1. **Evaluate: $3! + 4^2 \div 5 - (1 \times 6^0 + 7)$**

   (A) -3.6 (B) -0.4 (C) 1.2 (D) 5.8 (E) 10.4

2. **Ma Bell's telephone company charges a \$15.50 per month base fee plus 1.5¢ per minute for local calls and 25¢ per minute for long distance calls. What would the bill be if 250 minutes in local calls and 50 minutes in long distance calls were made during the month?**

   (A) \$17.80 (B) \$31.75 (C) \$54.25 (D) \$65.50 (E) \$78.75

3. **Let U (universal set) = {p, r, o, b, l, e, m, a, t, i, c}, R = {r, a, t, i, o}, and T = {t, a, b, l, e}. Let $S = T^C \cup R$. Set S contains how many distinct elements?**

   (A) 2 (B) 3 (C) 7 (D) 8 (E) 10

4. **Five-eighths is the same part of two-thirds as three-tenths is of _____.**

   (A) $\frac{9}{32}$ (B) $\frac{15}{16}$ (C) $\frac{18}{25}$ (D) $\frac{12}{19}$ (E) $\frac{8}{25}$

5. **Which of the following linear equations is perpendicular to the line through the points $(-3, 2)$ and $(5, -1)$?**

   (A) $8x + 3y = 10$ (B) $x - 2y = -6$ (C) $8x - 3y = 10$ (D) $3x - 8y = -22$ (E) $x + 2y = 6$

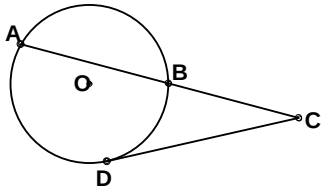
6. **Les Cash, Lotta Dough, and Noah Scents spent \$112.00 at the mall. Les spent half of the amount Noah spent and Lotta spent \$10.00 more than Les spent. How much did Noah spend?**

   (A) \$51.00 (B) \$50.00 (C) \$41.00 (D) \$35.50 (E) \$25.50

7. **If $3x^2 + 4x - 7 = (3x - a)(x - b)$, where a, b are integers, then $a + b = ?$.**

   (A) -8 (B) -6 (C) 6 (D) 7 (E) 8

8. **Given the circle with center O shown. Find AB if BC = 5 cm and CD = 6.5 cm.**



   (A) 12.25 cm (B) 11.5 cm (C) 8.45 cm (D) 3.45 cm (E) 3.05 cm

9. **If $\angle ABC$ and $\angle CBD$ are complementary and $\angle ABC$ and $\angle CBE$ are supplementary, find $m\angle CBE$ if $m\angle CBD = 42°$.**

   (A) 132° (B) 138° (C) 140° (D) 142° (E) 148°

10. Phil It-Upp has a gasoline tank in the shape of a right cylinder. The diameter of the tank is 4 feet. The capacity of the tank is 100 gallons, but it is only three-fourths full. How deep is the gasoline in the tank? (nearest 1/4 inch)

(A) 8.25 in (B) 8.75 in (C) 9 in (D) 9.25 in (E) 9.5 in

11. Find the 9^{th} term of the given the arithmetic sequence: { — 5, — 1.5, 2, a, b, c, ...}.

(A) 23 (B) 24.5 (C) 26.5 (D) 28 (E) 31.5

12. Using the following pattern of numbers, determine the median term of row 14.

1	(row 0)
1 1	(row 1)
1 2 1	(row 2)
1 3 3 1	(row 3)
1 4 5 4 1	(row 4)
1 5 7 7 5 1	(row 5)
1 6 9 10 9 6 1	(row 6)
...	(row ...)

(A) 52 (B) 50 (C) 47 (D) 46 (E) 47

13. If $\frac{2x-3}{4x-1} + \frac{Ax-B}{3x-2} = \frac{2x(13x+3)}{12x^2-11x+2}$, where A and B are constants, then A + B = ?

(A) 11 (B) 5 (C) 1 (D) — 1 (E) — 6

14. Kandy Korn bought 3 lbs of chocolates and 4 lbs of nuts for $24.00 at the *Saul T. Suite* store. Ima Nutt bought 1 lb of chocolates and 6 lbs of nuts for $15.00 at the same store. How much would it cost to buy 2 lbs of chocolates and 5 lbs of nuts at the *Saul T. Suite* store?

(A) $37.50 (B) $15.00 (C) $33.00 (D) $39.00 (E) $19.50

15. Let f(x) = 1 — 2sin(3π(x + 4). Find the sum of the vertical displacement, the amplitude, the period and the frequency.

(A) $5\frac{1}{6}$ (B) $1\frac{2}{3}$ (C) 6 (D) 10 (E) $1\frac{1}{6}$

16. Determine the range of(x) = 1 — 2cos[3π(x + 4)].

(A) [— 2, 2] (B) [0, 4] (C) [— 1 , 2] (D) [1, 4] (E) [— 1, 3]

17. Find a + b + c + d given the Fibonacci characteristic sequence: 6, a, b, 28, c, 73, d, ...

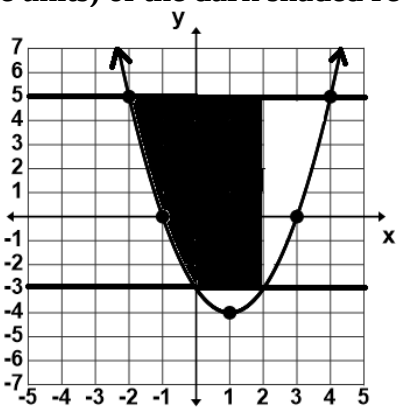
(A) 107 (B) 146 (C) 191 (D) 225 (E) 298

18. Omitted Problem.

19. Rose Thorn's assignment for her horticulture class was to plant birches and roses. While each girl planted 3 roses, each boy planted 1 birch. At the end of the day, 24 plants had been planted. How many girls were in the class if the number of boys in the class was the same as the number of girls?

(A) 4 **(B) 6** **(C) 8** **(D) 12** **(E) 18**

20. The area (in square units) of the dark shaded region shown is:



(A) $37\frac{1}{3}$ **(B)** $34\frac{2}{3}$ **(C) 28** **(D)** $26\frac{2}{3}$ **(E)** $25\frac{1}{3}$

21. A function, $g(x) = x^2 + bx + c$, exists such that $g(-1) = 1$ and $g(2) + g(-3) = -1$. Find $g(4)$.

(A) — 14 **(B) 24** **(C) — 54** **(D) 12** **(E) — 28**

22. The series 6, 4, 3, ... is a harmonic series. Find the sum of the 4^{th} and 6^{th} terms of the series.

(A) $2\frac{1}{4}$ **(B)** $3\frac{5}{7}$ **(C)** $4\frac{2}{5}$ **(D)** $4\frac{4}{35}$ **(E)** $6\frac{4}{35}$

23. Which of the following mathematicians is associated with the "Stepped Reckoner", the first hand cranked calculator that could perform all four arithmetic operations?

(A) Erastosthenes (B) Gottfried Leibniz (C) Diophantus (D) Benoit Mandelbrot (E) Theano

24. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{31}?

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

25. Reed Moore's book store is packaging five books to a bundle. He has history books, sports books, religious books, DIY books, children's books, and comic books. How many different bundles of books can Reed package?

(A) 126 **(B) 720** **(C) 252** **(D) 360** **(E) 462**

26. Find k if LCM(24, k) = 192 and GCF(24, k) = 8.

(A) 16 **(B) 32** **(C) 48** **(D) 64** **(E) 96**

27. Find the number of positive integral divisors of 2,025.

(A) 15 (B) 13 (C) 11 (D) 10 (E) 8

28. Determine the value of k so that $6x^2 - 4x + k = 0$ has one real root.

(A) 2 (B) $1\frac{1}{2}$ (C) 1 (D) $\frac{2}{3}$ (E) $\frac{1}{4}$

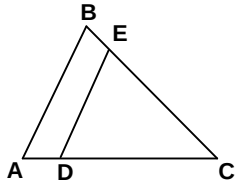
29. If $2x + 3y = 4$, $4x - y = 6$ and $x + ky = -1$, then k equals:

(A) -9 (B) -5 (C) $1\frac{6}{7}$ (D) $1\frac{2}{7}$ (E) 10

30. Rusty Yought sailed his boat from his dock 20 km in the direction N30°W. Then he changed course and sailed 14 km in the direction N45°E. How far was Rusty from his dock after sailing the 34 km? (nearest km)

(A) 17 km (B) 21 km (C) 27 km (D) 36 km (E) 42 km

31. $\triangle ABC$ and $\triangle CDE$ exist such that AB $\parallel$ DE, AB = 6", BC = 4" and BE = 1". Find DE.



(A) 1.5" (B) 4" (C) 4.5" (D) 5" (E) not enough information given

32. Point P (3, $-$ 2) is the midpoint of the line segment with endpoints Q ($-$ 1, y) and R (x, $-$ 11). Find $x + y$.

(A) 0.5 (B) 6.5 (C) 14 (D) 16 (E) 21

33. Which of the following points of concurrency does not lie on the line of Euler in a scalene triangle?

(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) all of them

34. Find the range of the function $y = |5 - 3x| + 2$ given that the domain is restricted to $\{x \in \text{Reals} \mid -2 \leq x \leq 5\}$.

(A) {all Reals} (B) $\{y \mid y \in \{\text{Reals}\}, 2.5 \leq y \leq 12\}$ (C) $\{y \mid y \in \{\text{Reals}\}, -2 \leq y \leq 2\}$
(D) $\{y : y > 2\}$ (E) $\{y \mid y \in \{\text{Reals}\}, 2 \leq y \leq 13\}$

35. Let $f(x) = 3x - 2$, $g(x) = 5x - 1$, $h(x) = 4x$, and $g(f(x)) - f(h(x)) = -27$. Find x.

(A) -12 (B) -9 (C) -6 (D) 3 (E) 6

36. Leo Pisa pulled the ace of hearts, ace of diamonds, two of spades, three of clubs, and the five of hearts from a standard deck of cards. He placed them face down and mixed them up. He randomly turns three cards over face up. What is the probability that the sum of the pip values of the three cards face up is a Fibonacci number? (nearest percent)

(A) 29% **(B)** $33\frac{1}{3}\%$ **(C) 57%** **(D)** $66\frac{2}{3}\%$ **(E) 70%**

37. A right triangle, $\triangle ABC$, exists such that $m\angle ACB = 90°$, M is the midpoint of $\overline{AB}$, BC = 5", and MC = 7.5". Find $m\angle BMC$. (nearest degree)

(A) 23° **(B) 29°** **(C) 39°** **(D) 43°** **(E) 45°**

38. Find the eccentricity of the ellipse, $\frac{(x-1)^2}{9} + \frac{y^2}{5} = 100$**. (nearest hundredth)**

(A) $\frac{1}{15}$ **(B)** $\frac{41}{50}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{2}{3}$ **(E) 1**

39. Let $f(x) = 6x^5 + 33x^4 - 30x^3 + 100$**. Find the sum of the x-values of the critical points of f(x).**

(A) −5.6 **(B) −4.4** **(C) 0** **(D) 0.6** **(E) 2.2**

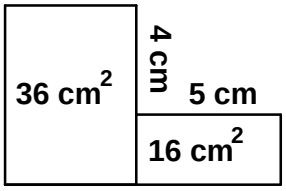
40. The odds of a Big 12 basketball team beating a SEC basketball team is $\frac{3}{7}$**. How many games can they expect to lose if the Big 12 teams play 14 games against the SEC?**

(A) 10 **(B) 9** **(C) 8** **(D) 6** **(E) 4**

41. $\int \left(\frac{1-x^2\sin(x)}{x^2}\right) dx =$ _____ $+ C$**, where C is some arbitrary constant and** $x > 0$**.**

(A) $\frac{2-\cos(x)}{x}$ **(B)** $\ln(x) + \cos(x)$ **(C)** $\cos(x) - \ln(x)$ **(D)** $\frac{-1+x\cos(x)}{x}$ **(E)** $\frac{-\ln(x)-\cos(x)}{x}$

42. Given the areas of the two rectangles shown find the perimeter of the hexagon. (The drawing is not drawn to scale.)



(A) 36 cm **(B) 34.4 cm** **(C) 32.5 cm** **(D) 31.2** **(E) not enough data**

43. Which of the following is/are true about the natural number 3?
1. prime 2. Germain prime 3. Mersenne prime 4. one of the primes of a pair of twin primes

(A) 1 & 3 but not 2 & 4 **(B) 1, 2, & 3 but not 4** **(C) 1, 3, & 4 but not 2**
(D) 1 only **(E) 1, 2, 3, & 4**

44. $2154_7 \div 6_7 \times 5_7 =$ _____ $_7$

(A) 2125_7 **(B)** 2233_7 **(C)** 2323_7 **(D)** 1611_7 **(E)** 1215_7

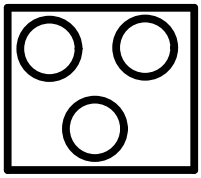
45. If $a_1 = -2$ $a_2 = -1$, $a_3 = 0$, and $a_n = (a_{n-2})(a_{n-3}) + (a_{n-1})$, where $n > 3$ then $a_7 = ?$

(A) -2 **(B)** 0 **(C)** 2 **(D)** 6 **(E)** 8

46. Let vector $u = (-1, 2)$ and vector $v = (3, -5)$. Find the measure of the larger angle with initial side u and terminal side v. (nearest minute)

(A) 184°39' **(B)** 94° 23' **(C)** 85° 36' **(D)** 175° 36' **(E)** 184° 24'

47. Willie Maykit tosses a bean bag at the target board. The board is a square with sides 3 feet long with 12 inch diameter circles cut out of it as shown. What is the probability he tosses the bag into one of the holes? (nearest whole percent)



(A) 20% **(B)** 25% **(C)** 26% **(D)** 30% **(E)** 33%

48. Find $m + n$ if $\begin{bmatrix} -1 & \frac{2}{3} \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

(A) -13 **(B)** -4 **(C)** -2 **(D)** 1 **(E)** 2

49. When $f(x) = x^3 + kx^2 + 5x + 1$ is divided by $x - 2$ the remainder is 7. Find the value of k.

(A) 4 **(B)** 3 **(C)** 1 **(D)** -2 **(E)** -3

50. How many five letter distinguishable code words can be created from the letters in the word ARITHMETIC such that the first letter is an I, the third letter is a consonant and the last letter is a T?

(A) 907,200 **(B)** 840 **(C)** 720 **(D)** 420 **(E)** 210

51. Let $\frac{dy}{dx} = 3x^2 - 6x + 2$, and $y = 4$ when $x = 0$. Find y when $x = -1$.

(A) -2 **(B)** 11 **(C)** -6 **(D)** 1 **(E)** 3

52. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Which of the following is a member of this sequence?

(A) 75,025 **(B)** 671,320 **(C)** 78,152 **(D)** 651,116 **(E)** 69,152

53. Let $f(x) = \begin{cases} x^2 + 2x & \text{if } x \geq 1 \\ 1 + 2\cos(x - 1) & \text{if } x < 1 \end{cases}$. Which of the following is/are true?

1. $\lim_{x \to 1^+} f(x)$ exists 2. $\lim_{x \to 1^-} f(x)$ exists 3. $f(x)$ is continuous

(A) none of these (B) 1 & 2 but not 3 (C) 1 only (D) 2 only (E) 1, 2, & 3

54. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \ldots + \frac{1}{45} + \frac{1}{55} + \frac{1}{66} = ?$

(A) $\frac{21}{22}$ (B) $\frac{5}{6}$ (C) $1\frac{1}{6}$ (D) $1\frac{23}{66}$ (E) $2\frac{1}{33}$

55. Let $f''(x) = -6x + 6$, $f'(2) = 0$, and $f(0) = -4$. Find $f(1)$.

(A) 8 (B) 2 (C) 0 (D) -2 (E) -6

56. Simplify: $\frac{(n-1)!}{(n-2)!} \times \frac{1}{(n)!} \div \frac{n}{(n+1)!}$

(A) $n - 1$ (B) $\frac{n-1}{n+2}$ (C) $n^2 - n$ (D) $\frac{(n-1)^2}{n}$ (E) $\frac{n^2-1}{n}$

57. If $\frac{x+12}{x-8} + \frac{x-8}{x+12}$ is written as the mixed number $A\frac{B}{C}$ then $B = ?$

(A) 96 (B) 32 (C) 400 (D) 16 (E) 200

58. Suppose A, B, and C are positive integers such that $\frac{41}{7} = A + \frac{1}{B + \frac{1}{C+1}}$.

The value of $3A + B + 4C$ equals:

(A) 41 (B) 36 (C) 34 (D) 28 (E) 11

59. If 4 Chops equal 3 Chips and 2 Chips equal 5 Chaps, then how many Chaps does it take to make one Chop?

(A) $1\frac{1}{5}$ (B) $1\frac{7}{8}$ (C) $2\frac{1}{8}$ (D) $1\frac{3}{5}$ (E) $2\frac{2}{3}$

60. How many ordered pairs (a, b) exist such that the four-digit number, a31b, is divisible by both 2 and 3?

(A) 18 (B) 15 (C) 12 (D) 9 (E) 6

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2017
Answer Key

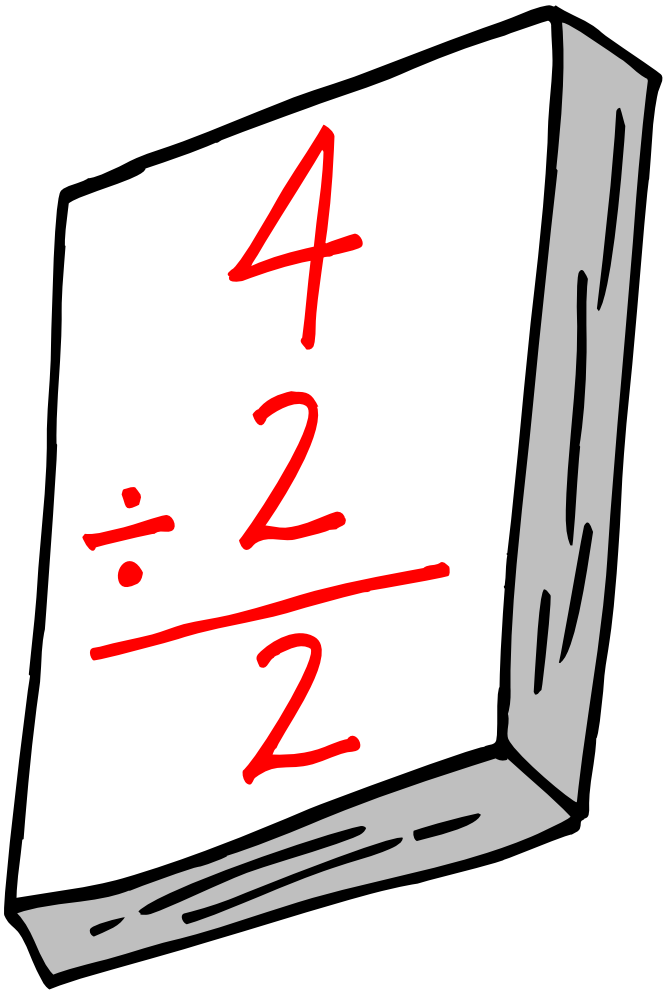
1. C	**21. C**	**41. D**
2. B	**22. D**	**42. B**
3. D	**23. B**	**43. E**
4. E	**24. E**	**44. D**
5. C	**25. C**	**45. D**
6. A	**26. D**	**46. E**
7. B	**27. A**	**47. C**
8. D	**28. D**	**48. B**
9. A	**29. A**	**49. E**
10. E	**30. C**	**50. E**
11. A	**31. C**	**51. A**
12. B	**32. C**	**52. A**
13. D	**33. A**	**53. E**
14. E	**34. E**	**54. B**
15. A	**35. C**	**55. D**
16. E	**36. A**	**56. E**
17. C	**37. C**	**57. C**
18. A	**38. D**	**58. B**
19. B	**39. B**	**59. B**
20. E	**40. A**	**60. B**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2017



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $5.4 \div (\frac{3}{2})^{-1} - (1)^{-2} \times 3! + 4.5$

   (A) -3.9 **(B)** $\frac{1}{6}$ **(C)** $\frac{3}{5}$ **(D)** 2.1 **(E)** 6.6

2. **If $\frac{3}{8}$ of A is 87.5% more than B, then A is what percent of B?**

   (A) 5% **(B)** 20% **(C)** 50% **(D)** $233\frac{1}{3}\%$ **(E)** 500%

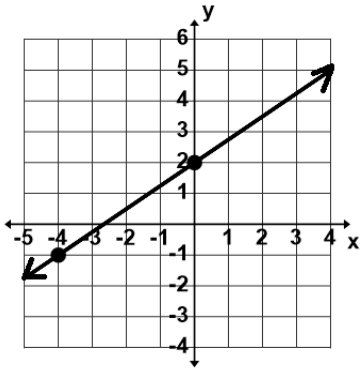
3. **One billion two and three-fourth million five hundred six is added to six million fifty-four thousand three hundred twenty-one. How many digits in the sum are twos?**

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. **Kookie Baykur baked some cookies. She took 20% of them to her grandmother. Then she ate 4 for lunch. After lunch she sold $\frac{1}{2}$ of what was left at her school's bake sale. She had 6 left to share with her parents for after supper. How many cookies did she bake originally?**

   (A) 18 **(B)** 20 **(C)** 24 **(D)** 30 **(E)** 36

5. **A line parallel to the line shown containing the point (6, 3) contains which of the following points?**



   (A) $(-3, -4)$ **(B)** $(-6, -6)$ **(C)** $(0, -2)$ **(D)** (9, 7) **(E)** (10, 6)

6. **Let p and q be the roots of $2x^2 + 3x - 5 = 0$. Find $p^3 + 3p^2q + 3pq^2 + q^3$.**

   (A) 15.625 **(B)** 6.5 **(C)** 3.625 **(D)** -1.125 **(E)** -3.375

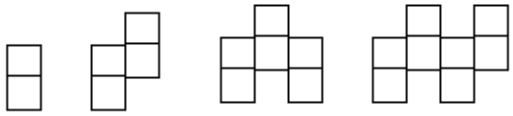
7. **Phil D. Belly budgets \$53.00 per week for lunch. He spends \$7.00 each day that he goes to McDee's Grill and \$9.00 each day that he goes to Queen's Burger. How much more does he spend at McDee's Grill than at Queen's Burger during a 7 day week?**

   (A) \$1.00 **(B)** \$15.00 **(C)** \$17.00 **(D)** \$33.00 **(E)** \$35.00

8. **$\angle A$ and $\angle B$ are supplementary. If m $\angle A = 3x + 4$ and m $\angle B = 2x + 1$, the measure of the larger angle is:**

   (A) 55° **(B)** 61° **(C)** 109° **(D)** 112° **(E)** 115°

9. **The four shapes below are made up of 1 cm squares. If the pattern continues, find the perimeter of the shape consisting of 16 squares.**



(A) 26 (B) 30 (C) 32 (D) 34 (E) 40

10. **M. T. Tank has a rectangular based water tank that is empty. The length of the tank is twice the width and the height is half of the width. How many gallons of water will he need to fill the tank if the height is 4 feet? (nearest gallon)**

(A) 3,830 gal (B) 3,456 gal (C) 3,192 gal (D) 2,608 gal (E) 2,095 gal

11. **The point (3 , — 4) is rotated 450° clockwise about the origin. The coordinates of the point after the rotation is _____.**

(A) (— 3, 4) (B) (4, — 3) (C) (— 3, — 4) (D) (3, 4) (E) (— 4, — 3)

12. **If $\frac{5x-2}{3x+1} + \frac{Ax-B}{x+4} = \frac{11x^2+5x-13}{3x^2+13x+4}$, where A and B are constants, then A × B equals:**

(A) — 3 (B) — 1 (C) 3 (D) 7 (E) 10

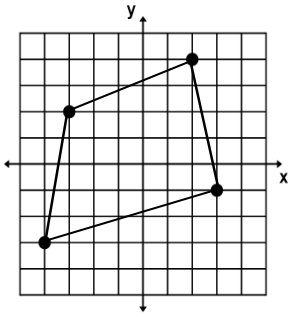
13. **The roots of the equation $2x^3 - x^2 - 5x - 2 = 0$ are —1, 2, and R. Find R.**

(A) 2.5 (B) 1 (C) — 0.5 (D) — 1 (E) — 1.5

14. **Let A = $\begin{bmatrix} 1 & 6 \\ -9 & -7 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 \\ 0 & -7 \end{bmatrix}$. Find $|A - B|$.**

(A) — 15 (B) — 3 (C) 33 (D) 61 (E) 63

15. **Find the area of the quadrilateral. The coordinates of the vertices are integers.**



(A) 31.5 sq. units (B) 30 sq. units (C) 28.5 sq. units (D) 28 sq. units (E) 26 sq. units

16. **Determine the frequency of $f(x) = 3 + 5\sin[4\pi(x - 2)]$.**

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) 4 (E) 5

17. A plane is 120 miles north and 85 miles east of an airport. What bearing should the plane take to fly directly to the airport?

(A) 65° (B) 55° (C) 45° (D) 35° (E) 25°

18. Given the arithmetic sequence 15, a, b, 37, c, ..., find $a + b + c$.

(A) $96\frac{1}{3}$ (B) 114 (C) $148\frac{2}{3}$ (D) $81\frac{2}{3}$ (E) $73\frac{1}{3}$

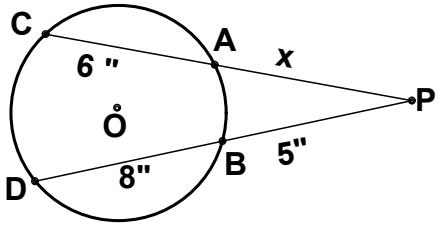
19. Find the remainder when $x^3 + 2x^2 - 3x + 4$ is divided by $x + 1$.

(A) 10 (B) 8 (C) 7 (D) 5 (E) 4

20. Find the eccentricity of the ellipse, $16x^2 + 100y^2 = 1600$. (nearest hundredth)

(A) 0.87 (B) 0.90 (C) 0.92 (D) 0.95 (E) 0.98

21. Given the circle with center O shown. Find x. (nearest tenth).



(A) 3.8 (B) 5.6 (C) 6.3 (D) 6.7 (E) 9.6

22. What is the sum of the digits in the tens place and the units place of $7^{(65)}$?

(A) 1 (B) 3 (C) 7 (D) 9 (E) 13

23. The function $f(x) = x^2$ is concave up on which of the following open intervals?
I. (0, 5) II. (— 5, 5) III. (— 5, 0)

(A) I only (B) II only (C) III only (D) I, II & III (E) none of them

24. The graph of $g(x) = (x^3 + 3x^2 + 3x + 1) \div (x^2 - 1)$ has vertical asymptote(s) at:

(A) $x = 1$ (B) $x = -1$ (C) $x = 1$ and -1 (D) $x = 0$ (E) g(x) has no vertical asymptotes

25. Let $f''(x) = 18x + 4$, $f'(-1) = 6$, and $f(1) = 6$. Find $f(-2)$.

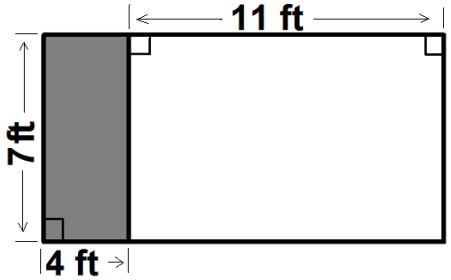
(A) — 34 (B) — 32 (C) — 18 (D) — 12 (E) 12

26. Suppose A, B, and C are positive integers such that $\frac{32}{5} = A + \frac{1}{B + \frac{1}{C+1}}$.

The value of $3A + 2B + 5C$ equals:

(A) 9 (B) $9\frac{2}{5}$ (C) $13\frac{1}{2}$ (D) 27 (E) 37

27. Spud Pharmer's son, Tater, buried his daddy's shovel in their rectangular garden. What is the probability that it was buried in the shaded section shown? (nearest whole percent)



(A) 7% (B) 20% (C) 25% (D) 27% (E) 36%

28. Lyn Koln flipped a penny four times and recorded the results. What are the odds of three or more consecutive heads occurring?

(A) $\frac{3}{16}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{5}{11}$ (E) $\frac{3}{13}$

29. Which of the following mathematicians are associated with for working with prime numbers?
I. Erastosthenes of Cyrene II. Sophie Germain III. Marin Mersenne

(A) I only (B) I & II (C) I & III (D) I, II & III (E) none of them

30. The number 13 is a member of which of the following sets of numbers?
(A)bundant (E)vil (L)ucas (P)rimeval

(A) L & P only (B) P only (C) E & L only (D) none of them (E) all of them

31. If 2 Babs equal 3 Bibs and 5 Bibs equal 7 Bobs, then how many Babs does it take to make 3 Bobs?

(A) $3\frac{1}{3}$ (B) $2\frac{5}{7}$ (C) $1\frac{3}{7}$ (D) $1\frac{1}{2}$ (E) $\frac{7}{10}$

32. Let U (universal set) = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {2, 3, 5, 7, 9}, and Q = {2, 1, 3, 4, 7}. Let $R = (P \cap Q)^C$. Set R contains how many distinct elements?

(A) 3 (B) 5 (C) 7 (D) 10 (E) none

33. Soh Yung is 7 years older than her sister Tu Yung. In 3 years Soh will be twice as old as Tu. How old will Tu be in 5 years ?

(A) 4 (B) 9 (C) 11 (D) 14 (E) 16

34. Seven students in Miss Work's math class had test scores of 75, 83, 85, 92, 95, 98, and 100. Three of her students haven't take the test yet. What will the remaining three students have to average so that the entire class average is 88?

(A) 84 (B) 85 (C) 86 (D) 87 (E) 88

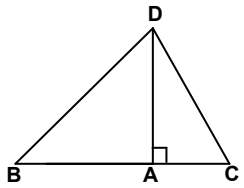
35. Rusty Yatt sailed his boat to Junk Yard Bay and back home. The trip took 12 hours going and 9 hours coming back. His average speed coming back was 20 kph. What was his average speed going?

(A) 8 kph (B) 11 kph (C) 15 kph (D) 21 kph (E) 28 kph

36. Dee Deeler has a standard deck of cards consisting of 4 Aces, 12 face cards, and 36 number cards. No Joker is allowed. Dee wants to see how many 5 card hands he can create such that each hand has 1 Ace, 2 face cards, and 2 number cards. How many such hands can he make?

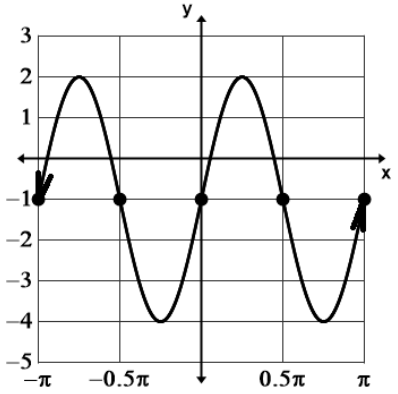
(A) 166,320 (B) 260 (C) 8,640 (D) 1,728 (E) 665,280

37. Find the perimeter of $\triangle BCD$ if AD = 3", $m\angle ADB = 45°$, and $m\angle ACD = 60°$. (nearest tenth)



(A) 7.6" (B) 8.2" (C) 10.2" (D) 12.4" (E) not enough information given

38. The equation y = _____ will produce this graph.



(A) $3\cos(2x + \pi)$ (B) $3\sin(2x) - 1$ (C) $3\cos(2x + \pi) - 1$
(D) $3\sin(2x - 1)$ (E) $2 - 3\sin(x)$

39. The function $f(x) = 3x^2 - 4x - 4$ crosses the x-axis at two points. Find the distance between the two points.

(A) $3\frac{1}{2}$ units (B) $1\frac{1}{3}$ units (C) 4 units (D) $2\frac{2}{3}$ units (E) $1\frac{1}{2}$ units

40. The point of concurrency of the angle bisectors of a triangle is called the:

(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler

41. If $a_1 = -1$, $a_2 = -2$, $a_3 = 3$, and $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$, where $n > 3$ then $a_6 = ?$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

42. Determine the range of $(x) = 3 + 5\sin[4\pi(x - 2)]$.

(A) $[-5, 5]$ (B) $[-5, 8]$ (C) $[-2, 8]$ (D) $[-2, 5]$ (E) $[-8, 8]$

43. Find the area (in square units) of the region bounded by $y = -x^2$ and $y = -4$.

(A) 16 (B) $11\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 8 (E) $5\frac{1}{3}$

44. $\frac{1+4+9+16+...+64+81}{1+3+6+10+...+36+45} =$ _____.

(A) $1\frac{4}{5}$ (B) $1\frac{9}{11}$ (C) $1\frac{64}{81}$ (D) $1\frac{74}{101}$ (E) $1\frac{8}{11}$

45. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_n = 121{,}393$ then n is:

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

46. Willie Pikette is going to randomly pick two different numbers from the set { 2, 1, 3, 4, 7, 11}. What is the probability that the sum of the two numbers he picks will be a prime number?

(A) 20% (B) $26\frac{2}{3}\%$ (C) $33\frac{1}{3}\%$ (D) 40% (E) $53\frac{1}{3}\%$

47. The function *f* is defined by $f(x) = 2 + \ln(x + 3)$. The inverse function of *f* is $f^{-1}(x) = ?$

(A) $(2 + \ln(x + 3))^{-1}$ (B) $\ln(x - 2)$ (C) $e^{(x+2)} - 3$
(D) $e^{(x-2)} - 3$ (E) $-(2 + \ln(x + 3))^{-1}$

48. Let $f(x) = \frac{x^3 - 3x^2}{x^2 - 1}$ and s(x) be the slant asymptote of f. Find the value of s(4).

(A) $1\frac{1}{15}$ (B) -1 (C) $\frac{15}{16}$ (D) 1 (E) 7

49. Alice, Bob, Charlie, Dan, and Edith sit randomly in a row of five chairs. What is the probability that Alice and Edith sit next to each other? (nearest percent)

(A) 3% (B) 7% (C) 20% (D) 35% (E) 40%

50. Find the slope of the line tangent to the curve $y = x^2 - 3x + 5$ at (3, 5).

(A) 2 (B) 3 (C) 5 (D) 6 (E) 10

51. If the three numbers 78, 169, and 246 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 7 (B) 5 (C) 3 (D) 2 (E) 1

52. Let $f(x) = x^3 + 2x^2 - 4x$. Find the sum of the x-values of the critical points of the function.

(A) 2 (B) 1 (C) $-\frac{2}{3}$ (D) $-1\frac{1}{3}$ (E) $-2\frac{2}{3}$

53. Let $g(x) = x^2 + 2x + 1$. Find k if $g(k + 1) - g(k) = 7$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

54. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{21}?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

55. $14_5 + 32_5 \times 23_5 =$ _____ $_5$

(A) 1410 (B) 1300 (C) 1113 (D) 2314 (E) 2323

56. If $15x^2 + cx - 12 = (5x + a)(bx - 4)$ then $a + b + c =$ _____.

(A) -5 (B) -2 (C) 3 (D) 6 (E) 17

57. Let $f(x) = 5x - 2$, $g(x) = x + 4$, $h(x) = 3x + 1$, and $g\Big(f\big(h(x)\big)\Big) = 10$. Find x.

(A) $-\frac{13}{15}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{7}{9}$ (E) $1\frac{2}{15}$

58. How many 3-digit numbers can be made using the digits 2, 1, 3, 4, and 7?

(A) 64 (B) 60 (C) 32 (D) 30 (E) 15

59. The sequence 6, p, q, 1.5 is a harmonic sequence. Find the value of p + q.

(A) $\frac{5}{6}$ (B) $1\frac{1}{5}$ (C) $3\frac{3}{4}$ (D) 5 (E) 7.5

60. A right triangle, $\triangle ABC$, with leg lengths 15" and 20" and the right angle at vertex B is congruent to right triangle, $\triangle BDE$, with the right angle at vertex D. Point C lies on segment BD and points A and E are on the same side of segment BD. Find the distance between points A and E. (nearest eighth of an inch).

(A) $20\frac{5}{8}$" (B) $20\frac{1}{4}$" (C) $19\frac{7}{8}$" (D) $18\frac{3}{8}$" (E) $17\frac{1}{2}$"

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

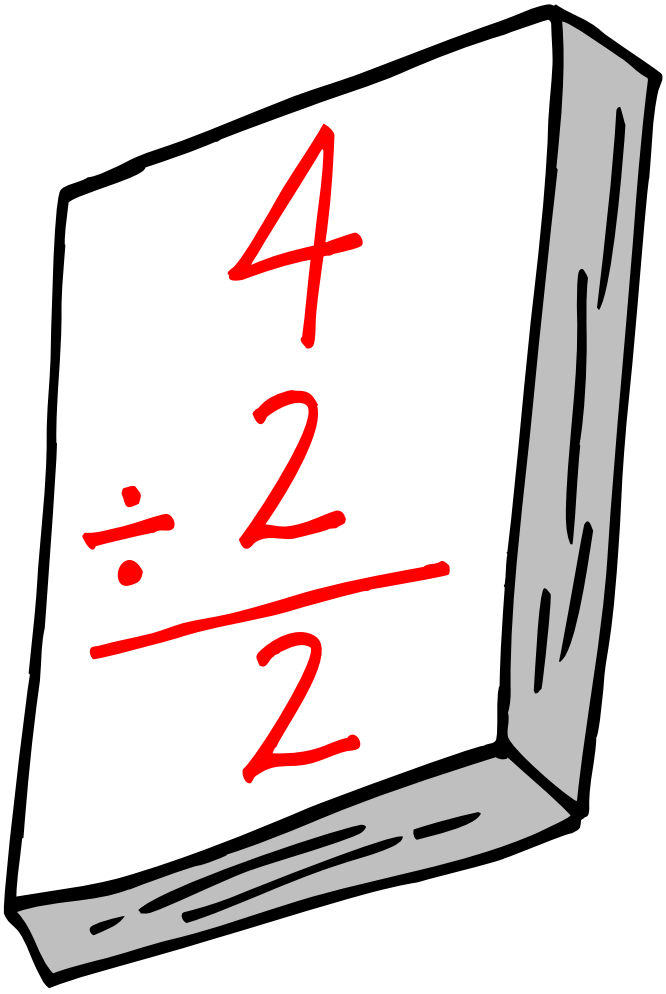
University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2017
Answer Key

1. E	**21. B**	**41. A**
2. E	**22. C**	**42. C**
3. B	**23. D**	**43. C**
4. B	**24. A**	**44. E**
5. B	**25. C**	**45. D**
6. E	**26. D**	**46. D**
7. C	**27. D**	**47. D**
8. C	**28. E**	**48. D**
9. D	**29. D**	**49. B**
10. A	**30. B**	**50. B**
11. E	**31. C**	**51. E**
12. E	**32. C**	**52. D**
13. C	**33. B**	**53. E**
14. E	**34. A**	**54. C**
15. B	**35. C**	**55. A**
16. B	**36. A**	**56. A**
17. D	**37. D**	**57. C**
18. A	**38. B**	**58. B**
19. B	**39. D**	**59. D**
20. C	**40. A**	**60. A**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2017



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $2 \div (10 - 20) + 17 \times 3 - 11 \times 20 \div (1 + 7)$

   (A) -48.5 **(B)** -23.3 **(C)** -0.4 **(D)** 22.9 **(E)** 98.5

2. **Rose Thorn's flower shop is having a spring plant sale. The regular price of a yellow rose bush is $6.95 and a red rose bush regularly sell for $8.50. Yellow roses are on sale for 20% off and red roses are on sale for 10% off. What would it cost Rose to buy 4 yellow rose bushes and 2 red rose bushes on sale before tax?**

   (A) $39.71 **(B)** $38.62 **(C)** $38.08 **(D)** $37.54 **(E)** $31.36

3. **If** $P = \{p, l, u, s\}$, $M = \{m, i, n, u, s\}$, $T = \{t, i, m, e, s\}$ **and** $O = \{o, p, e, r, a, t, i, o, n\}$ **then** $(P \cup M \cup T) \cap O$ **contains how many distinct elements?**

   (A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

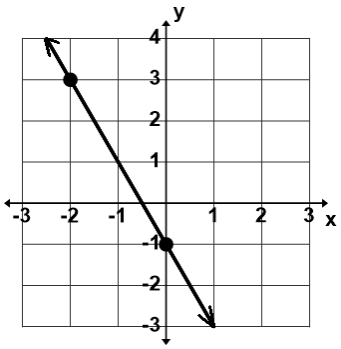
4. **Three-fourths is the same part of two-fifths as one-half is of _____.**

   (A) $3\frac{3}{4}$ **(B)** $3\frac{2}{5}$ **(C)** $2\frac{3}{10}$ **(D)** $\frac{15}{16}$ **(E)** $\frac{3}{20}$

5. **Which of the following multiples of 6 is the average of two consecutive prime numbers?**

   (A) 24 **(B)** 36 **(C)** 42 **(D)** 54 **(E)** 66

6. **Which of the following linear equations has the graph of a line perpendicular to the line shown and containing the point** $(-4, -1)$**?**



   (A) $2x + y = -9$ **(B)** $x - 2y = -7$ **(C)** $2x + y = -6$ **(D)** $x - y = -2$ **(E)** $x - 2y = -2$

7. **The set of positive composite numbers {4, 6, 8, 9, 10, 12, ...} is closed under how many of these operations:** $+$ **addition** $-$ **subtraction** $\times$ **multiplication** $\div$ **division**

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** none of them

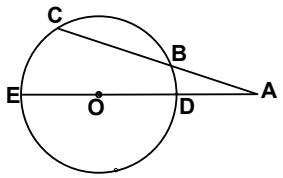
8. **Seymore Wirk can paint a picket fence in 8 hours. His brother Les Wirk can paint the same fence in 6 hours. How long would it take the Wirk brothers to paint the fence if they worked together? (nearest minute)**

   (A) 3 hrs 26 min **(B)** 3 hrs 30 min **(C)** 3 hrs 37 min **(D)** 3 hrs 43 min **(E)** 3 hrs 52 min

9. If $\frac{x+5}{x-4} + \frac{x-4}{x+5}$ is written as the mixed number $A\frac{B}{C}$ then B = ?

(A) 9 (B) 16 (C) 20 (D) 25 (E) 81

10. Given the circle with segment AE containing center O as shown. Find OE if AC = 12 cm, BC = 6 cm, and AD = 4 cm.



(A) 4 cm (B) 7 cm (C) 8 cm (D) 14 cm (E) 15 cm

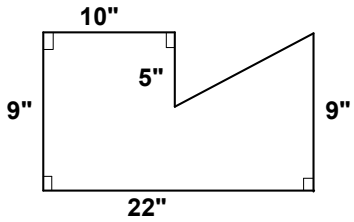
11. The sum of the measures of the interior angles of a single face of a regular octahedron is:

(A) 180° (B) 360° (C) 540° (D) 720° (E) 900°

12. The orthocenter of which of the following triangles lies outside the triangle?

(A) equilateral (B) acute isosceles (C) right scalene (D) acute scalene (E) obtuse scalene

13. Find the perimeter of the hexagon shown.



(A) 69 " (B) 68 " (C) 55 " (D) 52 " (E) not enough data

14. If $\frac{Ax+B}{2x-5} - \frac{3x+2}{5x-3} = \frac{14x^2+4x+7}{10x^2-31x+15}$, where A and B are constants, then A + B equals:7

(A) 3 (B) 2 (C) 5 (D) −1 (E) −3

15. Find the range of the function $y = 5 - 3|x+2|$ given that the domain is restricted to $\{x \in \text{Reals} \mid -5 \le x \le 1\}$.

(A) {all Reals} (B) $\{y \mid y \in \{\text{Reals}\}, -4 \le y \le 5\}$ (C) $\{y : y \le 5\}$
(D) $\{y : y > 5\}$ (E) $\{y \mid y \in \{\text{Reals}\}, -1 \le y \le 5\}$

16. Willis Quik flew his plane to Sumplace. The speed with a tailwind was 183 km/h. The speed on the return trip was 141 km/h going into the wind. Find the speed of the wind.

(A) 35 km/h (B) 28 km/h (C) 21 km/h (D) 14 km/h (E) 7 km/h

17. Find the value of $5A + 3B + 2C$, where A, B, and C are greater than zero and

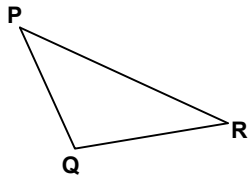
$$\frac{23}{5} = A + \left(\frac{1}{B + \left(\frac{1}{C+1}\right)}\right).$$

(A) 26 (B) 6.5 (C) 23.5 (D) 5.5 (E) 24

18. Determine the amplitude of $f(x) = 4 - 3\sin(2x + 1)$.

(A) 1 (B) 2 (C) 3 (D) π (E) 4

19. Find the height of $\triangle PQR$ from point P if $m\angle PQR = 110^o$, PQ = 20 cm, and QR = 15 cm. (nearest tenth)



(A) 19.3 cm (B) 18.8 cm (C) 17.5 cm (D) 14.1 cm (E) 13.2 cm

20. Sir Vayer places a stake in the ground. He walks 50 yards on a course heading 75^o west of south. Then he turns and walks 75 yards on a course heading 100^o east of north. How far will he have to walk to get back to his stake? (nearest yard)

(A) 36 yds (B) 59 yds (C) 30 yds (D) 38 yds (E) 45 yds

21. If $\log_3(x + 24) - \log_3(x + 2) = 2$ then x = ?

(A) $\frac{3}{4}$ (B) $\frac{7}{8}$ (C) $1\frac{1}{3}$ (D) $\frac{2}{3}$ (E) $1\frac{1}{7}$

22. Given the geometric sequence $-0.5, a, b, 0.0625, c, \ldots$, find $a + b + c$.

(A) $\frac{1}{2}$ (B) $\frac{13}{32}$ (C) $\frac{11}{32}$ (D) $\frac{1}{8}$ (E) $\frac{3}{32}$

23. Find mn if $\begin{bmatrix} -1 & m \\ n & 3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$

(A) -8 (B) -3 (C) -2 (D) $-1\frac{2}{3}$ (E) -1

24. Find k when $g(x) = 2x^2 + kx + 1$ divided by $h(x) = x - 2$ has a remainder of 2.

(A) -3.5 (B) -3 (C) 2 (D) 3.5 (E) 6

25. The numbers greater than 1 are arranged in the array below. In which column will 2017 fall.

(A)	(B)	(C)	(D)	(E)
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

26. Which of the following sequences are divergent?

I. $\left\{\frac{\ln(n)}{n^2}\right\}$ **II.** $\left\{\ln\left(\frac{1}{n-1}\right)\right\}$ **III.** $\left\{\frac{n^2+2}{2n^2-1}\right\}$

(A) I & III (B) II & III (C) III only (D) II only (E) none of them

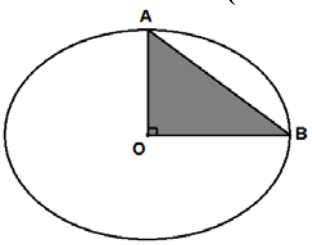
27. Evaluate: $\int_{1-a}^{a+1} (6x-2)\,dx$

(A) 8a (B) 10a + 2 (C) 12a (D) 16a + 10 (E) does not exist

28. Willie Pickette randomly chooses exactly one letter from each of the sets, {Q,U,I,C,K} and {P,I,C,K}. What is the probability of choosing two vowels?

(A) $22\frac{2}{9}\%$ **(B) 10% (C) 65% (D)** $33\frac{1}{3}\%$ **(E) 40%**

29. N. D. Shaid throws a dart that hits in the circle with center O and having a diameter of 12". What are the odds the dart hits in the shaded area? (nearest whole percent)



(A) 16% (B) 18% (C) 19% (D) 22% (E) 25%

30. The odds of the Ruff Ryders baseball team winning a game is $\frac{5}{11}$**. How many games can they expect to lose if there are 120 games in the season?**

(A) 54 (B) 66 (C) 75 (D) 82 (E) 85

31. Which of the following mathematicians first introduced the term "polytope" and had a good grasp of the concept of four dimensional geometry?

(A) Ada Byron (B) Benoit Mandelbrot (C) Agnesi (D) Hypatia (E) Alicia Stott

32. A happy number that is also perfect is:

(A) 6 **(B) 11** **(C) 28** **(D) 44** **(E) 100**

33. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{32}?

(A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

34. Willie Sawette had a log that was 12 feet long. He cut it into three smaller logs such that the ratio of the lengths was 2:3:5. How much longer was the longest piece than the shortest piece? (nearest inch)

(A) 1 ft 2 in **(B) 2 ft 5 in** **(C) 2 yds** **(D) 1 yd 2 ft** **(E) 1 yd 7 in**

35. There were 148 students competing at last year's state math contest. The number of young men competing was eight more than four times the number of the young ladies competing. How many young ladies competed?

(A) 18 **(B) 20** **(C) 24** **(D) 28** **(E) 36**

36. The ratio of the lateral area of a right cylinder to its total surface area is 5:8. Find the radius of the cylinder's base if it is 8 cm shorter than the height of the cylinder.

(A) $2\sqrt{3}$ cm **(B) $2\sqrt{10}$ cm** **(C) 6 cm** **(D) 12 cm** **(E) 20 cm**

37. If the probability that a student studies for an exam is 85%, and the probability that a student who studies passes the test is 90%, then the probability that a student both studies and passes the test is:

(A) 5% **(B) 76.5%** **(C) 80%** **(D) 87.5%** **(E) 95%**

38. Lotta Dough puts 3 one-dollar bills, 4 five-dollar bills, 2 ten-dollar bills, and 1 twenty-dollar bill in a box. She selects two bills randomly without replacement, what is the probability that the sum of the two bills selected is \$15.00 or more?

(A) 15% **(B) 20%** **(C) 40%** **(D) $44\frac{4}{9}\%$** **(E) $55\frac{5}{9}\%$**

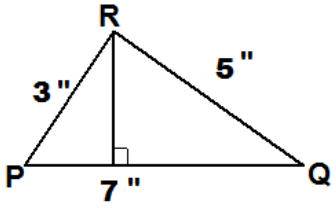
39. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find f_{23}.

(A) 17,711 **(B) 28,657** **(C) 23,765** **(D) 24,476** **(E) 46,368**

40. Find the least common multiple of 44, 60, and 76.

(A) 180 **(B) 3,135** **(C) 12,540** **(D) 50,160** **(E) 200,640**

41. Find the length of the altitude from point R to segment PQ. (nearest hundredth)



(A) 2.14" (B) 1.98" (C) 1.86" (D) 4.2" (E) not enough information given

42. If $18x^2 - 15x + 2 = (ax - 1)(bx - 2)$ then $a - b =$ _____, where a and b are integers.

(A) 2 (B) 3 (C) 5 (D) 6 (E) 9

43. Find the area of the circle, $x^2 + y^2 - 4x - 6y + 8 = 0$. (nearest tenth)

(A) 9.4 units^2 (B) 14.4 units^2 (C) 15.7 units^2 (D) 20.4 units^2 (E) 25.1 units^2

44. The equation of a parabola with its vertex at (3, 3) and its focus at (3, 2.5) is y = ?

(A) $-0.5x^2 + 3x - 15$ (B) $-0.5x^2 - 3x - 1.5$ (C) $-0.5x^2 + 3x - 1.5$
(D) $-0.5x^2 - 3x + 7.5$ (E) $-0.5x^2 + 3x + 1.5$

45. $1 + 3 + 6 + 10 + 15 + ... + 55 + 66 =$ _____.

(A) 286 (B) 298 (C) 306 (D) 368 (E) 398

46. $(110101_2 - 1001_2) \times 11_2 =$ _____ $_2$

(A) 1101100_2 (B) 10000100_2 (C) 1010101_2 (D) 1101110_2 (E) 110011_2

47. The function $f(x) = 5x^3 + 30x^2 + x + 1$ is concave down on which of the following open intervals?

(A) (1.1, 2.2) (B) (− 2.2, − 1.1) (C) (− 1.5, 1.5) (D) (− 4, 0) (E) (− 3.5, − 2.5)

48. $\{(x, y) \mid x,y \in \{\text{Integers}\}, -3 \le x \le 7, \text{ and } -1 \le y \le 9\}$ is the solution set of $2x - 5 = 3y$. How many such ordered pairs exist?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

49. Point M (− 1, 2) is the midpoint of the line segment with endpoints P (x , 1) and R (1, y). Find PR. (nearest tenth unit)

(A) 2.4 (B) 2.8 (C) 3.1 (D) 4.5 (E) 4.8

50. If $a_1 = -2$, $a_2 = 0$, $a_3 = 2$, and $a_n = (a_{n-3})^{(a_{n-2})} - (a_{n-1})$, where $n > 3$ then $a_7 = ?$

(A) $1\frac{1}{2}$ (B) 1 (C) 0 (D) $-\frac{1}{2}$ (E) -1

51. Let $\|V_1\| = 6$, $\|V_2\| = 8$, where the direction angles of V_1 and V_2 are 60° and 80°, respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

(A) 11° **(B)** 14° **(C)** 20° **(D)** 24° **(E)** 71°

52. Let $f(x) = ax - 5$ and $g(x) = bx + 3$, where a and b are positive integers. Find $a + b$ if $f(g(x)) = g(f(x))$.

(A) 5 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

53. The fraction $\frac{22}{30}$ in base 7 can be written as which of the following decimals in base 7?

(A) $0.4333..._7$ **(B)** $0.5222..._7$ **(C)** $0.6111..._7$ **(D)** $0.444..._7$ **(E)** $0.2555..._7$

54. Let $\frac{dy}{dx} = 4x^3 - 2x + 1$, and $y = 3$ when $x = 0$. Find y when $x = 3$.

(A) 103 **(B)** 4 **(C)** 66 **(D)** 96 **(E)** 78

55. Given the function $f(x) = 2x^2 + 1$, find the slope of the secant line between $x = 2$ and $x = 5$.

(A) 12 **(B)** 14 **(C)** 20 **(D)** 28 **(E)** 60

56. In the decimal number 2x3y4z, the letters x, y, and z represent digits where all six digits are distinct. If the number is divisible by 30 then $x + y + z$ could be:

(A) 12 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 18

57. How many 4-digit numbers can be created from the set of positive digits where the digits are not repeated?

(A) 6,561 **(B)** 6,480 **(C)** 5,040 **(D)** 4,536 **(E)** 3,024

58. The sum of the coefficients of the 3rd term in the expansion of $(x + 1)^3$, the 3rd term of $(x + 1)^4$, and the 3rd term of $(x + 1)^5$ is:

(A) 12 **(B)** 14 **(C)** 17 **(D)** 19 **(E)** 36

59. Let $f(x) = 4x$, $g(x) = 3 - x$, $h(x) = x + 2$, and $h\big(f(\,g(x)\,)\big) = 5$. Find x.

(A) $2\frac{1}{4}$ **(B)** $1\frac{3}{4}$ **(C)** 0 **(D)** -1 **(E)** -6

60. Points A,B, C, and D are coplanar. Point A lies on segment DC and point B exists such that DB = DC, BA = BC, and $m\angle ADB = \frac{\pi}{6}$. Find the ratio of DB to BC. (nearest tenth)

(A) 3.9 **(B)** 0.5 **(C)** 1.0 **(D)** 0.9 **(E)** 1.9

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

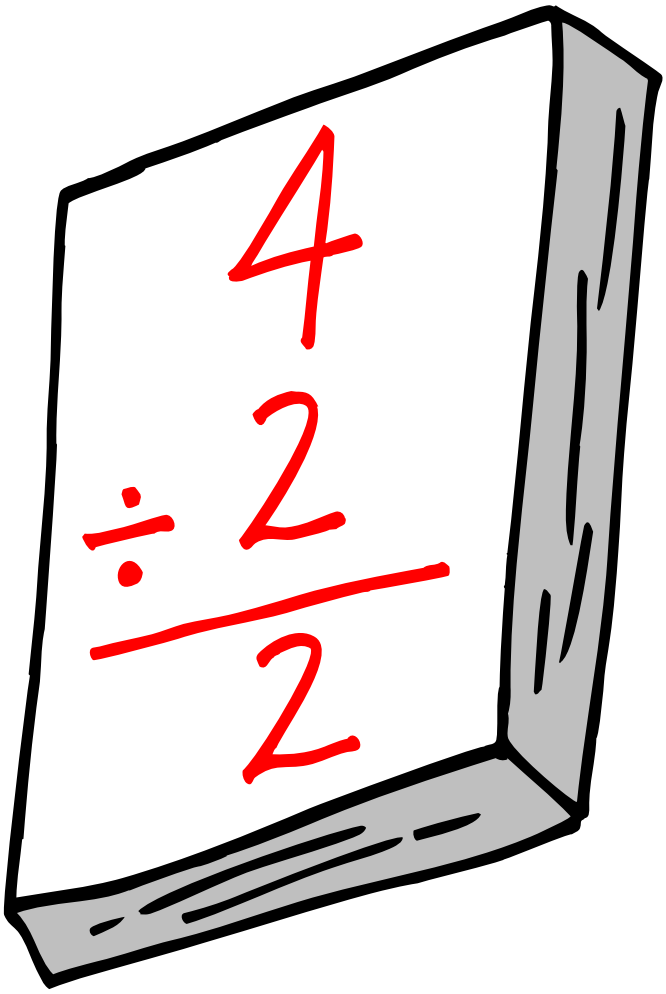
University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2017
Answer Key

1. B	**21. A**	**41. C**
2. D	**22. E**	**42. B**
3. C	**23. A**	**43. C**
4. A	**24. A**	**44. C**
5. C	**25. A**	**45. A**
6. E	**26. D**	**46. B**
7. A	**27. A**	**47. E**
8. A	**28. B**	**48. C**
9. E	**29. C**	**49. D**
10. B	**30. D**	**50. D**
11. A	**31. E**	**51. E**
12. E	**32. C**	**52. D**
13. B	**33. D**	**53. B**
14. C	**34. E**	**54. E**
15. B	**35. D**	**55. B**
16. C	**36. D**	**56. A**
17. E	**37. B**	**57. E**
18. C	**38. C**	**58. D**
19. B	**39. B**	**59. A**
20. A	**40. C**	**60. E**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Regional • 2017



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $0! + 1 \times 2^3 + 4 - 5 \div (6 - 7)$**

   (A) -15 **(B) -8** **(C) 8** **(D) 12** **(E) 18**

2. **Willie Shair had a bag of marbles. He picked out his 10 favorite ones and put them away in a lock box. He gave 40% of what was left to his best friend. Then he gave $\frac{1}{3}$ of what was left to his brother. He had 6 left to play with. How many marbles were in the bag originally?**

   (A) 32 **(B) 30** **(C) 25** **(D) 20** **(E) 15**

3. **If P is 15% less than Q and Q is 20% more than R, then R is what percent of P? (nearest whole percent)**

   (A) 135% **(B) 105%** **(C) 102%** **(D) 98%** **(E) 95%**

4. **If E= {e, u, c, l, i, d}, V = {v, i, e, t, a}, G = {g, e, r, m, a, i, n} and T = {t, h, e, a, n, o} then $(E \cap T) \cup (G \cap V)$ contains how many distinct elements?**

   (A) 1 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

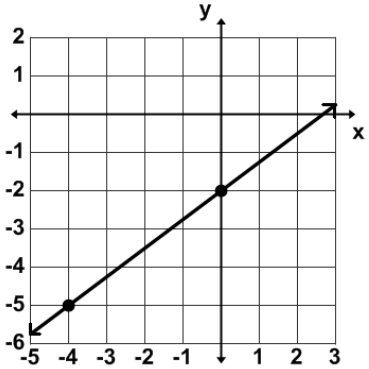
5. **Find the sum of the positive integral divisors of 1,488.**

   (A) 4,464 **(B) 4,188** **(C) 4,096** **(D) 3,968** **(E) 2,976**

6. **Andrew, Linh, and Zach worked a total of 141 problems on their last math test. Zach worked two-thirds of the number of problems Andrew worked and fifteen less problems than Linh did. How many problems did Zach work?**

   (A) 40 **(B) 36** **(C) 33** **(D) 31** **(E) 26**

7. **A line through point P (— 3, 5) and perpendicular to the line shown intersects at point Q (x, y). Find $x + y$.**



   (A) 2.36 **(B) 1.25** **(C) 0.75** **(D) 0.52** **(E) 0.28**

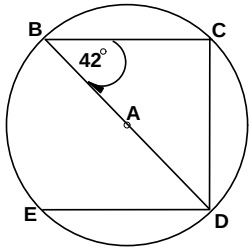
8. **The set of integers {... — 2, — 1, 0, 1, 2, ...} is closed under how many of these operations:**
   $+$ addition — subtraction $\times$ multiplication $\div$ division

   (A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) none of them**

9. If $18x^2 - 3x - 28 = (ax - 4)(bx + 7)$ then $(a + b)(a - b) =$ _____.

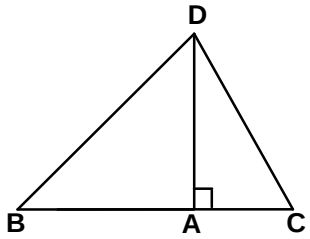
(A) — 27 (B) — 18 (C) — 3 (D) 9 (E) 18

10. Given the circle with center A shown with right angle CDE. Find $m\overset{\frown}{ED}$.



(A) 96° (B) 42° (C) 31° (D) 84° (E) 63°

11. Given △BCD with AD = 6", m∠ABD = 45°, and m∠ADC = 30°. Find the difference in the perimeters of triangles △BAD and △CAD. (nearest tenth)



(A) 7.9" (B) 7.0" (C) 6.6" (D) 5.8" (E) 4.1"

12. ∠A is supplementary to ∠B and ∠B complementary to ∠C. Let m ∠A = 4x — 3 and m ∠B = x + 2. Find m∠C.

(A) 36.2° (B) 38.2° (C) 51.8° (D) 53.8° (E) 57°

13. Poly Gone labeled thirty blank cards with the numbers 1 through 30. After mixing them up, Poly randomly drew out 1 card. What is the probability that the number on the card selected was a triangular number or a pentagonal number?

(A) 50 % (B) $40\frac{1}{3}$ % (C) $36\frac{2}{3}$ % (D) $33\frac{1}{3}$ % (E) 30 %

14. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_n = 317{,}811$ then n is:

(A) 21 (B) 23 (C) 26 (D) 28 (E) 31

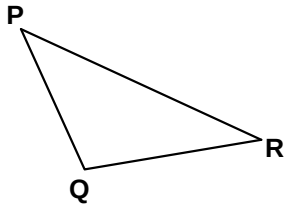
15. Let $A = \begin{bmatrix} 0 & 4 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ -1 & -7 \end{bmatrix}$. If $AB = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ then $w + x + y + z = ?$

(A) — 14 (B) — 26 (C) — 63 (D) — 82 (E) — 94

16. Find the area of the circle, $x^2 + y^2 - 6x - 10y = 2$. (nearest tenth)

(A) 119.4 units^2 (B) 104.7 units^2 (C) 131.1 units^2 (D) 106.8 units^2 (E) 113.1 units^2

17. Find the area of $\triangle PQR$ if $m\angle PQR = 95^o$, PQ = 16 cm, and QR = 12 cm. (nearest tenth)



(A) 96.0 cm^2 (B) 86.2 cm^2 (C) 95.0 cm^2 (D) 90.9 cm^2 (E) 95.6 cm^2

18. The bearing of the *Coral Princess* from the Panama Canal locks is 120º and the bearing of the *Pride of America* from the *Coral Princess* is 250º. The *Coral Princess* is 5 km from the locks and the *Pride of America* is 6.5 km from the locks . How far is the *Coral Princess* from the *Pride of America*? (nearest tenth)

(A) 2.0 km (B) 5.8 km (C) 7.0 km (D) 8.5 km (E) 10.4 km

19. Which of the following functions are considered to be odd?
I. $f(x) = x|x|$ II. $f(x) = |x| - 2$ III. $f(x) = \frac{x}{1 - x^2}$

(A) I only (B) I & III (C) II only (D) II & III (E) I, II, & III

20. If $a_1 = -5$, $a_2 = -3$, $a_3 = 2$ and $a_n = (a_{n-3}) + (a_{n-2})(a_{n-1})$ for $n \geq 4$, then a_6 equals:

(A) -275 (B) -273 (C) 275 (D) 277 (E) 286

21. Given the function $f(x) = 3x^2 - 4x$, find the slope of the secant line between $x = -1$ and $x = 3$.

(A) 8 (B) 4 (C) 2 (D) -1 (E) -3

22. Which of the following sequences are convergent?
I. $\left\{\frac{1}{\sqrt{n^2+1}-n}\right\}$ II. $\left\{\left(1+\frac{1}{3n}\right)^n\right\}$ III. $\left\{\frac{n+1}{2n-1}\right\}$

(A) I only (B) II only (C) II & III (D) I, II & III (E) none of them

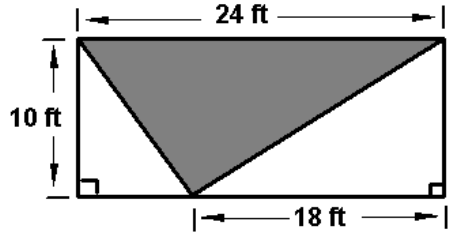
23. The three seniors on Cal Q. Later's four member math team scored 248, 288, and 268 on the UIL District math test. What would the only freshman on the team have to score so that the mean team score would be 252 and the median team score would be 258?

(A) 204 (B) 208 (C) 218 (D) 228 (E) 238

24. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $f_{20} + f_{21}$.

(A) 17,711 (B) 10,946 (C) 14,329 (D) 8,856 (E) 267,914,296

25. Doug Ittup lost his cell phone while tilling his garden. What is the probability that he lost it in shaded region shown? (nearest whole percent)



(A) 50% (B) 61% (C) 55% (D) 48% (E) 40%

26. Which of the following mathematicians was know for their contributions to abstract algebra and the development of the theories of rings, fields, and algebras?

(A) Karen Smith (B) Hypatia (C) Ada Byron (D) Alicia Stott (E) Emmy Noether

27. Noah Pennies has 34 coins consisting of nickels, dimes, and quarters worth $5.50. There are 6 more dimes than nickels. How many quarters does Noah have?

(A) 8 (B) 10 (C) 12 (D) 14 (E) 16

28. Mary Goround is putting lantern lights around her circular swimming pool. She wants to put a light every 4 feet. The diameter of the pool 28 feet. What will the cost of the lights be before taxes if the cost of each light is $12.50? (nearest quarter dollar)

(A) $250.00 (B) $262.50 (C) $268.25 (D) $275.00 (E) $281.50

29. The *Tuity Fruity* market sold a box containing 3 lbs of apples and 4 lbs of oranges for $5.15. They sold another box containing 5 lbs of apples and 2 lbs of oranges for $4.85. What would it cost to buy a box containing 7 lbs of apples and 6 lbs of oranges?

(A) $9.25 (B) $9.30 (C) $9.35 (D) $9.50 (E) $9.55

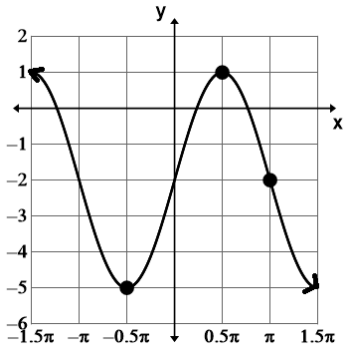
30. The probability that an applicant for a high-paying job has a college degree is 85%, and the probability that the applicant has experience in the field is 30%. If 25% of those with college degrees have experience, what is the probability that an applicant with experience has a college degree? (nearest degree)

(A) 60% (B) 64% (C) 67% (D) 71% (E) 79%

31. If the roots of $x^3 + cx^2 + dx - 28 = 0$ are 1, 4, and k, then $c + d$ equals:

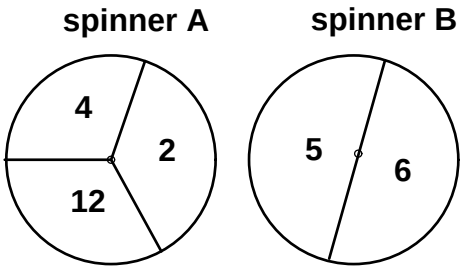
(A) 51 (B) 27 (C) 1 (D) 23 (E) 55

32. The equation y = _____ will produce this graph.



(A) 3cos(x — 2) (B) 3sin(x) — 2 (C) 3sin(x— 2) — 2 (D) 3cos(x) — 2 (E) 3sin(2x) — 2

33. Spinner A is divided into three equal sectors and spinner B into two halves. Betty Wont spins each spinner once. If A is bigger than B then Betty receives the sum of the A and B dollars. If A is smaller than B then Betty loses the sum of the A and B dollars. What is the mathematical expectation of playing the game many times?



(A) — $0.16 (B) + $0.17 (C) + $0.33 (D) + $0.84 (E) + $1.00

34. Point P (— 4, — 1) is the midpoint of the line segment with endpoints Q (x, 6) and R (— 11, y). Find x + y.

(A) — 15 (B) — 8 (C) — 5 (D) 3 (E) 5

35. Which of the following is an identity for $(\csc^2\theta)(\tan^2\theta) - 1$?

(A) $\tan\theta$ (B) $\cot\theta$ (C) $\cot^2\theta$ (D) $\sec^2\theta$ (E) $\tan^2\theta$

36. The function *f* is defined by *f*(x) = — 6 — ln(4 — x). The inverse function of *f* is $f^{-1}(x)$ = ?

(A) $(6 + \ln(4 - x))^{-1}$ (B) $e^{(6+x)} - 4$ (C) $4 + (\ln(6 + x))^{-1}$

(D) $e^{(x-2)}$ (E) $4 - e^{(-x-6)}$

37. The vertical asymptote and the oblique asymptote of $f(x) = \frac{2x^2 + 3x + 5}{x - 1}$ intersect at point (x , y). Find the value of y.

(A) 10 (B) 7 (C) 4 (D) 3 (E) 1

38. In the expansion of $(3x - 2y)^6$, the sum of the coefficients of the 2nd term, the 4th term, and the 6th term is:

(A) $-7{,}812$ **(B)** $-6{,}660$ **(C)** -192 **(D)** 720 **(E)** 32

39. Which of the following numbers is considered to be an evil and extravagant Lucas number?

(A) 76 **(B)** 46 **(C)** 18 **(D)** 4 **(E)** 123

40. $11011_2 + 1101_4 + 110_8 =$ _____ 16

(A) 114 **(B)** 180 **(C)** 16A28 **(D)** 10 **(E)** B4

41. Let f(x) = 2 — x, g(x) = 2x + 3, h(x) = 3x — 1, and $f\left(2g\left(3h(x)\right)\right) = 4$. Find x.

(A) $\frac{1}{9}$ **(B)** $\frac{35}{36}$ **(C)** $\frac{1}{18}$ **(D)** $-\frac{4}{9}$ **(E)** $-1\frac{7}{36}$

42. If $\sqrt{x^2\left(\sqrt{x^3\left(\sqrt{x^5}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime and $x > 1$, then k = ?

(A) 30 **(B)** 19 **(C)** 8 **(D)** 10 **(E)** 5

43. Find the slope of the line perpendicular to the curve $y = 2x^3 - 3x + 5$ at (— 1, 6).

(A) 3 **(B)** $\frac{1}{12}$ **(C)** $-\frac{1}{6}$ **(D)** $-\frac{1}{3}$ **(E)** $-\frac{1}{2}$

44. Let $f(x) = \frac{-5x+1}{x-1}$, where $x \neq 1$. Find $f^{-1}(x)$.

(A) $\frac{x+1}{x+5}$ **(B)** $\frac{x-1}{1+5x}$ **(C)** $\frac{1-x}{1-5x}$ **(D)** $\frac{x-5}{x-1}$ **(E)** $\frac{1-x}{x+5}$

45. Omit question #45. $^{2}\!/_{2}$,

(A) 24 **(B)** $27\frac{1}{3}$ **(C)** $30\frac{2}{3}$ **(D)** 34 **(E)** $37\frac{1}{3}$

46. The sequence 8, p, q, r, $\frac{8}{9}$ is a harmonic progression. Find the value of p + q + r. (nearest tenth)

(A) 5 **(B)** 5.4 **(C)** 6 **(D)** 6.3 **(E)** 6.7

47. Point P(2, 2) undergoes several transformations to point Q(x1, y1). First, it is rotated 90° clockwise about the origin. Then, it is reflected across the y-axis. Then it is translated 3 units horizontally in the positive direction and vertically 2 units in the negative direction. Find x1 + y1.

(A) — 3 **(B)** — 1 **(C)** 0 **(D)** 1 **(E)** 2

48. The numbers greater than 1 are arranged in the array below. In which column will 408 fall.

(A)	(B)	(C)	(D)	(E)
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

49. If $\frac{Ax+B}{5x+1} - \frac{2x+3}{3x-4} = \frac{11x^2-51x+5}{15x^2-17x-4}$, where A and B are constants, then $(A+B)(A-B)$ equals:

(A) 53 (B) 45 (C) 37 (D) 30 (E) 25

50. Let $\triangle PQR$ exist such that $m\angle QPR = 60°$. Point A lies on $\overline{PQ}$ and $\overline{AR}$ is the altitude $\triangle PQR$. Find $m\angle PQR$ if the area of $\triangle AQR$ is 75% of the area of $\triangle APR$. (nearest degree).

(A) 67° (B) 71° (C) 45° (D) 49° (E) 60°

51. Let $f(x) = ax + 8$ and $g(x) = bx - 11$, where a and b are positive integers. Find $a + b$ if $f(g(x)) = g(f(x))$.

(A) 2 (B) 3 (C) 8 (D) 11 (E) 19

52. Nick Ohl put 5 nickels in a bag, shook them up, then poured them on the table. What are the odds of four or more tails being face up?

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{8}$ (D) $\frac{5}{11}$ (E) $\frac{3}{13}$

53. Given the sequence, $\frac{13}{(1\times1+1)} - \frac{13}{(2\times2-1)} + \frac{13}{(3\times3+1)} - \frac{13}{(5\times5-1)} + \frac{13}{(8\times8+1)} - \ldots$, find the digit in the ten-thousandths place.

(A) 9 (B) 8 (C) 6 (D) 3 (E) 0

54. Let $x = 5 + \cfrac{2}{5 + \cfrac{2}{5 + \cfrac{2}{5 + \cfrac{2}{5 + \ldots}}}}$. Find x. (nearest hundredth)

(A) 5.20 (B) 5.25 (C) 5.37 (D) 5.40 (E) 5.42

55. 0.4717171... base 8 can be written as which of the following simplified fractions in base 8?

(A) $\frac{311}{504}_8$ (B) $\frac{65}{77}_8$ (C) $\frac{147}{250}_8$ (D) $\frac{309}{511}_8$ (E) $\frac{103}{170}_8$

56. If the three numbers 155, 227, and 344 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 7 **(B) 5** **(C) 3** **(D) 2** **(E) 1**

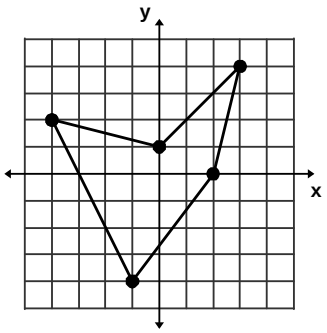
57. Mei Chado's height is 6' 2". She is walking at a rate of 5 ft/sec toward a street light that is 20 feet tall. At what rate is the tip of her shadow moving? (nearest tenth)

(A) 7.8 ft/sec **(B) 7.2 ft/sec** **(C) 6.2 ft/sec** **(D) 2.8 ft/sec** **(E) 2.2 ft/sec**

58. Simplify: $\left(\frac{(n-1)!}{(n-2)!} + \frac{(n+1)!}{n!}\right) \div \frac{(n)!}{(n-1)!}$

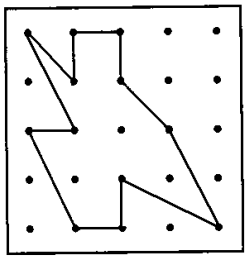
(A) 2 **(B)** $\frac{n-1}{n+1}$ **(C)** $2n^2 - 2n$ **(D)** $\frac{n^2+1}{n-1}$ **(E)** $2n^2$

59. Find the area of the pentagon. The coordinates of the vertices are integers.



(A) 21.5 sq. units **(B) 21 sq. units** **(C) 20.5 sq. units** **(D) 20 sq. units** **(E) 19.5 sq. units**

60. A rubber band was stretched on the geoboard to form this 12-sided figure. What is its area?



(A) 8 sq. units **(B) 8.5 sq. units** **(C) 9 sq. units** **(D) 9.5 sq. units** **(E) 10 sq. units**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

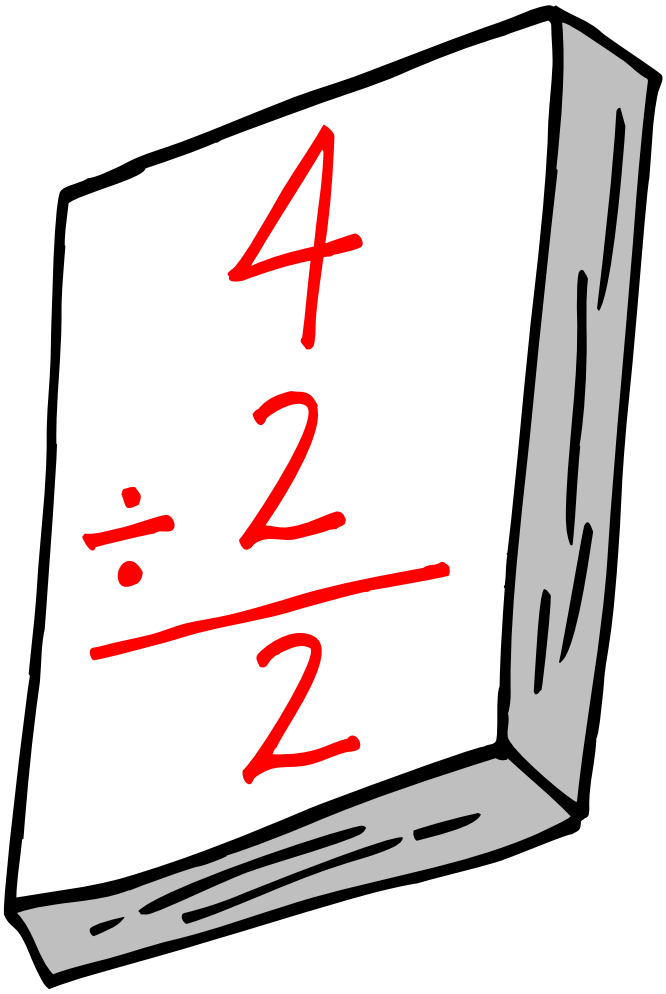
University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2017
Answer Key

1. E	**21. C**	**41. A**
2. C	**22. C**	**42. B**
3. D	**23. A**	**43. D**
4. B	**24. A**	**44. A**
5. D	**25. A**	**45. E**
6. B	**26. E**	**46. B**
7. D	**27. E**	**47. A**
8. C	**28. D**	**48. B**
9. A	**29. C**	**49. B**
10. A	**30. D**	**50. A**
11. E	**31. B**	**51. A**
12. C	**32. B**	**52. E**
13. D	**33. B**	**53. B**
14. D	**34. C**	**54. C**
15. D	**35. E**	**55. C**
16. E	**36. E**	**56. D**
17. E	**37. B**	**57. B**
18. D	**38. A**	**58. A**
19. B	**39. C**	**59. C**
20. D	**40. E**	**60. A**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2017



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $(1 - 2) + 3^4 - 5 \div 5 \times (4^3 + 2) - 1$

   (A) $-81\frac{1}{66}$ **(B)** -56 **(C)** 1 **(D)** 13 **(E)** 989

2. **Find the sum of the multiples of 8 that are greater than 20 and less than 200.**

   (A) 2,352 **(B)** 2,376 **(C)** 2,396 **(D)** 2,576 **(E)** 2,600

3. **Three million two hundred one thousand four hundred twenty-two is subtracted from one billion, two million, three hundred fifty-seven thousand eleven. What is the sum of the digits in the difference?**

   (A) 34 **(B)** 42 **(C)** 50 **(D)** 60 **(E)** 62

4. **On a map legend, 2.5 inches represents 500 miles. Big Sur, California is 1 foot 3 inches from Surfside, Florida on the map. What is the distance from Big Sur to Surfside?**

   (A) 3,250 miles **(B)** 3,750 miles **(C)** 2,600 miles **(D)** 2,750 miles **(E)** 3,000 miles

5. **If** $24x^2 + ax - 15 = (bx - 5)(4x + c)$ **then** $a + b + c =$ _____.

   (A) -1 **(B)** 1 **(C)** 5 **(D)** 7 **(E)** 11

6. **Let p and q be the roots of** $6x^2 - 3x - 1 = 0$ **. Find** $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.

   (A) 0.0625 **(B)** 1.23456... **(C)** 8. **(D)** 6.25 **(E)** 0.375

7. **The equation of a line through point P (2, 4) and perpendicular to the line** $3x - 5y = 1$ **is** $ax + by = c$. **Find** $a + b + c$.

   (A) -20 **(B)** -14 **(C)** 6 **(D)** 10 **(E)** 30

8. **What is the sum of the digits in the tens place and the units place of** $7^{(91)}$ **?**

   (A) 1 **(B)** 3 **(C)** 7 **(D)** 9 **(E)** 13

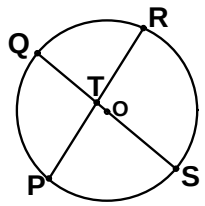
9. **Simplify:** $2\sqrt[3]{8w^5} \div \sqrt[4]{16w^8}$

   (A) $\frac{1}{2w}$ **(B)** $\frac{2}{\sqrt[3]{w}}$ **(C)** $2w$ **(D)** $\frac{1}{w}$ **(E)** $\sqrt[3]{w}$

10. **The sum of the measures of the interior angles of a single face of a regular convex dodecahedron is:**

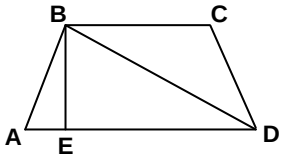
    (A) 180° **(B)** 360° **(C)** 540° **(D)** 720° **(E)** 900°

11. Given the circle with center O shown. Find $m\overset{\frown}{PQ}$ if $m\angle QTR = 75°$ and $m\overset{\frown}{RS} = 110°$.



(A) 110° (B) 105° (C) 95° (D) 75° (E) 70°

12. ABCD is an isosceles trapezoid with altitude BE = 20 cm and diagonal BD = 25 cm. What is the area of ABCD? (nearest cm)



(A) 187.5 cm^2 (B) 250 cm^2 (C) 300 cm^2 (D) 375 cm^2 (E) not enough information given

13. The point (— 5, 12) is rotated 630° clockwise about the origin. The coordinates of the point after the rotation is _____.

(A) (— 5, — 13) (B) (5, 12) (C) (5, — 12) (D) (5, 13) (E) (— 5, — 12)

14. If $\frac{Ax - 8}{3x - 7} + \frac{2x - B}{4x + 1} = \frac{34x^2 - 42x - 1}{12x^2 - 25x - 7}$, where A and B are constants, then (A + B)(A — B) equals:

(A) 64 (B) 36 (C) 42 (D) 56 (E) 48

15. Let f(x) = 2 — x, g(x) = x + 2, h(x) = 1 — 2x, and $f\big(g\big(h(x)\big)\big) = 3$. Find x.

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

16. If 7 QTs equal 5 MTs and 3 MTs equal 2 ETs, then how many ETs does it take to make 4 QTs?

(A) $2\frac{1}{10}$ (B) $\frac{10}{21}$ (C) $1\frac{19}{21}$ (D) $1\frac{2}{19}$ (E) $3\frac{11}{15}$

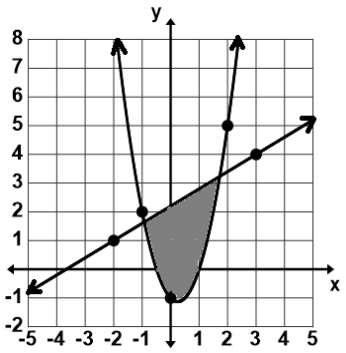
17. The roots of the equation $x^3 + bx^2 + cx + d = 0$ are $3 + i$, $3 - i$, and 2. Find b + c + d.

(A) — 6 (B) 10 (C) — 50 D) — 10 (E) 4

18. Which of the following is an identity for $(\tan^2\theta) \div (\tan^2\theta + 1)$?

(A) $1 + \cot^2\theta$ (B) $\sin^2\theta + 1$ (C) $\cot^2\theta$ (D) $\sin^2\theta$ (E) $\sec^2\theta$

19. Which of the following system of inequalities would be best represented by the shaded region?



(A) $3x - 5y \geq 11$, $y \leq 2x^2 - x - 1$ **(B)** $3x - 5y \leq -11$, $y \geq 2x^2 - x - 1$ **(C)** $3x - 5y \leq 11$, $y \leq 2x^2 - x + 1$

(D) $3x - 5y \geq -11$, $y \geq 2x^2 - x + 1$ **(E)** $3x - 5y \geq -11$, $y \geq 2x^2 - x - 1$

20. Which of the following expressions is not equal to 1?

(A) $\cot(\theta)\sin(\theta)\sec(\theta)$ **(B)** $\tan(\theta)\csc(\theta)\cos(\theta)$ **(C)** $\cos^2(\theta) + \sin^2(\theta)$
(D) $\sec^2(\theta) - \tan^2(\theta)$ **(E)** $\cot^2(\theta) - \sec^2$

21. Given the geometric sequence a, b, 45, c, d, 1215, ... find $a + b + c + d$.

(A) 410 **(B)** 560 **(C)** 605 **(D)** 1,260 **(E)** 1,812

22. Let $x^5 + 4x^4 - 3x^2 + x - 6 = 0$. According to Descartes' Rule of Signs, how many possible negative roots are there?

(A) 2 or 0 **(B)** 3 or 1 **(C)** 0 **(D)** 4, 2, or 0 **(E)** 5, 3, or 1

23. Find $m + n$ if $\begin{bmatrix} -1 & 2 \\ m & 3 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ n \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

(A) $2\frac{3}{8}$ **(B)** $-4\frac{1}{2}$ **(C)** $\frac{1}{2}$ **(D)** $-7\frac{1}{2}$ **(E)** $1\frac{7}{8}$

24. The figure shown is rotated 180° counter clockwise. Then it is reflected over its horizontal axis. Then it is rotated 90° clockwise. Finally, it is reflected over its negative diagonal. Which of the following figures is the result of these four transformations?

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

25. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, b, — 1, 1, c, d, 1, ...

(A) 6 **(B) — 3** **(C) 1** **(D) — 1** **(E) 0**

26. The graph of $g(x) = (x — 2) \div (2x^2 + 2x — 5)$ has how many asymptotes?

(A) 0 **(B) 1** **(C) 2** **(D) 3** **(E) 4**

27. Let $f(x) = x^3 — 2x^2 — 15x + 2$. Find the sum of the x-values of the critical points of the function.

(A) $4\frac{2}{3}$ **(B) 3** **(C) 2** **(D) $1\frac{2}{3}$** **(E) $1\frac{1}{3}$**

28. Evaluate: $\int_{-2a}^{2a} (9 — 5x)\, dx$

(A) 20a **(B) 18** **(C) 4a(9 — 5a)** **(D) 36a** **(E) does not exist**

29. The sequence 3, p, q, r, $\frac{1}{2}$ is a harmonic progression. Find the value of $p + q + r$. (nearest tenth)

(A) 2.8 **(B) 3.3** **(C) 5.8** **(D) 6.3** **(E) 6.5**

30. William Penn is putting together a 5-pack of colored pens. He has red pens, blue pens, black pens, and green pens. How many different 5-packs of pens can he make?

(A) 70 **(B) 625** **(C) 96** **(D) 2,880** **(E) 56**

31. $(422_7 — 124_7) \times 5_7 =$ _____ $_7$

(A) 2054 **(B) 1520** **(C) 1325** **(D) 1655** **(E) 2155**

32. An operation " $\odot$ " is defined by: $a \odot b = b^a + a^b$. What is the value of $(— 1 \odot 3)(2 \odot — 2)$?

(A) $— \frac{8}{51}$ **(B) $— 2\frac{5}{6}$** **(C) $— 3\frac{7}{12}$** **(D) $— 5\frac{2}{3}$** **(E) $— 4\frac{11}{12}$**

33. Let $f_1 = 4$, $f_2 = 9$, $f_3 = 13$, $f_4 = 22$, ... be the terms of a Fibonacci characteristic sequence. Find f_{15}.

(A) 4,378 **(B) 2,706** **(C) 4,305** **(D) 3,542** **(E) 4,325**

34. Given the sequence, $\frac{11}{(1\times1+1)} — \frac{11}{(2\times2—1)} + \frac{11}{(3\times3+1)} — \frac{11}{(5\times5—1)} + \frac{11}{(8\times8+1)} — \ldots$, find the digit in the ten-thousandths place.

(A) 2 **(B) 5** **(C) 6** **(D) 7** **(E) 9**

35. If $y^3 = 2 — 11i$, $y^2 = 3 — 4i$ and $y = a + bi$ then $a + b$ equals:

(A) — 2 **(B) — 1** **(C) 0** **(D) 1** **(E) 3**

36. Lotta Hare bought a ribbon that was 3 yards long. She cut it into four smaller ribbons such that the ratio of the lengths was 1:3:6:10. How much shorter was the shortest piece than the longest piece? (nearest inch)

(A) 1 ft 4 in (B) 1 ft 5 in (C) 1 ft 6 in (D) 1 ft 7 in (E) 1 ft 8 in

37. Otto Mobeal went to the Retread Discount Tire store to get some new tires. He got 20% off of the regular price of a set of 4 tires. It cost an extra $5.50 per tire for mounting and balancing and a disposal fee of $2.50 per tire. What was his final cost before taxes for a set of 4 tires if the regular price was $74.95 per tire?

(A) $247.84 (B) $265.44 (C) 271.84 (D) $299.80 (E) $331.80

38. David, Phyllis, and Jin scored a team total of 542 points at the TTU math camp. David scored thirty-two points less than three-fourths of the points Phyllis' scored. Jin scored two points more than half the sum of the points scored by David and Phyllis. What was the teams highest score?

(A) 212 (B) 214 (C) 224 (D) 242 (E) 254

39. Poly Gawn drew a rectangle with the length being 6 inches longer than the width. He drew a second rectangle with the length being half the original rectangle's length and the width being 2 inches shorter than the original rectangle's width. The perimeter of the second rectangle is 18 inches less than the perimeter of the original one. Find the perimeter of the original rectangle.

(A) 66" (B) 44" (C) 32" (D) 26" (E) 22 "

40. The *HAIR express* travels 50% faster than the *TERTAL coupe*. Both start from point A at the same time and reach point B 75 km away from point A at the same time. On the way, the *HAIR* stopped to rest for 12 minutes 30 seconds. Find the speed of the *TERTAL*.

(A) 150 kmph (B) 120 kmph (C) 100 kmph (D) 80 kmph (E) 75 kmph

41. Using the following array, determine the value of the last term of row 24.

1					(row 1)
2	3				(row 2)
4	5	6			(row 3)
7	8	9	10		(row 4)
11	12	13	14	15	(row 5)
...					(...)

(A) 293 (B) 296 (C) 300 (D) 305 (E) 311

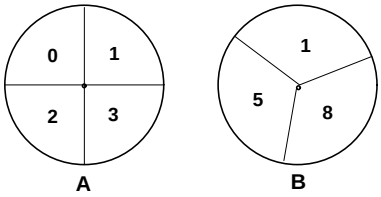
42. Willie Fall was skiing at Snow Bowl, Arizona last winter. He was at the top of Sunset Peak. He measured the angle of depression to the bottom of the run to be 14°. He read that the actual length of the run is 2675 feet. What is the change in altitude to the bottom of the run? (nearest foot)

(A) 2,596 ft (B) 187 ft (C) 647 ft (D) 2,599 ft (E) 667 ft

43. A rectangular prism water tank has a base width of 3 feet and a base length of 6 feet. The tank is being filled at a constant rate of 5 gallons per second. What is the rate of change of the height of the water in the tank? (nearest hundredth)

(A) 0.45 in/sec (B) 1.25 in/sec (C) 3.6 in/sec (D) 4.55 in/sec (E) 5.3 in/sec

44. Spinner A is divided into four equal sectors and spinner B into three equal sectors. Willie Whenn spins each spinner once. If the product of the two numbers is prime then Willie gets that number of points If the product in not prime then Willie loses that number of points. What is the mathematical expectation of spinning the spinners many times?



(A) $-4\frac{5}{6}$ pts (B) $-5\frac{1}{6}$ pts (C) $-5\frac{1}{3}$ pts (D) $-5\frac{2}{3}$ pts (E) $-6\frac{1}{6}$ pts

45. Poly Gawn wrote down the coordinates of a non-regular convex quadrilateral. She used a special technique to find the area of the quadrilateral called "Area the Easy Way". The technique is mostly associated with which of the following mathematicians?

(A) Archimedes (B) Descartes (C) Diophantus (D) Erastosthenes (E) Theano

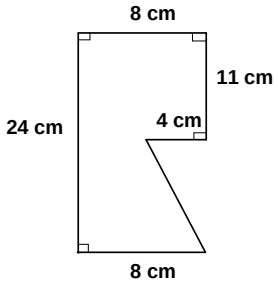
46. $\left\{(x, y) \middle| x,y \in \{\text{Integers}\}, -5 \leq x \leq 5, \text{and} -3 \leq y \leq 7\right\}$ is the solution set of $4x - 3y = 2$. How many such ordered pairs exist?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 7

47. Saul Dewould cut 4 dowel rods. The lengths of the rods are 3 feet, 4 feet, 5 feet and 6 feet. How many different acute triangles can he make using three rods at a time?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

48. Find the area of the hexagon shown.



(A) 140 in^2 (B) 152 in^2 (C) 166 in^2 (D) 175 in^2 (E) not enough data

49. $\angle PQR$ is an acute angle. Point A lies on segment PQ and point B lies on segment QR. AQ = 12", BQ = 10", and AB = 7". Find BR if segment AR bisects $\angle PAB$. (nearest inch)

(A) 15" (B) 14" (C) 11" (D) 9" (E) 5"

50. Let $\|V_1\| = 5$, $\|V_2\| = 12$, where the direction angles of V_1 and V_2 are 50° and 120°, respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

(A) 101° (B) 110° (C) 50° (D) 119° (E) 70°

51. Let f(x) = $\left(2x - 5 - \frac{3}{x}\right) \div \left(5x - 13 - \frac{6}{x}\right)$. The domain of f(x) is $\{x : x \neq a, b, c$, where x is a rational number$\}$. Find $a + b + c$.

(A) 5.1 (B) 4.3 (C) 3.4 (D) 2.6 (E) 2.4

52. If $(\log_k x)(\log_5 k) = 2.5$, find x. (nearest tenth)

(A) 97.7 (B) 2.0 (C) 76.8 (D) 1.7 (E) 55.9

53. Find the digit in the thousandth place of the series $\frac{7^0}{0!} - \frac{7^2}{2!} + \frac{7^4}{4!} - \frac{7^6}{6!} + \frac{7^8}{8!} - \ldots$.

(A) 0 (B) 3 (C) 5 (D) 7 (E) 9

54. The function $f(x) = x^4 - 3x^3 + 3x^2 + 1$ is concave down on which of the intervals?

I. $(\frac{3}{5}, \frac{7}{8})$ II. $(\frac{1}{9}, \frac{5}{6})$ III. $(\frac{3}{4}, 1\frac{1}{2})$ IV. $(\frac{2}{5}, 1\frac{1}{10})$

(A) I only (B) II only (C) II & III (D) I & IV (E) I, II, & III

55. Roland Bones tossed a fair 6-sided die 5 times. What is the probability that he rolled at least one 4? (nearest whole percent)

(A) 33% (B) 60% (C) 67% (D) 74% (E) 87%

56. Let T_n be the nth triangular number, S_n be the nth square number, and P_n be the nth pentagonal number. Then $T_n + S_{(n+1)}$ has the same value as:

(A) $P_{(n+1)}$ (B) $P_{(n+2)}$ (C) $T_{(2n+1)}$ (D) $S_{(2n+2)}$ (E) $P_{(2n)}$

57. Mark DeCard labels blank cards with the numbers from the set { 1, 3, 6, 10, 15, 21} with one number per card. He selects two cards at random. What are the odds that the absolute value difference is an odd number?

(A) $\frac{2}{3}$ (B) $\frac{8}{7}$ (C) $\frac{2}{5}$ (D) $\frac{2}{1}$ (E) $\frac{8}{15}$

58. The number 2017 is a member of which of the following sets of numbers?
I. Evil II. Unhappy III. Polite

(A) I, II, & III (B) I & II only (C) II only (D) II & III only (E) III only

59. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \ldots$ be the terms of the Fibonacci sequence. If $f_{(k)} = 832{,}040$ then $f_{(k\,—\,3)} = ?$

(A) 277,347 (B) 121,393 (C) 196,418 (D) 1,346,269 (E) 317,811

60. A lock's combination consists of three positive digits. The first digit is a Fibonacci number, the second digit is a composite number, and the last digit is a triangular number. How many unique combinations fit this criteria?

(A) 48 (B) 54 (C) 60 (D) 66 (E) 72

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2017
Answer Key

1. D	**21. B**	**41. C**
2. B	**22. A**	**42. C**
3. D	**23. A**	**43. A**
4. E	**24. D**	**44. C**
5. D	**25. E**	**45. B**
6. A	**26. D**	**46. B**
7. E	**27. E**	**47. C**
8. C	**28. D**	**48. C**
9. B	**29. A**	**49. B**
10. C	**30. E**	**50. A**
11. A	**31. A**	**51. D**
12. C	**32. B**	**52. E**
13. E	**33. E**	**53. B**
14. E	**34. D**	**54. A**
15. D	**35. D**	**55. B**
16. C	**36. A**	**56. A**
17. A	**37. C**	**57. B**
18. D	**38. C**	**58. D**
19. E	**39. B**	**59. C**
20. E	**40. B**	**60. C**