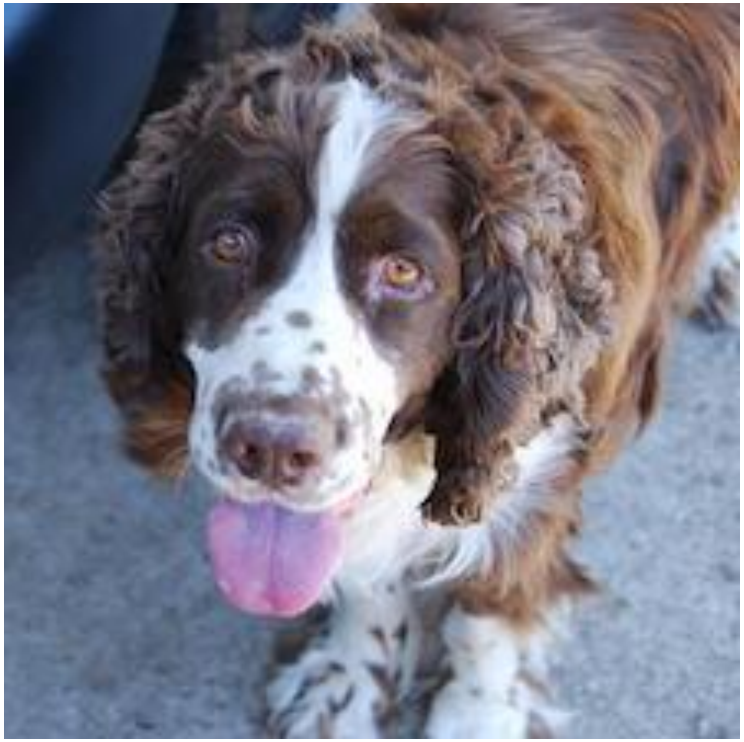
Frequency filtering

Semester 2, 2025

Kris Ehinger

Adversarial images



Network: MobileNet V2
Prediction: English springer (90.08%)

Model: MobileNet V2
Prediction: hot dog (68.88%)

Images: ImageNet, https://kennysong.github.io/adversarial.js/

Outline

- Intro to Fourier analysis in 1D
- Fourier analysis of images
- Filtering in frequency
- Applications

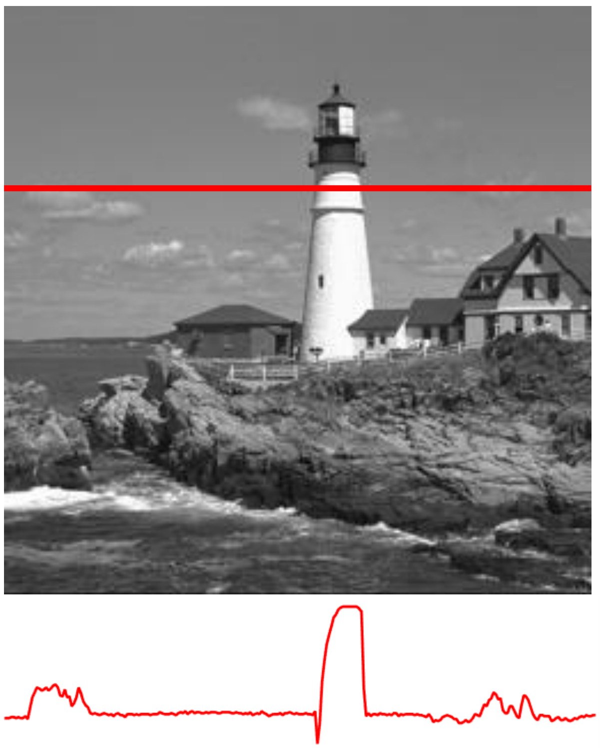
Learning outcomes

- Explain at a conceptual level how images are represented in the frequency domain
- Implement filters in the frequency domain
- Explain how frequency representations are used for image compression and analysis

Fourier analysis (in 1D)

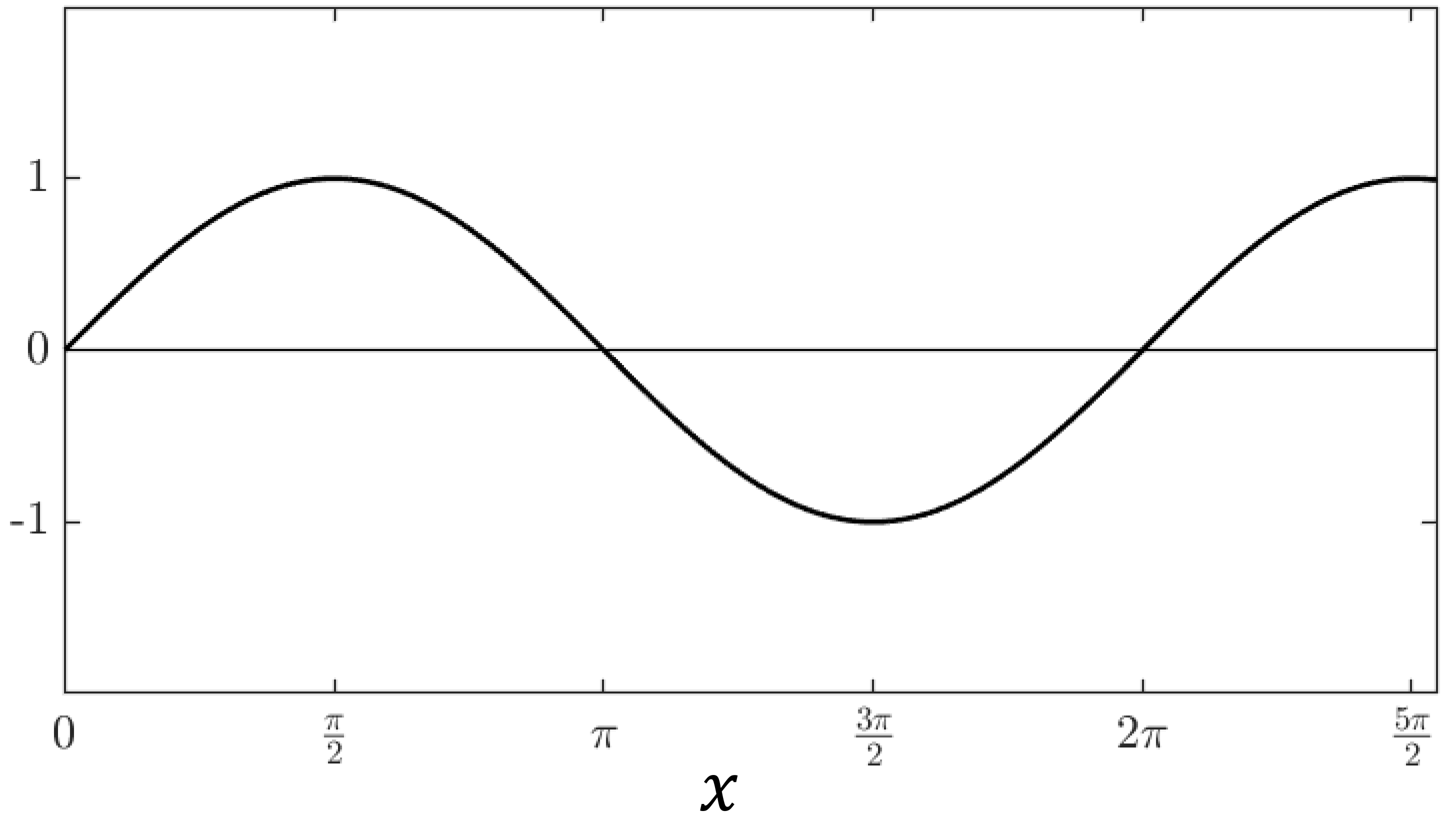
Signals

- Any signal or pattern can be described as a sum of sinusoids



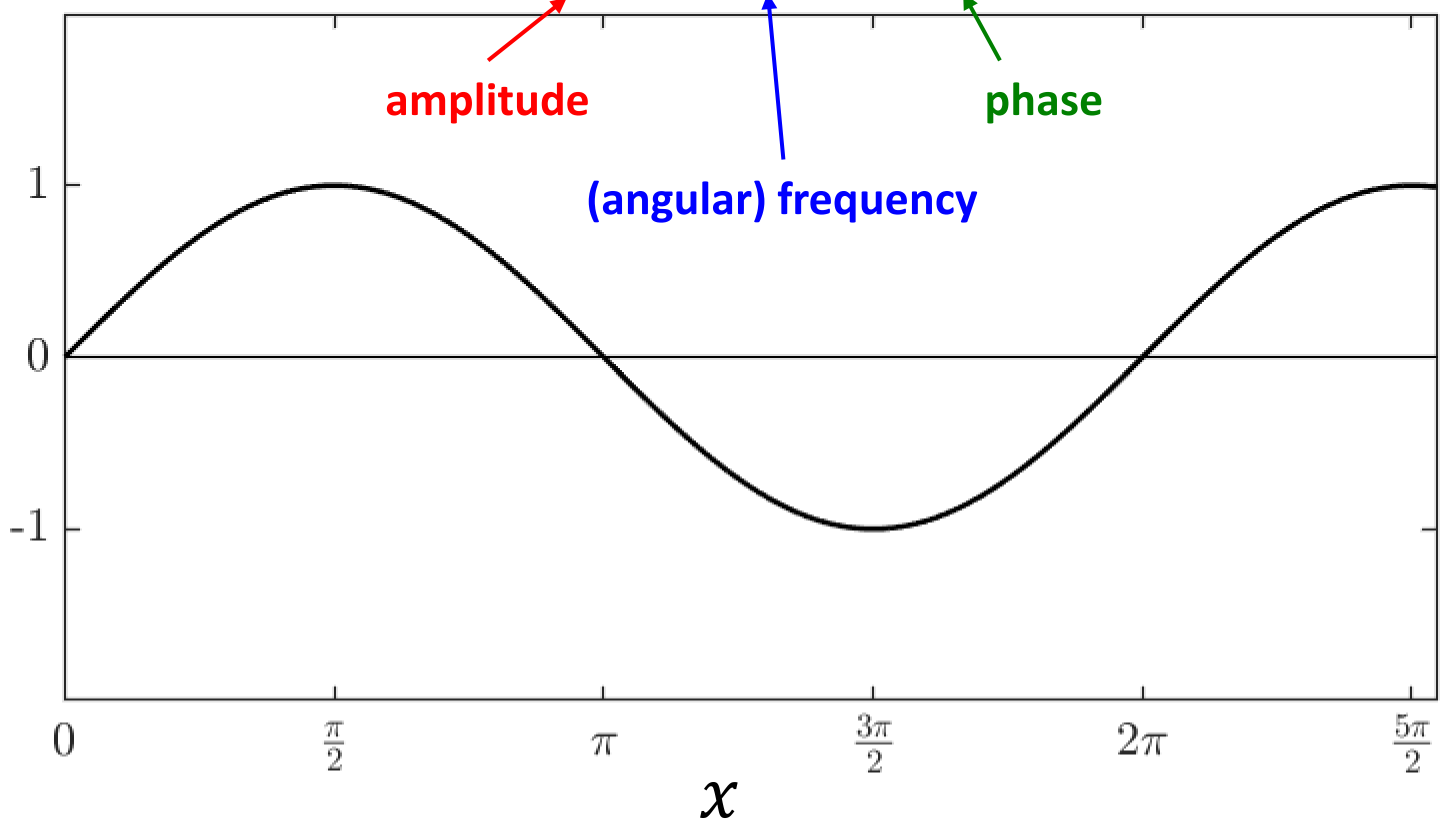
Sinusoids

$$y = \sin(x)$$



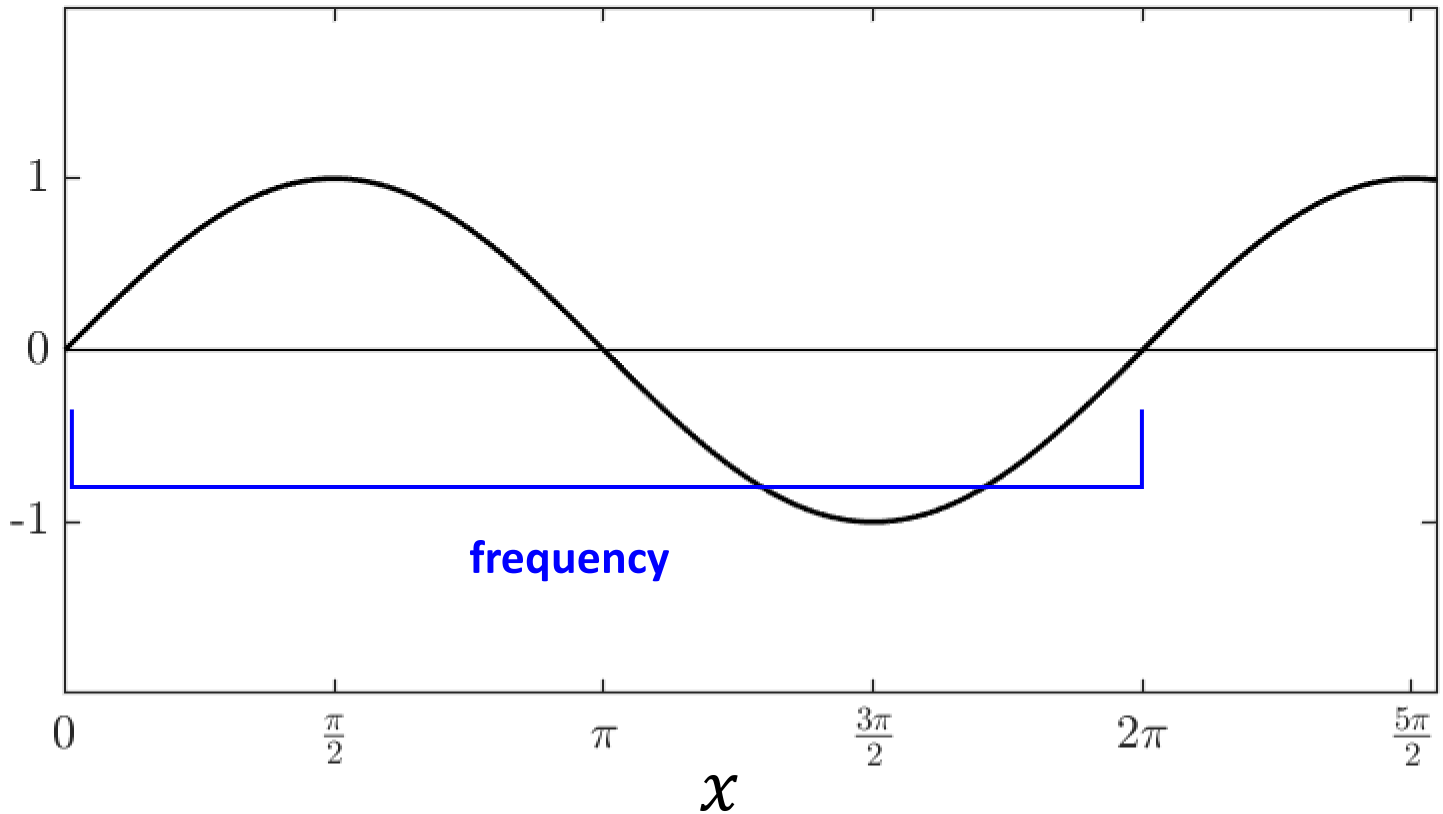
Sinusoids

$$y = A\sin(\omega x + \varphi)$$



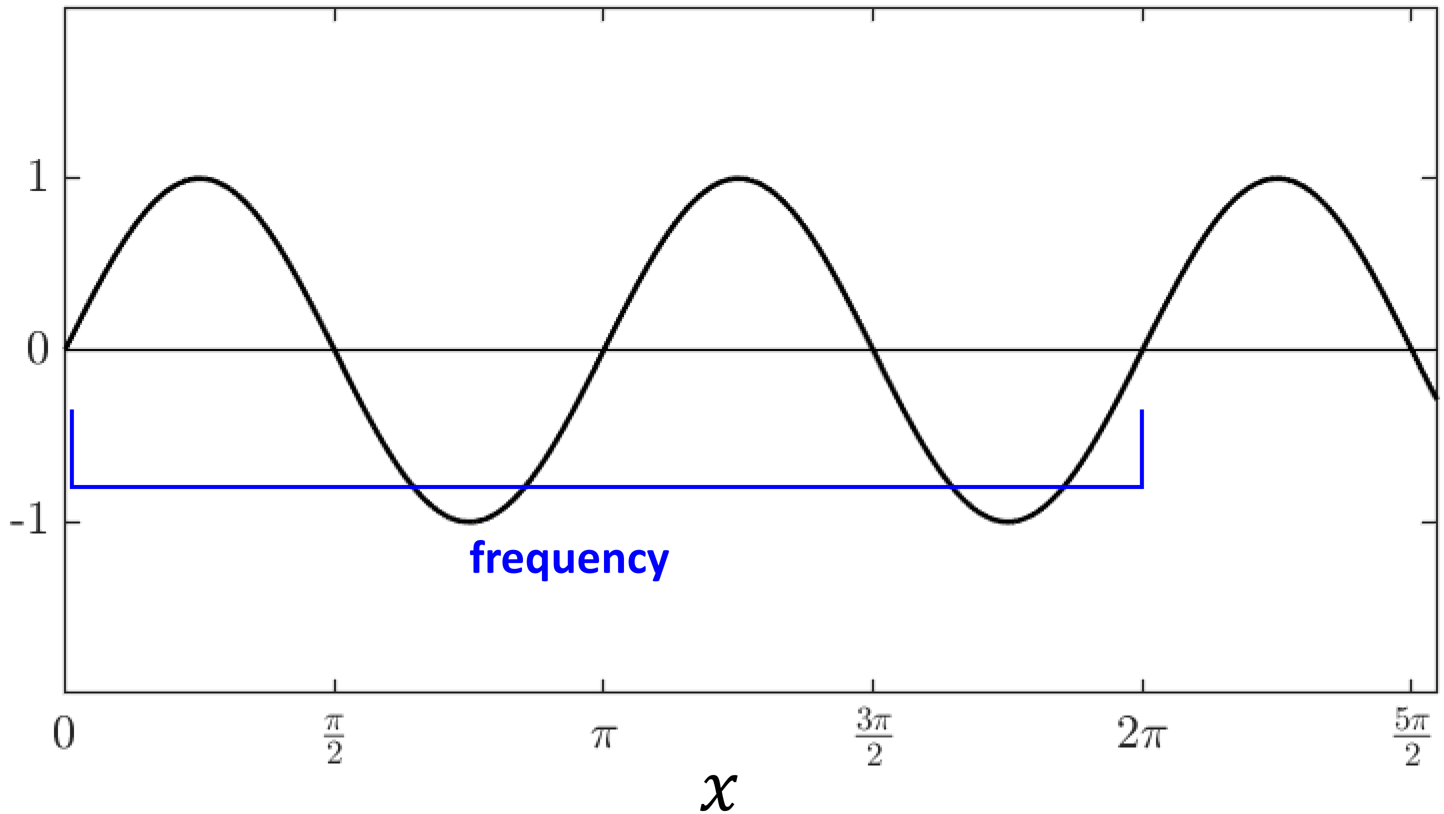
Sinusoids

$$y = 1\sin(1x + 0)$$



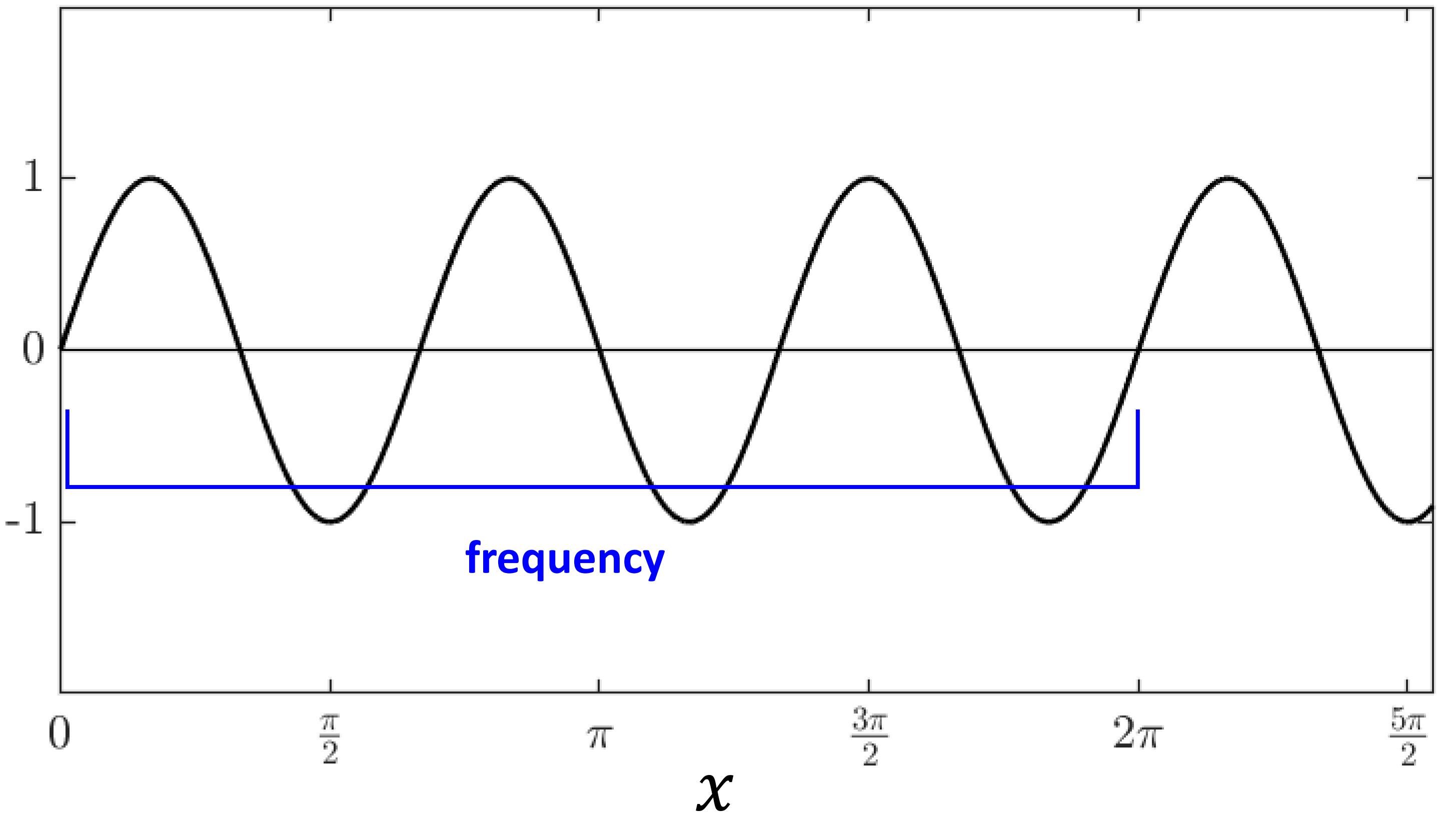
Sinusoids

$$y = 1\sin(2x + 0)$$



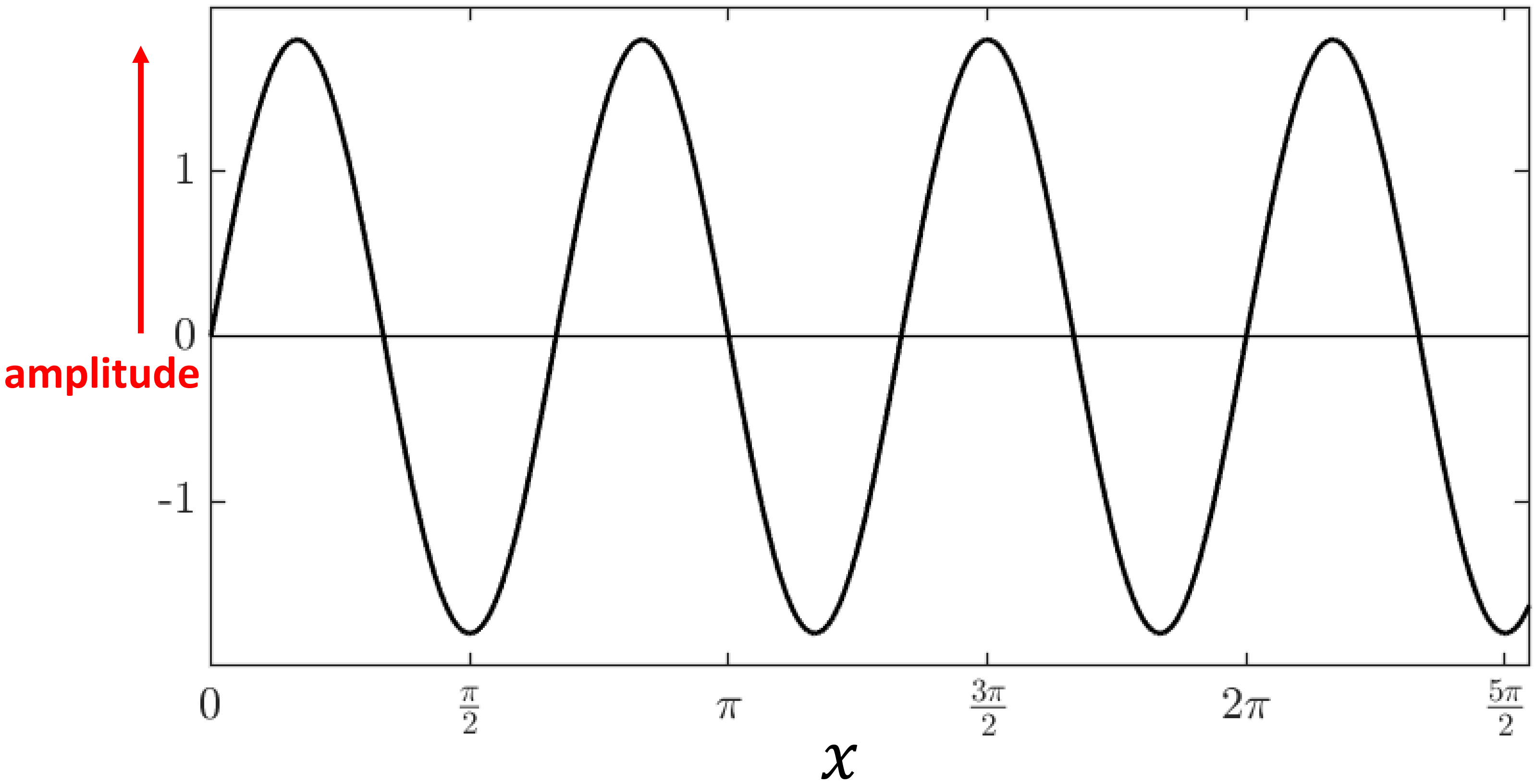
Sinusoids

$$y = 1\sin(3x + 0)$$



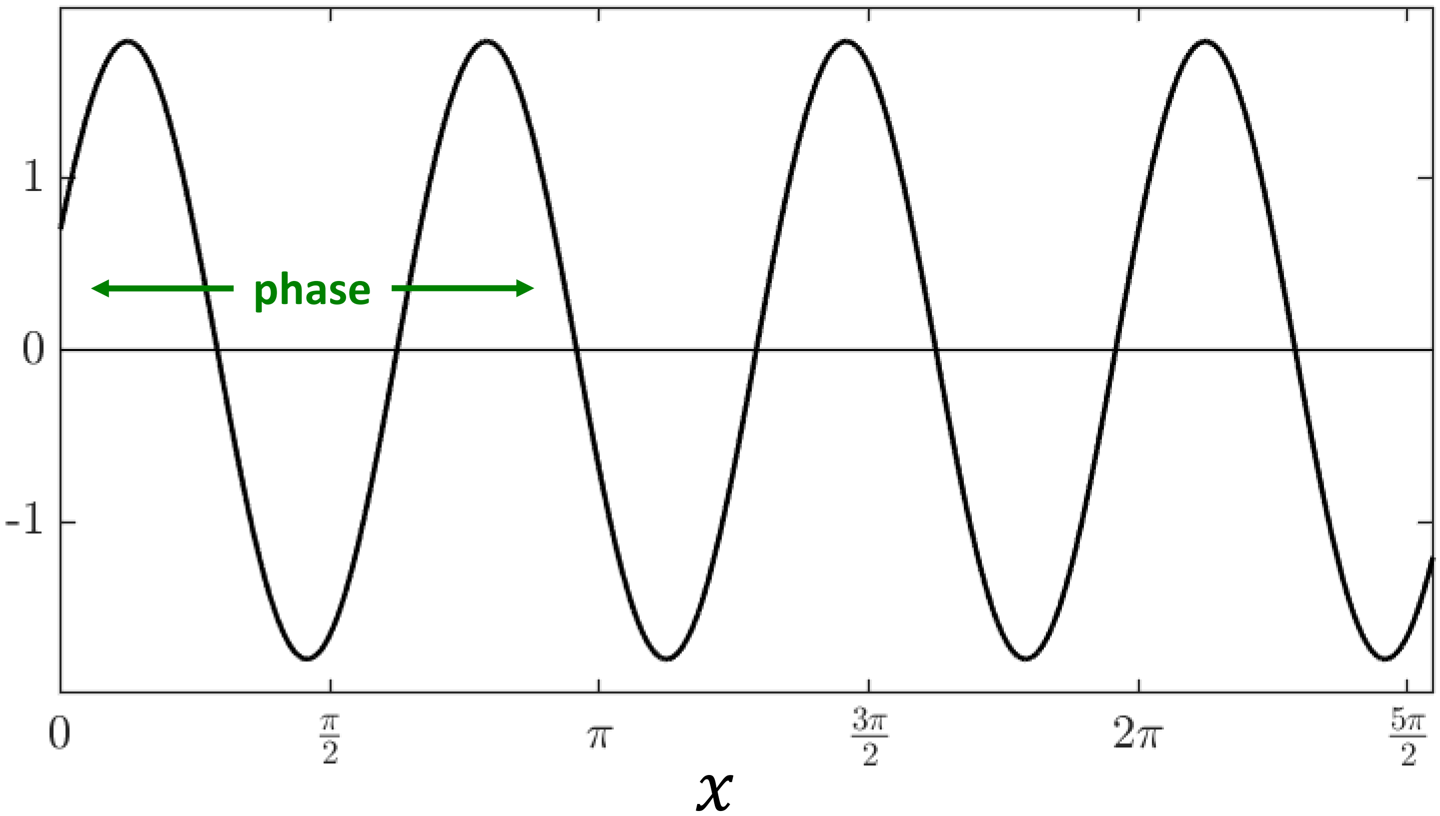
Sinusoids

$$y = 1.8\sin(3x + 0)$$



Sinusoids

$$y = 1.8\sin(3x + 0.4)$$



Fourier analysis

- Any signal can be expressed as a sum of sinusoids

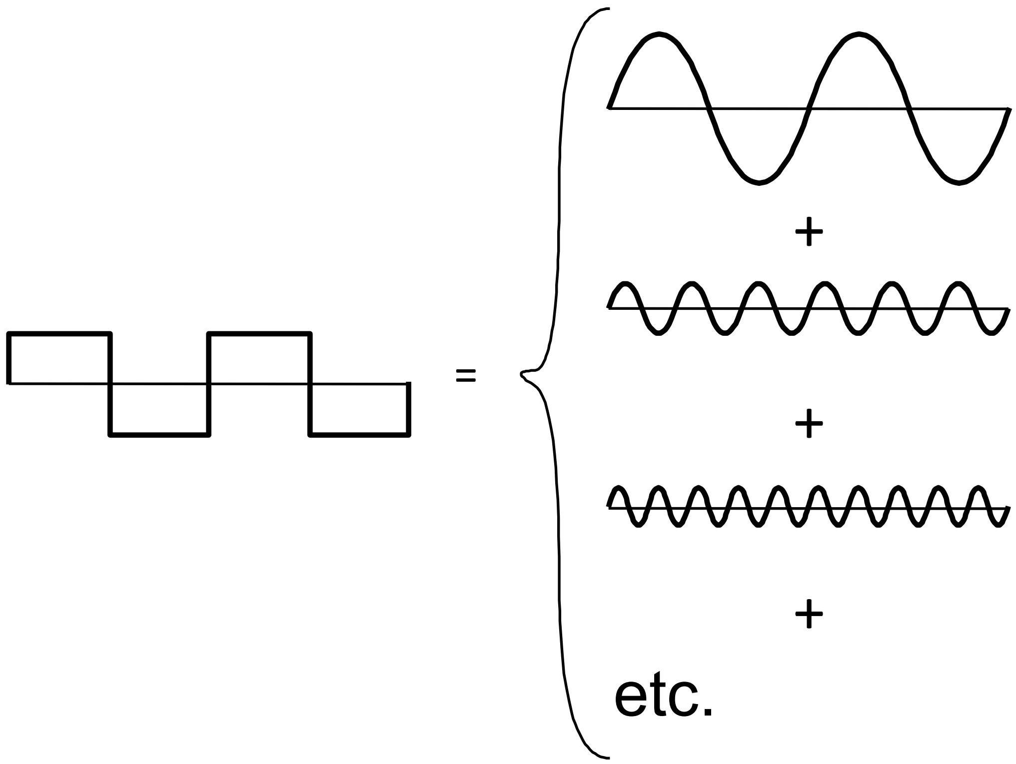


Image: R. Rosenholtz

Sum of sinusoids

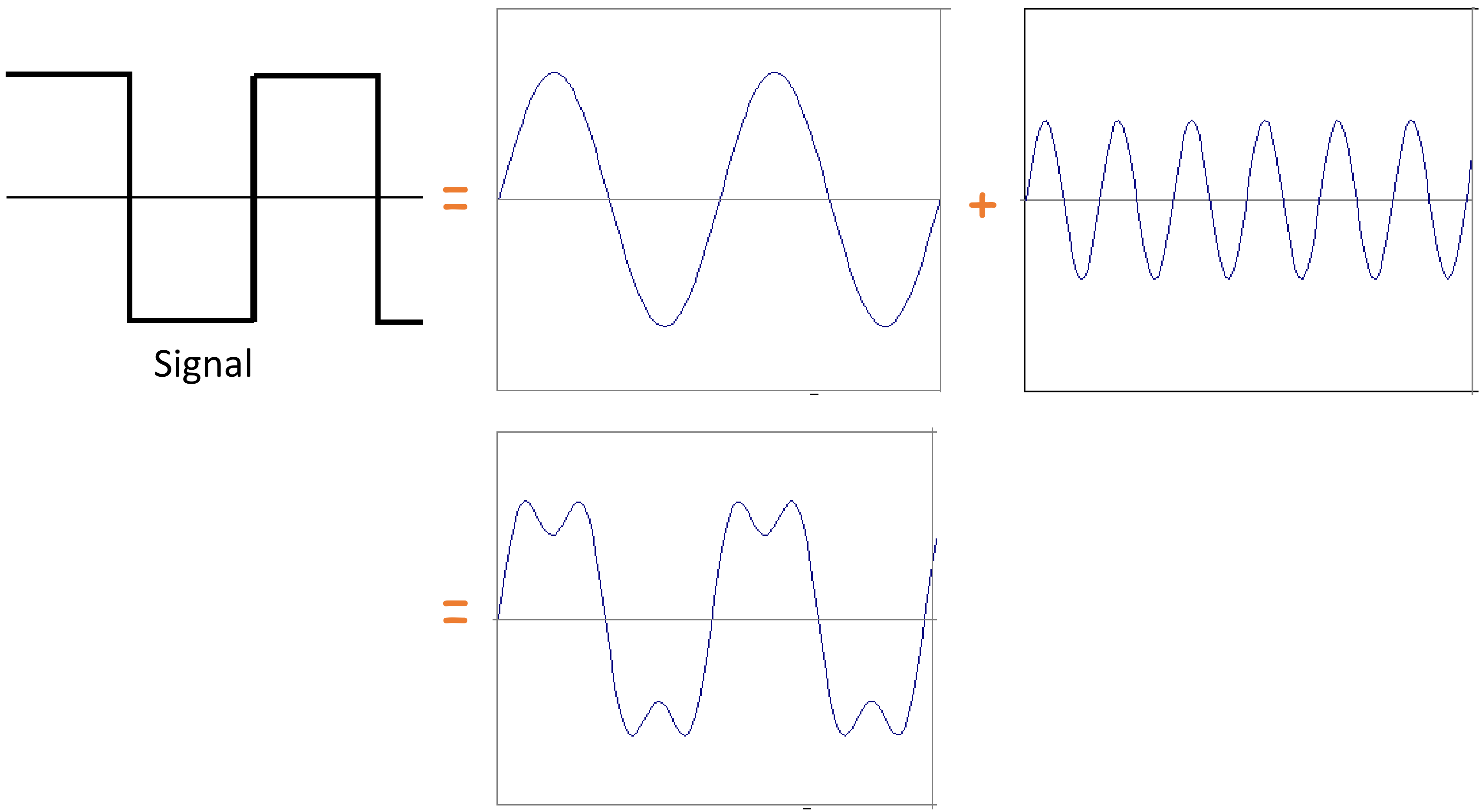


Image: J. Hays

Sum of sinusoids

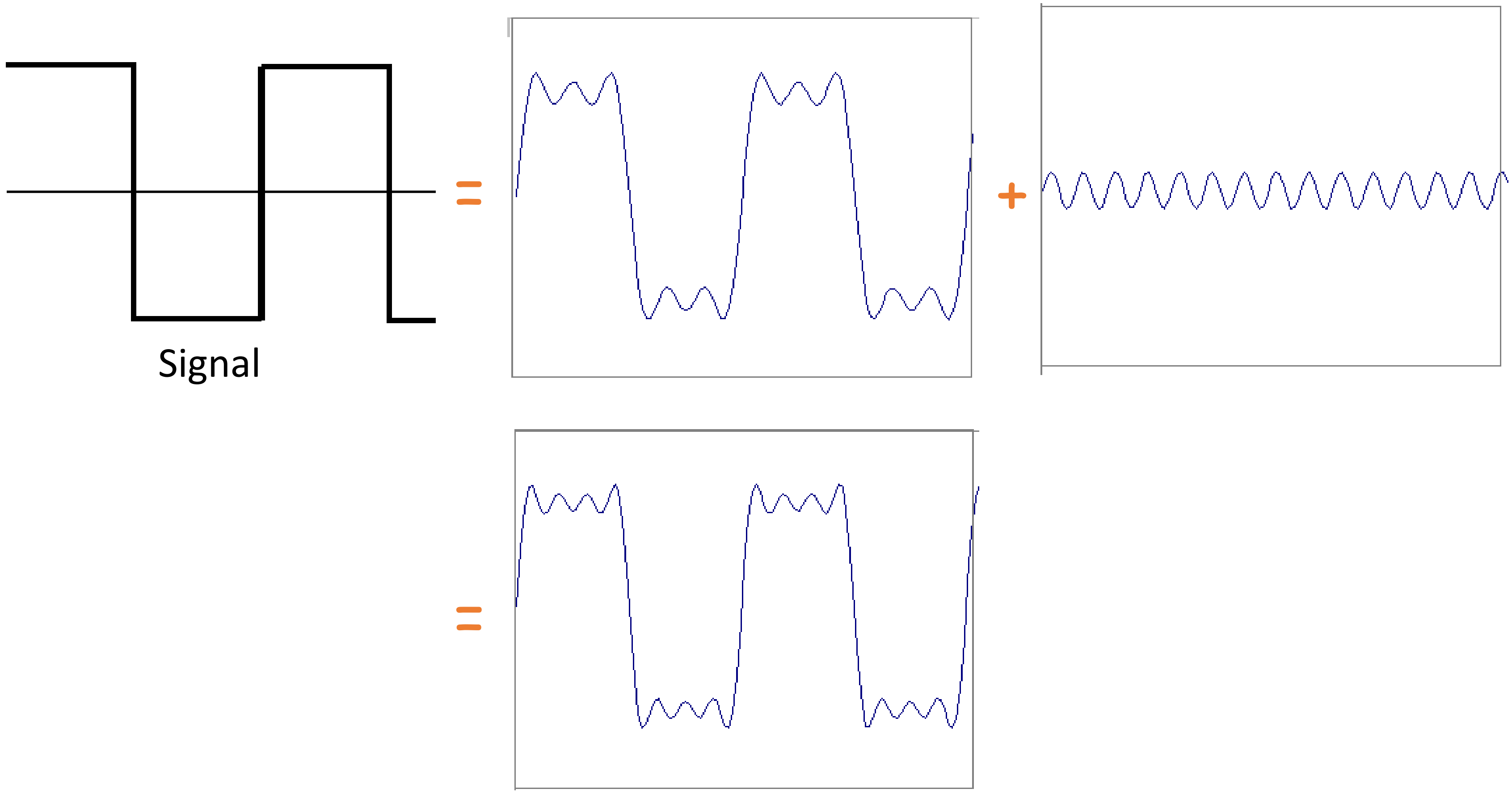


Image: J. Hays

Sum of sinusoids

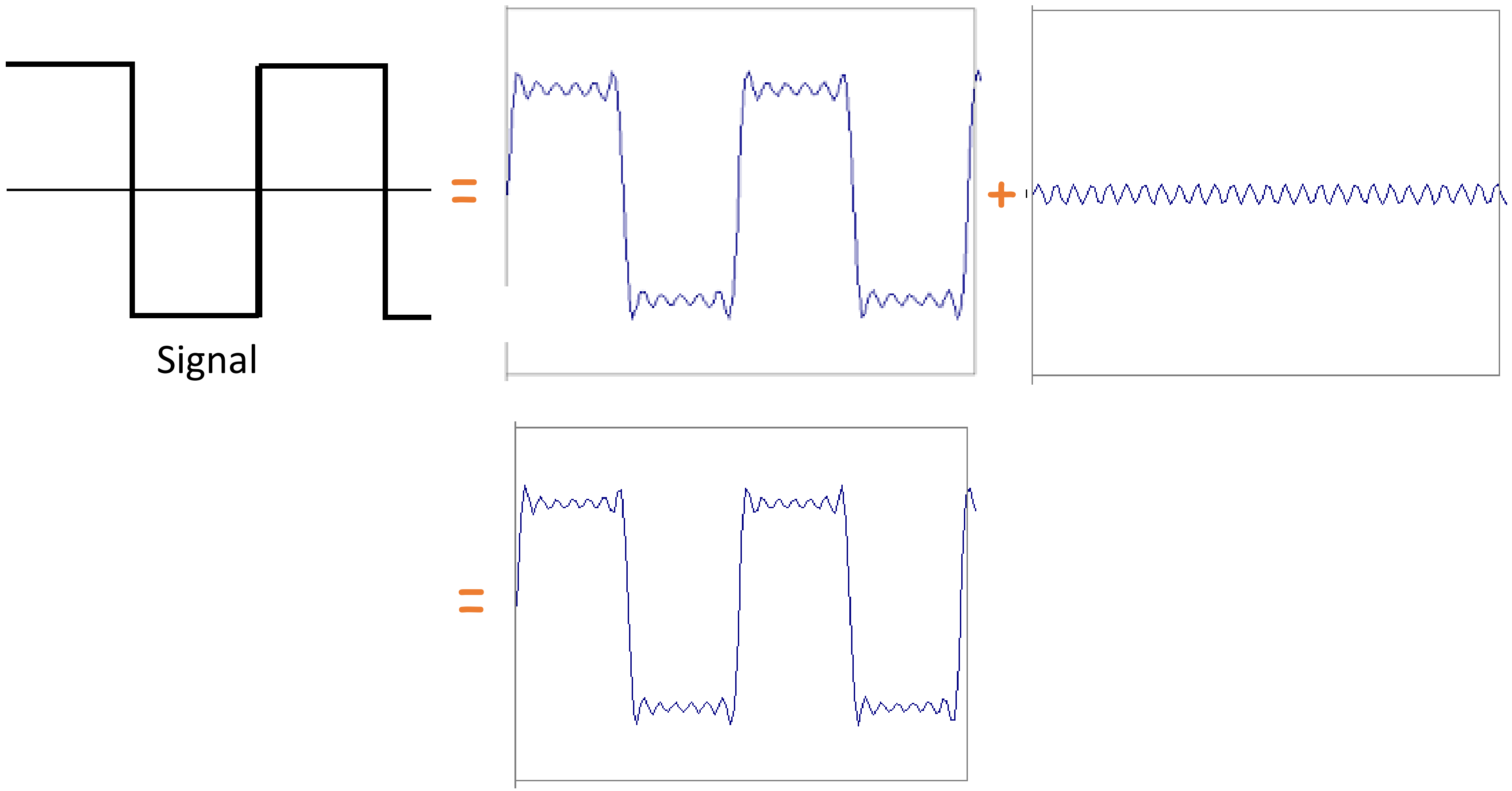


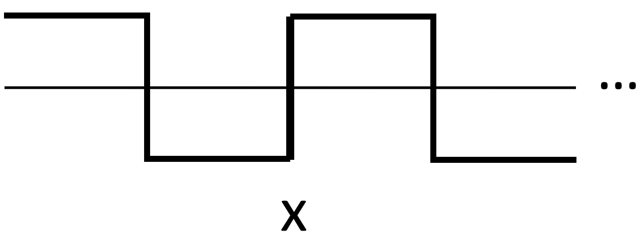
Image: J. Hays

Fourier transform

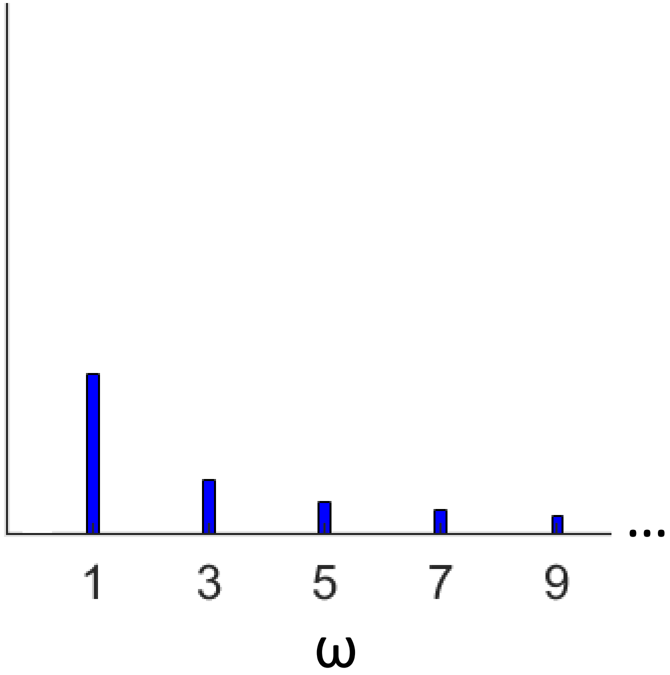
- Fourier transform decomposes signal into component frequencies
  - Values are complex numbers representing amplitude and phase of sinusoids
  - Time domain -> frequency domain (or, for images, spatial domain -> frequency domain)
- $F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2i\pi\omega x}\, dx$
- scipy.fft (1D), scipy.fft2 (2D), scipy.fftn (3D+)
- Inverse Fourier transform converts from frequency domain back to space domain

Frequency spectrum

Signal in spatial (or time) domain



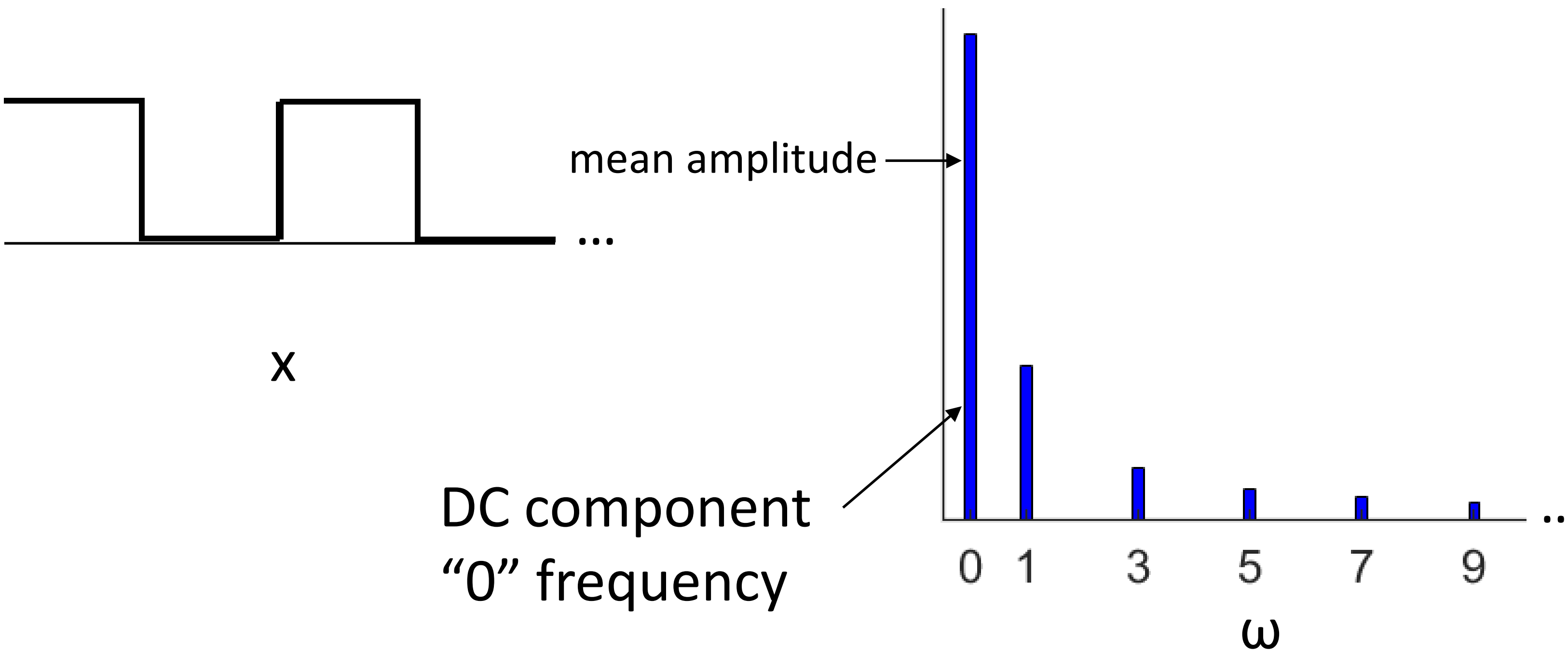
Signal in frequency domain (magnitude)



Frequency spectrum

Signal in spatial (or time) domain

Signal in frequency domain (magnitude)



Frequency spectrum

- Values in frequency domain are complex numbers
- For each frequency: magnitude (=amplitude) and angle (=phase)

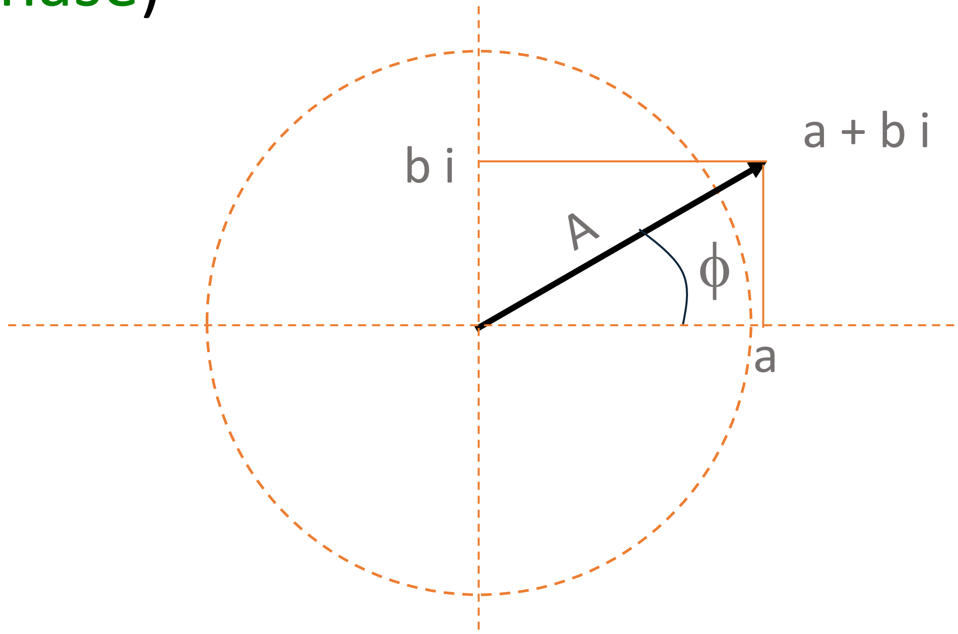


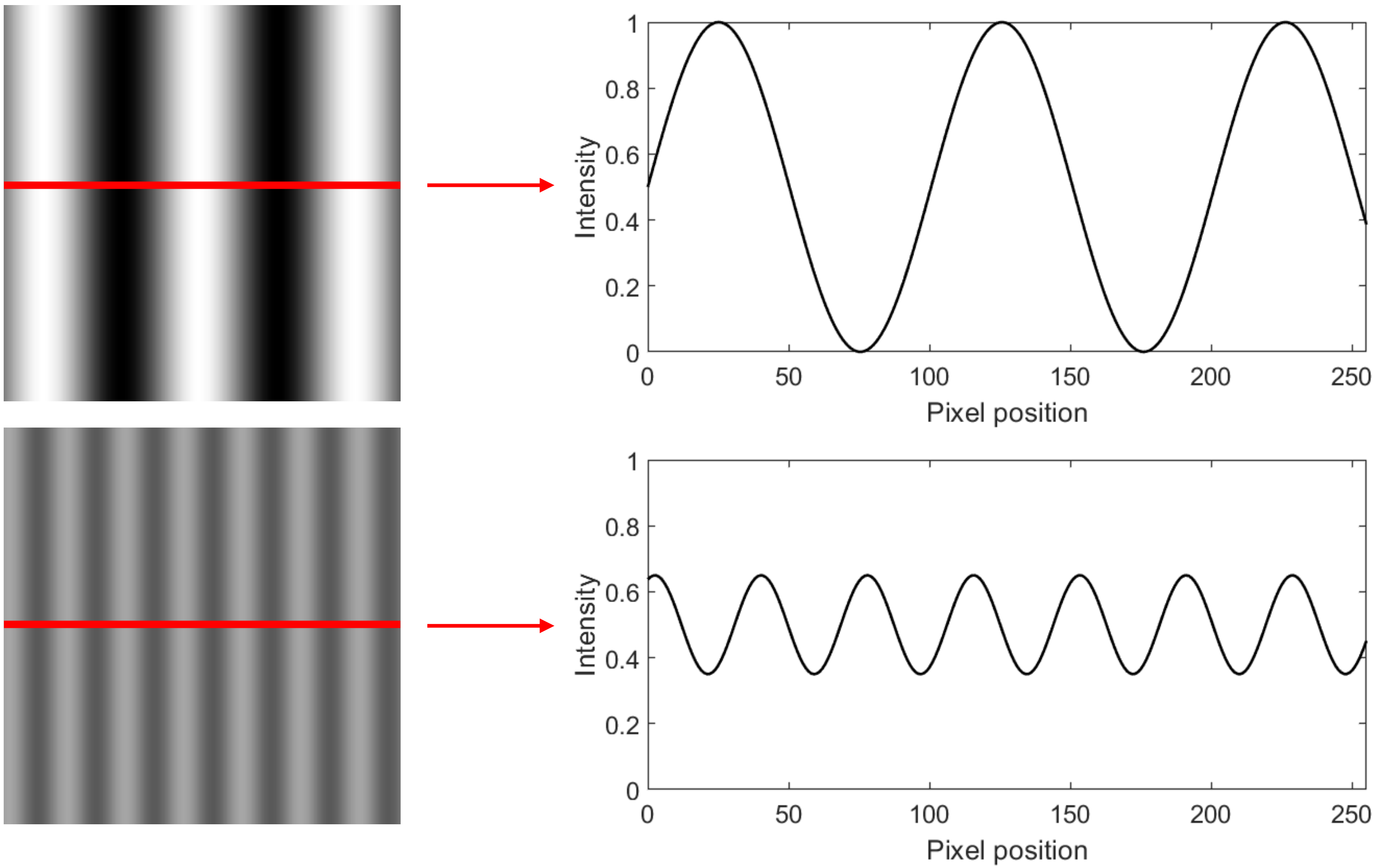
Image: R. Rosenholtz

Summary

- Any signal or pattern can be described as a sum of sinusoids
- Fourier transform decomposes a signal into its component sinusoids:
  - The axis is frequency
  - Values are complex numbers
  - Magnitude = amplitude of the sinusoid
  - Angle = phase of the sinusoid

Fourier analysis (images)

Images as sinusoids



Fourier transforms of images

Image

Fourier transform (magnitude)

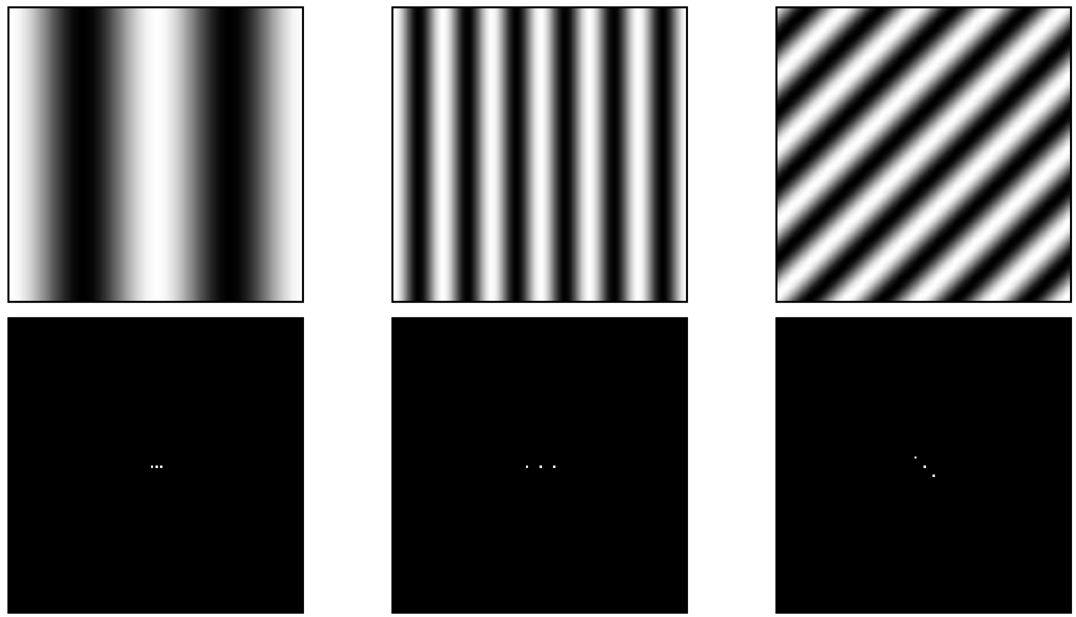
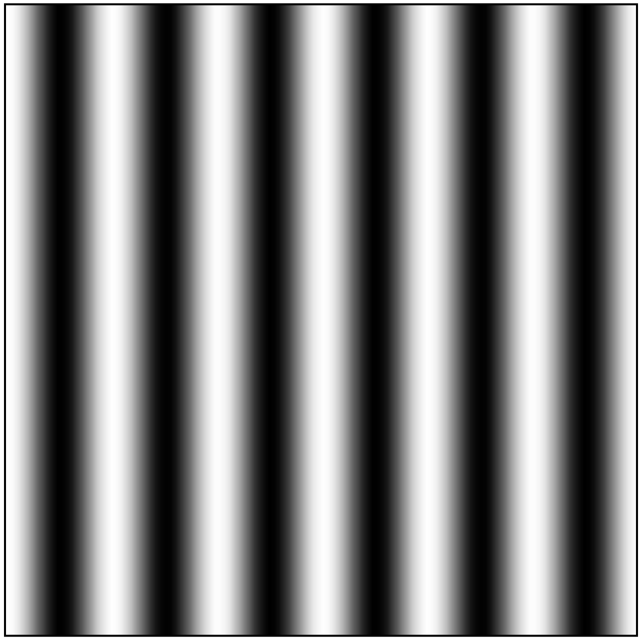
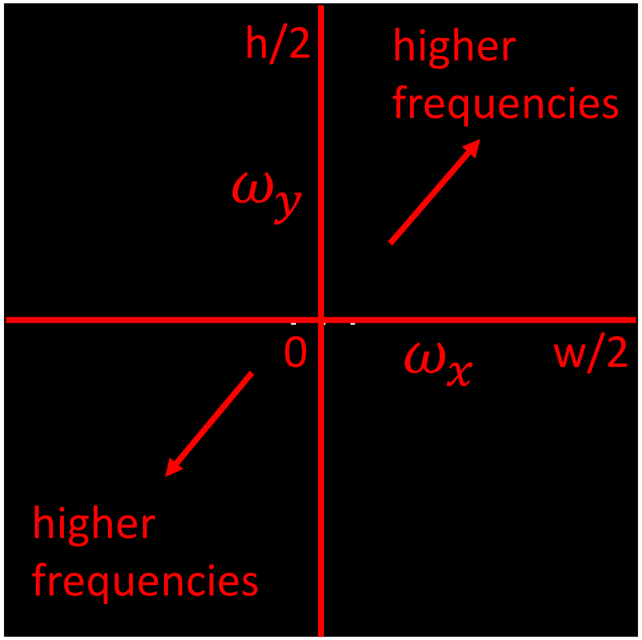


Image: J. Hays

How to interpret Fourier spectra



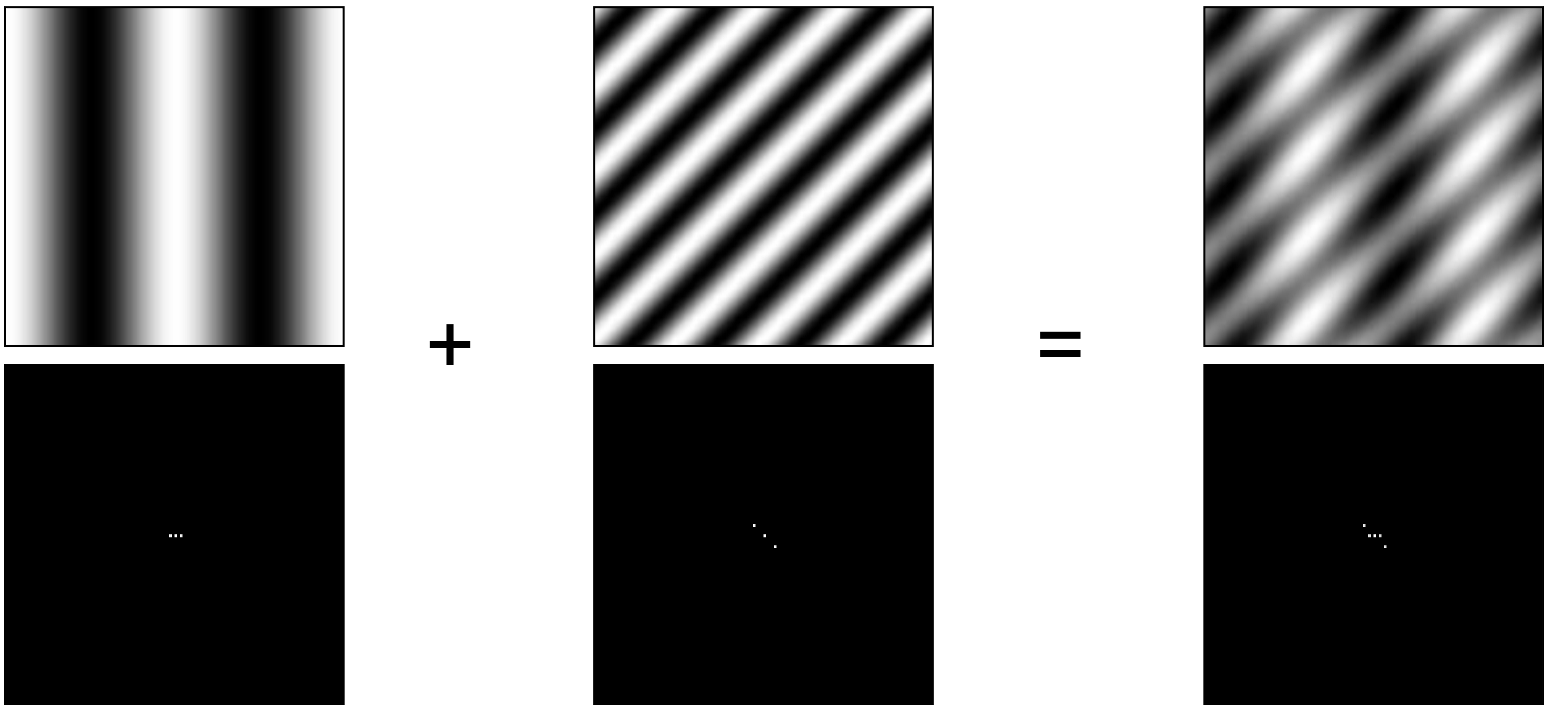
Image



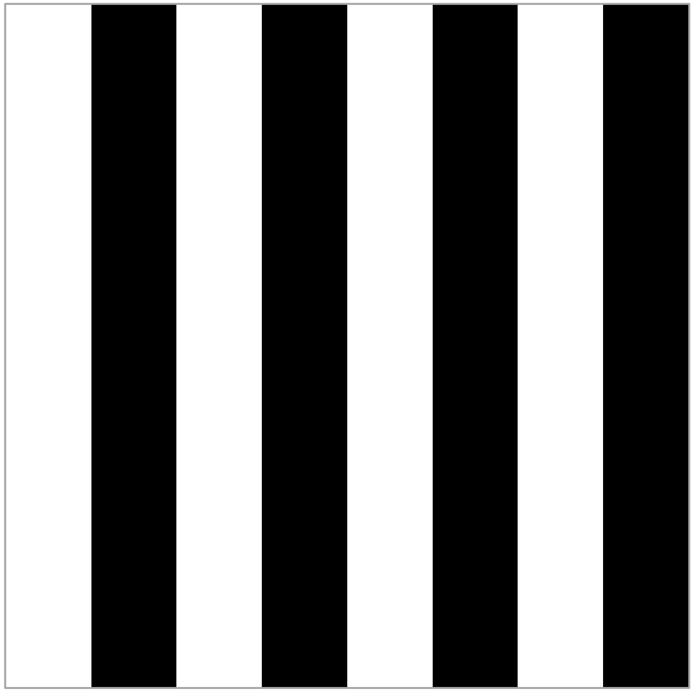
Fourier transform (magnitude)

Fourier transforms of images

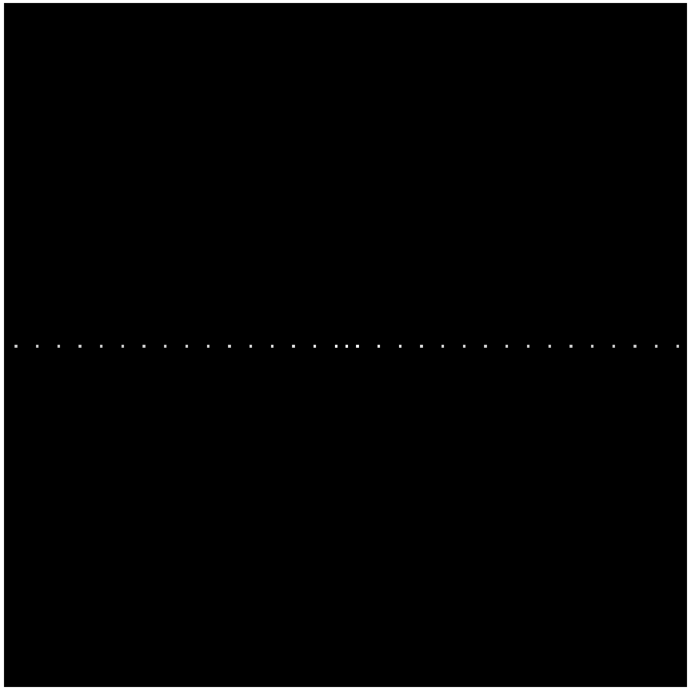
Image



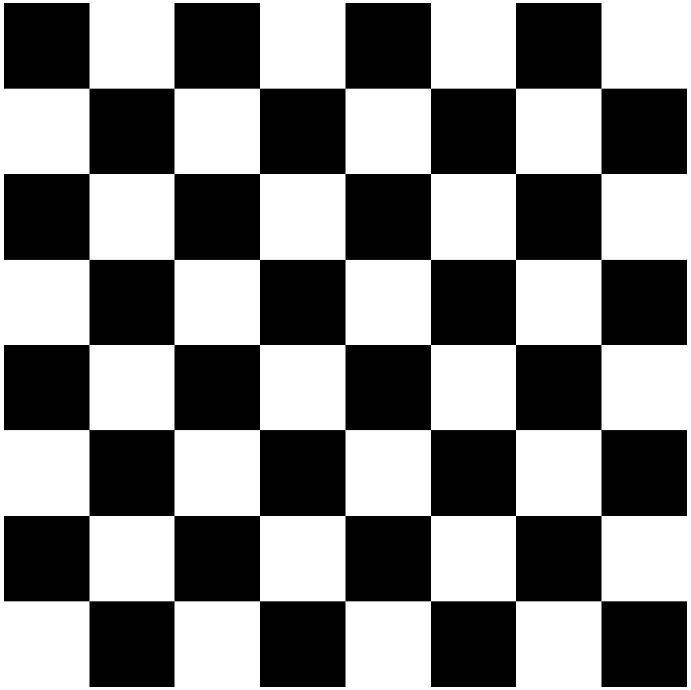
Fourier transform (magnitude)



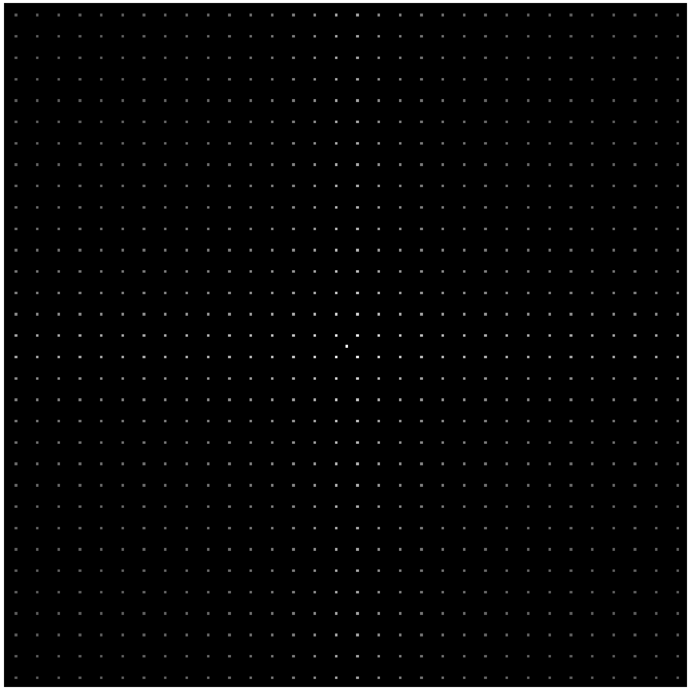
Image



Fourier transform (magnitude)



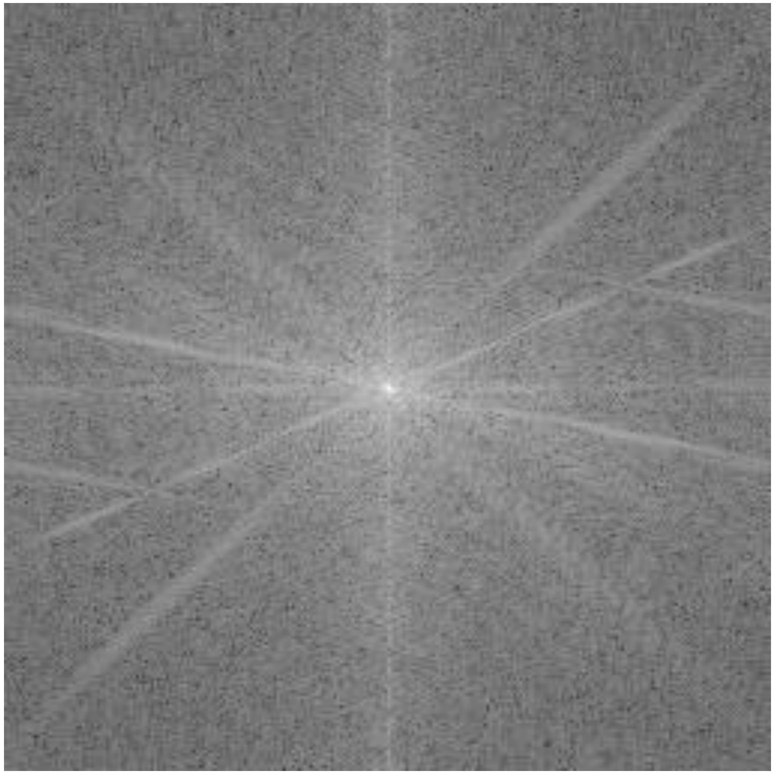
Image



Fourier transform (magnitude)

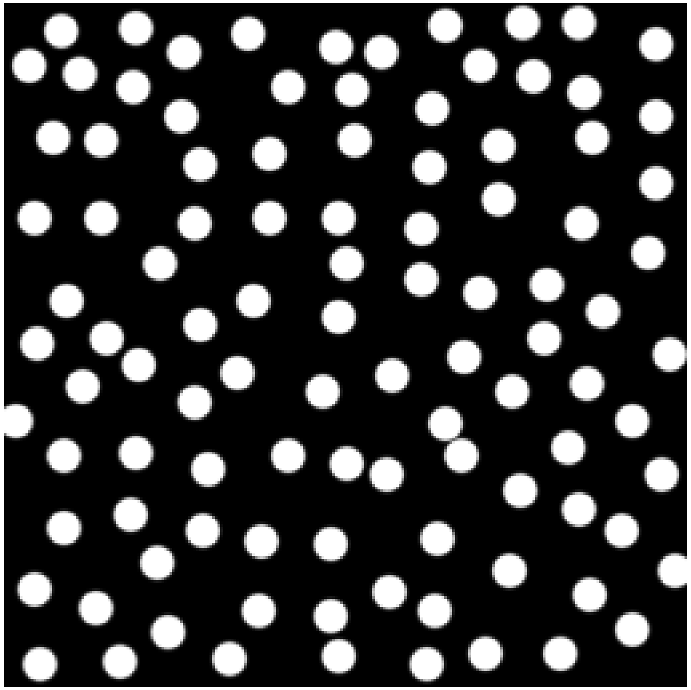
Fourier transforms of images

Image

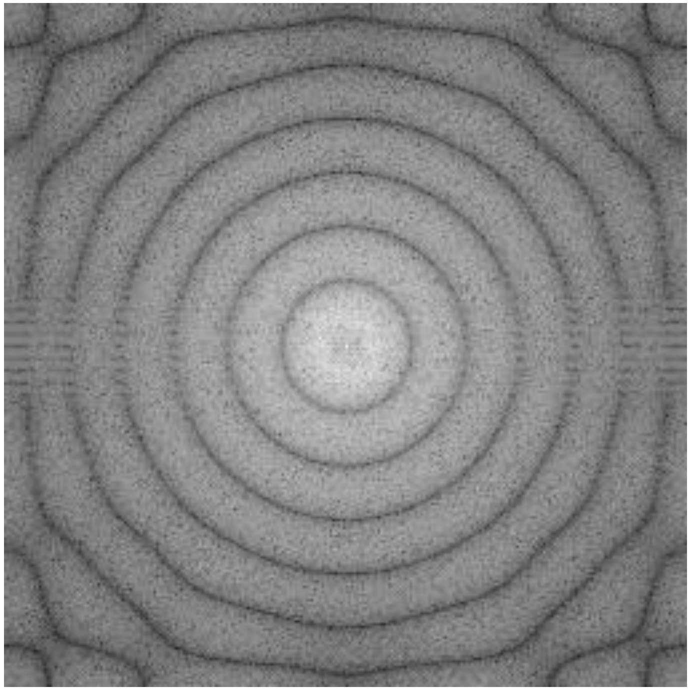


Fourier transform (magnitude)

Fourier transforms of images



Image



Fourier transform (magnitude)

Fourier analysis of images

- Any image can be represented by its Fourier transform
- Fourier transform = for each frequency, magnitude (amplitude) + phase
- Magnitude captures the holistic "texture" of an image, but the edges are mainly represented by Fourier phase

Frequency filtering

Operations in frequency domain

- Operations in the spatial domain have equivalent operations in frequency domain
- Convolution in spatial domain = multiplication in frequency domain

$$FT[h * f] = FT[h]FT[f]$$

- Inverse:

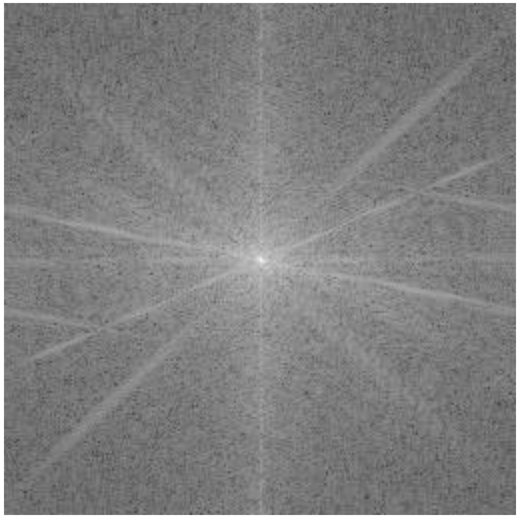
$$FT^{-1}[hf] = FT^{-1}[h] * FT^{-1}\,[f]$$

Bandpass filter

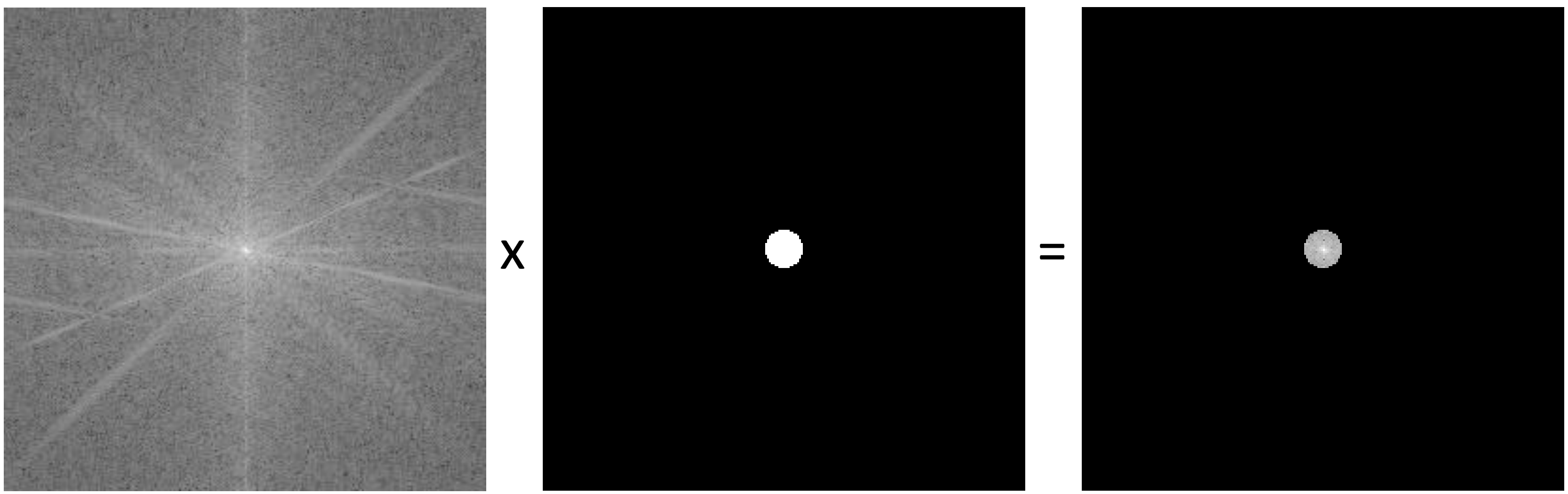
- Bandpass filter = a filter that removes a range of frequencies from a signal

Low pass filter

- Low pass filter = keep low spatial frequencies, remove high frequencies



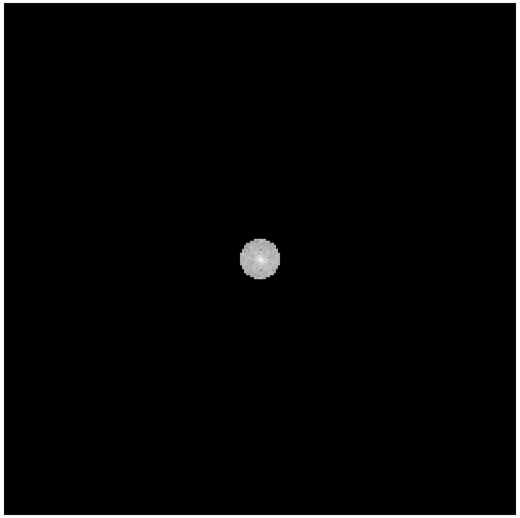
Low pass filter



Element-wise multiplication

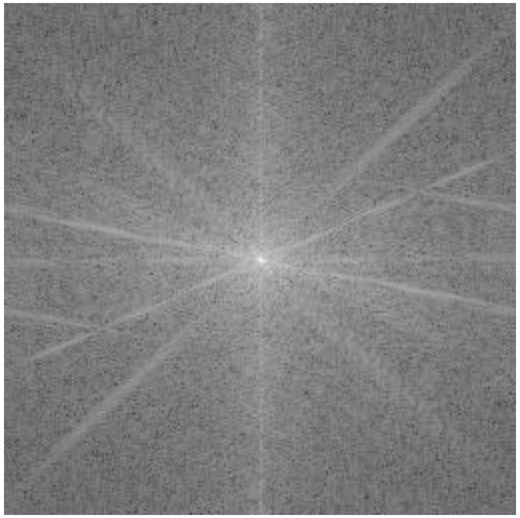
black = 0
white = 1

Low pass filter

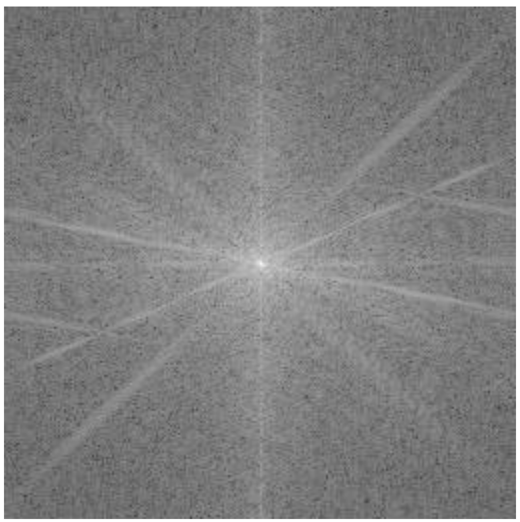


High pass filter

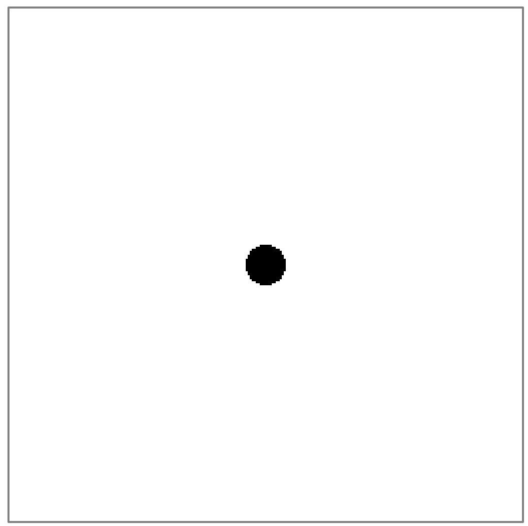
- High pass filter = keep high spatial frequencies, remove low frequencies



High pass filter



x



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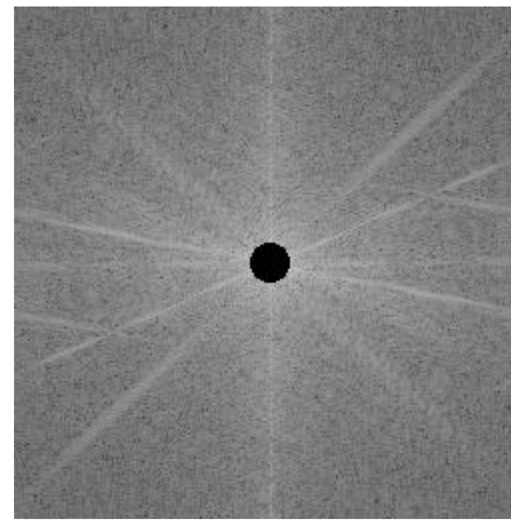

Element-wise multiplication

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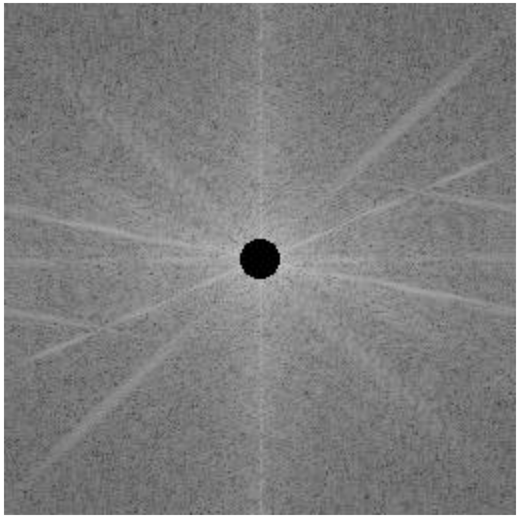
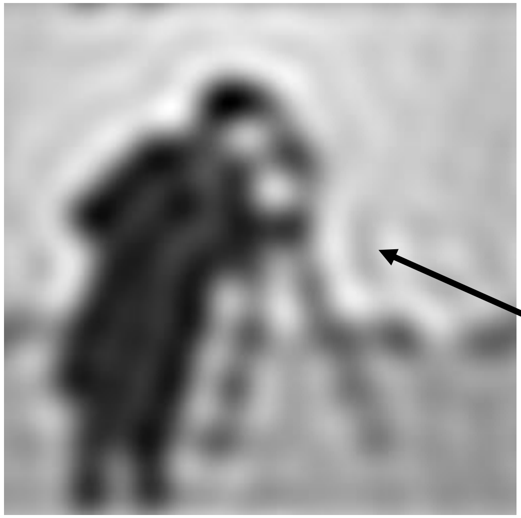
High pass filter



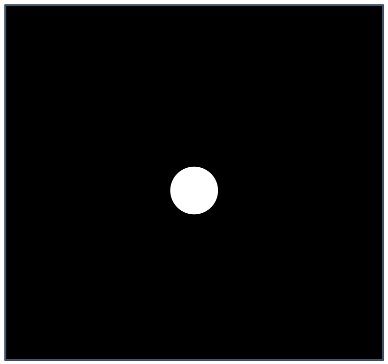
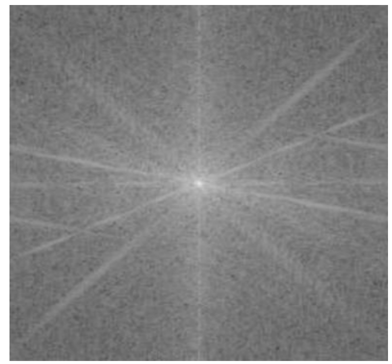
Filter artefacts



“Ringing”

Why does this happen?

Inverse convolution theorem

$FT^{-1}\Big($   $\Big) =$

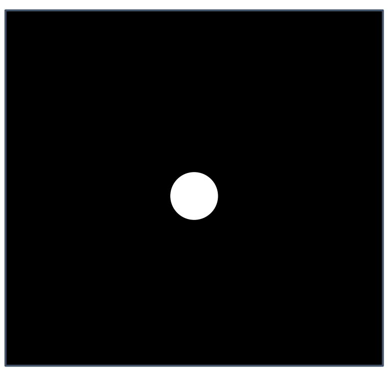
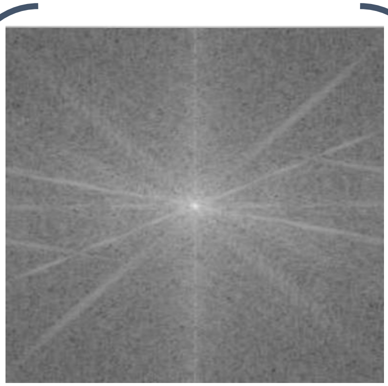
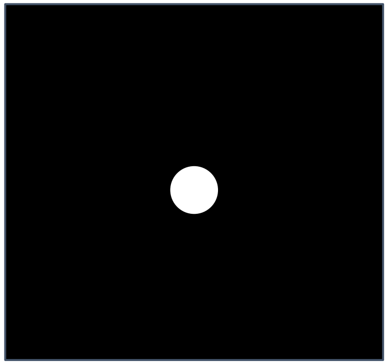
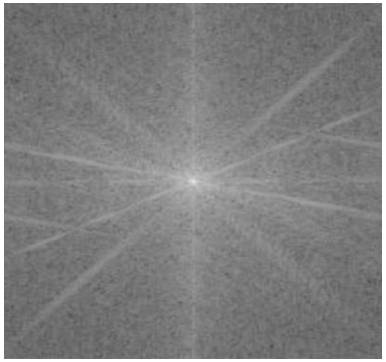
$FT^{-1}\Big($  $\Big) * FT^{-1}\Big($  $\Big)$

Image: S. Seitz

Inverse convolution theorem

FT^{-1} ( ) =

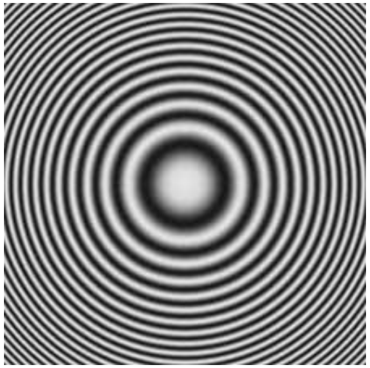
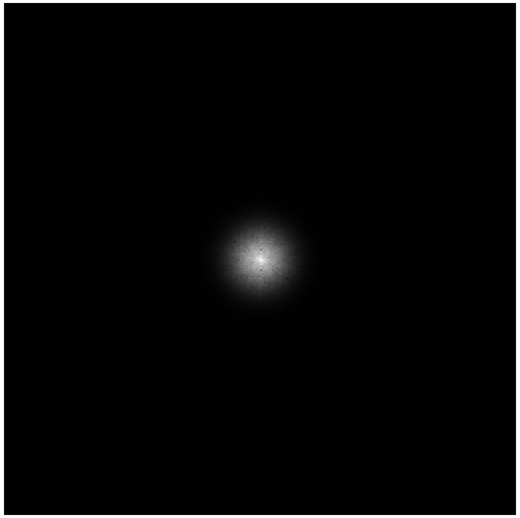
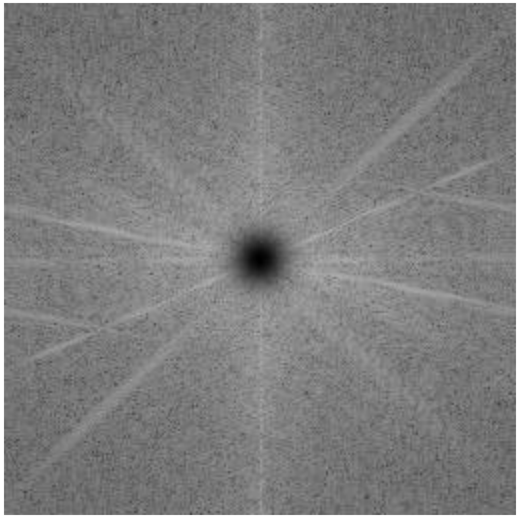
 $*$ 

Image: S. Seitz

Gaussian low pass filter



Gaussian high pass filter



Summary

- Images can be filtered in the spatial domain, or the frequency domain
- Operations in one domain have an equivalent in the other domain
  - Convolution in spatial domain = multiplication in Fourier domain
- Modelling filters in both domains can help understand/debug what a filter is doing

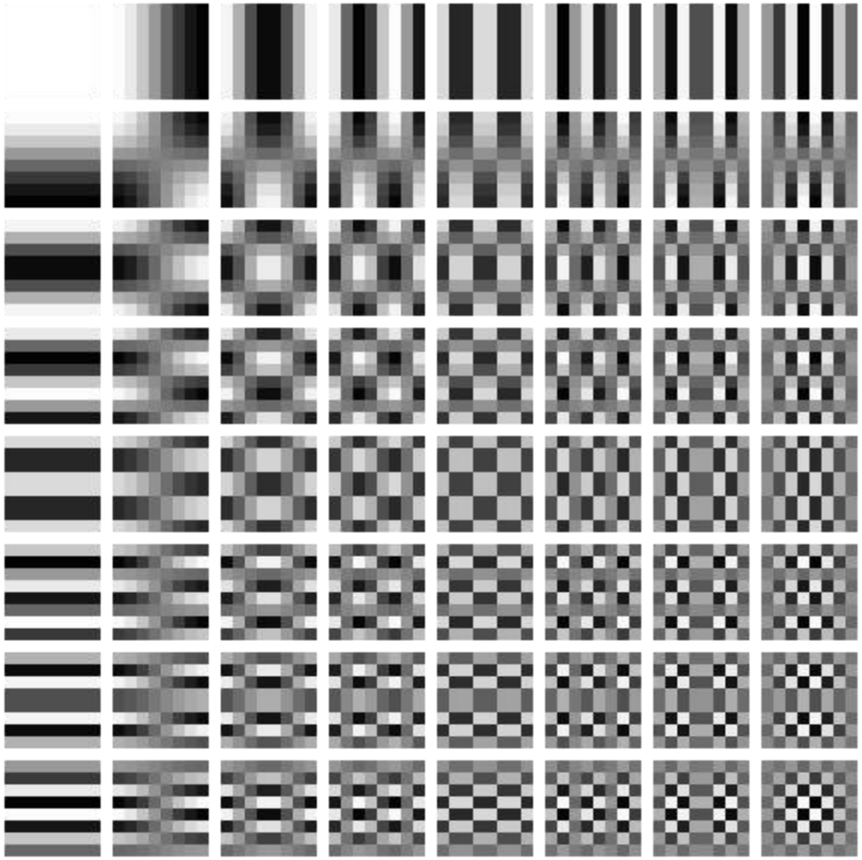
Applications

Image compression

- Frequency domain is a convenient space for image compression
- Why?
- Human visual system is not very sensitive to contrast in high spatial frequencies
- Discarding information in high spatial frequencies doesn’t change the “look” of an image

Image compression

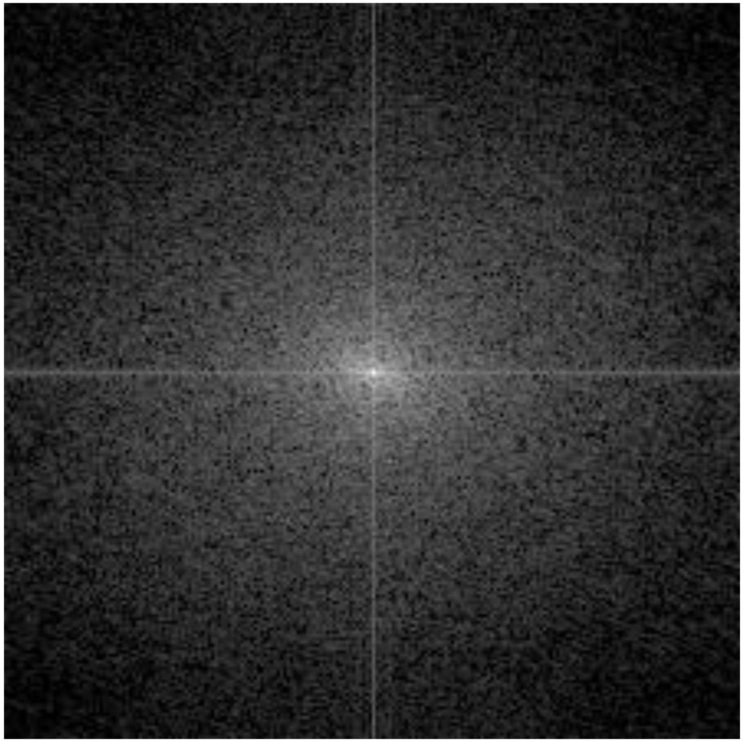
- JPEG compression: break image into 8x8 pixel blocks, each represented in frequency space
- Discrete cosine transform (DCT)
- High spatial frequency components are quantised



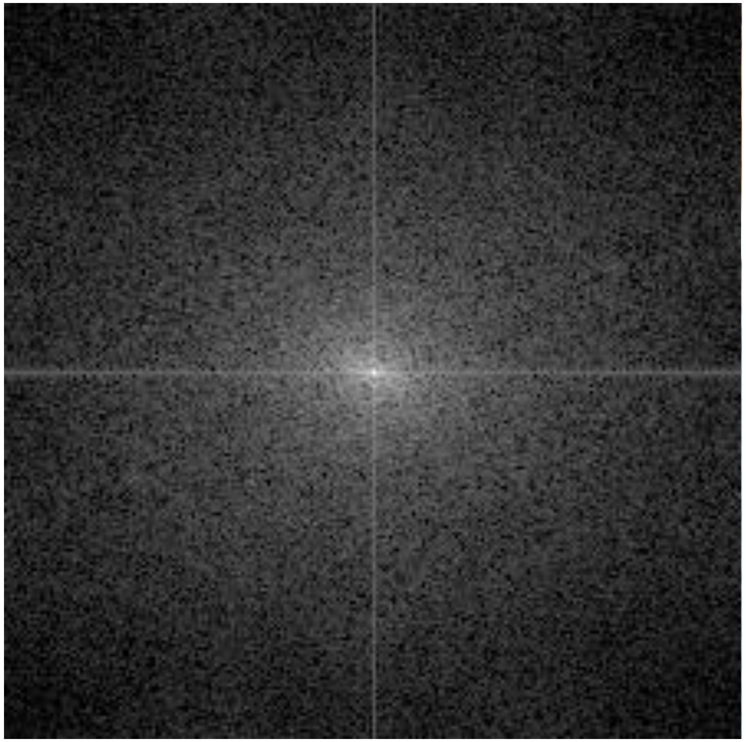
JPEG compression

Image: https://en.wikipedia.org/wiki/File:Felis_silvestris_silvestris_small_gradual_decrease_of_quality.png

Image forensics



Network: MobileNet V2
Prediction: English springer (90.08%)

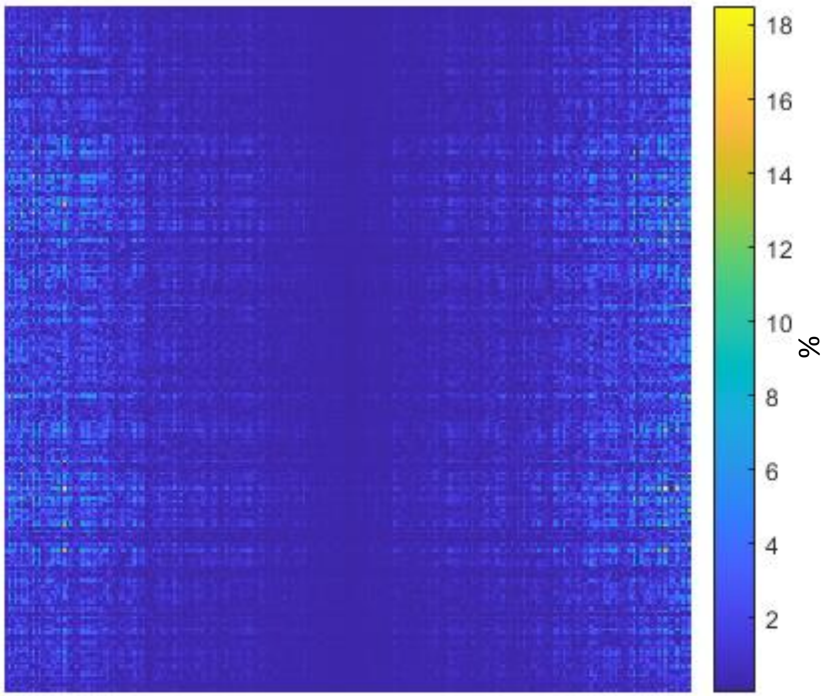


Model: MobileNet V2
Prediction: hot dog (68.88%)

Images: ImageNet, https://kennysong.github.io/adversarial.js/

Dog vs. hot dog – what changed?

Absolute difference in magnitude



Absolute difference in phase

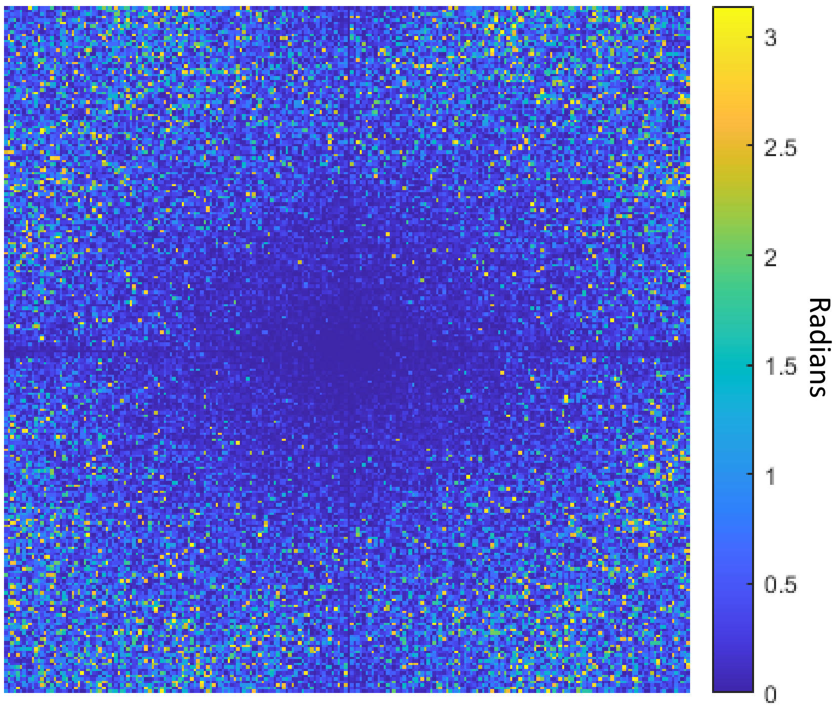
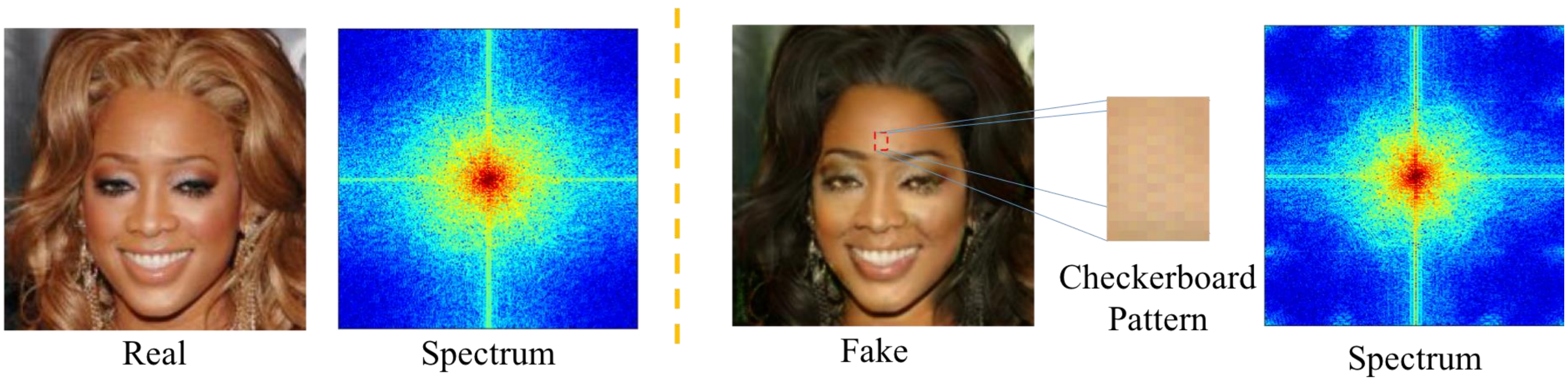


Image forensics

Which of these are real people vs. GAN-generated?



Dzanic, Shah, & Witherden (2019)
Zhang, Karaman, & Chang (2019)

Summary

- Any image can be represented in either the spatial or the frequency domain
- Frequency domain is a convenient space for many applications:
  - Filtering
  - Compression
  - Forensics
  - Frequency-based features