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GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $\sqrt[4]{104976} \div (81)^{3/4} + 6 \times 2^{-1} - 4$.
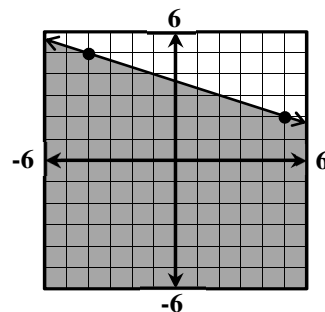
    (A) $-1\dfrac{5}{6}$ (B) 1 (C) 5 (D) $-\dfrac{1}{3}$ (E) $8\dfrac{2}{3}$

2. The city of Denton, TX grew from a population of 361 in 1870 to 113,383 in 2010. What was the average annual growth rate for the population of Denton during that 140-year period? (nearest tenth of a percent)

    (A) 2.2%   (B) 3.8%   (C) 4.8%   (D) 4.2%   (E) 5.1%

3. Which of the inequalities is best represented by the graph below?

    (A) $x + 3y \geq 11$ (B) $x - 3y \leq -11$ (C) $x + 3y \leq 11$

    (D) $x + 3y \leq 1$ (E) $x - 3y \geq 1$

4. Write this expression as a simplified proper fraction: $0 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{9}}}}}$

    (A) $\dfrac{2693}{4140}$ (B) $\dfrac{1051}{1380}$ (C) $\dfrac{913}{1380}$ (D) $\dfrac{329}{1051}$ (E) $\dfrac{1910}{9459}$

5. Holly's Holiday Ice Cream Shop serves 8 flavors of specialty ice cream and 3 flavors of cones. How many distinct 2-scoop ice cream cone orders are possible?

    (A) 108   (B) 135   (C) 165   (D) 36   (E) 45

6. Divide and simplify: $\dfrac{x^2 - 16}{x^2 - 6x + 9} \div \dfrac{3x - 12}{x^2 - 2x - 3}$

    (A) $\dfrac{x^2 + 5x + 4}{3x - 3}$ (B) $\dfrac{x + 4}{3x - 9}$ (C) $\dfrac{x^2 + 5x + 4}{3x - 9}$ (D) $\dfrac{x^2 + 5x + 4}{3x + 9}$ (E) $\dfrac{x^2 - 3x - 4}{3x + 9}$
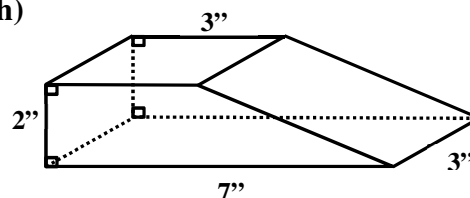
7. Let $O = \{1,3,5,7,...29\}$, $T = \{1,3,6,10,15,...66\}$ and $F = \{1,2,3,5,8,13,...89\}$. How many elements are in $(O \cup F) \cap T$?

    (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

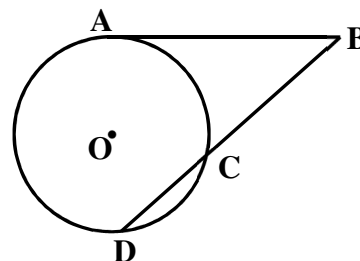8. If 48% of A is $2\dfrac{3}{4}$ of B, then B is what percent of A?

    (A) $16\dfrac{2}{3}\%$ (B) $15\dfrac{6}{11}\%$ (C) $17\dfrac{2}{3}\%$ (D) $17\dfrac{5}{11}\%$ (E) $17\dfrac{6}{11}\%$

9. Find the surface area of the prism shown. (nearest square inch)

(A)   56 in²   (B)   30 in²   (C)   96 in²   (D)   42 in²   (E) 69 in²

10. Given the circle with center O shown with BC = 12 cm and DC = 9 cm, find AB. (nearest tenth)

(A)   10.4 cm   (B)   15.9 cm   (C)   13.9 cm

(D)   16.2 cm   (E)   13.3 cm

11. If $\int_{-3}^{7} f(x)dx = 32$, evaluate $\int_{-3}^{7}\left[2f(x)+3x\right]dx$.

(A)   124   (B)   102   (C)   72   (D)   62   (E)   128

12. Two standard dice are rolled. What are the odds that the numbers on the two dice are different?

(A)   5:2   (B)   5:1   (C)   6:1   (D)   3:1   (E)   4:1

13. Simplify: $b^3\left(\dfrac{(b)^{-8}\sqrt{b^3}}{b^{-2}}\right)$

(A) $\left(\sqrt{b}\right)^{3}$ (B) $\left(\sqrt{b}\right)^{-1}$ (C) $\left(\sqrt{b}\right)^{-5}$ (D) $\left(\sqrt{b}\right)^{-3}$ (E) $\left(\sqrt{b}\right)$

14. If p and q are the roots of the function $f(x)=6x^2+11x-10$, then $p^3+3p^2q+3pq^2+q^3=?$

(A) $-\dfrac{1331}{1728}$ (B) $-\dfrac{1331}{216}$ (C) $-\dfrac{343}{216}$ (D) $-\dfrac{125}{27}$ (E) $\dfrac{6856}{216}$

15. The circle $x^2-10x+y^2+6y=-9$ has a radius r and center $\left(h,k\right)$. Find $r+h+k$.

(A)   13   (B) 7   (C)   3   (D) 4   (E)   27

16. Using the following pattern of numbers, which of the following numbers will NOT be in row 12?

```
                    1                        row 0
                 1     1                     row 1
              1     2     1                  row 2
           1     3     3     1               row 3
        1     4     5     4     1            row 4
     1     5     7     7     5     1         row 5
  1     6     9    10     9     6     1      row 6
                   ...                ...
```

(A)   12   (B)   65   (C)   220   (D)   495   (E)   792

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17. If p is a prime number and $2p+1$ is a prime number then $2p+1$ is called a *safe prime* and p is called a _____prime.

   (A)   Mersenne   (B) Pythagorean   (C)   Germain   (D)   Hypatian   (E)   Euclidean

18. The point of concurrency of the perpendicular bisectors of the sides of a of a triangle is called the:

   (A)   Incenter   (B)   Centroid   (C)   Orthocenter (D)   Origin   (E)   Circumcenter

19. The Real value solution set for $-2x^2 + 18 \geq 9x$ is:

   (A) $\left(-\infty,-6\right] \cup \left[\dfrac{3}{2},\infty\right)$ (B) $\left[-\dfrac{3}{2},6\right]$ (C)   No Solutions

   (D) $\left(-\infty,-\dfrac{3}{2}\right] \cup [6,\infty)$ (E) $\left[-6,\dfrac{3}{2}\right]$

20. How many positive proper fractions in lowest terms have a denominator of 52?
   (A)   22   (B)   24   (C)   25   (D)   26   (E) 20

21. Angela tossed a fair nickel until she got heads. What is the probability that she first got heads on the 6th toss?

   (A) $\dfrac{3}{32}$ (B) $\dfrac{1}{64}$ (C) $\dfrac{3}{64}$ (D) $\dfrac{1}{32}$ (E) $\dfrac{1}{16}$

22. If the dots on the grid shown below are 3 cm apart both vertically and horizontally, find the area of the shaded region.
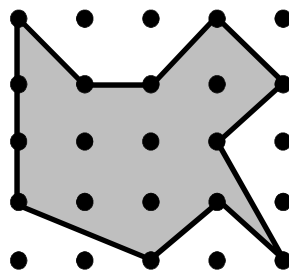
   (A)   74   (B)   72   (C)   99   (D)   90   (E)   81

23. Let $f(x) = x^3 - 27$ and $g(x) = x - 3$. Find $(f/g)(x)$ simplified where $g(x) \neq 0$.

   (A) $x^2 - 9$ (B) $x^2 + 3x + 9$ (C) $x^2 - 3x + 9$

   (D) $x^2 + 6x + 9$ (E) $x^2 - 6x + 9$

24. The vertex of a parabola is located at $(5,3)$ and the focus is located at $(8,3)$. Find the directrix of the parabola.

   (A) $x = 2$ (B) $x = -5$ (C) $y = 3$ (D) $x = 5$ (E) $x = 3$

25. Given that $z^5 = -38 - 41i$ and $z^4 = -7 - 24i$, where $z = a + bi$ and $a,b \in \mathbb{Z}$, find the value of $a + b$.

   (A)   1   (B)   −1   (C)   2   (D)   −2   (E)   0

26. Which of the following mathematicians is known as the first woman to write a mathematical handbook on calculus topics?

   (A) Erastothenes   (B) Euclid   (C) Diophantus   (D) Angnesi   (E) Fermat

27. How many three-digit numbers have all three digits as different even numbers?

   (A) 125   (B) 48   (C) 100   (D) 60   (E) 24

28. An icosidodecahedron has F faces, 30 vertices and 60 edges. Calculate $F + E + V$.

   (A) 118   (B) 124   (C) 120   (D) 92   (E) 122

29. The 2nd term of an infinite geometric sequence is $\dfrac{27}{40}$. The 5th term is $\dfrac{1}{5}$. Find the sum of the terms in the infinite geometric sequence.

   (A) $\dfrac{2}{3}$ (B) $\dfrac{81}{80}$ (C) $\dfrac{211}{80}$ (D) $\dfrac{13}{8}$ (E) $\dfrac{243}{80}$

30. If A represents a digit $0 - 9$ in the equation $3A7_9 + 5A3_6 = 200021_3$, find the value of A.

   (A) 5   (B) 4   (C) 3   (D) 2   (E) 1

31. The finite region enclosed by the curve $f(x) = 6x^2 + x - 35$ and the x-axis is rotated $360°$ around the x-axis. Calculate the volume of the solid generated. (nearest cubic unit)

   (A) 9944   (B) 2742   (C) 12056   (D) 909   (E) 8614

32. Given the sequence 2, 9, 18, 29, 42, 57, ... , find the 20th term.

   (A) 434   (B) 834   (C) 477   (D) 522   (E) 501

33. If $\log_2(x-7) + \log_2(x) = \log_2(6) + \log_2(5)$, then x equals____.

   (A) 10   (B) 5   (C) 3   (D) 6   (E) 7

34. $\displaystyle\sum_{k=1}^{3}\left[(-1)^k \left(kx - (k+1)y + k\right)\right] = ?$

   (A) $2x - 3y + 2$ (B) $-2x + 3y$ (C) $-2x + 3y - 2$ (D) $2x - 3y$ (E) $-4x + 6y - 4$

35. Given $f(x) = 4 - 5\cos\left[\dfrac{\pi}{3}(x+2)\right]$, find the sum of the period and amplitude.

   (A) 10   (B) 11   (C) 1   (D) 6   (E) 9

36. A function, $f(x)$, exists such that $f''(x) = 6x^2 + 18x - 8$, $f(2) = 21$ and $f(-2) = -39$. Find $f(4)$.

   (A) 256   (B) 208   (C) 267   (D) 285   (E) 251

37. $\dfrac{1}{6} + \dfrac{1}{10} + \dfrac{1}{15} + \dfrac{1}{21} + ... + \dfrac{1}{171} + \dfrac{1}{190} = ?$

   (A) $\dfrac{32}{57}$ (B) $\dfrac{9}{10}$ (C) $\dfrac{10}{11}$ (D) $\dfrac{17}{30}$ (E) $\dfrac{13}{15}$

38. When Jerri received her paycheck, she immediately paid 1/3 of it for rent and $150 to her phone company. The next day, she spent 25% of what was left for her car payment. Finally, she put half of the remaining money in savings and was left with $450. How much was she paid?

(A) $1710   (B) $2880   (C) $2160   (D) $2025   (E) $1265

39. Morgan folds the net shown into a cube. She rolls the cube as if it were a fair 6-sided die then adds the number on the top and bottom of the die. What is the expected value for the sum?

		7	4
	8	5	
4	5		
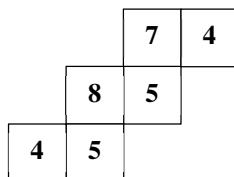
(A)   9   (B)   11   (C)   8   (D)   10   (E)   12

40. Point P $(3,-8)$ lies on the x-y plane. P is reflected across the line $y = 3$ to the point Q. Point Q is rotated 90° clockwise about the origin to point R. Point R is translated horizontally +6 units to point S. Point S is reflected over the x-axis to point T (x, y). Find $x + y$.

(A)   23   (B)   17   (C)   20   (D)   18   (E)   26

41. Let $2x^3 - 5x^2 y = 6y$. Find $\dfrac{dy}{dx}$.

(A) $\dfrac{6-10y}{5}$ (B) $\dfrac{x^2 - 2}{1 - x^2}$ (C) $6 - 2y$ (D) $\dfrac{6x^2 - 10xy}{6 + 5x^2}$ (E) $\dfrac{3x^2}{8}$

42. Given $f(x) = 2x - 3$ and $g(x) = 7 - x$ find $(f/g)^{-1}(x)$.

(A) $\dfrac{7x+3}{7}$ (B) $\dfrac{7x-3}{2+x}$ (C) $\dfrac{7x+3}{2+x}$ (D) $\dfrac{7x+3}{2-x}$ (E) $\dfrac{7x-3}{2-x}$

43. A caramel corn recipe calls for $6\dfrac{1}{3}$ cups of sugar and serves 20 people. How many cups of sugar would be needed to make enough for 50 people? (nearest tenth)

(A)   15.8 c.   (B)   16.2 c.   (C)   16.0 c.   (D)   16.7 c.   (E)   15.3 c.

44. Let $x + y = 18$ and $xy = 11$. Calculate $x^2 + y^2$.

(A)   324   (B)   313   (C)   335   (D)   346   (E)   302

45. Given the function $f(x) = x^3 - 5x^2 - 8$, find the slope of the secant line between the points on $f(x)$ where $x = -2$ and $x = 3$

(A)   15   (B)   2   (C)   18   (D) $\dfrac{1}{2}$ (E)   3

46. Find the sum of the coefficients of the 2nd and 3rd terms in the polynomial expansion of $(2x - 3)^9$.

(A)   41,472   (B)   −6912   (C)   324   (D)   34,560   (E)   −1944

47. Let $f(x) = \dfrac{3x^4 - 5x^2 + 8}{x^3 + 8x}$ and $s(x)$ be the slant asymptote of f. Find the value of $s(-1)$.

   (A) 1   (B) 2   (C) −3   (D) −2   (E) 3

48. The weather forecast calls for an 80% chance of rain for 5 days in a row. If the forecast is accurate, what is the probability that it will rain at least 3 of the days? (nearest hundredth)

   (A) 0.74   (B) 0.26   (C) 0.94   (D) 0.51   (E) 0.49

49. Let $f(x) = ax^5 - bx^3 + cx + 9$ where $f(3) = -32$, then $f(-3) =$

   (A) 50   (B) 25   (C) 32   (D) 59   (E) 41

50. Let $f(x) = \begin{cases} nx^4 + 3x & \text{if } x < 2 \\ mx^2 - 5x & \text{if } x \geq 2 \end{cases}$ be continuous and differentiable everywhere. Find n.

   (A) 5.25   (B) 0.5   (C) 3.25   (D) 1.5   (E) 0.75

51. What is the instantaneous rate of change at $x = -1$ of the function f given by $f(x) = \dfrac{4 - x^2}{3x + 2}$.

   (A) −3   (B) −6   (C) −4   (D) −11   (E) −5

52. An investor invested \$1000 for 5 years. The chart below shows the percentage growth each year.
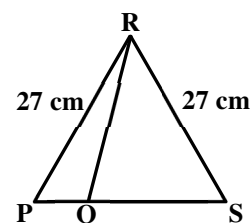   What was the average percentage growth for the 5-year period? (nearest hundredth percent)

Year	1	2	3	4	5
Growth %	3.7%	1.85%	2.31%	- 6.15%	0.75%

   (A) 2.95%   (B) 0.49%   (C) 0.43%   (D) 0.59%   (E) 2.17%

53. The graph of $f(x) = \dfrac{8x^3 - 125}{2x - 5}$ suggests that the *discontinuity* at $x = 2.5$ is *removable* by defining

   $f(2.5)$ to be_____.

   (A) 25   (B) 100   (C) 6.25   (D) 0   (E) 75

54. Given PQ = 8 cm, find m∠PRQ on the equilateral triangle shown below. (nearest degree)

   (A) 15°   (B) 12°   (C) 17°   (D) 22°   (E) 11°

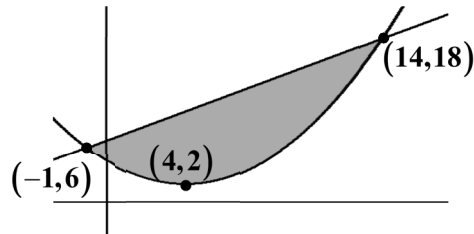55. How many ordered pairs (a, b) exist such that the four-digit number, 2a3b, is divisible 2 and 3 both?

   (A) 16   (B) 17   (C) 12   (D) 10   (E) 11

56. Which of the following words has exactly 362,880 unique permutations of its letters?

   (A) Rectangle   (B) Square   (C) Trapezoid (D) Parallelogram (E) Kite

57. Find the area of the shaded region enclosed by the graph of the line and parabola shown.

$(14,18)$

$(-1,6)$ $(4,2)$

   (A) 45   (B) 90   (C) 60   (D) 120   (E) 66

58. The infinite series $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\ldots$ **converges to _____.**

   (A) e (B) e^2 (C) 1   (D) $\ln 2$ (E) $\dfrac{2}{3}$

59. What is the digit in the tens place of 7^{4581}**.**

   (A) 0   (B) 7   (C) 6   (D) 3   (E) 4

60. If $f(x)=4\sin(3x)$**, then** $\displaystyle\lim_{h\to 0}\dfrac{f\left(\dfrac{\pi}{6}+h\right)-f\left(\dfrac{\pi}{6}-h\right)}{2h}=$

   (A) 3   (B) −6   (C) 0   (D) $-3\sqrt{3}$ (E) 6

2018-2019 TMSCA Mathematics Test One Answers

1. D	21. B	41. D
2. D	22. D	42. C
3. C	23. B	43. A
4. B	24. A	44. E
5. A	25. A	45. B
6. C	26. D	46. D
7. B	27. B	47. C
8. D	28. E	48. C
9. E	29. E	49. A
10. B	30. B	50. B
11. A	31. A	51. D
12. B	32. C	52. C
13. D	33. A	53. E
14. B	34. C	54. C
15. B	35. B	55. A
16. B	36. C	56. C
17. C	37. D	57. B
18. E	38. D	58. D
19. E	39. B	59. A
20. B	40. A	60. C

2018-2019 TMSCA Mathematics Test One Answers

2018-2019 TMSCA Mathematics Test One Solutions

2. $113383 = 361(1+r)^{140}$ for $1+r = \left(\dfrac{113383}{361}\right)^{1/140}$ for

$r \approx 4.2\%$

5. $_{8+2-1}C_2 \times 3 = 108$

10. $12(9+12) = (AB)^2$ for $AB \approx 15.9$

11. $2(32) + \left[\dfrac{3x^2}{2}\right]_{-3}^{7} = 124$

14. (sum of the roots)^3 for $\left(-\dfrac{11}{6}\right)^3 = -\dfrac{1331}{216}$

16. Binomial expansion of $(1+1)^{12}$ contains all of the numbers except 65.

22. $\dfrac{2I + P - 2}{2} = \dfrac{10 + 12 - 2}{2} = 10$ squares on the grid for the area, but each square on the grid represents an area of 9 square units for a total area of 90 square units.

25. $\dfrac{-38 - 41i}{-7 - 24i} = 2 - i$ for a sum of $a + b = 2 + (-1) = 1$

27. Even digits are 0, 2, 4, 6, and 8, but 0 cannot be the first digit, so $4 \times 4 \times 3 = 48$.

28. $V + F - E = 2$ for $30 + F - 60 = 2$ and $F = 32$ for a sum of 122.

31. $\pi \int_{-5/2}^{7/3} (f(x))^2 \, dx \approx 34686$

36. Take the anti-derivative twice for
$f(x) = \dfrac{x^4}{2} + 3x^3 - 4x^2 + Ax + B$, then substitute the values given to get the system $2A + B = 5$ and $-2A + B = -7$ for $B = -1$ and $A = 3$, then $f(4) = 267$.

39. $4 + 5 = 9$, $8 + 4 = 12$ and $5 + 7 = 12$ all of which are equally likely for an expected outcome of $\dfrac{1}{3}(9 + 12 + 12) = 11$

41. Solve: $6x^2 - 5x^2 \dfrac{dy}{dx} - 10xy = 6 \dfrac{dy}{dx}$ for $\dfrac{dy}{dx} = \dfrac{6x^2 - 10xy}{5x^2 + 6}$

42. solve $x = \dfrac{2y - 3}{7 - y}$ for $y = \dfrac{7x + 3}{2 + x}$

45. Evaluate the function at each of the x values to obtain the points $(-2, -36)$ and $(3, -26)$, then find the slope of the line containing those two points which is 2.

46. $9(2)^8(-3) + (_9C_2)(2)^7(-3)^2 = 34560$

50. The slope of each side and the function value of each side must match as they approach 2, so $32n + 3 = 4m - 5$ and $16n + 6 = 4m - 10$ then solve the system for $n = \dfrac{1}{2}$.

54. Use the law of cosines to get the length of the central segment:
$\sqrt{8^2 + 27^2 - 2(8)(27)\cos 60°} = \sqrt{577}$ then use the law of sines to find the missing angle:
$\dfrac{\sin 60°}{\sqrt{577}} = \dfrac{\sin \theta}{8}$ for $\theta \approx 17°$

55. b has to be even and the sum of the digits has to be divisible by 3, so

b	a
0	1, 4 or 7
2	2, 5 or 8
4	0, 3, 6, 9
6	1, 4 or 7
8	2, 5 or 8

for a total of 16 ordered pairs.

58. This is the Maclaurin series expansion of ln2

60. This is the definition of the derivative of the original function at $x = \dfrac{\pi}{6}$ which is 0.