T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 6 (U I L C) ©

D E C E M B E R 2 , 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate: $8 + 7 \times (6 - 5) \div (4 - 3) \times 2 + 1$

(A) 37.5 (B) 31 (C) 23 (D) 18 (E) 12.5

2. 10 yards per second = _____ miles per hour. (nearest tenth)

(A) 18.3 (B) 20.5 (C) 21.0 (D) 22.8 (E) 24.5

3. Let P = {2, 1, 3, 4, 7}, Q = {1, 3, 6, 10}, and R = {2, 3, 5, 7, 11}. How many elements are in $(P \cup R) \cap (P \cap Q)$?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

4. Macon Mony earns 15% commission on her monthly sales up to and including $800.00. She earns 20% commission on sales over $800.00 up to and including $1,500.00. Any sales over $1,500.00 will earn her a 25% commission. Macon's total sales for the month of May was $2,000.00. What was the total commission Macon earned for the month of May?

(A) $385.00 (B) $415.00 (C) $425.00 (D) $500.00 (E) $920.00

5. The line $2x - 5y = 7$ is perpendicular to the line $3x + ay = 5$. What is the value of a?

(A) -2.5 (B) -0.4 (C) 0.8333... (D) 1.2 (E) 7.5

6. Simplify: $(4)\left(\frac{4x^2 - 12x + 9}{8x + 12}\right)\left(\frac{4x^2 + 12x + 9}{4x^2 - 9}\right)$

(A) 1 (B) $2x + 3$ (C) $2x - 3$ (D) $\frac{2x - 3}{2x + 3}$ (E) $\frac{2x + 3}{2x - 3}$

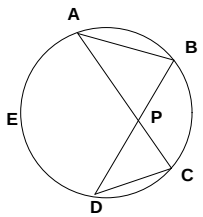
7. Find the least common multiple of $4x^4y^2$, $6x^3y^3$, and $8xy^5$.

(A) $12x^4y^5$ (B) $18x^8y^{10}$ (C) $24x^4y^5$ (D) $9x^3y^4$ (E) $24xy^2$

8. Jack and Jill have to go up a hill to fetch 10 buckets of water. Jack can fetch the 10 buckets in 5 hours by himself. If Jill helps him they can fetch the 10 buckets in 2 hours. How long would it take Jill to fetch the 10 buckets by herself?

(A) 2 hrs 30 min (B) 3 hrs (C) 3 hrs 20 min (D) 3 hrs 30 min (E) 3 hrs 40 min

9. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of the circle shown and point E lies on circle O. Find $m\angle BPC$ if $m\angle ABD = 80°$ and $m\angle BDC = 40°$.

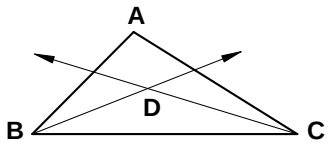


(A) 100° (B) 120° (C) 140° (D) 160° (E) not enough information

10. Point P(4 , — 5) lies on the x-y plane. P is translated vertically 2 units down to point Q. Then Q is translated horizontally 3 units to the right to point R. Then R is reflected across the x-axis to point S with coordinates (x, y). Find $x + y$

(A) 6 (B) 14 (C) 0 (D) 9 (E) 13

11. In $\triangle$ ABC, the bisectors of $\angle$B and $\angle$C meet at D. Find m$\angle$BAC if m$\angle$BDC = 140° .



(A) 70° (B) 80° (C) 90° (D) 100° (E) 120°

12. The Real value solution set for $1 + 2|3x - 4| < 5$ is?

(A) $\{x \mid \frac{2}{3} < x < 2\}$ (B) $\{x \mid \{x < \frac{1}{3}\} \cup \{x > 2\}\}$ (C) $\{x \mid \frac{1}{3} < x < 2\}$
(D) $\{x \mid \frac{1}{3} < x < \frac{2}{3}\}$ (E) $\{x \mid \{x > 2\} \cup \{x < \frac{2}{3}\}\}$

13. Find f(g(a — 1)) when g(x) = 4x — 2 and f(x) = 3 — 5x.

(A) 30 — 20a (B) 2 — 9a (C) 18 — 20a (D) 14 — 9a (E) 33 — 20a

14. If $4^{(x+y)} = 256$ and $8^{(x-y)} = 512$ then $16^{(xy)}$ equals _____.

(A) 128 (B) 64 (C) 4096 (D) 1024 (E) 256

15. Phil Deetank has a dirt water tank that can hold 2,640 gallons. He uses two pipes to fill the tank. One of the pipes can put in 265 gallons per hour. When both pipes are turned on they can fill the tank in 4 hours. How many gallons per hour can the other pipe put in?

(A) 594 gal (B) 530 gal (C) 395 gal (D) 375 gal (E) 160 gal

16. Which of the following is equivalent to $\frac{\cos^2\theta}{1+\sin\theta} + \sin\theta$?

(A) $1 - 2\sin\theta$ (B) $-2\sin\theta$ (C) $1 + 2\sin\theta$ (D) 1 (E) 0

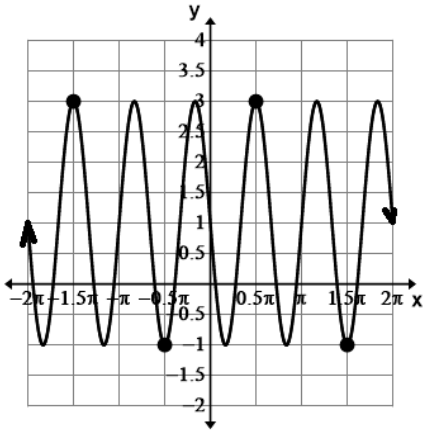
17. Find the sum of the x-values in $\left\{x \mid 2\cos^2(3x) + 5\cos(3x) - 3 = 0, x \in [\frac{2\pi}{3}, \frac{5\pi}{4})\right\}$?
(nearest hundredth)

(A) 8.03 (B) 3.84 (C) 6.28 (D) 4.71 (E) 3.14

18. The sum of the coefficients of the 2nd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 4th term of $(x + 1)^4$, and 5th term of $(x + 1)^5$ is:

(A) 24 (B) 21 (C) 16 (D) 14 (E) 12

19. T32. The equation y = _____ will produce this graph.



(A) $1 + 2\cos(3x + \pi)$ **(B)** $2\cos(3(x - \frac{\pi}{6})) - 1$ **(C)** $2\sin(3(x + \frac{\pi}{6})) - 1$

(D) $1 + 2\sin(3x)$ **(E)** $2\sin(3x + \pi) + 1$

20. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \ldots + \frac{1}{91} + \frac{1}{105} = ?$

(A) $\frac{7}{8}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{11}{12}$ **(D)** $\frac{13}{15}$ **(E)** $\frac{15}{17}$

21. Cole Muter drove 20 miles to a train station. He rode the train 60 miles to the big city where he works. The average rate of the train was 15 mph faster than the rate of the car. The trip from home to the big city takes 2 hours. Approximate the average rate of the car. (nearest tenth)

(A) 45.0 mph (B) 32.5 mph (C) 30.0 mph (D) 47.5 mph (E) 62.5 mph

22. The function f(x) = $5x^3 + 2x^2 - 3x$ **has an inflection point at (x , y). Find x.**

(A) $-\frac{2}{15}$ **(B)** $-\frac{3}{5}$ **(C)** $-\frac{13}{15}$ **(D)** $\frac{15}{49}$ **(E)** $\frac{1}{3}$

23. Let f(x) = $-2x + 1$**, g(x) =** $3x - 5$**, and h(x) =** $-4x$**. Find** $g\left(f\left(h\left(\frac{1}{2}x\right)\right)\right)$.

(A) $12x - 11$ **(B)** $12x - 2$ **(C)** $12x - 1$ **(D)** $12x + 2$ **(E)** $12x + 11$

24. Evaluate: $\int_{-a}^{a} (3x^2 + 1)\, dx$

(A) $2a^3$ **(B)** $a^2 + 1$ **(C)** $2a^3 + 2a$ **(D)** $2a$ **(E)** a

25. Jay Daman's science team will be made up of 6 students from his AP Physics class. There are 8 boys and 6 girls in his AP Physics class. How many different science teams consisting of 3 girls and 3 boys could he make up?

(A) 3,003 (B) 42 (C) 76 (D) 456 (E) 1,120

26. Which of the following mathematicians is known as the 'Father of Modern Algebra' and is associated with the formulas for finding the sum of the roots and the product of the roots, $-\frac{b}{a}$ and $\frac{c}{a}$?

(A) Marin Mersenne **(B) Alan Turing** **(C) Franciscus Vieta**
(D) Christian Goldbach **(E) John Napier**

27. An operation " $\odot$ " is defined by: $a \odot b = a(b + a) - b$. What is the value of $(-1 \odot 2)(2 \odot -1)$?

(A) -9 **(B) -6** **(C) -3** **(D) 0** **(E) 3**

28. Nick Olas has a bag of golf balls, 6 white ones, 4 yellow ones, and 3 green ones. What is the probability of randomly selecting two balls (without replacement) that are different colors? (nearest percent)

(A) 14% **(B) 35%** **(C) 65%** **(D) 76%** **(E) 69%**

29. How many non-negative proper fractions in lowest terms have a denominator of 60?

(A) 30 **(B) 25** **(C) 20** **(D) 16** **(E) 15**

30. Find the digit in the units place of the integer 3^{444}.

(A) 1 **(B) 3** **(C) 4** **(D) 7** **(E) 9**

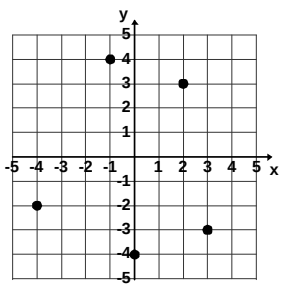
31. If $8x^2 + ax + 3 = (4x - 1)(bx + c)$, where a, b, and c are integers then $a + b + c =$ ____.

(A) -15 **(B) -13** **(C) -9** **(D) 13** **(E) 19**

32. How many integers, n, satisfy the inequality $\frac{1}{3} < \frac{n}{12} < \frac{4}{5}$

(A) 9 **(B) 7** **(C) 6** **(D) 5** **(E) 4**

33. Monsieur De' Kart plotted the points, whose coordinates are integers, on the coordinate plane below. If he starts in quadrant I and joins the dots in a clockwise direction he will form a pentagon. What is the area of his pentagon?



(A) 28.5 units^2 **(B) 31 units^2** **(C) 41.5 units^2** **(D) 34 units^2** **(E) 36 units^2**

34. A rectangle has an area of 500 sq. cm. If its length is increased by 20% and the width decreased by 10%. What is the area of the new rectangle?

(A) 620 cm^2 (B) 470 cm^2 (C) 520 cm^2 (D) 540 cm^2 (E) 350 cm^2

35. The two real roots of $f(x) = x^2 - 2x + k$ differ by 12. Find the value of k.

(A) -36 (B) -35 (C) -27 (D) -24 (E) -12

36. Let $A + B = 15$ and $A \times B = 30$. Find $B - A$, where $A < B$.

(A) $15 + \sqrt{105}$ (B) $\frac{\sqrt{105}}{2}$ (C) $\sqrt{105}$ (D) $15 - \sqrt{105}$ (E) $2\sqrt{105}$

37. Which of the following equations can be obtained from the graph of the parent function $y = \sin(x)$ by applying a vertical stretch of 3 units, a vertical shift of 2 units, and phase shift of -4? $y =$ _____.

(A) $2 + 3\sin(x + 4)$ (B) $3 + 2\sin(x - 4)$ (C) $-4 + 3\sin(x + 2)$
(D) $2 - 4\sin(x + 3)$ (E) $3 + 2\sin(x + 4)$

38. A guy wire runs from the ground to a point on a cell tower 150 feet above the ground. The angle formed by the ground and the wire is 40°. How long is the guy wire? (nearest ft)

(A) 300 ft (B) 179 ft (C) 173 ft (D) 196 ft (E) 233 ft

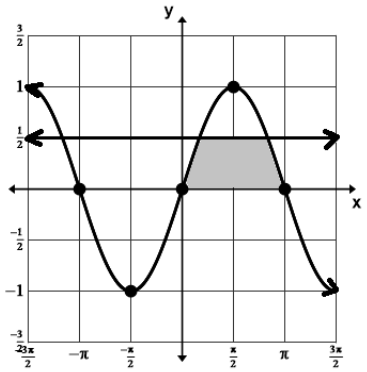
39. Let $f(x) = \frac{x^2 - 5x + 4}{x - 3}$ and s(x) be the slant asymptote of f. Find the value of s(1).

(A) -3 (B) -2 (C) -1 (D) 0 (E) 2

40. Let $f(x) = |x + 3| - |x - 1|$. Find the minimum value of f(x).

(A) -4 (B) -3 (C) -1 (D) 0 (E) 1

41. Find the area in square units of the shaded region. (nearest tenth)



(A) 1 (B) 0.5 (C) 1.0 (D) 1.3 (E) 1.6

42. Which of the following polar equations will produce the graph of a cardioid?

(A) $r = 2 + 2\sin(\theta)$ **(B)** $r = \sin(2\theta)$ **(C)** $r = 2\sin(\theta)$ **(D)** $r = 2\sin(\frac{\theta}{2})$ **(E)** $r = 2 + 3\sin(\theta)$

43. Let P = {a, b, c, d, e} and Q be (f, g, h, i, j, k}. Exactly one letter is chosen from each set. What is the probability of picking at least one vowel?

(A) $16\frac{2}{3}\%$ **(B)** $33\frac{1}{3}\%$ **(C)** 50% **(D)** $66\frac{2}{3}\%$ **(E)** $83\frac{1}{3}\%$

44. Mack N. Tosh needs to create a four digit pin to keep his laptop secure. The first digit has to be a triangular number, the second digit a prime number, the third digit a factor of 8, and the last digit an odd number. How many unique pins can Mack create?

(A) 240 **(B)** 256 **(C)** 180 **(D)** 320 **(E)** 16

45. The number 21 is a member of which of the following sets of numbers?
I. evil II. economical III. equidigital IV. extravagant

(A) I & III **(B)** I, II, & III **(C)** II & IV **(D)** III only **(E)** all of them

46. Let $545_b + 455_b = 1111_b$**. Find** 363_b **in base 10.**

(A) 243 **(B)** 729 **(C)** 354 **(D)** 108 **(E)** 300

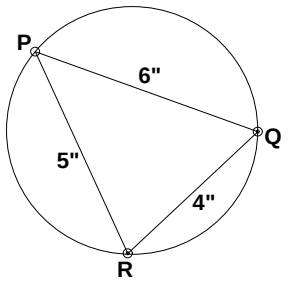
47. Cal Roper has a rope that is 50 feet long. He needs to cut it into three smaller ropes such that the ratio of the smaller ropes is 3:5:12. How much longer is the longest rope than the shortest? (nearest inch)

(A) 10 yds **(B)** 2 yds 1 ft 6 in **(C)** 7 yds 2 in **(D)** 5 yds 2 ft 6 in **(E)** 7 yds 1 ft 6 in

48. Find $f(-2) + f(1) + f(2)$ **if** $f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ 2x & \text{if } 0 \le x \le 2 \\ 1 - 2x & \text{if } x > 2 \end{cases}$

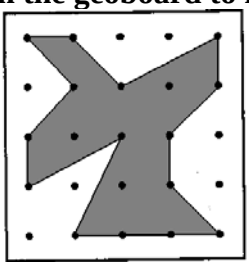
(A) -11 **(B)** 1 **(C)** -3 **(D)** 2 **(E)** -6

49. Find the diameter of the circle circumscribed about $\triangle PQR$**. (nearest inch)**



(A) 6" **(B)** 6.5 " **(C)** 5.3" **(D)** 7" **(E)** 7.2"

50. A rubber band was stretched on the geoboard to form this 13-sided figure. What is its area?



(A) 10 sq. units (B) 9.5 sq. units (C) 9 sq. units (D) 8.5 sq. units (E) 8 sq. units

51. Let function f = {(1, 3), (2, 5), (3, 7)} and function g = {(3, 7), (5, 5), (7, 3)}. Which of the following is a member of the function g(f)?

(A) (5, 2) (B) (7, 1) (C) (5, 7) (D) (3, 3) (E) (3, 2)

52. The bearing of the *FRIEND* ship from the *SCHOLAR* ship is 45°. The bearing of the *KIN* ship from the *FRIEND* ship is 120°. The *FRIEND* ship is 5 km from the *SCHOLAR* ship and 9 km from the *KIN* ship. How far is the *SCHOLAR* ship from the *KIN* ship? (nearest tenth)

(A) 13.0 km (B) 11.4 km (C) 7.5 km (D) 10.9 km (E) 13.9 km

53. Let x and y be integers such that $xy(x + y) = 84$ and $xy + x + y = 19$. Calculate $x^2 + y^2$.

(A) 50 (B) 17 (C) 65 (D) 58 (E) 25

54. Let $p + q = 3$ and $p^2 + q^2 = 6$. Find the numeric value of $p^3 + q^3$.

(A) 18 (B) 13.5 (C) 12 (D) 10.5 (E) 9

55. Max Space is fencing in a rectangular garden. The shed for his tools will form one side. He uses 40 meters of fencing. What is the maximum area possible for his garden?

(A) 100 m^2 (B) 120 m^2 (C) 160 m^2 (D) $177\frac{7}{9}$ m^2 (E) 200 m^2

56. Function f is defined by $\begin{cases} f(x) = \frac{\sqrt{4x+4} - \sqrt{2x+4}}{2x} \\ f(0) = C \end{cases}$ **where C is a constant.**

What must the value of C be equal to for function f to be continuous at x = 0?

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) 1 (E) Any real number

57. Leo Bigollo randomly selected two different numbers from the set {1, 2, 3, 5, 8}. What are the odds that the sum of the two numbers selected is a Fibonacci number?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{1}$ (D) $\frac{3}{7}$ (E) $\frac{2}{5}$

58. Roland Cubes pays $2.00 and rolls a fair die. If he rolls a 1, 2, or 3 he will receive half of his money back. If he rolls a 4 or 5 he gets 100% of his money back. If he rolls a 6 he gets 200% of his money back. What is the mathematical expectation value after playing many times? (nearest cent)

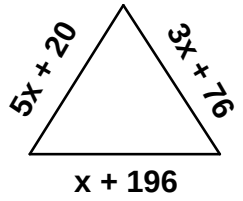
(A) — 33¢ (B) — 17¢ (C) $1.00 (D) $1.33 (E) $1.67

59. If the following pattern continues, find the median of row 9.

			1				row 1
		1		1			row 2
	1		2		1		row 3
1		3		3		1	row 4
	1		4		5		4		1		row 5
		1		5		7		7		5		1		row 6
1		6		9		12		9		6		1	row 7
						...							

(A) 20 (B) 23 (C) 24 (D) 28 (E) 30

60. Find the greatest possible perimeter of this triangle given that it is isosceles but not equilateral.



(A) 544 (B) 688 (C) 832 (D) 976 (E) 1,120

2017-18 TMSCA HS Math Test #6
Answer Key

1. C	**21. C**	**41. D**
2. B	**22. A**	**42. A**
3. D	**23. B**	**43. C**
4. A	**24. C**	**44. A**
5. D	**25. E**	**45. D**
6. C	**26. C**	**46. E**
7. C	**27. A**	**47. E**
8. C	**28. E**	**48. B**
9. B	**29. D**	**49. A**
10. B	**30. A**	**50. B**
11. D	**31. A**	**51. D**
12. A	**32. D**	**52. B**
13. E	**33. E**	**53. E**
14. A	**34. D**	**54. B**
15. C	**35. B**	**55. E**
16. D	**36. C**	**56. B**
17. C	**37. A**	**57. B**
18. D	**38. E**	**58. B**
19. E	**39. C**	**59. D**
20. D	**40. A**	**60. C**