COMS30127/COMSM2127 Computational Neuroscience

Lecture 14: Leaky integrate-and-fire neurons

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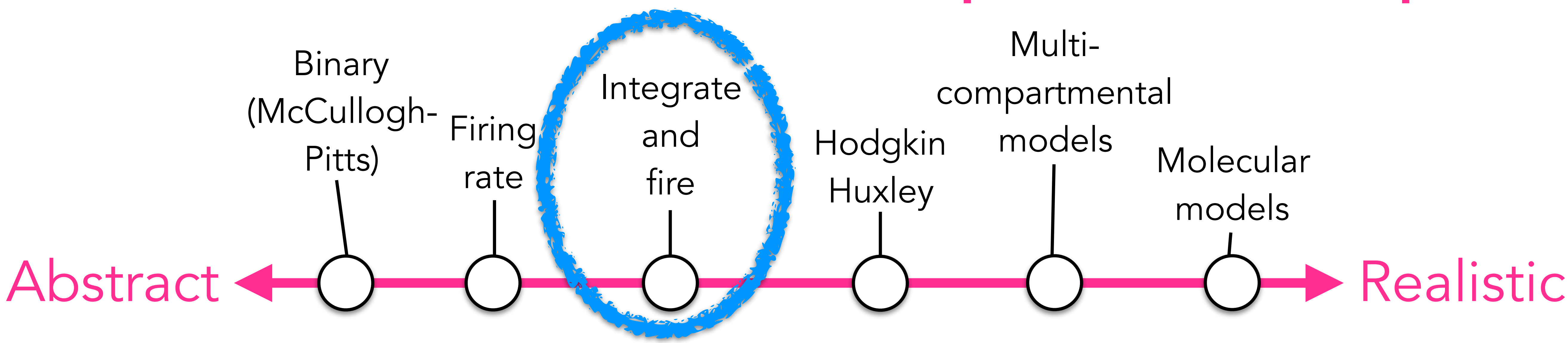
Link to lecture video:

https://mediasite.bris.ac.uk/Mediasite/Play/8024d985144744219f3da197967644981d

What we will cover today

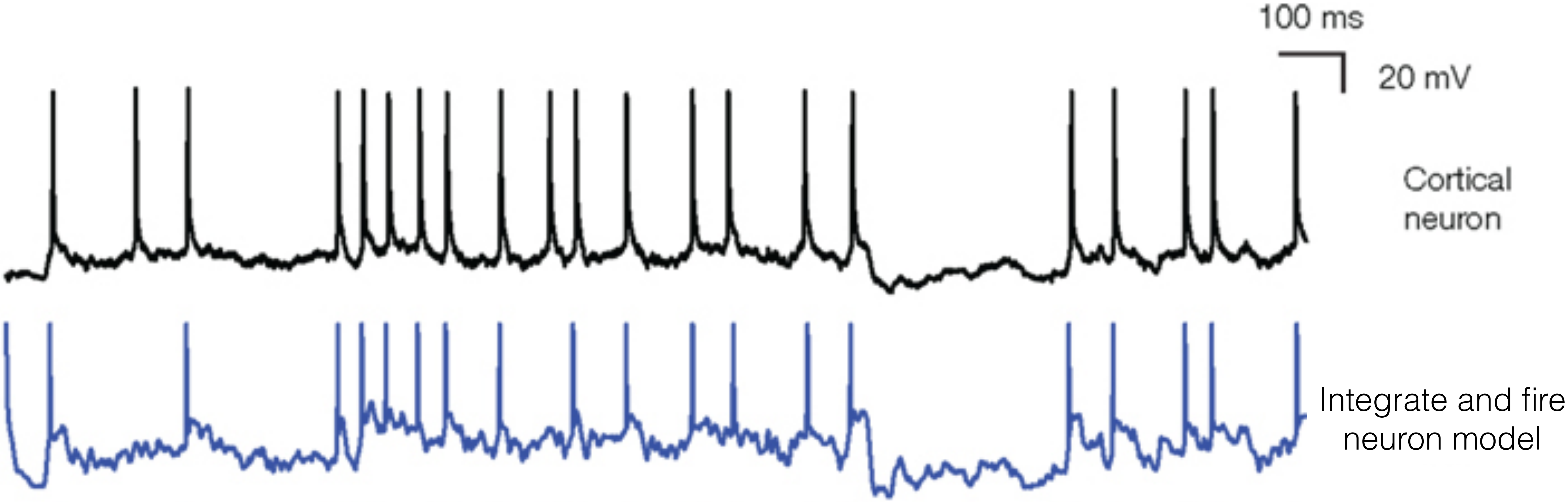
- Recap on model neuron types
- The leaky integrate-and-fire (LIF) neuron model
- The LIF's f-I curve
- The LIF's low-pass filtering of input signals

Model neuron types (recap)



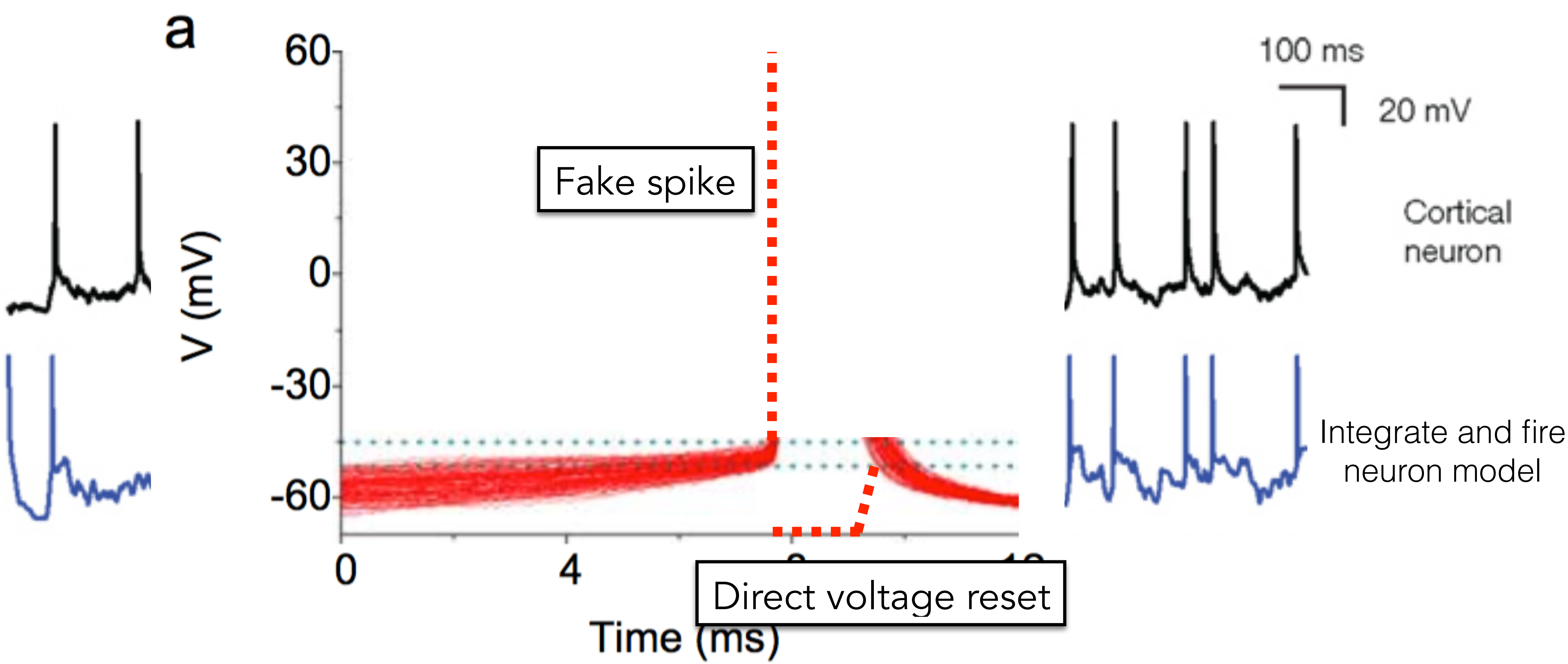
Abstract models		Realistic models
Simple	vs	Detailed
Hard to relate to biology	vs	Contains stuff you could measure
Few parameters	vs	Lots of parameters
Fast simulation	vs	Slow simulation
Mathematical analysis	vs	Intractable
Generic	vs	Specific

The basic idea



Rossant et al., *Frontiers in Neurosci* (2011)

The basic idea



Rossant et al., *Frontiers in Neurosci* (2011)

Yu et al., *J Neurosci* (2008)

Leaky integrate-and-fire neuron

- The leaky integrate-and-fire neuron model has two key components:
  1. An equation describing the voltage dynamics.
  2. A voltage-reset mechanism, mimicking a spike.

$$C_m \frac{dV}{dt} = (E_L - V)/R_m + I_e$$ voltage dynamics

if $V > V_{thresh} : V \leftarrow V_{reset}$ "spike has occurred"

- (The name is a bit misleading, the LIF model doesn't actually generate any spikes.)
- The LIF is heavily used in computational neuroscience because of its simplicity and analytical tractability.

The time constant

- From your earlier lecture on differential equations you may remember that the solution for this type model evolves over a typical timescale τ.

- For the integrate-and-fire neuron this time constant is $\tau_m = R_m C_m$
(You can verify yourself that this product has units of time).

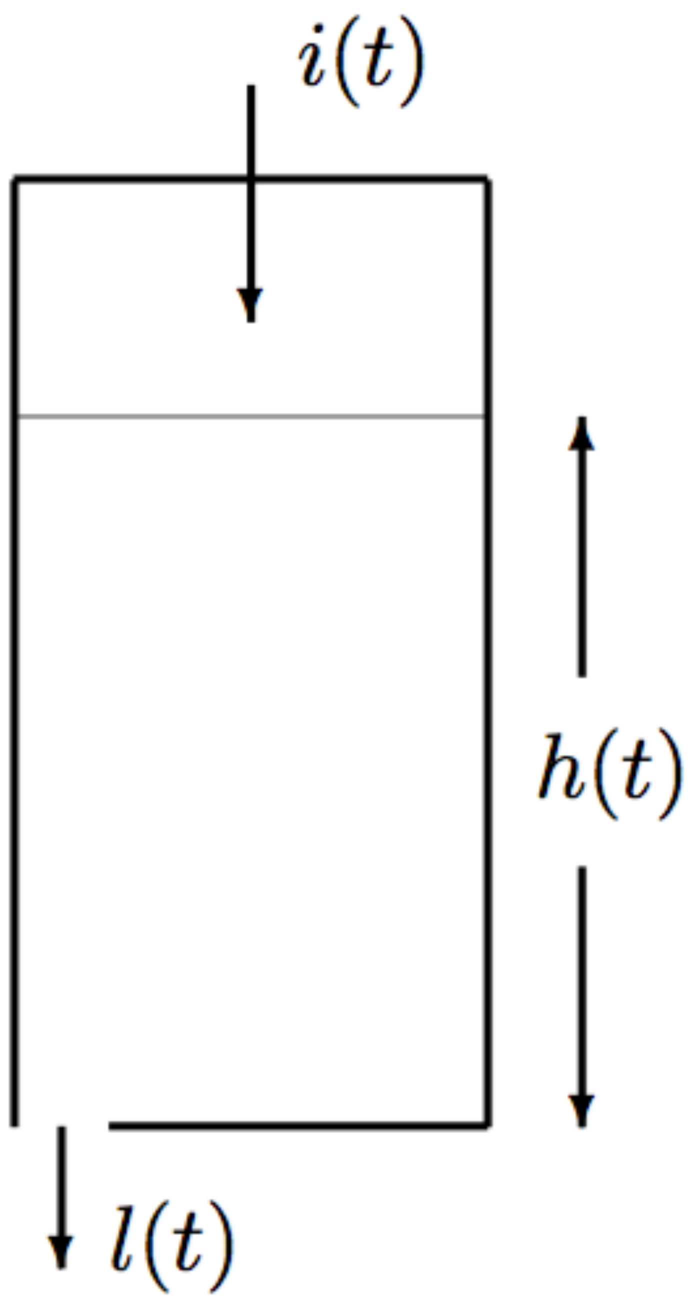
- We can see this directly in the voltage equation by multiplying across by R_m:

$$C_m \frac{dV}{dt} = (E_L - V)/R_m + I_e$$

$$R_m C_m \frac{dV}{dt} = (E_L - V) + R_m I_e$$

$$\tau_m \frac{dV}{dt} = (E_L - V) + R_m I_e$$

The leaky bucket analogy

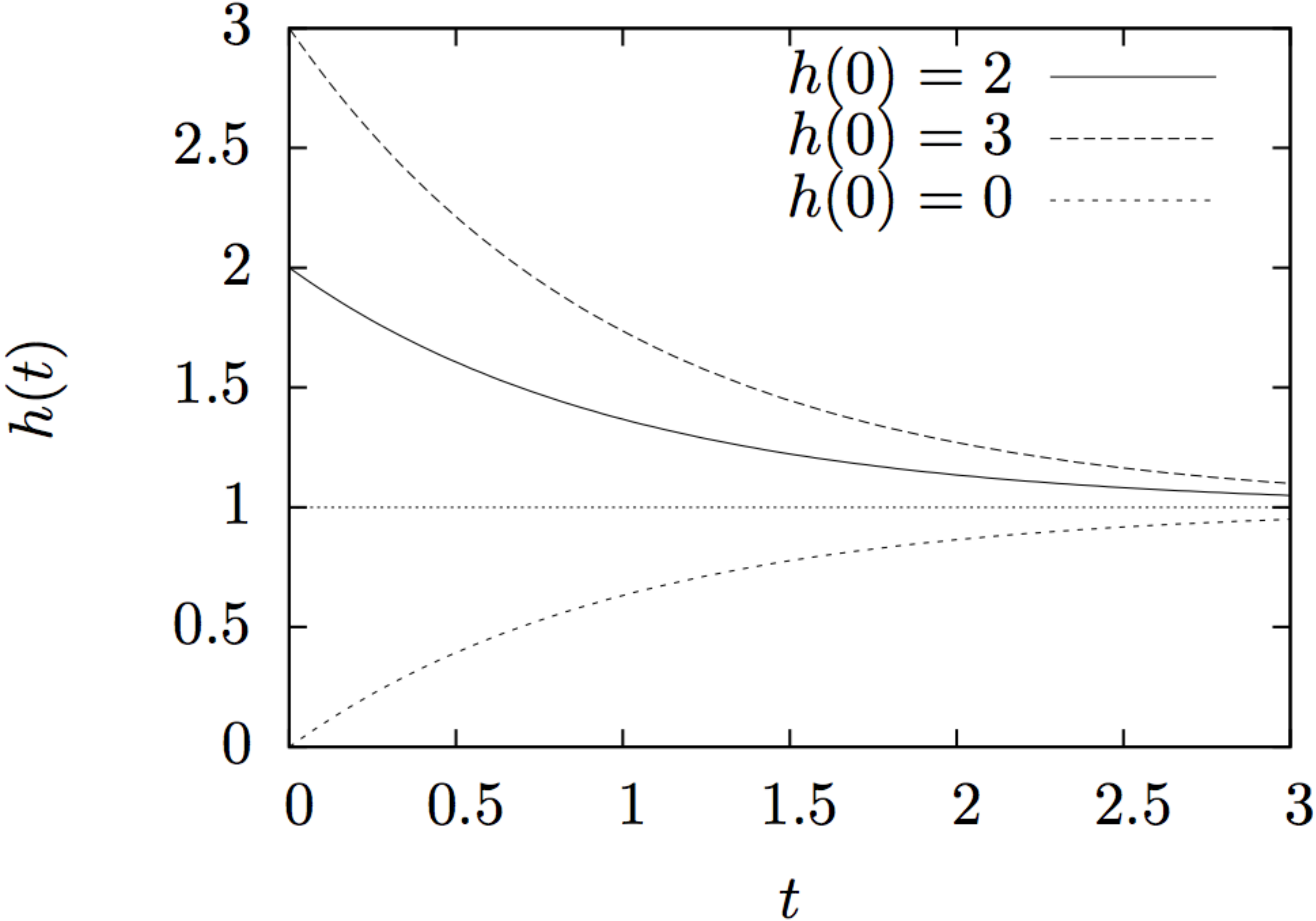


$$\frac{dh}{dt} = \frac{1}{C}(i - Gh)$$

C is cross-sectional area of bucket

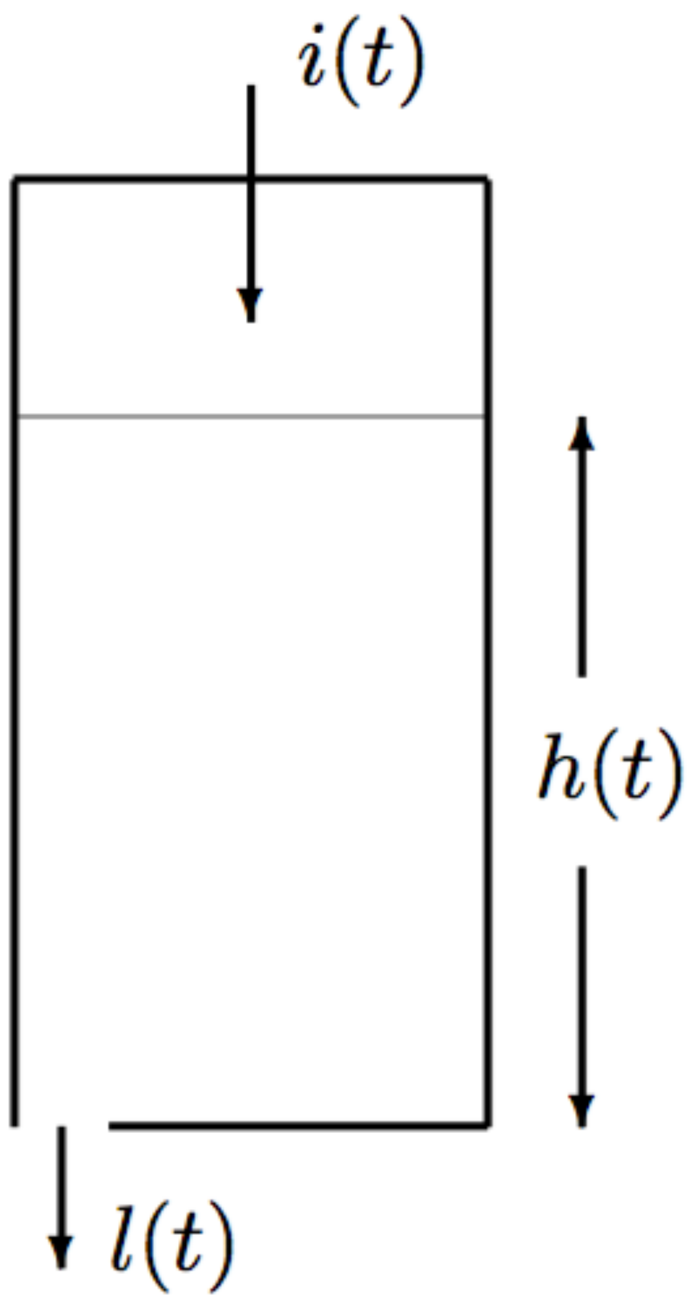
G is constant determining rate of water leak

[see Conor's notes for full derivation]



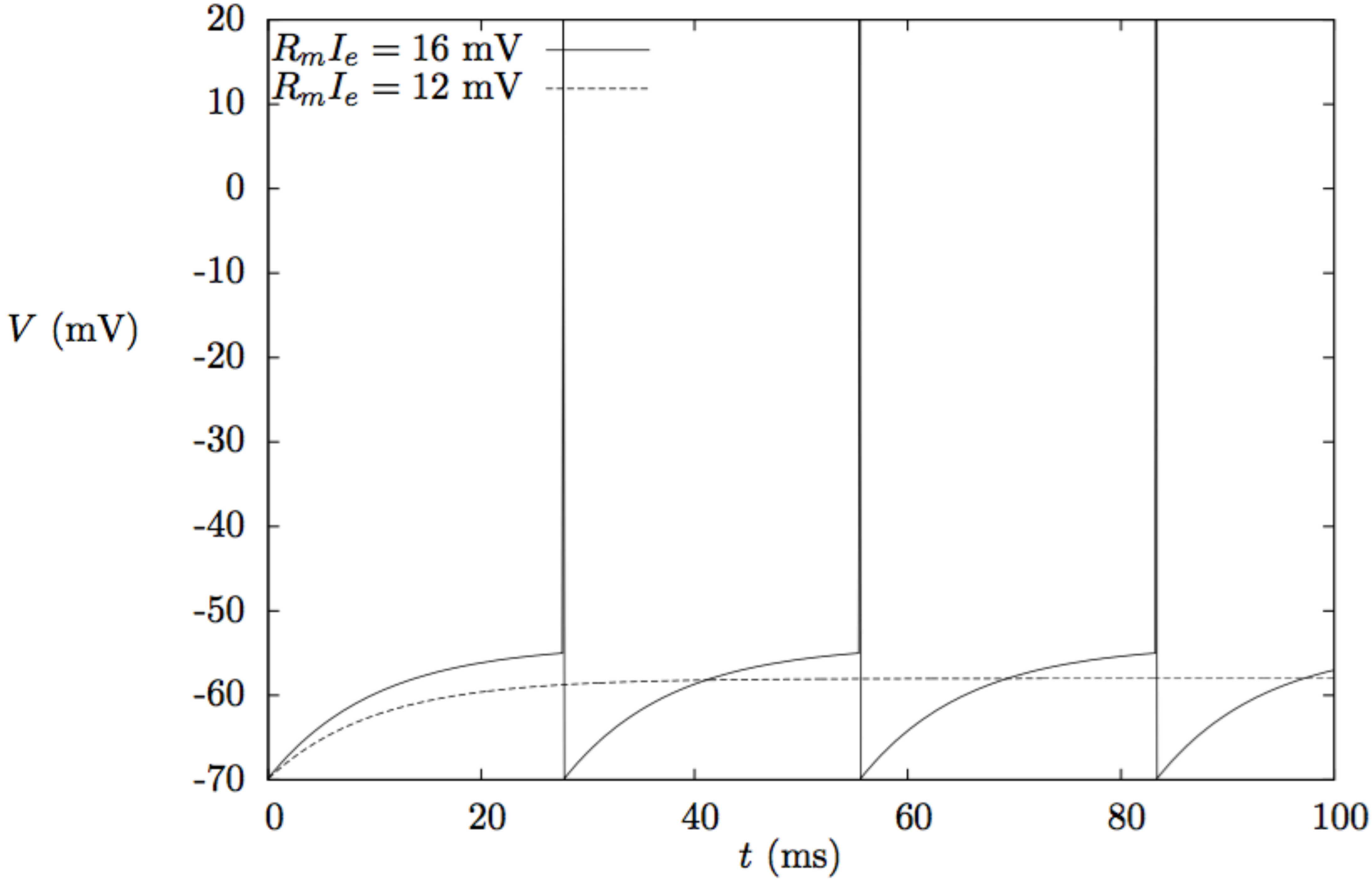
$h(0) = 2$
$h(0) = 3$
$h(0) = 0$
$h(t)$
t
0
0.5
1
1.5
2
2.5
3

The leaky bucket analogy



Height of water	=	Membrane voltage
Volume of water	=	Electrical charge
Water influx	=	Synaptic/electrode current
Water leak rate	=	Membrane leak current
Bucket circumference	=	Membrane capacitance

Leaky integrate-and-fire neuron



f-I curves

- Fundamentally, neurons are input-output devices:
  - Take synaptic inputs from other neurons
  - Output a series of spikes.

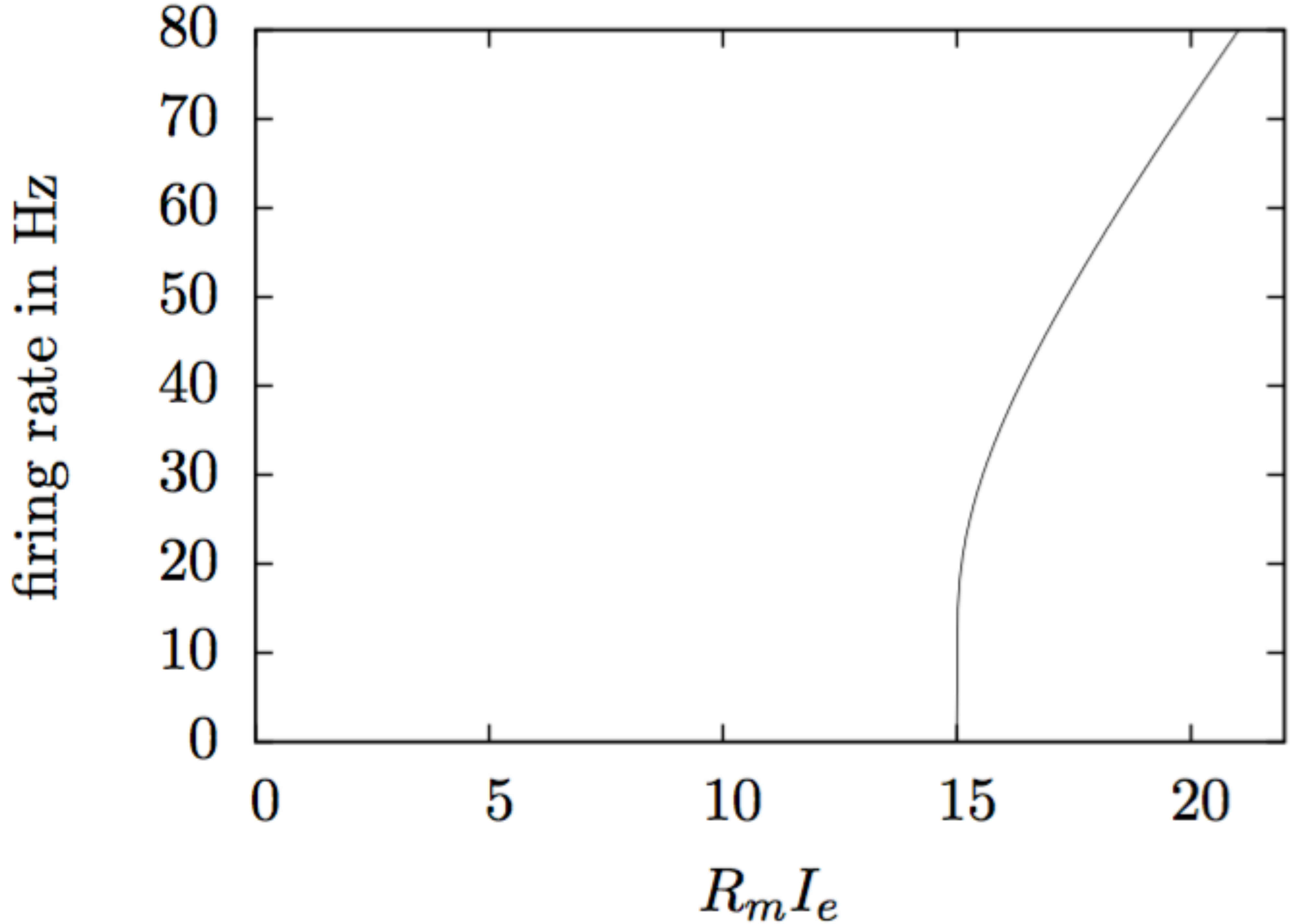
- One common way of characterising a neuron’s input-output function is the frequency-current (f-I) curve.

- The idea is that the experimenter injects current steps of various amplitudes to the neuron’s soma, then records the output firing rate of the neuron.

- Some real examples at: https://celltypes.brain-map.org/data

- For the LIF model we can analytically compute the time to spike, and therefore the spike frequency, as a function of the input current amplitude.

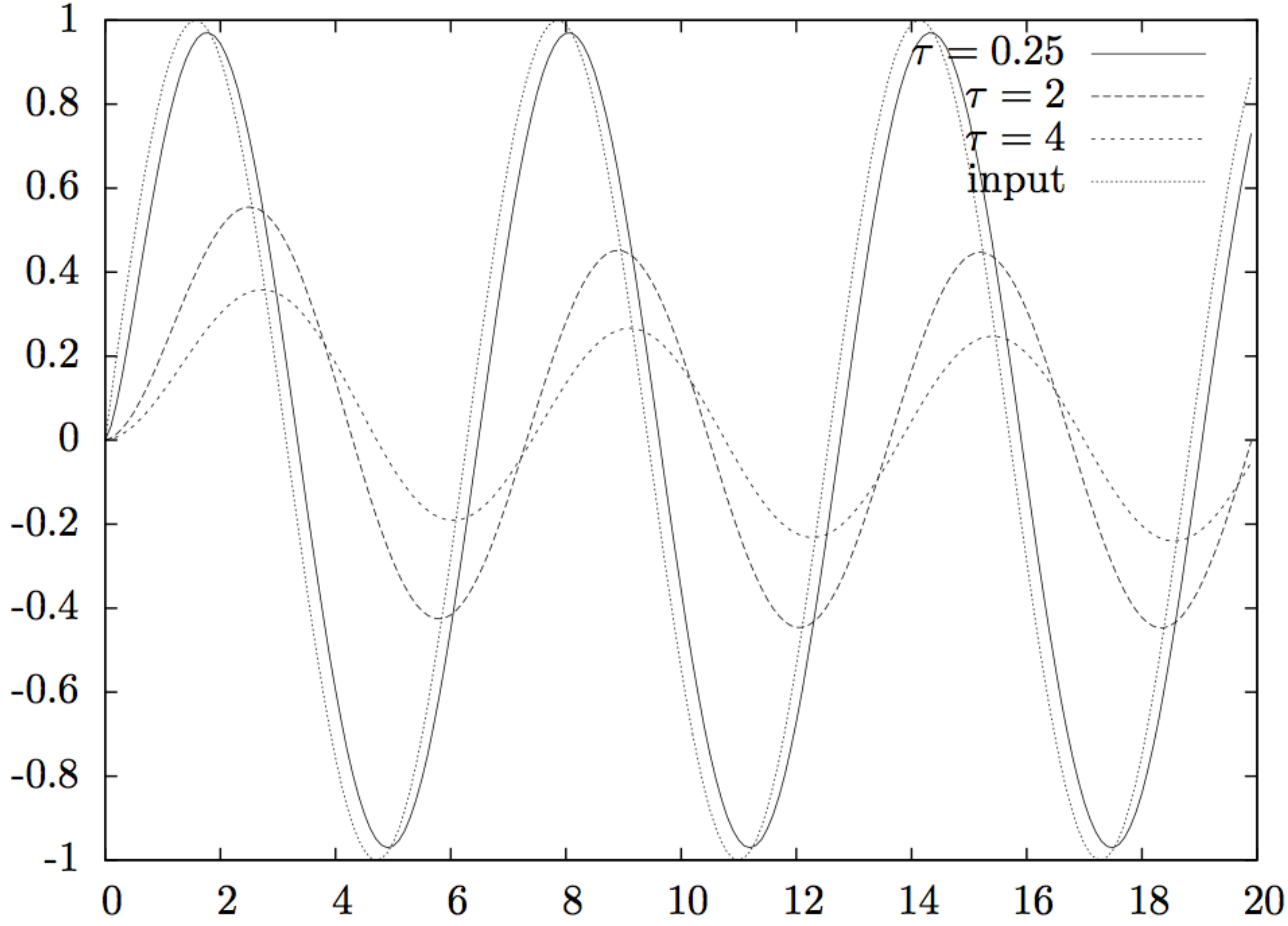
The LIF's f-I curve



Low-pass filtering by the LIF

- The membrane capacitance acts to slow down the voltage dynamics: it takes time to charge and discharge.
- Quickly changing input signals tend to get averaged out because the membrane voltage can't change quickly enough to track them.
- Slowly changing input signals, on the other hand, can be tracked by the membrane voltage.
- This implies that the LIF model filters high-frequency signals. In other words it is a "low-pass filter".

Low-pass filtering by the LIF



Low-pass filtering by the LIF

- We can summarise the input-output transform's frequency dependence with the *impedance*.

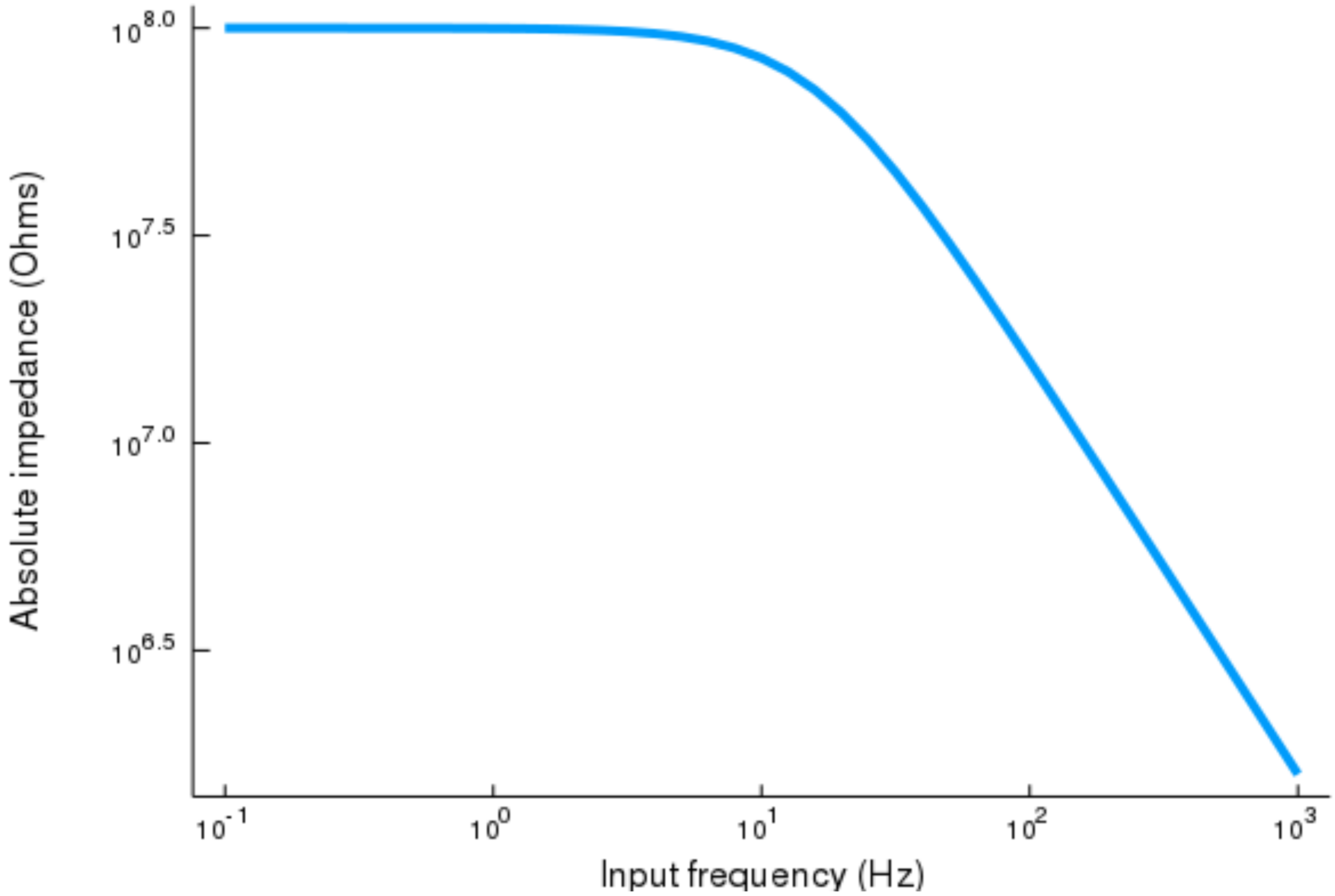
- The absolute value of the impedance is equal to the ratio of the voltage amplitude to the current amplitude.

- We can compute the impedance analytically by taking the Fourier transform of the voltage solution in response to a periodic input signal of frequency f.

$$|Z(f)| = \frac{R_m}{\sqrt{1 + (2\pi f \tau_m)^2}}$$

- For high frequencies the impedance is proportional to $1/f$.

LIF impedance



R_m = 100 MOhms, τ_m = 10 ms

Extensions to the LIF

The leaky integrate-and-fire neuron is a basic model. Over the years many extensions have been designed to make it more realistic:

- A refractory period.
- A mechanism for spike-frequency adaptation.
- A dynamic spike threshold value.
- A more realistic spiking mechanism.

Further reading

- Conor’s excellent LIF notes covering today’s maths: https://github.com/coms30127/2019_20/blob/master/notes/14_integrate_and_fire_cjh_notes.pdf