

TMSCA HIGH SCHOOL MATHEMATICS TEST #2 © OCTOBER 26, 2019

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

1. Evaluate: $5! \times (10)^{-1} \div \sqrt{3^4} + 6\frac{2}{3} - 16$
 (A) -12 (B) -8 (C) -4 (D) 0 (E) 4
2. For his birthday, Siddharth received a \$100 gift card to Academy Sports at Colony Square. He selected two Nike running shirts for \$18.85 each, three Under Armour running shirts for 17.50 each, and one Adidas running shirt for \$19.95. If the tax rate is 8.125%, how much of his own money did he need to make the purchase?
 (A) \$10.15 (B) \$13.13 (C) \$16.12 (D) \$19.10 (E) \$22.08
3. Find the least common multiple of 15, 57, and 95.
 (A) 95 (B) 190 (C) 285 (D) 380 (E) 475
4. Claire is the CEO of Sugarland Securities. Of her 280 female employees, 140 have a mathematics degree. Of her 220 male employees, 88 have a mathematics degree. If one of her employees is selected at random, what is the probability that the person selected is a male or has a math degree?
 (A) 0.60 (B) 0.70 (C) 0.72 (D) 0.75 (E) 0.80
5. Consider the sequence $\{2, 1, 3, 4, 7, 11, \dots\}$. What is the 12th term in the sequence?
 (A) 178 (B) 185 (C) 192 (D) 199 (E) 206
6. The line $ax + by = c$ contains the point (4, 5) and is perpendicular to the line $4x + 3y = 3$. Find c.
 (A) -8 (B) -4 (C) 2 (D) 4 (E) 8
7. If $A = \frac{1}{6}$ of B and B is 100% more than C, what percent of C is A?
 (A) $33\frac{1}{3}\%$ (B) 50% (C) $66\frac{2}{3}\%$ (D) 200% (E) 300%
8. Find the y-intercept of the line that contains the points (-3, -6) and (5, 12).
 (A) $\left(0, \frac{1}{2}\right)$ (B) $\left(0, \frac{5}{8}\right)$ (C) $\left(0, \frac{3}{4}\right)$ (D) $\left(0, \frac{7}{8}\right)$ (E) (0, 1)
9. Which of the following functions has an inverse?
 (A) $f(x) = x^3 - 2x - 3$ (B) $f(x) = x^2 - 6$ (C) $f(x) = |x + 3|$
 (D) $f(x) = -(x - 4)^2 + 3$ (E) $f(x) = (x - 32) \times \left(\frac{5}{9}\right)$

10. $6x - 5y = 10$ and $3x + 7y = 43$. $4xy =$

- (A) -20 (B) 20 (C) 40 (D) 60 (E) 80

11. Find the period of $f(x) = 3 - 5\cos\left(4x - \frac{\pi}{4}\right)$.

- (A) $\frac{2}{\pi}$ (B) 5 (C) $\frac{\pi}{4}$ (D) 3 (E) $\frac{\pi}{2}$

12. Find the sum of all the prime numbers between 60 and 80.

- (A) 349 (B) 351 (C) 353 (D) 355 (E) 357

13. The sketch of the line $3x - 2y = 12$ forms a triangle with the x-axis and y-axis. Find the area of this triangle.

- (A) 6 (B) 9 (C) 12 (D) 18 (E) 24

14. This mathematician is best remembered for her work in abstract algebra and theoretical physics. Einstein considered her the most important woman in the history of mathematics, in part because she was able to verify key parts of the general theory of relativity. In her later years, she was a professor at Bryn Mawr College. She was particularly interested in rings and fields.

- (A) Emmy Noether (B) Lise Meitner (C) Ada Lovelace (D) Sophie Germain (E) Maria Mitchell

15. Which of the following is the equation of a limaçon with an inner loop?

- (A) $r = 3 + \cos \theta$ (B) $r = 1.5 + \cos \theta$ (C) $r = 4 + 4\cos \theta$ (D) $r = 2 + 3\cos \theta$ (E) $r = 2 - \cos \theta$

16. What is the angle formed by the hour hand and minute hand at 3:25?

- (A) 42.5° (B) 45° (C) 47.5° (D) 50° (E) 52.5°

17. Consider the sketch of the hyperbola with equation $9x^2 - 25y^2 + 18x + 150y = 441$. One of the focal points is

- (A) $(-1, 3 + \sqrt{34})$ (B) $(-7, 3)$ (C) $(-1 + \sqrt{34}, -3)$ (D) $(-2, 3)$ (E) $(-1 - \sqrt{34}, 3)$

18. The expression $\frac{\csc(x) \cdot \cos(x)}{\tan(x) + \cot(x)}$ is equivalent to

- (A) $\sin^2(x)$ (B) $\cos(2x)$ (C) $\cos^2(x)$ (D) $\sin(2x)$ (E) $\sec(x)$

19. The eleventh polite number is 15. What is the politeness of 15?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

20. Assume that the license plates in Montana consists of 3 letters followed by 3 digits. How many different license plates are possible if no letters can be repeated and no digits can be repeated?
- (A) 9,936,000 (B) 17,576,000 (C) 13,646,000 (D) 11,232,000 (E) 15,438,000
21. The angle bisectors of the three angles of a triangle intersect at a point that is the center of the inscribed circle of the triangle. This point is called the
- (A) incenter (B) orthocenter (C) circumcenter (D) centroid (E) Spieker center
22. The position (in centimeters) of a particle moving along the x-axis is given by $s(t) = t^4 - 2t^3 + 3t^2 - 4t + 5$, $t \geq 0$, t in seconds. At what value of t does the particle come to rest?
- (A) 0.75 sec (B) 1.00 sec (C) 1.25 sec (D) 1.50 sec (E) 1.75 sec
23. Solve the equation $\sqrt{2x^2 - 7} = 3 - x$. What is the smallest solution?
- (A) -8 (B) -2 (C) 0 (D) 2 (E) 8
24. When the stock market collapsed, Ethan bought silver, Andrew bought gold, but Sameer bought land. He purchased a 28,000 acre ranch near Guthrie. The price of the ranch was \$16,606,500 per league of land. What was the purchase price of the ranch? (nearest dollar)
- (A) \$98,950,000 (B) \$100,875,625 (C) \$102,725,600 (D) \$105,000,000 (E) \$107,886,400
25. Consider the geometric sequence $64, a, b, c, \frac{81}{64}, \dots$. If $a > 0$, then $a + b + c =$
- (A) $36\frac{33}{128}$ (B) $36\frac{17}{64}$ (C) $36\frac{9}{32}$ (D) $36\frac{5}{16}$ (E) $36\frac{3}{8}$
26. $111111_2 + 3333_4 = \underline{\hspace{2cm}}_8$
- (A) 316 (B) 377 (C) 476 (D) 555 (E) 634
27. Consider $f(x) = Ax^3 + Bx^2 + Cx + D$, with $A = 1$, $C = -19$. The harmonic mean of the roots of $f(x) = 0$ is $4\frac{14}{19}$. Find the value of D .
- (A) 6 (B) 15 (C) 30 (D) 60 (E) 90
28. Consider $f(x) = -4x^3 + 3x^2 - 2x + 1$. Find the slope of the normal line to the sketch of $f(x)$ at $x = 2$.
- (A) $\frac{1}{4}$ (B) $\frac{1}{12}$ (C) $\frac{1}{27}$ (D) $\frac{1}{38}$ (E) $\frac{1}{50}$

29. The terms of the Fibonacci sequence are $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... Find $\frac{f_{17}}{f_{18}}$
- (A) $\frac{1+\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{1+\sqrt{5}}{3}$ (D) $\frac{\sqrt{5}-1}{3}$ (E) $-(1-\sqrt{5})$
30. Andrew received some cash for his birthday. He gave one-third of the cash to Luc. Then he gave one-fourth of what remained to Avi. Then he gave one-fifth of what he still had to Vallabh. Finally, he gave one-sixth of what still remained to William. If Andrew ended up with \$50, how much cash did he receive on his birthday?
- (A) \$120 (B) \$150 (C) \$180 (D) \$210 (E) \$240
31. Amy left Love Field in her private jet and flew 600 miles on a bearing of 40° . As the jet flew over Nashville, her pilot turned the plane and flew on a bearing of 150° all the way to a refueling stop in Florida. If Amy was now 991 miles from Love Field, how far is it from Nashville to her refueling stop? (nearest mile)
- (A) 851 mi (B) 878 mi (C) 922 mi (D) 967 mi (E) 1020 mi
32. Richard attends Harvard and he plays on a flag football team consisting of 15 math majors. Stacy, the coach, must select 11 starters for the final game. How many ways can she select the 11 starters if she must select Richard as one of the starters?
- (A) 336 (B) 864 (C) 1001 (D) 1365 (E) 3003
33. Michael worked for the Lowes on Inwood Road last summer. He earned \$12.00 per hour for the first 40 hours in a week. He made \$18.00 per hour for any hours worked over 40 and through 60 in a week. He earned \$24.00 per hour for any hours worked over 60 in a week. One busy week in July, Richard worked 84 hours. How much did he earn that week?
- (A) \$1336.00 (B) \$1356.00 (C) \$1376.00 (D) \$1396.00 (E) \$1416.00
34. Find the remainder when $(443344_7 \times 22_7)$ is divided by 6_7 .
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
35. If $(5x + c)(3x + 4) = 15x^2 + 14x + d$, then $c + d =$
- (A) -10 (B) -4 (C) -2 (D) 2 (E) 8
36. If $\frac{2}{x-2} + \frac{3}{x+1} = \frac{ax+b}{cx^2+dx+e}$, then $a + b + c + d + e =$
- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

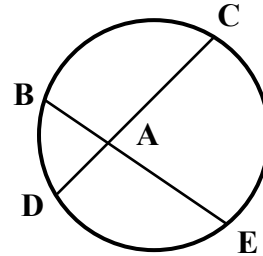
37. Consider the number 28. Which of the following statements are correct about this number?

- I. 28 is a perfect number II. 28 is a happy number III. 28 is an odious number
 (A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

38. Find the area between the curves $y_1 = .8x + 3$ and $y_2 = .2x^2 - 4$. (nearest tenth)

- (A) 56.1 (B) 58.3 (C) 60.5 (D) 62.7 (E) 64.9

39. Given: $m\widehat{BD} = 34^\circ$, $m\widehat{CE} = 136^\circ$. Find $m\angle CAE$.



- (A) 81° (B) 83° (C) 85° (D) 87° (E) 89°

40. The roots of $x^3 + x^2 - 14x - 24 = 0$ are p, q, and r. $(p + q)(q + r)(p + r) =$

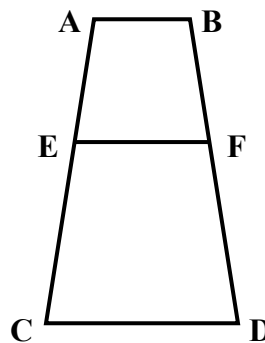
- (A) -16 (B) -10 (C) 12 (D) 18 (E) 36

41. The circle centered at (3, 6) is tangent to the line with equation $4x + 3y = 10$. The equation of the circle is $x^2 + y^2 - 6x - 12y + C = 9$. The value of C is

- (A) 17 (B) 24 (C) 31 (D) 38 (E) 45

42. $AB = 12$, $CD = 20$, $\overline{AB} \parallel \overline{EF} \parallel \overline{CD}$.

Find EF if \overline{EF} divides trapezoid ABDC into two similar trapezoids (nearest thousandth)



- (A) 14.667 (B) 15.000 (C) 15.246
 (D) 15.492 (E) 15.831

43. Consider the function $f(x) = -\sqrt{9 - x^2}$, $-3 \leq x \leq 0$. $f^{-1}(x) =$ _____, $-3 \leq x \leq 0$.

- (A) $\sqrt{x^2 - 9}$ (B) $x^2 - 9$ (C) $\sqrt{9 - x^2}$ (D) $9 - x^2$ (E) $-\sqrt{9 - x^2}$

44. Which of the following are true for the sequence $\left\{\frac{3}{n^2}\right\}_{n=2}^{\infty}$?
- I. the sequence is decreasing II. The sequence is monotonic III. The sequence is bounded below
- (A) I only (B) I and II only (C) I and III only (D) II only (E) I, II, and III
45. Tanner is 96 years old and he has a ranch near Boerne with 32 labors of prime ranch land. He plans to leave an equal amount of land to each of his 120 great-grand-children when his pilgrimage on Earth is completed. How much land will each person receive? (nearest acre)
- (A) 43 acres (B) 45 acres (C) 47 acres (D) 49 acres (E) 51 acres
46. Zane entered a bowling tournament and he completed six games before he was eliminated. His scores were 196, 202, 216, 204, 220 and 188. Find the absolute value difference between his mean score and his median score.
- (A) $0.\overline{6}$ (B) 1.0 (C) $1.\overline{3}$ (D) $1.\overline{6}$ (E) 2
47. Garrett answered 42 questions on the district UIL Mathematics test. He felt confident on 36 of the questions, but he admitted that he just guessed on six of the questions. What is the probability that Garrett got at least two of these six questions correct. (nearest thousandth)
- (A) 0.279 (B) 0.301 (C) 0.323 (D) 0.345 (E) 0.367
48. This geometric figure is a closed plane curve generated by a point moving in such a way that the sum of its distances from two fixed points is a constant.
- (A) ellipse (B) hyperbola (C) circle (D) parabola (E) angle
49. Simplify: $\ln(\ln(e)) + \log_3(81) + \log(\log(10)) - \log_6\left(\frac{1}{216}\right)$
- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9
50. How many distinct arrangements can be made if 4 red marbles, 5 green marbles and 3 blue marbles are arranged in a row?
- (A) 840 (B) 4320 (C) 27,720 (D) 166,320 (E) 479,001,600
51. Given: $x + y = 21$ and $xy = 108$. Find $x^3 + y^3$.
- (A) 2149 (B) 2251 (C) 2344 (D) 2457 (E) 2590
52. The medians of a triangle intersect at a point called the _____.
- (A) centroid (B) incenter (C) circumcenter (D) orthocenter (E) center

53. The area of a triangle is 48. The vertices are (3, 7), (a, 1), and (9, a). If $|a| < 10$, what is the value of a?
- (A) -6 (B) -3 (C) -1 (D) 2 (E) 4
54. The length of one edge of a regular tetrahedron is 12 mm. Find the total surface area of the tetrahedron. (nearest square millimeter)
- (A) 243 mm^2 (B) 249 mm^2 (C) 255 mm^2 (D) 261 mm^2 (E) 267 mm^2
55. Given isosceles trapezoid ABCD with BC parallel to AD and area = 504 in^2 . Point E lies on \overline{AD} and $\overline{BE} \perp \overline{AD}$. If $AB = 18.44 \text{ in}$ and $BE = 14 \text{ in}$, then $CE =$
- (A) 26.34 in (B) 26.70 in (C) 27.06 in (D) 27.42 in (E) 27.78 in
56. Consider the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{64} = 1$. Find the x-intercept of the line tangent to the sketch of the ellipse when $x = 3$.
- (A) 11 (B) $11\frac{3}{4}$ (C) 12 (D) $12\frac{1}{3}$ (E) 13
57. If p and q are zeros of $f(x) = 8x^2 - 46x + 65$, then $pq^2 + p^2q =$
- (A) $45\frac{21}{32}$ (B) $46\frac{23}{32}$ (C) $47\frac{25}{32}$ (D) $48\frac{27}{32}$ (E) $49\frac{29}{32}$
58. The pool in Preston's backyard is 36 feet long, 24 feet wide and 5 feet deep. How many gallons of water are required to completely fill his pool? (nearest gallon)
- (A) 30,602 (B) 30,976 (C) 31,464 (D) 31,988 (E) 32,316
59. The Ice Cream Shoppe in Athens offers 12 flavors of ice cream. Jacob decides to get two scoops in a cup. How many different ways can he order two scoops in a cup?
- (A) 66 (B) 78 (C) 110 (D) 132 (E) 144
60. On September 16, 2018, Eliud Kipchoge from Kenya set a new world record at the Berlin Marathon. His time was 2 hours 1 minute 39 seconds. The marathon distance is 42.195 kilometers. On average, how long did it take him to complete each mile? (nearest second)
- (A) 4 min 32 sec (B) 4 min 35 sec (C) 4 min 38 sec (D) 4 min 41 sec (E) 4 min 44 sec

2019 – 2020 TMSCA High School Mathematics Test # 2
Answer Key

1. B	21. A	41. D
2. D	22. B	42. D
3. C	23. A	43. E
4. C	24. D	44. E
5. D	25. E	45. C
6. A	26. C	46. C
7. A	27. C	47. D
8. C	28. D	48. A
9. E	29. B	49. D
10. E	30. B	50. C
11. E	31. E	51. D
12. B	32. C	52. A
13. C	33. E	53. B
14. A	34. D	54. B
15. D	35. A	55. E
16. C	36. B	56. C
17. E	37. E	57. B
18. C	38. E	58. E
19. C	39. C	59. B
20. D	40. B	60. C

19-20 TMSCA HSMA Test # 2 Selected Solutions

$$2. (2(18.85) + 3(17.50) + 19.95)(1.08125) - 100$$

$$\text{\$19.10}$$

$$4. \frac{220 + 140}{280 + 220} = 0.72$$

$$6. y - 5 = \frac{3}{4}(x - 4)$$

$$3x - 4y = -8$$

$$m = \frac{12 - -6}{5 - -3} = \frac{9}{4}$$

$$y = \frac{3}{2}x - 6$$

$$10(6) = 60$$

$$8. y - 12 = \frac{9}{4}(x - 5)$$

$$12. 61 + 67 + 71 + 73 + 79 = 351$$

$$13. A = .5(4)(6)$$

$$16. \left(\frac{25}{60}\right)(5)(6) = 12.5$$

$$y = \frac{9}{4}x + \frac{3}{4}$$

$$A = 12$$

$$60 - 12.5 = 47.5$$

$$9(x^2 + 2x + 1) - 25(y^2 - 6y + 9) =$$

$$441 + 9 - 225$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}}$$

$$17. \frac{(x+1)^2}{25} - \frac{(y-3)^2}{9} = 1$$

$$18. \frac{\cos x}{\sin x} \cdot \frac{\cos x \sin x}{1}$$

$$\cos^2 x$$

$$20. \frac{26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8}{11,232,000}$$

$$25 + 9 = c^2$$

$$(-1 - \sqrt{34}, 3)$$

$$\frac{81}{64} = 64r^4$$

$$22. s'(t) = 0 \text{ when } t = 1$$

$$24. \frac{28000}{4428.4}(16,606,500) =$$

$$\text{\$105,000,000}$$

$$25. r = \frac{3}{8}$$

$$26. 63 + 255 = 318_{10} = 476_8$$

$$24 + 9 + 3\frac{3}{8}$$

$$36\frac{3}{8}$$

$$27. \frac{-3D}{-19} = 4\frac{14}{19}$$

$$D = 30$$

$$28. f'(2) = -38$$

$$\text{slope of normal} = \frac{1}{38}$$

$$29. \frac{1597}{2584} = \frac{\sqrt{5} - 1}{2}$$

$$30. x \left(\frac{2}{3}\right) \left(\frac{3}{4}\right) \left(\frac{4}{5}\right) \left(\frac{5}{6}\right) = 50$$

$$x = 150$$

$$31. 991^2 = 600^2 + x^2 - 2(600)(x)\cos 70^\circ$$

$$x = 1020$$

$$32. {}_{14}C_{10} = 1001$$

$$33. 40(12) + 20(18) + 24(24) =$$

$$\text{\$1416}$$

$$3c + 20 = 14$$

$$35. c = -2, d = -8$$

$$c + d = -10$$

$$36. \frac{5x - 4}{x^2 - x - 2}$$

$$5 - 4 + 1 - 1 - 2 = -1$$

$$38. \int_{-4.2449979984}^{8.2449979984} (y1 - y2) dx = 64.9$$

$$d = \frac{|4(3) + 3(6) - 10|}{\sqrt{4^2 + 3^2}} = 4$$

$$39. \frac{136 + 34}{2} = 85$$

$$41. (x-3)^2 + (y-6)^2 = 16$$

$$42. \sqrt{12(20)} = 15.492$$

$$45. \frac{(32)(177.136)}{120} = 47$$

$$x^2 + y^2 - 6x - 12y + 38 = 9$$

$$46. \bar{x} = 204.\bar{3}, M = 203$$

$$47. \text{Binomial Cdf}(6, .2, 2, 6) = .34464$$

$$49. 0 + 4 + 0 - -3 = 7$$

$$50. \frac{12!}{4!5!3!} = 27,720$$

$$51. (21^2 - 3(108))(21) = 2457$$

$$53. 48 = \pm \frac{1}{2} \begin{vmatrix} 3 & 7 & 1 \\ a & 1 & 1 \\ 9 & a & 1 \end{vmatrix}$$

$$54. \frac{12^2 \sqrt{3}}{4} \cdot 4 = 249.415$$

$$a = -3$$

$$y = \sqrt{\frac{2304 - 64x^2}{36}}$$

$$w^2 + 14^2 = (18.44)^2$$

$$y'(3) = -\frac{4\sqrt{3}}{9}$$

$$55. 504 = \frac{1}{2} \cdot (14)(2m + 2w)$$

$$56. y(3) = 4\sqrt{3}$$

$$58. \frac{(36)(24)(5)(12)^3}{231} = 32,315.84$$

$$m^2 + 14^2 = x^2$$

$$x = 27.78$$

$$y - 4\sqrt{3} = -\frac{4\sqrt{3}}{9}(x - 3)$$

$$0 = \frac{1}{9}(x - 3)$$

$$x = 12$$

$$\frac{(42.195)(100000)}{(2.54)(12)(5280)} = A$$

$$59. {}_{12}C_2 + 12 = 78$$

$$60. \frac{12 + \frac{39}{60}}{A} = 4.6398$$

$$4 \text{ min } 38 \text{ sec}$$