T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 5 ©

N O VE M B E R 1 7 , 2 0 1 8

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2018 – 2019 TMSCA Mathematics Test Five

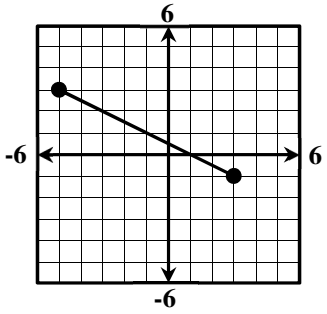
1. **Evaluate:** $\sqrt[4]{65536} \div (1296)^{3/4} + 9 \times 3^{-1}$.

   (A) $3\frac{2}{27}$ **(B)** 1 **(C)** $27\frac{2}{27}$ **(D)** 3459 **(E)** $\frac{2}{11}$

2. **The city of Denton, TX grew from a population of 4,732 in 1910 to 117,052 in 2010. What was the average annual growth rate for the population of Denton during that 100-year period? (nearest tenth of a percent)**

   (A) 2.2% **(B)** 3.8% **(C)** 4.8% **(D)** 4.2% **(E)** 3.3%

3. **What is the *y* – intercept of the perpendicular bisector of the line segment shown?**



   (A) $\left(0,\frac{1}{2}\right)$ **(B)** $(0,3)$ **(C)** $(1,0)$ **(D)** $\left(0,-\frac{3}{2}\right)$ **(E)** $(3,0)$

4. **Write this expression as a simplified improper fraction:** $1+\cfrac{1}{3+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{15}}}}$.

   (A) $\frac{921}{2914}$ **(B)** $\frac{65}{49}$ **(C)** $\frac{3835}{2914}$ **(D)** $\frac{85}{64}$ **(E)** $\frac{21}{64}$

5. **Coach Cleve has 9 girls on his tennis team, 3 of whom are seniors. He can take 4 of them to a special invitational training clinic. How many distinct groups of 4 could he form if he wants to take at least one senior?**

   (A) 126 **(B)** 111 **(C)** 14 **(D)** 180 **(E)** 60

6. **Solve:** $\frac{x-3}{x-9} = \frac{6}{x-9}$

   (A) 9 **(B)** –9 **(C)** 6 **(D)** No solution **(E)** infinitely many solutions

7. **Let** $Z = \{z,e,u,s\}$, $J = \{j,u,p,i,t,e,r\}$ **and** $M = \{m,i,n,e,r,v,a\}$. **How many elements are in** $(Z \cup M) \cap J$ **?**

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

8. **If A is** $66\frac{2}{3}\%$ **of B, and C is** $116\frac{2}{3}\%$ **of B, then A = _____% of C?**

   (A) $71\frac{3}{7}$ **(B)** $28\frac{4}{7}$ **(C)** $77\frac{7}{9}$ **(D)** $55\frac{5}{9}$ **(E)** $57\frac{1}{7}$

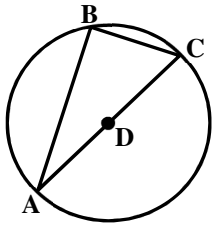
9. $\angle A$ **and** $\angle B$ **are complementary, and** $m\angle B$ **is 22° less than three times** $m\angle A$. **Find the measure of the supplementary angle to** $\angle A$ **.**

   (A) 148° **(B)** 62° **(C)** 152° **(D)** 116° **(E)** 114°

10. A large conical storage tank has an 8-foot base diameter and maximum 1000-gallon capacity. What is the vertex angle of the tank? (nearest degree)

(A) 27° **(B)** 53° **(C)** 60° **(D)** 31° **(E)** 62°

11. The circle with center D shown has an area of 25π cm^2 and BC = CD. The area of triangle ABC = _____cm^2.



(A) 25 **(B)** $\frac{25\sqrt{2}}{2}$ **(C)** 50 **(D)** $\frac{25}{2}$ **(E)** $\frac{25\sqrt{3}}{2}$

12. Two events, A and B are not independent and have probabilities such that $p(A)=0.2$, $p(A'\cap B)=0.22$ and $p(A\cap B)=0.18$. Find $p(A|B)$.

(A) $\frac{1}{11}$ **(B)** $\frac{9}{11}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{10}{21}$ **(E)** $\frac{9}{20}$

13. If $\int_2^k \frac{1}{x+8}dx = \ln 2$, find the value of k.

(A) 0 **(B)** 12 **(C)** –4 **(D)** 8 **(E)** 4

14. Simplify $\left(\sqrt[3]{2a^3b^2}\right)\left(\sqrt[6]{32a^3b^2}\right)$.

(A) $2ab\sqrt[3]{2a^3}$ **(B)** $2ab\sqrt[3]{2b^3}$ **(C)** $2ab\sqrt[6]{2a^3}$ **(D)** $2ab\sqrt[6]{2b^3}$ **(E)** $2ab\sqrt[6]{2a^3b^3}$

15. If p and q are the roots of the function $f(x)=6x^2+x-35$, then $p^3+3p^2q+3pq^2+q^3$=?

(A) $-\frac{1}{216}$ **(B)** $\frac{1}{36}$ **(C)** $\frac{1}{1728}$ **(D)** $\frac{1}{216}$ **(E)** $-\frac{1}{1728}$

16. The relation $x^2+y^2-14x+10y=-66$ is a circle. Find the area of the circle.

(A) 8π **(B)** 16π **(C)** 4π **(D)** 64π **(E)** 32π

17. Find K if the triangular pattern continues:

1
3 1
9 6 1
27 27 9 1
81 108 54 12 1
243 405 K 90 15 1

(A) 108 **(B)** 27 **(C)** 81 **(D)** 25 **(E)** 270

18. Who was the first known Greek mathematician who realized fractions as numbers; thus he allowed positive rational numbers for coefficients and solutions?

(A) Archimedes (B) Euclid (C) Diophantus (D) Hypatia (E) Agnesi

19. The incenter of a circle can be found by constructing the _____ of the triangle and finding the point of concurrency.

(A) Medians (B) Altitudes (C) Perpendicular Bisectors

(D) Angle Bisectors (E) Sides

20. The Real value solution set for $\left|\frac{1}{3}+4x\right| \geq 36$ is:

(A) $\left(-\infty,\frac{107}{12}\right]\cup\left[\frac{109}{12},\infty\right)$ **(B)** $\left(-\infty,-\frac{107}{12}\right]\cup\left[\frac{109}{12},\infty\right)$ **(C)** $\left[-\frac{109}{12},\frac{107}{12}\right]$

(D) $\left[-\frac{107}{12},\frac{109}{12}\right]$ **(E)** $\left(-\infty,-\frac{109}{12}\right]\cup\left[\frac{107}{12},\infty\right)$

21. How many positive proper fractions in lowest terms have a denominator of 78?

(A) 22 (B) 24 (C) 25 (D) 26 (E) 20

22. Angela tossed a fair nickel until she got 2 heads. What is the probability that she first got the second head on the 8th toss?

(A) $\frac{1}{256}$ **(B)** $\frac{3}{128}$ **(C)** $\frac{3}{64}$ **(D)** $\frac{7}{256}$ **(E)** $\frac{7}{64}$

23. $111_2+333_4+777_8=$ _____$_{10}$.

(A) 1221 (B) 1887 (C) 629 (D) 407 (E) 581

24. According to Descartes' rule of signs, $f(x)=7x^4-9x^2+20x-25$ has how many possible negative real roots?

(A) 2 or 0 (B) 0 (C) 1 (D) 1 or 3 (E) 4, 2 or 0

25. Given that $z^5=-1216-1312i$ and $z^4=-112-384i$, where $z=a+bi$ and $a,b\in\mathbb{Z}$, find the value of $a+b$.

(A) 1 (B) –1 (C) 2 (D) –2 (E) 0

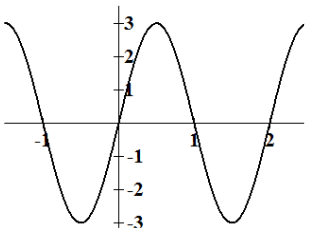
26. In how many distinct ways can Mr. Thompson's 8-member math team sit around a circular table if Alex and Joseph must sit next to each other?

(A) 1440 (B) 2880 (C) 5040 (D) 10080 (E) 720

27. The area of a sector of a circle with a central angle $\frac{3\pi}{8}$ in a circle with a diameter of 44 cm is _____cm^2. (nearest square centimeter)

(A) 1140 (B) 285 (C) 2281 (D) 570 (E) 71

28. If the equation of the function shown is $y = a\sin(bx)+c$, which of the following could be a value of b?



(A) 2π (B) $\frac{2}{\pi}$ (C) $\frac{1}{\pi}$ (D) π (E) $\frac{\pi}{2}$

29. The school day at Houston Elementary School begins at 8:35 am and ends at 3:45 pm. How many degrees has the minute hand on a clock travelled during the school day?

(A) 2550° (B) 2430° (C) 2310° (D) 2640° (E) 2580°

30. Two standard dice are rolled and the values on the top faces are added. What is the expected value of the sum?

(A) 7 (B) 6.5 (C) 7.5 (D) 7.25 (E) 6.75

31. $\frac{3x+5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$, where A and B are integers. Find the value of A.

(A) 2 (B) –2 (C) –1 (D) 1 (E) –4

32. What is the tenth harmonic number?

(A) $\frac{7129}{2520}$ (B) $\frac{4861}{2520}$ (C) $\frac{7381}{2520}$ (D) $\frac{4609}{2520}$ (E) $\frac{2341}{2520}$

33. If A represents a digit 0 – 9 in the equation $6A5_8 + 3A2_4 = 111011011_2$, find the value of A.

(A) 1 (B) 0 (C) 3 (D) 2 (E) cannot be determined

34. Paul has a 12-inch by 15-inch piece of cardstock that he would like to form into an open-top box in the shape of a rectangular prism by cutting a square out of each corner and folding up the sides. What is the largest possible volume box he can form? (nearest cubic inch)

(A) 176 in^3 (B) 162 in^3 (C) 130 in^3 (D) 400 in^3 (E) 177 in^3

35. Given the sequence $9,-19,-51,-69,-55,9,141,\ldots$, find the 20th term.

(A) 16,235 (B) 13,581 (C) 19,209 (D) –1207 (E) –1311

36. A function, $f(x)$, exists such that $f''(x) = 6x^2 + 18x - 8$, $f(-2) = 6$ and $f(2) = 10$. Find $f(4)$.

(A) –68 (B) 208 (C) 228 (D) 285 (E) 251

37. If $5^{x}\cdot 25^{2y}=1$ and $3^{5x}\cdot 9^{y}=\frac{1}{9}$, then $x+y$ equals____.

(A) $-\frac{4}{9}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{3}$ **(D)** $-\frac{1}{3}$ **(E)** $-\frac{1}{9}$

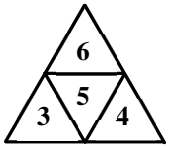
38. $\sum_{k=0}^{3}\left[kx+(k-1)y-k\right]=?\ =?$

(A) $6x+4y-6$ **(B)** $6x+2y-6$ **(C)** $6x+2y+6$ **(D)** $6x+3y-6$ **(E)** $6x+3y+6$

39. When Cora received her paycheck, she immediately paid 1/4 of it for rent and \$120 to her phone company. The next day, she spent 30% of what was left for her car payment. Finally, she put half of the remaining money in savings and was left with \$420. How much was she paid?

(A) \$1710 **(B)** \$3893 **(C)** \$2190 **(D)** \$2025 **(E)** \$1760

40. Morgan folds the net shown into a fair tetrahedral die. She rolls her tetrahedral die and adds the three visible sides. What is the expected value for the sum?



(A) 18 **(B)** 14.5 **(C)** 13.5 **(D)** 14 **(E)** 12.5

41. Let $5x^{2}y+8y^{2}=48$. Find the slope of the line normal to the graph of the relation at the point $(-2,-4)$.

(A) $-\frac{10}{3}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{20}{11}$ **(D)** $-\frac{11}{20}$ **(E)** $-\frac{4}{7}$

42. Given $f(x)=2x-3$ and $g(x)=4-2x$ find $(f/g)^{-1}(x)$.

(A) $\frac{3+4x}{2x-2}$ **(B)** $\frac{3+4x}{2x+2}$ **(C)** $\frac{3-4x}{2x-2}$ **(D)** $\frac{3-4x}{2x+2}$ **(E)** $\frac{7x}{2x+2}$

43. Working together, Aaron and Brandon can paint and trim a wall in half an hour. If Aaron works three times faster than Brandon, how long would it take Aaron to do the job alone?

(A) 40 min **(B)** 42 min **(C)** 45 min **(D)** 120 min **(E)** 135 min

44. Let $x+y=18$ and $xy=11$. Calculate $x^{3}+y^{3}$.

(A) 5238 **(B)** 5634 **(C)** 5436 **(D)** 6030 **(E)** 6228

45. A regular octagon has vertices A, B, C, D, E, F, G and H respectively. What is $m\angle AFD$?

(A) 60° **(B)** 45° **(C)** 57.5° **(D)** 67.5° **(E)** 112.5°

46. A set of positive integers has a mean of 36, a median of 30, a mode of 56 and a range of 38. If A, B, C, D and E are the integers arranged from least to greatest, what is the value of B?

(A) 18 (B) 24 (C) 20 (D) 30 (E) 22

47. Given the function $f(x)=5x^3-12x^2+4x-2$, find the slope of the secant line between the points on $f(x)$ where $x=-2$ and $x=3$

(A) 9 (B) 45 (C) 67 (D) 26 (E) 27

48. Find the sum of the coefficients of the 4th and 7th terms in the polynomial expansion of $(3x-5)^{10}$.

(A) 254,430 (B) 286,132,500 (C) 298,586,250 (D) 232,976,250 (E) 59,717,250

49. Let $f(x)=\dfrac{3x^4+5x^2-8x+3}{x^3+9x^2+2}$ and $s(x)$ be the slant asymptote of *f*. Find the value of $s(-5)$.

(A) −12 (B) −42 (C) 12 (D) 15 (E) −27

50. Let $f(x)=ax^4-bx^2+5x+6$ where, then $f(-9)=48$, find $f(9)$.

(A) 48 (B) 36 (C) 132 (D) −132 (E) 138

51. If $\begin{bmatrix}3 & 5\\ a & 6\end{bmatrix}\times\begin{bmatrix}3 & b\\ 1 & -8\end{bmatrix}=\begin{bmatrix}14 & -61\\ 0 & -34\end{bmatrix}$, find the value of $a+b$.

(A) 12 (B) −6 (C) −9 (D) 14 (E) −5

52. Let $f(x)=\begin{cases}mx^4+6x, & x\le 2\\ nx^2-4x & x>2\end{cases}$ be continuous and differentiable everywhere. Find *n*.

(A) $\dfrac{5}{8}$ (B) $-\dfrac{8}{5}$ (C) $\dfrac{15}{2}$ (D) $\dfrac{5}{4}$ (E) $-\dfrac{15}{16}$

53. If *f* is continuous on the closed interval $[a,b]$ and *k* is any number between $f(a)$ and $f(b)$, then there is at least one number *c* in $[a,b]$ such that $f(c)=k$. This is the _____.

(A) Rolle's Theorem (B) Sandwich Theorem (C) Fundamental Theorem of Calculus

(D) Intermediate Value Theorem (E) Fundamental Theorem of Algebra

54. A circle is inscribed in an equilateral triangle with perimeter 18 inches. The area of the circle is _____in^2.

(A) 9π (B) 6π (C) 12π (D) 3π (E) 12

55. The repeating decimal $0.363636..._8$ is equal to what reduced fraction in base 8?

(A) $\frac{10}{21_8}$ **(B)** $\frac{17}{34_8}$ **(C)** $\frac{12}{25_8}$ **(D)** $\frac{30}{56_8}$ **(E)** $\frac{36}{70_8}$

56. A metallurgist has an alloy with 5% titanium and an alloy with 30% titanium by mass. He needs 100 grams of an alloy with 15% titanium. How much of the 5% alloy should he use to obtain the new 100-gram alloy?

(A) 30 g **(B)** 40 g **(C)** 50 g **(D)** 60 g **(E)** 70 g

57. The graph of $f(x)=\frac{x^2+3x-10}{x^2+9x+20}$ suggests that the *discontinuity* at $x=-5$ is *removable* by defining $f(-5)$ to be_____.

(A) –7 **(B)** 7 **(C)** –1 **(D)** 1 **(E)** 5

58. What is the shortest distance between the line $4x-3y=18$ and the point? $(-6,5)$?

(A) 4.2 **(B)** 5.6 **(C)** 7.4 **(D)** 9.2 **(E)** 11.4

59. How many ordered pairs (a,b) exist such that the four-digit number, 5a7b, is divisible 2 and 3 both?

(A) 16 **(B)** 20 **(C)** 12 **(D)** 17 **(E)** 11

60. If $f(x)=3\cos^2(2\theta)$, then $\lim_{h\to 0}\frac{f\left(\frac{\pi}{6}+h\right)-f\left(\frac{\pi}{6}-h\right)}{2h}=$

(A) 3 **(B)** –6 **(C)** 0 **(D)** $-3\sqrt{3}$ **(E)** 6

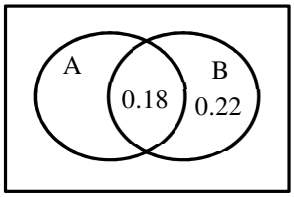
2018-2019 TMSCA Mathematics Test Five Answers

1. A	**21. B**	**41. D**
2. E	**22. D**	**42. B**
3. B	**23. E**	**43. A**
4. C	**24. C**	**44. A**
5. B	**25. C**	**45. D**
6. D	**26. A**	**46. C**
7. C	**27. B**	**47. E**
8. E	**28. D**	**48. D**
9. C	**29. E**	**49. B**
10. B	**30. A**	**50. E**
11. E	**31. D**	**51. C**
12. E	**32. C**	**52. C**
13. B	**33. C**	**53. D**
14. C	**34. E**	**54. D**
15. A	**35. A**	**55. C**
16. A	**36. C**	**56. D**
17. E	**37. D**	**57. B**
18. C	**38. B**	**58. E**
19. D	**39. E**	**59. D**
20. E	**40. C**	**60. D**

2018-2019 TMSCA Mathematics Test Five Solutions

11. Any triangle inscribed in a semicircle is a right triangle with the right angle across from the diameter. Also, since the short leg is ½ the length of the hypotenuse, this is 30-60-90 special triangle. So, $r = 5 = BC$ **and** $AB = 5\sqrt{3}$ **for a triangle are of** $\frac{1}{2}(5)\left(5\sqrt{3}\right) = \frac{25\sqrt{3}}{2}$.

12. For conditional events, either use the formula: $p(A|B) = \frac{p(A \text{ and } B)}{p(B)} = \frac{0.18}{0.40} = \frac{9}{20}$ **or use a Venn diagram to visualize the situation:**



13. $\left[\ln(x+8)\right]_2^k = \ln(k+8) - \ln 10 = \ln 2$ **for** $\frac{k+8}{10} = 2$ **and** $k = 12$.

14. $\left(2^{1/3}a^{3/3}b^{2/3}\right)\left(2^{5/6}a^{3/6}b^{2/6}\right) = 2^{7/6}a^{9/6}b^{6/6} = 2ab\sqrt[6]{2a^3}$

15. The expression is the polynomial expansion of $(p+q)^3 = \left(-\frac{1}{6}\right)^3 = -\frac{1}{216}$

17. Each row is the coefficients of the polynomial expansion of $(3x+1)^n$ **where *n* is the row number beginning with 0 for the top row, so the third term of the expansion of** $(3x+1)^5$ **is** ${}_5C_3(3)^3(1)^2 = 270$

22. There are seven different possibilities:
HTTTTTTH, THTTTTTH, TTHTTTTH, TTTHTTTH, TTTTHTTH, TTTTTHTH, TTTTTTHH
for each toss of a fair coin, $p(H) = p(T) = \frac{1}{2}$**, so the probability of each of the seven is** $\left(\frac{1}{2}\right)^8 = \frac{1}{216}$ **and a total probability of** $\frac{7}{216}$

26. Treat Alex and Joseph as one item when seating the students for $\frac{7!}{7} = 720$ **possible arrangements at a circular table, then multiply by two to adjust for the different orders that Alex and Joseph can sit in and still be next to each other for 1440 possible arrangements.**

27. The area of a sector when the central angle measure is given in radians is $A = \frac{1}{2}r^2\theta = \frac{1}{2}\left(22^2\right)\left(\frac{3\pi}{8}\right)$ **or about 285 cm².**

31. Multiply both sides by the least common multiple of the denominators for $3x+5 = A(x-1) + B(x+3)$ **, then let** $x = -3$ **for** $-9+5 = -4A$ **and** $A = 1$.

34. Let the side length of the square corner cutouts be *x*, then find the maximum of the function $V = (12-2x)(15-2x)x$ **on the domain** $[0,6]$ **because the cutout side can not be more than ½ the shortest side. For a volume of 177 in³.**

36. Find $f'(x) = 2x^3 + 9x^2 - 8x + A$ **and** $f(x) = \frac{1}{2}x^4 + 3x^3 - 4x^2 + Ax + B$ **where *A* and *B* are unknown constants. Then use the known function values to find** $A = -11$, $B = 16$ **and** $f(4) = 228$.

41. Use the product rule and implicit differentiation to get $5x^2\frac{dy}{dx} + y(10x) + 16y\frac{dy}{dx} = 0$ **then** $5(-2)^2\frac{dy}{dx} + (-4)10(-2) + 16(-4)\frac{dy}{dx} = 0$ **and** $\frac{dy}{dx} = \frac{20}{11}$ **and a slope for the normal line of** $-\frac{11}{20}$.

43. Let A be the time it would take Aaron on his own and B be the time it would take Brandon on his own. Solve the equation $\left(\frac{1}{B} + \frac{3}{B}\right)(30) = 1$ **for Brandon's time 120 minutes and Aaron's time 40 minutes.**

44.
$x^3 + y^3 = (x+y)\left[(x+y)^2 - 3xy\right] = (18)\left(18^2 - 3\cdot 11\right) = 5238$

Think of the octagon as if it was inscribed in a circle. The intercepted arc of the angle is 135°, so the measure of the angle is $135° \div 2 = 67.5°$. **52. If the function is continuous and differentiable at every point, then the function values and derivative values at 2 for each piece of the function must be the same, so $4mx + 6 = 2nx - 4$ and $mx^4 + 6x = nx^2 - 4x$ when $x = 2$ and $n = \frac{15}{2}$.** **57. $f(x) = \frac{(x+5)(x-2)}{(x+4)(x+5)} \approx \frac{x-2}{x+4}$ and if there wasn't a hole in the graph, the function value would be $\frac{-5-2}{-5+4} = 7$**	