GENERAL TEST

This test consists of 10 short-answer problems to be solved individually in 50 minutes. Problems are unequally weighted with point values as shown in brackets on the answer form. The maximum possible score is 50 points. There is no point penalty for guessing, though in the case of a tie it is slightly more advantageous not to answer than to answer incorrectly.

No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted. Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.

Answers should be simplified as much as is reasonably possible and must be exact unless otherwise specified. Rational numbers should be written in lowest terms, although denominators of irrationals need not be rationalized. An nth root should be simplified so that the radicand is not divisible by the nth power of any prime.

Correct mathematical notation must be used. No partial credit will be given unless otherwise specified.

If you believe the test contains an error, please submit your protest in writing to the Science Center Lobby during lunchtime.

Enjoy!

HMMT November 2012

Saturday 10 November 2012

General Test

1. [**3**] What is the sum of all of the distinct prime factors of $25^3 - 27^2$?

2. [**3**] Let $Q(x) = x^2 + 2x + 3$, and suppose that $P(x)$ is a polynomial such that

$$P(Q(x)) = x^6 + 6x^5 + 18x^4 + 32x^3 + 35x^2 + 22x + 8.$$

Compute $P(2)$.

3. [**3**] $ABCD$ is a rectangle with $AB = 20$ and $BC = 3$. A circle with radius 5, centered at the midpoint of DC, meets the rectangle at four points: W, X, Y, and Z. Find the area of quadrilateral $WXYZ$.

4. [**4**] If you roll four fair 6-sided dice, what is the probability that at least three of them will show the same value?

5. [**4**] How many ways are there to arrange three indistinguishable rooks on a 6×6 board such that no two rooks are attacking each other? (Two rooks are attacking each other if and only if they are in the same row or the same column.)

6. [**5**] $ABCD$ is a parallelogram satisfying $AB = 7$, $BC = 2$, and $\angle DAB = 120°$. Parallelogram $ECFA$ is contained in $ABCD$ and is similar to it. Find the ratio of the area of $ECFA$ to the area of $ABCD$.

7. [**6**] Find the number of ordered 2012-tuples of integers $(x_1, x_2, \ldots, x_{2012})$, with each integer between 0 and 2011 inclusive, such that the sum $x_1 + 2x_2 + 3x_3 + \cdots + 2012x_{2012}$ is divisible by 2012.

8. [**7**] Let n be the 200th smallest positive real solution to the equation $x - \frac{\pi}{2} = \tan x$. Find the greatest integer that does not exceed $\frac{n}{2}$.

9. [**7**] Consider triangle ABC where $BC = 7$, $CA = 8$, and $AB = 9$. D and E are the midpoints of BC and CA, respectively, and AD and BE meet at G. The reflection of G across D is G', and $G'E$ meets CG at P. Find the length PG.

10. [**8**] Let α and β be reals. Find the least possible value of

$$(2\cos\alpha + 5\sin\beta - 8)^2 + (2\sin\alpha + 5\cos\beta - 15)^2.$$

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Name ________________________________ Team ID# ________

School ______________________ Team ______________________

1. [**3**] ______________
2. [**3**] ______________
3. [**3**] ______________
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9. [**7**] ______________
10. [**8**] ______________

Score: __________