COMS30127/COMSM2127 Computational Neuroscience

Lecture 4: Differential equations

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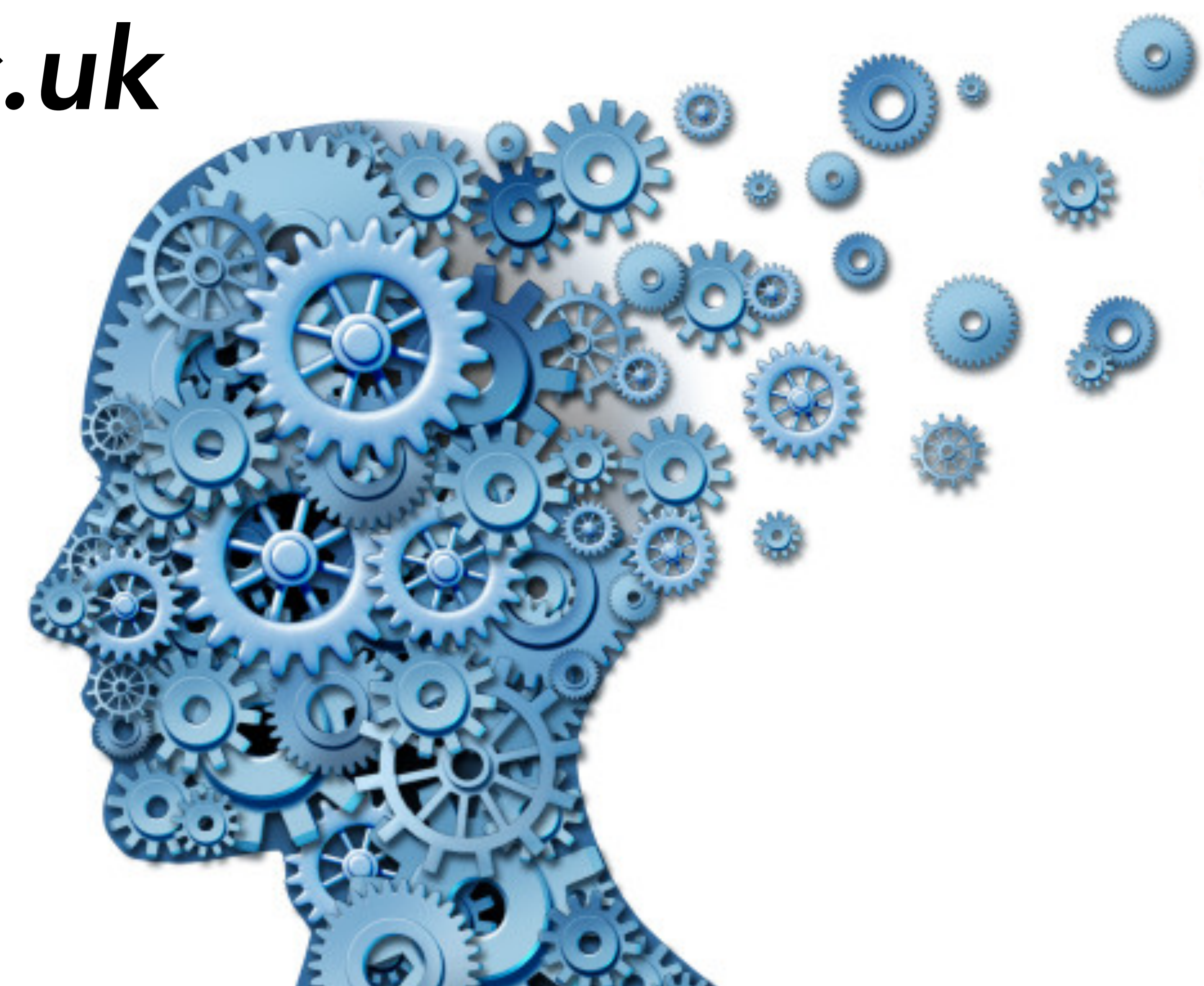


Intended learning outcomes

- Understand differential equations as tools for for modelling real-world systems.
- How to solve easy ordinary differential equations.
- Get an intuition for the typical dynamics in linear ODEs.

Differential equation models of real-world systems

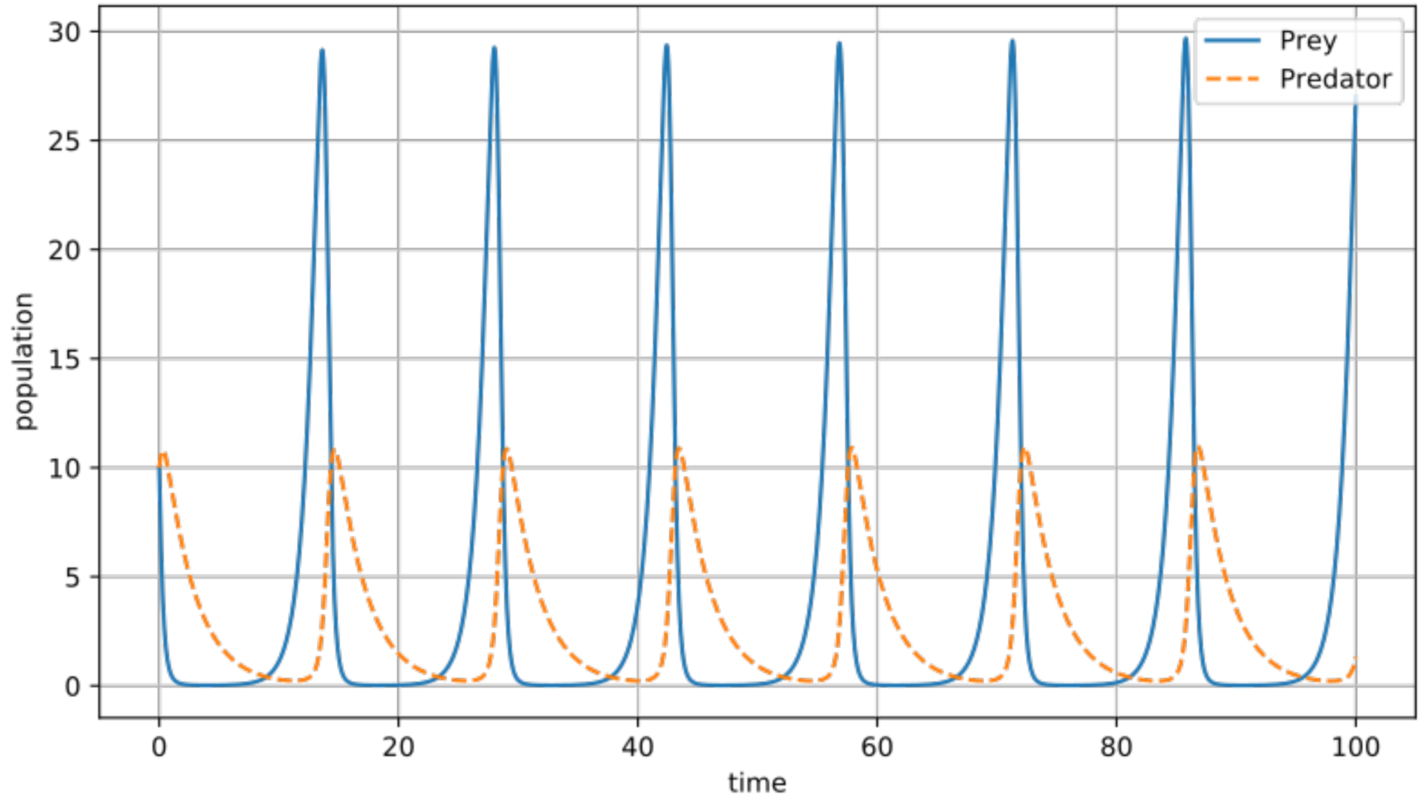
- Differential equations (DEs) are by far the most common formalism for modelling real-word systems.
- Ubiquitous in physics, chemistry, engineering, geoscience, biology.
- Useful for modelling things changing over time, and when the variables of interest can be treated as continuous quantities.
- However not universal! For discrete and/or stochastic systems we need other mathematical models.

Lotka-Volterra model example

- Famous simple mathematical model of a predator-prey system from ecology.
- Consists of a system of two nonlinear ODEs:

$$\frac{dx}{dt} = \alpha x - \beta xy$$
$$\frac{dy}{dt} = \delta xy - \gamma y$$

- x and y are variables that represent the number of rabbits and wolves, respectively.
- α, β, γ and δ are parameters.



https://commons.wikimedia.org/wiki/File:Lotka-Volterra_model_(1.1,_0.4,_0.4,_0.1).svg

Types of differential equations

- Common type we will study are ordinary, linear, first-order, inhomogeneous differential equations, which can often can be written in form

$$\frac{df}{dt} = -f(t) + g(t)$$

- *Ordinary* (vs partial) refers to the fact that there is only one dependent variable (f in the above).
- *Linear* (vs nonlinear) means there are no f^2 or anything.
- *First-order* (vs second or third-order etc) means that the highest derivative is $\frac{df}{dt}$, there is no $\frac{d^2f}{dt^2}$, etc.
- *Inhomogeneous* because the solution depends on something else in addition to f. Usually in models of physical systems this will correspond to some external signal.

Differential equation models in neuroscience

- In computational neuroscience the dynamic variables of interest are commonly things like:
  - Voltage and current flow in a single neuron.
  - Firing rates of a population of neurons.
  - EEG or fMRI signals (if modelling an experiment)
- For example here is a common differential equation for the voltage of a neuron:

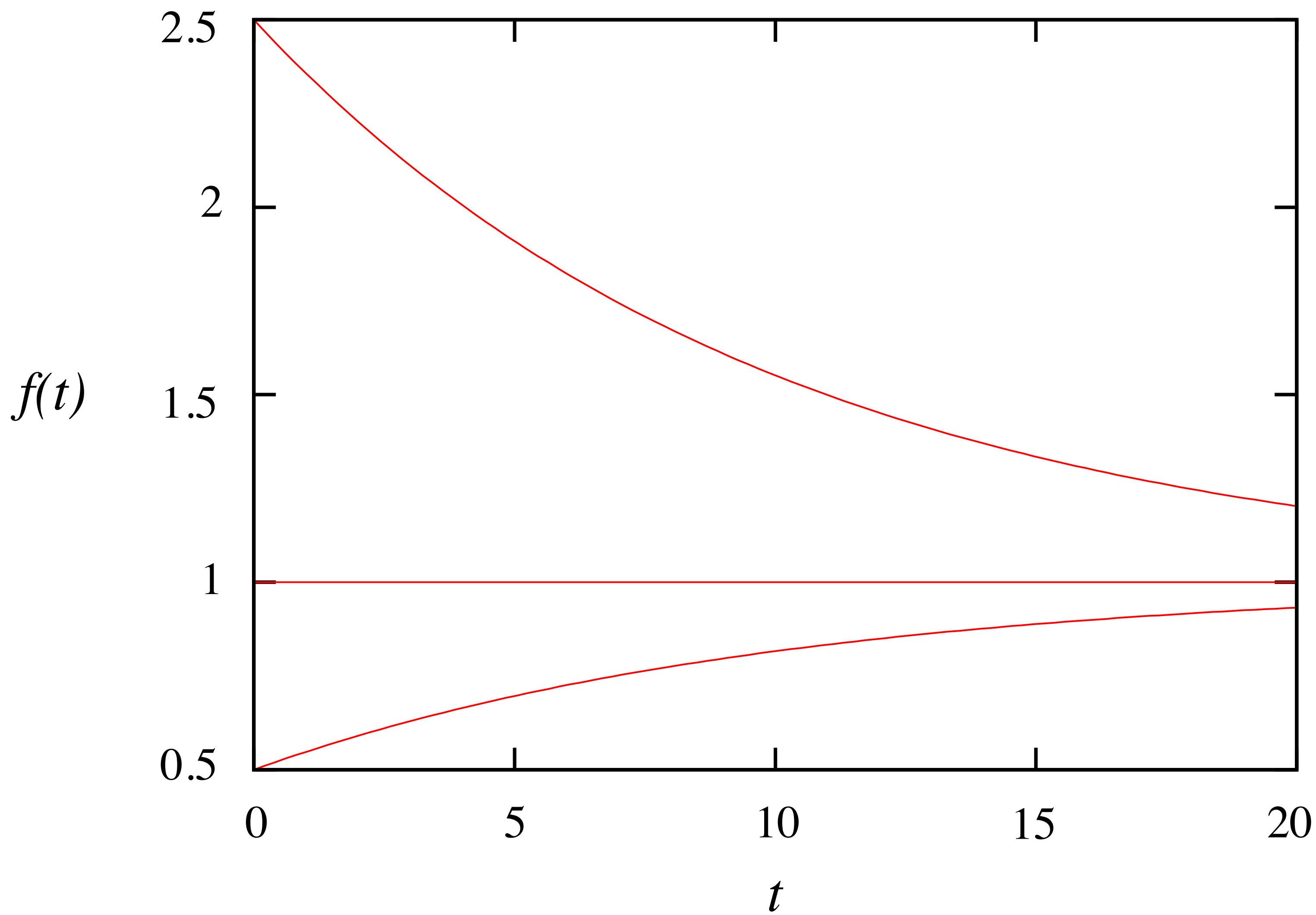
$$\tau_m \frac{dV}{dt} = E_l - V(t) + R_m I(t)$$

Solving easy differential equations

- "Solving" a differential equation means finding a function that when differentiated returns the original DE.
- In this case we want to find a formula for $f(t)$.
- So how do we solve $\dfrac{df}{dt} = -f(t) + g(t)$?
- See Conor Houghton's notes on the GitHub repo: https://github.com/coms30127/2019_20/notes/maths

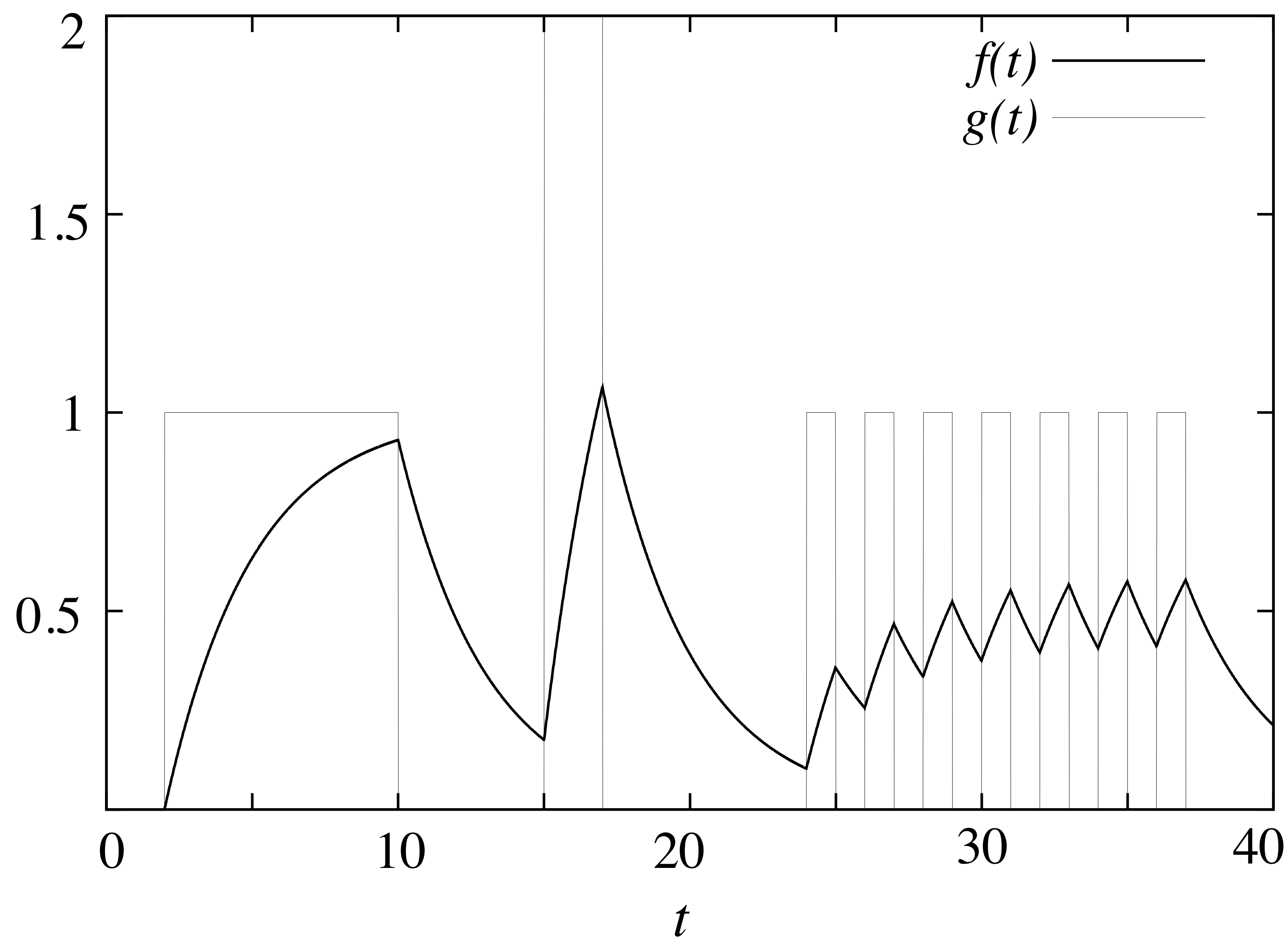
The solution for constant g

$$f(t) = \bar{g} + [f(0) - \bar{g}]e^{-t/\tau}$$



- Note that the trajectories all eventually converge towards the same value, no matter what the initial condition.

The solution for time-varying g



- Again see Conor Houghton's notes on the GitHub repo: https://github.com/coms30127/2019_20/notes/maths
- Note that $f(t)$ "tries" to track $g(t)$.

Intuition for ODE dynamics

- Linear differential equations with a constant input do one of two things:
  1. Converge to a steady-state value.
  2. "Explode” to positive or negative ∞.
- Linear differential equations with a time varying input “try” to track the input signal.
- If the time constant(s) in the DEs are fast compared to the timescale of the input signal, the variables can track the input almost perfectly.
- In the converse case, if DE time constants are slow compared to the timescale of the input signal, then the variables will low-pass filter the input - they “average out” the high frequency components of the input.

Next Tuesday

- In most models of practical interest, we can't solve the differential equations analytically.
- HOWEVER we can use computers to calculate an approximate answer numerically.
- Next Tuesday's lecture will introduce basic numerical methods for solving ODEs, and mention some of the practical issues.