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T E S T # 1 2 ©

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GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2018 – 2019 TMSCA Mathematics Test Twelve

1. Evaluate: $(3-8)+2^3\times11\div(3^2+1)-2$

(A) 1.8 **(B)** 11.8 **(C)** 5.6 **(D)** 2.8 **(E)** 7.8

2. Find the sum of the multiples of six that are greater than 0 and less than 226.

(A) 4212 **(B)** 4107 **(C)** 4292 **(D)** 4218 **(E)** 4256

3. $(x+3)(x-7)=x^2-7x+3x-21$ **and** $9(2x+7)=18x+63$ **are examples of the _____ property of equality.**

(A) associative **(B)** commutative **(C)** addition **(D)** distributive **(E)** multiplication

4. Which of the following is an equation of a line parallel to $7x-3y=17$ **that passes through** $(3,-1)$**?**

(A) $7x-3y=18$ **(B)** $3x+7y=2$ **(C)** $3x+7y=16$

(D) $7x-3y=6$ **(E)** $7x-3y=24$

5. Let the pattern 2, 6, 12, 20, 30, … continue. Find the sum of the 18th and 19th terms.

(A) 648 **(B)** 800 **(C)** 130 **(D)** 722 **(E)** 144

6. In a survey of 50 juniors, 32 voted in favor of a Broadway prom theme and 6 were neutral. If the whole junior class was 234 students, what is the best estimate of how many students did not want the Broadway theme?

(A) 12 **(B)** 150 **(C)** 59 **(D)** 156 **(E)** 56

7. Which of the following multiples of 5 is the mean of two consecutive prime numbers?

(A) 60 **(B)** 35 **(C)** 25 **(D)** 40 **(E)** 65

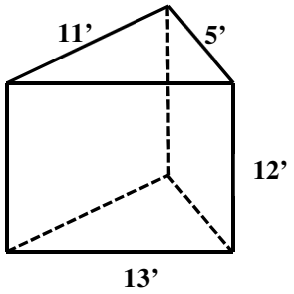
8. If $B=\{b,a,c,h\}$ **,** $M=\{m,o,z,a,r,t\}$ **and** $H=\{h,a,n,d,e,l\}$ **, then** $(B\cap M)\cap(H\cap M)=$ **?**

(A) $\{a\}$ **(B)** $\{d,e,h,l,n\}$ **(C)** $\{a,d,l,n\}$ **(D)** $\{a,d,e,h,l,n\}$ **(E)** $\{d,e,l,n\}$

9. Topsy weighs 90 pounds and is sitting on a seesaw 6.3 feet from the middle. Turvy weighs 81 pounds. How far from the middle must Turvy sit in order to balance the seesaw?

(A) 7 ft. 8 in. **(B)** 8 ft. 2 in. **(C)** 7 ft. 9 in. **(D)** 8 ft. 0 in. **(E)** 7 ft

10. A custom tank at an aquarium has the dimensions of the triangular prism shown. If the tank is designed to be filled to 80% of the total volume of the prism, what is the capacity? (nearest gallon)



(A) 2414 **(B)** 1931 **(C)** 2469 **(D)** 1975 **(E)** 2208

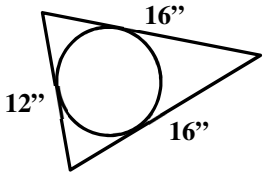
11. The measure of one central angle of a regular dodecagon is _____°.

(A) 36 (B) 20 (C) 30 (D) 40 (E) 60

12. Given that quadrilateral ABCD is inscribed in circle O, $\angle A$ and $\angle C$ are not consecutive angles, where $m\angle A, m\angle C \in \mathbb{Z}$, $m\angle A = \left(3x^2+10\right)^\circ$ and $m\angle C = \left(7x^2+2x+2\right)^\circ$, find $m\angle A$.

(A) 58° (B) 144° (C) 34° (D) 122° (E) 92°

13. The circle in the illustration is inscribed in the triangle. Find the circle circumference. (nearest tenth)



A) 25.4 in (B) 12.9 in (C) 51.4 in (D) 12.7 in (E) 5.1 in

14. Simplify: $a^3\left(\dfrac{(a^{-6})\sqrt{a^5}}{a^{-5}}\right)$

(A) $\left(\sqrt{a}\right)^{-1}$ **(B)** $\left(\sqrt{a}\right)^{7}$ **(C)** $\left(\sqrt{a}\right)^{-4}$ **(D)** $\left(\sqrt{a}\right)^{9}$ **(E)** $\left(\sqrt{a}\right)^{3}$

15. Consider the point (a,b) in the Cartesian plane. The transformation $\begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$ **results in a rotation of _____about the origin.**

(A) 60° counter-clockwise (B) 120° clockwise (C) 30° clockwise

(D) 120° counter-clockwise (E) 60° clockwise

16. A metallurgist has an alloy with 15% titanium and an alloy with 30% titanium. He needs 100 grams of an alloy with 21% titanium. How much of the 15% titanium alloy should he use in to make the new alloy?

(A) 40 g (B) 32 g (C) 60 g (D) 25 g (E) 75 g

17. The point (x,y) is a point of inflection on $f(x)=\dfrac{\sin x}{1+\cos x}$, where $3\pi < x < 6\pi$. Find the value of x.

(A) $\dfrac{9\pi}{2}$ **(B)** $\dfrac{11\pi}{2}$ **(C)** $\dfrac{17\pi}{3}$ **(D)** 4π **(E)** $\dfrac{11\pi}{3}$

18. What is the area of enclosed by the graph of the relation $|x|+|y|=12$? (nearest square unit)

(A) 576 (B) 204 (C) 144 (D) 216 (E) 288

19. If $\log_6(x+7)+\log_6(x+2)=2$, then $x=$?

(A) 11 (B) 13 (C) 2 (D) 4 (E) No solution

20. Find the range of the function $y = 7 + 2|x - 3|$ given that the domain is restricted to $\{x \in \text{Reals} | -2 \le x \le 5\}$.

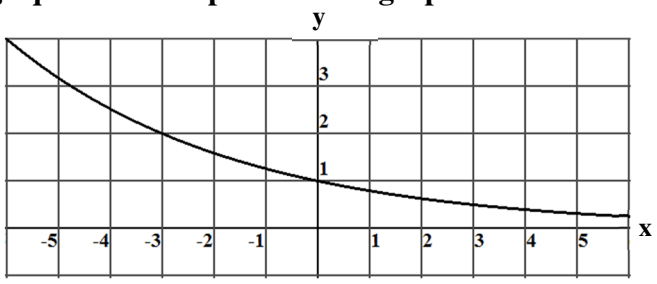
(A) $\{y \in \text{Reals} | 11 \le y \le 17\}$ **(B)** $\{y \in \text{Reals} | 7 \le y \le 17\}$ **(C)** $\{y \in \text{Reals} | y \ge 7\}$

(D) $\{y \in \text{Reals} | y \le 17\}$ **(E)** $\{y \in \text{Reals} | y \ge 11\}$

21. Which of the following is a reference angle for $\frac{29\pi}{6}$?

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{4}$

22. Which of the following equations will produce the graph shown here?



(A) $y = e^{3x}$ **(B)** $y = 2\log x$ **(C)** $y = 2^{-x/3}$ **(D)** $y = 2^{-3x}$ **(E)** $y = \sqrt{2^x}$

23. If *f* is continuous on the closed interval $[a,b]$ and *k* is any number between $f(a)$ and $f(b)$, then there is at least one number *c* in $[a,b]$ such that $f(c) = k$. This is the _____.

(A) Sandwich Theorem **(B)** Rolle's Theorem **(C)** Fundamental Theorem of Calculus

(D) Intermediate Value Theorem **(E)** Fundamental Theorem of Algebra

24. A number is defined as *highly composite* if it has more positive divisors than all smaller whole numbers. Forty-eight is a highly composite number. What is the smallest highly composite number greater than 48?

(A) 72 **(B)** 80 **(C)** 60 **(D)** 96 **(E)** 54

25. $\frac{x^3 - 7x^2 - 4x + 28}{x^3 + 2x^2 - 49x - 98} \div \frac{x + 2}{x^2 - 49} =$

(A) $\frac{x^2 - 5x - 14}{x + 7}$ **(B)** $\frac{x^2 + 5x - 14}{x - 2}$ **(C)** $\frac{x^2 + 9x + 14}{x - 7}$

(D) $x^2 - 5x - 14$ **(E)** $\frac{x^2 - 9x + 14}{x + 2}$

26. Let $(6 + 5i)(2 - i) \div (2 + 3i) = a + bi$. Find $a + b$.

(A) $\frac{81}{13}$ **(B)** $\frac{37}{13}$ **(C)** $\frac{89}{13}$ **(D)** $\frac{3}{13}$ **(E)** $\frac{41}{13}$

27. $15^3+16^3+17^3+...+28^3=$ **?**

(A) 153,811 (B) 174,825 (C) 164,836 (D) 141,008 (E) 159,323

28. Stephen wants to find three-digit numbers such that the hundreds digit is twice the units digit and the tens digit is the square of the units digit. What is the sum of all such numbers?

(A) 11525 (B) 1616 (C) 1328 (D) 752 (E) 1346

29. Evaluate $\lim_{h\to 0}\dfrac{\tan\left[2\left(\frac{\pi}{3}+h\right)\right]-\tan\left[2\left(\frac{\pi}{3}-h\right)\right]}{2h}$ **.**

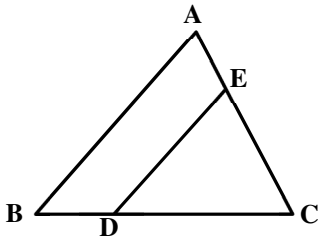
(A) $\frac{1}{2}$ **(B) 2 (C) 4 (D)** $\frac{1}{4}$ **(E) 8**

30. Which of the following mathematicians is noted for his work with polynomial equations that allow variables to be integers only?

(A) Erastosthenes (B) Leibniz (C) Descartes

(D) Diophantus (E) Archimedes

31. Triangle ABC and triangle CDE exist such that $AB \parallel DE$**, AB = 20", AC = 16" and AE = 6". Find DE.**



(A) 11.5" (B) 12.5" (C) 13" (D) 14.5" (E) 14"

32. If $\int_{-8}^{12} f(x)dx = 17$ **evaluate** $\int_{-8}^{12}\left[3+5f(x)\right]dx$ **.**

(A) 85 (B) 88 (C) 109 (D) 95 (E) 145

33. Let $f_0=0, f_1=1, f_2=1, f_3=2, f_4=3,...$ **be the terms of the Fibonacci sequence. How many digits are in** f_{30}**?**

(A) 8 (B) 6 (C) 7 (D) 9 (E) 10

34. If *p, q* and *r* are non-negative integers, determine the number solutions to $p+q+r=18$**.**

(A) 171 (B) 162 (C) 190 (D) 153 (E) 181

35. Chords $\overline{BD}$ **and** $\overline{AC}$ **intersect inside circle O at point P. If PA = 13.5 cm, AC = 22.5 cm and PB = 3 cm, then PD =?**

(A) 31.5 cm (B) 101.25 cm (C) 18 cm (D) 40.5 cm (E) 67.5 cm

36. How many distinct 4-letter arrangements can be made using all of the letters in the words "HUMPTY DUMPTY"?

(A) 1800 (B) 1740 (C) 960 (D) 1020 (E) 900

37. The functions $f(x)$ and $g(x)$ are defined as $f(x)=x^2$ and $g(x)=2x+11$. Calculate $f(g(3))-g(f(3))$

(A) 153 (B) 260 (C) 225 (D) 318 (E) 167

38. The first term of an infinite geometric sequence is 36, while the third term is 16. What is the smallest possible value for the sum of all the terms in the sequence?

(A) 108 (B) 66 (C) 21.6 (D) 96 (E) 43.2

39. $\prod_{k=2}^{5}(2k+1)=$

(A) 9009 (B) 945 (C) 32 (D) 3465 (E) 315

40. $\sin^4\theta+2\sin^2\theta\cos\theta+\cos^4\theta=$

(A) $\sin\theta\sec\theta$ (B) $\sin\theta\csc\theta$ (C) $\cos\theta\csc\theta$ (D) $\tan^2\theta$ (E) $\sin\theta\cos\theta$

41. If $\int_{13}^{k}\frac{1}{2x-1}dx=\ln(1.4)$, calculate the value of k.

(A) 49 (B) 50 (C) 25 (D) 26 (E) 18

42. Given $f(x)=ax^5+bx^3+cx-12$ and $f(3)=19$, calculate $f(-3)$.

(A) −43 (B) −31 (C) 31 (D) −9 (E) −39

43. Chloe has a large bucket of nickels, dimes, quarters and half-dollar coins. She is going to hide sets of 4 coins in eggs for an Easter egg hunt. How many distinct sets of 4 coins could she make?

(A) 70 (B) 15 (C) 112 (D) 56 (E) 35

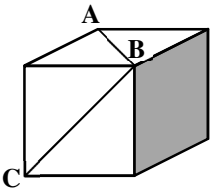
44. Suppose that $f(a)=g(a)=0$, $f'(a)$ and $g'(a)$ exist, and that $g'(a)\neq 0$, then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=$

(A) $f'(a)g'(a)$ (B) $\frac{f(a)}{g(a)}$ (C) 0 (D) $\frac{f'(a)}{g'(a)}$ (E) $f'(a)-g'(a)$

45. Find k when $g(x)=3x^3+kx^2+kx+8$ divided by $h(x)=x-1$ has a remainder of 5.

(A) −2 (B) −3 (C) 2 (D) 1 (E) 5

46. A, B, and C are each vertices of the cube shown. Find $m\angle ABC$.



(A) 75° (B) 45° (C) 60° (D) 67.5° (E) 90°

47. Find the slope to the line tangent to $3x^2+2xy+y^2=12$ at the point $(-2,4)$.

(A) −1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) $-\frac{5}{2}$

48. Point $M(-1,19)$ is the midpoint of the line segment with endpoints $P(-11,y)$ and $Q(x,25)$. Find PQ. (nearest whole number)

(A) 23 (B) 43 (C) 28 (D) 46 (E) 12

49. Clive hiked 6 miles on a bearing of 135° then turned and went 7 more miles on a bearing of 148°. Finally, he hiked 12 miles on a bearing of 82° how far was his final destination from his starting point? (nearest mile)

(A) 19 mi (B) 25 mi (C) 17 mi (D) 22 mi (E) 2 mi

50. Given $y = -2\sin x$, find the value of x for which $\frac{dy}{dx} = \frac{dx}{dy}$, where $0 \le x \le \frac{\pi}{2}$

(A) π (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$ (E) 0

51. Given $f(x) = 11 - 3\sin\left[\frac{\pi}{2}(x-2)\right]$, find the sum of the period and amplitude.

(A) 7 (B) 8 (C) −3 (D) 4 (E) 3

52. Find the value of $2A - 2B + 2C$, where A, B and C are greater than zero and

$$\frac{346}{31} = A + \left(\frac{1}{B + \left(\frac{1}{C+1}\right)}\right).$$

(A) 16 (B) 18 (C) 14 (D) 20 (E) 21

53. Find the distance between the line $6x + 5y = 42$ and the point $(-5,1)$. (nearest tenth)

(A) 9.7 (B) 10.5 (C) 9.1 (D) 11.2 (E) 8.6

54. The graph of $h(x) = \frac{5x^2 + 37x + 14}{x+7}$ suggests that the *discontinuity* at $x = -7$ is *removable* by defining $h(-7)$ to be_____.

(A) 37 (B) −14 (C) −19 (D) −33 (E) 14

55. Let $f(x) = 7x^5 + 13x^4 - 3x^3 - 4x - 12$. According to Descartes' Rule of Signs, how many possible negative real roots are there?

(A) 0 or 1 (B) 0 or 2 (C) 1 or 3 (D) 0 (E) 1 only

56. $111000_2 + 1110_4 + 10_8 =$ _____$_{16}$

(A) 94 (B) 82 (C) 90 (D) 88 (E) 98

57. Barry can dig a cellar in 20 hours. Carl can dig a cellar the same size in 30 hours. How long would it take the two of them working together to dig a cellar twice as wide, long and deep?

(A) 48 hr (B) 24 hr (C) 96 hr (D) 12 hr (E) 50 hr

58. What is the area of the largest isosceles triangle that can be inscribed in the circle with the equation $x^2 + y^2 + 10x - 6y - 47 = 0$? (nearest unit)

(A) 35 **(B) 105** **(C) 61** **(D) 70** **(E) 86**

59. The repeating decimal 0.414141… in base 5 can be written as which of the following simplified base 10 fractions?

(A) $\frac{41}{44}$ **(B)** $\frac{21}{22}$ **(C)** $\frac{13}{16}$ **(D)** $\frac{15}{16}$ **(E)** $\frac{7}{8}$

60. Find $f(2) - f(6) + f(8)$ **if** $f(x) = \begin{cases} x^2 + 1, & x < 6 \\ x - 3, & x = 6 \\ x + 2, & x > 6 \end{cases}$

(A) 12 **(B) 11** **(C) 4** **(D) 9** **(E) 5**

2018-2019 TMSCA Mathematics Test Twelve Answers

1. A	**21. A**	**41. C**
2. D	**22. C**	**42. A**
3. D	**23. D**	**43. E**
4. E	**24. C**	**44. D**
5. D	**25. E**	**45. B**
6. E	**26. D**	**46. C**
7. A	**27. A**	**47. D**
8. A	**28. E**	**48. A**
9. E	**29. E**	**49. D**
10. B	**30. D**	**50. C**
11. C	**31. B**	**51. A**
12. A	**32. E**	**52. B**
13. A	**33. B**	**53. E**
14. D	**34. C**	**54. D**
15. D	**35. D**	**55. B**
16. C	**36. A**	**56. A**
17. D	**37. B**	**57. C**
18. E	**38. C**	**58. B**
19. C	**39. D**	**59. E**
20. B	**40. B**	**60. A**

2018-2019 TMSCA Mathematics Test Twelve Solutions

10. Find the included angle between the 11' and 5' side, $\arccos\left(\frac{13^2-11^2-5^2}{-2(11)(5)}\right)=\theta$, **then find the area of the base,** $B=\frac{1}{2}(11)(5)\sin\theta$, **then finally,** $0.8B\ ft^3\left(\frac{12in}{1ft}\right)^3\left(\frac{1gal}{231in^3}\right)$ **or about 1931.**

12. The opposite angles in a quadrilateral inscribed in a circle are supplementary, so solve $3x^2+10+7x^2+2x+2=180$ **for** $x=4$ **and** $m\angle A=58°$

13. $r=\sqrt{\frac{(22-16)(22-16)(22-12)}{22}}$ **and the area is** $2\pi r\approx 25.4$

18. This makes a square with diagonals of 24, so the area is $A=\frac{1}{2}(24)^2=288$

27. The sum of the first *n* cubes is $\left(\frac{n(n+1)}{2}\right)^2$ **, so the sum shown is** $\left(\frac{28\times 29}{2}\right)^2-\left(\frac{14\times 15}{2}\right)^2=153{,}811$

32. First $12-^{-}8=20$**, and the integral is** $3(20)+5(17)=145$

34. The number of positive integer solutions is the $(18+1)^{th}$ **triangular number or** $\frac{19(20)}{2}=190$

39. The non-repeated letters are just the H and D the other 5 letters are doubles. There are 7 distinct letters:
No repeats: ${}_7P_4=840$
1 repeat: $5\times{}_6C_2\times\frac{4!}{2!}=900$
2 repeats: ${}_5C_2\times\frac{4!}{2!\times 2!}=60$
for a total of 1800 distinct possibilities.

38. The two possible common ratios that fit the given terms are $\pm\frac{2}{3}$ **with the smaller possible sum** $S=\frac{36}{1+2/3}=21.6$

39. This is the product $(5)(7)(9)(11)=3465$

41. $\left[\frac{\ln(2x-1)}{2}\right]_{13}^{k}$ **for** $\frac{\ln(2k-1)}{2}-\frac{\ln 25}{2}=\ln\left(\frac{7}{5}\right)$ **and** $\frac{\sqrt{2k-1}}{\sqrt{25}}=\frac{7}{5}$ **and** $k=25$

48. The two points are $P(-11,13)$ **and** $Q(9,25)$ **for a midpoint of** $M(-1,19)$ **, then**
$PQ=\sqrt{(9+11)^2+(25-13)^2}\approx 23$

50. Take the derivative and solve $-2\cos x=\pm 1$ **for** $x=\frac{\pi}{3}$

53. $d=\frac{|-5(6)+1(5)-42|}{\sqrt{36+25}}\approx 8.6$