coins consisting of pennies, nickels, and quarters. He has four times as many nickels as pennies and half as many quarters as nickels. How much money does he have?

(A) \$.86 (B) \$ 2.36 (C) \$ 2.84 (D) \$ 1.88 (E) \$.78

11. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the top view of this figure has a perimeter of:



(A) 30 cm (B) 18 cm (C) 16 cm (D) 15 cm (E) 12 cm

12. If $8^{(k-1)} = 16^{(3k)}$, then $4^{(k-1)} = ?$

(A) 256 (B) $\frac{1}{64}$ (C) 2 (D) $\sqrt[3]{4}$ (E) $1\frac{1}{3}$

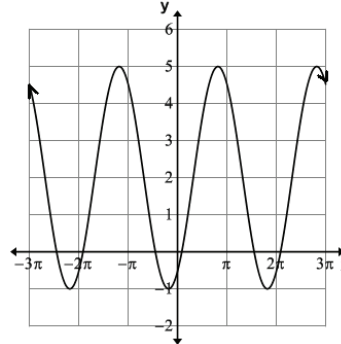
13. Babe, Dizzy, and Yogi are playing "toss and catch" with a baseball. The bearing from Babe to Dizzy is 254° . The bearing from Yogi to Dizzy is 344° . The bearing from Yogi to Babe is 32° . The distance from Yogi to Dizzy is 20 feet. How far is it from Yogi to Babe? (nearest inch)

(A) 29 ' 11 " (B) 28 ' 3 " (C) 26 ' 11 " (D) 22 ' 3 " (E) 18 ' 1 "

14. Determine the frequency of $y = 2 - 3 \cos 10x$. (nearest tenth)

(A) 0.1 (B) 0.6 (C) 1.6 (D) 5 (E) 15.7

15. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



(A) $2 + 3\sin(x - 1)$ (B) $3 + 2\cos(x - 1)$ (C) $2 - 3\sin(x + 1)$
 (D) $2 - 3\cos(x - 1)$ (E) $2 + 3\sin(x + 1)$

16. Which of the following is a reference angle for 456° ?

- (A) 6° (B) 16° (C) 44° (D) 84° (E) 89°

17. Find the determinant of the 2×2 matrix $\begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$.

- (A) 29 (B) 15 (C) 14 (D) 1 (E) -1

18. Given the arithmetic sequence 15, a, b, c, 47, ..., find $a + b + c$.

- (A) 24 (B) 32 (C) 78 (D) 93 (E) 108

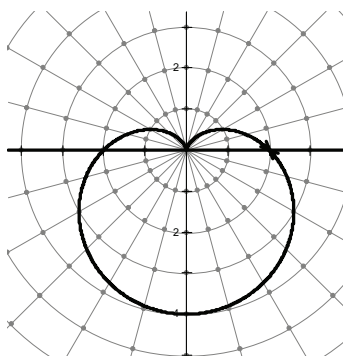
19. In the binomial expansion of $(3x - 1)^5$, the coefficient of the fourth term is:

- (A) -90 (B) -45 (C) -10 (D) 30 (E) 270

20. Find an equation of the line tangent to the curve $y = x^3 - 2x^2$ at the point $(1, -1)$.

- (A) $y = -x$ (B) $y = 6x - 4$ (C) $y = -\frac{2x}{3}$ (D) $y = 3$ (E) $y = 2 - x$

21. Which of the following polar equations will produce this graph on a polar grid?



- (A) $r = 2\sin \theta - 2$ (B) $r = 4 + 2\sin \theta$ (C) $r = 2 - 2\sin \theta$

- (D) $r = 2 - 4\cos \theta$ (E) $r = 2 + 2\cos \theta$

22. The area (in square units) of the region bounded by $y = -x^2 - 4x$ and $y = 0$ is:

- (A) $9\frac{1}{4}$ (B) $9\frac{5}{6}$ (C) $10\frac{1}{2}$ (D) $10\frac{2}{3}$ (E) $10\frac{3}{4}$

23. A box contains circular poker chips that are congruent in shape but not color. There are red ones, white ones, and blue ones. Drew Goode randomly draws out a chip. He gets 5 points if it is a blue one, 1 point for a white one, and he loses 3 points for a red one. The probability of drawing out a red one is 25%, a blue one is 60%, and a white one is 15%. What is his mathematical expectation on any one draw?

- (A) 5.0 (B) 3.9 (C) 3.0 (D) 2.4 (E) 2.1

24. What are the odds that a factor of 2010 is a prime number?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) 1

25. The number of integers that satisfy the inequality $\frac{4}{15} \leq \frac{n}{5} \leq 1\frac{1}{30}$ is:

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

26. Simplify: $\frac{(n+1)! - (n-1)!}{(n-2)!}$

- (A) $n^2 + 1$ (B) $n^3 + 2n^2 - 2n + 1$ (C) $\frac{2(n+1)}{n-2}$ (D) $\frac{2}{n(n-1)}$ (E) $n^3 - 2n + 1$

27. The formula $e^{ix} = \cos x + i \sin x$, where e is the base of the natural logarithm and i is the imaginary unit, is named after:

- (A) Rene Descartes (B) Claudius Ptolemy (C) Theano of Crotona
(D) Leonard Euler (E) Eratosthenes of Cyrene

28. The odd numbers from 1 to 17 are to be placed in this magic square in which the rows and columns have the same sum. Find the value of x .

	1	
5		13
x		

- (A) 3 (B) 7 (C) 9 (D) 11 (E) 15

29. $P = \{p, l, u, s\}$, $Q = \{m, i, n, u, s\}$, and $R = \{t, i, m, e, s\}$. How many elements are in $(P \cup Q) \cap (P \cup R)$?

- (A) 10 (B) 6 (C) 5 (D) 4 (E) 2

30. The number 12010 in base 3 is equivalent to the number $wxyz$ in base 5, where w , x , y , and z are digits. Find $w + x + y + z$.

- (A) 10 (B) 9 (C) 8 (D) 6 (E) 3

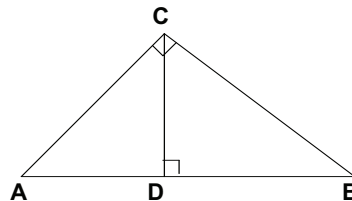
31. Simplify: $a^5 \div b^{-4} \times a^{-4} \times b^5 \div a^3 \times b^{-3}$

- (A) $a^{-2} b^6$ (B) $a^4 b^{-2}$ (C) $a^{-2} b^{12}$ (D) $a^4 b^2$ (E) $a^2 b^{-6}$

32. Simplify: $\frac{x^2 - 9}{4x + 12} \div \frac{x^2 - x - 6}{x^2 + 2x}$

- (A) $\frac{x}{4}$ (B) $\frac{4}{x+3}$ (C) $\frac{x-3}{4(x+3)}$ (D) $\frac{4}{x}$ (E) $\frac{x+3}{4}$

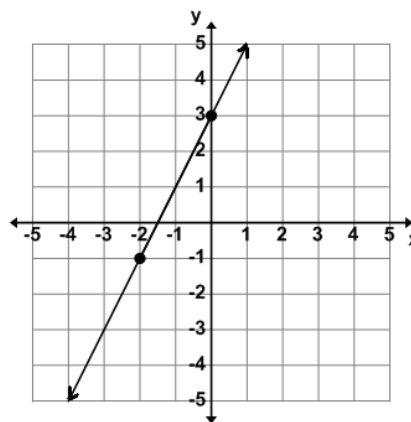
33. The distance from Abilene to Dallas by way of I30 is 185 miles. Ima Slow is leaving Abilene on I30 at 9:00 a.m. driving toward Dallas at 55 mph. Ura Quick is leaving Dallas on I30 at 9:00 a.m. driving toward Abilene at 70 mph. What time will they meet? (nearest minute)
- (A) 10:48 a.m. (B) 10:41 a.m. (C) 10:29 a.m. (D) 10:19 a.m. (E) 10:10 a.m.
34. $\angle A$ and $\angle B$ are complementary angles. $\angle A$ and $\angle C$ are supplementary angles. Find $m\angle C$ if $m\angle A = 2x - 5$ and $m\angle B = x + 2$.
- (A) 121° (B) 149° (C) 135° (D) 123° (E) 147°
35. If $a_1 = 2$, $a_2 = 4.5$, and $a_3 = 7$ are the first 3 terms of an arithmetic sequence, then $a_9 = ?$
- (A) 17 (B) 19.5 (C) 21 (D) 22 (E) 24.5
36. The graph of $4x^2 + 9y^2 - 16x + 18y = 2$ is a(n):
- (A) parabola (B) line (C) hyperbola (D) ellipse (E) circle
37. The eccentricity of the hyperbola $4x^2 - y^2 = 4$ is:
- (A) $\sqrt{5}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\sqrt{15}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{\sqrt{5}}{2}$
38. If $\cos \theta < 0$ and $\tan \theta > 0$ which quadrant will θ terminate in?
- (A) QI or QII (B) QI only (C) QII only (D) QIII only (E) QII or QIII
39. Let $\|V_1\| = 15$ and $\|V_2\| = 9$, where the direction angles of V_1 and V_2 are 20° and 80° , respectively. Find $\|V_1 + V_2\|$. (nearest tenth)
- (A) 23.6 (B) 17.5 (C) 20.7 (D) 12.0 (E) 21.0
40. Find AD if AB = 90 cm. and AC = 50 cm. (nearest cm)



- (A) 67 cm (B) 19 cm (C) 28 cm (D) 60 cm (E) 45 cm
41. $\int (-x \sin x) \, dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.
- (A) $\cos x^2$ (B) $x \cos x - \sin x$ (C) $-\sin x \cos x$ (D) $-x \cos x$ (E) $x \sin x - \cos x$

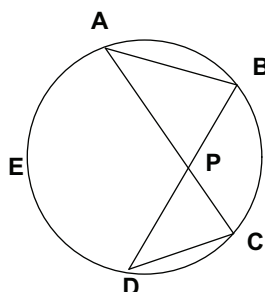
42. If $f''(x) = 6$ and $f'(-1) = -8$ and $f(1) = 2$, then $f(-2) = \underline{\hspace{2cm}}$.
- (A) 20 (B) 17 (C) 8 (D) -7 (E) -14
43. Find the instantaneous rate of change of the reciprocal of a number with respect to the number when the number is 4.
- (A) $-\frac{1}{16}$ (B) $-\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{1}{16}$
44. How many different letter arrangements can be made by rearranging the letters in the word 'LETTER'?
- (A) 180 (B) 21 (C) 120 (D) 24 (E) 360
45. Willie Lawkit can't remember the combination to the padlock shown. He knows that the first number is greater than 30, the second number is a positive Fibonacci number, and the third number is a factor of 30. How many combinations can he try to open the lock?
-
- (A) 25 (B) 378 (C) 576 (D) 72 (E) 480
46. The operation " \triangle " is defined by: $a \triangle b = a^b - b^a$. What is the value of $(0 \triangle 1) \triangle (2 \triangle 3)$?
- (A) -1 (B) 0 (C) 1 (D) 2 (E) 4
47. $3(x + 4) = 5$ and $3(4 + x) = 5$ is an example of the _____ property.
- (A) reflexive (B) commutative (C) identity (D) associative (E) distributive
48. Slim Sails rents kayaks and life vests for white water rafting. The kayak rental fee last year was \$40 and the life vest rental fee last year was \$12. This year, the kayak rental fee increased 15% and the life vest fee decreased 25%. What is the overall percent increase in rental fees for the kayak and vest from last year to this year? (nearest tenth)
- (A) 10.0% (B) 9.1% (C) 8.3% (D) 6.5% (E) 5.8%
49. If $-3(2 - x) = 2(x + 3)$ then $(2x - 3)$ equals:
- (A) 12 (B) -9 (C) 21 (D) -3.4 (E) 1.8
50. The area of a right isosceles triangle is 12.5 cm^2 . Its perimeter is: (nearest tenth).
- (A) 18.7 cm (B) 11.4 cm (C) 21.2 cm (D) 11.7 cm (E) 17.1 cm

51. Find the slope of a line perpendicular to the line drawn in the graph below.



- (A) -2 (B) -1.5 (C) $-.5$ (D) $.5$ (E) 2

52. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O and point E lies on circle O. Which of following is a true statement?



- (A) $m\angle ABD = \frac{1}{2} \times m\widehat{AED}$ (B) $m\angle BPC = \frac{1}{2} \times m\widehat{CB}$ (C) $m\angle ACD = 2 \times m\widehat{AED}$
 (D) $m\angle APD = m\angle ABP + m\angle DCP$ (E) $m\angle ABP + m\angle BDC$

53. A regular polygon has S sides and D diagonals. If the polygon had one more side, $S + 1$, it would have $D + 10$ diagonals. The polygon is a:

- (A) octagon (B) nonagon (C) decagon (D) undecagon (E) dodecagon

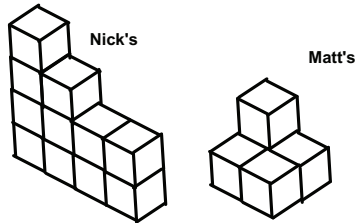
54. Let $f(x) = 2 - 5x$ and $g(x) = 3x + 5$. If $h(x)$ is the inverse function of $\frac{f(x)}{g(x)}$, then $h(-4) = ?$

- (A) $-\frac{22}{7}$ (B) $-\frac{18}{17}$ (C) $\frac{7}{22}$ (D) $\frac{17}{18}$ (E) 4

55. $\sin \theta \sec \theta + \cos \theta \csc \theta$ is equivalent to:

- (A) $\frac{\csc \theta}{\cot \theta}$ (B) $\frac{\tan^2 \theta}{\csc \theta}$ (C) $\tan^2 \theta + 1$ (D) $\frac{\sec^2 \theta}{\tan \theta}$ (E) $\sec^2 \theta - 1$

56. Willie Ketchit drops a golfball from a height of 10 meters. Each time it hits the ground it rebounds to a height of 50% of the distance it fell. Find the total distance the golfball travels when it reaches the ground the third time. (nearest tenth)
- (A) 35.0 m (B) 32.5 m (C) 30.0 m (D) 28.5 m (E) 25.0 m
57. The polynomial $2x^4 - 8x^2 + x + 5$ has at most ____ negative zeros.
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
58. Coach Winters has 4 seniors, 5 juniors, 3 sophomores, and 4 freshmen on her math team. How many ways can she form practice groups of four members consisting of one member from each of the grade levels?
- (A) 16 (B) 81 (C) 108 (D) 240 (E) 256
59. Romeo, Juliet, and three classmates are randomly assigned seats in a row of five chairs. What is the probability that Romeo and Juliet will be seated next to each other?
- (A) 20% (B) 25% (C) 30% (D) 35% (E) 40%
60. Matt and Nick constructed two buildings using identical cubes. Matt's building weighs 200 g, and Nick's building weighs 600 g. How many of the cubes in Nick's building are hidden and cannot be seen in the figure?



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation A • 2010
Answer Key**

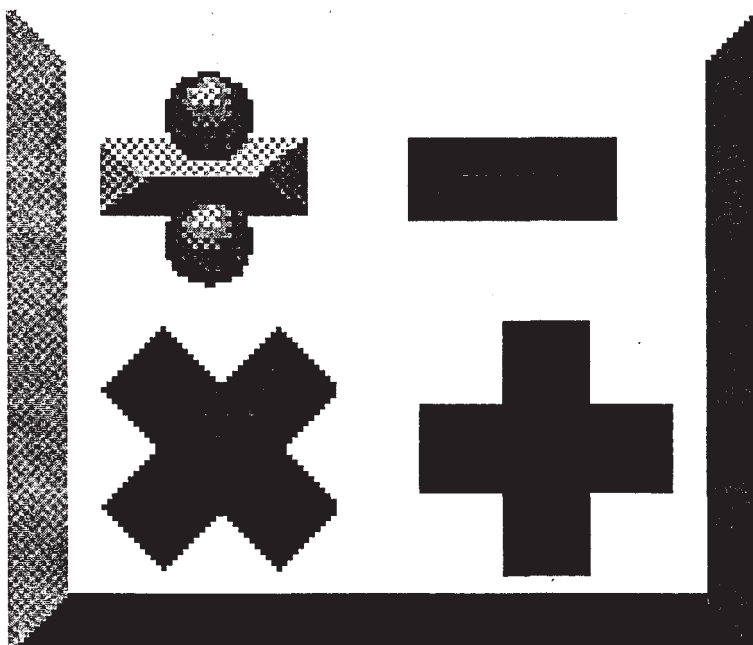
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|-------|-------|-------|
| 1. C | 21. C | 41. B |
| 2. E | 22. D | 42. B |
| 3. D | 23. D | 43. A |
| 4. C | 24. B | 44. A |
| 5. B | 25. B | 45. C |
| 6. A | 26. E | 46. B |
| 7. B | 27. D | 47. B |
| 8. E | 28. B | 48. E |
| 9. D | 29. B | 49. C |
| 10. C | 30. D | 50. E |
| 11. B | 31. A | 51. C |
| 12. B | 32. A | 52. A |
| 13. A | 33. C | 53. D |
| 14. B | 34. D | 54. A |
| 15. A | 35. D | 55. D |
| 16. D | 36. D | 56. E |
| 17. E | 37. A | 57. C |
| 18. D | 38. D | 58. D |
| 19. A | 39. E | 59. E |
| 20. A | 40. C | 60. D |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2010



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1. Evaluate: $\frac{7}{8} + \frac{3}{4} \div (\frac{5}{8} - \frac{1}{2}) \times \frac{3}{8} + \frac{1}{4} - \frac{1}{8}$

(A) $3\frac{1}{4}$

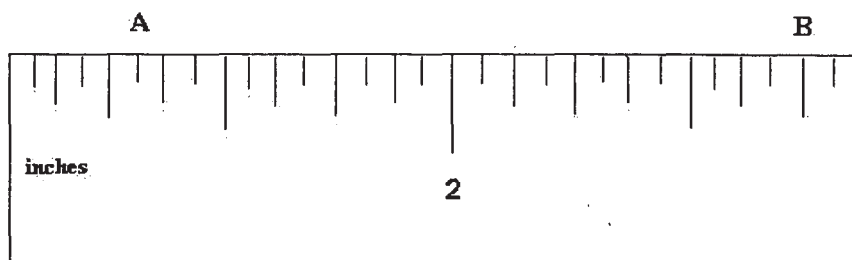
(B) $2\frac{45}{64}$

(C) $2\frac{9}{16}$

(D) $1\frac{3}{32}$

(E) $\frac{73}{80}$

2. Using the partial ruler shown below, find the distance from A to B.



(A) $1\frac{7}{8}$ "

(B) $1\frac{1}{2}$ "

(C) $1\frac{7}{16}$ "

(D) $1\frac{9}{16}$ "

(E) $1\frac{3}{4}$ "

3. If $ax + b = c$ and $c = dx + e$, then $ax + b = dx + e$ is an example of the _____ property.

(A) reflexive

(B) associative

(C) symmetric

(D) distributive

(E) transitive

4. May B. Fishy has a salt water aquarium. She mixes 5 gallons of water with some salt to make a 20% saline solution. The fish require a 16% solution. How much water will she have to add to make the required 16% saline solution?

(A) 200 oz

(B) 160 oz

(C) 128 oz

(D) 120 oz

(E) 96 oz

5. Find $f(5) + f(-1) + f(2)$ if $f(x) = \begin{cases} x - 3 & \text{if } x < 0 \\ 3x & \text{if } 0 < x < 3 \\ 3 - x & \text{if } x > 3 \end{cases}$

(A) -3

(B) 0

(C) 1

(D) 3

(E) 6

6. If $y = 1 - x$ and $y = \frac{2}{x}$ then $(x + y)(x^2 - xy + y^2) = ?$

(A) 7

(B) -2

(C) -5

(D) 8

(E) 3

7. Which of the following are the side lengths of a scalene acute triangle?

(A) 9, 40, 41

(B) 4, 7, 11

(C) 9, 10, 11

(D) 5, 5, 8

(E) 8, 7, 14

8. The point (6 , - 6) is rotated 60 degrees clockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

(A) (- 6.7 , - 2.2)

(B) (- 8.2 , - 2.2)

(C) (5.1 , - 8.2)

(D) (- 8.2 , - 1.1)

(E) (8.2 , 2.2)

9. Find the quotient: $(x^4 + 2x^3 - 10x^2 + 22x - 15) \div (x^2 - 2x + 3)$

(A) $x^2 + 4x - 5$

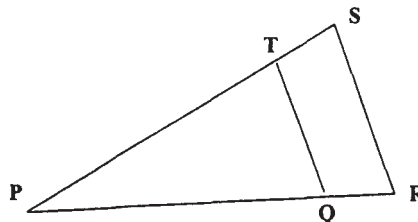
(B) $x^2 + 5x - 6$

(C) $x^2 + 4x + 5$

(D) $x^2 - 4x + 3$

(E) $x^2 - 4x - 5$

10. In $\triangle PRS$, $QT \parallel RS$, $RS = 4$, $QT = 3$, $ST = x$, and $PT = x + 5$. Find PS .



- (A) 12 (B) 10 (C) 7.5 (D) 5 (E) not enough information given

11. The graph of $x^2 - 2xy + y^2 + 0x + 0y + 0 = 0$ is a _____.

- (A) point (B) line (C) pair of lines (D) ellipse (E) hyperbola

12. Les Moolah has 28 coins. The coins are nickels and quarters and have a total value of \$ 4.00. How many more nickels than quarters does Les have?

- (A) 5 (B) 4 (C) 3 (D) 2 (E) 1

13. Which of the following is equivalent to $\frac{\sin \theta \tan \theta}{\sin (90^\circ - \theta)} + \frac{\cot \theta}{\tan (90^\circ - \theta)}$?

- (A) $\sec^2 \theta$ (B) $\cos \theta$ (C) $\tan^2 \theta$ (D) $\sin \theta$ (E) 1

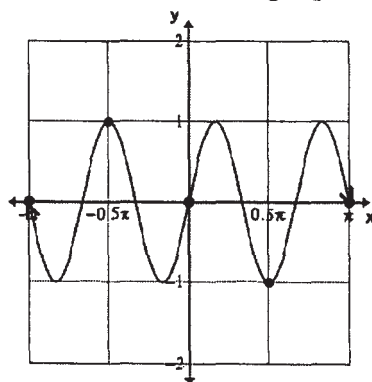
14. If $\cos x - \sin x = a$ and $\cos x + \sin x = b$, then $\cos^2 2x = ?$

- (A) $2ab$ (B) $a - b$ (C) $a + b$ (D) $a^2 - 2ab + b^2$ (E) $a^2 b^2$

15. Let $\|V_1\| = 9$, $\|V_2\| = 8$, where the direction angles of V_1 and V_2 are 60° and 150° , respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

- (A) 42° (B) 48° (C) 102° (D) 108° (E) 120°

16. Which of the following is true about the relation graphed below?



- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
(D) It is neither an even nor an odd function (E) It is a one-to-one function.

17. Integers x & y exist such that $x = 2y$ and the arithmetic mean of x & y is 1 more than the harmonic mean of x & y . Find the geometric mean of x & y .

- (A) $3\sqrt{2}$ (B) 9 (C) $2\sqrt{3}$ (D) 6 (E) $6\sqrt{2}$

18. Find k if $x + 4$ is a factor of $x^3 - x^2 + kx + 12$.

- (A) 3 (B) -17 (C) 8 (D) -20 (E) 12

19. The focus of the figure given by the equation $x^2 + 6x - 12y + 57 = 0$ is (x, y) . Find x .

- (A) $(-3, 4)$ (B) $(0, 1)$ (C) $(-7, 4)$ (D) $(4, -3)$ (E) $(-3, 7)$

20. Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of $f(x)$ over the interval $[2, 5]$.

- (A) $-\frac{3}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{2}{7}$ (D) $1\frac{1}{3}$ (E) $3\frac{1}{2}$

21. Roland Bones found a die with 6 blank faces on it. He painted the numbers 1, 1, 2, 3, 5, & 8, one number per face, on the die. He created a game such that he gets 10 points if he rolls a composite number, he gets 5 points if he rolls a prime number, and he loses 7 points if he rolls a unit. What would the mathematical expectation be for any given roll?

- (A) -7 pts (B) $-2\frac{1}{3}$ pts (C) 1 pt (D) $1\frac{5}{6}$ pts (E) 11 pts

22. Two distinct numbers are selected randomly from the set $\{2, 1, 3, 4, 7, 11\}$. What is the probability that their sum is an odd number?

- (A) $53\frac{1}{3}\%$ (B) $51\frac{1}{4}\%$ (C) 50% (D) $43\frac{3}{4}\%$ (E) $46\frac{2}{3}\%$

23. The figure shown is reflected over a negative diagonal. Which of the following figures is the result of that single transformation?



24. A recent visit to the planet Strangebase discovered that the equation, $3S^2 - 25S + 66 = 0$, has two solutions, 4 and 9. What base was being used for the number system on planet Strangebase?

- (A) base 5 (B) base 11 (C) base 13 (D) base 17 (E) base 36

25. On the map legend, 1 inch represents 120 miles. Beautiful downtown Millersview is 45 miles from San Angelo. How far is it on the map?

- (A) $\frac{5}{16}$ " (B) $\frac{3}{8}$ " (C) $\frac{3}{4}$ " (D) $1\frac{1}{8}$ " (E) $2\frac{2}{3}$ "

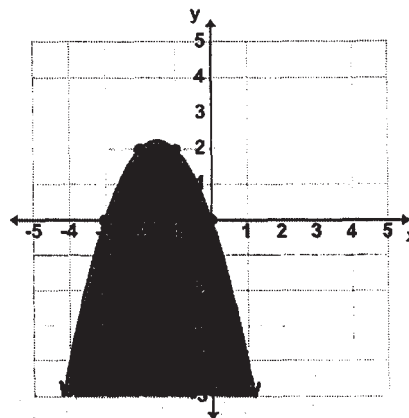
26. Which of the following is not a solution of $3 + 2|5x - 1| \leq 4$?

- (A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{1}{6}$ (D) $\frac{2}{7}$ (E) $\frac{1}{8}$

27. If two parallel lines are intersected by a transversal, then the alternate interior angles are ____.

- (A) acute (B) complementary (C) congruent (D) obtuse (E) supplementary

28. Which of the equations will produce the shaded portion of the graph shown?



- (A) $y \leq -x^2 + 3x - 3$ (B) $y \leq -x^2 + 3x$ (C) $y \geq x^2 - 3x$
(D) $y \leq -(x^2 + 3x)$ (E) $y \geq -x^2 - 3x + 3$

29. Sir Vayor is trying to find the height of a flagpole. His eyes are 1.7 meters above the ground and he is standing 10 meters from the base of the pole. The angle of elevation from his eyes to the top of the pole is 60° . Using this information Sir Vayor computes the top of the flagpole to be: (nearest meter)

- (A) 10 m (B) 13 m (C) 15 m (D) 17 m (E) 19 m

30. Find the first term of the geometric sequence: $a, b, 44, c, 19\frac{5}{9}, \dots$

- (A) $69\frac{19}{45}$ (B) $88\frac{2}{3}$ (C) 99 (D) 132 (E) $222\frac{3}{4}$

31. Evaluate: $\prod_{n=2}^6 (1 + \frac{1}{n})$

- (A) 1.45 (B) 3.5 (C) 6.1666... (D) 6.45 (E) 11.39

32. Coach Fuhrmann has 8 boys and 6 girls in his math and science club. He needs to send a delegation to a UIL planning conference. How many possible delegations can he send if each delegation must contain exactly 2 boys and exactly 2 girls?

- (A) 182 (B) 420 (C) 1,001 (D) 1,680 (E) 24,024

33. Which of the following mathematicians created an abacus for calculating products and quotients and extracting square roots that was based on Arab mathematics and lattice multiplication?

(A) John Venn

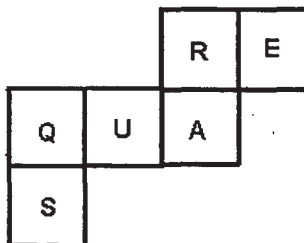
(B) Sophie Germain

(C) George Boole

(D) John Napier

(E) Leonard Euler

34. Polly Euler folds the net shown into a cube. What letter will be on the opposite side of side S?



(A) Q

(B) U

(C) A

(D) R

(E) E

35. The Azusa Aztec band is selling band calendars to make money for their trip. They get 30% of the sales for the first 100 sold, 40% of the sales above 100 but less than or equal to 200, and 50% of the sales over 200. How much will the band make if they sell 275 calendars if each calendar sells for \$10?

(A) \$2062.5

(B) \$1375.00

(C) \$1100.00

(D) \$1075.00

(E) \$825.00

36. Simplify: $a^{-2} \times b^2 \div a^3 \div b^{-3} \times a \div b$

(A) $a^{-4}b^4$

(B) a^2b^0

(C) a^0b^{-2}

(D) a^4b^2

(E) $a^{-4}b^{-2}$

37. The points (2, 3) and (−4, k) lie on the line $5x - 6y = C$. Find k.

(A) −8

(B) −3

(C) −2

(D) 1

(E) 4

38. Les Quik, Moe Fass, and Willie Makit run in a 100 meter race. Les beat Moe by 10 meters and Moe beat Willie by 20 meters. If the runners ran at a constant speed, by how much did Les beat Willie?

(A) 8 meters

(B) 10 meters

(C) 18 meters

(D) 28 meters

(E) 30 meters

39. Point P(−3, 2) and point Q(4, −5) lie on the x-y plane. P is translated horizontally 2 units to the left. Q is reflected across the y-axis. What is the distance between the points after the translations? (nearest tenth of a unit)

(A) 6.1

(B) 7.1

(C) 7.6

(D) 9.5

(E) 11.4

40. If $a_1 = 2$, $a_2 = 3$, $a_3 = 5$ and $a_n = a_{n-1} + a_{n-2} - a_{n-3}$, where $n \geq 4$, then a_8 equals:

(A) 14

(B) 12

(C) 11

(D) 9

(E) 0

41. Find $f(g(1 - x))$ when $f(x) = 3x - 1$ and $g(x) = x - 3$.

- (A) $-3x - 1$ (B) $5 - 3x$ (C) $-7 - 3x$ (D) $4x - 4$ (E) $5 - 4x$

42. If $\sqrt{x \sqrt[3]{x} \sqrt[4]{x}} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $k = ?$

- (A) 26 (B) 24 (C) 17 (D) 12 (E) 8

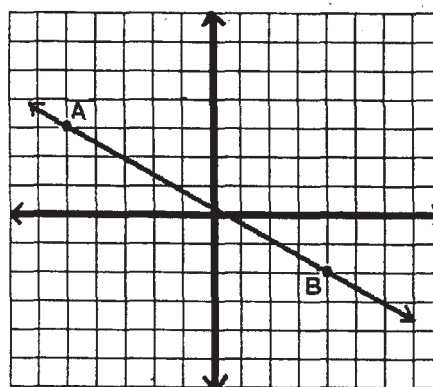
43. Using the equation $y = \frac{3}{4}\cos(2x - \frac{\pi}{3}) - 1$ which of the following has the largest numeric value?

- (A) amplitude (B) displacement (C) frequency (D) period (E) phase shift

44. $\triangle ABC$ and $\triangle PQR$ exist such that $AB = BC = PQ = PR$, $m\angle ABC = 2x^\circ$, $m\angle QPR = x^\circ$, and they have equal areas. Find x .

- (A) 15 (B) 30 (C) 45 (D) 60 (E) 75

45. Find an equation of a line parallel to line AB and passing through point $(-2, -3)$.



- (A) $y = \frac{-5x-37}{9}$ (B) $y = \frac{9x-3}{5}$ (C) $y = \frac{-x-8}{2}$ (D) $y = 2x + 1$ (E) $y = \frac{5x-17}{9}$

46. A circle with the center at C has a radius of 9 cm. A chord AB of the circle is 6 cm long. Find the distance from the chord to the center C .

- (A) $6\sqrt{2}$ cm (B) $3\sqrt{13}$ cm (C) $3\sqrt{10}$ cm (D) $3\sqrt{5}$ cm (E) $2\sqrt{14}$ cm

47. Find the determinant of the 3×3 matrix.

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

- (A) -2 (B) -1 (C) 1 (D) 3 (E) 4

48. If $f'(x) = 15x^2 - 6x + 2$ and $f(-1) = -9$, find $f(1)$.

- (A) 5 (B) 3 (C) -6 (D) 11 (E) -3

49. R_1 , R_2 and R_3 are the roots of the equation $24x^3 + 26x^2 - 19x - 6 = 0$.
 R_1 and R_2 are the roots of the equation $12x^2 - 5x - 2 = 0$ as well. Find R_3 .

(A) 2 (B) $\frac{2}{3}$ (C) $-\frac{1}{4}$ (D) $-1\frac{1}{2}$ (E) -4

50. $\int \sin(2x) \cos(2x) \, dx = \underline{\hspace{2cm}} + C$, where C is an arbitrary constant.

(A) $-\frac{1}{4}\sin(2x)$ (B) $-2\sin(x)\cos(x)$ (C) $-\frac{1}{8}\cos(4x)$ (D) $-4\cos(4x)$ (E) $-\frac{1}{2}\sin(4x)$

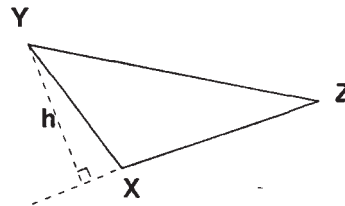
51. Willie Bettit has 5 plain red poker chips, 3 plain white poker chips, and 2 plain blue poker chips.
 How many ways can he line all of them up in a row?

(A) 1,440 (B) 3,628,800 (C) 120,960 (D) 5,040 (E) 2,520

52. Let $x = \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \dots}}}}$ be the continued fraction. Find x .

(A) $\frac{\sqrt{15}+1}{2}$ (B) $\frac{\sqrt{2}+1}{3}$ (C) $\frac{\sqrt{15}-3}{2}$ (D) $\frac{-\sqrt{15}-3}{2}$ (E) $\frac{\sqrt{2}-1}{3}$

53. Find the perimeter of $\triangle XYZ$ if $XY = 8''$, $XZ = 11''$ and $m\angle YXZ = 120^\circ$. (nearest tenth)



(A) 22.7" (B) 35.5" (C) 33.4" (D) 32.6" (E) 30.4"

54. Elmoor Fudd is building a rectangular shaped pen for his porkie pigs. It will have 4 parallel fences dividing the pen into 5 sections as shown. If he has 600 feet of fencing, what is the maximum area of his pig pen?



(A) 6562.5 sq. ft. (B) 7000 sq. ft. (C) 7250 sq. ft. (D) 7500 sq. ft. (E) 8437.5 sq. ft.

55. How many 5 digit numbers can be made using the digits 1, 2, 3, 4 & 5 where the digits in the tens place and the hundreds place must be a prime number. Each digit can only be used once in a number.

(A) 18 (B) 24 (C) 36 (D) 42 (E) 48

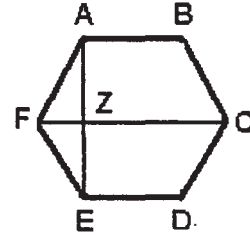
56. The Cowboys and the Texans will play twice this season. The Cowboys are twice as likely to win any game as the Texans. What is the probability that they will each win one of the two games?

- (A) $33\frac{1}{3}\%$ (B) $44\frac{4}{9}\%$ (C) 50% (D) $55\frac{5}{9}\%$ (E) $66\frac{2}{3}\%$

57. Simplify: $\frac{(n+1)!}{(n-1)!} \div \frac{(n+2)!}{n!}$

- (A) $\frac{n}{n+2}$ (B) $n^2 + 2n$ (C) $n(n+1)(n+2)$ (D) $\frac{n+1}{n}$ (E) $n^2 - 2n + 1$

58. Polygon ABCDEF is a regular hexagon and segments AE and CF intersect at point Z. The ratio of the area of triangle EFZ to the area of the quadrilateral ABCZ is:



- (A) $1 : 2.5$ (B) $1 : 3$ (C) $1 : 4$ (D) $1 : 5$ (E) $1 : 6$

59. The universal set $U = \{1, 2, 3, 5, 8, 13, 21, 34\}$. Subset $A = \{1, 3, 8, 21, 34\}$ and subset $B = \{2, 3, 5, 13, 21\}$. How many elements are in the complement set of $A \cap B$?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

60. Which of the following numbers is an unhappy and evil number?

- (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

**UIL HIGH SCHOOL SCIENCE CONTEST
ANSWER KEY**

INVITATIONAL B • 2010

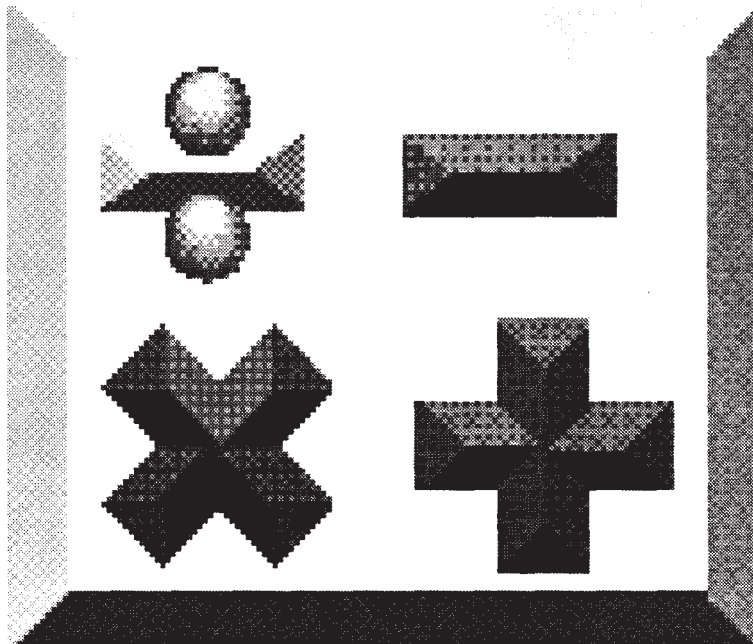
1.	C	21.	B	41.	C
2.	D	22.	E	42.	D
3.	D	23.	D	43.	D
4.	D	24.	A	44.	B
5.	D	25.	D	45.	E
6.	D	26.	C	46.	A
7.	B	27.	E	47.	C
8.	C	28.	D	48.	C
9.	B	29.	B	49.	E
10.	B	30.	E	50.	C
11.	E	31.	E	51.	B
12.	D	32.	D	52.	A
13.	A	33.	B	53.	A
14.	A	34.	C	54.	B
15.	B	35.	B	55.	E
16.	E	36.	C	56.	E
17.	A	37.	C	57.	D
18.	C	38.	B	58.	B
19.	A	39.	D	59.	E
20.	D	40.	D	60.	C



UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District 1 • 2010



**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

**DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!**

1. Evaluate: $10 \times 8 + 6 \div (4 - 2) \div 3 - 5 \times 7 + 9$

- (A) 19.5 (B) 37 (C) 55 (D) $74\frac{1}{3}$ (E) 125

2. Bill Meelater is buying a set of 4 tires for his car. The tires cost \$49.50 each. There is a "buy 3 and get the 4th at half price" sale going on. If Bill pays with cash instead of credit card, he gets 8% off of his total bill before taxes. What is Bill's final cost before taxes if he pays with cash?

- (A) \$182.16 (B) \$161.00 (C) \$273.24 (D) \$173.17 (E) \$159.39

3. Which of the following is not a solution to $3|5 - 7x| < 9$?

- (A) 1 (B) $\frac{5}{7}$ (C) 1.1 (D) $\frac{1}{4}$ (E) 0.4

4. Ye Olde Nut Shoppe mixes pecans that cost \$1.85 per pound with some walnuts that cost \$2.45 per pound to make 20 pounds of mixed nuts that costs \$2.00 per pound. How many pounds of pecans are in the mixture?

- (A) 6 lb (B) 10 lb (C) 15 lb (D) 9 lb (E) 5 lb

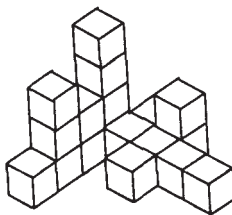
5. Let $\triangle JKL$ be a right triangle where $\angle JKL = 90^\circ$. Which of the following is the same point as K?

- (A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) midcenter

6. A water tank in the shape of a rectangular prism contains approximately 718 gallons of water, which is 80% of the tanks maximum capacity. The length of the tanks base is 6 feet and its width is 5 feet. How tall is the tank? (nearest foot)

- (A) 3 ft (B) 4 ft (C) 5 ft (D) 6 ft (E) 7 ft

7. One-centimeter cubes are glued together to form the object in the figure shown. If the side of each cube has a length of 2 cm the two-dimensional perspective of the top view of this figure has a surface area of:



- (A) 11 cm^2 (B) 22 cm^2 (C) 36 cm^2 (D) 44 cm^2 (E) 60 cm^2

8. If you slice a complete cone (double cone) with a plane that is parallel to the axis of the cone, the intersection is a(n) _____.

- (A) parabola (B) ellipse (C) circle (D) hyperbola (E) line

9. Determine the period of $y = 1 + 2\tan(3\pi(x) + 4)$.

- (A) $\frac{1}{4}$ (B) 3 (C) $\frac{1}{2}$ (D) 2 (E) $\frac{1}{3}$

10. How many elements are in $\left\{x \mid 4\sin(x)\cos(x) = \sqrt{3}, x \in [0, \pi)\right\}$?

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 8

11. Let $A = \begin{bmatrix} -2 & -3 \\ 5 & 8 \end{bmatrix}$. Find $\det(A + A^T)$.

- (A) -68 (B) -40 (C) -30 (D) -2 (E) -1

12. Les Dough received the following 5 year stock report. Based on the report. What was his average gain/loss over the 5 year period for the Sure2Make\$\$\$ stock? (nearest tenth)

Sure2Make\$\$\$ Stock Report

Year	1	2	3	4	5
Gain/Loss	+ 8%	+ 5%	- 7%	- 3%	+ 1%

- (A) + 0.3% (B) + 0.7% (C) + 0.8% (D) + 1.0% (E) + 1.1%

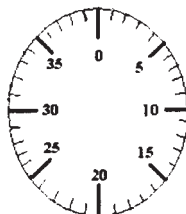
13. $\int \left(\frac{3}{x^2 - 2x + 1} \right) dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.

- (A) $6 \log(x - 1)$ (B) $\log(x - 1)^2$ (C) $-\frac{3}{x-1}$ (D) $\frac{(x-1)^2}{3}$ (E) $-\frac{6}{x-1}$

14. Find the slope of the line tangent to the curve $y = x^2 - 2x + 1$ at the point (3, 4).

- (A) 6 (B) 4 (C) 2 (D) -1 (E) -3

15. Albe Darned lost the 3 number combination to the padlock shown. He knows that the first number is a prime number greater than 20, the second number is a composite number less than 20, and the third number is a perfect square greater than 10, but less than 39. How many combinations can he try to open the lock?



- (A) 60 (B) 90 (C) 99 (D) 120 (E) 132

16. Coach Newberry's math team is working on probability problems. His team consists of 4 boys and 2 girls. Coach randomly selects two students names without replacement. What is the probability that the first name drawn is a girl if it is known that the second will be a girl?

(A) $\frac{1}{15}$ (B) $\frac{1}{14}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$

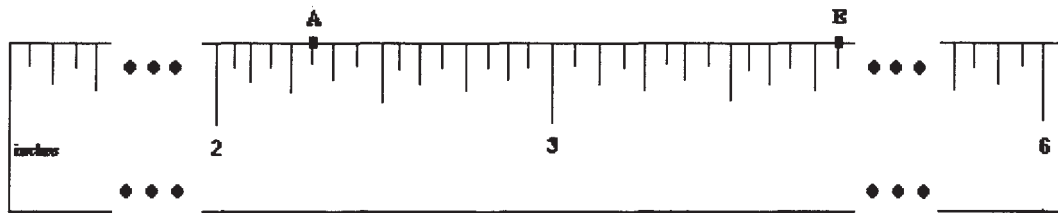
17. Which of the following mathematicians is best know for their work with 4-D POLYTOPES ?

(A) Karen E. Smith (B) George Boole (C) Alicia Stott
(D) Leonard Euler (E) Freda Porter

18. The operation \odot is defined as $x \odot y = y^2 - 2yx + x^2$, where x, y are integers and $x \neq y$. Find the value of $(2 \odot -3)(4 \odot -5)$.

(A) 2025 (B) 436 (C) 106 (D) 1 (E) 0

19. Dee Zar uses a 6" ruler to measure the lengths of two pieces of string. One piece of string has a length of A and another piece of string has a length of B. What is the total length of the two pieces of string?



(A) $1\frac{5}{8}$ " (B) $6\frac{1}{8}$ " (C) $5\frac{11}{16}$ " (D) $6\frac{1}{4}$ " (E) $1\frac{1}{2}$ "

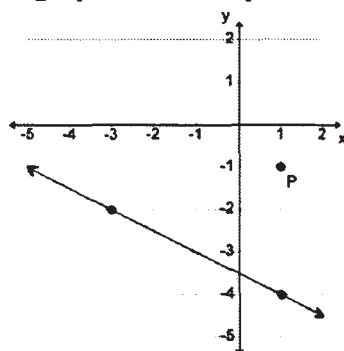
20. Which of the following is an example of the identity property of real numbers?

(A) $a + 0 = a$ (B) $b = b$ (C) $c - c = 0$ (D) $d \times \frac{1}{d} = 1$ (E) $0 \div e = 0$

21. $111_2 + 111_3 + 111_4 = \underline{\hspace{2cm}}_5$

(A) 2313 (B) 40 (C) 111 (D) 333 (E) 131

22. Find an equation of a line through point P and parallel to the line shown.



(A) $x + 2y = -1$ (B) $x - 2y = 3$ (C) $x - 2y = 1$ (D) $x + 2y = 3$ (E) $-x - 2y = -1$

23. The solution set of $4x + 7y = 56$ is $\{(x, y) \mid x, y \in \{\text{Integers}\} \text{ and } x, y \geq 0\}$. How many such ordered pairs exist?

- (A) 14 (B) 8 (C) 7 (D) 4 (E) 3

24. The points $(-1, 1)$ and $(2, -A)$ lie on the line $3x + By = -5$. Find A.

- (A) -0.5 (B) 1.375 (C) -0.125 (D) -5.5 (E) 2

25. A triangle with side lengths of 7 cm, 11 cm, and 11 cm is a(n) _____ triangle.

- (A) isosceles acute (B) scalene obtuse (C) isosceles obtuse (D) scalene acute (E) right

26. A circle with the center at C has a radius of 11 inches. A chord AB of the circle is 8 inches long. Find the distance, nearest quarter inch, from the chord to the center C.

- (A) $9\frac{1}{2}$ in (B) $9\frac{3}{4}$ in (C) $10\frac{1}{4}$ in (D) $10\frac{3}{4}$ in (E) $11\frac{1}{2}$ in

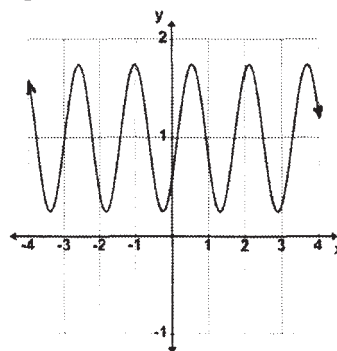
27. If $a_1 = -1$, $a_2 = 3$, $a_3 = -5$ and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_7 equals:

- (A) -9 (B) 7 (C) -5 (D) 3 (E) -1

28. If $27^{(k)} = 9^{(2k+1)}$, then $3^{(3k+2)} = ?$

- (A) $\frac{1}{81}$ (B) 343 (C) $-\frac{1}{3}$ (D) -81 (E) 6561



29. The equation $y =$ _____ will produce this graph.




- (A) $\frac{3\sin(x+1) - 3\cos(x+1)}{4}$ (B) $\frac{4 - 3\cos(4x+1)}{4}$ (C) $\frac{3\cos(x+1) + 3\sin(x+1)}{4}$
 (D) $\frac{3 + 4\sin(3x+1)}{3}$ (E) $\frac{3 - 3\cos(4x-1)}{4}$

30. Which of the following is true about $f(x) = (x)^{-1}$?


- (A) It is an odd function. (B) It is an even function. (C) It has no asymptotes.
 (D) It is neither an even nor an odd function (E) It is not one-to-one function.

31. Betty Lobs drops a tennis ball from a height of 12 meters. Each time it hits the ground it rebounds to a height of 60% of the distance it fell. Find the total distance the tennis ball travels when it reaches the ground the fourth time. (nearest tenth)
- (A) 13.2 m (B) 26.1 m (C) 38.1 m (D) 40.2 m (E) 52.2 m
32. Find the sum of the series $0.5 - \frac{(0.5)^2}{2} + \frac{(0.5)^3}{3} - \frac{(0.5)^4}{4} + \frac{(0.5)^5}{5} - \dots$ (nearest thousandth)
- (A) 0.385 (B) 0.401 (C) 0.403 (D) 0.405 (E) 0.407
33. The eccentricity of the figure given by the equation $x^2 + 4y^2 - 6x - 16y = 11$ is:
- (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{3}$ (C) 2 (D) $\frac{1}{2}$ (E) $2\sqrt{3}$
34. The Millersview Gifted School has 2 administrators, 5 teachers, 10 boys, and 10 girls. An advisory committee containing 1 administrator, 2 teachers, 3 boys, and 4 girls is to be created. How many different committees can be created?
- (A) 342 (B) 15,000 (C) 504,000 (D) 288,000 (E) 270
35. Roland Bones tosses a pair of dice. What are the odds that he won't roll a 2, 3, 7, 11, or 12?
- (A) 3 to 1 (B) 2 to 5 (C) 2 to 1 (D) 5 to 7 (E) 1 to 3
36. The figure shown is rotated 180° . Then it is reflected over its negative diagonal. Finally it is reflected over its horizontal axis of symmetry. Which of the following figures is the result of these three transformations?
- 
- 


(A)




(B)




(C)

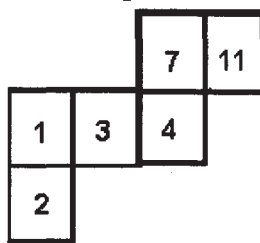


(D)



(E)
37. C. N. Blurry glued 5 congruent cubes together to make this special figure. The total surface area of the figure is $11\frac{1}{4}$ sq. cm. What is the length of each cube's edge?
- 
- (A) $\frac{1}{8}$ cm (B) $\frac{3}{16}$ cm (C) $\frac{1}{4}$ cm (D) $\frac{3}{4}$ cm (E) $\frac{7}{8}$ cm
38. Two billion two hundred thousand two plus three million three thousand three minus five thousand five hundred five results in the number X. How many zeros are in the number X?
- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2

39. Mr. Lou Kuss folds the net shown into a cube. He adds the numbers on each pair of opposite sides. What is the largest sum Lou comes up with?

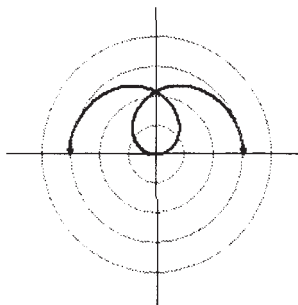


- (A) 14 (B) 13 (C) 11 (D) 9 (E) 6
40. Jan has 18 red marbles and 12 blue marbles. Dean has 20 marbles, all of them either red or blue. If the ratio of the red marbles to the blue marbles is the same for both Jan and Dean, then Jan has how many more red marbles than Dean?
- (A) 8 (B) 6 (C) 4 (D) 3 (E) 2
41. The coordinates of point A are $(0, -4)$. Point A is reflected across the line $y = -x$ to point B. Then point B is rotated 90° counterclockwise about the origin to point C. Then point C is translated vertically -4 units to point D. Find the coordinates of point D.
- (A) $(0, -8)$ (B) $(4, 4)$ (C) $(4, 0)$ (D) $(-4, 4)$ (E) $(0, 0)$
42. If two parallel lines are intersected by a transversal, then the exterior angles on the same side of the transversal are ____.
- (A) acute (B) complementary (C) congruent (D) obtuse (E) supplementary
43. Let $f(x) = x^3 - 1$ and $g(x) = x^2 - 4$ and $h(x) = x - 8$. Find $g(h(f(2)))$.
- (A) 37 (B) -513 (C) 13 (D) -3 (E) -5
44. The graph of $x^2 + 0xy - y^2 + 0x + 0y + 0 = 0$ is a _____.
- (A) point (B) line (C) pair of intersecting lines (D) pair of parallel lines (E) hyperbola
45. Cy Figh points telescope A towards the northeast at an angle of elevation of 72° to see the Nova Star. He positions telescope B 2.5 miles due east of A and points it toward the northwest at an angle of elevation of 77° to see the same Nova Star. How far is the Nova Star from telescope A? (nearest tenth of a mile).
- (A) 4.3 mi (B) 4.4 mi (C) 4.5 mi (D) 4.6 mi (E) 4.7 mi
46. Which of the following is equivalent to $\sin(x) = \cos(\frac{x}{2})$?
- (A) $2\sin(x)\cos(x) = 0$ (B) $\sin^2(x) - 2\sin(x) + 1 = 0$ (C) $\sin(\frac{x}{2}) = 0$
- (D) $2\cos^2(x) + \cos(x) - 1 = 0$ (E) $\cos(2x) = 0$

47. If the three numbers 114, 170, and 233 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

- (A) 2 (B) 3 (C) 5 (D) 7 (E) 9

48. Which of the following polar equations will produce this graph on a polar grid?



- (A) $r = 2\sin(\frac{\theta}{3})$ (B) $r = 3\sin(2\theta)$ (C) $r = \frac{3\cos(\theta)}{2}$ (D) $r = 2\cos(3\theta)$ (E) $r = 3\cos(\frac{\theta}{2})$

49. The instantaneous rate of change of the volume of a right-circular cone with a radius of 4 cm and an altitude twice its radius is $K \text{ cm}^3/\text{cm}$. Find K. (nearest tenth)

- (A) 100.5 (B) 33.5 (C) 16.8 (D) 50.3 (E) 301.6

50. The *Teechur Stor* sells red, blue, green, and black ink pens. They package the pens in sets of 3. How many different sets of 3 can they package?

- (A) 35 (B) 30 (C) 24 (D) 20 (E) 12

51. Suppose A, B, and C are positive integers such that $\frac{44}{9} = A + \frac{1}{B + \frac{1}{C}}$.

The value of $\frac{1}{A} - \frac{1}{B} + \frac{1}{C}$ equals:

- (A) $-0.444\ldots$ (B) -0.625 (C) -0.25 (D) 1.125 (E) 2.375

52. Which of the following is not a frugal number?

- (A) 125 (B) 128 (C) 243 (D) 256 (E) They are all frugal numbers

53. Herr Gauss has a box that contains the four letters of the word NINE. If he randomly selects one letter at a time and lines them up from left to right, what is the probability that the lined up letters will spell the word NEIN?

- (A) $\frac{1}{16}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

54. In the binomial expansion of $(2x - 1)^5$, the coefficient of the x^2y^3 term is:

- (A) -10 (B) -20 (C) -40 (D) 20 (E) 10

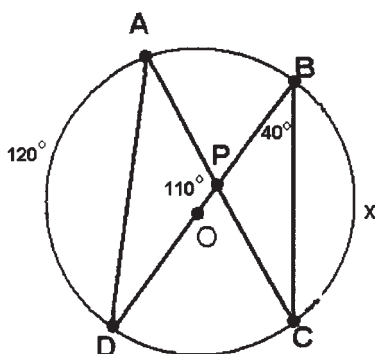
55. Let $\|V_1\|=12$ and $\|V_2\|=18$. The angle between V_1 and V_2 is 60° . Find $\|V_1 + V_2\|$. (nearest tenth)

- (A) 20.5 (B) 26.2 (C) 16.8 (D) 15.9 (E) 38.5

56. Willie Spendett will have saved \$260.00 from his allowance by the end of this year. He plans to withdraw $\frac{1}{10}$ of his savings balance at the end of each week next year. At this rate, what will his balance be at the end of the 52nd week? (nearest cent)

- (A) \$1.09 (B) \$1.26 (C) \$1.34 (D) \$2.60 (E) \$3.86

57. \overline{AC} , \overline{AD} , \overline{BD} , and \overline{BC} are chords of circle O and intersect at point P. Find x, the measure of arc BC.



- (A) 110° (B) 100° (C) 80° (D) 75° (E) 50°

58. If $18x^3 - 3x^2 - 6x = (ax)(ax - b)(bx + c)$ then $a + b + c =$ _____.

- (A) 1 (B) 2 (C) 3 (D) 5 (E) 6

59. Simplify: $(a)^{-3} \times (ab)^{-2} \div (a^{-1}b^2)^{-1} \times (a^2b^3)^0$

- (A) $a^{-4}b^4$ (B) a^4b^3 (C) $a^{-6}b^{-4}$ (D) $a^{-4}b^3$ (E) a^{-6}

60. Lorena hit the golf ball with her driver so that its initial speed was 180 feet per second. The ball left the club at an angle of 11° . Find the ball's initial speed in the vertical direction. (nearest foot)

- (A) 177 ft/s (B) 16 ft/s (C) 101 ft/s (D) 146 ft/s (E) 34 ft/s

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2010
Answer Key**

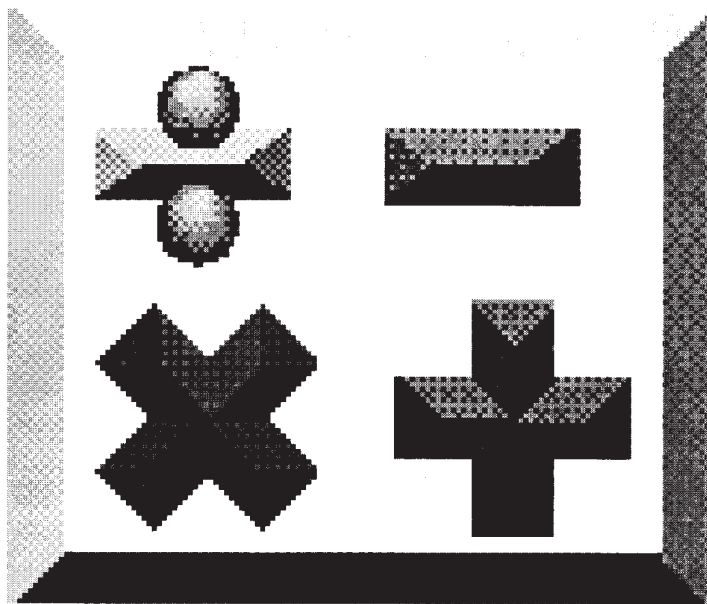
1. C	21. E	41. E
2. E	22. A	42. E
3. D	23. E	43. D
4. C	24. D	44. C
5. D	25. A	45. E
6. B	26. C	46. D
7. D	27. C	47. A
8. D	28. A	48. E
9. E	29. B	49. A
10. B	30. A	50. D
11. A	31. D	51. B
12. B	32. D	52. E
13. C	33. A	53. B
14. B	34. C	54. C
15. D	35. C	55. B
16. E	36. B	56. A
17. C	37. D	57. B
18. A	38. C	58. E
19. B	39. A	59. E
20. A	40. B	60. E



UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District 2 • 2010



**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

**DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!**

1. Evaluate: $6 \times 1\frac{1}{3} + 8 \div 1\frac{3}{5} - 10 \div 1\frac{5}{7}$

(A) $4\frac{1}{6}$

(B) $6\frac{4}{5}$

(C) $7\frac{1}{6}$

(D) $10\frac{3}{5}$

(E) $18\frac{5}{6}$

2. The original price of a suit at the *Everything-Must-Go* thrift store was \$89.99. It was successively discounted by 20%, then 10%, and finally 5%. What was the price of the suit after the three discounts? (nearest cent)

(A) \$45.00

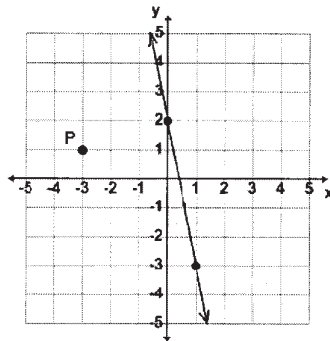
(B) \$55.01

(C) \$58.49

(D) \$61.55

(E) \$74.62

3. Find an equation of the line through point P and perpendicular to the line shown.



(A) $x + 5y = 2$

(B) $x - 5y = -8$

(C) $2x - 5y = -11$

(D) $2x + 5y = -1$

(E) $5x - 2y = 1$

4. The distance from El Paso to San Antonio by way of I-10 is 545 miles. Speedy Karr is leaving El Paso on I-10 at 7:00 a.m. driving toward San Antonio at 80 mph. Wes Bound is leaving San Antonio on I-10 at the same time driving toward El Paso at 65 mph. How much farther will Speedy have driven than Wes when they meet? (nearest mile)

(A) 56 miles

(B) 73 miles

(C) 15 miles

(D) 45 miles

(E) 36 miles

5. The length of a rectangle is decreased by 25%. The width of the rectangle is increased by 25%. The area of the original rectangle is what percent less than the area of new rectangle?

(A) $12\frac{1}{2}\%$

(B) $9\frac{3}{8}\%$

(C) $8\frac{3}{4}\%$

(D) 6%

(E) $6\frac{1}{4}\%$

6. A segment whose length is the shortest distance from the center to the side of a regular polygon is called a(n):

(A) tangent

(B) apothem

(C) arc length

(D) secant

(E) altitude

7. The roots of the equation $6x^3 + kx^2 - 5x + 6 = 0$ are 0.5, 3, and R. Find k.

(A) -3

(B) -12

(C) 7.5

(D) -17

(E) 15

8. If $\frac{11}{(x-2)(x-3)} - \frac{7}{(x-3)(x-5)} = \frac{k}{(x-2)(x-3)(x-5)}$, then k equals:

(A) $4x + 13$

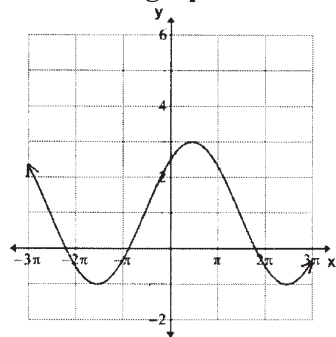
(B) $4x - 9$

(C) $4x + 1$

(D) $4x - 41$

(E) $4x - 19$

9. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $1 + 2\cos(\frac{x}{2} - 4)$ (B) $1 - 2\sin(\frac{x}{4} + 2)$ (C) $1 + 2\cos(\frac{x}{4} - 2)$
 (D) $2\sin(\frac{x}{2} - 4) + 1$ (E) $1 - 2\sin(\frac{x}{2} + 4)$

10. Determine the frequency of $y = 4 + 3\tan(2\pi x + 1)$.

- (A) 4 (B) 3 (C) 2 (D) $1\frac{1}{2}$ (E) $\frac{1}{2}$

11. The function $f(x) = 4x^3 - 12x^2 - 5x + 1$ has at most ____ negative real zeros.

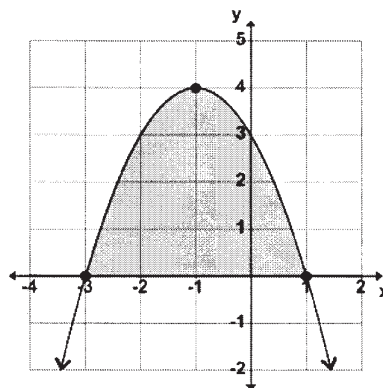
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. Find the determinant of the 3×3 matrix.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 3 \\ -3 & 2 & 1 \end{bmatrix}$$

- (A) 0 (B) -6 (C) -14 (D) -35 (E) -42

13. Find the area of the shaded region in square units.



- (A) 11 (B) $10\frac{2}{3}$ (C) $10\frac{1}{2}$ (D) $10\frac{1}{3}$ (E) 10

14. If $f''(x) = 10$ and $f'(1) = 7$ and $f(-1) = 10$, then $f(2) = \underline{\hspace{2cm}}$.

- (A) 28 (B) 21 (C) 17 (D) 16 (E) 13

15. How many ways can a father, mother, son, and daughter be seated around a circular table with 4 chairs?

(A) 6 (B) 10 (C) 12 (D) 18 (E) 24

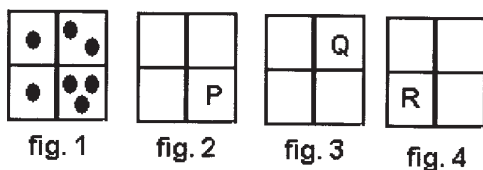
16. All of the aces, treys, fives, sevens, and nines are removed from a standard 52-card deck. The remaining cards are shuffled. Two cards are dealt face up. What is the probability that the first card is a face card and the second card is a factor of 8? (nearest %)

(A) 38% (B) 25% (C) 20% (D) 15% (E) 13%

17. $1100_2 - 123_4 + 67_8 = \underline{\hspace{2cm}}_{16}$

(A) 15 (B) 28 (C) 32 (D) 40 (E) 65

18. Fig. 1 is rotated clockwise 270° to fig. 2. Then, fig. 2 is reflected over its positive diagonal to fig. 3. Then fig. 3 is translated to the right to fig. 4. What is the total number of dots in squares P, Q, & R?



(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

19. The number 11111 in base 2 is equivalent to the number X in base 5. Find the sum of the digits in the number X.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

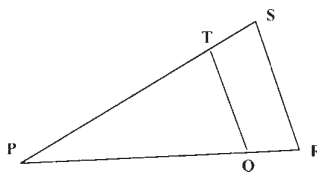
20. On the map legend, $\frac{1}{4}$ inch represents 11 miles. The town of Fife is 143 miles from San Angelo. How far is it on the map?

(A) $2\frac{3}{4}$ " (B) $2\frac{7}{8}$ " (C) 3 " (D) $3\frac{1}{4}$ " (E) $3\frac{5}{8}$ "

21. Which of the following is an example of the inverse property of real numbers?

(A) $a - 0 = a$ (B) $b \times \frac{1}{b} = 1$ (C) $c + 0 = c \times 1$ (D) $0 \div d = 0$ (E) $e = e$

22. $\triangle PRS$ is similar to $\triangle PQT$. $PT = 6$, $ST = 3$, $RS = x + 1$, and $QT = x - 2$. Find x .

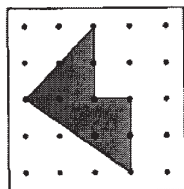


(A) 2 (B) 4 (C) 8 (D) 9 (E) not enough information given

23. Eve N. Steevan bought 2 television sets. Each set cost the same price. She sold one of them for 80% of her purchase price. What percentage of the cost of the other set must she sell it for in order to make a 20% profit of the total original cost of both sets?

(A) 110% (B) 120% (C) 140% (D) 150% (E) 160%

24. The adjacent dots on the grid are 5 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown. (nearest tenth)



(A) 91.0 cm² (B) 112.5 cm² (C) 122.5 cm² (D) 125.0 cm² (E) 137.5 cm²

25. Find $f(-1) + f(0) - f(2)$ if $f(x) = \begin{cases} x-1 & \text{if } x < 0 \\ x & \text{if } x = 0 \\ 1+x & \text{if } x > 0 \end{cases}$

(A) -5 (B) -2 (C) 0 (D) 1 (E) 3

26. Let p and q be the roots of $x^2 - 9x + 20 = 0$, where $p > q$. Find $p^5 - 5p^4q + 10p^3q^2 - 10p^2q^3 + 5pq^4 - q^5$.

(A) -5 (B) -1 (C) 1 (D) 5 (E) 9

27. A water tank in the shape of a right cylinder is 6 feet high and has an inside diameter of 8 feet. The tank is not full. It contains 1000 gallons of water. How deep is the water? (nearest inch)

(A) 2' 3" (B) 2' 6" (C) 2' 8" (D) 3' 1" (E) 3' 7"

28. If $a_1 = 2$, $a_2 = -3$, $a_3 = 5$ and $a_n = (a_{n-2})(a_{n-3}) + a_{n-1}$, where $n \geq 4$, then a_7 equals:

(A) -21 (B) -16 (C) -11 (D) -5 (E) -1

29. Let $f(x) = 1 + 2x$ and $g(x) = 3 - 4x$. If $h(x)$ is the inverse function of $\frac{f(x)}{g(x)}$, then $h(5) = ?$

(A) $-1\frac{6}{11}$ (B) $-1\frac{1}{9}$ (C) $-\frac{11}{17}$ (D) $\frac{8}{9}$ (E) $\frac{7}{11}$

30. If $\sqrt{x^3 \left(\sqrt[3]{x^2 (\sqrt[4]{x})} \right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $k = ?$

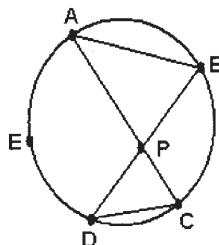
(A) 18 (B) 15 (C) 8 (D) 5 (E) 4

31. How many elements are in $\left\{x \mid 3\sin(x)\cos\left(\frac{\pi}{3}\right) + 3\cos(x)\sin\left(\frac{\pi}{3}\right) = 0, x \in (-\pi, 2\pi)\right\}$?
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
32. Clark and Lois are camped on opposite sides of a small lake. Straight paths from each of their camps lead to the Meet & Greet Bait store. Clark walks 600 feet to the store and Lois walks 870 feet to the store. The angle between the two paths is 35° . What is the distance between the two camps straight across the lake? (nearest foot)
- (A) 512 ft (B) 540 ft (C) 605 ft (D) 718 ft (E) 735 ft
33. Find the sum of the series $5 - \frac{(5)^3}{3!} + \frac{(5)^5}{5!} - \frac{(5)^7}{7!} + \frac{(5)^9}{9!} - \dots$ (nearest ten-thousandth)
- (A) -1.1336 (B) -0.9376 (C) -0.9589 (D) 0.0896 (E) 0.9753
34. The directrix of the conic given by the equation $y = -2x^2 + 5x - 3$ is:
- (A) $y = \frac{1}{8}$ (B) $y = \frac{1}{4}$ (C) $y = \frac{5}{16}$ (D) $y = \frac{11}{16}$ (E) $y = \frac{5}{4}$
35. Let $p + q = 12$, where $p, q > 0$. Find p and q such that pq^2 is a maximum product.
- (A) 128 (B) 200 (C) 243 (D) 245 (E) 256
36. Eight 400 meter relay teams are competing at the Eden Spring Track Meet. In how many ways can the first, second, and third place trophies be awarded?
- (A) 42 (B) 56 (C) 171 (D) 336 (E) 512
37. Ma Bell's telephone company assigns 10 digit phone numbers (i.e. 934-227-1458). What are the odds that the last digit of a randomly chosen phone number is 9 or 0?
- (A) 1 to 5 (B) 1 to 4 (C) 1 to 10 (D) 5 to 1 (E) 4 to 1
38. The odd numbers from 1 to 17 are to be placed in this magic square in which the rows, the columns, and the diagonals have the same sum. Find the value of x .

	13	
17		1
		x

- (A) 7 (B) 9 (C) 11 (D) 15 (E) 19

39. Lotta Sense has 6 coins, a penny, a nickel, a dime, a quarter, a half-dollar, and a silver dollar. How many different sets of three coins can she create?
- (A) 20 (B) 18 (C) 15 (D) 12 (E) 9
40. Let $\triangle ABC$ be an isosceles right triangle and $m\angle BAC = 90^\circ$. If the orthocenter of the triangle is the vertex point A, which of the following lies on segment BC?
- (A) centroid (B) circumcenter (C) incenter (D) outcenter (E) midcenter
41. The mathematician who came up with a type of linguistic algebra using the three most basic operations AND, OR, and NOT and developed a logic system based on a binary approach (yes-no, on-off, true-false, 0-1) is:
- (A) John Venn (B) George Boole (C) John Napier
(D) Leonard Euler (E) Claudius Ptolemy
42. The value of $58\frac{1}{3}\%$ of $(0.777 \dots)^{-1}$ is the same as the value of 25% of _____.
- (A) 3 (B) $1\frac{22}{27}$ (C) 1.5 (D) .1875 (E) $\frac{1}{48}$
43. Let $L = \{2, 1, 3, 4, 7, 11\}$, $P = \{2, 3, 5, 7, 11\}$, and $O = \{1, 3, 5, 7, 9, 11\}$. How many elements are in $(L \cap P) \cup (L \cap O) \cup (P \cap O)$?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10
44. $\{(x, y) \mid x, y \in \{\text{Integers}\}, -10 \leq x \leq 10, \text{ and } -5 \leq y \leq 5\}$ is the solution set of $3x - 5y = 7$. How many such ordered pairs exist?
- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2
45. James the butler was offered a job for \$8000 and a new car for a year of service. James quit after 7 months of service. He was given the car and \$1600 for the 7 months of service. What was the dollar value of the car?
- (A) \$6400 (B) \$7200 (C) \$7360 (D) \$8000 (E) \$15360
46. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O and point E lies on circle O. Find $m\widehat{AED}$ given $m\angle BPC = 110^\circ$ and $m\angle BAP = 30^\circ$.

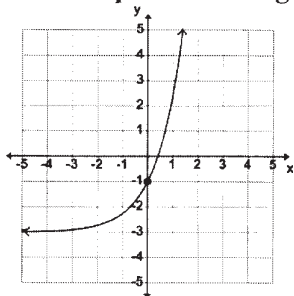


- (A) 160° (B) 150° (C) 140° (D) 110° (E) 80°

47. Bear Foote High student body is made up of 60% boys and 40% girls. All of the boys wear sneakers. Some of the girls wear flip flops and some wear sneakers. A student is standing behind their locker and Principal Smokie can only see a pair of sneakers. What is the probability that the student is a girl?

(A) $33\frac{1}{3}\%$ (B) 30% (C) 25% (D) 20% (E) $16\frac{2}{3}\%$

48. Which of the following equations will produce the graph shown here?



(A) $y = 2e^x - 3$ (B) $y = 2\ln(x) - 3$ (C) $y = \sqrt{e^x} - 3$ (D) $y = 2e^{(-x)} - 3$ (E) $y = \ln(x) - 3$

49. Cy Phy's physics class is trying to determine the magnitude of the sum if two forces, 220 and 180 lb, act at on an object at an angle of 46° . What is the magnitude of their sum? (nearest lb).

(A) 89 lb (B) 161 lb (C) 223 lb (D) 369 lb (E) 381 lb

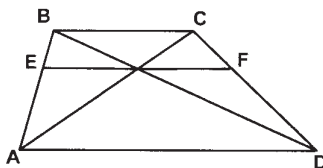
50. The type of graph of the polar equation $r = 1 + 2\theta$ is called a:

(A) Archimedian spiral (B) cardioid (C) lemniscate (D) limacon (E) rose

51. The polar graph of $r^2 = 4\sin(2\theta)$ is symmetric to the pole and to the:

(A) polar axis (B) line $\theta = \pi$ (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{3}$ (E) line $\theta = \frac{\pi}{4}$

52. The trapezoid shown exists such that $BC = 10$ cm, $AD = 16$ cm, $\overline{BC} \parallel \overline{EF} \parallel \overline{AD}$, and \overline{EF} and the two diagonals intersect at a common point. Find EF. (nearest tenth)

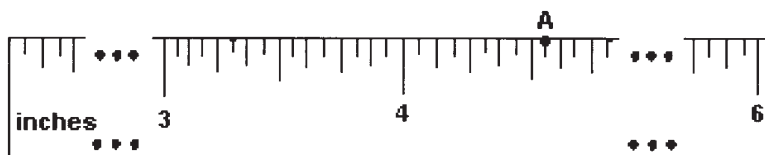


(A) 12.0 cm (B) 12.3 cm (C) 12.6 cm (D) 13.0 cm (E) 13.3 cm

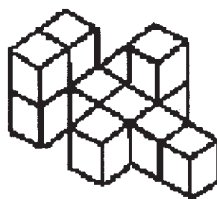
53. Let $A = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$. Find $\det(A + A^{-1})$.

(A) -1 (B) -4 (C) -32 (D) -40 (E) -48

54. Bo Kutter uses a 6" ruler to measure the length of his ribbon. The ribbon has a length of A. He wants to cut the ribbon into four equal parts. How long will each part be?



- (A) $\frac{25}{64}$ " (B) $\frac{13}{16}$ " (C) $1\frac{3}{32}$ " (D) $1\frac{1}{8}$ " (E) $1\frac{9}{64}$ "
55. I. C. Water skates around the edge of a circular pond at a linear velocity of 7.3 meters/second. The edge of the pond is 6 meters from the center of the pond. How many revolutions can I. C. make per minute? (nearest tenth)
- (A) 7.6 rpm (B) 11.6 rpm (C) 12.4 rpm (D) 13.5 rpm (E) 19.4 rpm
56. How many asymptotes exist of $h(x) = \frac{x^2 + 3x + 1}{4x^2 - 9}$?
- (A) none exists (B) 1 (C) 2 (D) 3 (E) 4
57. $\int [\sin(x)\cos(x)] dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.
- (A) $\cos^2(x) - 1$ (B) $\frac{1}{2}\cos^2(x)\sin(x)$ (C) $-\cos^2(x)\sin(x)$ (D) $-\sin(x)\cos(x)$ (E) $-\frac{1}{2}\cos^2(x)$
58. Wally World Discount Mart collected the following data about customer complaints.
- | Number of complaints in a day: | 5 | 10 | 15 | 20 |
|---|-----|-----|-----|-----|
| Probability of that number of complaints happening: | 15% | 30% | 40% | 15% |
- Based on this data, what is the expected number of complaints per day?
- (A) 12 (B) 12.5 (C) 12.75 (D) 13 (E) 13.75
59. The number 2010 is a(n) _____ number.
- (A) deficient (B) happy (C) lucky (D) evil (E) prime
60. One-centimeter cubes are glued together to form the object in the figure shown. Dee Z. Marker picks up the figure and puts a "Z" on all of the cubes' faces that can be marked on. How many of the cubes' faces will not be marked with a "Z"?



- (A) 20 (B) 24 (C) 26 (D) 28 (E) 32

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 2 • 2010
Answer Key**

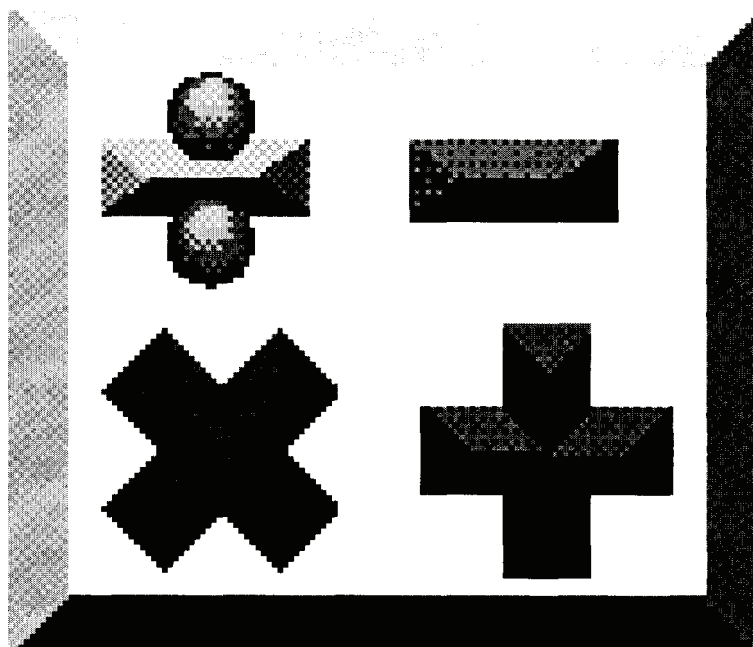
1. C	21. B	41. B
2. D	22. C	42. A
3. B	23. E	43. C
4. A	24. D	44. C
5. E	25. A	45. C
6. B	26. C	46. A
7. D	27. C	47. C
8. D	28. D	48. A
9. E	29. E	49. D
10. C	30. B	50. A
11. B	31. B	51. E
12. E	32. A	52. B
13. B	33. C	53. D
14. D	34. B	54. E
15. A	35. E	55. B
16. D	36. D	56. D
17. B	37. B	57. E
18. C	38. D	58. C
19. B	39. A	59. D
20. D	40. B	60. C



UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Regional • 2010



**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

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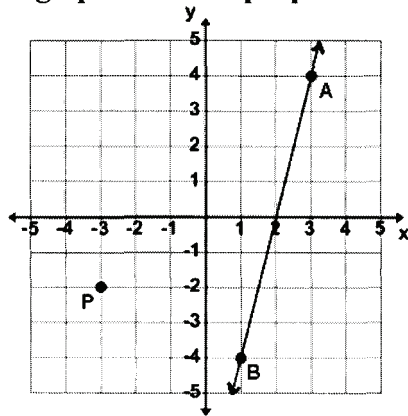
1. Evaluate: $[4! - (3)^3] + 2^{-2} \times \sqrt{2^4 \div 3^4}$

- (A) $-3\frac{1}{9}$ (B) $-2\frac{8}{9}$ (C) $-1\frac{2}{9}$ (D) $6\frac{7}{9}$ (E) $15\frac{1}{9}$

2. Will Itkosmoor wants to buy 4 new calculators for his math team. He can buy 2 at the regular price, 2 at half price, and pay 8% of the total price for shipping and handling. He can get 16% off and pay no shipping if he buys 4 at the regular price. If the regular price is \$89.95, how much will he save if he takes the best deal? (tax exempt)

- (A) \$10.79 (B) \$10.30 (C) \$9.59 (D) \$7.20 (E) \$5.40

3. Find an equation of a line through point P and perpendicular to the line shown.



- (A) $x + 4y = -14$ (B) $4x - y = 14$ (C) $x - 4y = 5$ (D) $x + 4y = -11$ (E) $4x + y = 5$

4. The statement "If $x = y + 1$ then $y + 1 = x$ " is an example of the ____ property of real numbers?

- (A) commutative (B) inverse (C) reflexive (D) identity (E) symmetric

5. The length of a rectangle is increased 40%. The width of the rectangle is decreased by 60%. The area of the new rectangle is what fractional part of the area of the original rectangle?

- (A) $\frac{21}{25}$ (B) $\frac{14}{25}$ (C) $\frac{11}{25}$ (D) $\frac{9}{25}$ (E) $\frac{7}{25}$

6. Find the area, in square units, of the quadrilateral whose vertices are $(-3, -3)$, $(3, -2)$, $(0, 2)$, and $(-2, 1)$.

- (A) 14 (B) 15.5 (C) 16 (D) 17 (E) 19.5

7. Noah Sense has 42 coins consisting of pennies, nickels, dimes, and quarters. He has twice as many nickels as pennies, three less dimes than nickels, and three more quarters than pennies. How much money does he have?

- (A) \$ 5.31 (B) \$ 4.37 (C) \$ 4.20 (D) \$ 4.07 (E) \$ 3.81

8. The graph of $x^2 + y^2 - 10x + 6y + 9 = 0$ is a circle with a center (h, k) and a radius r . Find $h + k + r$.

- (A) 1 (B) 3 (C) 7 (D) 13 (E) 15

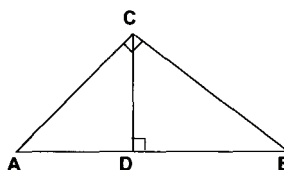
9. Using the equation $y = 4 + 3\sin(2x + 1)$, which of the following has the smallest numeric value?

- (A) amplitude (B) displacement (C) frequency (D) period (E) phase shift

10. Which of the following is equivalent to $\frac{\sin(\theta)}{1 + \cos(\theta)} + \frac{1 + \cos(\theta)}{\sin(\theta)}$?

- (A) $\csc(\theta) + 1$ (B) $\frac{\sec(\theta)}{2}$ (C) $\cot(\theta)$ (D) $\tan(\theta)$ (E) $2\csc(\theta)$

11. Find BC if AD = 24 cm. and DB = 30 cm. (nearest tenth)



- (A) 49.4 cm (B) 36.0 cm (C) 44.9 cm (D) 26.8 cm (E) 40.2 cm

12. Let $f(x) = 3x - 2$, and $g(x) = 2x + 1$. Find the $f(f(-x)) - g(g(-x))$.

- (A) $-7 - 13x$ (B) $-11 - 5x$ (C) $7 - 5x$ (D) $4 - 9x$ (E) $3 - 4x$

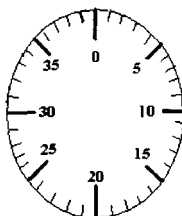
13. Find the angle of rotation, θ (nearest tenth degree), where $0^\circ < \theta < 90^\circ$, such that the conic $2x^2 + 12xy + 18y^2 - 3y = 5$ contains no xy term in its equation.

- (A) 74.5° (B) 71.6° (C) 63.4° (D) 60.5° (E) 58.6°

14. $\int \left(\frac{3-x}{x+2} \right) dx = \text{_____} + C$, where C is some arbitrary constant.

- (A) $x - 3\ln(x + 2)$ (B) $5\ln(x - 2) + x$ (C) $x + 3\ln(x - 2)$
(D) $5\ln(x + 2) - x$ (E) $5\ln(x + 2) + x$

15. E. Z. Lockett forgot her 3 number combination to the padlock shown. She knows that all of the numbers have a 3 as one of its digits and all 3 numbers of the combination are different. How many combinations can she try to open the lock?

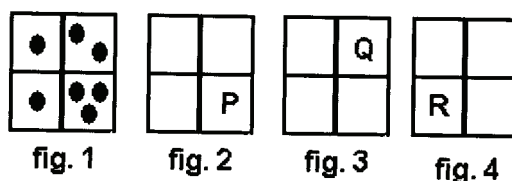


- (A) 1,320 (B) 2,184 (C) 1,716 (D) 1,872 (E) 2,197

16. The *Play Ball* Association is putting 5 balls in each gift box for kids to play with. The association has golf balls, baseballs, basketballs, footballs, ping pong balls, tennis balls, and dodge balls. How many different gift boxes of 5 balls can they put together?

(A) 13,860 (B) 5,544 (C) 792 (D) 462 (E) 330

17. Figure 1 is a square with four smaller squares, each containing a number of dots. Reflect fig. 1 over its vertical axis. The reflected figure will be fig. 2. Then, translate fig. 2 horizontally. The translated figure will be fig. 3. Finally, rotate fig. 3 180° clockwise. The rotated figure will be fig. 4. What is the total number of dots in the smaller squares denoted by P, Q, & R?



(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

18. Which of the following mathematicians is known for their work explaining and clarifying the the number $\frac{1+\sqrt{5}}{2}$?

(A) Theano of Crotona (B) Grace Alele Williams (C) Hypatia
(D) Agnesi (E) Freda Porter

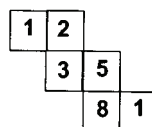
19. Ester Bunnee had a bag of chicken peeps. She hid 4 of them under a bush. She gave 25% of what was left to her cousin Dee Hair. The she gave $\frac{5}{6}$ of what was left to the little boys and girls. She had 2 left for herself. How many peeps were in the bag to begin with?

(A) 32 (B) 24 (C) 40 (D) 36 (E) 20

20. The number 4321 in base 5 is equivalent to the number wxyz in base 7, where w, x, y, and z are digits. Find $w + x + y + z$.

(A) 18 (B) 16 (C) 11 (D) 10 (E) 9

21. When the net below is folded into a cube the sum of the faces opposite the faces containing the number 1 will be?



(A) 5 (B) 7 (C) 8 (D) 10 (E) 11

22. Let p and q be the real roots of $x^2 - 2x - 8 = 0$, where $p > q$. Find $p^3q + 2p^2q^2 + pq^3$.

(A) -32 (B) -16 (C) 4 (D) 6 (E) 8

23. Bea Debest, Ima Slo, and Betsy Luzes run in a 200 meter race. When Bea crosses the finish line, Ima is 10 meters behind Bea. When Ima crosses the finish line, Betsy is 10 meters behind Ima. If all 3 runners ran at a constant speed, how far was Betsy from the finish line when Bea won the race?

(A) 18 meters (B) 19 meters (C) 19.5 meters (D) 20 meters (E) 21.5 meters

24. If y varies directly as x and inversely as z , then $z = 4$ when $x = 2$ and $y = 5$. Find z when $x = 5$ and $y = 3$.

(A) $33\frac{1}{3}$ (B) $16\frac{2}{3}$ (C) 6 (D) $4\frac{1}{6}$ (E) $\frac{1}{6}$

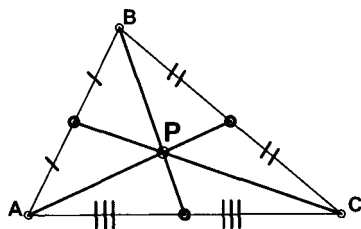
25. Points A, B, and D are on circle O. \overline{CA} is secant to O through point B. \overrightarrow{CD} is tangent to O at D. If $m\widehat{AD} = 80^\circ$ and $m\widehat{BD} = 30^\circ$, then $m\angle BCD = ?$

(A) 15° (B) 25° (C) 40° (D) 55° (E) 110°

26. A box contains five rods whose lengths are 4", 5", 7", 11" and 12". How many different acute triangles can be made using only three rods at a time.

(A) 7 (B) 5 (C) 4 (D) 2 (E) 0

27. Point P is the _____ of $\triangle ABC$ shown below.



(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) center

28. Let $f(x) = x^3 + 3$ and $g(x) = x^2 - 2$ and $h(x) = x + 1$. Find $f(g(h(-2)))$.

(A) 2 (B) 4 (C) 12 (D) 24 (E) 30

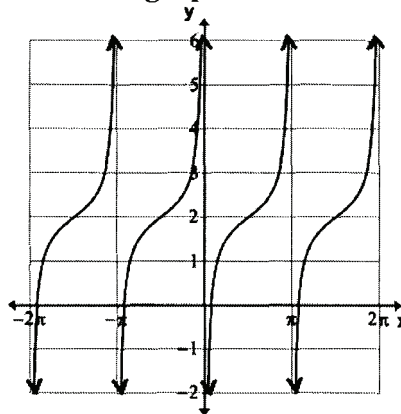
29. If you slice a complete cone (double cone) with a plane through the diameter of the cone and its vertex point, the intersection is a _____.

(A) line (B) pair of intersecting lines (C) pair of parallel lines
(D) pair of perpendicular lines (E) point

30. If $a_1 = 2$, $a_2 = -1$, $a_3 = 1$ and $a_n = (a_{n-3})(a_{n-2}) - a_{n-1}$, where $n \geq 4$, then a_7 equals:

(A) 9 (B) 5 (C) 2 (D) -1 (E) -11

31. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $1 + 2\tan(x)$ (B) $\frac{4 - \cot(x)}{2}$ (C) $\frac{1 - 2\tan(x)}{2}$ (D) $2 - \cot(x)$ (E) $\frac{2 + 4\cot(x)}{2}$

32. The type of graph of the polar equation $r^2 = 25\sin(2\theta)$ is called a:

- (A) Archimedian spiral (B) cardioid (C) lemniscate (D) limaçon (E) rose

33. How many elements are in $\left\{ x \mid 2 + \csc(2x - \pi) = 0, x \in [-\pi, 0] \cup [\pi, 2\pi] \right\}$?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

34. Let $f(x) = \cos(x)\sin(x)$ for all Real numbers. Which of the following is true about $f(x)$?

- (A) It is an odd function. (B) It is an even function. (C) It has two asymptotes.
(D) It is neither an even nor an odd function (E) It is a one-to-one function.

35. Let $A = \begin{bmatrix} 1 & -2 \\ 0 & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 4 \\ y & 2 \end{bmatrix}$. Find $x + y$.

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) 1 (E) -4

36. How many asymptotes exist of $h(x) = \frac{x+10}{|x|}$?

- (A) none exists (B) 1 (C) 2 (D) 3 (E) 4

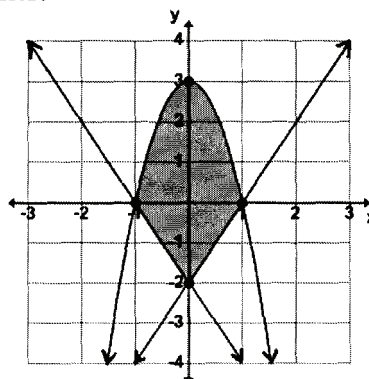
37. Let $f(x) = 3x^2 - 4x + 3$. A line tangent to $f(x)$ at $x = 0$ intersects with a line tangent to $f(x)$ at $x = 2$ at point (x, y) . Find y .

- (A) 0.8 (B) 0.5 (C) 0 (D) -0.2 (E) -1

38. If $f''(x) = 12x + 2$ and $f'(1) = 5$ and $f(1) = 4$, then $f(-1) = \underline{\hspace{2cm}}$.

- (A) 6 (B) 3 (C) 1 (D) -4 (E) -10

39. Find the area of the shaded region in square units.



- (A) 6 (B) $5\frac{3}{4}$ (C) $6\frac{1}{2}$ (D) $5\frac{1}{4}$ (E) 5

40. The Brite Lite Company produced 5000 100-watt bulbs of which 50 were defective. The Brite Bulb Company produced 3000 100-watt bulbs of which 100 were defective. A bulb was chosen at random from the 8000 bulbs and turns out to be defective. What is the probability that the bulb came from the Brite Lite Company?

- (A) $33\frac{1}{3}\%$ (B) $18\frac{3}{4}\%$ (C) 10% (D) $3\frac{1}{3}\%$ (E) 1%

41. A pair of dice are rolled. What are the odds that the roll comes up a 2, 5, 6, 10, or 12?

- (A) 7 to 18 (B) 5 to 7 (C) 5 to 12 (D) 7 to 12 (E) 7 to 11

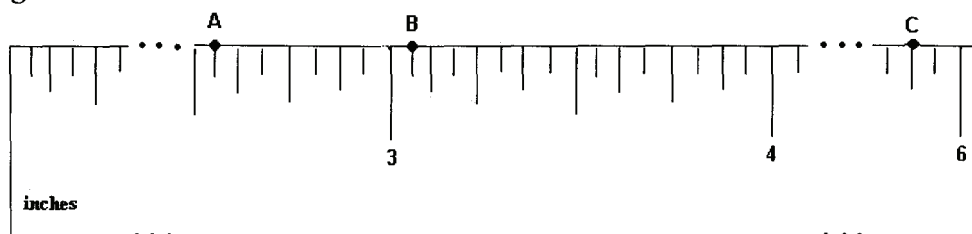
42. Let $x = \frac{2}{3 + \frac{2}{4 + \frac{2}{3 + \frac{2}{4 + \dots}}}}$ be the continued fraction. Find x. (nearest tenth)

- (A) 4.6 (B) 1.7 (C) 1.2 (D) 0.6 (E) 0.3

43. The operation $m \star n$ is defined as $(m + n) \div (m \times n)$. Compute $(1 \star 9) \star (3 \star 3)$.

- (A) 6 (B) 0.444... (C) 2.111 (D) 2.4 (E) 1

44. May Juror uses a 6" ruler to find the lengths of three pieces of string. One piece has a length of A, a second piece has length B, and a third piece has a length of C. What is the average length of the three pieces of string?



- (A) $3\frac{1}{3}$ " (B) $3\frac{3}{8}$ " (C) $3\frac{5}{6}$ " (D) $3\frac{15}{16}$ " (E) $3\frac{27}{32}$ "

45. Let $F = \{1, 2, 3, 5, 8, 13, 21\}$, $P = \{1, 3, 4, 7, 11, 18\}$, and $H = \{2, 3, 4, 5, 6, 8, 9\}$. How many elements are in $P \cup (F \cap H)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

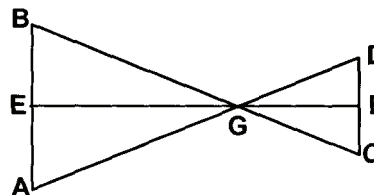
46. Anne Surr's final exam is worth 120 points. The exam consists of 45 problems of which some problems are worth 2 points and the others are worth 3 points. Find Anne's score on the test if she got all of the 3-pointers correct and missed all of the 2-pointers.

(A) 108 (B) 105 (C) 99 (D) 90 (E) 84

47. Simplify: $\frac{9x^2 - 1}{3x^2 + 4x + 1} \div \frac{3x^2 - 10x + 3}{9x^2 + 6x + 1}$

(A) $\frac{3x+1}{x+1}$ (B) $\frac{x^2-2x-3}{3x+1}$ (C) $\frac{3x+1}{x-3}$ (D) $\frac{x+1}{(3x+1)^2}$ (E) $\frac{(3x+1)^2}{x^2-2x-3}$

48. $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \perp \overline{EG}$, and $\overline{CD} \perp \overline{FG}$. If $AB = 27$, $EG = 21$, and $FG = 14$, then $CD = ?$



(A) 40.5 (B) 20 (C) 18 (D) 10.888... (E) not enough information given

49. Let $\triangle PQR$ be a right triangle with \overline{QR} being the hypotenuse and point M the midpoint of \overline{QR} . Which of the following is a true statement?

(A) $MQ + MR = (PQ + PR)^2$ (B) $MP = PQ \div 2$ (C) $MP = MQ = MR$
 (D) $MQ + MR = (PQ)(PR) \div 2$ (E) $MP = (PQ + PR) \div 2$

50. If $\sqrt[4]{x^3 \left(\sqrt[5]{x^4 \left(\sqrt[3]{x^2} \right)} \right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $n + k = ?$

(A) 21 (B) 38 (C) 69 (D) 84 (E) 119

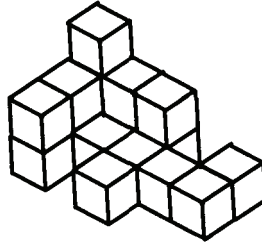
51. Juan Weeler rides his unicycle at 10 mph in the local parade. The radius of the wheel is 18 inches. What is the angular velocity of the unicycle wheel in radians per minute? (nearest tenth)

(A) 586.7 (B) 560.2 (C) 186.8 (D) 176.0 (E) 93.4

52. If the three numbers 259, 223, and 196 are each divided by the number D , each of their quotients will have the same remainder R . Find R .

(A) 2 (B) 3 (C) 4 (D) 7 (E) 9

53. One-centimeter cubes are glued together to form the object in the figure shown. Art Payntor picks up the figure and paints each of the cubes' faces red that are not glued together. How many of the cubes' faces will he paint red?



- (A) 56 (B) 57 (C) 58 (D) 59 (E) 60
54. Find the direction of a resultant vector whose vertical component has a magnitude of 9 and a direction of 90° and a horizontal component having a magnitude of 6 and a direction of 180° . (nearest tenth)
- (A) 156.3° (B) 146.3° (C) 133.7° (D) 123.7° (E) 112.6°
55. The polar graph of $r = 2\sin(3\theta)$ is symmetric to the:
- (A) polar axis (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{4}$ (E) line $\theta = \pi$
56. The eccentricity of the ellipse $\frac{(x-3)^2}{49} + \frac{(y+1)^2}{25} = 1$ is:
- (A) $\frac{\sqrt{24}}{5}$ (B) $\frac{24}{25}$ (C) $\frac{\sqrt{24}}{25}$ (D) $\frac{5}{7}$ (E) $\frac{\sqrt{24}}{7}$
57. Evaluate: $\prod_{n=2}^6 (n - \frac{1}{n})$
- (A) 420 (B) 74.2 (C) 72 (D) 36 (E) 18.55
58. The president wants to form a finance committee consisting of 3 Democrats, 2 Republicans, 1 Libertarian, and 1 Independent. He can choose this committee from a group of 9 Democrats, 7 Republicans, 5 Libertarians, and 3 Independents. How many different committees can he form?
- (A) 346,104 (B) 26,460 (C) 168 (D) 29,400 (E) 113
59. Simplify: $\frac{n!(n-1)!(n+2)!}{(n+1)!(n-2)!}$
- (A) $(n+1)!$ (B) $n!(n^2+n-2)$ (C) $n!(n-1)$ (D) $n!$ (E) $n!(n+2)$
60. How many of the elements in the set $\{6, 28, 496, 8128, 12468\}$ are considered to be both odious and perfect numbers?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2010
Answer Key**

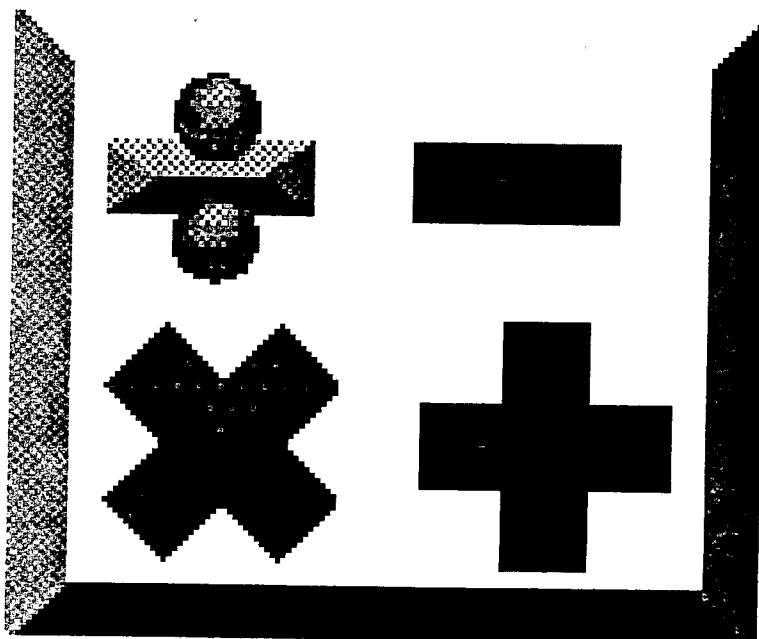
- | | | |
|-------|-------|-------|
| 1. B | 21. C | 41. E |
| 2. A | 22. A | 42. D |
| 3. D | 23. C | 43. D |
| 4. E | 24. B | 44. C |
| 5. B | 25. B | 45. D |
| 6. D | 26. D | 46. D |
| 7. B | 27. A | 47. E |
| 8. C | 28. A | 48. C |
| 9. E | 29. B | 49. C |
| 10. E | 30. D | 50. E |
| 11. E | 31. B | 51. A |
| 12. B | 32. C | 52. D |
| 13. B | 33. C | 53. C |
| 14. D | 34. A | 54. D |
| 15. C | 35. A | 55. C |
| 16. D | 36. D | 56. E |
| 17. E | 37. E | 57. A |
| 18. A | 38. A | 58. B |
| 19. E | 39. A | 59. B |
| 20. B | 40. A | 60. C |



UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2010



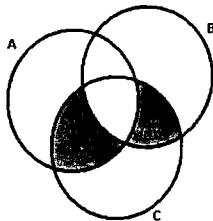
**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $9 + 1 \times 8 - 2 \div (7 - 3) \times (6 + 4) \div 5$

- (A) 1.875 (B) 7.5 (C) 16 (D) 30 (E) 39

2. Let $A = \{0, 2, 1, 3, 4, 7\}$, $B = \{0, 1, 2, 3, 5, 6, 8\}$, and $C = \{0, 1, 3, 5, 6, 7, 9\}$. How many elements are in the shaded areas of the Venn diagram shown?



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3. On a map legend, $\frac{3}{4}$ inch represents 15 miles. The distance on the map from Ft. Stockton to Van Horn is 6 inches. How far is that in miles?

- (A) 120 (B) 105.75 (C) 90 (D) 75.25 (E) 67.5

4. Dr. Saul Tee had a bucket that contained 4 ounces of salt and 6 ounces of water. If he poured 70 ounces of water into the bucket, what percent of the mixture would be salt?

- (A) $1\frac{1}{7}\%$ (B) $3\frac{3}{7}\%$ (C) 5% (D) $5\frac{2}{3}\%$ (E) 7%

5. Noah Dough has a bag of nickels and dimes. He has a total of 42 coins. Les Cash has 8 dimes and 6 nickels in his pocket. The ratio of nickels to dimes is the same for both Les and Noah. How much more money does Noah have than Les?

- (A) \$3.30 (B) \$3.00 (C) \$2.20 (D) \$2.00 (E) \$1.10

6. Find an equation of a line through point $(2, -1)$ that is parallel to a line that contains point $(1, -2)$ and point $(2, 1)$.

- (A) $x - 3y = -1$ (B) $y = 3x - 7$ (C) $x + 3y = 1$ (D) $y = 3x + 5$ (E) $x - 2y = -3$

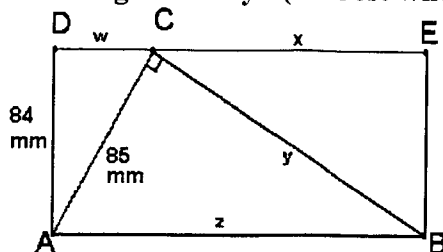
7. Two coplanar angles that have a common vertex, a common side, and have no common interior points are called _____ angles.

- (A) alternate (B) vertical (C) corresponding (D) adjacent (E) dihedral

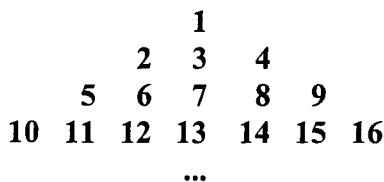
8. $\angle A$ and $\angle B$ are complementary angles. $\angle B$ and $\angle C$ are supplementary angles. Find $m\angle C$ if $m\angle A = 4x - 5$ and $m\angle B = 3x + 4$.

- (A) 137° (B) 133° (C) 127° (D) 123° (E) 117°

9. $\triangle ABC$ is drawn in the rectangle. Find y . (nearest whole number)



- (A) 536 mm (B) 549 mm (C) 562 mm (D) 583 mm (E) 598 mm
10. The *Local Snail* leaves the train station and runs at an average rate of 35 mph. An hour and a half later, the *Local Express* leaves the station and runs at an average rate of 56 mph on a parallel track. How long will it take the *Express* to overtake the *Snail*?
- (A) 4 hrs (B) 3.5 hrs (C) 2.5 hrs (D) 2 hrs (E) 1.6 hrs
11. Let $f(x) = 2x - 3$ and $g(x) = 3x + 1$ and $h(x) = 4 - x$. Find $g(f(h(5x)))$.
- (A) $30x - 4$ (B) $14 - 30x$ (C) $30x - 6$ (D) $23 - 30x$ (E) $16 - 30x$
12. If $y^2 = -3 - 4i$ and $y^3 = 11 - 2i$ where $y = a + bi$ then $a + b$ equals:
- (A) 4 (B) 3 (C) 1 (D) 0 (E) -1
13. Determine the range of $f(x) = 5\cos(4x - 3) - 2$.
- (A) $[-5, 5]$ (B) $[-2, 5]$ (C) $[-7, 3]$ (D) $[2, -5]$ (E) $[7, -3]$
14. How many elements are in $\left\{x \mid \sin^2(x) + \sin(x) - 6 = 0, x \in [0, 2\pi)\right\}$?
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
15. The Red Baron leaves Snoopy airfield flying at a ground speed of 160 mph for 1 hour and 45 minutes on a bearing of 30° . Then he changes course and flies at a ground speed of 185 mph for 1 hour on a bearing of 130° . How far is the Red Baron from the airfield at this time? (nearest mile)
- (A) 278 mi (B) 308 mi (C) 124 mi (D) 345 mi (E) 103 mi
16. If the set of numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, then the median of the 9th row would be?



- (A) 56 (B) 63 (C) 71 (D) 73 (E) 76

17. In the binomial expansion of $(2x - y)^4$, the sum of the coefficients of the terms x^3y and xy^3 is:

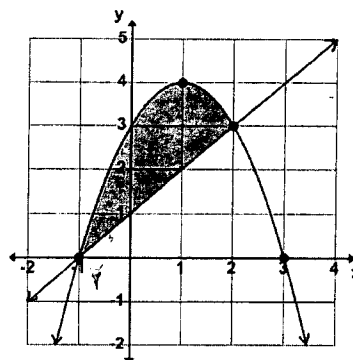
- (A) -24 (B) 40 (C) -10 (D) 24 (E) -40

18. Find the determinant of the 3×3 matrix.

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

- (A) 6 (B) 5 (C) 2 (D) -8 (E) -10

19. Find the area of the shaded region in square units.



- (A) 5 (B) $4\frac{3}{4}$ (C) $4\frac{1}{2}$ (D) $4\frac{1}{4}$ (E) $3\frac{1}{2}$

20. The directrix of the conic given by the equation $9x^2 - 16y^2 = 144$ is:

- (A) $x = 3.2$ (B) $y = 1.8$ (C) $x = 1\frac{1}{3}$ (D) $y = 1\frac{7}{9}$ (E) $x = 0.3125$

21. The point $(-1, -4)$ is rotated $\frac{17\pi}{6}$ radians counterclockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

- (A) $(-3.0, 2.9)$ (B) $(2.9, 3.8)$ (C) $(3.8, -1.6)$ (D) $(2.9, 3.0)$ (E) $(-1.6, -3.8)$

22. Find the angle of rotation, θ (nearest degree), where $0^\circ < \theta < 90^\circ$, such that the conic $4x^2 - 6xy - 9y^2 = 36$ contains no xy term in its equation.

- (A) 78° (B) 71° (C) 61° (D) 22° (E) 12°

23. There are 5 boys and 8 girls in the senior class at Millersview High School. Principal White wants an advisory committee made up of 4 seniors. He wants at least 1 girl and at least 1 boy. How many committees can be formed from the senior class?

- (A) 275 (B) 280 (C) 440 (D) 640 (E) 714

24. Mr. Lou Kuss bought a pair of blank dice. He wrote the numbers 2, 1, 3, 4, 7, and 11 on each die such that each face had only one number on it. He rolled the dice and added the two top faces. What are the odds that the sum was a prime number?

- (A) 5 to 7 (B) 7 to 23 (C) 3 to 10 (D) 7 to 29 (E) 13 to 23

25. Find the tens digit of $(2011)^{2010}$.

- (A) 0 (B) 1 (C) 4 (D) 8 (E) 9

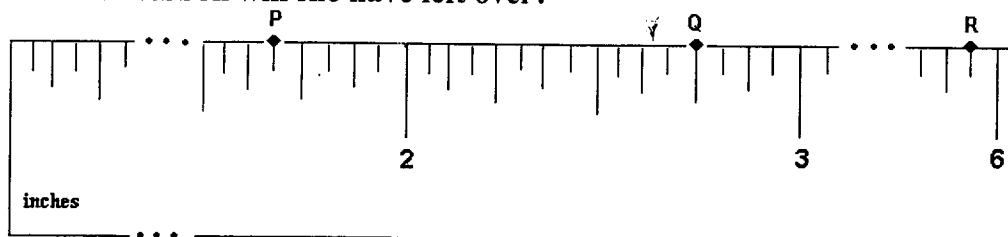
26. Yaps, Yeps, and Yips are three types of precious stones. Six Yaps has the same value as eight Yips. Three Yeps has the same value as five Yips. Rank the three stones in order of their value from highest to lowest.

- (A) Yap, Yep, Yip (B) Yip, Yap, Yep (C) Yep, Yip, Yap (D) Yep, Yap, Yip (E) Yap, Yip, Yep

27. One trillion minus two billion plus three million minus four thousand plus five hundred minus sixty plus seven equals K. How many digits in the number K only appear once?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

28. Pinkie Riban bought a foot of ribbon to use on her class project. She needs three lengths of ribbon. One piece has a length of P, a second piece has length Q, and a third piece has a length of R. How much ribbon will she have left over?



- (A) $\frac{3}{8}$ " (B) $1\frac{5}{8}$ " (C) $1\frac{3}{4}$ " (D) $1\frac{7}{8}$ " (E) $2\frac{1}{8}$ "

29. $44_5 + 333_4 + 2222_3 + 11111_2 = \underline{\hspace{2cm}}_{10}$

- (A) 198 (B) 196 (C) 137 (D) 110 (E) 99

30. Five car washers can wash 8 cars in 3 hours. How long will it take 4 car washers to wash 6 cars if the length of time it takes the washers to wash the cars varies directly as the number of cars and inversely as the number of washers? (nearest minute)

- (A) 1 hr 17 min (B) 5 hrs (C) 2 hrs 20 min (D) 5 hrs 7 min (E) 2 hrs 49 min

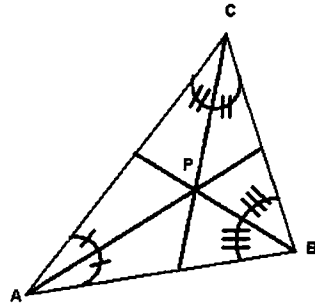
31. Find $f(-3) + f(-2) + f(-1)$ if $f(x) = \begin{cases} x-1 & \text{if } |x-1| > 3 \\ x & \text{if } |x-1| = 3 \\ x+1 & \text{if } |x-1| < 3 \end{cases}$

- (A) 6 (B) 3 (C) -5 (D) -6 (E) -9

32. Points P and Q lie on the x-y plane. Point P(3,0) is reflected across the line $y = -x$ to P'. Point Q(0, -2) is rotated 90° counterclockwise to Q'. Find the shortest distance between P' and Q'. (nearest tenth of a unit)

- (A) 2.2 (B) 2.4 (C) 3.1 (D) 3.3 (E) 3.6

33. Point P is the _____ of $\triangle ABC$ shown below.



- (A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

34. Ima Hogg has \$240.00 in her piggy jar. She decides to spend $\frac{1}{12}$ of the money remaining in the jar on the first day of the month, starting on June 1. At this rate, how much money will she have spent by May 30 the next year? (nearest cent)

- (A) \$20.00 (B) \$77.44 (C) \$147.84 (D) \$155.52 (E) \$162.56

35. If $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_n = a_{n-2} - a_{n-1} + a_{n-3}$, where $n \geq 4$, then a_8 equals:

- (A) -8 (B) -5 (C) -2 (D) 6 (E) 11

36. Find $\sin\left(\frac{\pi}{6}\right) \times \cos\left(\frac{\pi}{3}\right) \times \tan\left(\frac{\pi}{4}\right) \times \csc\left(\frac{\pi}{2}\right) \times \sec\left(\frac{2\pi}{3}\right) \times \cot\left(\frac{3\pi}{4}\right)$.

- (A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

37. A forest ranger marks a point x feet from the base of a cliff and a second point 500 feet farther from the first point. The angles of elevation from the points to the top of the cliff are 70° and 59° , respectively. Find the height of the cliff. (nearest foot)

- (A) 2111 ft (B) 3404 ft (C) 4880 ft (D) 4221 ft (E) 2571 ft

38. Find the harmonic mean of the real roots of $x^3 - 7.5x^2 + 15.5x - 6 = 0$.

- (A) $2\frac{9}{10}$ (B) $2\frac{1}{2}$ (C) $1\frac{15}{16}$ (D) $1\frac{4}{5}$ (E) $1\frac{5}{31}$

39. Let $f(x) = \frac{x^3 - 2x^2 - 8x}{x^2 - 9}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(2)$.

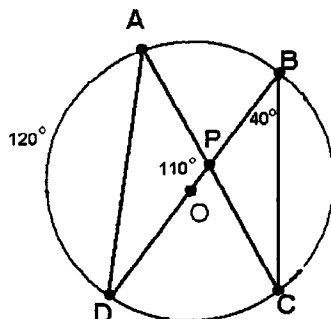
- (A) 3 (B) 2 (C) 0 (D) -2 (E) -3

40. The graph of $x^2 - 2xy + y^2 - 7x + 7y + 10 = 0$ is a(n) _____.

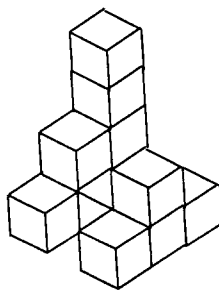
- (A) point (B) line (C) pair of parallel lines (D) ellipse (E) hyperbola

41. Robin D. Hood shoots an arrow from an 80 ft castle wall at a velocity of 64 ft/sec. Its height, H in feet, above the ground at S seconds can be found by the equation $H = 80 + 64S - 16S^2$. Find the instantaneous velocity at 1.5 seconds.
- (A) 140 ft/sec (B) 112 ft/sec (C) 48 ft/sec (D) 20 ft/sec (E) 16 ft/sec
42. The following cards are removed from a standard deck of 52 cards: $2\spadesuit, 4\spadesuit, 6\spadesuit, 8\spadesuit, 10\spadesuit, 3\clubsuit, 5\clubsuit, 7\clubsuit$, and $9\clubsuit$. The remaining cards are shuffled. Two cards are dealt face up. What is the probability that the first card is a $J\spadesuit, Q\spadesuit$, or $K\spadesuit$ and the second card is a club? (nearest tenth)
- (A) 5.0% (B) 3.5% (C) 2.8% (D) 2.3% (E) 1.5%
43. Lotta Moolaw went to the bank and got some \$1 bills, \$5 bills, \$10 bills, \$20 bills, \$50 bills, and \$100 bills. She is stuffing donation envelopes with 4 bills. How many different envelopes containing 4 bills can Lotta make?
- (A) 21 (B) 840 (C) 126 (D) 84 (E) 210
44. Which of the following numbers is considered to be a *lucky prime* number that is *odious*?
- (A) 2 (B) 3 (C) 9 (D) 11 (E) 13
45. Willie Proffett bought 3 used computers at a garage sale for \$200 each. He sold his neighbor one for a 30% profit and sold another one to his favorite uncle for half of his cost. How much will he have to sell the third one for to make a net profit of 25%?
- (A) \$240.00 (B) \$255.00 (C) \$305.00 (D) \$345.00 (E) \$390.00
46. If Lois Skor scores 246 on her next UIL math test, her average score will be 264. If she scores 276, her average score will be 267. How many tests had she already taken?
- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
47. The Real value solution set for $|5 - 3x| > 2$ is?
- (A) $\{x \mid \{2\frac{2}{3} > x\} \cup \{1 > x\}\}$ (B) $\{x \mid 1 < x < 2\frac{2}{3}\}$ (C) $\{x \mid \{x > 1\} \cup \{x < 2\frac{1}{3}\}\}$
- (D) $\{x \mid \{x \mid 1 > x > 2\frac{2}{3}\}\}$ (E) $\{x \mid \{x < 1\} \cup \{x > 2\frac{1}{3}\}\}$
48. The solution set of $6x - 5y = 30$ is $\{(x, y) \mid x, y \in \{\text{Integers}\}, x \geq 0, \text{ and } y \leq 0\}$. How many such ordered pairs exist?
- (A) 6 (B) 5 (C) 3 (D) 2 (E) 0

49. \overline{AC} , \overline{AD} , \overline{BD} , and \overline{BC} are chords of circle O and intersect at point P. Find $m\widehat{AB}$.

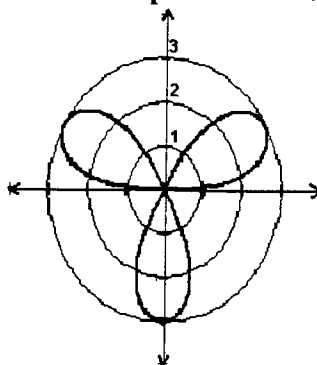


- (A) 35° (B) 50° (C) 60° (D) 70° (E) 80°
50. Find the area of a circumscribed circle of a triangle of sides 5", 12", and 13". (nearest tenth)
- (A) 265.5 sq. in (B) 113.1 sq. in (C) 153.9 sq. in (D) 115.5 sq. in (E) 132.7 sq. in
51. Simplify: $[\log_a(6k) - \log_a(4)] + [\log_a(3) - \log_a(5k)]$
- (A) $\log_a(9) - \log_a(10)$ (B) $2\log_a(22.5k)$ (C) $\log_a(1.6)$
- (D) $\log_a(k) - \log_a(1)$ (E) $\log_a(11k)$
52. Thirteen congruent cubes are glued together to form the figure shown. Dee Artiste picks up the figure and paints each of the cubes' faces blue that are not glued together. How many of the cubes' faces will she paint blue?



- (A) 49 (B) 48 (C) 47 (D) 46 (E) 45
53. A sine function, $f(x)$, has a maximum value of 7, a minimum value of 3, a period of $\frac{2\pi}{3}$ and a horizontal phase shift of $\frac{\pi}{2}$. Determine the value of $f(\frac{\pi}{3})$. (nearest tenth)
- (A) 5.2 (B) 5.0 (C) 4.5 (D) 3.4 (E) 3.0
54. If the three numbers 433, 342, and 290 are each divided by the number D, each of their quotients will have the same remainder R. Find R.
- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

55. Which of the following polar equations will produce this graph on a polar grid?



- (A) $r = 3\cos(\theta) + 3$ (B) $r = \sin(3\theta) + 3$ (C) $r = 3\cos(3\theta)$
 (D) $r = 3\sin(3\theta)$ (E) $r = 3\sin(\theta) - 3$

56. Find C if the remainder when $x^3 - 9x^2 + 15x + C$ is divided by $x - 3$ is -14 .

- (A) -12 (B) -9 (C) -5 (D) 3 (E) 15

57. Evaluate: $\prod_{n=2}^5 \left(\frac{1}{n} - n\right)$

- (A) $129\frac{3}{5}$ (B) 72 (C) 15 (D) $-12\frac{43}{60}$ (E) -15

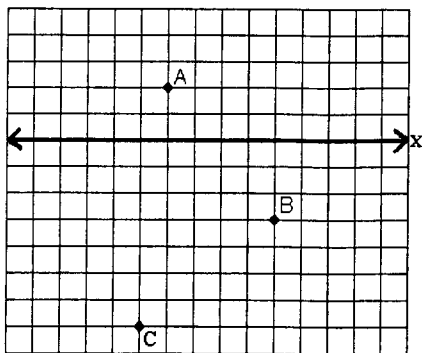
58. Larry, Moe, Curly, and four of the little rascals are randomly assigned seats in a row of seven chairs. What is the probability that Larry, Moe, and Curly will be seated in no specific order next to each other? (nearest tenth)

- (A) 42.9% (B) 25.0% (C) 14.3% (D) 7.1% (E) 2.4%

59. The U-Nee-Cue tribe uses a special base to solve their math problems. Using their special base they found that $16Q - 70 = 0$ and $13Q - 60 = 0$. What base are they using?

- (A) base 5 (B) base 8 (C) base 12 (D) base 15 (E) base 16

60. The x-axis of a Cartesian Coordinate plane is shown. Point A is in QII, the coordinates of point B is $(1, -3)$, and the coordinates of point C is (x, y) . Find $x - y$.



- (A) 10 (B) 4 (C) 3 (D) -5 (E) -11

**University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2010
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. D | 41. E |
| 2. B | 22. A | 42. E |
| 3. A | 23. D | 43. C |
| 4. C | 24. E | 44. E |
| 5. C | 25. A | 45. E |
| 6. B | 26. D | 46. D |
| 7. D | 27. C | 47. E |
| 8. A | 28. B | 48. D |
| 9. B | 29. A | 49. C |
| 10. C | 30. E | 50. E |
| 11. E | 31. D | 51. A |
| 12. C | 32. E | 52. B |
| 13. C | 33. D | 53. E |
| 14. E | 34. D | 54. C |
| 15. B | 35. C | 55. D |
| 16. D | 36. D | 56. C |
| 17. E | 37. A | 57. B |
| 18. A | 38. E | 58. C |
| 19. C | 39. C | 59. D |
| 20. A | 40. C | 60. C |