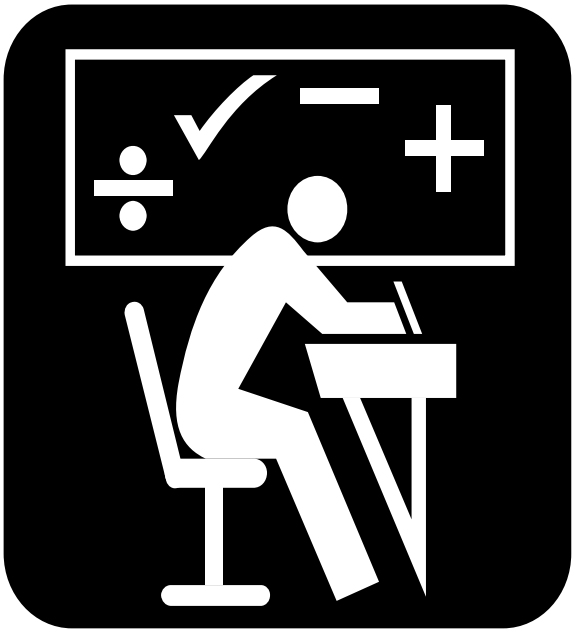
UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

SAC • 2011



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

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1. **Evaluate:** $15 - 12 \div 9 + 6 \times 3$

   (A) $-4\frac{1}{3}$ **(B)** $\frac{3}{5}$ **(C)** 19 **(D)** $31\frac{2}{3}$ **(E)** 59

2. **Brad D. Cruncher needs 4 calculators for his calculator team. The regular price is $78.90 each. A special summer sale is going on. He can buy the first two at the regular price, the third one at 30% off the regular price, and the fourth one at 40% off the regular price. What will the total cost be excluding tax? (to the nearest cent)**

   (A) $ 213.03 **(B)** $ 220.92 **(C)** $ 228.81 **(D)** $ 244.59 **(E)** $ 260.37

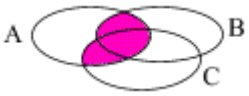
3. **Two hundred thousand three hundred four plus one million two thousand thirty has how many zeros in the sum when written out using digits instead of words?**

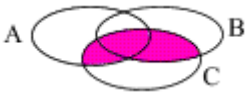
   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

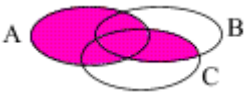
4. **If 80% of A is $\frac{5}{8}$ of B, then B is what percent of A?**

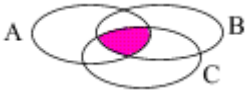
   (A) $37\frac{1}{2}\%$ **(B)** 50% **(C)** $78\frac{1}{8}\%$ **(D)** 128% **(E)** 200%

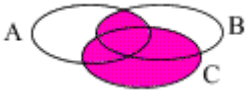
5. **In which of the following Venn diagrams does the shaded regions represent the set $(A \cap B) \cup C$?**

   (A) 

   (B) 

   (C) 

   (D) 

   (E) 

6. **Simplify:** $(x - 3)(x - 3)(x + 3)$

   (A) $x^3 - 3x^2 - 9x + 27$ **(B)** $x^3 - 9x^2 - 27x + 27$ **(C)** $x^3 + 3x^2 - 9x + 27$

   (D) $x^3 + 9x^2 - 27x + 27$ **(E)** $x^3 - 3x^2 + 9x + 27$

7. **Solve for x:** $3(2x - 1) - 2(x + 3) = 3x - 2$

   (A) 2 **(B)** 4 **(C)** 7 **(D)** -4 **(E)** -11

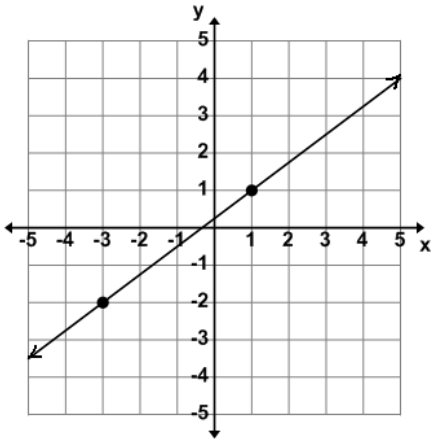
8. **An equation of a line with a y-intercept of -2 and a slope of $\frac{1}{2}$ is _____.**

   (A) $x - 2y = 2$ **(B)** $x - 2y = 4$ **(C)** $2x + y = 1$ **(D)** $x + 2y = -2$ **(E)** $\frac{1}{2}x + y = -2$

9. **$\angle A$ and $\angle B$ are complementary angles. $\angle B$ and $\angle C$ are supplementary angles. If $m\angle C = 112°$ then $m\angle A$ = ?**

   (A) 32° **(B)** 12 ° **(C)** 68° **(D)** 28° **(E)** 22°

10. Find the slope of a line perpendicular to the line drawn in the graph below.



(A) $-1\frac{1}{3}$ **(B)** $-\frac{3}{4}$ **(C)** $-\frac{1}{4}$ **(D)** $\frac{2}{3}$ **(E)** $1\frac{1}{4}$

11. $\triangle ABC$ is inscribed in the circle shown such that $m\angle A = 80°$ and $m\overset{\frown}{AC} = 140°$. Find $m\angle C$.

(A) 15° **(B)** 20° **(C)** 30° **(D)** 40° **(E)** 60°

12. Find the perimeter of the pentagon shown. (nearest cm).

(A) 103 cm. **(B)** 110 cm **(C)** 116 cm **(D)** 119 cm **(E)** 130 cm

13. Let $f(x) = x - 2$ and $g(x) = 3 - x$ and $h(x) = 5x + 1$. Find $g\big(h\big(f(1)\big)\big)$.

(A) 21 **(B)** -2 **(C)** 1 **(D)** -5 **(E)** 7

14. Simplify: $\left(1 - \frac{1}{x^2}\right) \div \left(1 + \frac{1}{x}\right)$

(A) $\frac{1}{x} - 1$ **(B)** -1 **(C)** $\frac{x-1}{x+1}$ **(D)** $\frac{x-1}{x}$ **(E)** $\frac{x^2 - x - 1}{x}$

15. How many values of θ satisfy $2\cos^2\theta + 7\cos\theta - 4 = 0$, where $0 \le \theta \le 2\pi$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. **Juan Dorround stands in the middle of a field. He walks 25 steps on a bearing of 300°. The he walks 40 steps on a bearing of 60°. What is the least number of steps he will have to take to return to the spot he started from?**

(A) 65 **(B) 35** **(C) 48** **(D) 33** **(E) 30**

17. **How many negative real roots does $f(x) = x^3 + 3x^2 - x - 3$ have?**

(A) 4 **(B) 3** **(C) 2** **(D) 1** **(E) 0**

18. **Determine the sum of the infinite series $1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots$. (nearest thousandth)**

(A) 2.094 **(B) 2.618** **(C) 2.708** **(D) 2.718** **(E) 2.727**

19. **If $f'(x) = 6x^2 + 6x - 4$ and $f(1) = 6$, then $f(-1) = ?$**

(A) 5 **(B) -6** **(C) 10** **(D) -4** **(E) 14**

20. **Find the y-intercept of the line tangent to the curve $y = x^2 - 1$ at the point (2,3).**

(A) $(0, -5)$ **(B) $(0, -3)$** **(C) $(0, -1)$** **(D) (0,2)** **(E) (0,3)**

21. **Ima Washen has a laundry bag that contains 8 blue socks, 6 white socks, and 4 red socks. She reaches in her bag and draws out 2 socks, one at a time, without replacement. What is the probability that both are red? (nearest tenth percent)**

(A) 5.2% **(B) 4.9%** **(C) 4.4%** **(D) 4.0%** **(E) 3.9%**

22. **A lock's combination consists of three non-zero digits. The first digit is a prime number, the second digit is a Fibonacci number, and the last digit is a triangular number. How many unique combinations fit this criteria?**

(A) 9 **(B) 12** **(C) 45** **(D) 60** **(E) 75**

23. **Find k given the sequence 2, 3, 10, 15, 26, k, 50, 63, 82, ...**

(A) 33 **(B) 35** **(C) 37** **(D) 40** **(E) 48**

24. **$361_7 + 613_7 + 136_7 =$ _____ $_7$**

(A) 1110 **(B) 3210** **(C) 1443** **(D) 4131** **(E) 3144**

Mathematicians (No new ones this year.)

Agnesi	Archimedes	Boole, George	Byron, Ada (Lady Lovelace)
Cantor, Georg	Descartes, Rene	Diophantus	Erastosthenes
Euclid	Euler, Leonard	Germain, Sophie	Goldbach, Christian
Hypatia	Kovalevsky, Sonya	Leibniz, Gottfried	Mandelbrot, Benoit
Napier, John	Noether, Emmy	Porter, Freda	Ptolemy, Claudius
Smith, Karen E.	Stott, Alicia	Theano	Venn, John
Williams, Grace			

Types of Numbers (No new ones this year.)

Complex	Real	Imaginary	Rational	Irrational
Transcendental	Integer	Whole	Natural	Even
Odd	Prime	Composite	Unit	Deficient
Frugal	Economical	Perfect	Equidigital	Abundant
Extravagant	Wasteful	Fibonacci	Lucas	Happy
Unhappy	Lucky	Unlucky	Evil	Odious
Polite	Primeval			

2011-12 Special Emphasis Concept: Patterns, Sequences, and Series

Possible questions (but not limited to) might include:

1. The 8th term of the arithmetic sequence — 9, — 3, 3, 9, ... is ?

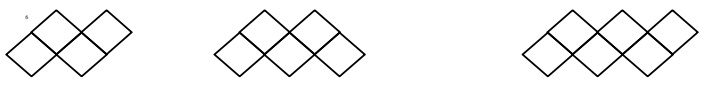
2. The integers greater than 0 are arranged in five columns as shown. If this pattern continues which column would contain the number 2012?

A	B	C	D	E
1	2	3	4	
	8	7	6	5
9	10	11	12	
	16	15	14	13

3. The 7th Lucas number is ?

4. If $a_1 = -3$, $a_2 = 2$, and $a_n = a_{n-2} - a_{n-1}$, then a_7 equals ?

5. The three zig-zag shapes are made up of little squares. The side length of each little square is 1 cm. What is the perimeter of a zig-zag shape made up of 17 little squares ?



6. $\sum_{n=1}^{8} n^2 - n = ?$

***** See #18 and #23 on the 2011SAC test.**

University Interscholastic League
MATHEMATICS CONTEST
HS ● SAC ● 2011
Answer Key

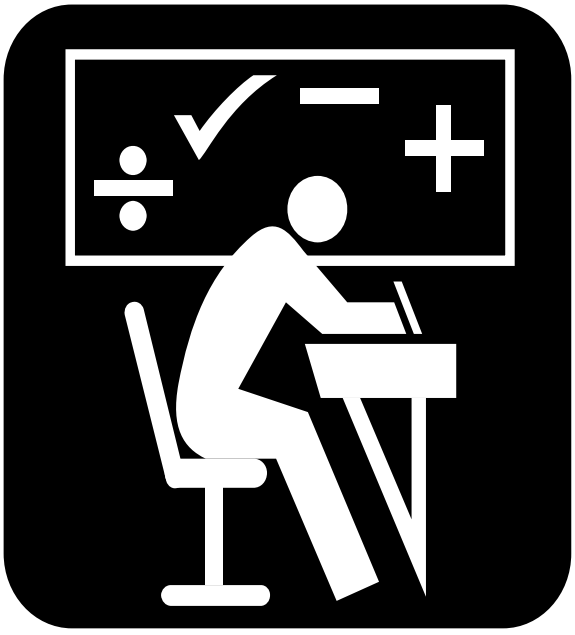
1. **D**
2. **E**
3. **B**
4. **D**
5. **E**
6. **A**
7. **C**
8. **B**
9. **E**
10. **A**
11. **C**
12. **D**
13. **E**
14. **D**
15. **C**
16. **B**
17. **C**
18. **D**
19. **C**
20. **A**
21. **E**
22. **D**
23. **B**
24. **C**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2012



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1. **Evaluate:** $64 \div 32 + (16)^{\frac{1}{2}} \times (8 - 4^2)$

   (A) -30 **(B)** -16 **(C)** 0 **(D)** 6 **(E)** 32

2. **1.2 thousand is subtracted from the sum of 3.4 million and 5.6 billion. How many zeros will the answer contain when written out using digits instead of words?**

   (A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

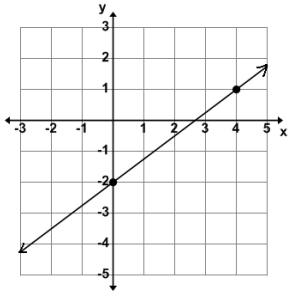
3. **I. M. Broke borrows $1000.00 from his uncle Sam to buy college textbooks. He will pay back the $1000.00 plus the amount of simple interest in 8 equal monthly payments. What will his monthly payments be if the interest rate is 3%?**

   (A) \$113.33 **(B)** \$120.00 **(C)** \$125.21 **(D)** \$127.50 **(E)** \$128.75

4. **The statement $5(4 - 3) = 5(4) - 5(3)$ is an example of which of the following properties?**

   (A) Associative **(B)** Commutative **(C)** Distributive **(D)** Mult. Inverse **(E)** Subtraction

5. **Find an equation of the line shown using the two points shown.**



   (A) $4x + 3y = 6$ **(B)** $3x - 4y = 8$ **(C)** $4x + 3y = -2$ **(D)** $3x - 4y = -2$ **(E)** $3x + 4y = 8$

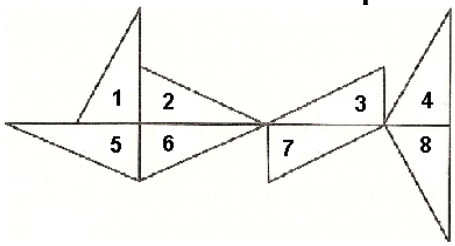
6. **If $6 - 7x - 20x^2 = (ax + b)(cx + d)$ then $a + b + c + d =$ _____.**

   (A) 14 **(B)** 10 **(C)** 4 **(D)** $-\frac{7}{20}$ **(E)** -1

7. **$\angle P$ and $\angle Q$ are complementary. $\angle Q$ and $\angle R$ are supplementary. If $m\angle P = 3x + 8$ and $m\angle Q = 2x - 3$ then $m\angle R = ?$**

   (A) 121° **(B)** 131° **(C)** 149° **(D)** 159° **(E)** 163°

8. **Which of the transformations will map triangle 3 to triangle 6?**



   (A) glide reflection **(B)** half-turn **(C)** reflection **(D)** rotation **(E)** translation

9. **Dim, Nit, and Half looked at their father's circular clock. The time shown was 4:44 PM. They asked their father, Mr. Wit, to find the measure of the smaller angle between the big hand and the little hand. What should Mr. Wit find the measure to be?**

(A) 142 ° **(B) 112 °** **(C) 102 °** **(D) 132 °** **(E) 122 °**

10. **If x varies directly as 2y — 1, and x = 9 when y = 2, find y when x = 15.**

(A) 23 **(B)** $1\frac{1}{2}$ **(C)** $13\frac{1}{2}$ **(D)** $3\frac{3}{17}$ **(E) 3**

11. **Given the sequence 3, 9, 18, 30, 45, 63, 84, k, 135, 165, ... find k.**

(A) 93 **(B) 102** **(C) 108** **(D) 111** **(E) 117**

12. **Three window washers can wash 6 windows in 60 minutes. How many windows could 5 window washers wash in 90 minutes if they work at the same rate as the 3 window washers?**

(A) 9 **(B) 12** **(C) 15** **(D) 16** **(E) 20**

13. **Determine the range of** $f(x) = 4\cos(3x + 2\pi) - 1$.

(A) $[-5, 3]$ **(B)** $[1, 5]$ **(C)** $[-5, 5]$ **(D)** $[-1, 4]$ **(E)** $[2, 3]$

14. **Willie B. Quik flies his jet on a bearing of 25° at a speed of 540 mph. How far north of his starting point will Willie be after an hour and a half? (nearest mile)**

(A) 608 mi **(B) 684 mi** **(C) 734 mi** **(D) 755 mi** **(E) 810 mi**

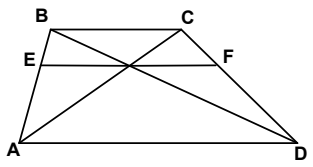
15. **Determine the period of** $y = 1 - 2\cos\left[3\pi(x + 4)\right]$.

(A) 2 **(B)** $1\frac{1}{3}$ **(C) 1** **(D)** $\frac{2}{3}$ **(E)** $\frac{1}{3}$

16. **How many solutions are there for the equation** $4x + 3y = 72$ **such that both x and y are non-negative integers?**

(A) 6 **(B) 7** **(C) 8** **(D) 9** **(E) 12**

17. **The trapezoid shown exists such that BC = 8 cm, AD = 12 cm,** $\overline{BC} \parallel \overline{EF} \parallel \overline{AD}$, **and** $\overline{EF}$ **and the two diagonals intersect at a common point. Find EF. (nearest tenth)**



(A) 8.2 cm **(B) 9.8 cm** **(C) 9.6 cm** **(D) 10.2 cm** **(E) 10.0 cm**

18. Simplify $\left(6+\sqrt{-18}\right)\left(3-\sqrt{-8}\right)$ to the form $a+bi$.

(A) $6+3\sqrt{2}\,i$ **(B)** $12+6\sqrt{2}\,i$ **(C)** $30+3\sqrt{2}\,i$ **(D)** $6-6\sqrt{2}\,i$ **(E)** $30-3\sqrt{2}\,i$

19. Let $f(x) = x^5 - 2x^4 + 2x^3 - 3x^2 + x - 3$. Find the coefficient of the quadratic term of $f''(x)$.

(A) -24 **(B)** -6 **(C)** 0 **(D)** 12 **(E)** 20

20. A particle is moving in a straight line according to the function $f(t) = t^3 + 3t^2 - 9t + 4$. For which of the following values of t will the particle be moving to the left?

(A) 3.5 **(B)** 3 **(C)** 1.5 **(D)** 1 **(E)** 0.5

21. How many distinguishable permutations can be made from the letters of the word 'DIVISION' ?

(A) 6720 **(B)** 120 **(C)** 40320 **(D)** 720 **(E)** 5040

22. The IB2 Smart club has 15 members of which 4 are geniuses, 5 are computer geeks, and 6 are puzzle solvers. The club needs a 5-member committee consisting of 2 geniuses, a geek, and 2 solvers. How many 5-member committees can be formed from the club membership?

(A) 3003 **(B)** 450 **(C)** 180 **(D)** 120 **(E)** 26

23. Coach Garza asks each of his 4 number sense team members what their date of birth is. He calculates what day of the week each of them was born on. What is the probability that all 4 of them were born on a different day of the week? (nearest whole percent)

(A) 14% **(B)** 28% **(C)** 35% **(D)** 43% **(E)** 57%

24. The four shapes below are made up of 1 cm squares. If the pattern continues, fine the perimeter of the shape consisting of 14 squares.

(A) 26 **(B)** 30 **(C)** 32 **(D)** 36 **(E)** 38

25. Simplify: $\frac{(n+1)!}{(n-2)!} \times \frac{(n)!}{(n-1)!}$

(A) $2n^2$ **(B)** $n^2(n^2-1)$ **(C)** 1 **(D)** n^4-1 **(E)** $-2n$

26. Let P = {p,r,i,m,e}, N = {n,u,m,b,e,r}, and M = {m,a,k,e,r}. The number of distinct elements in $(M \cap N) \cup P$ is _____.

(A) 3 **(B)** 5 **(C)** 7 **(D)** 8 **(E)** 9

27. In the algebraic expression, $5xy + 6(x - y) - \frac{x+y}{7}$, there are how many terms?

(A) 8 (B) 6 (C) 5 (D) 4 (E) 3

28. Which of the following side lengths form an obtuse isosceles triangle?

(A) 7, 7, 10 (B) 16, 30, 34 (C) 9, 10, 10 (D) 3, 6, 10 (E) 15, 15, 21

29. What is the digit in the tens place of $(1!) + (1!) + (2!) + (3!) + (5!) + (8!) + ... + (34!)$?

(A) 8 (B) 5 (C) 3 (D) 2 (E) 0

30. $\sin(\theta - \frac{\pi}{2})$ equals :

(A) $\cos(\theta + \pi)$ (B) $\sin(\theta + \pi)$ (C) $\cos(\theta + \frac{\pi}{2})$ (D) $\sin(\theta + \frac{\pi}{2})$ (E) $\cos(\theta - \frac{\pi}{2})$

31. If $A = \begin{bmatrix} 1 & 1 \\ 2 & x \end{bmatrix}$ and $B = \begin{bmatrix} y & 1 \\ 3 & 4 \end{bmatrix}$ then $AB = \begin{bmatrix} 5 & 5 \\ 13 & 14 \end{bmatrix}$. Find $x + y$.

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

32. If $f(\theta) = \sin 2\theta$ then $f'(\frac{\pi}{12}) =$ _____.

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{3\sqrt{3}}{2}$ (E) $\sqrt{3}$

33. The following chart shows the percentage growth of the price of a gallon of gasoline each quarter for one year. What was the average percentage growth for that year? (nearest hundredth)

Quarter	Percentage Growth
1	increased 10%
2	increased 5%
3	increased 12%
4	decreased 8%

(A) 4.13% (B) 4.35% (C) 4.41% (D) 4.45% (E) 4.75%

34. The set of numbers, {3, 6, 9, 12, 15, 18} are _____ numbers.

(A) Odious (B) Abundant (C) Perfect (D) Deficient (E) Evil

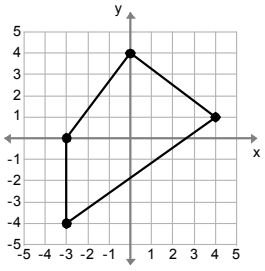
35. What is $6\frac{1}{4}\%$ of $(\frac{1}{6} \div (0.08333...)$?

(A) $\frac{2}{9}$ (B) 0.25 (C) $\frac{1}{3}$ (D) 0.41666... (E) $\frac{1}{8}$

36. $234_5 + 314_5 + 123_5 =$ _____ $_5$

(A) 1231 (B) 2131 (C) 1321 (D) 1141 (E) 2111

37. Dee Kartez drew this quadrilateral on the coordinate plane below. The coordinates of the vertices are integers. What is the area of his quadrilateral?



(A) 27 units^2 **(B)** $26\frac{1}{2} \text{ units}^2$ **(C)** 26 units^2 **(D)** $25\frac{1}{2} \text{ units}^2$ **(E)** 25 units^2

38. Simplify: $\left(\frac{6x^2 - 2x}{2x^2 + x - 6}\right)\left(\frac{3x^2 + 7x + 2}{6x - 2}\right)$

(A) $\frac{3x^2 + x}{2x - 3}$ **(B)** $\frac{3x + 1}{-6}$ **(C)** $\frac{6x^2 + 2x}{x - 6}$ **(D)** $\frac{3x + 1}{2x^2 - 3x}$ **(E)** $\frac{3x^2 - 1}{3x - 2}$

39. Which of the following is NOT a member of the solution set for $1 + 3|6 - 10x| \geq 13$ **?**

(A) -1 **(B)** -0.6 **(C)** 0.1 **(D)** 0.5 **(E)** 1

40. If $a_1 = 8$, $a_2 = 5$ **and** $a_n = a_{n-1} - a_{n-2}$, **then** a_8 **equals:**

(A) -8 **(B)** -5 **(C)** -3 **(D)** 5 **(E)** 8

41. Let $f(x) = 2x + 3$ **and** $g(x) = 4x - 5$. **Find** $f\left(g\left(\frac{a+1}{4}\right)\right)$.

(A) $2a + 9$ **(B)** $a - 6$ **(C)** $a + 6$ **(D)** $2a - 5$ **(E)** $1.5a$

42. $\sum_{k=0}^{2} \left(kx - (k-1)y + k\right) = ?$

(A) $3x + 3$ **(B)** $x - 2y + 1$ **(C)** $3x + 2y + 1$ **(D)** $3x - y + 3$ **(E)** $x + 2$

43. The directrix of the parabola $y = x^2 - 5$ **is:**

(A) $x = 5$ **(B)** $y = \frac{19}{4}$ **(C)** $x = \frac{5}{4}$ **(D)** $y = -\frac{21}{4}$ **(E)** $y = -\frac{5}{4}$

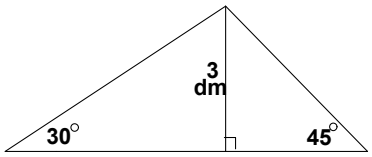
44. Willie When and Mae Beknott each have a bag containing a red chip, a blue chip, and a white chip. If they randomly draw out a chip what are the odds that at least one of them draws out a red chip?

(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{4}{5}$ **(E)** $\frac{5}{4}$

45. The sequence 3, 0, 2, 3, 2, 5, ... , where $P_0 = 3$, $P_1 = 0$, and $P_2 = 2$, is a recursive sequence for all P_n where $n \geq 3$. Find P_8.

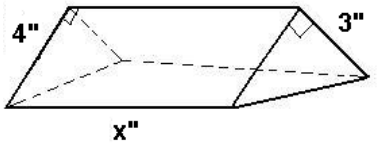
(A) 3 (B) 5 (C) 7 (D) 10 (E) 12

46. Find the perimeter of the figure shown. (nearest tenth).



(A) 18.4 dm (B) 17.2 dm (C) 16.2 dm (D) 15.4 dm (E) 15.0 dm

47. The volume of the right prism shown is 48 cubic inches. Find x.



(A) 4 (B) 5 (C) 7 (D) 8 (E) 12

48. Find the number of positive integral divisors of 900.

(A) 6 (B) 8 (C) 9 (D) 27 (E) 30

49. Les Scents has a bag containing nickels and dimes. If Les spends a dime, 20% of the remaining coins would be dimes. If Les had spent 2 nickels instead of a dime, 25% of the remaining coins would be dimes. How many coins were in his bag originally?

(A) 18 (B) 24 (C) 26 (D) 30 (E) 36

50. A child's ticket at the Cheap Flick movie theater cost \$4.00 and an adult ticket cost \$6.50. How many adult tickets were sold if a total of \$432.50 was collected for 80 tickets?

(A) 30 (B) 35 (C) 40 (D) 45 (E) 50

51. Two elements of the set { 2 , 1 , 3 , 4 , 7, 11 } are randomly selected. What is the probability that their sum is an element of the set?

(A) 25% (B) $26\frac{2}{3}\%$ (C) $36\frac{4}{11}\%$ (D) 50% (E) $66\frac{2}{3}\%$

52. Evaluate: $(\log_4 64) + (\log_3 9) - (\log_2 8)$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

53. Romeo leans a twelve-foot ladder from the ground to Juliet's window. The angle of inclination is 60 °. How high is Juliet's window from the ground? (nearest inch)

(A) 7' 2" **(B) 11' 3"** **(C) 10' 8"** **(D) 9' 4"** **(E) 10' 5"**

54. Let $\cot A = -\frac{11}{60}$, where A is in QII. Find $\sin A + \cos A$.

(A) $1\frac{10}{61}$ **(B) $1\frac{11}{60}$** **(C) $\frac{51}{60}$** **(D) $\frac{49}{61}$** **(E) $\frac{60}{61}$**

55. Let $f(x) = x^3 - 2x^2 + x - 3$, $g(x) = 2x^2 + x - 3$, and $h(x) = x - 3$. Find $h\Big(g\big(f(1)\big)\Big)$

(A) 3 **(B) 6** **(C) 9** **(D) -3** **(E) -6**

56. Let $f(x) = 2x^2 + 3x + 1$, $g(x) = x - 1$, and s(x) be the slant (oblique) asymptote of $\frac{f(x)}{g(x)}$. Find $S(-4)$.

(A) -5 **(B) -3** **(C) 1** **(D) 5** **(E) 6**

57. Let $f(x) = x^5 - 5x^4 + 5x^3 + 20$. The sum of the x-values of the critical points of the function is:

(A) 21 **(B) 11** **(C) 5** **(D) 4** **(E) 3**

58. Find the area (in square units) of the region bounded by the curves $y = 2\sqrt{x}$ and $y = \frac{x^2}{4}$.

(A) $5\frac{1}{6}$ **(B) $5\frac{1}{4}$** **(C) $5\frac{1}{3}$** **(D) $5\frac{3}{4}$** **(E) $5\frac{2}{3}$**

59. $\triangle ABC$ is a right triangle with $\angle ACB$ being the right angle. Point D lies on $\overline{AC}$ such that AD = 3" and CD = 5". Find the area of $\triangle ABD$ if BC = 6". (nearest square inch)

(A) 6 sq. in. **(B) 9 sq. in.** **(C) 12 sq. in.** **(D) 15 sq. in.** **(E) 24 sq. in.**

60. $11_2 + 111_4 + 1111_8 =$ _____ $_{16}$

(A) 1111 **(B) F22** **(C) 667** **(D) 261** **(E) 25D**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2012
Answer Key

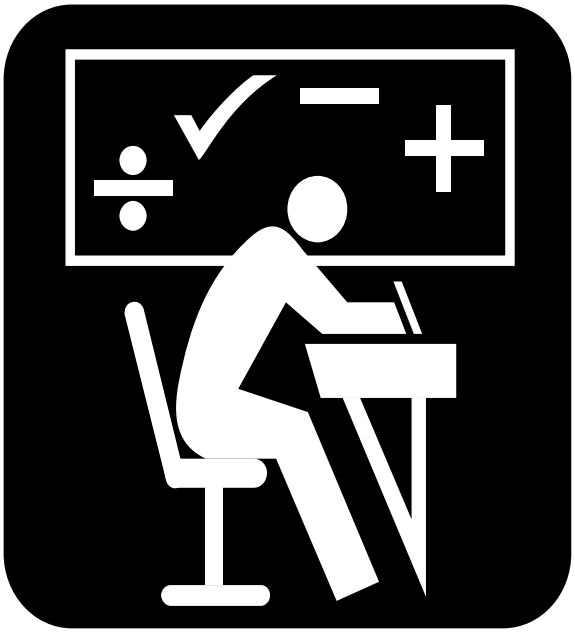
1. A	**21. A**	**41. D**
2. C	**22. B**	**42. A**
3. D	**23. C**	**43. D**
4. C	**24. B**	**44. E**
5. B	**25. B**	**45. D**
6. C	**26. B**	**46. A**
7. C	**27. E**	**47. D**
8. D	**28. A**	**48. D**
9. E	**29. B**	**49. C**
10. E	**30. A**	**50. D**
11. C	**31. C**	**51. B**
12. C	**32. E**	**52. A**
13. A	**33. D**	**53. E**
14. C	**34. E**	**54. D**
15. D	**35. E**	**55. C**
16. B	**36. A**	**56. B**
17. C	**37. B**	**57. D**
18. E	**38. A**	**58. C**
19. A	**39. D**	**59. B**
20. E	**40. D**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational B • 2012



WRITE ALL ANSWERS WITH
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1. **Evaluate:** $\frac{1}{16} \div 0.08333... \times (\frac{1}{8} + 0.25) - \frac{1}{2}$

   (A) $-\frac{7}{32}$ **(B)** -0.154 **(C)** $\frac{1}{32}$ **(D)** .041666... **(E)** $\frac{11}{32}$

2. **Phil Detank has a car that gets 24 miles per gallon of gas. How much will it cost him to drive 320 miles if a gallon of gas costs $3.60?**

   (A) $64.80 **(B) $46.80** **(C) $52.40** **(D) $60.00** **(E) $48.00**

3. **What is $2\frac{2}{3}$ of 0.3125 plus 25% of 0.8333... ?**

   (A) $\frac{1}{4}$ **(B)** $\frac{41}{48}$ **(C)** $1\frac{1}{24}$ **(D)** $1\frac{2}{3}$ **(E)** $\frac{5}{8}$

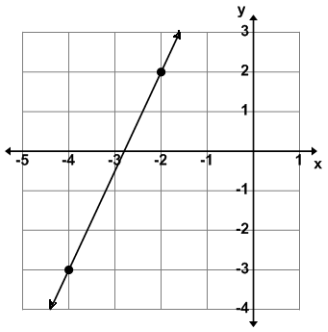
4. **The statement $(5 + 4) + 3 = (4 + 5) + 3$ is an example of which of the following properties?**

   (A) Associative **(B) Commutative** **(C) Distributive** **(D) Additive Inverse** **(E) Identity**

5. **Y varies directly with the reciprocal of X and Y = 3 when X = 2. Find Y if X = 15.**

   (A) $\frac{2}{45}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{2}{5}$ **(D)** $2\frac{1}{2}$ **(E) 6**

6. **Find the equation of the line shown.**



   (A) $2x + 5y = 7$ **(B) $5x + 2y = 35$** **(C) $5x - 2y = 7$** **(D) $2x - 5y = 14$** **(E) $5x - 2y = -14$**

7. **Which of the following mathematicians concluded that Saturn's rings are egg-shaped rather than elliptical, with only one line of symmetry?**

   (A) Freda Porter **(B) Christian Goldbach** **(C) Alicia Stott**
   (D) Gottfried Leibniz **(E) Sonya Kovalevsky**

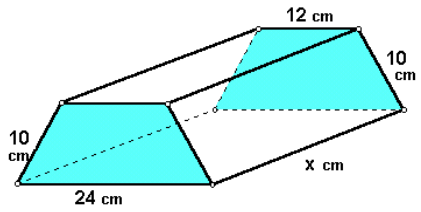
8. **Two chords, $\overline{WX}$ and $\overline{YZ}$, are in circle O and do not intersect. If $WX > YZ$ then $\overline{YZ}$ _____ the center of the circle.**

   (A) intersects **(B) ends at** **(C) begins at** **(D) is nearer to** **(E) is farther from**

9. **Given the sequence 1, 3, 7, 13, 21, 31, ..., 91, 111, k, 157, 183, ... find k.**

   (A) 121 **(B) 123** **(C) 131** **(D) 133** **(E) 141**

10. The volume of the trapezoidal prism shown is 2,160 cm^3. Find x, in centimeters. Drawing is not to scale.



(A) 8 (B) 10 (C) 13 (D) 15 (E) 18

11. Point A (1, 2) is reflected across the x-axis to point B. Then point B is rotated 180° clockwise around the origin to point C. Then point C is reflected across the y-axis to point D. Then point D is translated vertically 3 units down to point E (x, y). Find $x + y$.

(A) -3 (B) -1 (C) 0 (D) 1 (E) 3

12. If $\frac{x-1}{x+2} - \frac{x-3}{x+4} = \frac{ax^2+bx+c}{px^2+qx+r}$, then $\frac{a+b+c}{p+q+r}$ equals:

(A) $\frac{2}{5}$ (B) $\frac{4}{7}$ (C) $\frac{1}{2}$ (D) $\frac{14}{15}$ (E) $\frac{1}{3}$

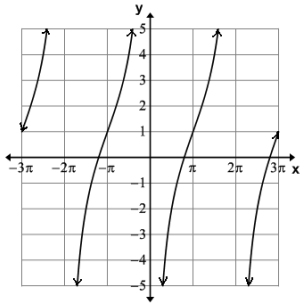
13. Les Spede leaves the barn and rides his horse down the country road at 15 mph. Forty- five minutes later, his brother, Moe Spede, leaves the same barn and rides his horse down the same country road at 20 mph. How long will it take for Moe to catch Les?

(A) 3 hrs (B) 2.75 hrs (C) 2.5 hrs (D) 2.25 hrs (E) 1.75 hrs

14. Simplify: $(\sin\theta)(\tan\theta) + \cos\theta$

(A) $\sec\theta$ (B) $\csc\theta$ (C) $\sin\theta + \cos\theta$ (D) $\cot\theta$ (E) $(\sec\theta)(\csc\theta)$

15. The equation y = _____ will produce this graph.



(A) $\tan(\frac{x+\pi}{2}) + 1$ (B) $2\tan(x - \frac{\pi}{3}) + 1$ (C) $3\cot(x - \pi) + 1$

(D) $\cot(x - \frac{\pi}{2}) + 1$ (E) $3\tan(\frac{x-\pi}{2}) + 1$

16. Determine the range of $f(x) = 3 + 5\sin(2x - \frac{\pi}{4})$.

(A) $[-4, 4]$ (B) $[4, 6]$ (C) $[2, 4]$ (D) $[-2, 8]$ (E) $[-5, 5]$

17. If the three numbers 131, 227, and 355 are each divided by the number D, each of their quotients will have the same remainder R. Find R where $R > 1$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 7

18. If $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 \\ 13 & 21 \end{bmatrix}$ then $AB = \begin{bmatrix} r & s \\ t & u \end{bmatrix}$. Find ru — st.

(A) — 1 (B) 0 (C) 1 (D) 2 (E) 34

19. Evaluate: $\prod_{n=1}^{3} \left(n^{(n-1)} + n\right)$

(A) 18 (B) 48 (C) 70 (D) 96 (E) 144

20. Nine pingpong balls are numbered with a unique single digit starting with 1 and ending with 9 and placed in a paper bag. Two balls are randomly selected without replacement. What is the probability that the first ball is has an odd digit on it and the second ball has an even digit on it?

(A) $19\frac{61}{81}\%$ (B) $22\frac{2}{9}\%$ (C) $24\frac{56}{81}\%$ (D) $27\frac{7}{9}\%$ (E) $30\frac{70}{81}\%$

21. $\int (2 - 5x)\ dx =$ _____ + C, where C is some arbitrary constant.

(A) $2 - 2.5x$ (B) $2x^2 - 5x$ (C) $\frac{(4-5x)x}{2}$ (D) $4x - 5x^2$ (E) $\frac{4x-5}{2}$

22. Let $f(x) = 3x - 5$ and $g(x) = 5x + 3$. Find $f\big(g(1)\big) + g\big(f(-1)\big)$

(A) — 20 (B) — 18 (C) — 10 (D) — 6 (E) — 2

23. Let $f(x) = 2x^5 - 3x^4 + 4x^3 - 5x^2 + 6x - 1$. Find the coefficient of the constant term of $f''(x)$.

(A) — 36 (B) — 10 (C) 0 (D) 24 (E) 40

24. $A1_{16} + 234_8 + 3210_4 =$ _____ $_2$

(A) 1000100001 (B) 11000100001 (C) 10100100001 (D) 110001011 (E) 100010001

25. The length of a piece of string is 15 yards. Betty Kuttitt snipped the string into four smaller pieces such that the ratio of the lengths was 1:2:3:4. How much longer was the longest piece than the shortest piece?

(A) 4 yds 2 ft 3 in (B) 4 yds 1 ft 6 in (C) 3 yds (D) 1 yd 1 ft 6 in (E) 1 yd 1 ft $\frac{1}{2}$ in

26. Let P and Q be the roots of $2x^2 - 3x - 2 = 0$. Find $P^4 + 4P^3Q + 6P^2Q^2 + 4PQ^3 + Q^4$.

(A) 16 (B) $5\frac{1}{16}$ (C) $3\frac{13}{81}$ (D) 1 (E) $\frac{1}{81}$

27. Point P is the _____ of △ABC shown below.

(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) center

28. How many terms of the arithmetic sequence, — 18, — 13, — 8, — 3, ..., will be needed to get a sum of 18?

(A) 3 (B) 8 (C) 9 (D) 10 (E) 15

29. If $\cos x + \sin x = \frac{1-\sqrt{3}}{2}$ and $\cos x - \sin x = \frac{-1-\sqrt{3}}{2}$ find the numerical value of cos 2x, where x ∈ QII.

A) $-\frac{\sqrt{3}}{2}$ (B) -1 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{3}}{2}$

30. Robin Banks invested some money with her stock broker. In the first year her investment increased 10%, in the second year it went down 4%, and in the third year it increased 14%. What was Robin's average rate of return over the three year period? (nearest tenth)

(A) 6.7% (B) 9.3% (C) 6.1% (D) 11.8% (E) 6.4%

31. If a triangle has two sides of lengths 10 cm and 15 cm, and the included angle between these two sides is increasing at the rate of 10° per hour, at what rate is the area of the triangle changing when the included angle is 60°? (nearest tenth)

(A) 13.7 cm^2/hr (B) 11.3 cm^2/hr (C) 10.8 cm^2/hr (D) 7.5 cm^2/hr (E) 6.5 cm^2/hr

32. The arithmetic mean of all of the positive integral divisors, not including the number one nor itself, of the number 48 is:

(A) 7.5 (B) 9.375 (C) 12.4 (D) 13.666... (E) 15.5

33. Using the following array, determine the value of the median of the 12th row.

2
4 6
8 10 12
14 16 18 20
22 24 26 28 30
⋮

(A) 145 (B) 149 (C) 141 (D) 147 (E) 143

34. $11235_8 - 2134_8 - 136_8 =$ _____ $_8$

(A) 6743 **(B)** 7101 **(C)** 1677 **(D)** 6373 **(E)** 7765

35. Simplify: $\left(a^2 \div b^{-3} \times a^{-4} \div b^5\right)^{-1}$

(A) $\frac{b^2}{a^2}$ **(B)** a^2b^2 **(C)** a^6b^8 **(D)** $\frac{1}{a^2b^2}$ **(E)** 1

36. Tuff Guy competed in a 30 mile race, half the time running and half the time on bicycle. He averaged 15 mph running and 25 mph on his bicycle. How long did it take him to complete the race?

(A) 40 min **(B)** 45 min **(C)** 1 hr **(D)** 1 hr 30 min **(E)** 1 hr 45 min

37. If $\frac{x+1}{x-2} + \frac{x-2}{x+1}$ **is written as the mixed number** $A\frac{B}{C}$ **then B = ?**

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

38. Find AB if CD = $4\sqrt{3}$ **cm.**

(A) $12\sqrt{3}$ cm **(B)** $6 + \sqrt{2}$ cm **(C)** 6 cm **(D)** $3\sqrt{3} + 2\sqrt{2}$ cm **(E)** 12 cm

39. Two spray painters can paint four cars in three hours. How long would it take five spray painters to paint twelve cars if they work at the same rate as the two spray painters?

(A) 2 hrs 30 min **(B)** 3 hrs 45 min **(C)** 3 hrs 12 min **(D)** 4 hrs 24 min **(E)** 3 hrs 36 min

40. The set of numbers, $\{2^{\sqrt{2}}, e, \Omega, \pi, \sin(5)\}$ **are** _____ **numbers.**

(A) extravagant **(B)** transcendental **(C)** primeval **(D)** unhappy **(E)** evil

41. An equilateral triangle has a side length *S* and an apothem *A*. The area of the triangle is:

(A) $\frac{3SA}{2}$ **(B)** $\frac{\sqrt{3}S^2A}{3}$ **(C)** $3SA$ **(D)** $\frac{\sqrt{3}S^2A}{4}$ **(E)** $\frac{SA}{2}$

42. If $(x+2)(2x-3)(3x) = ax^3 + bx^2 + cx + d$ **then** $a + b + c + d =$ _____.

(A) -9 **(B)** -3 **(C)** 0 **(D)** 6 **(E)** 15

43. Willie Shair had a box of candy hearts. He gave 40% of them to his girlfriend. Then he gave $\frac{1}{3}$ of the remaining candy hearts to his best friend. He ate 5 of the remaining ones leaving him with 25 candy hearts. How many candy hearts were in the original box?

(A) 50 (B) 60 (C) 65 (D) 75 (E) 80

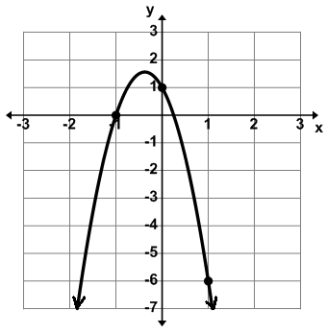
44. A waiter received a $6 tip from table 1 and a $6 tip from table 2. The tip from table 1 was 10% of their bill and the tip from table 2 was 20% of their bill. What was the difference, in dollars, between the two bills?

(A) $6.00 (B) $10.00 (C) $12.00 (D) $15.00 (E) $30.00

45. Two similar polygons have areas of 147 sq. inches and 48 sq. inches. The larger polygon has a perimeter of 35 inches. What it the perimeter of the smaller polygon?

(A) $11\frac{3}{7}$" (B) 12" (C) 20" (D) $22\frac{6}{7}$" (E) 24"

46. The vertex of the graph shown is (x , y). Find x.



(A) $-\frac{1}{4}$ (B) $-\frac{3}{8}$ (C) $-\frac{1}{3}$ (D) $-\frac{2}{5}$ (E) $-\frac{3}{7}$

47. Mr. Lou Kaas puts each of the elements from this set {2, 1, 3, 4, 7, 11, 18} on a different blank card. He shuffles the cards and turns over the first card which is 18. What are the odds that the next card is a factor of the first card?

(A) 1:3 (B) 4:7 (C) 3:7 (D) 1:2 (E) 1:1

48. If $y = x^2 - 4$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when x = ?

(A) $\frac{1}{4}$ (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{2}}{2}$

49. If 1,140 degrees is equal to $k\pi$ radians, then k is:

(A) $6\frac{1}{2}$ (B) $6\frac{1}{3}$ (C) 6 (D) $5\frac{2}{3}$ (E) $5\frac{1}{2}$

50. Use the Fibonacci characteristic sequence ... — 2, p, q, 7, r, ... to Find $p + q + r$.

(A) 16.5 (B) 14 (C) 12.5 (D) 11 (E) 9.5

51. The probability of scoring above 240 on this test is $\frac{2}{9}$. How many students took this test if 36 students scored above 240?

(A) 80 (B) 108 (C) 156 (D) 162 (E) 168

52. Chris P. Beycon starts at point X and walks 300 feet to point Y on a bearing of 150°. Saul T. Kraaker starts at point X and walks 150 feet on a bearing of 300° to point Z. How far is point Y from point Z? (nearest foot)

(A) 397 ft (B) 450 ft (C) 436 ft (D) 425 ft (E) 260 ft

53. In the expansion of $(3x + 2y)^6$, the absolute value difference of the coefficients of the 3rd and the 5th term is:

(A) 4,320 (B) 4,050 (C) 3,420 (D) 2,700 (E) 1,980

54. Find the equation of the line tangent to the curve $y = 3x^2 - 2x - 1$ at the point $(-1, 4)$.

(A) $8x + y = -4$ (B) $6x + y = -8$ (C) $2x + y = 1$ (D) $8x + y = 12$ (E) $6x + y = -2$

55. I. C. Cleerly has 6 boys in his class and 8 girls in his class. Two of the boys and 3 of the girls wear glasses. He randomly selects two students to pass out papers. What is the probability that both are boys or both wear glasses? (nearest percent)

(A) 26% (B) 11% (C) 35% (D) 16% (E) 50%

56. How many solutions are there for the equation $4x + 5y = 2012$ such that both x and y are non-negative integers?

(A) 20 (B) 41 (C) 54 (D) 80 (E) 101

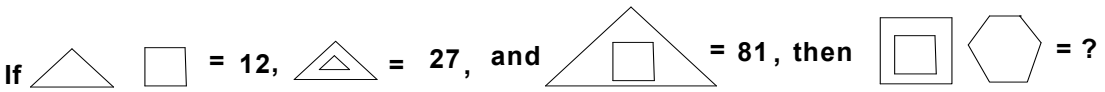
57. Three couples triple date and go to the movie theater. Their seats are next to each other in the same row. Naturally, each couple wants to sit next to each other. How many seating arrangements are possible for the three couples?

(A) 12 (B) 24 (C) 36 (D) 48 (E) 72

58. Point M is the midpoint of $\overline{AB}$ and points P and Q exist such that $\overline{AP} \perp \overline{AB}$, $\overline{BQ} \perp \overline{AB}$, $AB = 4"$, $AP = BQ = 1"$, $\overline{PQ}$ intersects $\overline{AB}$ at point M, and all 5 points are coplanar. Find $m\angle BMQ$. (nearest tenth)

(A) 22.5 ° (B) 26.6 ° (C) 30 ° (D) 45 ° (E) 63.4 °

59.



(A) 486 **(B) 384** **(C) 1,536** **(D) 262** **(E) 1,280**

60. How many positive cubes divide 3! x 5! x 7! ?

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2012
Answer Key

1. A	**21. C**	**41. A**
2. E	**22. B**	**42. A**
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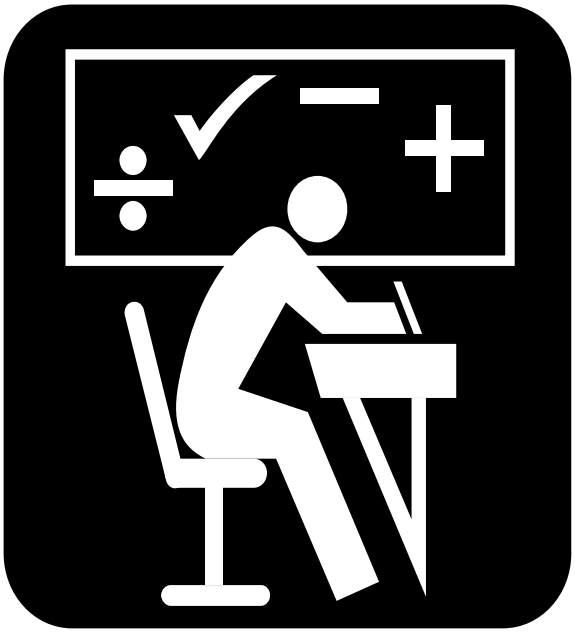
4. B	**24. A**	**44. E**
5. C	**25. B**	**45. C**
6. E	**26. B**	**46. B**
7. E	**27. B**	**47. E**
8. E	**28. C**	**48. D**
9. D	**29. D**	**49. B**
10. D	**30. E**	**50. A**
11. C	**31. E**	**51. D**
12. A	**32. B**	**52. C**
13. D	**33. A**	**53. D**
14. A	**34. A**	**54. A**
15. E	**35. B**	**55. A**
16. D	**36. D**	**56. E**
17. B	**37. E**	**57. D**
18. C	**38. E**	**58. B**
19. D	**39. E**	**59. C**
20. D	**40. B**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 1 • 2012



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

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1. **Evaluate:** $(1+1)\times 2\div 3+5\div(8-13)$

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $1\frac{4}{15}$ **(D)** $-1\frac{1}{6}$ **(E)** $-11\frac{1}{24}$

2. **What is** $33\frac{1}{3}\%$ **of 0.1666...divided by** $\frac{1}{9}$**?**

(A) 0.08333... **(B)** 0.222... **(C)** $\frac{1}{3}$ **(D)** 0.5 **(E)** $\frac{1}{162}$

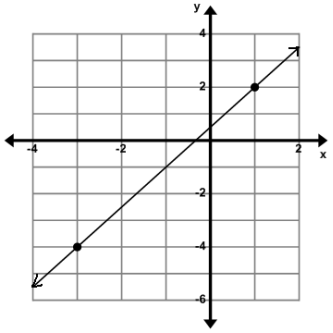
3. **Shelby and Osias were cutting ribbons to put around their birthday presents. They had a ribbon that was 9 feet long. They needed two ribbons such that the ratio of the length of Shelby's ribbon to Osias's ribbon was 3:5 with 8 inches left over. How long should Shelby's ribbon be?**

(A) 5 ft 7.5 in **(B)** 5 ft 2.5 in **(C)** 4 ft 2 in **(D)** 3 ft 4.5 in **(E)** 3 ft 1.5 in

4. **Let P = {p,l,u,s}, M = {m,i,n,u,s}, and T = {t,i,m,e,s}. The number of distinct elements in** $(P\cup M)\cap T$ **is _____.**

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. **Find an equation of the line shown using the two points shown.**



(A) $3x+2y=1$ **(B)** $2x+3y=-6$ **(C)** $2x-3y=-4$ **(D)** $3x-2y=-1$ **(E)** $3x-2y=7$

6. **Simplify:** $\left(\frac{5x^2-5x-30}{x^2-6x+9}\right)\left(\frac{x^2+2x-15}{x^2+7x+10}\right)$

(A) $\frac{5x+10}{x-3}$ **(B)** $\frac{5x^2+10x-75}{x^2-x-6}$ **(C)** $\frac{5(x+2)^2}{(x-3)^2}$ **(D)** 5 **(E)** $\frac{5x-15}{x+2}$

7. **Three spray painters can paint five identical cylinder tanks in two hours. How many of these cylinders could five sprayers paint in six hours?**

(A) 30 **(B)** 25 **(C)** 20 **(D)** 18 **(E)** 12

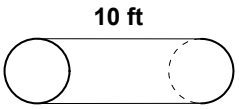
8. **Simplify:** $(3\sqrt{5}+\sqrt{3})(5\sqrt{3}-\sqrt{5})$

(A) $5\sqrt{3}-3\sqrt{5}$ **(B)** $14\sqrt{15}$ **(C)** $15-\sqrt{15}$ **(D)** $7\sqrt{15}$ **(E)** $30-\sqrt{15}$

9. $\angle A$ and $\angle B$ are complementary. $\angle B$ and $\angle C$ are supplementary. If $m\angle A = 2x - 5$ and $m\angle C = 4x + 3$ then $m\angle B = ?$

(A) 3° (B) 41° (C) 77° (D) 49° (E) 13°

10. Find the lateral surface area of the 8" diameter pipe shown. (nearest tenth)

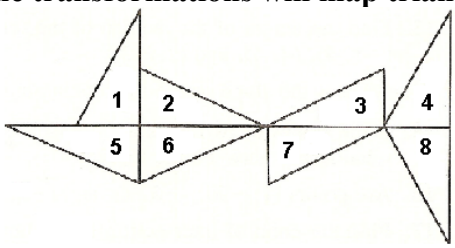


(A) 15.9 sq. ft. (B) 41.9 sq. ft. (C) 20.9 sq. ft. (D) 25.1 sq. ft. (E) 21.6 sq. ft.

11. An obtuse scalene triangle has side lengths of 6", 9", and k". How many possible triangles exist where k is an integer?

(A) 10 (B) 8 (C) 6 (D) 4 (E) 1

12. Which of the transformations will map triangle 4 to triangle 8?



(A) glide reflection (B) half-turn (C) reflection (D) rotation (E) translation

13. Let $a_1 = -2$, $a_2 = 1$ and $a_n = (a_{n-1})(a_{n-2})$ for $n \geq 3$. Find a_6.

(A) -8 (B) -3 (C) -1 (D) 4 (E) 16

14. Let $f(x) = x + 6$, $g(x) = -2x + 5$, and $h(x) = 3x - 4$ then $g\left[f\left(h[0]\right)\right]$ equals:

(A) -1 (B) 1 (C) 3 (D) 7 (E) 17

15. If y varies directly as $3x + 5$, and $y = 14$ when $x = 3$, find x when $y = -7$.

(A) 6 (B) $4\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) -4 (E) -6

16. If $\tan\theta = .75$ then the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$ is:

(A) -7 (B) $-1\frac{1}{3}$ (C) $-\frac{1}{4}$ (D) 1 (E) $3\frac{1}{2}$

17. Saul T. Water leaves port and sails 15 miles on a bearing of 85° then turns and sails 20 miles on a new bearing of 190°. How far from port is he after sailing 35 miles? (nearest tent mile)

(A) 25.0 mi (B) 23.9 mi (C) 22.7 mi (D) 21.7 mi (E) 17.5 mi

18. Determine the range of $f(x) = 1 - 2\cos(\frac{\pi}{3}x - \frac{\pi}{4})$.

(A) $[-2, 2]$ (B) $[-1, 3]$ (C) $[0, 2]$ (D) $[-3, 1]$ (E) $[-1, 2]$

19. Find $m\angle DCE$, nearest degree, if $BC = \sqrt{2}$ inches.

(A) 22° (B) 52° (C) 25° (D) 38° (E) 41°

20. In the expansion of $(3x + 4y)^5$, the absolute value difference of the coefficients of the 3rd and the 4th term is:

(A) 720 (B) 1,440 (C) 2,220 (D) 3,600 (E) 4,140

21. The mean of 25 negative numbers is -25. The mean of 45 positive numbers is 45. The mean of all 70 of these numbers is:

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

22. Use the Fibonacci characteristic sequence ..., p, -2, q, 5, r, ... to Find $p + q + r$.

(A) 4 (B) 10 (C) 12 (D) 14 (E) 28

23. A particle is moving in a straight line according to the function $f(t) = 2t^3 - 3t^2 - 12t + 8$. For which of the following values of t will the particle be changing direction from left to right?

(A) -2 (B) -1 (C) 0 (D) 2 (E) 3

24. If $y = \frac{x^2}{6}$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when x = ?

(A) $\frac{\sqrt{6}}{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{6}$ (D) $\frac{1}{3}$ (E) 3

25. Let $f(x) = 2x^5 - 6x^4 + 4x^3 + x^2 - 3x + 2$. Find the coefficient of the linear term of $f''(x)$.

(A) 40 (B) 24 (C) 10 (D) -25 (E) -75

26. Three sheets of paper are marked with an X and two sheets are marked with an O. The sheets of paper are arranged randomly in a row. What are the odds that the arrangement is XOXOX?

(A) 3:5 (B) 3:2 (C) 2:3 (D) 1:9 (E) 1:10

27. The radius of the largest circle is 5 cm. The radius of the smallest circle is 2 cm. The radius of the other circle is 3 cm. Betty Misses throws a dart and hits inside of the three circles. What is the probability that the dart hit the non-shaded section? (nearest percent)

(A) 80% **(B) 70%** **(C) 64%** **(D) 36%** **(E) 16%**

28. Find the digit in the 10^{-6} place of the series $\frac{2^0}{1} - \frac{2^2}{2} + \frac{2^4}{24} - \frac{2^6}{720} + \frac{2^8}{40320} - \ldots$.

(A) 9 **(B) 8** **(C) 6** **(D) 4** **(E) 0**

29. Willie Drawett has a new box of colored pencils, 3 blue ones, 4 red ones, 2 green ones, and 5 brown ones. Willie randomly selects 3 pencils, one at a time, without replacement. What is the probability that all 3 pencils are brown? (nearest tenth percent)

(A) 2.1% **(B) 2.7%** **(C) 3.6%** **(D) 6.0%** **(E) 6.9%**

30. $8F_{16} + 47_8 + 23_4 =$ _____ 2

(A) 11 **(B) 100111** **(C) 11000001** **(D) 101** **(E) 1100011**

31. Using the following array, determine the value of the mean of the 9th row.

2
4 6
8 10 12
14 16 18 20
22 24 26 28 30
⋮

(A) 65 **(B) 74** **(C) 82** **(D) 90** **(E) 101**

32. Four million thirty-two thousand one plus one hundred thousand two hundred thirty minus two thousand three hundred fourteen equals K. What is the sum of the digits in K?

(A) 26 **(B) 29** **(C) 33** **(D) 36** **(E) 41**

33. Papa Bear's Cubs bought 500 candy bars at a price of ten for $3.00 and sold all of them at a price of two for $1.00. Mama Bear's Brownies bought 50 boxes of cookies at a price of five boxes for $20.00 and sold all of them at a price of two boxes for $10.00. How much profit did the two groups make for the Bear Club?

(A) $50.00 **(B) $100.00** **(C) $150.00** **(D) $200.00** **(E) $250.00**

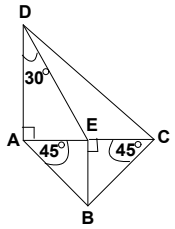
34. If the roots of $x^3 + bx^2 + cx + d = 0$ are $-3, -1$, and 2, then $b + c + d$ equals:

(A) -9 (B) -3 (C) -2 (D) 1 (E) 6

35. In the algebraic expression, $2x(x + y) + 3y(x - y) - \frac{x + y}{5}$, there are how many terms?

(A) 8 (B) 6 (C) 5 (D) 4 (E) 3

36. Find the perimeter of quadrilateral ABCD if AE = 1 inch. (nearest quarter inch).



(A) $3\frac{3}{4}$" (B) 7" (C) $9\frac{1}{4}$" (D) $7\frac{1}{4}$" (E) $3\frac{1}{2}$"

37. The secant to a circle contains the center. Find the measure of the angle formed by the secant and a tangent if the larger intercepted arc measures 130°.

(A) 40° (B) 65° (C) 20° (D) 50° (E) 115°

38. Simplify: $\log_p(x - 1) + \log_p(x + 1) - 2\log_p(x + 1)$

(A) $\log_p(x^2 - 1)$ (B) $\log_p(x^2 - 2x + 1)$ (C) $\log_p(\frac{x - 1}{x + 1})$

(D) $\log_p(x^2 + 2x + 1$ (E) $\log_p(-1)$

39. If $\frac{2x + 1}{x + 3} - \frac{3x - 1}{x - 2} = \frac{Ax^2 + Bx + C}{Px^2 + Qx + R}$, then $\frac{A + B + C}{P + Q + R}$ equals:

(A) -3 (B) -1.25 (C) 0.5 (D) 2 (E) 2.75

40. Les Hite is a circus cable walker. He connects a 30 feet long cable from the ground to a telephone pole at a point 8 feet above the ground. Find the angle of elevation of the cable from the ground to the platform pole. (nearest minute)

(A) 14° 28′ (B) 14° 56′ (C) 15° 28′ (D) 15° 47′ (E) 15° 56′

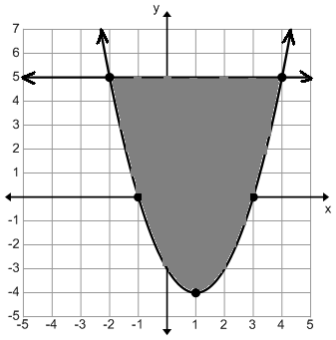
41. $\sum_{k=0}^{2} \left((3 - k)^k x + (k - 3)^k y \right) = ?$

(A) $2x - 2y$ (B) $4y$ (C) $4x - 2y$ (D) $4x$ (E) $4x + 4y$

42. How many values of θ satisfy $2\cos(3\theta - 1) = 0$, where $-\pi \leq \theta \leq \pi$?

(A) 8 (B) 7 (C) 6 (D) 4 (E) 3

43. Find the area of the shaded region in square units.



(A) $30\frac{2}{3}$ **(B)** $39\frac{1}{3}$ **(C)** 34 **(D)** $33\frac{1}{3}$ **(E)** 36

44. Kandy Packer bought little peeps to hand out to the children. She bought blue ones, white ones, yellow ones, and pink ones. How many different sets of 4 peeps balls can she package to give to the children?

(A) 40 **(B)** 35 **(C)** 24 **(D)** 16 **(E)** 4

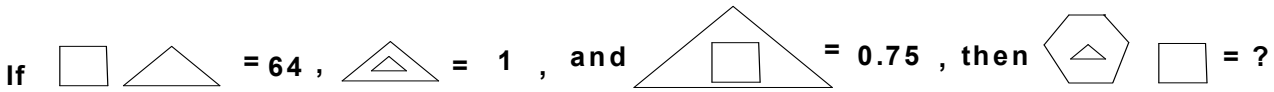
45. What is the probability that a factor of 120 is a multiple of 6?

(A) 25% **(B)** 31.25% **(C)** 37.5% **(D)** 50% **(E)** 60%

46. If you lived in the late 1600's and was working on a base 2 math problem, which of the following mathematicians could best assist you?

(A) Archimedes **(B)** Leonard Euler **(C)** Freda Porter **(D)** Gottfried Leibniz **(E)** Hypatia

47.



(A) 16 **(B)** 8 **(C)** 4.5 **(D)** 2 **(E)** 0.5

48. Eight schools are competing in number sense at their district meet. Each school enters three students. First, second, and third place winners receive medals and qualify for the regional meet. How many different arrangement of students can be awarded medals at the district meet?

(A) 336 **(B)** 2,016 **(C)** 2,024 **(D)** 4,048 **(E)** 12,144

49. Simplify: $a^3 \times b^{-2} \div a^{-1} \times b \div b^3 \div a^2 \times b^{-3} \div a^{-3}$

(A) a^2b^3 **(B)** a^5b^{-7} **(C)** a^2b^{-5} **(D)** a^5b^{-3} **(E)** a^9b^{-10}

50. An equation of a line with a y-intercept of $\frac{3}{4}$ and going through the point (— 1, 2) is ____.

(A) $5x + 4y = 3$ **(B)** $3x — 4y = 2$ **(C)** $4x + 2y = 3$ **(D)** $2x — 5y = 3$ **(E)** $4x — 5y = 2$

51. Simplify: $\frac{(n-3)!}{(n-1)!} \times \frac{(n)!}{n} \div \frac{(n-2)!}{n+1}$

(A) $n^2 - n - 2$ **(B)** $n + 1$ **(C)** 1 **(D)** $\frac{n+1}{n-2}$ **(E)** $\frac{n-3}{n+1}$

52. If $A = \begin{bmatrix} 2 & 5 \\ x & 3 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & 5 \\ 3 & y \end{bmatrix}$ **then** $AB = \begin{bmatrix} 23 & 0 \\ 5 & -11 \end{bmatrix}$. **Find** $x + y$.

(A) 2 **(B)** 0 **(C)** -3 **(D)** 6 **(E)** -1

53. Let $f(x) = \frac{1}{x+1}$. **Find** $f\big(f(f(x))\big)$

(A) $\frac{x+2}{2x+3}$ **(B)** $\frac{x+1}{x+2}$ **(C)** $\frac{1}{x+1}$ **(D)** $\frac{x+2}{x+3}$ **(E)** $x + 1$

54. Two chords, $\overline{PQ}$ **and** $\overline{RS}$, **are in a circle with center O such that** $\overline{OX} \perp \overline{PQ}$ **and** $PQ < RS$. **Which of the following is a false statement.**

(A) $\overline{RS}$ **can be a diameter** **(B)** $\overline{PX} \cong \overline{XQ}$ **(C)** $\overline{RS}$ **is closer to O**

(D) $\overline{OX}$ **can intersect** $\overline{RS}$ **(E)** $\overline{PQ} \parallel \overline{RS}$

55. How many solutions are there for the equation $5x + 8y = 610$ **such that both x and y are non-negative integers?**

(A) 10 **(B)** 13 **(C)** 15 **(D)** 16 **(E)** 20

56. Simplify to the nearest ten-thousandths place if $x = 1.5$: $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$.

(A) 0.4055 **(B)** 0.4167 **(C)** 0.4185 **(D)** 0.4047 **(E)** 0.4073

57. $\triangle ABC$ **is an equilateral triangle. Point E lies on segment AB such that AE = 2" and BE = 1". Point D lies on segment AC such that AD = 2" and CD = 1". Point F lies on segment BC such that BF = 2" and CF = 1". Find EF if ED = 2".**

(A) 2" **(B)** 1" **(C)** $\sqrt{2}$" **(D)** 3" **(E)** $\sqrt{3}$"

58. How many positive cubes divide 6! x 4! x 2! ?

(A) 10 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 2

59. $321_6 \times 5_6 + 44_6 =$ _____ $_6$

(A) 2053 **(B)** 404 **(C)** 3204 **(D)** 1053 **(E)** 2533

60. Dee Pedler and Cy Kohl live 6.5 miles apart. After lunch, Dee starts riding toward Cy's house. A little time later Cy starts riding toward Dee's house. When they meet, Dee had ridden 1.5 times as long as Cy and at 1.5 of Cy's rate. How far had Cy ridden when they met?

(A) 0.5 mi **(B)** 1 mi **(C)** 1.5 mi **(D)** 2 mi **(E)** 2.5 mi

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 1 ● 2012
Answer Key

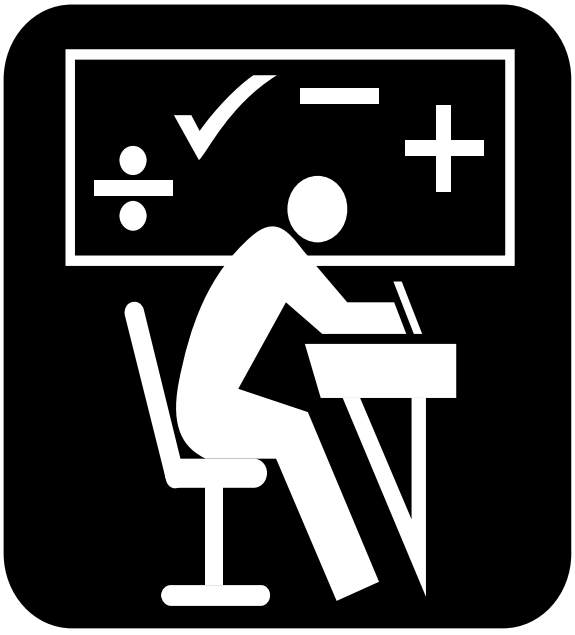
1. A	**21. D**	**41. D**
2. D	**22. E**	**42. C**
3. E	**23. D**	**43. E**
4. C	**24. E**	**44. B**
5. D	**25. B**	**45. C**
6. D	**26. D**	**46. D**
7. B	**27. A**	**47. A**
8. B	**28. C**	**48. E**
9. E	**29. B**	**49. B**
10. C	**30. C**	**50. A**
11. C	**31. C**	**51. D**
12. C	**32. C**	**52. C**
13. A	**33. C**	**53. A**
14. B	**34. A**	**54. E**
15. D	**35. C**	**55. D**
16. A	**36. D**	**56. A**
17. D	**37. A**	**57. E**
18. B	**38. C**	**58. C**
19. E	**39. E**	**59. E**
20. B	**40. C**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 2 • 2012



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $2 + 1 \times 3! + 4 \div (7 - 11) \times (18 - 29)$**

   (A) -128 (B) -7 (C) 19 (D) 27.5 (E) 29

2. **Mei Suhn borrowed $2500.00 to pay for his first semester college tuition. He agreed to pay back the loan plus the amount of simple interest in 5 equal monthly payments. What will his monthly payments be if the annual simple interest rate is 6%?**

   (A) $512.50 (B) $520.00 (C) $530.50 (D) $550.00 (E) $562.50

3. **Evaluate: $18\frac{3}{4}\% \times 0.375 \div \frac{3}{4}$?**

   (A) 0.5 (B) 0.046875 (C) 1.5 (D) 0.09375 (E) 0.625

4. **The shaded region of which of the Venn diagrams represents the set $(P \cap Q) \cup (Q \cap R)$?**

   (A) (B) (C) (D) (E)

5. **The set {— 1, 0, 1} is closed under which of these operations?**

   A - addition S - subtraction M - multiplication D - division

   (A) A, S, M, & D (B) D only (C) M only (D) A & M (E) M & D

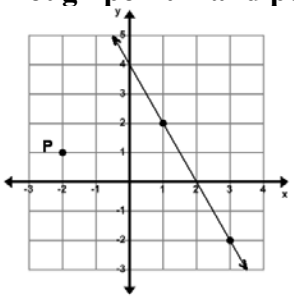
6. **Willie Maykett competed in an endurance race. He swam for 30 minutes at 8 mph, then ran for 45 minutes at 16 mph, and finished the race by riding his bicycle one hour at 24 mph. How far did Willie race?**

   (A) 32 mi (B) 36 mi (C) 40 mi (D) 44 mi (E) 48 mi

7. **If $6x^2 + 11x - 10 = (ax + b)(cx + d)$ then $a + b + c + d =$ _____.**

   (A) -10 (B) -4 (C) 7 (D) 8 (E) 12

8. **Find an equation of the line through point P and perpendicular to the line shown.**



   (A) $x + 2y = 2$ (B) $2x - 4y = 1$ (C) $x + 2y = 4$ (D) $2x - y = -2$ (E) $x - 2y = -4$

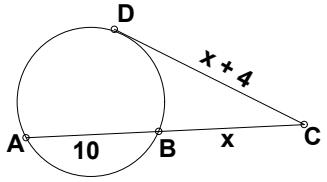
9. The measure of an interior angle of a regular nonagon is ___. (nearest degree)

(A) 120° (B) 129° (C) 135° (D) 140° (E) 144°

10. Point P(2, 0) lies on the x-y plane. Point P is rotated 90° counter clockwise to point Q. Point Q is translated vertically 3 units down to point R. Point R is reflected across the line y = x to point S. The coordinate of S is (x, y). Find $x + y$.

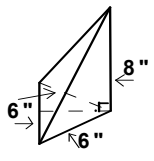
(A) -5 (B) -2 (C) -1 (D) 1 (E) 2

11. Given the circle, determine how much longer the secant is than the tangent.



(A) 6 (B) 3 (C) 7 (D) 4 (E) 8

12. What is the volume of the triangular pyramid shown? (nearest cubic inch)



(A) 32 in^3 (B) 42 in^3 (C) 55 in^3 (D) 125 in^3 (E) 166 in^3

13. Given the sequence 3, 7, 13, 21, 31, ..., 183, k, 241, ... find k.

(A) 201 (B) 203 (C) 207 (D) 211 (E) 223

14. Two house painters can paint 3 houses in 40 hours. How long would it take 5 house painters to paint 2 houses if they work at the same rate as the 2 house painters?

(A) 10 hrs 40 min (B) 12 hrs (C) 16 hrs (D) 9 hrs 36 min (E) 21 hrs 20 min

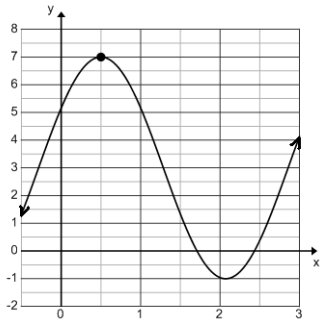
15. Which of the following is a function:

(A) $x = -2$ (B) $x = |3y - 2|$ (C) $x = 3 + 2y - y^2$ (D) $x = (y^2 - 1)^{(\frac{1}{3})}$ (E) $x = \frac{3}{y}$

16. If $\sqrt[2]{x^5\left(\sqrt[3]{x^4\left(\sqrt[4]{x^3}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 5 (B) 24 (C) 32 (D) 60 (E) 79

17. The equation $y =$ _____ will produce this graph.



(A) $3 - 4\cos(2x - 1)$ **(B)** $4\sin(2x + 1) - 3$ **(C)** $4\cos(2x - 1) + 3$

(D) $3 + 4\sin(2x - 1)$ **(E)** $4\sin(x - 2) + 3$

18. Determine the frequency of $y = 1 + 2\sin\left[3\pi(x - 4)\right]$.

(A) 0.5 **(B)** 0.666... **(C)** 1 **(D)** 1.5 **(E)** 2

19. Simplify: $(\tan\theta)(1 + \cos 2\theta)$

(A) $\sin 2\theta$ **(B)** $\csc 2\theta$ **(C)** $\sin\theta\cos\theta$ **(D)** $\tan\frac{\theta}{2}$ **(E)** $(\sec\theta)(\csc\theta)$

20. Let $3x^5 - 4x^4 + x^3 - 5x^2 = -15$. **According to Descartes' Rule of Signs how many possible negative roots are there?**

(A) 1, 3, or 5 **(B)** 1 or 3 **(C)** 0, 2, or 4 **(D)** 1 **(E)** 3

21. Find A if the remainder of $Ax^5 - 3x^3 + 2x^2 - x + 3$ **divided by** $x + 1$ **is 4.**

(A) -2 **(B)** -1 **(C)** 3 **(D)** 5 **(E)** 9

22. If $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, **then** $a \times d - b \times c = ?$

(A) 40 **(B)** 25 **(C)** 13 **(D)** 9 **(E)** 5

23. Let $f(x) = x^2 - 5x + 7$. **The absolute minimum of f(x) on** $[-1, 3]$ **is found at** $y = ?$

(A) .25 **(B)** .75 **(C)** 1 **(D)** 2.5 **(E)** 3

24. If $f(\theta) = \sin\theta\cos\theta$ **then** $f'(\frac{5\pi}{6}) =$ _____.

(A) $-\frac{\sqrt{3}}{2}$ **(B)** $-\frac{1}{2}$ **(C)** $-\frac{\sqrt{3}}{4}$ **(D)** $\frac{1}{2}$ **(E)** 1

25. Let $f(x) = x^3 + 3x^2 + 3x + 1$ **and** $g(x) = x^2 - 2x + 1$. **Find** $g\left(f''(-1)\right)$

(A) -12 **(B)** 0 **(C)** 1 **(D)** 4 **(E)** 6

26. A small box of Strike-It matches contains 36 matches. The odds that a match will light is $\frac{7}{9}$. What is the probability that they won't light?

(A) 15.75% **(B) 28%** **(C) 56.25%** **(D) 84.25%** **(E) 43.75%**

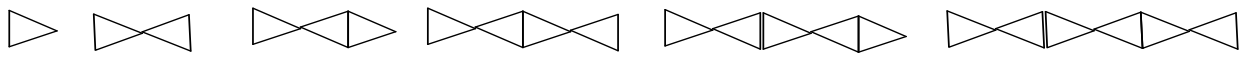
27. I. M. Kannfused has a multiple dial lock that has four rotating discs. He doesn't know the combination but he does know it consists of four non-zero digits. The first digit is an even number, the second digit is divisible by 3, the third digit is a perfect square, and the last digit is a Fibonacci number. How many unique combinations can he try to open the lock?

(A) 180 **(B) 144** **(C) 135** **(D) 120** **(E) 108**

28. The fruit market has apples, oranges, bananas, grapes and melons. Each bag they sell contains 4 fruits. How many different bags of fruit could they make to sell?

(A) 70 **(B) 20** **(C) 630** **(D) 56** **(E) 30**

29. The six shapes below are made up of equilateral triangles with side length 1 cm. If the pattern continues, fine the perimeter of the shape consisting of 15 triangles.



(A) 34 cm **(B) 33 cm** **(C) 32 cm** **(D) 31 cm** **(E) 30 cm**

30. How many integers are in the domain of the real-valued function $f(x) = \sqrt{3 - \sqrt{6x - 10}}$?

(A) infinite **(B) 3** **(C) 2** **(D) 1** **(E) none**

31. Find the number of positive integral divisors of 600.

(A) 30 **(B) 24** **(C) 20** **(D) 8** **(E) 6**

32. $421_8 + 532_8 - 347_8 =$ _____ $_8$

(A) 606 **(B) 604** **(C) 704** **(D) 616** **(E) 706**

33. Find the mean of the median, mode, and range of $2\frac{3}{4}, 5\frac{1}{2}, 3\frac{3}{4}, 4\frac{1}{8}, 5\frac{1}{2}$, and $1\frac{1}{16}$.

(A) $2\frac{5}{16}$ **(B) $2\frac{31}{40}$** **(C) $3\frac{7}{8}$** **(D) $4\frac{1}{4}$** **(E) $4\frac{5}{8}$**

34. If $a_1 = 2$, $a_2 = -1$, $a_3 = -3$ and $a_n = (a_{n-2})(a_{n-1}) - (a_{n-3})$, where $n \geq 4$ then a_8 equals:

(A) -3 **(B) -2** **(C) -1** **(D) 1** **(E) 2**

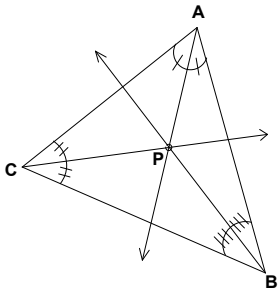
35. If $x^3 - 3x^2 - x + 3 = (ax + b)(cx + d)(ex + f)$ then $(a + c + e) - (b + d + f) =$ _____.

(A) 8 **(B)** 6 **(C)** 3 **(D)** 0 **(E)** -3

36. The coordinate of points P and Q are plotted on the Cartesian plane. The abscissa of point P is less than zero the ordinate of P is less than zero. The abscissa of point Q is greater than zero and the ordinate of Q is less than zero. Which quadrants will the segment connecting P and Q be in?

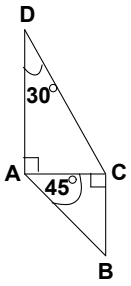
(A) I, II, & III **(B)** III & IV **(C)** II & III **(D)** I, III, & IV **(E)** I & IV

37. The point of concurrency, P, is the _____ of $\triangle ABC$ shown below.



(A) centroid **(B)** circumcenter **(C)** incenter **(D)** orthocenter **(E)** center

38. Find the area of quadrilateral ABCD if AB = 3 cm. (nearest tenth).



(A) 1.8 cm^2 **(B)** 10.3 cm^2 **(C)** 5.0 cm^2 **(D)** 6.1 cm^2 **(E)** 13.0 cm^2

39. The ellipse $x^2 + 2y^2 - 2x + 8y = 5$ has a center at (h, k). Find $h + k$.

(A) -3 **(B)** -1 **(C)** 3 **(D)** 4 **(E)** 5

40. How many elements are in $\left\{x \mid 3\tan(x)\sin(x) + \sin(x) = 0, x \in [-\pi, 2\pi]\right\}$?

(A) 8 **(B)** 7 **(C)** 4 **(D)** 3 **(E)** 2

41. Chris B. Kritter sees two fires from his ranger station. One fire is about 12 miles on a bearing of 60° degrees and the other is 10 miles on a bearing of 300°. How far apart are the two fires? (nearest mile)

(A) 19 mi **(B)** 18 mi **(C)** 17 mi **(D)** 15 mi **(E)** 14 mi

42. Simplify to the nearest ten-thousandths place if $x = \frac{1}{2}$: $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

(A) 0.4185 **(B)** 0.4167 **(C)** 0.4073 **(D)** 0.4047 **(E)** 0.4055

43. How many solutions are there for the equation $6x + 7y = 111$ such that both x and y are positive integers?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 13

44. Simplify $\left(1 + \sqrt{-12}\right)\left(2 - \sqrt{-75}\right)$ to the form $a + bi$.

(A) $-28 - \sqrt{3}\,i$ **(B)** $2 - 10\sqrt{3}\,i$ **(C)** $32 - \sqrt{3}\,i$ **(D)** $28 - 3\sqrt{3}\,i$ **(E)** $32 - 9\sqrt{3}\,i$

45. Find the area (in square units) of the region bounded by the curves $y = 3\sqrt{x}$ and $y = \frac{3x^2}{8}$.

(A) $6\frac{7}{8}$ **(B)** 7 **(C)** $7\frac{3}{4}$ **(D)** 8 **(E)** $9\frac{1}{8}$

46. $\int \left(2x\cos(2x) + \sin(2x)\right)\,dx =$ _____ $+ C$, where C is some arbitrary constant.

(A) $\cos(2x)$ **(B)** $\sin^2(2x)$ **(C)** $\cos(2x) - \sin(2x)$ **(D)** $\cos^2(2x)$ **(E)** $x\sin(2x)$

47. Bill Counter has a money bag containing a \$20 bill, two \$10 bills, two \$5 bills and a \$1 bill. If he randomly draws out three bills what is the probability that the sum of the bills is greater than \$25.00? (nearest percent)

(A) 25% **(B)** 40% **(C)** 44% **(D)** 50% **(E)** 61%

48. The odds of scoring above 150 on this test is $\frac{5}{8}$. How many students took this test if 16 students scored below 150?

(A) 26 **(B)** 24 **(C)** 16 **(D)** 13 **(E)** 10

49. The sequence $-2, -1, 0, -1, -2, -1, \ldots$, where $P_0 = -2$, $P_1 = -1$, and $P_2 = 0$, is a recursive sequence for all P_n where $n \geq 3$. Find P_8.

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

50. The operation " $\oplus$ " is defined as $P \oplus M = P + M - MP$. Compute $(2 \oplus 3) \oplus 5$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 7 **(E)** 9

51. If $\frac{x-4}{x+3} + \frac{x+3}{x-4}$ is written as the mixed number $A\frac{B}{C}$ then B = ?

(A) 49 **(B)** 24 **(C)** 14 **(D)** 7 **(E)** 1

52. Anne Surwrite needs to earn 90 or higher on at least 75% of her 40 daily homework assignments to make an A in the class. If she earned a 90 or higher on 20 of the first 25 assignments, how many of the remaining assignments can she make less than 90 and still make an A?

(A) 15 (B) 10 (C) 9 (D) 5 (E) 1

53. The area of a regular pentagon is 61.8 sq. cm. Find the length of the apothem if the length of the side of the pentagon is 6 cm. (nearest hundredth)

(A) 1.72cm (B) 2.24 cm (C) 3.21 cm (D) 3.84 cm (E) 4.12 cm

54. Find the sum of the coefficients of the quotient: $(x^3 - 3x^2 + 4x + 8) \div (x + 1)$

(A) 4 (B) 5 (C) 7 (D) 10 (E) 13

55. Find an identity for $\sec^4\theta - 2\sec^2\theta \tan^2\theta + \tan^4\theta$ for all defined values of θ.

(A) -1 (B) $\csc^4\theta$ (C) $\cot^4\theta$ (D) $\csc^2\theta + \cot^2\theta$ (E) 1

56. Point B lies on $\overline{AC}$ and points D and E exist such that $\overline{DB} \perp \overline{AC}$, $\overline{EC} \perp \overline{BC}$, AB = 1", BC = 2", CE = 3", BD = 4", $\overline{DB}$ does not intersect $\overline{AE}$, and all 5 points are coplanar. Find m∠DAE. (nearest degree).

(A) 124 ° (B) 121 ° (C) 104 ° (D) 76 ° (E) 59

57. May Kingkash inherited $500.00 and invested it for a 5 year period. The five annual returns were + 5%, + 3%, — 2%, + 7%, and — 4%. What was May's average rate of return over the five year period? (nearest tenth)

(A) 3.3% (B) 3.0% (C) 2.5% (D) 1.8% (E) 1.7%

58. A right triangle has a hypotenuse of length 10". If one of the acute angles is decreasing at the rate of 5° per second, how fast is the area of the triangle decreasing when this acute angle is 30°? (nearest tenth)

(A) 0.8 in^2/sec (B) 2.2 in^2/sec (C) 2.5 in^2/sec (D) 3.8 in^2/sec (E) 4.3 in^2/sec

59. Seymore Wirk's math class has 9 boys and 8 girls. 4 of the boys and 4 of the girls need tutoring after school. He randomly selects two students to work a problem on the board. What is the probability that both are girls or both need tutoring after school? (nearest percent)

(A) 50% (B) 47% (C) 44% (D) 40% (E) 37%

60. The set of numbers, {1, 10, 13, 23, 31} are _____ numbers.

(A) Abundant (B) Happy (C) Lucky (D) Odious (E) Prime

University Interscholastic League
MATHEMATICS CONTEST
HS ● District 2 ● 2012
Answer Key

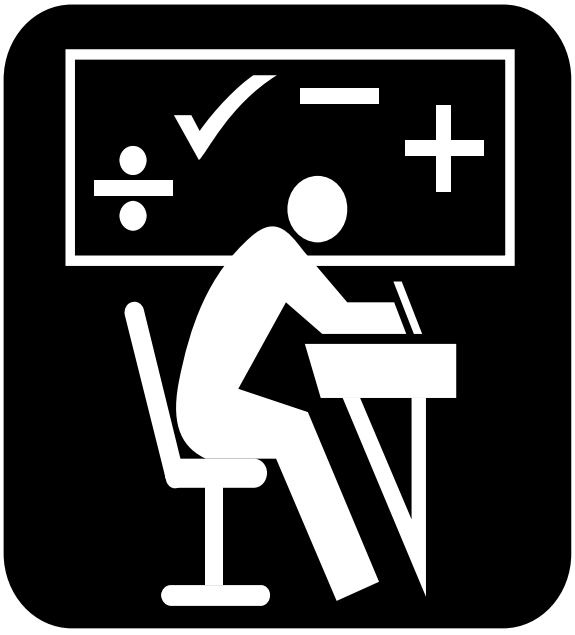
1. C	**21. D**	**41. A**
2. A	**22. B**	**42. E**
3. D	**23. B**	**43. A**
4. E	**24. D**	**44. C**
5. C	**25. C**	**45. D**
6. C	**26. C**	**46. E**
7. D	**27. A**	**47. D**
8. E	**28. A**	**48. A**
9. D	**29. D**	**49. A**
10. C	**30. C**	**50. E**
11. A	**31. B**	**51. A**
12. B	**32. B**	**52. D**
13. D	**33. E**	**53. E**
14. A	**34. C**	**54. B**
15. E	**35. B**	**55. E**
16. E	**36. B**	**56. B**
17. C	**37. C**	**57. E**
18. D	**38. D**	**58. B**
19. A	**39. B**	**59. E**
20. D	**40. B**	**60. B**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Regional • 2012



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $2 + 3! \times 5 + 7 + 11 \div (13 - 17)$**

   (A) 44.25 (B) 36.25 (C) 19 (D) -7 (E) -9

2. **Let S = {s,u,n,d,a,y}, M = {m,o,n,d,a,y}, T = {t,u,e,s,d,a,y} and F = {f,r,i,d,a,y}. The number of distinct elements in $(F \cup S) \cap (M \cup T)$ is _____.**

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

3. **What is $87\frac{1}{2}\%$ of $\frac{5}{6} \div 0.75$?**

   (A) 2 (B) 0.91666... (C) $\frac{35}{36}$ (D) 0.7875 (E) $\frac{9}{10}$

4. **Les Iron picked 3 steel rods out of his pile of scrap steel. When he measured them he found out that the longest rod was 8 inches longer than twice the shortest rod and the middle length rod was 1 foot 4 inches shorter than the longest rod. If he lined them up end to end the total length was 20 feet. How long was the longest rod?**

   (A) 10 ft 8 in (B) 9 ft 6 in (C) 8 ft (D) 8 ft 8 in (E) 7 ft 4 in

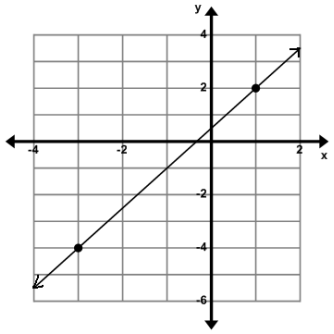
5. **Let X vary inversely with the square of Y and Y = 5 when X = 3. Find X if Y = 3.**

   (A) 15 (B) $1\frac{2}{3}$ (C) 5 (D) $1\frac{1}{4}$ (E) $8\frac{1}{3}$

6. **If $x^4 - 13x^2 + 36 = (ax + b)(cx + d)(ex + f)(gx + h)$, where a, b, c, d, e, f, and g do not have to be unique, then $(a + c + e + g) - (b + d + f + h)$ = _____.**

   (A) 14 (B) 10 (C) 6 (D) 4 (E) 1

7. **The y-intercept of the line containing the point $(-3, -1)$ and parallel to the line shown is (x, y). Find y.**

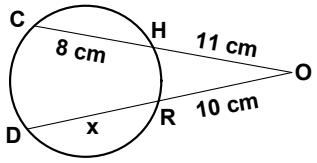


   (A) $-2\frac{1}{3}$ (B) 0 (C) $\frac{1}{2}$ (D) $3\frac{1}{2}$ (E) 7

8. **Miss White hires Sleepy and Dopey to pick apples from her orchard. The two of them can pick three crates of apples in four hours. If she hires five more pickers, how many crates of apples could the seven pickers pick in eight hours working at the same rate as Sleepy and Dopey?**

   (A) 28 (B) 21 (C) 20 (D) 18 (E) 15

9. Given the circle, determine the value of x to the nearest tenth.



(A) 7.3 cm (B) 8.9 cm (C) 9.0 cm (D) 10.9 cm (E) 13.0 cm

10. A triangle has side lengths of 6", 10", and k". How many acute scalene or obtuse scalene triangles exist where k is an integer?

(A) 5 (B) 8 (C) 9 (D) 11 (E) 15

11. Given the diameter and height of the pipe shown, determine how many gallons of water will it hold before overflowing. (nearest gallon)

(A) 15 gals (B) 62 gals (C) 47 gals (D) 39 gals (E) 188 gals

12. Bill Meelator is broke. He borrows $50.00 from Les Cash and $120.00 from Ima Rich. He promises to pay them back at the end of 6 months time at an annual simple interest rate of 5% for Les and 12% for Ima. How much will he need to pay them off at the end of 6 months?

(A) $176.00 (B) $178.45 (C) $184.45 (D) $186.90 (E) $187.14

13. Given the sequence $-1, 1, 5, 11, 19, ..., 505, k, 599, ...$ find k.

(A) 571 (B) 569 (C) 555 (D) 549 (E) 551

14. Find the sum of the coefficients of quadratic term and the linear term of the quotient: $(6x^4 + 3x^2 - 9) \div (x - 1)$

(A) 3 (B) 12 (C) 15 (D) 18 (E) 30

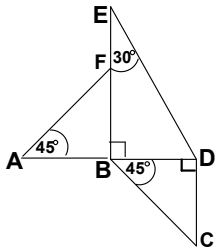
15. Let $f(x) = \frac{2x+3}{4}$ and $g(x) = \frac{4x-5}{6}$. Find $g(f(a+1))$.

(A) $\frac{a}{3}$ (B) $\frac{a+2}{3}$ (C) $\frac{4a+1}{12}$ (D) $\frac{a-1}{3}$ (E) $\frac{2a-3}{12}$

16. Thomas Bayes used a magic marker to label 11 golf balls with the letters from the word "PROBABILITY". He randomly selected 4 balls, one at a time, without replacement. What is the probability that the letter on the first ball was a vowel, the second one a consonant, the third one a vowel and the fourth one a consonant? Let "Y" be a vowel. (nearest tenth percent)

(A) 7.6% (B) 2.7% (C) 3.6% (D) 7.9% (E) 5.1%

17. Find the perimeter of the hexagon, if CD = 3 inches and EF = 1 inch. (nearest half inch)

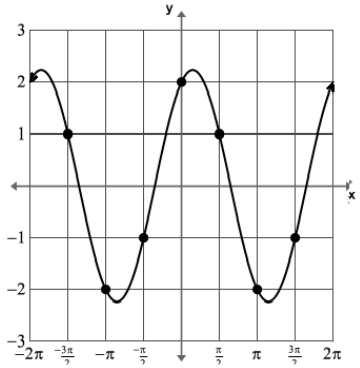


(A) 25.0 in (B) 24.5 in (C) 22.5 in (D) 21.5 in (E) 20.0 in

18. Determine the phase shift of $f(x) = 1 - 2\cos(\frac{\pi}{3}x - \frac{\pi}{4})$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$ (E) $\frac{1}{12}$

19. The equation y = _____ will produce this graph.



(A) $\sin(x) + 2\cos(x)$ (B) $2\cos^2(x) - 2\sin^2(x)$ (C) $\cos(x) - 2\sin(x)$
(D) $2\sin(x)$ (E) $2.25\cos(x - 0.25\pi)$

20. Willie B. Lossed stands in the middle of a field. He starts walking on a bearing of 150°. After walking 75 yards, he turns and walks 100 yards on a new bearing of 300°. How far is Willie from the middle of the field where he started? (nearest yard)

(A) 25 yds (B) 44 yds (C) 51 yds (D) 71 yds (E) 90 yards

21. If $(4 - 3i) \div (2 - i) - (3 + 2i) = a + bi$ then $a + b$ = ?

(A) — 3.2 (B) — 1.6 (C) — 0.8 (D) 2 (E) 2.666...

22. Use the infinite geometric sequence P, — 4, 2, Q, R, ... to find PQR.

(A) — 8 (B) — 4 (C) 5.5 (D) 7.5 (E) 32

23. Let $x^5 - x^4 - 3x^3 + 2x^2 - x = 1$. According to Descartes' Rule of Signs how many possible negative roots are there?

(A) 1, 3, or 5 (B) 1 or 3 (C) 0 or 2 (D) 1 (E) 0

24. M. T. Bank has $1 bills, $5 bills, $10 bills, $20 bills, $50 bills, and $100 bills. He is giving away envelopes containing 3 bills to people going into the grocery store. How many different envelopes of 3 bills could he give away?

(A) 84 (B) 28 (C) 720 (D) 56 (E) 20

25. May B. Luckie tosses a dart at the April calendar on the wall. What are the odds that the dart hits a date that is a prime number? Each date has an equal chance of being hit.

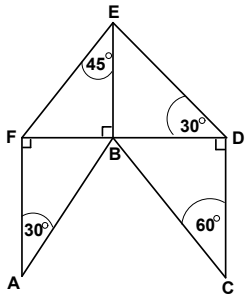
(A) 1:3 (B) 3:7 (C) 1:1 (D) 3:10 (E) 1:2

26. Using the following array, determine the value of the last integer of the 30th row.

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
⋮

(A) 911 (B) 915 (C) 919 (D) 921 (E) 929

27. Find the perimeter of hexagon BF = 3 cm. (nearest cm).



(A) 23 cm (B) 25 cm (C) 28 cm (D) 30 cm (E) 33 cm

28. What is the digit in the tens place of $\left((11)^{10}\right)^{9}$?

(A) 0 (B) 1 (C) 2 (D) 8 (E) 9

29. How many integers are in the domain of the real-valued function f(x) = $\sqrt{7-\sqrt{5x+3}}$?

(A) 10 (B) 8 (C) 6 (D) 4 (E) 2

30. Betty Fallsoff leans a 30 foot ladder against a wall. When Betty starts to climb on the ladder its upper end starts sliding down the wall at the rate of $\frac{1}{2}$ ft/sec. What is the rate of change of the acute angle made by the ladder with the ground when the upper end is 18 ft above the ground?

(A) $\frac{1}{24}$ rads/sec (B) $\frac{2}{3}$ rads/sec (C) $\frac{7}{30}$ rads/sec (D) $\frac{1}{48}$ rads/sec (E) $\frac{1}{2}$ rads/sec

31. $(311_4 - 113_4) \div 3_4 =$ _____ $_4$

(A) 132 (B) 231 (C) 110 (D) 31 (E) 22

32. If $\frac{x-11}{x+12} + \frac{x+12}{x-11}$ is written as the mixed number $A\frac{B}{C}$ then B = ?

(A) 1 (B) 23 (C) 264 (D) 265 (E) 529

33. Let $\triangle ABC$ exist such that D is the midpoint of segment AC, $m\angle CAB = m\angle CBD$, and AB = 12. Find the square of the length of BD.

(A) 576 (B) 144 (C) 72 (D) 36 (E) 24

34. The diameter of the largest circle is 6 inches. The diameter of the smallest circle is 2 inches. The diameter of the other circle is 4 inches. Miss Mi Target shoots an arrow at the target. She gets 10 points if she hits the shaded area or loses 15 points if she hits the non-shaded area. What is the mathematical expectation of shooting one arrow? (nearest tenth)

(A) -12.0 pts (B) -7.9 pts (C) -6.7 pts (D) -5.0 pts (E) -3.9 pts

35. Let $a_1 = 2$, $a_2 = 3$, $a_3 = 5$ and $a_n = (a_{n-1}) - [(a_{n-2}) + (a_{n-3})]$ for $n \geq 4$. Find a_7.

(A) -13 (B) -5 (C) 0 (D) 7 (E) 11

36. $\sin(\theta + \frac{\pi}{4})$ equals :

(A) $\cos(\theta - \frac{3\pi}{4})$ (B) $\sin(\theta + \frac{5\pi}{4})$ (C) $\cos(\theta + \frac{3\pi}{4})$ (D) $\sin(\theta - \frac{\pi}{4})$ (E) $\cos(\theta + \frac{7\pi}{4})$

37. $\sum_{k=1}^{3} \left((2-k)^k x + (k-2)^k y \right) = ?$

(A) $-2x - 2y$ (B) $2x + 2y$ (C) 0 (D) $2x$ (E) $-2y$

38. Ima Digger has 168 feet of fence to enclose a rectangular shaped garden and partition it into 3 sections to keep the deer out. The length of the garden is 5 times as long as the width. She divides the length of the garden into 3 sections such that the lengths of each section is in the ratio of 1:3:6. What is the area of the largest section?

(A) 480 sq. ft. (B) 432 sq. ft. (C) 360 sq. ft. (D) 288 sq. ft. (E) 216 sq. ft.

39. Let $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ x^2 - 6x + 7 & \text{if } 1 \le x \end{cases}$. Find the distance between the points where the absolute minimum and the absolute maximum of f(x) occur on [— 5, 4].

(A) $\sqrt{2}$ **(B)** $2\sqrt{17}$ **(C)** $3\sqrt{10}$ **(D)** $4\sqrt{5}$ **(E)** $6\sqrt{2}$

40. How many distinguishable permutations can be made from the letters 'MATHCHAMP' ?

(A) 181,440 **(B)** 5,760 **(C)** 45,360 **(D)** 720 **(E)** 60,480

41. $11011101_2 + 1234_8 + 3210_4 =$ _____ $_{16}$

(A) 8AB **(B)** 45D **(C)** 911 **(D)** 2DD **(E)** ABD

42. Point A(2, — 1) lies on the x-y plane. Point A is reflected across the line x = — 1 to point B. Point B is rotated 90° clockwise about the origin to point C. Point C is translated vertically 5 units down to point D. Point D is reflected across the origin to point E. The coordinate of E is (x, y). Find $x + y$.

(A) — 6 **(B)** — 2 **(C)** 0 **(D)** 1 **(E)** 2

43. Find the mean of the median, mode, and range of 4.5, 1.2, 3.4, 4.5, 2.3, and 0.6.

(A) 3.75 **(B)** 2.4 **(C)** 4.11666... **(D)** 2.75 **(E)** 4.15

44. Which of the following is NOT a member of the solution set for $2 + 3|5 - 7x| < 11$?

(A) 0.1428 **(B)** 0.2858 **(C)** 0.4285 **(D)** 0.8571 **(E)** 1.1428

45. Find the equation of an ellipse whose foci are (— 1,0) and (1,0) and the sum of the focal radii is 4.

(A) $x^2 + y^2 = 2$ **(B)** $4x^2 + 3y^2 = 1$ **(C)** $x^2 + y^2 = 16$

(D) $3x^2 + 4y^2 = 12$ **(E)** $3x^2 + 4y^2 = 7$

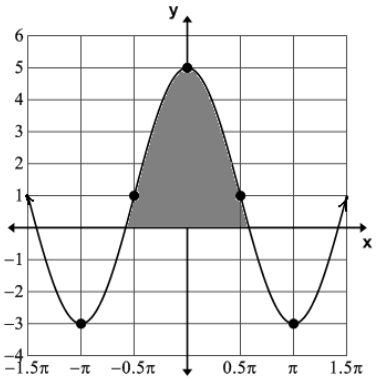
46. Mary Goround looked at her circular clock and saw that the time was 8:24 AM. The next time she looked at the clock the time was 8:50 AM. How many degrees had the big hand moved?

(A) 143 ° **(B)** 150 ° **(C)** 156 ° **(D)** 175 ° **(E)** 186 °

47. I. C. Kold used the graph of $T = 37 \sin\left[\frac{2\pi}{365}(D - 101)\right] + 25$, where T is the temperature and D is the day of the year to find the estimated mean daily Fahrenheit temperature in Kold Canyon, North Dakota. What day in April, 1988 was the estimated mean temperature 32°F?

(A) 1st **(B)** 7th **(C)** 15th **(D)** 21st **(E)** 23rd

48. Find the area of the shaded region in square units. (nearest tenth)



(A) 12.5 (B) 11.9 (C) 11.4 (D) 10.8 (E) 10.5

49. Find the digit in the 10^{-4} place of the series

$$\frac{(0.5\ln(3))^0}{0!} + \frac{(0.5\ln(3))^1}{1!} + \frac{(0.5\ln(3))^2}{2!} + \frac{(0.5\ln(3))^3}{3!} + \frac{(0.5\ln(3))^4}{4!} + \dots .$$

(A) 0 (B) 2 (C) 3 (D) 5 (E) 7

50. The sequence — 2, 3, — 5, —10, — 2, ... , where $P_0 = -2$, $P_1 = 3$, and $P_2 = -5$, is a recursive sequence for all P_n where $n \geq 3$. Find P_{13}.

(A) — 5 (B) — 3 (C) — 2 (D) 3 (E) 5

51. Harry Heds has a store that sells wigs. He has 7 blonde wigs, 6 brunette wigs, 5 black wigs and 4 red wigs for sale. He randomly selects three wigs to put in the showcase window. What is the probability that he selects a blonde wig, then a brunette wig, then a red wig? (nearest percent)

(A) 10% (B) 8% (C) 6% (D) 4% (E) 2%

52. Simplify: $\left(a \div b^{-2} \times a^{-3} \div b^4\right)^{-2} \times \left(b^{-1} \times a^2 \div b^3 \times a^{-4}\right)^2$

(A) $(ab)^{-8}$ (B) $a^{-1}b^4$ (C) ab^{-4} (D) b^{-4} (E) a^{-8}

53. How many positive cubes divide 5! x 4! x 3! ?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

54. If (2, 3), (3, 5), and (k, — 2) are members of the function {(x, y) | 3ax + 2y = b}, then k = ?

(A) — 5 (B) — 3 (C) — 2.5 (D) — 1 (E) — 0.5

55. If the three numbers 417, 354, and 249 are each divided by the number D, each of their quotients will have the same remainder R. Find R where $R > 1$.

(A) 3 (B) 6 (C) 7 (D) 14 (E) 18

56. How many solutions are there for the equation $4x + 21y = 2012$ such that both x and y are non-negative integers?

(A) 25 (B) 24 (C) 23 (D) 20 (E) 12

57. Let $f(x) = \frac{x-1}{x+1}$. Find $f\Big(f\big(f(-x)\big)\Big)$

(A) $x - 1$ (B) $\frac{x+1}{1-x}$ (C) $\frac{1}{x}$ (D) $\frac{1-x}{1+x}$ (E) $x + 1$

58. Let $f(x) = x^3 - 2x^2 - 15x + 2$. Find the sum of the x-values of the critical points of f(x).

(A) $-1.666...$ (B) -1 (C) 1.333... (D) 3 (E) 4.666...

59. The function $f(x) = \frac{x^2 - 2x - 8}{x + 4}$ has two asymptotes that intersect at (x, y). Find y.

(A) -10 (B) -6 (C) -4 (D) 2 (E) 4

60. $\triangle ABC$ is an acute triangle. Point D lies on segment AB such that CD is the length of a median of $\triangle ABC$, $m\angle ACD = 30°$, $m\angle BCD = 20°$, and AC = 10 cm. Find BC. (nearest tenth)

(A) 22.4 cm (B) 17.1 cm (C) 15.3 cm (D) 14.6 cm (E) 13.7 cm

University Interscholastic League
MATHEMATICS CONTEST
HS ● Regional ● 2012
Answer Key

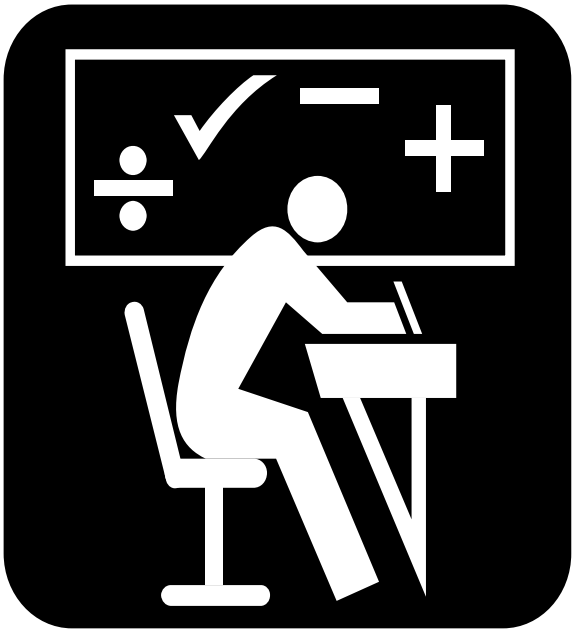
1. B	**21. A**	**41. B**
2. D	**22. B**	**42. E**
3. C	**23. C**	**43. A**
4. D	**24. D**	**44. A**
5. E	**25. E**	**45. D**
6. D	**26. E**	**46. C**
7. D	**27. D**	**47. D**
8. B	**28. A**	**48. C**
9. D	**29. A**	**49. A**
10. B	**30. D**	**50. D**
11. C	**31. E**	**51. E**
12. B	**32. E**	**52. D**
13. E	**33. C**	**53. D**
14. C	**34. C**	**54. E**
15. A	**35. B**	**55. E**
16. A	**36. E**	**56. B**
17. B	**37. C**	**57. D**
18. C	**38. B**	**58. C**
19. A	**39. E**	**59. A**
20. C	**40. C**	**60. D**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

State • 2012



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $(1 - 2^3 + 4! - 5) \div 6 \times 7 + 8 - 9^0$

   (A) 21 **(B) 23.333...** **(C) 4.1666...** **(D) 22** **(E) 13**

2. **Find the number of positive integral divisors of 1,250.**

   (A) 6 **(B) 8** **(C) 10** **(D) 12** **(E) 16**

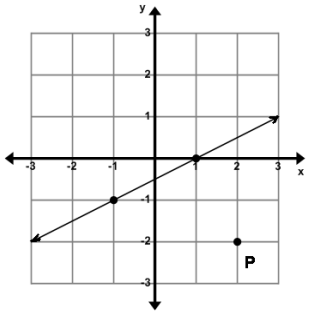
3. **If P is 20% less than Q and Q is 60% more than R, then R is what percent of P?**

   (A) 128% **(B)** $83\frac{1}{3}\%$ **(C) 120%** **(D)** $31\frac{1}{4}\%$ **(E)** $78\frac{1}{8}\%$

4. **Megan Degrade has a home work average of 96 and a test average of 85. Her grade for the class is computed by totaling 25% of her home work average, 60% of her test average, and 15% of her final exam. What does she need to make on her final exam to have a class average of 90?**

   (A) 100 **(B) 98** **(C) 96** **(D) 94** **(E) 92**

5. **The x-intercept and y-intercept of the line containing point P and parallel to the line shown are (x, 0) and (0, y). Find x + y.**



   (A) −6 **(B) −3** **(C) 0** **(D) 3** **(E) 6**

6. **Simplify:** $\left(\frac{12x^2 - 5x - 2}{2x^3 + 4x^2 + 2x}\right) \div \left(\frac{3x^2 - 5x + 2}{-4x^3 + 4x}\right)$

   (A) $\frac{-4x^2 + 3x + 1}{-x + 1}$ **(B)** $\frac{-8x - 2}{x + 1}$ **(C)** $\frac{4x + 1}{1 + x}$

   (D) $\frac{(4x + 1)(3x - 2)^2}{-2}$ **(E)** $\frac{-24x^2 + 10x + 4}{x^2 - 1}$

7. **If Y varies jointly as X and Z^2, and Y is 48 when X is 8 and Z is 3, what is the value of Y when X is 12 and Z is 2.**

   (A) 2 **(B) 12** **(C) 24** **(D) 32** **(E) 36**

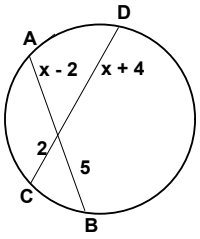
8. **Millie weighs 6 kg more than half of Hector's weight. Dezi weighs 4 kg more than the sum of Millie's weight and Hector's weight. How much does Millie weigh if Dezi weighs 70 kg?**

   (A) 20 kg **(B) 26 kg** **(C) 47 kg** **(D) 56 kg** **(E) 113 kg**

9. Let $\angle PQS$ and $\angle SQT$ be complementary and $\angle PQS$ and $\angle RQS$ be supplementary. If $m\angle PQS = 2x - 3$ and $m\angle RQS = 3x + 8$ then $m\angle SQT = ?$

(A) 113° (B) 70° (C) 67° (D) 35° (E) 23°

10. Given the length of the two chords of the circle shown, find AB + CD.

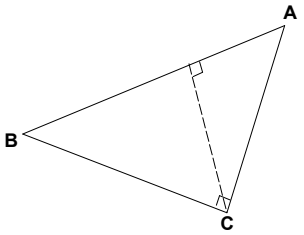


(A) 18 (B) 19 (C) 21 (D) 22 (E) 25

11. Find the total surface area of a right circular cylinder with a radius of 3" and a height of 6". (nearest square inch)

(A) 170 sq. in. (B) 254 sq. in. (C) 75 sq. in. (D) 113 sq. in. (E) 141 sq. in.

12. The point of concurrency, C, is the _____ of $\triangle ABC$ shown below.



(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) foci

13. Given the sequence 3, 11, 27, 51, ..., 227, k, 363, ..., find k.

(A) 273 (B) 287 (C) 291 (D) 293 (E) 321

14. If $\frac{x-3}{x+2} + \frac{2x-1}{3x+1} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

(A) $-1\frac{1}{4}$ (B) $-\frac{11}{12}$ (C) $-\frac{1}{12}$ (D) $-\frac{3}{4}$ (E) $-\frac{5}{12}$

15. If $\sqrt[2]{x^4\left(\sqrt[3]{x^6\left(\sqrt[5]{x^8}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 49 (B) 32 (C) 18 (D) 16 (E) 15

16. Simplify: $\frac{1}{1-\sin(\theta)} + \frac{1}{1+\sin(\theta)}$

(A) 2 (B) $2\csc^2(\theta)$ (C) $2\sin(\theta)$ (D) $2\cos(\theta)$ (E) $2\sec^2(\theta)$

17. Two ranch hands working for 3 hours without a break can dig 10 post holes. How many ranch hands would it take to dig 50 post holes in 6 hours if they work at the same rate as the 2 ranch hands?

(A) 10 (B) 9 (C) 6 (D) 5 (E) 3

18. Find the sum of the amplitude, the vertical shift, and the frequency of $y = 2 + 4\sin(4\pi x + 3)$.

(A) 6.5 (B) 7.5 (C) 8 (D) 9 (E) 11

19. How many elements are in $\left\{x \mid 4\sin(2x) - 2\cos(x) = 0, x \in \left[-\frac{\pi}{4}, \frac{5\pi}{4}\right]\right\}$?

(A) 8 (B) 7 (C) 4 (D) 3 (E) 2

20. The directrix of the parabola $8x + y^2 + 8y + 8 = 0$ is:

(A) $x = 3$ (B) $y = 1$ (C) $x = 2$ (D) $y = -4$ (E) $x = -2$

21. The inverse matrix, if it exists, of the matrix $\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find $a + b + c + d$.

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ (E) doesn't exist

22. Use the Fibonacci characteristic sequence ... p, −3, q, r, 7, ... to Find $(q + r) - p$.

(A) 15 (B) 11 (C) 2 (D) 0 (E) −1

23. Rusty Karr is trying to qualify for the first mini-NASCAR race of the year. The average of his 4 laps will determine if he qualifies or not. His lap speeds are 75 mph, 82 mph, 90 mph, and 84 mph. What was Rusty's average speed for the four laps? (nearest hundredth)

(A) 82.92 mph (B) 82.75 mph (C) 82.58 mph (D) 82.40 mph (E) 82.28 mph

24. Let $f(x) = 2x^3 + 5x^2 - 14x - 8$ and $g(x) = 3x^2 + 2x - 1$. Find $g'(f''(x - 1))$

(A) $72x - 38$ (B) $12x - 19$ (C) $72x - 7$ (D) $12x - 5$ (E) $72x - 10$

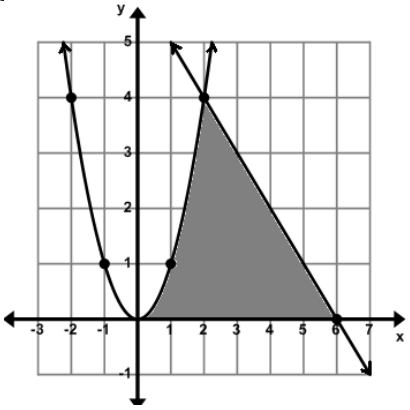
25. $\int [\sin^2(x) - \cos^2(x)]\ dx =$ _____ $+ C$, where C is some arbitrary constant.

(A) $-x$ (B) $2[\sin(x) + \cos(x)]$ (C) $2\cos(x)\sin(x)$ (D) 1 (E) $-\cos(x)\sin(x)$

26. In how many indistinguishable ways can all of the letters from the word "CIRCLE" be arranged on the circumference of a circle?

(A) 24 (B) 60 (C) 120 (D) 300 (E) 360

27. Find the area of the shaded region in square units.



(A) $9\frac{1}{3}$ **(B)** $10\frac{2}{3}$ **(C)** 12 **(D)** $15\frac{1}{3}$ **(E)** 18

28. Roland Bones rolls a pair of standard dice. What are the odds that the sum of the top faces is a triangular number?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{11}{25}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{13}$

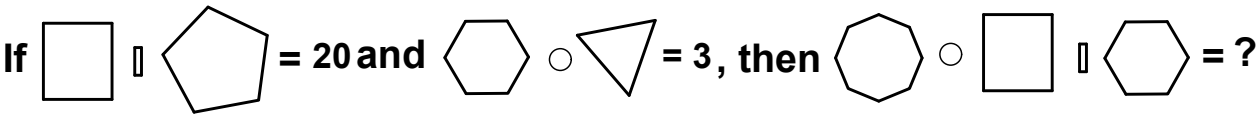
29. The probability of scoring above 200 on this test is 40%. Based on this probability, how many students are expected to score 200 or less on this test if 125 students are taking the test?

(A) 100 **(B)** 80 **(C)** 75 **(D)** 50 **(E)** 45

30. If you lived in the late 1800's and needed some assistance with working on infinite sets of numbers, which of the following mathematicians could best assist you?

(A) Georg Cantor **(B)** George Boole **(C)** Emmy Noether **(D)** Alicia Stott **(E)** John Venn

31.



(A) 26 **(B)** 24 **(C)** 12 **(D)** — 16 **(E)** — 20

32. Using Blaise Pascal's triangle and letting the 1 at the top be row 1, determine which of the following numbers will be in the 12th row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
...

(A) 252 **(B)** 311 **(C)** 337 **(D)** 420 **(E)** 462

33. On a map legend, $\frac{3}{4}$ inch represents 75 miles. How far is it in miles, from Nearhear to Overderr, if the distance on the map is $3\frac{1}{4}$ inches? (nearest mile)

(A) 183 mi (B) 244 mi (C) 275 mi (D) 300 mi (E) 325 mi

34. One billion two hundred thirty million four thousand five hundred sixty-seven minus seventy-six million five hundred forty thousand three hundred twenty-one equals D. What is the sum of the digits in D?

(A) 62 (B) 56 (C) 47 (D) 36 (E) 28

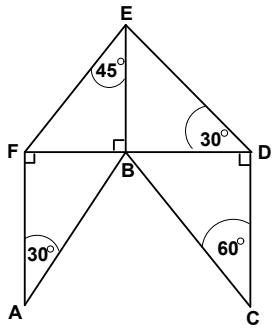
35. If the roots of $x^3 + bx^2 + cx + d = 0$ are — 5, — 1, and 3, then $b \times c \times d$ equals:

(A) — 25 (B) — 3 (C) 15 (D) 255 (E) 585

36. Which of the following sets is closed under both addition and multiplication?
C = {composite numbers} E = {even numbers} N = {natural numbers}

(A) C, E, & N (B) C & E (C) E & N (D) C & N (E) E only

37. Find the area of the hexagon if BF = 4 cm. (nearest cm).



(A) 58 cm (B) 50 cm (C) 29 cm (D) 18 cm (E) 13 cm

38. Let $\triangle ABC$ exist such that D is the midpoint of segment AC, $m\angle CAB = m\angle CBD$, and AB = 8. Find BD.

(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) 6

39. If (—1, 2), (3, — 4), and (— 5, k) are members of the function {(x, y) | y = ax — b}, then k = ?

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

40. According to the Kartesia National Park map, ranger station #2 is 20 miles from ranger station #1 on a bearing of 115° and ranger station #3 is 30 miles from ranger station #2 on a bearing of 215°. What is the bearing from the ranger station #1 to ranger station #3? (nearest degree)

(A) 143° (B) 153° (C) 178° (D) 333° (E) 358°

41. Let $a_1 = -3$, $a_2 = 2$, $a_3 = -1$ and $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$ for $n \geq 4$. Find a_7.

(A) 1 **(B)** 0 **(C)** -4 **(D)** -7 **(E)** -13

42. If $\frac{x+11}{x-7} + \frac{x-7}{x+11}$ is written as the mixed number $A\frac{B}{C}$ then B = ?

(A) 324 **(B)** 154 **(C)** 72 **(D)** 36 **(E)** 16

43. Simplify $\left(5 + \sqrt{-112}\right)\left(6 - \sqrt{-175}\right)$ to the form $a + bi$.

(A) $50 - \sqrt{7}\,i$ **(B)** $170 - \sqrt{7}\,i$ **(C)** $50 - 49\sqrt{7}\,i$ **(D)** $170 - 7\sqrt{7}\,i$ **(E)** $22 - \sqrt{7}\,i$

44. $(FACE_{16} - CAFE_{16}) \div 8_{16} =$ _____ $_{16}$

(A) 217 **(B)** BAD **(C)** 17E5 **(D)** 5FA **(E)** 437D

45. Let $f(x) = 2x^3 - 5x^2 + 4x + 10$. Find the distance between the points where the relative minimum and the relative maximum of f(x) occur.

(A) $\frac{\sqrt{82}}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{\sqrt{3}}{3}$ **(D)** $\frac{4\sqrt{5}}{27}$ **(E)** $\frac{1}{3}$

46. Find the digit in the 10^{-7} place of the series $-\frac{1}{1!} + \frac{1}{3!} - \frac{1}{5!} + \frac{1}{7!} - \frac{1}{9!} + \ldots$.

(A) 9 **(B)** 8 **(C)** 7 **(D)** 5 **(E)** 0

47. Dee Sketcher's pencil box contains five red pencils, three blue pencils, three green pencils and one brown pencil. She randomly selects two pencils. What is the probability that at least one of them is red? (nearest per cent)

(A) 42% **(B)** 56% **(C)** 68% **(D)** 74% **(E)** 80%

48. In how many ways can twelve different colored crayons be divided among a boy, a girl, and a teacher if the teacher gets three crayons, the boy gets four, and the girl gets five?

(A) 27,720 **(B)** 935 **(C)** 86,248,800 **(D)** 4,464 **(E)** 347

49. All of the elements of the set {1, 2, 13, 37, 107, 113} are _____ numbers.

(A) Abundant **(B)** Evil **(C)** Happy **(D)** Lucky **(E)** Primeval

50. Cher Whitall received roses for her senior prom. She gave $\frac{1}{5}$ of them to her 2 best buddies. Then she gave 80% of the remaining roses to her other classmates. Then she gave 2 of the remaining ones to her parents leaving her with 6 roses. How many roses did she receive originally?

(A) 12 **(B)** 25 **(C)** 36 **(D)** 50 **(E)** 60

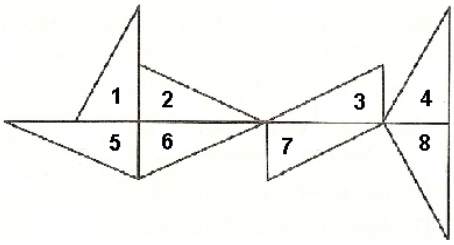
51. $(543_6 - 345_6) \times (12_6 + 34_6) =$ _____ $_6$

(A) 9108 **(B) 70100** **(C) 13420** **(D) 52444** **(E) 2100**

52. If $2x^3 + 5x^2 - 14x - 8 = (ax + b)(cx + d)(ex + f)$ then $(a + c + e) - (b + d + f) =$ ____.

(A) 1 **(B) 0** **(C) -7** **(D) -12** **(E) -15**

53. **Mary Goround maps triangle 4 to triangle 5 by using which of the groups of transformations in the order given?**



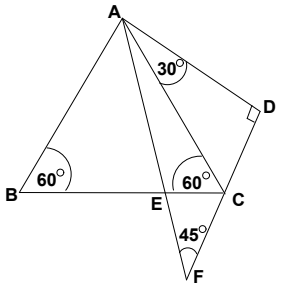
(w) half-turn **(x) reflection** **(y) rotation** **(z) translation**

(A) x, z, & y **(B) w, z, & x** **(C) z, x, y, & x** **(D) w, y, z, & x** **(E) y, z, w, & x**

54. **How many positive cubes divide 2! x 5! x 8! ?**

(A) 12 **(B) 9** **(C) 8** **(D) 7** **(E) 5**

55. **Find EF, if AB = 40 cm. (nearest cm)**



(A) 9 cm **(B) 11 cm** **(C) 13 cm** **(D) 15 cm** **(E) 18 cm**

56. **$\triangle ABC$ is a right triangle with $\angle ACB$ being the right angle. Point D lies on AC such that AD = 8" and CD = 4". Find the area of $\triangle ABD$ if BC = 5".**

(A) 10 sq. in. **(B) 11.5 sq. in.** **(C) 16 sq. in.** **(D) 20 sq. in.** **(E) 32.5 sq. in.**

57. **If a triangle has two sides of lengths 5" and 9", and the included angle between these two sides is increasing at the rate of 7° per hour, what would the approximate measure of the included angle be if the rate the area of the triangle is changing is 1 in^2/hr? (nearest degree)**

(A) 69° **(B) 56°** **(C) 45°** **(D) 23°** **(E) 21°**

58. Let $f(x) = 2x^4 - 2x^3 - 8.5x^2 - 6x + 1.5$. Find the sum of the x-values of the critical points of f(x).

(A) 2.25 (B) 0.75 (C) — 0.5 (D) — 1.75 (E) — 3.25

59. If the following pattern continues, find C + D.

```
                1
              2   1
            4   4   1
          8   12   6   1
        16   32  24   8   1
      32   B   C   D   E   1
```

(A) 132 (B) 120 (C) 114 (D) 90 (E) 88

60. How many solutions are there for the equation $8x + 12y = 2012$ such that both x and y are positive integers?

(A) 96 (B) 84 (C) 80 (D) 64 (E) 48

University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2012
Answer Key

1. A	**21. B**	**41. D**
2. C	**22. E**	**42. A**
3. E	**23. D**	**43. B**
4. A	**24. E**	**44. D**
5. D	**25. E**	**45. A**
6. B	**26. B**	**46. A**
7. D	**27. B**	**47. C**
8. B	**28. E**	**48. A**
9. E	**29. C**	**49. E**
10. C	**30. A**	**50. D**
11. A	**31. D**	**51. C**
12. D	**32. E**	**52. A**
13. C	**33. E**	**53. C**
14. E	**34. D**	**54. C**
15. A	**35. E**	**55. C**
16. E	**36. C**	**56. D**
17. D	**37. B**	**57. A**
18. C	**38. D**	**58. B**
19. D	**39. B**	**59. B**
20. A	**40. C**	**60. B**