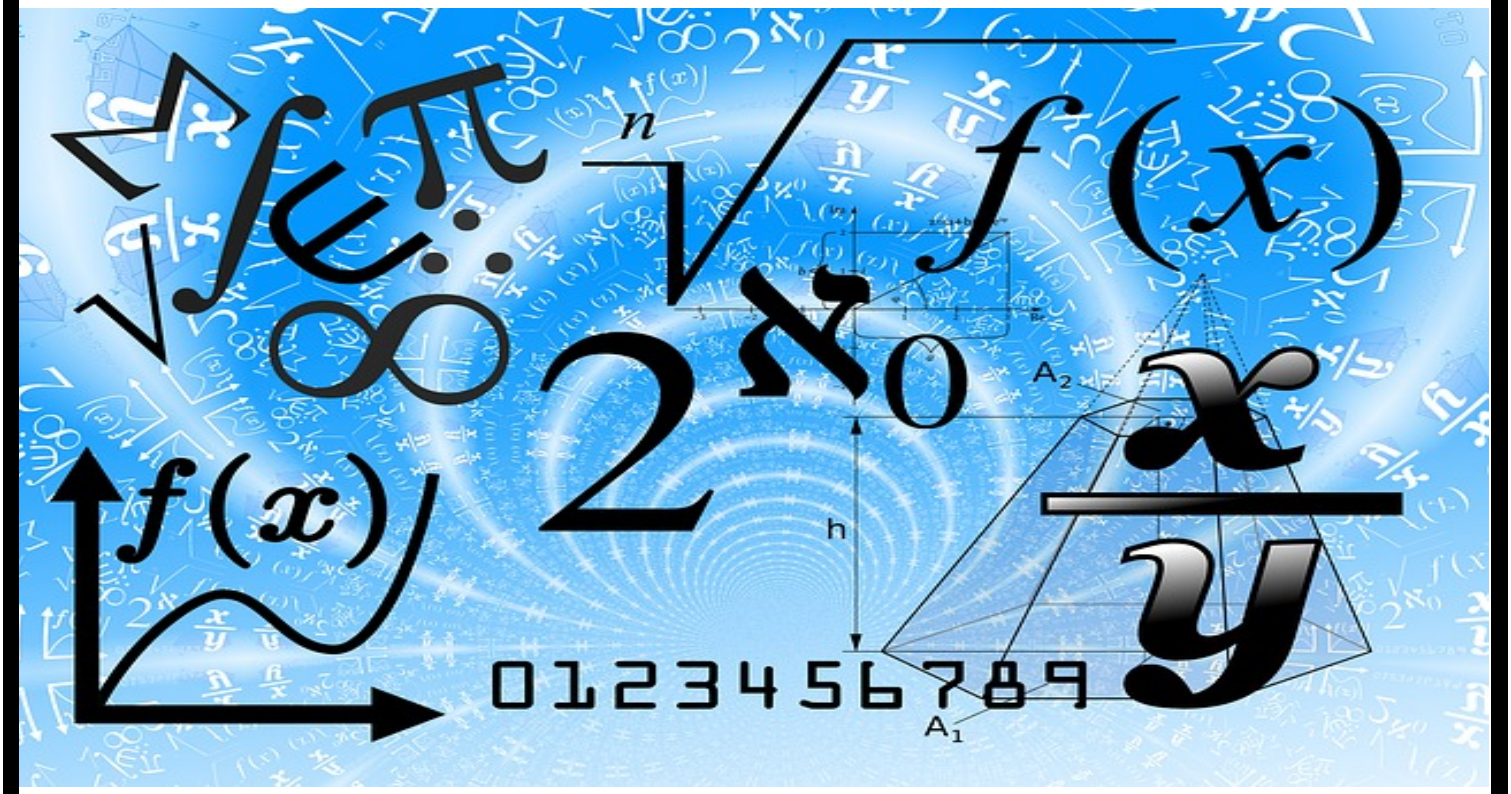
THE VIRTUAL MEET EXPERIENCE

2021 - 2022

HS VIRTUAL CHALLENGE REGIONAL QUALIFIERS' MEET



MATHEMATICS

DO NOT OPEN TEST UNTIL TOLD TO DO SO

The Virtual Challenge Meets™

1. **Solve:** $\sqrt[3]{3x+4}+8=6$

   (A) -3 **(B)** -4 **(C)** -5 **(D)** -6 **(E)** -7

2. **The drama club at Bridgeport HS charges \$3.00 for student tickets and \$7.50 for adult tickets. They sold 180 tickets for their last production and brought in \$877.50. How many adult tickets did the club sell?**

   (A) 105 **(B)** 110 **(C)** 70 **(D)** 75 **(E)** 72

3. **A department store is having a special sale where customers get 15% off the first \$20 worth of purchases, 25% off the next \$50 worth of purchases and 40% off purchases over \$70. How much will a customer's final bill be if he buys \$110 worth of merchandise?**

   (A) \$53.50 **(B)** \$66.00 **(C)** \$78.50 **(D)** \$70.50 **(E)** \$64.50

4. **It takes three workers 5 hours to plaster a wall. How long will it take 4 workers to plaster a wall that is the same length but twice as high?**

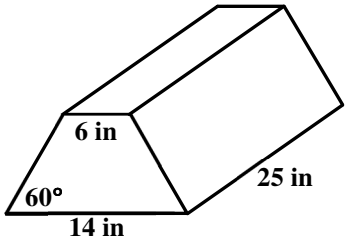
   (A) 9 hr **(B)** 7.5 hr **(C)** 8.5 hr **(D)** 3 hr **(E)** 10 hr

5. **The line segment** $\overline{AB}$ **has endpoints** $(-3,5)$ **and** $(7,1)$**. Which of the following points lies on the perpendicular bisector of** $\overline{AB}$ **?**

   (A) $(5,-6)$ **(B)** $(7,1)$ **(C)** $(-3,-5)$ **(D)** $(-4,-12)$ **(E)** $(6,-2)$

6. **The volume of the isosceles trapezoidal prism shown is ___in^3. (nearest in^3)**

   (A) 1732 **(B)** 2000 **(C)** 2598 **(D)** 1500 **(E)** 1414



7. **Simplify:** $\dfrac{x^2+10x+25}{x^2-25}\times\dfrac{x^2-10x+25}{x-5}$

   (A) x^2+25 **(B)** $x^2+10x+25$ **(C)** x^2-25 **(D)** $x-5$ **(E)** $x+5$

8. **Tasty Serve Food Truck serves vanilla or chocolate ice cream with the following optional mix-ins: graham cracker crumbs, chocolate chips, chocolate cookie crumbles, chopped pecans, peanut butter cups or strawberry pieces. How many different ways can a customer order a vanilla or chocolate cup of ice cream?**

   (A) 63 **(B)** 128 **(C)** 64 **(D)** 126 **(E)** 120

9. **Carl's bicycle tires have a diameter of 56 cm. One morning he set up a counter for the number of rotations one of his tires made on the way to school. When he arrived, the counter read 1628 revolutions. How many miles did Carl ride on his way to school? (nearest hundredth)**

   (A) 1.78 mi **(B)** 0.89 mi **(C)** 1.62 mi **(D)** 0.81 mi **(E)** 1.57 mi

10. The length of the second leg of a right triangle is 1 cm more than the length of the first leg, and the hypotenuse is 11 cm less than twice the length of the first leg. The perimeter of the triangle is ____cm.

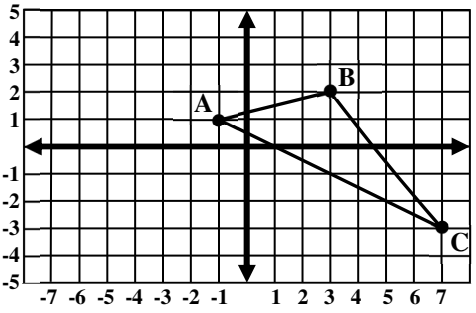
(A) 20 (B) 210 (C) 70 (D) 24 (E) 120

11. If $f(x) = 2x + 1$ and $g(x) = x^2$, then $g(f(x+2)) =$

(A) $x^2 - 25$ (B) $4x^2 + 9$ (C) $4x^2 + 20x + 25$

(D) $4x^2 + 12x + 9$ (E) $4x^2 + 25$

For problems 12, 13 and 14, use the graph to the right.



12. Find the perimeter of triangle ABC. (nearest whole number)

(A) 16 (B) 19 (C) 21 (D) 23 (E) 240

13. Find the measure of angle $\angle ACB$. (nearest degree)

(A) 22° (B) 26° (C) 28° (D) 25° (E) 32°

14. Find the area of triangle ABC. (nearest whole number)

(A) 11 (B) 12 (C) 15 (D) 17 (E) 18

15. $A = \begin{bmatrix} a & 4 \\ -5 & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} -16 & 16 \\ -20 & -16 \end{bmatrix}$. Find the value of *a*.

(A) −2 (B) 4 (C) −5 (D) 2 (E) −4

16. Consider the statement "If a quadrilateral is a square, then it is a rectangle." Which of the following is true?

I. Converse II. Inverse III. Contrapositive

(A) I, II, III (B) I, III only (C) I, II only (D) III only (E) I only

17. Soybean meal is 18% protein by weight; cornmeal is 9% protein by weight. How many pounds of cornmeal should be combined with soybean meal if produce 360 pounds of a mixture that is 17% protein?

(A) 320 (B) 48 (C) 312 (D) 40 (E) 38

18. $14_{16} + 12_8 + 10_4 =$ _____$_2$

(A) 101001 (B) 100001 (C) 100010 (D) 100101 (E) 100000

19. Find the number that is $\frac{4}{11}$ of the way from $-2\frac{3}{4}$ and 11?

(A) $1\frac{3}{4}$ **(B)** $2\frac{1}{4}$ **(C)** 2 **(D)** $2\frac{3}{4}$ **(E)** $2\frac{1}{2}$

20. Which of the following types of function is the best model for the sunrise times in a fixed location every day for a year?

(A) Logistic (B) Exponential (C) Sinusoidal (D) Quadratic (E) Logarithmic

21. Chuck deposits $18,000 in an account that earns 3.75% annual interest compounded monthly. At the same time, Larry deposits $17,200 in an account that earns 4.25% annual interest compounded quarterly. How many years are required for the balance in Larry's account to equal the balance in Chuck's account? (nearest tenth)

(A) 8.8 (B) 8.9 (C) 9.1 (D) 9.2 (E) 9.4

22. $\cot(2x)-\cot(x)$ simplifies to_____.

(A) $\csc(2x)$ **(B)** $\csc(x)$ **(C)** $-\sec^2 x$ **(D)** $-\csc(2x)$ **(E)** $\sec(2x)$

23. Find the domain of $f(x)=\frac{\sqrt{2x-10}}{4x^2-31x+21}$

(A) $x\neq 7$ **(B)** $x\geq 5$ **(C)** $x>5$ **(D)** $x\neq\frac{3}{4}, x\neq 7$ **(E)** $x\geq 5, x\neq 7$

24. You commute 56 miles one way to work. The trip to work takes 10 minutes longer than the return. Your average speed on the trip home is 8 mph faster. How long does it take you to get home?

(A) 60 min (B) 66 min (C) 48 min (D) 50 min (E) 70 min

25. Let *w* and *m* be the complex solutions to the equation $x^2-x+8=0$. If $w^2-m^2=\frac{a\sqrt{b}}{c}i$, where *a, b* and *c* are positive integers then the value of $a+b+c$ = ____.

(A) 33 (B) 34 (C) 30 (D) 38 (E) 35

26. The point $(2,-7)$ is reflected over the *x*-axis, reflected over the line $y=x$, rotated 270° clockwise around the origin, then shifted down three units to the point (a,b). $a+b=$?

(A) 2 (B) –9 (C) –12 (D) –5 (E) –7

27. At 8 AM on Monday morning, Carrie's plasma level of a pain medication was 1200 mg/L. The medication has a half-life of 6 hours. If no further doses were administered, what was the plasma level of the medication at 9:30 PM on Monday? (nearest mg/L)

(A) 300 mg/L (B) 278 mg/L (C) 267 mg/L (D) 252 mg/L (E) 259 mg/L

28. A full cylindrical water tank with a radius of 1 yard and a height of 3 yards springs a steady leak. If it is empty four hours after developing the leak, what is the rate of loss? (nearest gallon per minute)

(A) 3 gal/min (B) 2 gal/min (C) 6 gal/min (D) 8 gal/min (E) 7 gal/min

29. A baker can produce 2 sizes of scones, mini and regular. Her oven can bake at most 200 scones per day. Each regular scone requires 2 oz of flour, each mini scone requires 1 oz of flour, and there are 300 oz of flour available for scones. The income from each regular scone is \$0.13 and from each mini is \$0.06. What is the maximum income the baker can make selling scones?

(A) \$19.50 (B) \$26.00 (C) \$22.50 (D) \$14.25 (E) \$17.50

30. How many distinct arrangements can be made with the letters "SPRINGTIME"?

(A) 302,400 (B) 3,628,800 (C) 1,209,600 (D) 1,814,400 (E) 403,200

31. The period of $h(x)=\frac{1}{k}\cos\left(4\pi kx\right)$ is 3. Find the amplitude of $h(x)$.

(A) 12 (B) 6 (C) 4 (D) 3 (E) 1

32. How many points of intersection occur when $r=3\sin\theta-1$ and $r=3$, $[0,2\pi]$, are graphed on the polar coordinate system?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

33. Find the angle between the vectors $u=\langle 3,-5,1\rangle$ and $v=\langle -2,1,3\rangle$. (nearest tenth)

(A) 110.2° (B) 110.9° (C) 111.0° (D) 111.2° (E) 111.7°

34. Convert the rectangular equation to polar form: $3x-7y-8=0$.

(A) $r=8\csc\theta$ **(B)** $r=\frac{8}{3\sin\theta-1\cos\theta}$ **(C)** $r=8\sec\theta$

(D) $r=\frac{8}{3\cos\theta+7\sin\theta}$ **(E)** $r=\frac{8}{3\cos\theta-7\sin\theta}$

35. Find the distance between the plane $4x+2y+z=4$ and the point $(1,3,-2)$. (nearest tenth)

(A) 1.1 (B) 0.9 (C) 2.6 (D) 1.9 (E) 1.6

36. Find the sum of all three-digit numbers whose digits have a sum of eight and whose digits can all be arranged to form a perfect cube.

(A) 1420 (B) 915 (C) 861 (D) 1925 (E) 1776

37. Express $\log\left(10\sqrt{ab}\right)$ in terms of P and Q if $P=\log a$ and $Q=\log b$.

(A) $\frac{1}{2}(P+Q)+1$ **(B)** $\frac{PQ}{2}$ **(C)** $5P+\frac{1}{2}Q$ **(D)** $10P+\frac{1}{2}Q$ **(E)** $\frac{1}{2}(P+Q)+10$

38. Find the sum of the solutions in the interval $[0,2\pi]$ for $\cos^2\theta-\sin^2\theta=\cos\theta$.

(A) 2π **(B)** $\frac{10\pi}{3}$ **(C)** $\frac{14\pi}{3}$ **(D)** 4π **(E)** 3π

39. If $f(x)=x^{\sin x}$, find $f'(x)$.

(A) $x^{\sin x}\left(\frac{\sin x}{x}+\ln x\cos x\right)$ **(B)** $\left(\frac{\sin x}{x}+\ln x\cos x\right)$ **(C)** $x^{\sin x}\left(\frac{\sin x+\cos x}{x}\right)$

(D) $x^{\sin x}\left(\frac{\sin x}{x}+\cos x\right)$ **(E)** $\left(\frac{\sin x+\ln x\cos x}{x}\right)$

40. Consider the function $g(x)$, which is continuous and differentiable on $[2,6]$, and with $g(2)=4$ and $g(4)=4$. If $g''(x)$ is continuous and positive on $[2,6]$, then which of the following must be true?

(A) $g'(3)=0$ **(B)** $g'(3)>0$ **(C)** $g'(3)<0$ **(D)** $g'(5)<0$ **(E)** $g'(5)>0$

41. Use four rectangles of equal width to approximate the area in the first quadrant bounded by the curves $y_1=\frac{2}{x}$, $x=1$ and $x=2$. Use right endpoints of each interval to find a lower sum. (nearest hundredth)

(A) 1.25 **(B)** 1.15 **(C)** 1.31 **(D)** 1.27 **(E)** 1.22

42. Consider the function $f(x)=x^3-3x+5$. Which value(s) satisfy the Mean Value Theorem for Derivatives in the interval $[-2,2]$?

(A) $\frac{2\sqrt{3}}{3}$ only **(B)** $-\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$ **(C)** $-\frac{\sqrt{3}}{2}$ only **(D)** $-\frac{2\sqrt{3}}{3}$ only **(E)** $-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}$

43. Find all of the values of *K* such that $f(x)=Kx^2-4x+5-K$ has two distinct real roots.

(A) $(-6,2)$ **(B)** $\left(-\infty,\frac{4}{5}\right)$ **(C)** $(-\infty,-6)\cup(2,\infty)$ **(D)** $(-\infty,1)\cup(4,\infty)$ **(E)** $(1,4)$

44. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ **is a definition of what constant:**

(A) π **(B)** φ **(C)** e **(D)** σ **(E)** λ

45. Find the area of the region bounded by the curves $y_1=x+8$ and $y_2=12+x-x^2$.

(A) $10\frac{2}{3}$ **(B)** $9\frac{1}{3}$ **(C)** 10 **(D)** $10\frac{1}{3}$ **(E)** 11

46. Find the volume of the solid generated by revolving the region bounded by the graphs of $x = y^2$ and $x = 4$ about the line $x = 6$.

(A) $\frac{384\pi}{15}$ **(B)** $\frac{354\pi}{5}$ **(C)** $\frac{384\pi}{5}$ **(D)** $\frac{354\pi}{2}$ **(E)** 128π

47. If $\frac{A}{x+3}+\frac{B}{x+2}=\frac{2x+36}{x^2+x-6}$**, then** $A+B=?$

(A) 10 **(B)** 2 **(C)** 8 **(D)** –2 **(E)** 6

48. Find the interval of convergence of $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$

(A) $\left[-\frac{1}{3},0\right)$ **(B)** $(-\infty,\infty)$ **(C)** $(-1,1)$ **(D)** $\left(-\frac{1}{3},\frac{1}{3}\right]$ **(E)** $(1,\infty)$

49. The top five 2021 finishers in the NCAA DI Women's 10,000 M in Eugene, OR had times of 32:16.13, 32:22.11, 32:34.05, 32:34.16 and 32:35.59. What is the positive difference between the mean and median times? (nearest hundredth)

(A) 0.08 **(B)** 0.09 **(C)** 0.10 **(D)** 0.11 **(E)** 0.13

50. Evaluate $\int_{-1}^{8}\left(3+4g'(x)\right)dx$ **using the table below.**

x	-1	0	2	5	7	8
$g(x)$	17	12	14	47	89	116

(A) 2 **(B)** 23 **(C)** 293 **(D)** 423 **(E)** 475

51. What is the area of a regular hexagon in terms of the length, *s*, of one side?

(A) $\frac{3s^2\sqrt{3}}{2}$ **(B)** $\frac{3s^2\sqrt{3}}{3}$ **(C)** $\frac{s^2\sqrt{3}}{4}$ **(D)** $\frac{s^2\sqrt{3}}{6}$ **(E)** $\frac{3s^2\sqrt{3}}{4}$

52. Meredith has bins containing 9 flavors of candies. In how many ways can she package 5 to sell if she can repeat flavors?

(A) 3003 **(B)** 715 **(C)** 2002 **(D)** 1001 **(E)** 1287

53. A statistics class at a large school with 3500 students took an SRS of 150 students and determined that 120 of the students were taking a math class while 30 were not. Construct a 95% confidence interval for p, the proportion of the students at the school taking a math class. (nearest ten-thousandth)

(A) (0.718,0.882) **(B)** (0.732,0.868) **(C)** (0.702,0.890) **(D)** (0.736, 0.864) **(E)** (0.720,0.880)

54. The faces of a fair tetrahedral die are labeled with the numbers 1, 2, 3 and 4. If the die is rolled five times, find the probability that at least one roll was a 2. (nearest thousandth)

(A) 0.999 (B) 0.598 (C) 0.763 (D) 0.673 (E) 0.727

55. If $y^2 = 5 - 12i$ and $y^3 = -9 - 46i$, where $y = a + bi$ then $a + b =$ _____.

(A) 6 (B) 1 (C) –62 (D) 5 (E) –68

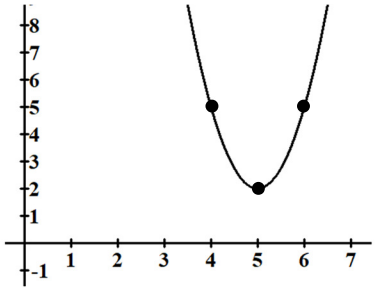
56. Which of the following mathematicians introduced $f(x)$ notation for functions, the use of the e symbol for natural logs and the use of i as the symbol for $\sqrt{-1}$.

(A) Newton (B) Leibniz (C) Napier (D) Euler (E) Descarte

57. A linear regression was performed using the following data points:
A(1,11), B(3,31), C(5,59), D(7,95) and E(9,139).
The absolute value of the residual for point D is _____. (nearest tenth)

(A) 2.8 (B) 3.1 (C) 3.7 (D) 4.0 (E) 4.9

58. The graph of $f'(x)$ is shown on the right. The points marked have integer coordinates. If $f(1) = 82$, then $f(6) =$ ____.



(A) 138 (B) 166 (C) 154

(D) 135 (E) 157

59. Simplify to the nearest ten-thousandth place: $1 + 1.5 + \frac{1.5^2}{2!} + \frac{1.5^3}{3!} + \frac{1.5^4}{4!} + \ldots$

(A) 4.4005 (B) 4.0707 (C) 4.9975 (D) 4.9975 (E) 4.4817

60. A company produces bags of flour labeled 10 pounds. In reality, the mean weight of the bags is 10.1 pounds with a standard deviation of 0.15 pounds. If two bags are selected at random, what is the probability that both will weigh more than 10.2 pounds? (nearest thousandth)

(A) 0.064 (B) 0.008 (C) 0.252 (D) 0.249 (E) 0.062

2021-2022
Virtual Challenge Regional Qualifiers' Meet
Mathematics – Student Answer Sheet

Contestant Name_____Grade_____

Score 1:_____	Score 2: _____
Score 3:_____	**Final Score:** _____

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