THE HARVARD-MIT MATHEMATICS TOURNAMENT

GEOMETRY

This test consists of 10 short-answer problems to be solved individually in 50 minutes. Problems will be weighted with point values after the contest based on how many competitors solve each problem. There is no penalty for guessing.

No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted. Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.

Answers should be simplified as much as is reasonably possible and must be exact unless otherwise specified. Rational numbers should be written in lowest terms, although denominators of irrationals need not be rationalized. An nth root should be simplified so that the radicand is not divisible by the nth power of any prime.

Correct mathematical notation must be used. No partial credit will be given unless otherwise specified.

If you believe the test contains an error, please submit your protest in writing to the Science Center Lobby during lunchtime.

Enjoy!

HMMT 2014

Saturday 22 February 2014

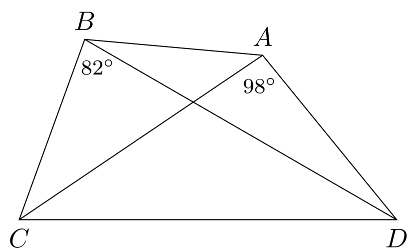
Geometry

1. Let O_1 and O_2 be concentric circles with radii 4 and 6, respectively. A chord AB is drawn in O_1 with length 2. Extend AB to intersect O_2 in points C and D. Find CD.

2. Point P and line ℓ are such that the distance from P to ℓ is 12. Given that T is a point on ℓ such that $PT = 13$, find the radius of the circle passing through P and tangent to ℓ at T.

3. ABC is a triangle such that $BC = 10$, $CA = 12$. Let M be the midpoint of side AC. Given that BM is parallel to the external bisector of $\angle A$, find area of triangle ABC. (Lines AB and AC form two angles, one of which is $\angle BAC$. The *external bisector* of $\angle A$ is the line that bisects the other angle.)

4. In quadrilateral $ABCD$, $\angle DAC = 98°, \angle DBC = 82°, \angle BCD = 70°$, and $BC = AD$. Find $\angle ACD$.



5. Let $\mathcal{C}$ be a circle in the xy plane with radius 1 and center $(0, 0, 0)$, and let P be a point in space with coordinates $(3, 4, 8)$. Find the largest possible radius of a sphere that is contained entirely in the slanted cone with base $\mathcal{C}$ and vertex P.

6. In quadrilateral $ABCD$, we have $AB = 5, BC = 6, CD = 5, DA = 4$, and $\angle ABC = 90°$. Let AC and BD meet at E. Compute $\frac{BE}{ED}$.

7. Triangle ABC has sides $AB = 14, BC = 13$, and $CA = 15$. It is inscribed in circle Γ, which has center O. Let M be the midpoint of AB, let B' be the point on Γ diametrically opposite B, and let X be the intersection of AO and MB'. Find the length of AX.

8. Let ABC be a triangle with sides $AB = 6$, $BC = 10$, and $CA = 8$. Let M and N be the midpoints of BA and BC, respectively. Choose the point Y on ray CM so that the circumcircle of triangle AMY is tangent to AN. Find the area of triangle NAY.

9. Two circles are said to be *orthogonal* if they intersect in two points, and their tangents at either point of intersection are perpendicular. Two circles ω_1 and ω_2 with radii 10 and 13, respectively, are externally tangent at point P. Another circle ω_3 with radius $2\sqrt{2}$ passes through P and is orthogonal to both ω_1 and ω_2. A fourth circle ω_4, orthogonal to ω_3, is externally tangent to ω_1 and ω_2. Compute the radius of ω_4.

10. Let ABC be a triangle with $AB = 13, BC = 14$, and $CA = 15$. Let Γ be the circumcircle of ABC, let O be its circumcenter, and let M be the midpoint of minor arc $\widehat{BC}$. Circle ω_1 is internally tangent to Γ at A, and circle ω_2, centered at M, is externally tangent to ω_1 at a point T. Ray AT meets segment BC at point S, such that $BS - CS = 4/15$. Find the radius of ω_2.

HMMT 2014

Saturday 22 February 2014

Geometry

PUT LABEL HERE

Name ____ Team ID# ________

Organization __________________________ Team ____________________

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Score: ____________