



TMSCA HIGH SCHOOL MATHEMATICS TEST # 1 © OCTOBER 21, 2017

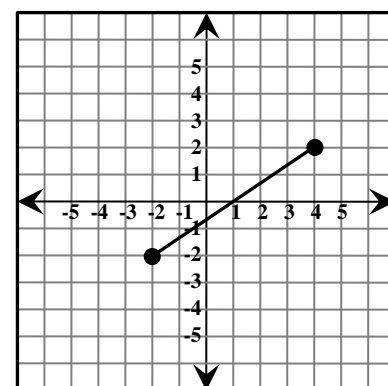
GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

2017-2018 TMSCA Mathematics Test One

- Evaluate $5.4 \div \left(\frac{5}{2}\right)^{-1} - 4! + 7.8$.
 (A) -14.0 (B) 5.3 (C) -6.0 (D) 10.8 (E) -2.7
- Seven billion, eighty-seven is subtracted from sixteen billion, three hundred sixty-one thousand, eight hundred seventy-two. What is the sum of the digits in the difference?
 (A) 47 (B) 35 (C) 26 (D) 63 (E) 39
- Find the sum of the multiples of 5 that are greater than -27 and less than 148.
 (A) 2220 (B) 1925 (C) 2100 (D) 2975 (E) 2240
- Given that $(3x - 2)$ is a factor of $3x^3 + ax^2 - 27x + 2a$, find the value of a .
 (A) 5 (B) 3 (C) -5 (D) 7 (E) -15
- Which of the following is an equation of the perpendicular bisector of the line segment illustrated?

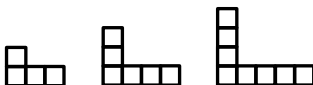


- (A) $3x + 2y = 3$ (B) $2x - 3y = 2$ (C) $3x - 2y = -3$
- (D) $3x - 2y = -1$ (E) $3x + 2y = -8$
- Simplify: $\frac{(n+2)!}{(n-1)!} \times \frac{1}{n} \div \frac{(n+1)!}{n!}$
 (A) $\frac{n+2}{n}$ (B) $n+2$ (C) $n^2 + n - 2$ (D) $n^2 + 2n$ (E) $\frac{n^2 + n - 2}{n}$
 - Given that $(2x + 3)$ is a factor of $2x^3 + ax^2 - 22x + 3a$, find the value of a .
 (A) 5 (B) 3 (C) -5 (D) 7 (E) -15
 - Carol bought Columbian coffee that cost \$13.00 per pound and Guatemalan coffee that cost \$15.00 per pound. How much Guatemalan coffee did she buy if she bought 3 pounds of coffee for \$42.50?
 (A) 24 oz. (B) 28 oz. (C) 20 oz. (D) 16 oz. (E) 18 oz.
 - A box contains 6 black marbles, 7 red marbles and 8 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 red marbles?
 (A) 1:38 (B) 49:1091 (C) 1:37 (D) 49:1140 (E) 7:221
 - $\angle A$ and $\angle B$ are supplementary. If $m\angle A = 8x^2 - 3x$ and $m\angle B = 17x - 4$, find the measure of the larger angle.
 (A) 64° (B) 48° (C) 132° (D) 116° (E) 41°

11. If $\frac{Ax+B}{3x+2} + \frac{x-3}{2x+1} = \frac{7x^2+5x-1}{6x^2+7x+2}$, where A and B are constants, then $A+B = ?$

- (A) 12 (B) 7 (C) 5 (D) 10 (E) 15

12. The three shapes below are made up of one-unit squares. If the pattern continues, the area of the figure with a perimeter of 114 units is _____ square units.



- (A) 56 (B) 48 (C) 60 (D) 230 (E) 234

13. A square has side lengths of 8 cm. If the square's width is quadrupled and the length is halved, the area of the square is what percent of the area of the new rectangle?

- (A) 75% (B) $12\frac{1}{2}\%$ (C) 25% (D) 50% (E) $33\frac{1}{3}\%$

14. A tank in the form of a rectangular prism is completely empty. The length of the tank is twice the width and the height is triple the width. How many gallons of water will fill the tank to 80% capacity if the height of the tank is 5 feet? (nearest gallon)

- (A) 120 gal (B) 22 gal (C) 208 gal (D) 147 gal (E) 166 gal

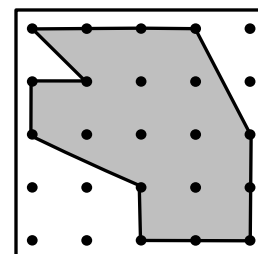
15. Let $A = \begin{bmatrix} 2 & 7 \\ -13 & -9 \end{bmatrix}$, $B = \begin{bmatrix} -7 & 0 \\ 1 & -11 \end{bmatrix}$ and $C = AB$. Find $C_{1,2}$.

- (A) -77 (B) -12 (C) -13 (D) 82 (E) 7

16. An operation " Ω " is defined by $a\Omega b = b^a - ab$. What is the value of $(3\Omega 2)\Omega(2\Omega 3)$?

- (A) 2 (B) 6 (C) 9 (D) 3 (E) 7

17. A rubber band was stretched on the geoboard to form this 9-sided figure. What is the area?



- (A) 11 units² (B) 10 units² (C) 9 units²
(D) 11.5 units² (E) 10.5 units²

18. Determine the frequency of $f(x) = 7 + 4\cos[6\pi(x-3)]$.

- (A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) 3 (D) $\frac{2}{3}$ (E) $\frac{1}{3}$

19. Given the geometric sequence $-48, a, b, c, -3, \dots$, find the largest possible value of $a+b+c$.

- (A) -9 (B) -42 (C) -6 (D) 18 (E) 3

20. Find the remainder when $x^4 - 3x^3 + 8x - 5$ is divided by $x - 5$.

- (A) 955 (B) 285 (C) 505 (D) -9 (E) 585

21. Which of the following functions is neither even nor odd? $f(x) = \underline{\hspace{2cm}}$.

- (A) $\sin(3x)$ (B) $|x-5|$ (C) $|x|-5$ (D) $4x^2-7$ (E) $\cos(2x)$

22. Let $f'(x) = 12x^2 + 10x - 2$ and $f(2) = 60$. Find $f(-2)$.

- (A) -12 (B) 12 (C) 0 (D) 4 (E) -4

23. If the height of a right conical container is doubled and the radius of the base is cut in half, then what is the ratio of the volume of the original container to the volume of the new container?

- (A) $1:2$ (B) $1:1$ (C) $4:1$ (D) $1:4$ (E) $2:1$

24. Let $f(x) = x^2 + 1$ and $g(x) = x^3$. Calculate $f(g^{-1}(27))$.

- (A) $19,683$ (B) 4 (C) 730 (D) 82 (E) 10

25. What is the angle between the minute and hour hands on a circular clock at 3:27 pm?

- (A) 58.5° (B) 60° (C) 56.5° (D) 57° (E) 57.5°

26. Caroline's Ice Creams offers 6 flavors of ice cream, 4 toppings and three container options. How many different 2-scoop, two different topping desserts can be ordered?

- (A) 126 (B) 504 (C) 165 (D) 378 (E) 168

27. Let the "1" at the top of Pascal's triangle be row 0. Determine the fifth number in row 42.

- (A) $111,930$ (B) $850,668$ (C) $962,598$ (D) $123,410$ (E) $101,270$

28. Coach Euclid's UIL team consists of 7 boys and 9 girls. How many different 4-member teams consisting of at least one girl could he make up?

- (A) 246 (B) $1,785$ (C) 126 (D) $1,330$ (E) $2,345$

29. Which of the following mathematicians is/are known for working with prime numbers?

I. Sophie Germain

II. Marin Mersenne

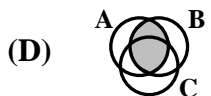
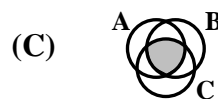
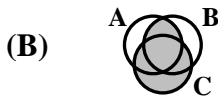
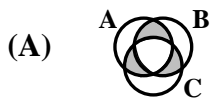
III. Eratosthenes

- (A) I only (B) III only (C) I & III (D) II & III (E) all of them

30. Given that $x - y = -12$ and $xy = 28$, find $x^3 - y^3$.

- (A) -2400 (B) -2064 (C) -8976 (D) -9264 (E) -2736

31. The shaded region in which of the following Venn diagrams represents the set $(A \cap C) \cup (B \cap C)$?



32. The statement $(5x - 3)(3x - 7) = 15x^2 - 35x - 9x + 21$ is an example of _____ property.

- (A) Associative (B) Commutative (C) Distributive
(D) Transitive (E) Identity

33. Use the table of values to create a function and find K.

X	1	2	3	4	5	...	18
Y	-25	-15	13	71	171		K

- (A) 10,753 (B) 1,841 (C) 4,105 (D) 760 (E) 498

34. Find $\lim_{x \rightarrow -\infty} \frac{2x^3 - x - 3}{2x^3 - 9x + 9}$

- (A) 0 (B) 1 (C) $-\infty$ (D) ∞ (E) -1

35. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$ be the terms of the Fibonacci sequence. How many digits are in f_{47} ?

- (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

36. If $[(5 + 3i)(3 - 2i)] \div (4 - i) = a + bi$, then $a + b = ?$

- (A) 3.8 (B) 4.2 (C) 6.0 (D) 8.0 (E) 9.2

37. The chart shows the losses and gains in an investment over the course of five years. What was the average growth rate over the course of the five years? (nearest hundredth)

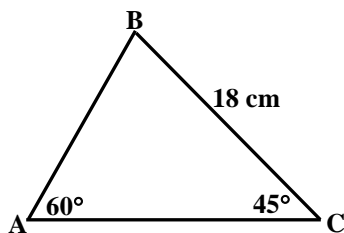
Year	1	2	3	4	5
Percent Growth	+ 10%	-4%	-8%	+7%	+6%

- (A) +2.46% (B) +1.96% (C) +1.59% (D) +2.22% (E) +5.01%

38. Which of the following are the side lengths of a scalene obtuse triangle?

- (A) 5, 6, 6 (B) 4, 5, 6 (C) 5, 6, 8 (D) 6, 8, 10 (E) 8, 10, 12

39. Find the perimeter of triangle ABC. (nearest tenth)



- (A) 45.5 (B) 52.8 (C) 56.4 (D) 54.6 (E) 96.9

40. If $\frac{11}{(x-2)(x+4)} - \frac{2}{(x-5)(x+4)} = \frac{k}{(x-2)(x-5)(x+4)}$, then k equals:

- (A) $9x - 52$ (B) $9x - 59$ (C) $13x - 59$ (D) $13x - 52$ (E) $9x - 51$

41. The intersection of the altitudes of a triangle of a triangle is called the_____.

- (A) Incenter (B) Orthocenter (C) Center (D) Centroid (E) Circumcenter

42. Which of the following statements is a false statement for $f(x) = \begin{cases} 3x^2 - 2 & \text{if } x < 3 \\ 20 & \text{if } x = 3 \\ 6x + 7 & \text{if } x > 3 \end{cases}$.

- (A) $f(3)$ exists (B) f is continuous at 3 (C) $\lim_{x \rightarrow 3^+} f(x)$ exists
(D) $\lim_{x \rightarrow 3^-} f(x)$ exists (E) none of them

43. If p and q are the zeros of the function $f(x) = 10x^2 - 53x + 63$ then $pq^2 + p^2q =$

- (A) $\frac{1113}{50}$ (B) $\frac{3339}{100}$ (C) $\frac{2809}{200}$ (D) $\frac{53}{10}$ (E) $\frac{3969}{100}$

44. Find the mean value of the function $f(x) = 2x^3 - 8x$ over the interval $[-4, 6]$.

- (A) 44 (B) 59 (C) 592 (D) 440 (E) 22

45. Given $y = \ln(5x - 4)$, find the positive value of x for which $\frac{dy}{dx} = \frac{dx}{dy}$.

- (A) 0.2 (B) 1 (C) 0.6 (D) 1.2 (E) 1.8

46. A polyhedron has 30 faces and 60 edges, how many vertices does it have?

- (A) 92 (B) 52 (C) 30 (D) 32 (E) 10

47. Let $f(x) = \frac{(x+2)^3}{x^2}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(-2)$.

- (A) 32 (B) 4 (C) -32 (D) 24 (E) 16

48. What is the constant term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{15}$?

- (A) 3003 (B) 30,030 (C) 96,096 (D) 480 (E) 205,920

49. $(12121_3 + 21212_3) \times 2_3 = \underline{\hspace{2cm}}_9$.

- (A) 222,220 (B) 886 (C) 1086 (D) 880 (E) 726

50. The length of one edge of a regular tetrahedron is $9\sqrt{3}$ cm. The surface area of the tetrahedron is _____cm². (nearest square centimeter)

- (A) 508 (B) 191 (C) 303 (D) 255 (E) 421

51. If $a_0 = -2$, $a_1 = 3$, $a_2 = 2$ and $a_n = (a_{n-3})(a_{n-2}) + a_{n-1}$ for $n \geq 3$, then $a_6 =$:

- (A) 9 (B) -26 (C) -6 (D) -14 (E) 2

52. There are two values of k for which $\det \begin{bmatrix} 6 & 1 & 2 \\ 4 & -k & 7 \\ k & 0 & 5 \end{bmatrix} = -71$. What is the smallest value of k ?

- (A) 3 (B) -2 (C) 2 (D) 8.5 (E) -3

53. Find the area in square units of a scalene triangle with vertices $(-3,1)$, $(4,2)$ and $(0,-5)$.

- (A) 22.5 (B) 16.5 (C) 18.5 (D) 20 (E) 20.5

54. The function $f(x) = 2x^4 - 5x^2$ is decreasing over which of the following intervals?

- (A) $(-\infty, 0]$ (B) $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$ (C) $[0, \infty)$ (D) $\left[0, \frac{\sqrt{5}}{2}\right]$ (E) $\left[\frac{\sqrt{3}}{2}, 0\right]$

55. If $27^{(x+1)} = 3^{-3x}$, then $x = ?$

- (A) $-\frac{1}{2}$ (B) -2 (C) $-\frac{2}{3}$ (D) $-\frac{1}{3}$ (E) 1

56. The graph of $f(x) = x^3 - x^2 - x + 1$ has a point of inflection at x equals?

- (A) 1 (B) -1 (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{3}$

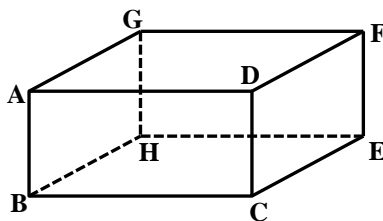
57. $f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} \dots$. Find the digit in the 10^{-8} place of $f(3)$.

- (A) 4 (B) 5 (C) 8 (D) 2 (E) 0

58. If $2e^{2x} - 7e^x = -3$, then x equals:

- (A) $-\ln 3, \ln 2$ (B) $-\log 2, \log 3$ (C) $-\log 3, \log 2$ (D) $-\ln 2, \ln 3$ (E) $\log 2, \log 3$

59. Given the rectangular solid shown, find BF if $DC = 12''$, $CE = 20''$ and $BC = 18''$. (nearest tenth)



- (A) 27.4" (B) 29.5" (C) 24.0" (D) 22.6" (E) 26.2"

60. If $16^{x+y} = 256$ and $4^{x-y} = 256$ then $x^2 - y^2$ equals_____.

- (A) 8 (B) 6 (C) 4 (D) 10 (E) -7

2017 – 2018 TMSCA Mathematics Test One Answers

1. E	21. B	41. B
2. E	22. D	42. B
3. C	23. E	43. B
4. D	24. E	44. A
5. A	25. A	45. E
6. B	26. D	46. D
7. C	27. A	47. B
8. B	28. B	48. C
9. C	29. E	49. B
10. D	30. E	50. E
11. B	31. E	51. D
12. A	32. C	52. A
13. D	33. A	53. A
14. E	34. B	54. D
15. A	35. D	55. A
16. D	36. C	56. C
17. D	37. B	57. C
18. C	38. C	58. D
19. D	39. B	59. B
20. B	40. E	60. A

2017-2018 TMSCA Mathematics Test One Select Solutions

<p>9. $p(3red) = \frac{7}{21} \times \frac{6}{20} \times \frac{5}{19} = \frac{1}{38}$ so the odds are $\frac{1}{38-1} = \frac{1}{37}$ or 1:37</p> <p>12. The first three perimeters are 10, 14 and 18, while the first three areas are 4, 6 and 8. Both are increasing by a set value, so find the linear function that relates the two sets for $A = \frac{1}{2}p - 1$ and an area of 56 when $p = 114$.</p> <p>13. Solve $64 = (32)(4)x$ for $x = 0.5 = 50\%$</p> <p>14. $V = 0.8(60in)(20in)(40in) = 38400in^3 \times \frac{1gal}{231in^3} \approx 166gal$</p> <p>17. Let I = the number of interior points and P = the number of points on the perimeter, then $A = \frac{2I + P}{2} - 1 = 11.5$</p> <p>18. The frequency is $\frac{b}{2\pi} = \frac{6\pi}{2\pi} = 3$ b is the coefficient of x.</p> <p>19. $r^4 = \frac{-3}{-48}$ for $r = \pm \frac{1}{2}$, then use the negative value for a sum of 18.</p> <p>26. ${}_{(6+2-1)}C_2 \times {}_4C_2 \times 3 = 378$</p> <p>28. All -no girls $= {}_{16}C_4 - {}_7C_4 = 1785$</p> <p>30. $x^3 - y^3 = (x - y)(x^2 + xy + y^2) = (x - y)[(x - y)^2 + 3xy] = -12[(-12)^2 + 3(28)] = -2736$</p> <p>37. $\sqrt[3]{(1.10)(0.96)(0.92)(1.07)(1.06)} \approx 1.0196$ for an average growth rate of 1.96%.</p> <p>42. f is not continuous at 3 because 20 does not equal the values of the other two expressions evaluated at 3.</p> <p>43. This is the product of the sum of the roots and the product of the roots for $\left(\frac{53}{10}\right)\left(\frac{63}{10}\right) = \frac{3339}{100}$</p> <p>45. Either graph or use solver to find the solutions to $\pm 1 = \frac{5}{5x-4}$ for $x = 0.2$ or 1.8. The 0.2 is outside of the domain of the original function, so only 1.8 is an acceptable answer.</p> <p>46. $V + F - E = 2$, so $V + 30 - 60 = 2$ for $V = 32$</p>	<p>47. $(x^3 + 6x^2 + 12x + 8) \div x^2 = x + 6 + \frac{12x-8}{x^2}$, so $s(x) = x + 6$ and $s(-2) = 4$</p> <p>48. Use binomial theorem for $\binom{15}{5}(2x^2)^5\left(-\frac{1}{x}\right)^{10} = 96096$</p> <p>53. Find the area of any polygon using coordinates of vertices by $A = \left \frac{(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \dots + (x_ny_1 - y_nx_1)}{2} \right$ where the points are arranged in either clockwise or counter-clockwise order. So here $A = \left \frac{(-3 \cdot 2 - 4 \cdot 1) + (-5 \cdot 4 - 0 \cdot 2) + (0 \cdot 1 - (-3)(-5))}{2} \right = 22.5$</p> <p>55. $3^{3(x+1)} = 3^{-3x}$, so solve $3x + 3 = -3x$ for $x = -\frac{1}{2}$</p> <p>57. This is the McClaurin series polynomial for $f(x) = \cos x + \sin x$ making sure to be in radian mode, evaluate $\cos 3 + \sin 3$ for an 8 in the requested decimal place</p> <p>58. Factor $0 = 2e^{2x} - 7e^x + 3 = (2e^x - 1)(e^x - 3)$ then solve $2e^x - 1 = 0$ and $e^x - 3 = 0$ for $-\ln 2$ and $\ln 3$</p> <p>60. Solve $\begin{matrix} x + y = 2 \\ x - y = 4 \end{matrix}$ for $x = 3$ and $y = -1$, then $9 - 1 = 8$</p>
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