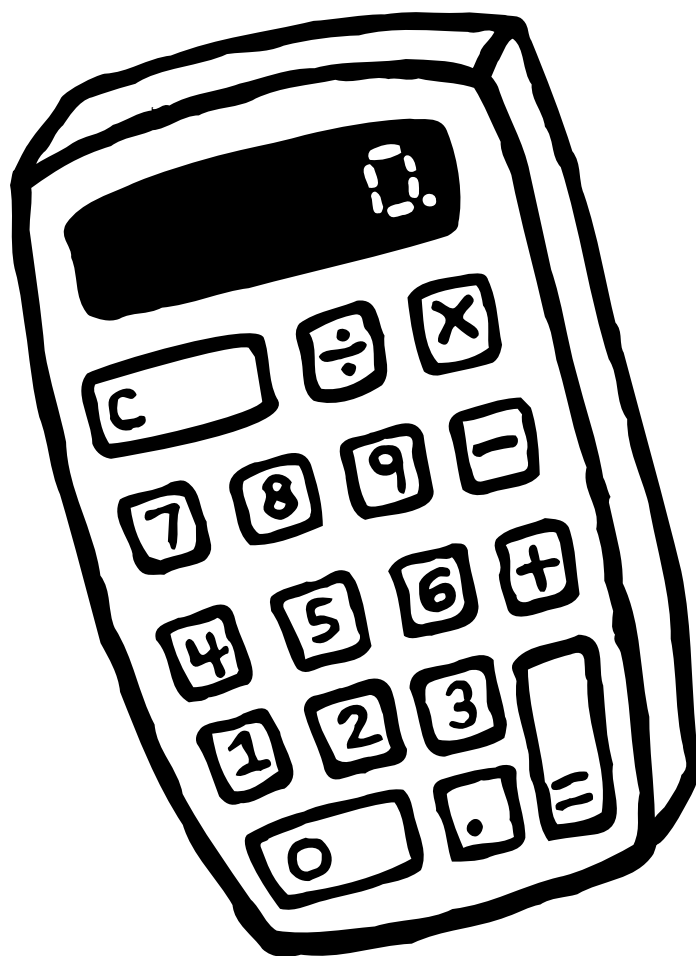




UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

Invitational A • 2019



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1. Evaluate:  $1 + 11^2 \div (2 + 9) + 1 \times 9$

- (A) 79                      (B)  $27\frac{9}{11}$                       (C) 21                      (D)  $20\frac{2}{11}$                       (E) 11

2. Let  $A = \{a, c, u, t, e\}$ ,  $O = \{o, b, t, u, s, e\}$ , and  $R = \{r, i, g, h, t\}$ . The number of elements in  $(A \cup R) \cap O$  is:

- (A) 1                      (B) 3                      (C) 4                      (D) 5                      (E) 6

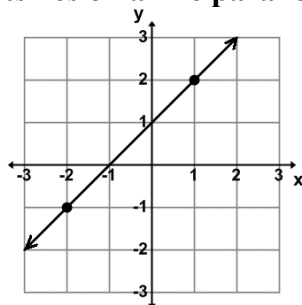
3.  $111A09201B \div 9$  has a remainder of 5. Find the least value of  $A + B$ .

- (A) 8                      (B) 7                      (C) 6                      (D) 5                      (E) 4

4. Mae B. Tulong had twelve yards of rope. She cut off a length of rope that was 2 yards 1 foot 8 inches long. Then she divided the remaining length of rope into four equal parts. How long was each of the four equal parts of rope?

- (A) 1 yd 1' 4"    (B) 1 yd 2' 8"    (C) 2 yds 0' 11"    (D) 2 yds 2' 1"    (E) 2 yds 1' 1"

5. Which of the following points lies on a line parallel to the line shown and containing point (0, 3)?



- (A) (9, 6)                      (B) (7, 11)                      (C) (11, 15)                      (D)  $(-7, -4)$                       (E)  $(-12, -12)$

6. Let  $4x^2 + 17x - 15 = (ax + b)(cx + d)$ . Find  $a + b + c + d$ .

- (A) 3                      (B) 6                      (C) 7                      (D) 2                      (E) 12

7. Let  $(2x - 1)^2(2x + 1) = ax^3 + bx^2 + cx + d$ . Find  $a + b + c + d$ .

- (A) 1                      (B) 3                      (C) 7                      (D) 11                      (E) 15

8. Simplify:  $\left(\frac{x^2 - 3x - 10}{x^2 + 2x - 35}\right) \div \left(\frac{x^2 + 9x + 14}{x^2 + 4x - 21}\right)$

- (A)  $x - 3$                       (B)  $\frac{x+7}{x-5}$                       (C)  $\frac{x-3}{x+7}$                       (D)  $\frac{x^2+4x+4}{x^2+4x-21}$                       (E)  $\frac{x-7}{x+3}$

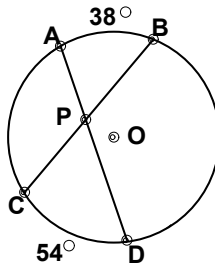
9. Leo Oiler drew a polyhedron with 7 faces and 11 edges. How many vertices does it have?

- (A) 2                      (B) 6                      (C) 13                      (D) 16                      (E) 20

10. Two lines are \_\_\_\_\_? \_\_\_\_\_ if and only if the product of their slopes is  $-1$ .

- (A) parallel      (B) skew      (C) collinear      (D) perpendicular      (E) intersecting

11. Find  $m\angle APB$ . (drawing is not to scale)



- (A)  $76^\circ$       (B)  $46^\circ$       (C)  $44^\circ$       (D)  $16^\circ$       (E)  $80^\circ$

12. A right cylinder can of *PaPi* Spinach has a diameter length of 4" and a height of 5". What is the total surface area of the spinach can? (nearest tenth)

- (A)  $88.0 \text{ in}^2$       (B)  $56.5 \text{ in}^2$       (C)  $75.4 \text{ in}^2$       (D)  $44.0 \text{ in}^2$       (E)  $62.8 \text{ in}^2$

13. If  $\frac{3x+2}{x-1} - \frac{x-3}{2x+1} = \frac{ax^2+bx+c}{dx^2+ex+f}$ , then  $a + b + c + d + e + f$  equals:

- (A) 21      (B) 19      (C) 18      (D) 16      (E) 15

14. If  $a_1 = 1$ ,  $a_2 = 3$ ,  $a_3 = -5$  and  $a_n = a_{n-1} + a_{n-3} - a_{n-2}$ , where  $n \geq 4$ , then  $a_6$  equals:

- (A) 11      (B) 9      (C) 3      (D)  $-1$       (E)  $-7$

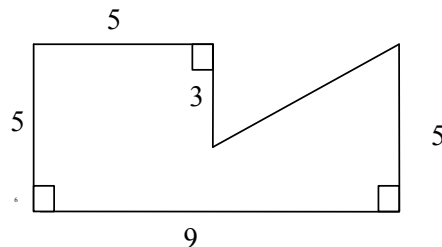
15. Let  $x - 3y = 5$  and  $2y + z = 3$  and  $2 - z = x$ . Find  $x + yz$ .

- (A)  $-103$       (B)  $-63$       (C)  $-8$       (D) 13      (E) 77

16. Which of the following mathematicians is noted for his work with sets, probability, and logic?

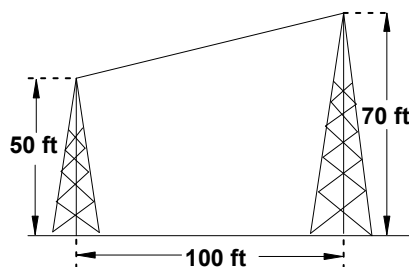
- (A) John Venn      (B) Leonard Euler      (C) Euclid      (D) Alan Turing      (E) John Napier

17. Find the perimeter this hexagon? All lengths are in cm.



- (A) 27 cm      (B) 31 cm      (C) 32 cm      (D) 36 cm      (E) not enough data

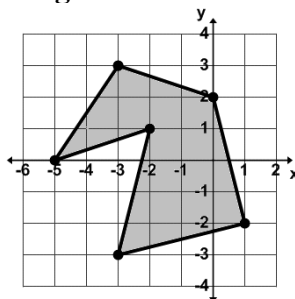
18. Thirty seniors took the state math test last year. Twenty-two of them were boys and eight were girls. All of them had an equal chance to win one of the top three medals. What was the probability that two girls and one boy won one of the top three medals? (nearest whole percent)
- (A) 1%      (B) 2%      (C) 15%      (D) 30%      (E) 46%
19. Given:  $f(x) = 3 - 2\sin(x + 4)$ , where the domain is  $\{x \mid x \in \text{Reals}\}$  and the range is  $\{f(x) \mid a \leq f(x) \leq b \text{ and } y \in \text{Reals}\}$ . Which of the following is not in the range?
- (A) 1.5      (B) 3.124      (C) 2.04      (D) 5.333...      (E) 4.75
20. The expression  $(1 - \cos \theta)(1 + \cos \theta)(1 + \cot^2 \theta)$  is equivalent to:
- (A)  $\csc^2 \theta$       (B)  $\frac{1}{\sec \theta}$       (C) 1      (D)  $\frac{1}{\csc \theta}$       (E)  $\sin^2 \theta + 1$
21.  $e^{3i} = \cos(3) + i\sin(3)$  is an example of \_\_\_\_\_ formula.
- (A) Aryabhata's      (B) Bayes'      (C) Cantor's      (D) Diophantus'      (E) Euler's
22. If  $f(x) = x^2 - 3x + 2$  and  $g(x) = 2x^2 - x + 3$ , then  $g(f(4)) = ?$
- (A) 69      (B) 182      (C) 4      (D) 13      (E) 870
23.  $(8x^3 - 4x^2 - 2x + 1) \div (2x + 1)$  has a remainder of \_\_\_\_\_.
- (A) -1      (B) -0.5      (C) 0      (D) 0.5      (E) 1
24. Find the absolute value difference between coefficients of the  $x^2y^3$  term and the  $x^3y^2$  term in the expansion of  $(3x + 2y)^5$ .
- (A) 36      (B) 144      (C) 330      (D) 360      (E) 720
25. A cable is connected from the shorter tower to the taller tower. What is the minimum length of the cable? (nearest inch)



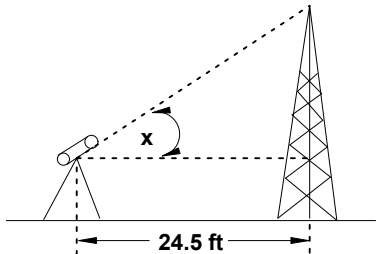
- (A) 102'3"      (B) 102'0"      (C) 101'8"      (D) 101'6"      (E) 101'0"
26. Find the area bounded by  $f(x) = x^3$ ,  $f(y) = -2$ , and  $f(y) = 1$ . (square units)
- (A) -3.75      (B) -3      (C) -1      (D) 4      (E) 4.25

27. What is the slope of the secant line to the graph of  $f(x) = 2x^2 + 3x - 4$  passing through the points  $(1, m)$  and  $(-3, n)$ ?
- (A) 1.5                      (B) 1                      (C)  $-1$                       (D)  $-1.5$                       (E)  $-2$
28. Find the 20<sup>th</sup> term given the sequence: 3, 8, 15, 24, 35, 48, ...
- (A) 483                      (B) 420                      (C) 380                      (D) 399                      (E) 440
29. A box of golf balls contains 6 white ones, 4 pink ones, and 2 blue ones. Three balls are randomly drawn from the box, without replacement. What are the odds that they are all the same color?
- (A) 12%                      (B) 60%                      (C) 10%                      (D) 6%                      (E) 55%
30. The Shawk Electric Company charges a monthly base fee of \$10.50 and a usage fee of 8¢ per kilowatt hour used. The company offers a \$25.00 credit if the kilowatt usage is over 1200 kWh. How much would the bill be before taxes if the monthly usage was 1450 kWh.
- (A) \$197.50                      (B) \$124.50                      (C) \$121.50                      (D) \$101.50                      (E) \$81.50
31. Two billion three hundred four million five thousand sixty-seven is added to twenty-three million four hundred fifty-two thousand six hundred seven. Which of the following digits appears the most in the sum?
- (A) 2                      (B) 4                      (C) 5                      (D) 6                      (E) 7
32. Soh Yung is 3 times as old as her sister Tu Yung. In 4 years Soh will only be twice as old as Tu. What will the sum of their ages be in 10 years?
- (A) 14                      (B) 16                      (C) 22                      (D) 26                      (E) 36

33. Find the area of the shaded figure.



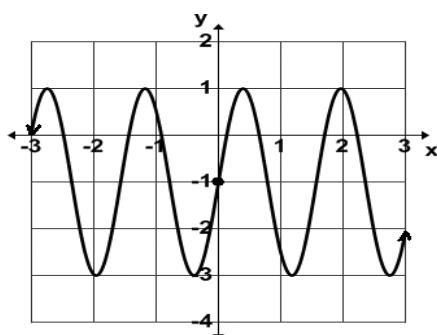
- (A) 11.5 units<sup>2</sup>                      (B) 14 units<sup>2</sup>                      (C) 17.5 units<sup>2</sup>                      (D) 18 units<sup>2</sup>                      (E) 19 units<sup>2</sup>
34. The length of the base of  $\triangle PQR$  is 40 cm. and the height is 60 cm.  $\triangle ICU$  is formed by cutting off 25% of the base of  $\triangle PQR$  and adding 20% of the height of  $\triangle PQR$ . The area of  $\triangle ICU$  is what percent of  $\triangle PQR$ ?
- (A) 55%                      (B)  $69\frac{4}{9}\%$                       (C) 80%                      (D)  $83\frac{1}{3}\%$                       (E) 90%

35. *PurtyDurty* detergent contains 80% soap and 20% bleach. *WishyWashy* detergent contains 55% soap and 45% bleach. If *PurtyDurty* is mixed with *WishyWashy*, what percent of the mixture should be *PurtyDurty* if the final mixture is 35% bleach?
- (A) 60%      (B) 40%      (C) 35%      (D) 32%      (E) 10%
36. A circle with its center at the origin of the Cartesian x-y coordinate system has a radius of 3 units. If you start at  $(-3, 0)$  and travel on the circle  $\frac{8\pi}{3}$  radians in a clockwise direction, where on the x-y coordinate plane will you stop at?
- (A) Quadrant III    (B) Quadrant IV    (C) Quadrant I    (D) Quadrant II    (E) y-axis
37. Rusty Pipes has a leaky pipe dripping water onto the floor forming a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm? (nearest  $\text{cm}^2/\text{min}$ )
- (A)  $126 \text{ cm}^2/\text{min}$     (B)  $20 \text{ cm}^2/\text{min}$     (C)  $314 \text{ cm}^2/\text{min}$     (D)  $157 \text{ cm}^2/\text{min}$     (E)  $63 \text{ cm}^2/\text{min}$
38. Sir Vayor used his theodolite to measure the height of the tower to be 17 ft 4" tall. His theodolite was 5 ft from the level ground. What angle  $x$  did he use to compute the height? (nearest second)
- 
- (A)  $42^\circ 21' 04''$     (B)  $26^\circ 43' 15''$     (C)  $35^\circ 16' 44''$     (D)  $41^\circ 38' 01''$     (E)  $24^\circ 30' 00''$
39. Betty Chuzrite selects one letter from each of the sets  $\{a, c, u, t, e\}$  and  $\{o, t, u, s, e\}$ . What is the probability she selects one vowel? (nearest whole percent)
- (A) 92%      (B) 84%      (C) 67%      (D) 60%      (E) 48%
40. Betty Chuzrite selects one letter from each of the sets  $\{a, c, u, t, e\}$  and  $\{o, t, u, s, e\}$ . What is the probability she selects at least one vowel? (nearest whole percent)
- (A) 92%      (B) 84%      (C) 67%      (D) 60%      (E) 48%
41. Betty Chuzrite selects one letter from each of the sets  $\{a, c, u, t, e\}$  and  $\{o, t, u, s, e\}$ . What is the probability she selects two vowels given that a vowel is chosen from the first set? (nearest whole percent)
- (A) 92%      (B) 84%      (C) 67%      (D) 60%      (E) 48%
42.  $236_7 + 453_7 - 165_7 = \underline{\hspace{2cm}}_7$ .
- (A) 263      (B) 512      (C) 524      (D) 412      (E) 563

43. Let P and Q be the roots of  $4x^2 + 17x = 15$ . Find  $(P + Q)(PQ)$ .
- (A)  $-63.75$  (B) 8 (C)  $-81.25$  (D) 0.5 (E) 15.9375
44. Let  $\begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ . Find  $a + b + c + d$ .
- (A)  $-19$  (B)  $-7$  (C)  $-4$  (D) 5 (E) 7
45. Let  $f(x) = ax^2 + bx + 5$  where a and b are integers. If  $f(1) = 2$  and  $f(2) = 3$ , then  $f(3) = ?$
- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9
46. Find  $a + b + c + d$  given the Fibonacci characteristic sequence: a,  $-3$ , b,  $-1$ , c, 0, d, 1, ...
- (A) 9 (B) 8 (C) 7 (D) 6 (E) 4
47. Evaluate:  $\prod_{n=2}^6 (-1)^{n-2} - n$
- (A) 0 (B)  $-5$  (C)  $-6$  (D)  $-19$  (E)  $-360$
48. Let  $f''(x) = 12x - 6$ ,  $f'(0) = 4$ , and  $f(0) = -5$ . Find  $f(1)$ .
- (A)  $-2$  (B)  $-1$  (C) 3 (D) 4 (E) 6
49. How many distinct 4-letter code words can be made from the letters in the word ALGEBRA?
- (A) 500 (B) 480 (C) 420 (D) 360 (E) 70
50. Find the sum of the first three Mersenne Primes such that the sum is a prime but is not a Mersenne Prime.
- (A) 29 (B) 31 (C) 37 (D) 41 (E) 43
51. 8,051 is the product of the two prime factors. The sum of these two prime factors is?
- (A) 86 (B) 131 (C) 164 (D) 177 (E) 180
52. Eratosthenes sifted out all of the prime numbers less than 100. How many of these primes contained the digit 9?
- (A) 6 (B) 9 (C) 5 (D) 8 (E) 7
53. Which of the following is/are not function(s)?
- I.  $\{(2,6), (-3,6), (4,9), (2,10)\}$  II.  $\{(1,3), (2,3), (3,3), (4,3)\}$  III.  $\{(-2,2), (-1,1), (0,0), (1,1)\}$
- (A) I only (B) II only (C) I & II (D) II & III (E) all of them

54. Which of the following points does not lie on the line containing the point  $(-2, -3)$  and having a slope of  $-1.5$ ?
- (A)  $(-5, 1.5)$     (B)  $(-8, 6)$     (C)  $(7, -15.5)$     (D)  $(4, -12)$     (E)  $(9, -19.5)$

55. The graph of  $f(x)$  shown below has a frequency of  $0.6366197\dots$ . Find  $f(5.7)$ . (nearest tenth)



- (A)  $-2.1$     (B)  $-2.4$     (C)  $-2.5$     (D)  $-2.6$     (E)  $-2.8$
56. Let  $S = \{7, 11, 18\}$ . The arithmetic mean of  $S$  is  $A$ . The geometric mean of  $S$  is  $G$ . And the harmonic mean of  $S$  is  $H$ . Which of the following is the correct order of the means  $A$ ,  $G$ , and  $H$  from least to greatest?
- (A)  $H, A, G$     (B)  $A, G, H$     (C)  $G, A, H$     (D)  $H, G, A$     (E)  $A, H, G$
57. Les Avridge had quiz grades of 75, 83, 66, 90, 83, 50, 65, and 83. The average of the arithmetic mean, median, mode, and range of his quiz grades is? (nearest whole number)
- (A) 62    (B) 69    (C) 71    (D) 72    (E) 80
58. Let function  $f$  be defined as  $f(x) = 2x - 6$  for all real numbers.  
Let function  $g$  be defined as follows for all integers such that  $-3 \leq x \leq 3$ :

$x$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$
$g(x)$	12	15	16	15	12	7	0

Which of the following is true about both functions  $f$  and  $g$ ?

- (A) They reach their maximum value at the same domain value  
 (B) They have the same y-intercept    (C) They are both odd functions  
 (D) They share an x-intercept    (E) none of these are true
59. Let  $f(x) = |x - 5|$ . How many of the following statements are always true?
- a.  $\lim_{x \rightarrow 5^+} f(x)$  exists    b.  $\lim_{x \rightarrow 5^-} f(x)$  exists    c.  $f(x)$  is continuous    d.  $f(x)$  is differentiable

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

60. Expand  $10^B \div (10^{(2B)} - 10^B - 1)$  for  $B = 2$ . What is the 20<sup>th</sup> digit after the decimal place?

- (A) 4    (B) 5    (C) 7    (D) 8    (E) 9



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**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitation A • 2019  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. C  | 21. E | 41. D |
| 2. B  | 22. A | 42. C |
| 3. A  | 23. C | 43. E |
| 4. E  | 24. D | 44. B |
| 5. D  | 25. B | 45. D |
| 6. C  | 26. E | 46. A |
| 7. B  | 27. C | 47. E |
| 8. C  | 28. E | 48. A |
| 9. B  | 29. A | 49. B |
| 10. D | 30. D | 50. D |
| 11. B | 31. E | 51. E |
| 12. A | 32. E | 52. A |
| 13. E | 33. D | 53. A |
| 14. C | 34. E | 54. C |
| 15. A | 35. B | 55. B |
| 16. A | 36. C | 56. D |
| 17. C | 37. A | 57. B |
| 18. C | 38. B | 58. D |
| 19. D | 39. E | 59. D |
| 20. C | 40. B | 60. B |

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
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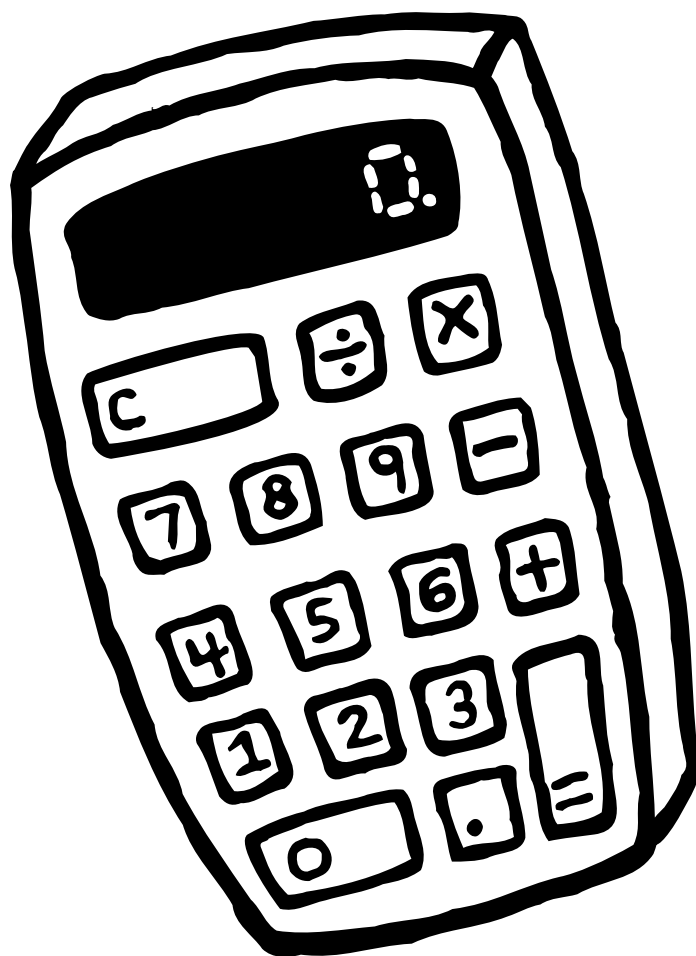
- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

Invitational B • 2019



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1. Evaluate:  $2 + 15 \div 3 - 16 \div 20 + 1 \times 9$

- (A)  $-9.2$       (B)  $-4.8$       (C)  $4.35$       (D)  $15.2$       (E)  $63.2$

2. Les Cash bought five radios at the local flea market at a cost of \$50.00 each. He sold three of them making a 20% profit and the other two were sold at a 10% loss. How much did Les net from his sales?

- (A) \$30.00      (B) \$20.00      (C) \$15.00      (D) \$10.00      (E) \$5.00

3. Let  $U = \{0, 1, 2, 3, 5, 6, 9\}$  be the universal set,  $J = \{1, 2, 5\}$ , and  $F = \{1, 3, 6\}$ . The number of elements in  $\{J' \cap F'\}$ , where  $J'$  and  $F'$  denote the complement sets.

- (A) 0      (B) 1      (C) 2      (D) 3      (E) 7

4. Let  $(2x - 3)^3 = ax^3 + bx^2 + cx + d$ . Find  $a + b + c + d$ .

- (A)  $-17$       (B)  $-1$       (C) 3      (D) 125      (E) 216

5. Let  $27x^3 + 27x^2 + 9x + 1 = (3x + 1)(ax^2 + bx + c)$ . Find  $a + b + c$ .

- (A) 27      (B) 22      (C) 18      (D) 16      (E) 4

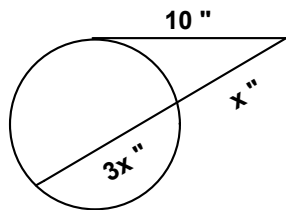
6. Mr. Ruiz sold tickets for the local one-act play. He sold 15 more adult tickets than children tickets and he sold three times as many senior tickets as children tickets. In total, he sold 300 tickets. How many adult tickets did Mr. Ruiz sell?

- (A) 24      (B) 57      (C) 72      (D) 129      (E) 171

7. If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is/are:

- (A) supplementary      (B) equal      (C) complementary      (D) acute      (E) obtuse

8. Given the tangent and secant shown, find  $x$ . (nearest tenth)



- (A) 2.5 "      (B) 1.6"      (C) 5.0 "      (D) 5.8 "      (E) 1.8 "

9. Horace Troff bought a water tank for his cattle. The tank was in the shape of a rectangular prism without the top. It was 3 feet deep, 2 feet wide, and 8 feet long. How many gallons of water would it take to fill it to the top without spilling over?

- (A) 279 gal      (B) 299 gal      (C) 311 gal      (D) 359 gal      (E) 478 gal

10. Find the 5<sup>th</sup> term of row 10.

				1						row 0
				1		1				row 1
			1		2		1			row 2
		1		3		3		1		row 3
	1		4		6		4		1	row 4
					...					

- (A) 120      (B) 126      (C) 210      (D) 252      (E) 330

11. The *Lick'em Slow* lollipop company package 5 lollipops per pack. The company has chocolate, raspberry, coconut, grape, lime, and licorice lollipops. How many different packs of 5 lollipops can they package?

- (A) 252      (B) 720      (C) 42      (D) 720      (E) 210

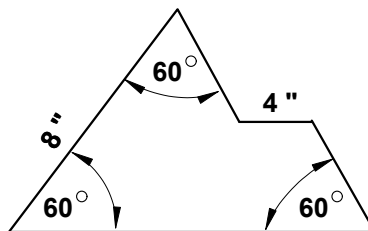
12. Nicole Taas is going to flip a coin three times and record the results. What is the probability she gets at least one head? (nearest whole percent)

- (A) 38%      (B) 50%      (C) 67%      (D) 75%      (E) 88%

13. Which of the following mathematicians is noted for work on conic sections and the construction of astrolabes used for navigation?

- (A) Aryabhata      (B) Noether      (C) Agnesi      (D) Theano      (E) Hypatia

14. Find the perimeter this pentagon?



- (A) 24"      (B) 28"      (C) 32"      (D) 36"      (E) not enough data

15. 4 bushels + 2 pecks + 1 gallon + 3 quarts = \_\_\_\_\_ pints.

- (A) 128      (B) 256      (C) 286      (D) 302      (E) 512

16. Find the range of the function  $f(x) = 2 - 3|x + 4|$ .

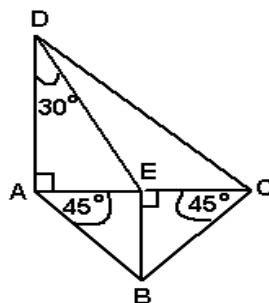
- (A)  $-4 \leq y \leq 2$       (B)  $y \geq 2$       (C)  $-4 \geq y \geq 4$       (D)  $y \leq 2$       (E)  $y \leq -3$

17. Which of the following are the side lengths of an obtuse triangle?

- (A) 6, 8, 9      (B) 5, 6, 7      (C) 4, 4,  $4\sqrt{2}$       (D)  $3, 3\sqrt{3}, 6$       (E) 8, 8, 12

18. Let  $2x - y = 5$  and  $3x + y = 6$ . Find  $20x + 19y$ .
- (A) 32.6      (B) 5      (C) 19.5      (D) 63      (E) 1.6
19. The expression  $(\sin \theta + \cos \theta)^2 - 1$  is equivalent to:
- (A)  $\sin 2\theta$       (B)  $\sin \theta \cos \theta$       (C) 0      (D)  $\cos^2 \theta - 1$       (E)  $\sin^2 \theta + 1$
20. Let  $f(x) = |x^2 - 7x + 10|$ . Find the sum of the local maximum and minimum values.
- (A)  $5\frac{3}{4}$       (B)  $5\frac{1}{2}$       (C)  $4\frac{1}{4}$       (D)  $3\frac{1}{2}$       (E)  $2\frac{1}{4}$
21. Let  $f''(x) = 6x + 12$ ,  $f'(-1) = 0$ , and  $f(1) = 12$ . Find  $f(-2)$ .
- (A) 10      (B) 2      (C) 0      (D) -2      (E) -6
22.  $(613_8 - 316_8) \times 4_8 = \underline{\hspace{2cm}}_8$ .
- (A) 1100      (B) 1110      (C) 1210      (D) 1332      (E) 1364
23. Find the greatest common divisor of 270, 504, and 882.
- (A) 21      (B) 18      (C) 9      (D) 6      (E) 2
24.  $2153A16B19 \div 11$  has a remainder of 6. Find  $A - B$ .
- (A) 2      (B) 3      (C) 5      (D) 7      (E) 9
25. Which of the following points of concurrency is on a side of a right triangle but not a vertex point, on the interior of an acute triangle, and on the exterior of an obtuse triangle?
- (A) centroid      (B) circumcenter      (C) incenter      (D) orthocenter      (E) none of these
26. The roots of  $x^4 + x^3 - 7x^2 - x + 6 = 0$  are p, q, r, and s.  
Find  $(p + q + r + s) + (pqr + pqs + prs + qrs) - (pqrs)$ .
- (A) 7      (B) -1      (C) -6      (D) -8      (E) -14
27. Andy Foundette knows of an odd integer between 600 and 800 that is divisible by both 7 and 9. He added the digits of this number. What sum did Andy get?
- (A) 7      (B) 9      (C) 14      (D) 16      (E) 18
28. Which point is the reflection of the point  $(-7, 5)$  over  $y = -x$ ?
- (A)  $(-5, 7)$       (B)  $(-7, 5)$       (C)  $(5, -7)$       (D)  $(7, -5)$       (E)  $(7, 5)$

29. Find DC if  $CE = 5''$ .



- (A)  $5\sqrt{7}$  in      (B) 10 in      (C)  $7\sqrt{5}$  in      (D)  $5\sqrt{3}$  in      (E)  $3\sqrt{7}$  in

30. Points P and R are on a circle with center C such that  $m\angle PCR = 94^\circ$ . Point Q lies outside of the circle such that QP and QR are tangent to the circle. Find  $m\angle PQR$ .

- (A)  $94^\circ$       (B)  $90^\circ$       (C)  $88^\circ$       (D)  $86^\circ$       (E)  $84^\circ$

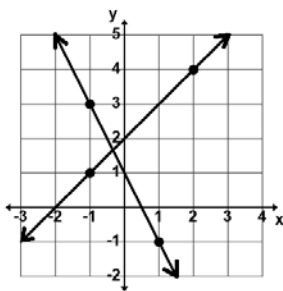
31. Nicole Taas is going to flip a coin three times and record the results. What are the odds against her getting exactly two heads?

- (A) 5:3      (B) 4:4      (C) 5:8      (D) 3:8      (E) 3:5

32. The number 215 is a member of which of the following sets of special types of numbers:  
(E)vil      (H)appy      (O)dious      (U)nhappy

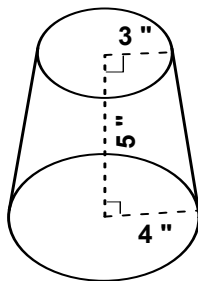
- (A) H & O      (B) E & U      (C) O & U      (D) E & H      (E) none of these

33. The point of intersection of the two lines shown is (h, k). Find  $h + k$ .



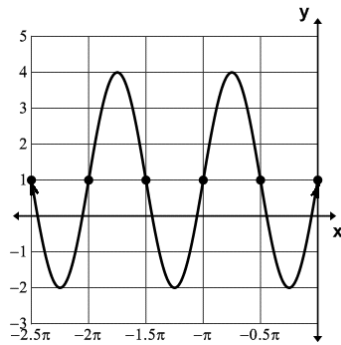
- (A)  $1\frac{1}{3}$       (B)  $1\frac{5}{12}$       (C)  $1\frac{1}{2}$       (D)  $1\frac{7}{12}$       (E) 2

34. Find the volume of the figure shown. (nearest tenth)



- (A)  $754.0 \text{ in}^3$       (B)  $36.7 \text{ in}^3$       (C)  $185.0 \text{ in}^3$       (D)  $581.2 \text{ in}^3$       (E)  $193.7 \text{ in}^3$

35. Which of the following equations is represented by the graph shown?



(A)  $\cos(2x + \frac{\pi}{2}) + 3$

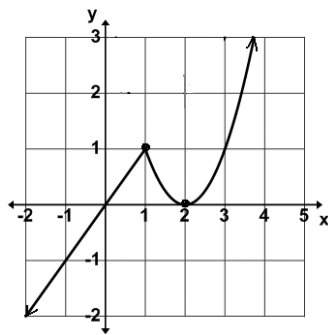
(B)  $3\sin(\frac{\pi}{2}x - \pi) + 1$

(C)  $3\cos(2x - \frac{\pi}{2}) + 1$

(D)  $3\sin(2x - \pi) - 1$

(E)  $\cos(2x - \pi) - 3$

36. The graph of  $f(x)$  is shown. For what values of  $x$  is  $f(x)$  differentiable?



(A)  $1 < x < 2$

(B)  $x < 1$  and  $x > 1$

(C)  $x < 2$  and  $x > 2$

(D) all values of  $x$

(E) no values of  $x$

37. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 7<sup>th</sup> number in row 8 and the 8<sup>th</sup> number in row 9.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				( ... )

(A) 128

(B) 130

(C) 132

(D) 134

(E) 136

38. Find  $f(4) - f(0) + f(1)$  if  $f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ x + 4 & \text{if } 1 \leq x \leq 4 \\ 1 - x & \text{if } x > 4 \end{cases}$

(A) 8

(B) 9

(C) 12

(D) 14

(E) 17



39. Let  $\begin{bmatrix} a & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & b \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 33 & 46 \end{bmatrix}$ . Find  $a + b$ .
- (A) 61                      (B) 52                      (C) 18                      (D) 10                      (E) 11
40. Find the sum of the  $x$ -values in  $\left\{x \mid 2\cos^4(x) - 3\cos^2(x) + 1 = 0, x \in \left[-\pi, \frac{3\pi}{2}\right]\right\}$ .  
(nearest hundredth)
- (A) 3.14                      (B) 3.93                      (C) 4.71                      (D) 5.50                      (E) 8.64
41. Which of these trapezoidal means are used for finding the volume of a frustrum of a cone?
- (A) Geometric              (B) Heronian              (C) Centroidal              (D) Arithmetic              (E) Harmonic
42. A parabola has a vertical axis of symmetry with a vertex at  $(1, 4)$  and focus at  $(1, 2)$ . Find the equation of the directrix of this parabola.
- (A)  $y = 3$                       (B)  $y = \frac{1}{4}$                       (C)  $y = 6$                       (D)  $y = \frac{1}{2}$                       (E)  $y = 8$
43. N. A. Hurry stops at a convenience store. The probability that she buys a loaf of bread is 60%, the probability she buys a gallon of milk is 50%, and the probability she buys both bread and milk is 30%. What is the probability she will buy either bread or milk or both?
- (A) 20%                      (B) 40%                      (C) 70%                      (D) 80%                      (E) 100%
44. Find the digit in the hundred-thousandth place of the sum of  $1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \dots$ .
- (A) 8                      (B) 6                      (C) 5                      (D) 3                      (E) 0
45. Nicole Taas is going to flip a coin three times and record the results. What is the probability she gets at least two tails given that the first flip was a tail.      (nearest whole percent)
- (A) 38%                      (B) 50%                      (C) 67%                      (D) 75%                      (E) 88%
46. Les Stickum has \$60.00 to buy some 48-cent stamps and some 1-dollar stamps. He has to buy twice as many 48-cent stamps as 1-dollar stamps. What is the greatest number of 48-cent stamps can he buy?
- (A) 96                      (B) 62                      (C) 60                      (D) 48                      (E) 30
47. The *Wind Gust* electric company erected a wind turbine 1500 yards from the main station on a bearing of  $110^\circ$ . They erected a second wind turbine 800 yards from the main station on a bearing of  $300^\circ$ . How far apart were the two turbines? (nearest yard)
- (A) 2,105 yds              (B) 2,229 yds              (C) 2,292 yds              (D) 2,300 yds              (E) 2,490 yds

48. Kanyu Emahjun changed the rectangular point  $(-1, 4)$  to the polar point  $(r, \theta)$ . Kanyu found the sum of  $r$  and  $\theta$  to be: (nearest whole number)
- (A) 6                      (B) 5                      (C) 76                      (D) 3                      (E) 80
49. Bill Defense is fencing in a non-square rectangular area of 3,200 square feet. The cost of the fencing for two sides of the rectangle will cost \$1.00 per foot and the other two sides will cost \$2.00 per foot. What is the lowest possible cost for the fence?
- (A) \$80.00              (B) \$160.00              (C) \$320.00              (D) \$356.00              (E) \$400.00
50. Let  $e^{(3x+2)} = 4e^{(x-5)}$ . Find  $e^{(x)}$ . (nearest hundredth)
- (A) 0.02                      (B) 0.06                      (C) 0.07                      (D) 0.22                      (E) 0.24
51. If  $A + B = 14$  and  $A \times B = 26$ , then  $|B - A| = \underline{\hspace{2cm}}$ .
- (A)  $7 + \sqrt{23}$               (B)  $4\sqrt{23}$                       (C)  $7 - \sqrt{23}$                       (D)  $\sqrt{23}$                       (E)  $2\sqrt{23}$
52. How many points of intersection are there for the curves  $r = 1 + 3\cos \theta$  and  $\theta = \frac{1}{3}$ ?
- (A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4
53.  $(2x^2 + kx + 1) \div (x + 3)$  has a remainder of 43. Find  $k$ .
- (A)  $-8$                       (B)  $-3$                       (C) 5                      (D)  $14\frac{1}{3}$                       (E)  $21\frac{1}{3}$
54. Let  $f(x) = \frac{2x^3 + x^2 + 11x + 5}{x^2 + 5}$  and  $s(x)$  be the slant asymptote of  $f$ . Find the value of  $s(-3)$ .
- (A)  $-163$                       (B)  $-9\frac{5}{6}$                       (C)  $-5\frac{3}{14}$                       (D)  $-5$                       (E)  $-3$
55. Given:  $f$  is a continuous function on the interval  $[0, 2]$  such that  $\int_0^2 f(x) dx = 5$ . Find  $\int_0^1 f(2y) dy$ .
- (A) 10                      (B) 5                      (C) 2.5                      (D) 1.25                      (E) 0.625
56. How many distinct 4-letter code words can be made from the letters in the words "PIZZA PIE" if the first letter must be a vowel and the second letter must be a consonant?
- (A) 54                      (B) 88                      (C) 98                      (D) 120                      (E) 354
57. Expand  $10^B \div (10^{(2B)} - 10^B - 1)$  for  $B = 3$ . What is the 21<sup>st</sup> digit after the decimal place?
- (A) 0                      (B) 1                      (C) 3                      (D) 5                      (E) 8

58. Given:  $x^2y + xy^2 + x + y = 63$  and  $xy = 6$ . Find  $x^2 + y^2$

- (A) 99                      (B) 69                      (C) 57                      (D) 54                      (E) 7

59. Three integers,  $p$ ,  $q$ , and  $r$  exist such that they form an arithmetic progression and their product is a prime number. Find the absolute value difference of the smallest and largest of the three integers.

- (A) 0                      (B) 2                      (C) 3                      (D) 4                      (E) 5

60. Given:  $25! \div 5^k$  is an integer. What is the greatest value of  $k$ ?

- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

**DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST**

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitation B • 2019  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. D  | 21. E | 41. B |
| 2. B  | 22. E | 42. C |
| 3. C  | 23. B | 43. D |
| 4. B  | 24. C | 44. D |
| 5. D  | 25. B | 45. D |
| 6. C  | 26. C | 46. C |
| 7. A  | 27. E | 47. C |
| 8. C  | 28. A | 48. A |
| 9. D  | 29. A | 49. C |
| 10. C | 30. D | 50. B |
| 11. A | 31. A | 51. E |
| 12. E | 32. B | 52. D |
| 13. E | 33. A | 53. A |
| 14. C | 34. E | 54. D |
| 15. D | 35. C | 55. C |
| 16. D | 36. B | 56. C |
| 17. E | 37. A | 57. C |
| 18. A | 38. D | 58. B |
| 19. A | 39. D | 59. D |
| 20. E | 40. B | 60. E |

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
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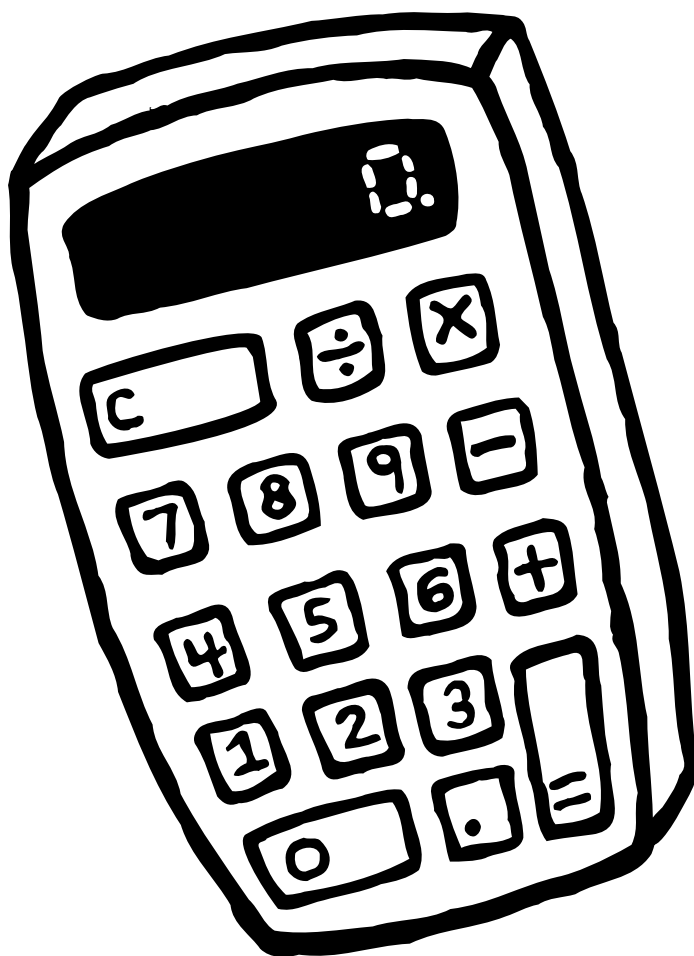
- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

District • 2019



DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $3 + 2 \times 5 - 3 + 0! \times 2^0 \div (1 + 9)$

- (A) 2.3                      (B) 10                      (C) 10.1                      (D) 10.2                      (E) 32

2. The local Country Bumpkin grocery is having a sale on all canned vegetables. Willie Byette needs to purchase 10 cans of vegetables. Which of the following is the least expensive deal, assuming all of the cans are of the same size and quality?

- (A) regular price at 59¢ each                      (B) \$2.75 for a 4-pack and get one free  
(C) 65¢ each less 15% off coupon                      (D) \$8.00 for a 12-pack  
(E) 3 for \$1.75

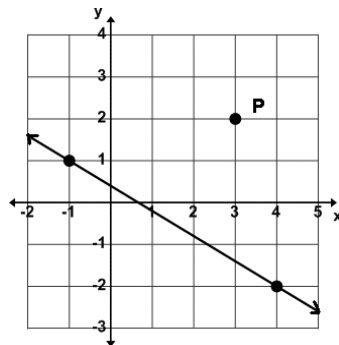
3. The sales tax on \$24.00 is \$1.98. What would the total cost including sales tax be on \$118.50? (nearest cent)

- (A) \$130.62                      (B) \$128.28                      (C) \$126.75                      (D) \$119.58                      (E) \$119.33

4. Let  $m = \{m, a, r, c, h\}$ ,  $M = \{m, o, n, d, a, y\}$ , and  $F = \{f, r, i, d, a, y\}$ . The number of elements in  $(m \cap M) \cup (m \cap F)$  is:

- (A) 1                      (B) 3                      (C) 4                      (D) 5                      (E) 6

5. Line  $m$  through point P is perpendicular to the line shown. Which of the following points lie on line  $m$  in quadrant III



- (A)  $(-3, -8)$                       (B)  $(-1, -5)$                       (C)  $(-1, -2)$                       (D)  $(-5, -2)$                       (E)  $(-3, -2)$

6. Let  $(5x - 1)^3 = ax^3 + bx^2 + cx + d$ . Find  $a + b + c + d$ .

- (A) 216                      (B) 184                      (C) 114                      (D) 65                      (E) 64

7. Which of the following is/are examples of a function?

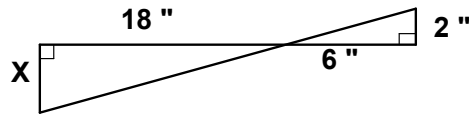
I.  $\{(2,3), (3,4), (4,5), (5,6)\}$     II.  $\{(1,1), (2,3), (3,5), (5,8)\}$     III.  $\{(-2, -1), (-1, 2), (0,0), (3,3)\}$

- (A) I only                      (B) II only                      (C) I & II                      (D) II & III                      (E) all of them

8. Ester Buney divided 4 dozen eggs among Huey, Duey, and Luey. She gave Luey three times as many eggs as Huey, and she gave Duey 3 more eggs than Huey. How many eggs did Duey get?

- (A) 16                      (B) 15                      (C) 13                      (D) 12                      (E) 9

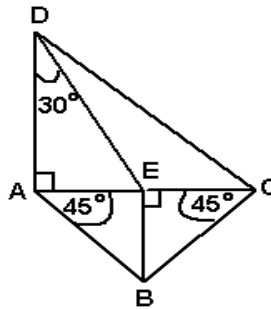
9. Find  $x$ . (drawing is not to scale)



- (A) 9 "      (B) 6 "      (C) 1.5 "      (D)  $\frac{2}{3}$  "      (E) cannot be determined
10. Which of the following points of concurrency is on the right triangle vertex point, on the interior of an acute triangle, and on the exterior of an obtuse triangle?
- (A) centroid    (B) circumcenter    (C) incenter    (D) orthocenter    (E) none of these
11. A graph of a relation has a line of symmetry of  $x = 3$ . The graph passes through the point  $(-1, 7)$ . What is the  $x$ -coordinate of another point that must have a  $y$ -coordinate of 7?
- (A)  $-6$       (B)  $-5$       (C) 2      (D) 7      (E) 10
12. A right cylinder can of *Bugs* carrots has a lateral surface area of  $72 \text{ in}^2$  and a height of 4 in. What is the surface area of the base of the can of carrots? (nearest tenth)
- (A)  $32.8 \text{ in}^2$     (B)  $31.4 \text{ in}^2$     (C)  $28.3 \text{ in}^2$     (D)  $26.7 \text{ in}^2$     (E)  $25.8 \text{ in}^2$
13. Let  $3x - 2y = 4$  and  $x + 3y = 5$ . Find  $3x - 3y$ .
- (A) 0      (B)  $-17\frac{4}{7}$       (C) 3      (D)  $-5\frac{8}{11}$       (E) 6
14. Saul Would can cut a cord of wood in 2 hours. Tim Burr can cut a cord of wood in 1 hour and 20 minutes. How long would it take them to cut a cord if they work together?
- (A) 40 min    (B) 48 min    (C) 52 min    (D) 60 min    (E) 100 min
15. Let function  $f = \{(8, 5), (2, 8), (11, 4), (5, 6)\}$  and function  $g = \{(4, 5), (8, 0), (6, 3), (0, 8)\}$ . Find  $(f \circ g)(4) - (g \circ f)(5)$  ?
- (A)  $-1$       (B) 1      (C) 3      (D) 5      (E) 9
16. Determine the range of  $f(x) = 2 - 3\sin(5x + 7)$ .
- (A)  $[-1, 1]$     (B)  $[-5, -1]$     (C)  $[-3, 2]$     (D)  $[-2, 3]$     (E)  $[-1, 5]$
17. If  $f(x) = 3x^2 - 2x + 5$  and  $g(x) = 5x^2 - 3x + 2$ , then  $g(f(-1)) = ?$
- (A) 472      (B) 5      (C) 559      (D) 4      (E) 346
18.  $(3x^2 + kx + 2) \div (x + 5)$  has a remainder of 32. Find  $k$ .
- (A)  $-5$       (B)  $-3$       (C) 5      (D) 7      (E) 9

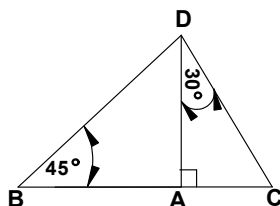


19. Find the perimeter of the quadrilateral ABED if  $AE = 20$  in. (nearest inch)



- (A) 96 in      (B) 117 in      (C) 120 in      (D) 123 in      (E) 126 in
20. Find the sum of the coefficients of the  $x^4y^2$  term,  $x^3y^3$  term, and  $x^2y^4$  term in the expansion of  $(2x + y)^6$ .
- (A) 460      (B) 520      (C) 60      (D) 180      (E) 2,560
21. Let  $f''(x) = -6$ ,  $f'(2) = -1$ , and  $f(-1) = -6$ . Find  $f(0)$ .
- (A) 16      (B)  $-1$       (C) 8      (D) 0      (E)  $-13$
22. Max E. Mumm is fencing in a non-square rectangular area of 1,800 square feet. The cost of the fencing is \$7.00 per foot for the two short sides and \$5.00 per foot for the two long sides. What is the lowest possible cost for the fence?
- (A) \$1004.00      (B) \$1018.23      (C) \$1020.00      (D) \$1030.00      (E) \$1180.00
23. Cash Droor has five \$20.00 bills, three \$10.00 bills, and two \$5.00 bills. If he selects two bills at random what is the probability that one is a \$20.00 bill and one is a \$10.00 bill? (nearest whole percent)
- (A) 6%      (B) 11%      (C) 17%      (D) 33%      (E) 67%
24. Principal Rewler selected 5 teachers, 8 students, and 4 parents at his school to form an advisory committee. How many different 6-member committees can he form if 2 are teachers, 3 are students, and 1 is a parent?
- (A) 960      (B) 1,120      (C) 2,016      (D) 2,240      (E) 12,376
25. 113 is an emirp prime. How many emirp primes less than 113 exist?
- (A) 7      (B) 8      (C) 9      (D) 10      (E) 11
26. Which of the following mathematicians proved that the number of elements of the infinite set of real numbers is greater than the number of elements of the infinite set of natural numbers?
- (A) Georg Cantor      (B) Marin Mersenne      (C) Franciscus Vieta  
(D) Leonard Euler      (E) Aryabhata

27. Using dry volume measurements: 3 bushels — 1 peck + 2 gallon — 1 quarts = \_\_\_\_\_ pints.
- (A) 46                      (B) 92                      (C) 174                      (D) 190                      (E) 200
28. The least common multiple of 315, 525, and 735 minus the greatest common factor of 315, 525, and 735 is?
- (A) 1,470                      (B) 2,205                      (C) 1,675                      (D) 10,920                      (E) 11,130
29. Let  $9x^2 - 9x - 4 = (ax + b)(cx + d)$ . Find  $a + b - c + d$ .
- (A) -5                      (B) -4                      (C) -3                      (D) 3                      (E) 10
30. Find  $f(2) + f(3) + f(-5)$  if  $f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ 3x & \text{if } 0 \leq x \leq 3 \\ 1 - 4x & \text{if } x > 3 \end{cases}$
- (A) 14                      (B) 4                      (C) 0                      (D) -11                      (E) -16
31. The area of  $\triangle ABD$  contains how many more square feet than the area of  $\triangle ACD$  if  $AC = 3$  ft. (nearest tenth)

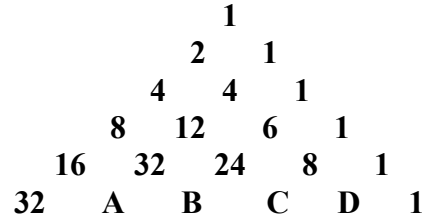


- (A) 8.4 ft<sup>2</sup>                      (B) 1.8 ft<sup>2</sup>                      (C) 5.7 ft<sup>2</sup>                      (D) 7.8 ft<sup>2</sup>                      (E) 13.5 ft<sup>2</sup>
32. An obtuse angle is bisected. One of the acute angles created by bisecting the original obtuse angle is trisected. One of the smallest acute angles is bisected. The measure of the smallest angle is 12°. What was the angle measure of the original obtuse angle?
- (A) 144°                      (B) 132°                      (C) 120°                      (D) 112°                      (E) 108°
33. Let  $A = \begin{bmatrix} -1 & 4 \\ 3 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 7 \\ 5 & -8 \end{bmatrix}$ . If  $AB = \begin{bmatrix} d & e \\ f & g \end{bmatrix}$  then  $ef = ?$
- (A) -1,404                      (B) -486                      (C) 648                      (D) 918                      (E) 1,053
34. If  $a_1 = -2$ ,  $a_2 = 0$ ,  $a_3 = 2$  and  $a_n = a_{n-2} - a_{n-1} + a_{n-3}$ , where  $n \geq 4$ , then  $a_7$  equals:
- (A) 10                      (B) 2                      (C) 0                      (D) -4                      (E) -8
35. The expression  $\cos \theta \div (1 - \sin \theta) - \tan \theta$  is equivalent to:
- (A)  $\sec \theta$                       (B)  $\frac{\cos \theta}{1 - \sin \theta}$                       (C)  $\cot \theta$                       (D)  $\frac{1}{\csc \theta}$                       (E)  $\frac{1}{\sec \theta}$

36. Saul T. Water anchored a buoy 200 feet from his pier on a bearing of  $320^\circ$ . He places a second buoy 150 feet from his pier on a bearing less than  $90^\circ$ . What bearing would the second buoy have to be placed in order for the buoys to be 300 feet apart? (nearest degree)

(A)  $84^\circ$       (B)  $77^\circ$       (C)  $67^\circ$       (D)  $50^\circ$       (E)  $40^\circ$

37. Find  $A + B + C + D$  if the triangular pattern shown below continues.



(A) 248      (B) 236      (C) 224      (D) 210      (E) 202

38. Les Cash invested his inheritance in a mutual fund. At the end of the first year the fund decreased 7%. The fund increased 2% at the end of the second year, then decreased 5% at the end year three. What was Les' average rate of return over the four year period if the fund increased 3% the fourth year? (nearest tenth)

(A)  $-1.8\%$       (B)  $-1.0\%$       (C)  $+1.0\%$       (D)  $+2.2\%$       (E)  $+3.4\%$

39. Find the slope of the line tangent to  $f(x) = 2x^2 - 13x + 5$  at  $x = 3$ .

(A) 4      (B) 3      (C) 0      (D)  $-1$       (E)  $-4$

40. Let  $f(x) = \frac{x^2 - 5}{x + 1}$ . How many of the following statements are always true?

I.  $\lim_{x \rightarrow -1^+} f(x)$  exists

II.  $\lim_{x \rightarrow -1^-} f(x)$  exists

III.  $f(x)$  is continuous

IV.  $f(x)$  is differentiable at  $x = 1$

(A) 0      (B) 1      (C) 2      (D) 3      (E) 4

41. Willie Chuzrite selects one number from each of the sets  $\{1, 2, 3, 5, 7\}$  and  $\{0, 4, 6, 8, 9\}$ . What is the probability he selects two numbers totaling a sum greater than 10 given that a prime number is chosen from the first set? (nearest whole percent)

(A) 33%      (B) 40%      (C) 50%      (D) 60%      (E) 67%

42. Roland Bones tosses a pair of dice. What are the odds the sum of the dots (pips) on the top faces is greater than seven?

(A) 5:7      (B) 5:11      (C) 1:7      (D) 7:12      (E) 5:12

43. 1,088 is the  $k^{\text{th}}$  term of the sequence 0, 3, 8, 15, 24, 35, 48, ... . Find  $k$ .

(A) 23      (B) 27      (C) 31      (D) 33      (E) 37

44.  $(642_7 - 135_7) \times 6_7 = \underline{\hspace{2cm}}_7$ .
- (A) 4303      (B) 4233      (C) 3313      (D) 3042      (E) 3024
45. If  $\sqrt[5]{x} \sqrt[3]{x} \sqrt{x} = \sqrt[n]{x^k}$ , where  $k$  and  $n$  are relatively prime, then  $k + n = ?$
- (A) 24      (B) 22      (C) 19      (D) 15      (E) 13
46. Let  $(-1 + \sqrt{3}i)^{-8} = (a + bi)$ . Find  $a + b$ . (nearest ten-thousandth)
- (A) .0082      (B) .0054      (C) .0032      (D) .0018      (E) .0014
47. An odd number  $N$  exists such that  $500 < N < 600$  and  $N$  is divisible by both 7 and 11. The sum of the digits of the number  $N$  is?
- (A) 18      (B) 17      (C) 16      (D) 15      (E) 14
48.  $\int \sin(x) \cos(x) dx = ?$
- (A)  $-\cos(x) \sin(x) + C$       (B)  $\frac{\sin^2(x)}{4} + C$       (C)  $-\frac{\cos(2x)}{4} + C$   
 (D)  $\cos^2(x) + C$       (E)  $-\frac{\cos^2(2x)}{4} + C$
49. The *Mixed Fruit* shop sells fruit bowls consisting of 6 pieces of fruit. The shop has oranges, bananas, apples, grapes, peaches, pears, and plums. How many different fruit bowls can they make to sell?
- (A) 1,716      (B) 132      (C) 5,040      (D) 720      (E) 924
50. Suppose the radius of a right circular cylinder is decreasing at the rate of 4 feet per minute, while the height is increasing at the rate of 2 feet per minute. Find the rate of change in the volume when the radius is 2 feet and the height is 6 feet. (nearest  $\frac{\text{ft}^3}{\text{min}}$ )
- (A)  $-75 \frac{\text{ft}^3}{\text{min}}$       (B)  $-101 \frac{\text{ft}^3}{\text{min}}$       (C)  $-125 \frac{\text{ft}^3}{\text{min}}$       (D)  $-151 \frac{\text{ft}^3}{\text{min}}$       (E)  $-276 \frac{\text{ft}^3}{\text{min}}$
51. The sequence 1, 5, 6, 11, 17, ... is a Fibonacci characteristic sequence, where 1 is the first term. Find the sum of the first 11 terms.
- (A) 789      (B) 798      (C) 799      (D) 804      (E) 880
52. Find the area of the region bounded by the curve  $y = x^3$ ,  $x = -2$ , and  $x = 1$ .
- (A) 4.75 sq. units      (B) 4.25 sq. units      (C) 4.125 sq. units      (D) 3.875 sq. units      (E) 3.75 sq. units
53. The arithmetic mean of 3, 25, and 30, is 19.3333. Which of the following trapezoidal means of 3, 25, and 30 would be 7.4 if rounded to the nearest tenth?
- (A) Geometric      (B) Heronian      (C) Centroidal      (D) Contraharmonic      (E) Harmonic

54. Find the sum of the  $x$ -values in  $\left\{x \mid 5\cos^2(x) = 4 - 3\sin^2(x), x \in [0, \frac{3\pi}{2}] \right\}$ . (nearest hundredth)
- (A) 6.44            (B) 7.07            (C) 7.23            (D) 7.854            (E) 8.02
55. The harmonic mean of the roots of  $x^4 - 13x^3 + 56x^2 - 92x + 48 = 0$  is ? (nearest tenth)
- (A) 3.4            (B) 3.3            (C) 2.6            (D) 2.3            (E) 2.1
56. The medians of a triangle are concurrent. The length of one of the medians is 24 cm. What is the length of the segment joining the midpoint of a side of the triangle to the point of concurrency?
- (A) 16 cm            (B) 12 cm            (C) 8 cm            (D) 6 cm            (E) 4 cm
57. Let  $\frac{a-b}{3} = 1.5$ ,  $\frac{a-c}{2} = 3$ , and  $\frac{b+c}{5} = \frac{2}{5}$ . Find the arithmetic mean of  $a$ ,  $b$ , and  $c$ .
- (A)  $2\frac{3}{4}$             (B)  $2\frac{7}{12}$             (C)  $1\frac{1}{4}$             (D)  $1\frac{5}{12}$             (E)  $1\frac{19}{30}$
58.  $32P3Q2019 \div 9$  has a remainder of 7. Find the least value of  $P + Q$ .
- (A) 9            (B) 8            (C) 7            (D) 6            (E) 5
59. Expand  $10^B \div (10^{(2B)} - 10^B - 1)$  for  $B = 4$ . What is the 24<sup>th</sup> digit after the decimal place?
- (A) 0            (B) 1            (C) 3            (D) 5            (E) 8
60. How many distinct 5-letter code words can be made from the letters in the words HYPATIA?
- (A) 2,520            (B) 1,320            (C) 1,020            (D) 720            (E) 120

**DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST**

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • District • 2019  
Answer Key**

<b>1. C</b>	<b>21. C</b>	<b>41. C</b>
<b>2. B</b>	<b>22. No Correct Answer</b>	<b>42. A</b>
<b>3. B</b>	<b>23. D</b>	<b>43. D</b>
<b>4. B</b>	<b>24. D</b>	<b>44. B</b>
<b>5. A</b>	<b>25. C</b>	<b>45. E</b>
<b>6. E</b>	<b>26. A</b>	<b>46. E</b>
<b>7. E</b>	<b>27. D</b>	<b>47. B</b>
<b>8. D</b>	<b>28. D</b>	<b>48. C</b>
<b>9. B</b>	<b>29. C</b>	<b>49. E</b>
<b>10. D</b>	<b>30. B</b>	<b>50. E</b>
<b>11. D</b>	<b>31. C</b>	<b>51. D</b>
<b>12. E</b>	<b>32. A</b>	<b>52. B</b>
<b>13. C</b>	<b>33. A</b>	<b>53. E</b>
<b>14. B</b>	<b>34. A</b>	<b>54. B</b>
<b>15. C</b>	<b>35. A</b>	<b>55. E</b>
<b>16. E</b>	<b>36. B</b>	<b>56. C</b>
<b>17. A</b>	<b>37. D</b>	<b>57. A</b>
<b>18. E</b>	<b>38. A</b>	<b>58. E</b>
<b>19. D</b>	<b>39. D</b>	<b>59. E</b>
<b>20. A</b>	<b>40. B</b>	<b>60. B</b>

University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final	_____	_____
2nd	_____	_____
1st	_____	_____
Score		Initials

Contestant # _____	Conference _____
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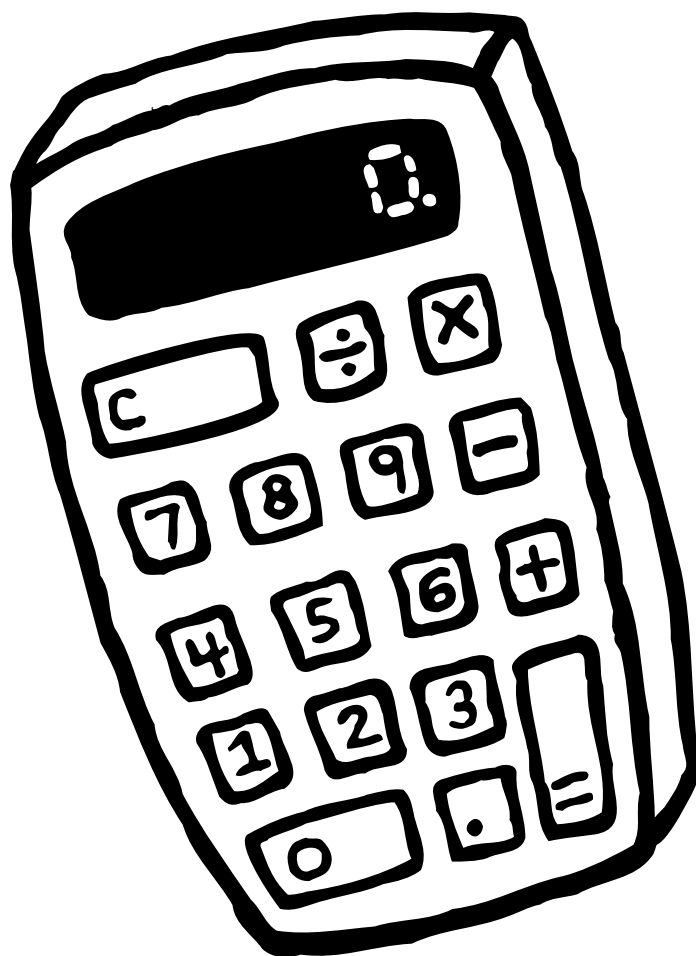
- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

Region • 2019



DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!



1. Evaluate:  $4 - 1 \times 3 + (2 + 0!) \div 1 \times 9$

- (A)  $1\frac{1}{3}$       (B)  $1\frac{2}{9}$       (C) 19      (D) 28      (E) 36

2. 1 rod + 12 yards — 5.5 feet + 3 inches = \_\_\_\_\_ inches.

- (A) 963      (B) 825      (C) 729      (D) 567      (E) 477

3. Travlin Tex's weekly salary consists of a base salary plus commission. He gets \$500.00 base pay, 20% commission on sales up to and including \$500.00, and 25% on sales over \$500.00. His total sales for the last week of March was \$2019.00. How much was his salary for last week?

- (A) \$979.75      (B) \$997.75      (C) \$1008.55      (D) \$1,104.75      (E) \$1,133.55

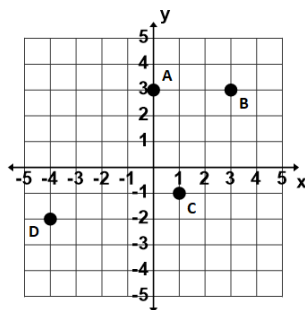
4. The digits 4, 1, 3, 1, and 9 are each used once to form the smallest possible 5-digit odd number. What is the digit in the hundreds place?

- (A) 9      (B) 4      (C) 3      (D) 1      (E) 0

5. The sequence 3, 7, 10, 17, 27, ... is a Fibonacci characteristic sequence, where 3 is the first term. Find the sum of the first ten terms.

- (A) 759      (B) 770      (C) 781      (D) 792      (E) 797

6. The slope of the line containing the points A and C is  $m$ . The slope of the line containing the points B and D is  $n$ . Find the absolute value difference in the slopes.



- (A)  $\frac{3}{4}$       (B)  $\frac{1}{5}$       (C)  $2\frac{3}{5}$       (D)  $3\frac{2}{7}$       (E)  $4\frac{5}{7}$

7. Let  $14x^2 + x - 4 = (ax + b)(cx + d)$ . Find  $a + b + c + d$ .

- (A) 14      (B) 12      (C) 11      (D) 6      (E) 5

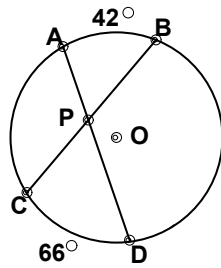
8. The *Cheep Seets* ticket outlet sells adult tickets for \$3.50 each and student tickets for \$1.50 each. 250 tickets were sold totaling \$505.00. How many more student tickets were sold than adult tickets?

- (A) 120      (B) 185      (C) 130      (D) \$0.60      (E) 65

9.  $4 + (1 + 3) = (4 + 1) + 3$  is an example of the \_\_\_\_\_ property of addition.

- (A) associative      (B) commutative      (C) distributive      (D) identity      (E) inverse

10. Given the circle with center O, find  $m\angle BPD$ . (drawing is not to scale)



- (A)  $108^\circ$       (B)  $138^\circ$       (C)  $136^\circ$       (D)  $114^\circ$       (E)  $126^\circ$

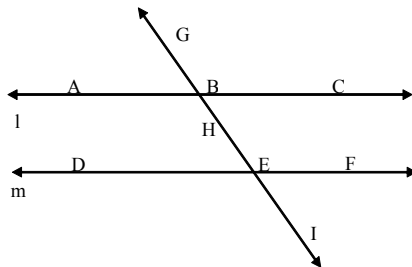
11. The Euler line is a line that cannot be determined in which of the following types of triangles?

- (A) scalene      (B) obtuse      (C) right      (D) isosceles      (E) equilateral

12. A rectangle with integral sides has an area of 80 sq. inches and a perimeter of 42 inches. Two inches are cut from both the length and the width. The area of the new rectangle is how much less than the original rectangle?

- (A)  $30 \text{ in}^2$       (B)  $34 \text{ in}^2$       (C)  $38 \text{ in}^2$       (D)  $40 \text{ in}^2$       (E)  $42 \text{ in}^2$

13. The lines in the figure are coplanar with  $m \parallel l$ . Which of the following are true statements?



1.  $\angle BEF$  &  $\angle BED$  are supplementary  
3.  $\angle CBH \cong \angle DEH$

2.  $m\angle GBA = m\angle IEF$   
4.  $\angle ABG$  &  $\angle DEH$  are complementary

- (A) 1 only      (B) 1 & 2      (C) 2 & 3      (D) 1, 2, & 3      (E) 1, 2, 3, & 4

14. If  $\frac{2x+3}{x-4} + \frac{5x-7}{2x+3} = \frac{ax^2+bx+c}{dx^2+ex+f}$ , then  $a - b + c - d + e - f$  equals:

- (A) 12      (B) 14      (C) 33      (D) 42      (E) 66

15. If  $a_1 = 4$ ,  $a_2 = -1$ ,  $a_3 = 3$  and  $a_n = (a_{n-3} + a_{n-1})(a_{n-2})$ , where  $n \geq 4$ , then  $a_6$  equals:

- (A)  $-189$       (B)  $-24$       (C) 147      (D) 216      (E) 243

16. Dee Loper and Les Spede workout on the same track. Dee runs at an average rate of 5 miles per hour. Les runs at an average rate of 3.5 miles per hour. Les starts to run at 8:00 AM and Dee starts to run at 8:30 AM. At what time will Dee catch up to Les?

- (A) 9:30 AM      (B) 9:40 AM      (C) 10:00 AM      (D) 10:10 AM      (E) 10:30 AM

17. Penney Flipper tossed a penny four times and record the results, heads or tails. What are the odds that she got precisely three heads or three tails?

- (A) 1:2      (B) 5:3      (C) 2:1      (D) 1:1      (E) 3:8

18. The expression  $\frac{\sin \theta}{1 - \cos \theta} - \cot \theta$  is equivalent to:

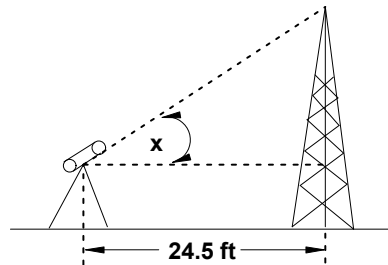
- (A)  $\csc \theta$       (B)  $\tan \theta$       (C)  $\sec \theta$       (D)  $\frac{1}{2} \sin \theta$       (E)  $\sin 2\theta$

19. Given:  $\cos \alpha = -\frac{3}{4}$ ,  $\frac{\pi}{2} < \alpha < \pi$  and  $\cos \beta = \frac{2}{3}$ ,  $\frac{3\pi}{2} < \beta < 2\pi$ .

Evaluate:  $\sin(\alpha + \beta)$  (nearest thousandth)

- (A)  $-0.118$       (B)  $-0.401$       (C)  $0.450$       (D)  $0.599$       (E)  $1.000$

20. Sir Vayor used his theodolite to measure the height of the tower to be 20 ft 3 inches tall. His theodolite was 5 ft 6 inches from the level ground. What angle  $x$  did he use to compute the height? (nearest minute)



- (A)  $15^\circ 20'$       (B)  $31^\circ 3'$       (C)  $37^\circ 0'$       (D)  $39^\circ 34'$       (E)  $46^\circ 25'$

21. The harmonic mean of the roots of  $x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120 = 0$  is? (nearest tenth)

- (A) 1.6      (B) 2.2      (C) 2.7      (D) 3.0      (E) 3.5

22. Find the sum of the 3<sup>rd</sup> term and the 11<sup>th</sup> term of row 12.

			1				row 0
		1		1			row 1
	1		2		1		row 2
	1	3		3	1		row 3
	1	4	6	4	1		row 4

...

- (A) 132      (B) 136      (C) 144      (D) 168      (E) 180

23.  $(214_7 \times 5_7 - 413_7) \div 6_7$  has a remainder of \_\_\_\_\_.

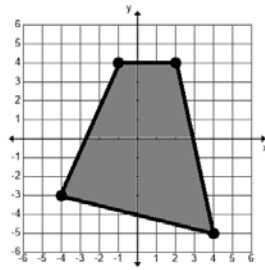
- (A) 0      (B) 1      (C) 2      (D) 3      (E) 5

24. Find the area bounded by the curves  $y = \sin(x)$  and  $y = \cos(x)$ , where  $0 \leq x \leq 2\pi$ .

- (A)  $2\sqrt{2}$       (B)  $\frac{\sqrt{3}}{2}$       (C)  $\sqrt{2}$       (D) 1      (E) 0

25. Let  $f(x) = \frac{3}{\sqrt{2x+1}}$ . Find  $f'(12)$ .
- (A)  $-0.024$       (B)  $-0.012$       (C)  $-0.006$       (D)  $0.6$       (E)  $1.666\dots$
26. What is the slope of the secant line to the graph of  $f(x) = 2 - 3x - 4x^2$  passing through the points  $(-2, p)$  and  $(2, q)$ ?
- (A)  $7$       (B)  $4$       (C)  $-1$       (D)  $-2$       (E)  $-3$
27. How many distinct 4-letter code words can be made from the letters in the words "ODIOUS"?
- (A)  $360$       (B)  $216$       (C)  $192$       (D)  $180$       (E)  $120$
28. Find the 30<sup>th</sup> term given the sequence:  $3, 8, 15, 24, 35, 48, \dots$
- (A)  $899$       (B)  $915$       (C)  $930$       (D)  $960$       (E)  $1,030$
29. Let  $U = \{r, e, g, i, o, n, m, a, t, h\}$  be the universal set,  $G = \{g, r, e, a, t\}$ , and  $T = \{t, i, m, e\}$ . The number of elements in  $\{G \cup T\}'$ , where the symbol,  $'$ , denotes the complement set.
- (A)  $1$       (B)  $3$       (C)  $5$       (D)  $6$       (E)  $7$
30. How many teaspoons will it take to get  $\frac{3}{4}$  cup?
- (A)  $16$  tsp      (B)  $18$  tsp      (C)  $32$  tsp      (D)  $36$  tsp      (E)  $48$  tsp
31. Let  $(ax + d)^3 = 8x^3 + bx^2 + cx + 1$ . Find  $a + b + c + d$ .
- (A)  $17$       (B)  $18$       (C)  $21$       (D)  $27$       (E)  $81$
32. Given:  $x^2y - xy^2 - x + y = 56$  and  $xy = 8$ . Find  $x^2 + y^2$
- (A)  $48$       (B)  $64$       (C)  $65$       (D)  $80$       (E)  $81$
33. Which of the following's most notable accomplishment was conceiving the ideas of differential and integral calculus?
- (A) Franciscus Vieta      (B) Christian Goldbach      (C) Leonardo Bigollo  
(D) Claudius Ptolemy      (E) Gottfried Leibniz
34. Let  $x + 2y = 3$ ,  $3y - z = 2$ , and  $2x + z = 3$ . Find  $z$ .
- (A)  $1$       (B)  $2\frac{1}{3}$       (C)  $3$       (D)  $5\frac{2}{3}$       (E)  $7$
35. Let  $f(x) = x + 1$  and  $g(x) = 2x - 1$  and  $h(x) = -x - 2$ . Find  $h(f(g(x - 1) + 1) - x)$
- (A)  $2x - 1$       (B)  $3x$       (C)  $3x - 1$       (D)  $-2x + 1$       (E)  $-x - 1$

36. Find the area of the shaded quadrilateral with integral vertices.



- (A)  $44.5 \text{ units}^2$  (B)  $43 \text{ units}^2$  (C)  $42.5 \text{ units}^2$  (D)  $41.5 \text{ units}^2$  (E)  $40 \text{ units}^2$

37. Brad flew his new plane 87 miles from Dumas to Perryton on a bearing of  $70^\circ$ . Then he flew 98 miles from Perryton to Shamrock on a bearing of  $115^\circ$ . What distance would he fly to return directly to Dumas? (nearest mile)

- (A) 171 miles (B) 152 miles (C) 138 miles (D) 131 miles (E) 125 miles

38. Find the sum of the x-values of the points where  $y = \cos 2x$  intersects with  $y = 2\cos x$  and  $0 \leq x \leq 2\pi$ . (nearest hundredth)

- (A) 2.39 (B) 4.33 (C) 6.28 (D) 7.48 (E) 8.29

39. Let  $f(x) = |2x - 1| + |x - 2|$ . Find the minimum value of  $f(x)$ .

- (A)  $-2.5$  (B)  $-2$  (C) 0.5 (D) 1.5 (E) 2

40. Determine the value of  $5 + 5(\frac{2}{3}) + 5(\frac{2}{3})^2 + 5(\frac{2}{3})^3 + \dots + 5(\frac{2}{3})^{15}$  to the nearest thousandths.

- (A) 14.977 (B) 14.965 (C) 14.949 (D) 14.923 (E) 14.917

41. Roland Bones tosses a pair of dice. What is the probability the sum of the top faces is greater than 7 and the number of dots (pips) on each die is a Fibonacci number? (nearest whole percent)

- (A) 6% (B) 8% (C) 13% (D) 15% (E) 31%

42.  $(413_8 - 312_4) \times 11_2 = \underline{\hspace{2cm}}_{10}$ .

- (A) 639 (B) 303 (C) 23 (D) 248 (E) 1,111

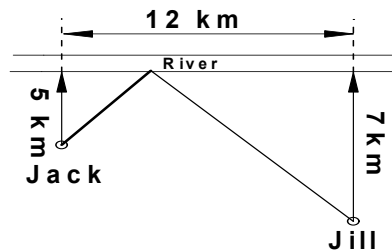
43. Les Arria inherited 3 "leagues" of land. He had to sell 2 "labors" of land to pay the inheritance tax. How many total acres did he have left? (nearest integer)

- (A) 13,639 acres (B) 12,931 acres (C) 12,754 acres (D) 12,657 acres (E) 8,502 acres

44. Find the range of the function  $f(x) = 4 - 3|2x - 4|$ , where  $0 \leq x \leq 4$ .

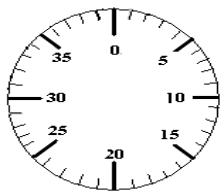
- (A)  $2 \geq y \geq -2$  (B)  $y \geq -4$  (C)  $-8 \leq y \leq 4$  (D)  $y \leq 4$  (E)  $y \geq -8$

45. Jack flew his drone to the river to fill a pail of water and delivered it to Jill. The drone flew the minimum distance possible. How much shorter would the trip be if he had water at his house and the drone took it directly to Jill? (nearest hundredth)



- (A) 4.24 km      (B) 4.81 km      (C) 5.92 km      (D) 6.00 km      (E) 6.72 km
46. A baseball pitcher's earned-run average varies directly with the number of earned runs and inversely as the number of innings pitched. Willy Tossett had an earned-run average 1.8, giving up 18 runs in 90 innings. Based on his average, how many earned runs would he give up having pitched 240 inning?
- (A) 36      (B) 42      (C) 45      (D) 48      (E) 54
47. Point P (0, 3) lies in the x-y plane. Point P is rotated  $90^\circ$  clockwise about the point  $(-2, 1)$  to Point Q. Point Q is reflected across the line  $x = 3$  to point R. Point R is translated vertically 4 units up to point S (x, y). Find  $x + y$ .
- (A) 10 units      (B) 9 units      (C) 7 units      (D) 5 units      (E) 3 units
48. How many of the following are the side lengths of an acute triangle?  
 I. 3,  $3\sqrt{3}$ , 6    II. 7, 9, 11    III. 8, 9, 20    IV. 12, 13, 14
- (A) 1      (B) 2      (C) 3      (D) all of them      (E) none of them
49. The geometric mean of 4, 13, and N is 9.96 (rounded off to the nearest hundredth). The harmonic mean of 4, 13, and N is? (rounded off to the nearest hundredth)
- (A) 12.00      (B) 9.04      (C) 8.72      (D) 7.96      (E) 7.90
50. Given:  $f(x) = 4 - 13\sin(20x + 19)$ . If the amplitude is decreased by 5, the phase shift is doubled, the displacement is increased by 2, and the frequency is divided by 2 then the sum of the amplitude, phase shift, displacement and frequency is? (nearest integer)
- (A) 9      (B) 12      (C) 17      (D) 20      (E) 26
51. Three bags labeled 1, 2, and 3 each contain four ping pong balls labeled 1, 2, 3, and 4.  
 I. M. Kahnfuzed randomly selects one ball from each of the bags. What is the probability of getting three different numbered balls given that the first ball chosen is an even number? (nearest whole percent)
- (A) 50%      (B) 38%      (C)  $33\frac{1}{3}\%$       (D) 22%      (E) 11%

52. Let  $A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 6 \\ -4 & k \end{bmatrix}$ . Find  $k$  if the determinant of  $A - B$  is 33.
- (A)  $-23$  (B)  $-22$  (C)  $8$  (D)  $9$  (E)  $45$
53. Let  $P$  and  $Q$  be two prime numbers such that  $P + Q = 150$  and  $PQ = 5,561$ . Find  $|P - Q|$ .
- (A)  $8$  (B)  $12$  (C)  $16$  (D)  $24$  (E)  $28$
54. Which of the following functions have a horizontal, a vertical, and an oblique asymptote?
- I.  $f(x) = \frac{x^2 - 1}{x^2 + 3x + 2}$  II.  $f(x) = \frac{x^2 - 3x - 4}{x + 3}$  III.  $f(x) = \frac{x^3 + 2x^2 - x - 3}{x^2 - 2x + 1}$
- (A) I & III (B) III only (C) II & III (D) I, II, & III (E) none of these
55. Willie Skor had 6-wk test grades of 88, 91, 65, 73, and 84. He wants to make a high enough grade on his last 6-wk test to average 80. What will the median of his six 6-wk grades be if his average is exactly 80?
- (A)  $79$  (B)  $80$  (C)  $81.5$  (D)  $83.5$  (E)  $85$
56. The function  $g(x) = x^3 + x^2 - 8x - 2$  has a minimum at  $(x, y)$  and a maximum at  $(x, z)$  on the interval  $[-2, 2]$ . Find  $y + z$ . (nearest tenth)
- (A)  $8.0$  (B)  $2.5$  (C)  $0$  (D)  $1.5$  (E)  $6.0$
57. For what values of  $x$  does the series  $x^n(n^2 + 3)^{-\left(\frac{1}{2}\right)}$  converge?
- (A)  $-1 \leq x < 3$  (B)  $0 \leq x < 3$  (C)  $-.5 \leq x < 3$  (D)  $1 \leq x < 3$  (E)  $-1 \leq x < 1$
58. Forty students took the regional math test last year. Twelve of them were 9th graders. All of them had an equal chance to go to state as one of the three medal winners or as one of the two alternates. What was the probability that at least one 9th grader made it to state? (nearest whole percent)
- (A)  $1\%$  (B)  $13\%$  (C)  $30\%$  (D)  $67\%$  (E)  $85\%$
59. What is the sum of the digits of the least emirp prime that is greater than 100?
- (A)  $7$  (B)  $8$  (C)  $9$  (D)  $10$  (E)  $11$
60. T-S-P is the combination needed to open the lock box with the combination dial shown below. How many distinct combinations exist if T is a triangular number, S is a square number, and P is a pentagonal number?



- (A)  $384$  (B)  $240$  (C)  $117$  (D)  $21$  (E)  $19$

**DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST**

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Regional • 2019  
Answer Key**

1. D	21. B	41. B
2. D	22. A	42. A
3. A	23. D	43. B
4. C	24. A	44. C
5. C	25. A	45. B
6. E	26. E	46. D
7. B	27. C	47. B
8. A	28. D	48. B
9. A	29. B	49. E
10. E	30. D	50. C
11. E	31. C	51. B
12. C	32. D	52. D
13. D	33. E	53. C
14. E	34. A	54. E
15. C	35. E	55. C
16. B	36. A	56. D
17. D	37. A	57. E
18. A	38. C	58. E
19. E	39. D	59. B
20. B	40. A	60. B



University Interscholastic League  
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH  
CAPITAL LETTERS

Final \_\_\_\_\_  
2nd \_\_\_\_\_  
1st \_\_\_\_\_  
Score \_\_\_\_\_  
Initials \_\_\_\_\_

Contestant # \_\_\_\_\_ Conference \_\_\_\_\_

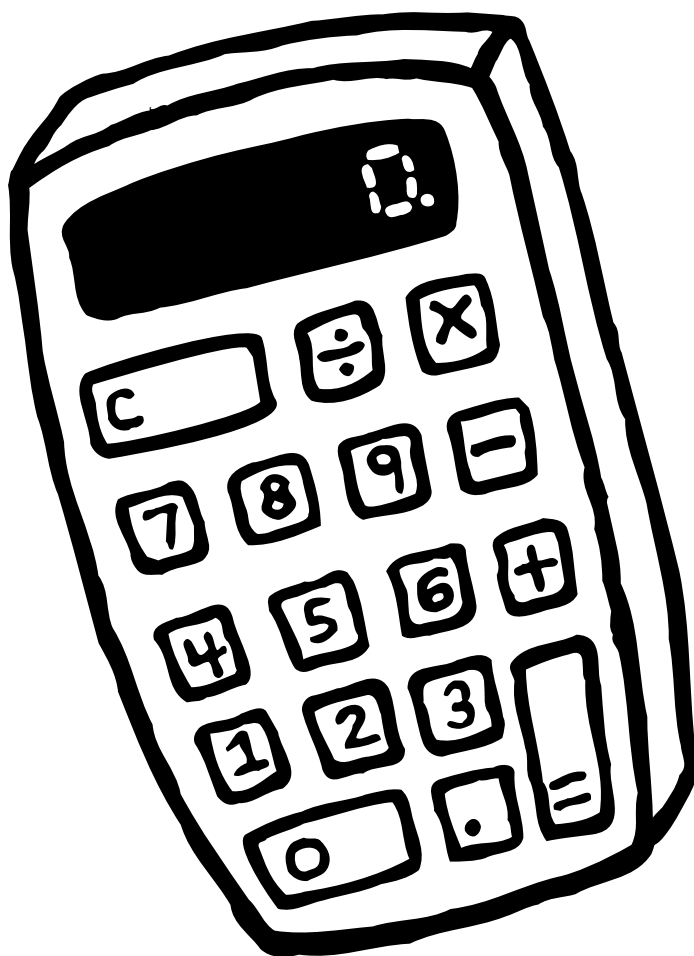
- |           |           |           |
|-----------|-----------|-----------|
| 1. _____  | 21. _____ | 41. _____ |
| 2. _____  | 22. _____ | 42. _____ |
| 3. _____  | 23. _____ | 43. _____ |
| 4. _____  | 24. _____ | 44. _____ |
| 5. _____  | 25. _____ | 45. _____ |
| 6. _____  | 26. _____ | 46. _____ |
| 7. _____  | 27. _____ | 47. _____ |
| 8. _____  | 28. _____ | 48. _____ |
| 9. _____  | 29. _____ | 49. _____ |
| 10. _____ | 30. _____ | 50. _____ |
| 11. _____ | 31. _____ | 51. _____ |
| 12. _____ | 32. _____ | 52. _____ |
| 13. _____ | 33. _____ | 53. _____ |
| 14. _____ | 34. _____ | 54. _____ |
| 15. _____ | 35. _____ | 55. _____ |
| 16. _____ | 36. _____ | 56. _____ |
| 17. _____ | 37. _____ | 57. _____ |
| 18. _____ | 38. _____ | 58. _____ |
| 19. _____ | 39. _____ | 59. _____ |
| 20. _____ | 40. _____ | 60. _____ |



UNIVERSITY INTERSCHOLASTIC LEAGUE

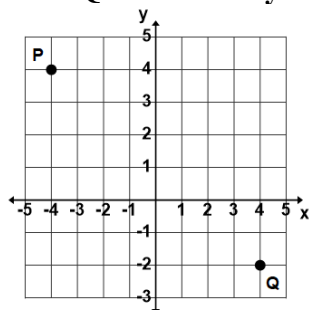
# Mathematics

State • 2019



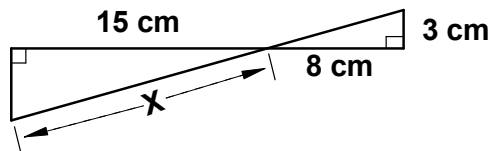
DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!

- Evaluate:  $5 - 4 \times (2! + 1) + 9 \div 3 - 5$   
 (A)  $-9$  (B)  $-1$  (C)  $0$  (D)  $1$  (E)  $9$
- Nine million five hundred two thousand three hundred five is subtracted from five hundred four million two thousand nineteen. Which of the following digits appears the most in the difference?  
 (A)  $9$  (B)  $7$  (C)  $4$  (D)  $1$  (E)  $0$
- Lotta Dowe bought four calculators at a cost of \$35.00 each while shopping at a garage sale. She sold them at her garage sale for \$60.00, \$50.00, \$45.00, and \$30.00. What percent profit did she net from these transactions? (nearest whole percent)  
 (A)  $30\%$  (B)  $32\%$  (C)  $45\%$  (D)  $54\%$  (E)  $68\%$
- If the sales tax on \$34.75 is \$2.52, what would it cost, including tax for an item that sells for \$504.00? (nearest cent)  
 (A) \$540.55 (B) \$536.55 (C) \$517.80 (D) \$511.25 (E) \$506.52
- The line containing points P and Q crosses the y-axis at  $(0, y)$  and the x-axis at  $(x, 0)$ . Find  $x + y$ .

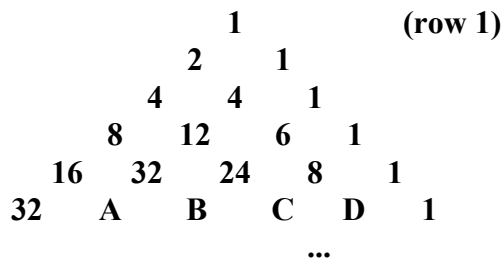


- (A)  $-1\frac{1}{3}$  (B)  $-\frac{2}{3}$  (C)  $\frac{1}{3}$  (D)  $1\frac{2}{3}$  (E)  $2\frac{1}{3}$
- $\frac{3-5}{4} = \frac{3}{4} - \frac{5}{4}$  is an example of the \_\_\_\_\_ property of addition.  
 (A) associative (B) commutative (C) distributive (D) identity (E) inverse
- Let  $(5x + 3)(4x - 1)(2x) = ax^3 + bx^2 + cx + d$ . Find  $a + b + c + d$ .  
 (A) 20 (B) 48 (C) 54 (D) 60 (E) not enough information
- Simplify:  $\left(\frac{x^2 - 2x - 15}{x^2 + 2x - 3}\right) \div \left(\frac{x^2 - 2x - 8}{x^2 + x - 2}\right)$ .  
 (A)  $\frac{x-5}{x-4}$  (B)  $\frac{x+5}{x-4}$  (C)  $x + 4$  (D)  $x - 5$  (E)  $\frac{x+5}{x+4}$

9. Find  $x$  to the nearest tenth. (drawing is not to scale)



- (A) 15.6 cm    (B) 16.0 cm    (C) 16.7 cm    (D) 17.0 cm    (E) cannot be determined
10. If two chords of a circle are unequal in length and neither chord is the diameter, then the longer chord is \_\_\_\_\_ the center of the circle.
- (A) nearer to    (B) equal distance to    (C) through    (D) below    (E) farther from
11. Line  $m$  contains the points A  $(-5, 3)$  and B  $(4, -5)$ . Which of the following points would be the midpoint of segment AB?
- (A)  $(-1, 0)$     (B)  $(-0.5, -0.5)$     (C)  $(-1, -1)$     (D)  $(-0.5, -1)$     (E)  $(0, -1.5)$
12. Horace Tank bought a rectangular prism shaped trough to water his sheep. It was 2 feet deep, 3 feet wide, and 12 feet long. He used a water hose to put 350 gallons into the trough. What percent of the trough's capacity was filled with water? (nearest %)
- (A) 36%    (B) 49%    (C) 53%    (D) 60%    (E) 65%
13. Which of the following mathematicians replaced the operation of multiplication by the word "and" and addition by the word "or"?
- (A) Boole, George    (B) Cantor, Georg    (C) Descartes, Rene  
(D) Euler, Leonard    (E) Goldbach, Christian
14. Given the table of values : find  $p + q$ , where  $p, q \geq 0$ .
- |        |    |   |    |    |    |     |     |
|--------|----|---|----|----|----|-----|-----|
| $x$    | -1 | 2 | 1  | 3  | 4  | 7   | $q$ |
| $f(x)$ | -2 | 7 | -2 | 22 | 43 | $p$ | 70  |
- (A) 124    (B) 134    (C) 137    (D) 147    (E) 152
15. Given the triangular pattern shown, find the sum of the terms in row 8 minus the first and last terms.

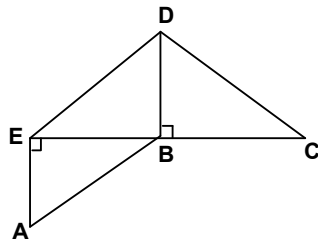


- (A) 2,058    (B) 664    (C) 1,930    (D) 2,187    (E) 6,304

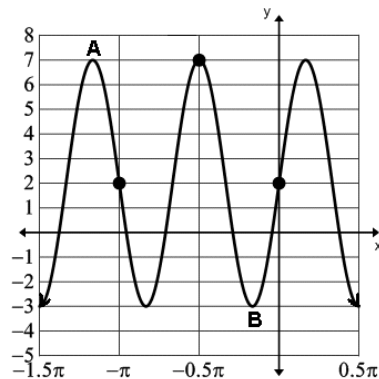
16. If the following pattern continues, find the sum of the terms on the left side of the equality sign of row 20.

$$\begin{array}{rcl} 1 + 2 = 3 & & \text{row 1} \\ 4 + 5 + 6 = 7 + 8 & & \text{row 2} \\ 9 + 10 + 11 + 12 = 13 + 14 + 15 & & \text{row 3} \end{array}$$

- (A) 8,610      (B) 8,810      (C) 8,400      (D) 8,390      (E) 8,000
17. The sequence 2, 4, 6, 10, 16, ... is a Fibonacci characteristic sequence. Find the ratio of the 22nd term to the 20th term given that 2 is the first term. (nearest tenth)
- (A) 1.62      (B) 1.89      (C) 2.62      (D) 3.43      (E) 5.24
18.  $101111101_2 + 10100100_2 - 1111001_2 = \underline{\hspace{2cm}}_8$ .
- (A) 1232      (B) 1212      (C) 1210      (D) 650      (E) 522
19.  $(503_8 \times 11_8 - 504_8) \div 7_8$  has a remainder of         .
- (A) 0      (B) 2      (C) 4      (D) 5      (E) 6
20. Given:  $m\angle BAE = 60^\circ$ ,  $m\angle BCD = 45^\circ$ ,  $BD = 5''$ , and  $\triangle ABE \cong \triangle DBE$ . Find the perimeter of the pentagon ABCDE. (nearest inch)

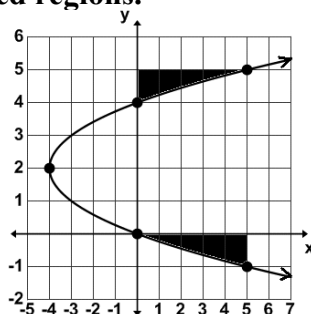


- (A) 42 in      (B) 38 in      (C) 37 in      (D) 35 in      (E) 34 in
21. The sinusoidal graph below reaches a maximum at point  $A(x_1, y_1)$  and a minimum at point  $B(x_2, y_2)$ . Find  $x_1 + x_2$ . (nearest tenth)



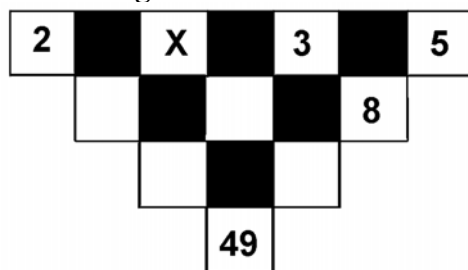
- (A) -1.6      (B) -2.1      (C) -3.1      (D) -4.2      (E) -4.7

22. Find the area of the shaded regions.



- (A)  $4\frac{5}{6}$  sq. units   (B) 5 sq. units   (C)  $5\frac{1}{6}$  sq. units   (D)  $5\frac{1}{3}$  sq. units   (E)  $5\frac{2}{3}$  sq. unit

23. The number in the unshaded box is found by adding the numbers connected with it from the row above it. (ex. 8 is found using 3 and 5. What is the value of X?)



30. If  $\sqrt{x^5 \sqrt{x^4 \sqrt{x^3}}} = \sqrt[n]{x^k}$ , where  $k$  and  $n$  are relatively prime, then  $k = ?$
- (A) 60                      (B) 39                      (C) 31                      (D) 12                      (E) 8
31. The expression  $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta$  is equivalent to:
- (A)  $\csc \theta$                       (B)  $\tan \theta$                       (C)  $\sec \theta$                       (D)  $\cot \theta$                       (E)  $\sin \theta \cos \theta$
32. Evaluate:  $\prod_{n=1}^4 [(-n)^{(n-1)} + (n-1)]$
- (A) 737                      (B) 671                      (C) 600                      (D)  $-50$                       (E)  $-74$
33. The first Germain prime of the form  $4k + 3$  is  $G$ . Find  $k$ .
- (A) 11                      (B) 7                      (C) 5                      (D) 4                      (E) 2
34. If  $\frac{d}{dx} \left( \frac{5x-4}{2x+3} \right) = \frac{a(bx+c) - b(ax-d)}{(bx+c)^2}$ , then  $a + b + c + d = ?$
- (A) 5                      (B) 6                      (C) 14                      (D) 18                      (E) 23
35. Let  $\begin{bmatrix} -2 & -3 \\ 1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} d & e \\ f & g \end{bmatrix}$  then  $f + g = ?$
- (A)  $-1$                       (B)  $-0.2$                       (C) 0.6                      (D) 0.8                      (E) 1.5
36. Let  $(1 - 2i)^5 = (a + bi)$ . Find  $a + b$ . (nearest whole number)
- (A) 35                      (B) 41                      (C) 57                      (D) 78                      (E) 79
37. Expand  $10^B \div (10^{(2B)} - 10^B - 1)$  for  $B = 3$ . What is the 35<sup>th</sup> digit after the decimal place?
- (A) 1                      (B) 3                      (C) 4                      (D) 5                      (E) 8
38. The points A, B, C, D, and E are collinear and arranged alphabetically from left to right. Find BD given the following:  $AE = 24$  cm,  $AC = 8$  cm,  $DE = 6$  cm, and  $AB = \frac{2}{3}DE$ .
- (A) 4 cm                      (B) 8 cm                      (C) 10 cm                      (D) 12cm                      (E) 14 cm
39. One hundred twenty-five students attended the invitational math/science/robotics competition. Eighty took the math test, seventy took the science test, and fifty took both math and science. How many did not take the math or science test?
- (A) 5                      (B) 15                      (C) 20                      (D) 25                      (E) 50

40. Currently, Les Cash has \$75.00 and his sister, Lotta Cash, has \$105.00. Starting next week, Les will save \$10.00 a week and Lotta will save \$6.00 a week. How many weeks will it take until Les has more money than Lotta?
- (A) 5                      (B) 7                      (C) 8                      (D) 9                      (E) 11
41. Point P (4,  $-4$ ) lies in the x-y plane. Point P is reflected across the y-axis to point Q. Point Q is rotated  $90^\circ$  clockwise about the origin to point R. Point R is translated horizontally  $+3$  units and vertically  $-5$  units to point S. Point S is reflected across the x-axis to point T (x, y). Find  $x + y$ .
- (A) 8                      (B)  $-2$                       (C)  $-5$                       (D) 2                      (E) 0
42. The *PeaKann* company sells shelled pecans for \$7.95 a pound. The *PeaNutt* company sells shelled peanuts for \$3.75 a pound. The two companies come together to create a mixture of pecans and peanuts that will sell for \$5.50 a pound. What percent of the mixture would be pecans if the bag of mixture held 3 pounds? (nearest percent)
- (A) 87%                      (B) 69%                      (C) 48%                      (D) 42%                      (E) 25%
43. Saul T. Water sailed his yacht from his Port Aransas dock 30 miles on a bearing of  $280^\circ$ . Then he turned and sailed on a bearing of  $75^\circ$ . After two hours he turned to head to his home dock on a bearing of  $150^\circ$ . How far was he from his dock when he turned for home? (nearest tenth)
- (A) 12.8 miles      (B) 13.1 miles      (C) 15.0 miles      (D) 23.8 miles      (E) 26.1 miles
44. Nicole Koin flips a nickel five times and records the results, heads or tails. What are the odds that she got at least three tails?
- (A) 1:2                      (B) 3:2                      (C) 1:1                      (D) 1:3                      (E) 2:1
45. Ken Dahl has 6 different romance novels, 4 different historical novels, and 5 different mystery novels. How many different sets of 7 novels can he form if 3 are mystery novels, 3 are romance novels, and 1 is a historical novel?
- (A) 29                      (B) 800                      (C) 6,435                      (D) 400                      (E) 14,400
46. The hour hand and the minute hand on a circular clock form a  $160^\circ$  angle at which of the following times?
- (A) 3:20                      (B) 11:44                      (C) 7:20                      (D) 5:32                      (E) 2:40
47. The roots of  $x^4 - 13x^3 + 56x^2 - 92x + 48 = 0$  are p, q, r, and s. Find  $(pqrs) + (pqr + pqs + prs + qrs) - (pq + pr + ps + qr + qs + rs)$ .
- (A) 196                      (B) 84                      (C) 12                      (D)  $-12$                       (E)  $-100$
48. How many x-values in  $\left\{x \mid 3\sin(x) - 2\cos^2(x) = 0, x \in \left[-\frac{10\pi}{3}, \frac{8\pi}{3}\right]\right\}$  exist?
- (A) 3                      (B) 5                      (C) 6                      (D) 7                      (E) 8



49. Find the sum of the coefficients of the  $x^5y^3$  term and the  $x^4y^4$  term in the expansion of  $(2x - y)^8$ .
- (A)  $-1792$       (B)  $-672$       (C)  $2,912$       (D)  $1,334$       (E)  $672$
50. The function  $f(x) = x^3 - 5x + 3$  has a minimum at  $(m, n)$  and a maximum at  $(M, N)$  on the interval  $[0, 3]$ . Find  $(m + M) - (n + N)$ . (nearest tenth)
- (A)  $-9.4$       (B)  $-12.0$       (C)  $-13.7$       (D)  $-14.6$       (E)  $-16.3$
51. Roland Decubes tosses a pair of dice twice. What is the probability the sum of the dots on the top faces of the first toss is the same as the sum on the second toss? (nearest whole percent)
- (A)  $11\%$       (B)  $14\%$       (C)  $22\%$       (D)  $36\%$       (E)  $65\%$
52. Given that the number of minutes needed to solve a set of problems varies directly as the number of problems and inversely as the number of people working on the solutions. A problem solving committee consisting of 4 people took 24 minutes to solve 12 problems. How many minutes will it take to solve 30 problems if 2 more people are added to the committee?
- (A) 32 min      (B) 40 min      (C) 48 min      (D) 56 min      (E) 60 min
53. Given:  $A = 5$  and  $B = 6$ . Which of the following trapezoidal means of A and B have the least numerical value?
- (A) Geometric      (B) Heronian      (C) Centroidal      (D) Arithmetic      (E) Harmonic
54. Points A and B are on a circle with center O and point C lies outside of the circle such that segments CA and CB are tangent to the circle. Find  $m\angle BOA$  if the  $m\angle ACB = 77^\circ$ .
- (A)  $113^\circ$       (B)  $38.5^\circ$       (C)  $123^\circ$       (D)  $83.5^\circ$       (E)  $103^\circ$
55. The *Get The Lead Out* pencil shop packages four map pencils in each pack. The shop has red leaded ones, black leaded ones, blue leaded ones, green leaded ones, and purple leaded ones. How many different packs of four pencils can they package?
- (A) 20      (B) 70      (C) 105      (D) 120      (E) 280
56. For what values of  $x$  is the expression  $\sqrt{2x - \sqrt{6 - 5x}}$  defined over the real numbers?
- (A)  $\frac{1}{2} \leq x \leq \frac{5}{6}$       (B)  $-2 \leq x \leq 1$       (C)  $\frac{1}{2} \leq x \leq 1\frac{1}{5}$       (D)  $\frac{3}{4} \leq x \leq 1\frac{1}{5}$       (E)  $x \leq \frac{3}{4}$
57. Let  $f(x) = \begin{cases} ax + b, & \text{if } x < 0 \\ 2\sin(x) + 3\cos(x) & \text{if } x \geq 0 \end{cases}$ .
- Find the value of  $a + b$  where the values of  $a$  and  $b$  exist so that  $f(x)$  is differentiable at  $x = 0$ .
- (A) 0      (B) 2      (C) 3      (D) 5      (E) 6

58. Which of the following sequences converge to 1?

1.  $a_n = \frac{\ln(n+1)}{n}$     2.  $a_n = (8)^{\frac{1}{n}}$     3.  $a_n = \frac{\ln(n)}{(n)^{\frac{1}{n}}}$     4.  $a_n = \sqrt[n]{10n}$

(A) 2, 3, & 4    (B) 2 & 4    (C) 1 & 3    (D) 1, 2, & 3    (E) none of these

59. A package of Mathlink cubes contains 5 green ones, 4 yellow ones, and 3 orange ones. Three cubes are randomly drawn from the package, without replacement. What is the probability that all are green given that the first one is not orange? (nearest whole percent)

(A) 22%    (B) 11%    (C) 10%    (D) 6%    (E) 5%

60. How many distinct 6-letter code words can be made from the letters in the word TEENAGER ?

(A) 20,188    (B) 20,160    (C) 36    (D) 3,720    (E) 12,120

**DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST**

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • State • 2019  
Answer Key**

<b>1. A</b>	<b>21. D</b>	<b>41. E</b>
<b>2. C</b>	<b>22. B</b>	<b>42. D</b>
<b>3. B</b>	<b>23. D</b>	<b>43. B</b>
<b>4. A</b>	<b>24. B</b>	<b>44. C</b>
<b>5. E</b>	<b>25. D</b>	<b>45. B</b>
<b>6. C</b>	<b>26. A</b>	<b>46. E</b>
<b>7. B</b>	<b>27. D</b>	<b>47. B</b>
<b>8. A</b>	<b>28. E</b>	<b>48. C</b>
<b>9. B</b>	<b>29. B</b>	<b>49. B</b>
<b>10. A</b>	<b>30. C</b>	<b>50. A</b>
<b>11. D</b>	<b>31. C</b>	<b>51. A</b>
<b>12. E</b>	<b>32. B</b>	<b>52. B</b>
<b>13. A</b>	<b>33. E</b>	<b>53. E</b>
<b>14. D</b>	<b>34. C</b>	<b>54. E</b>
<b>15. A</b>	<b>35. C</b>	<b>55. B</b>
<b>16. A</b>	<b>36. E</b>	<b>56. D</b>
<b>17. C</b>	<b>37. C</b>	<b>57. D</b>
<b>18. D</b>	<b>38. E</b>	<b>58. B</b>
<b>19. A</b>	<b>39. D</b>	<b>59. D</b>
<b>20. C</b>	<b>40. C</b>	<b>60. E</b>