DYNAMICS OF NON-LINEAR ROBOTIC SYSTEMS

Assignment 3

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Task 1 :- Calculate $q, \dot{q}, \ddot{q}$ trajectories

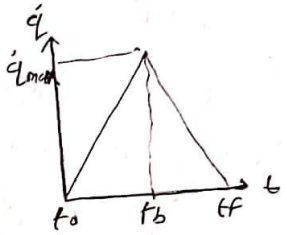
Formula

Triangular profile

$$\Delta q = t_b \cdot \dot{q}_{max}$$

$$t_b = \frac{\Delta q}{\dot{q}_{max}}$$

$$\ddot{q}_{max} = \frac{\dot{q}_{max}}{t_b}$$



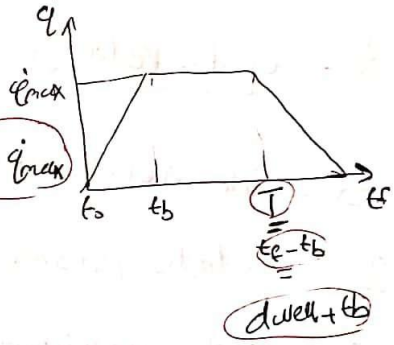
Trapezoidal profile

$$\Delta q = t_f - \frac{t_f - 2t_b}{2} \times \dot{q}_{max}$$

$$\Delta q = (t_f - t_b) \times \dot{q}_{max} = T \times \dot{q}_{max}$$

$$t_b = \frac{q_0 - q_f + \dot{q}_{max} t_f}{\dot{q}_{max}}$$

$$\ddot{q}_{max} = \frac{\dot{q}_{max}}{t_b}$$



Check

$$\sqrt{\ddot{q}_{max} \Delta q}$$

$< = \dot{q}_{max}$

$$t_b = \frac{\Delta q}{\dot{q}_{max}} = \sqrt{\frac{\Delta q}{\ddot{q}_{max}}}$$

$$T = t_b$$

$$t_f = 2t_b$$

Triangular

$> \dot{q}_{max}$

$$t_b = \frac{\dot{q}_{max}}{\ddot{q}_{max}}$$

$$T = \frac{\Delta q}{\dot{q}_{max}}$$

$$t_f = T + t_b$$

Trapezoidal

All joints are trapezoidal profile, then:

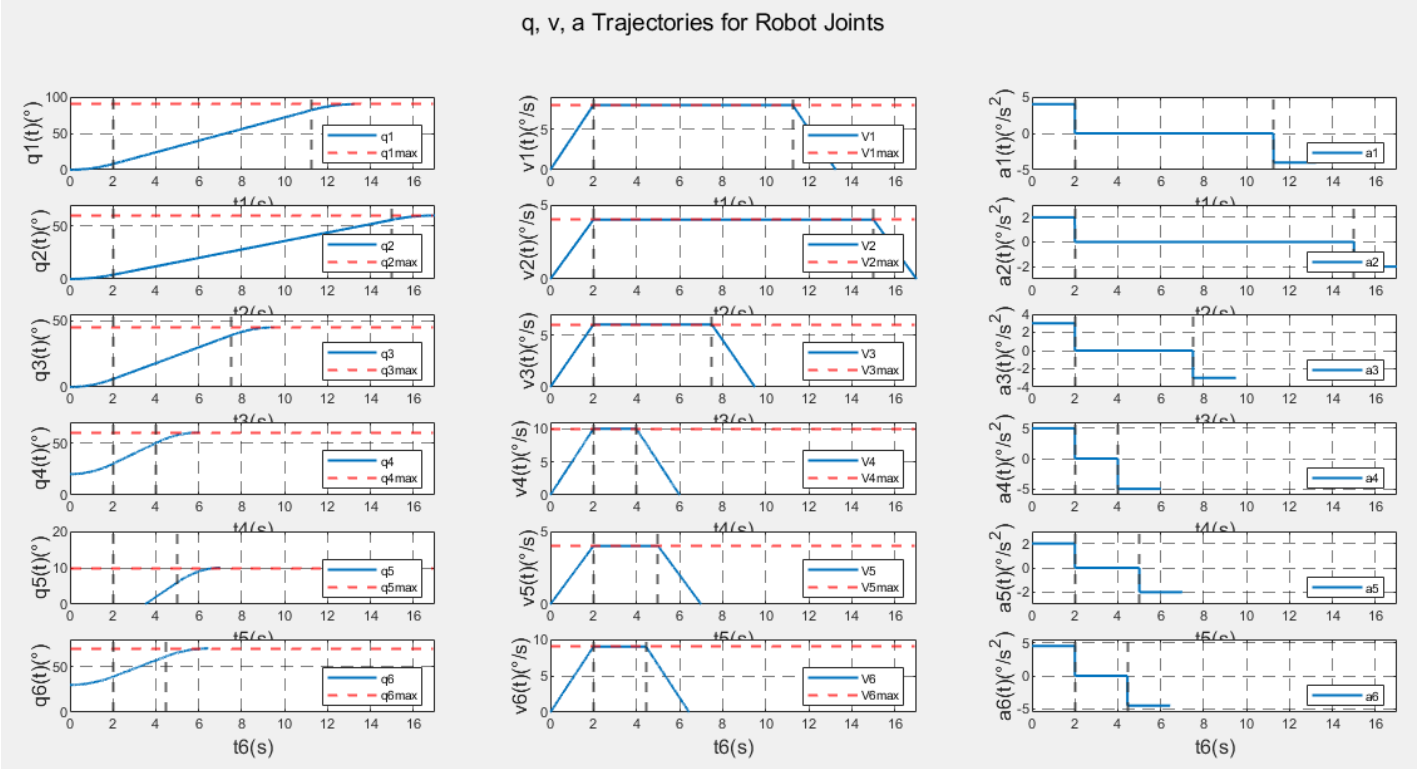
$$q(t)=\begin{cases} q_0+\frac{1}{2}\ddot{q}_{max}t^2 & ,0<t\le t_b \\ q_0+\frac{1}{2}\ddot{q}_{max}t_b^2+\dot{q}_{max}\left(t-t_b\right) & ,t_b<t\le t_f-t_b \\ q_f-\frac{1}{2}\ddot{q}_{max}(t-t_f)^2 & ,t_f-t_b<t\le t_f \end{cases}$$

t-t0

Code is performed and below are the results:

```
Task1
...Calculation of q, v, a Trajectories...
J1: Trapezoidal Profile>> rise time: 2.000, dwell time: 9.250, drop time: 2.000, total time: 13.250
J2: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J3: Trapezoidal Profile>> rise time: 2.000, dwell time: 5.500, drop time: 2.000, total time: 9.500
J4: Trapezoidal Profile>> rise time: 2.000, dwell time: 2.000, drop time: 2.000, total time: 6.000
J5: Trapezoidal Profile>> rise time: 2.000, dwell time: 3.000, drop time: 2.000, total time: 7.000
J6: Trapezoidal Profile>> rise time: 2.000, dwell time: 2.444, drop time: 2.000, total time: 6.444
```

It calculates rise time, dwell time, drop time, total time, and plot q, v, a for each joint.



Task [2] : Synchronization

Concept

$J = [q_0, q_f, dq\text{-}m, ddq\text{-}m]$

q_0 : initial angle, q_f : final angle, $dq\text{-}m$: max vel, $ddq\text{-}m$: max acc

① → Find maximum rise time : $tb = \max([tb_1, \ldots, tb_8])$

② → Find maximum dwell time : $dwell = \max([T_1 - tb_1, \ldots, T_8 - tb_8])$

③ → $T = dwell_{max} + rise_{max}$

④ → $tf = T + rise_{max}(tb)$

⑤ → recalc velocity : $dq\text{-}m = \frac{J(2) - J(1)}{T}$

⑥ → recalc acceleration : $ddq\text{-}m = dq\text{-}m / tb$

⑦ → update parameters : $J(3:4) = [dq\text{-}m, ddq\text{-}m]$

⑧ → get maximum total time : $tf = \max([tf_1, \ldots, tf_8])$

⑨ → get q_{max} , V_{max} , a_{max}

⑩ → plotting

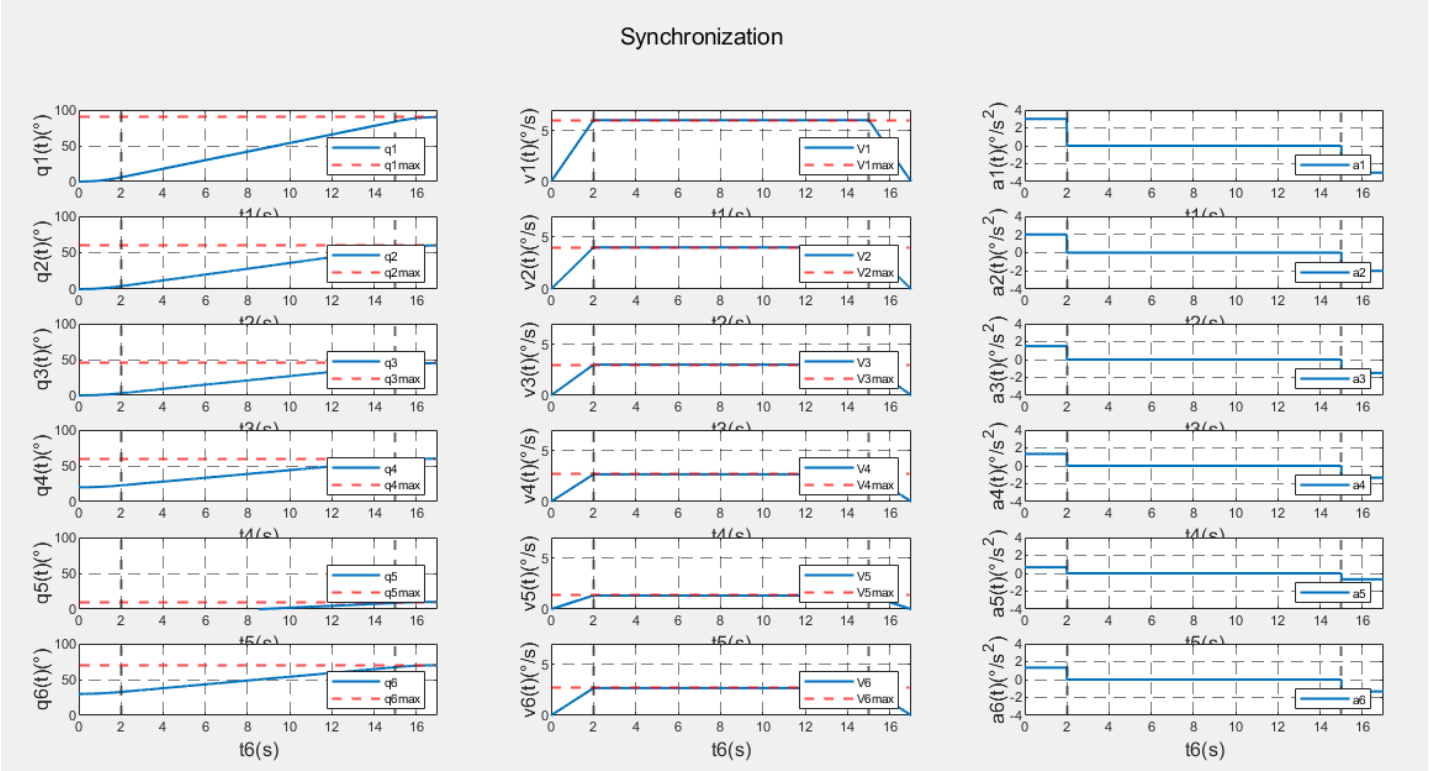
→ assumption : $t_0 = 0$

Code is performed and below are the results:

```
Task2
...Synchronization...
Synchronized Trajectory Time>> rise time: 2.000, dwell time: 13.000, drop time: 2.000,total time: 17.000
Joint1 velocity modified from 8 to 6.0000 and acceleration modified from 4 to 3.0000
Joint2 velocity modified from 4 to 4.0000 and acceleration modified from 2 to 2.0000
Joint3 velocity modified from 6 to 3.0000 and acceleration modified from 3 to 1.5000
Joint4 velocity modified from 10 to 2.6667 and acceleration modified from 5 to 1.3333
Joint5 velocity modified from 4 to 1.3333 and acceleration modified from 2 to 0.6667
Joint6 velocity modified from 9 to 2.6667 and acceleration modified from 5 to 1.3333

After Calculating New Trajectory:
J1: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J2: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J3: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J4: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J5: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
J6: Trapezoidal Profile>> rise time: 2.000, dwell time: 13.000, drop time: 2.000, total time: 17.000
```

It calculates synchronized rise time, dwell time, drop time, total time, and plot q, v, a for each joint after recalculating new trajectories.



Task[3] : Redefine sync motion for numerical control

Concept

① $f = \frac{1}{\Delta t} \rightarrow \Delta t = \frac{1}{f}$, f : controller frequency, Δt = sampling time

② → sync time $[t_{b_{sync}}, T_{sync}, t_{f_{sync}}]$

③ → $t_{b\_num} = \left(floor\left[\frac{t_b}{\Delta t}\right] + 1\right) * \Delta t$ (round to nearest integer values)

④ → $T_{num} = \left(floor\left[\frac{T}{\Delta t}\right] + 1\right) * \Delta t$

⑤ → $t_{f\_num} = T_{num} + t_{b\_num}$

⑥ → Calc $q_{num}, v_{num}, a_{num}$

⑦ → error in angles = $J(2) - q_{num}(end)$

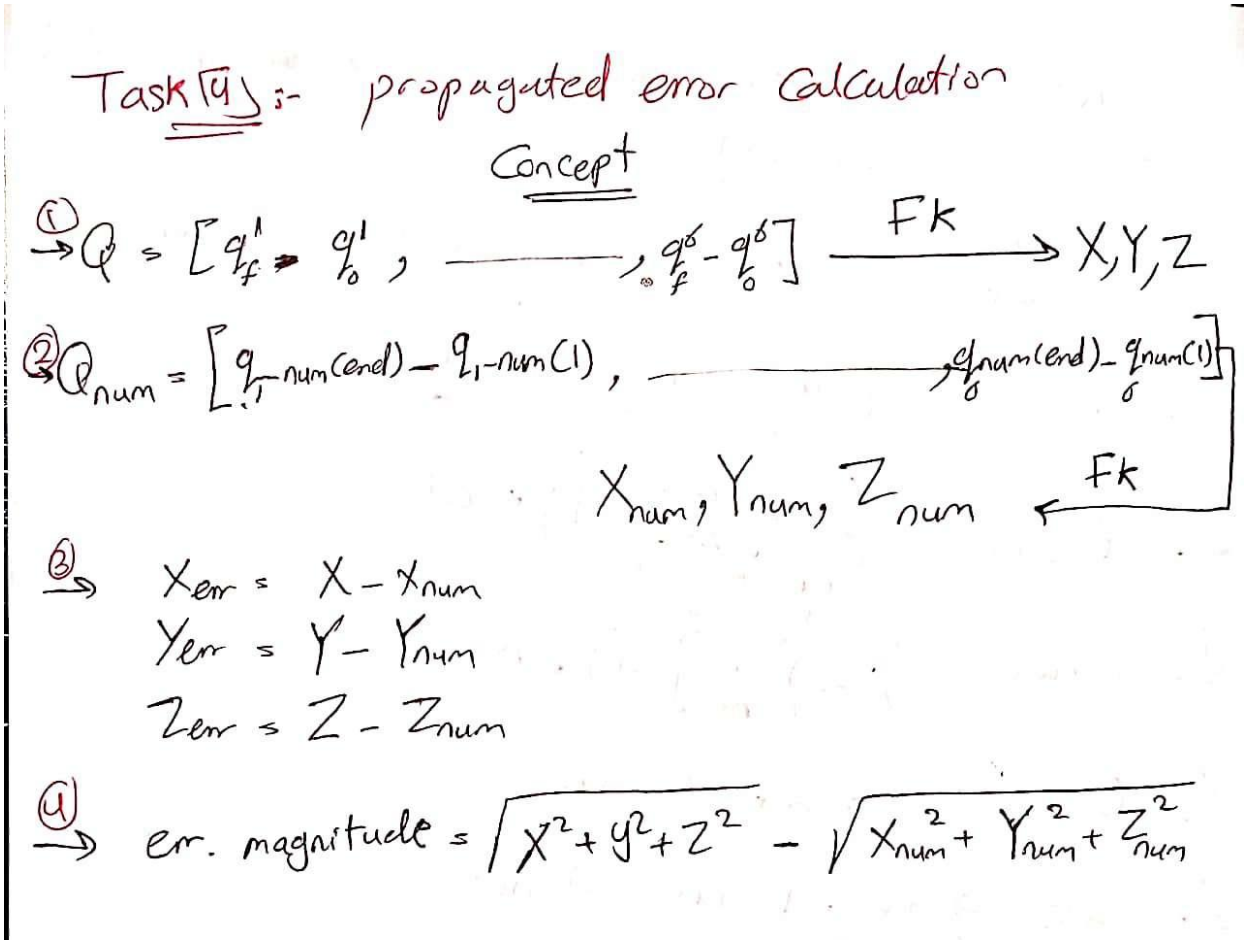
Code is performed and below are the results:

```
Task3
...Numerical Control for Synchronized Trajectories...
J1: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400
J2: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400
J3: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400
J4: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400
J5: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400
J6: Numerical>> rise time: 2.200, dwell time: 13.000, drop time: 2.200, total time: 17.400

Calculating Angles Error:
error in q1 = 0.2400 as q1 = 90.0000 and q1_num = 89.7600
error in q2 = 0.1600 as q2 = 60.0000 and q2_num = 59.8400
error in q3 = 0.1200 as q3 = 45.0000 and q3_num = 44.8800
error in q4 = 0.1067 as q4 = 60.0000 and q4_num = 59.8933
error in q5 = 0.0533 as q5 = 10.0000 and q5_num = 9.9467
error in q6 = 0.1067 as q6 = 70.0000 and q6_num = 69.8933
```

Task [4] :- propagated error calculation

Concept



Code is performed and below are the results:

```
Task4
...Propagated Error in End-effector Position using FK...
X = -0.4084, X_num = -0.4063
Y = 0.3289, Y_num = 0.3314
Z = -44.6879, Z_num = -44.5680
X_err = -0.0021, Y_err = -0.0025, Z_err = -0.1199
error magnitude = 0.1199
```

Task [5]: polynomial solution for 3 consecutive points

Concept (3rd polynomial)

① → Divide the problem into 2 trajectories problems.

② → for each trajectory: function $p=[q_0, q_f, t_0, t_f, v_0, v_f]$

$$\Rightarrow M = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}$$

$$\Rightarrow b = \begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix}$$

$$\Rightarrow a = inv(M) * b = \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$\Rightarrow q(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$$\Rightarrow \dot{q}(t) = 3a_3 t^2 + 2a_2 t + a_1$$

$$\Rightarrow \ddot{q}(t) = 6a_3 t + 2a_2$$

Code is performed and below are the results:

Task5
...Poly
J1: Tra
from po
q(t)= -
v(t)= -
a(t)= -
J2: Tra
from po
q(t)= -
v(t)= -
a(t)= -

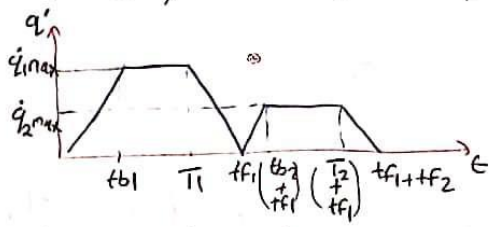
7.625

9.500

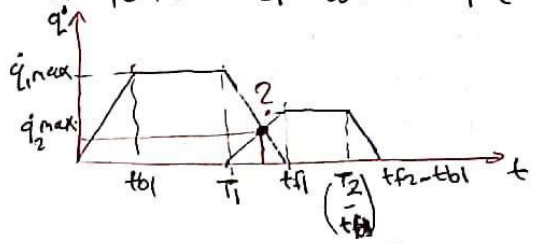
Task 5 :- Trajectory Junction

Concept (velocity profile)

① → get V_1, V_2 profile and plot them



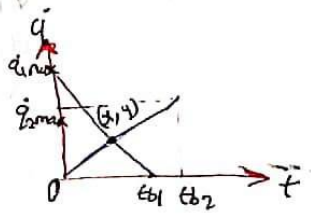
② → move V_2 to intersect with V_1 (shift by $+t_{b1}$) ←



③ → get the coordinate for the intersection point

- get equation for the straight line

$y = mx + b$



$$y_1 = m_1 x + b_1$$

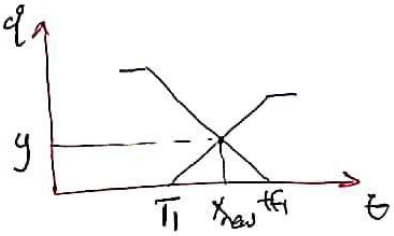
$$m_1 = \frac{-\dot{q}_{1max}}{t_{b1}}, \quad b_1 = \dot{q}_{1max} \quad \text{(i)}$$

$$y_2 = m_2 x + b_2$$

$$m_2 = \frac{\dot{q}_{max}}{t_{b2}}, \quad b_2 = 0 \quad \text{(ii)}$$

Solve (i), (ii) → (x, y) of intersection point

but you need to shift x as it is calculated at ①, so $(x_{new} = x + T_1)$



④ → update V_1, remove all values after y.

⑤ → update V_2, remove all values before y.

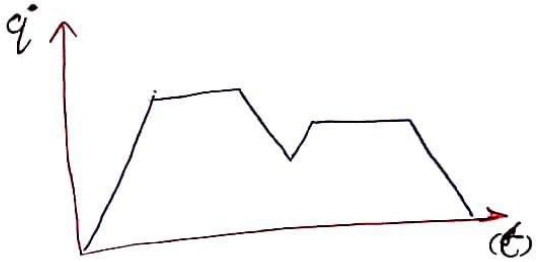
⑥ → Concatenate V_1, V_2 in V.

⑦ → adjust t_1 as linspace(0, x_{new}, size of updated V_1)

⑧ → adjust t_2 as linspace(x_{new}, $T_1 + t_{f_2}$, size of updated V_2)

⑨ → Concatenate t_1, t_2 in t.

⑩ → plot t, V



Code is performed and below are the results: