Analysis of Algorithms - Notes

Philip Warton

November 5, 2020

1 Graph Search Algorithms

We begin with this simple example of the "whatever first search". This is an algorithm that will brute force find some path from $v \to s$ for any s-reachble vertex v.

Whatever-first-search algorithm with path rememberance:

```
WFS(G, s) {
    Parent(s) = Ø
    Bag = {(s, Ø)}
    while Bag ≠ Ø {
        (v, p) = any vertex from Bag
            (remove v from Bag)
        if v is not marked {
            mark v
            Parent(v) = p
            for all (v, w) ∈ E {
                add (w, v) to Bag
            }
        }
    }
}
```

Once the bag is eventually empty, we can find the path from v to s by

$$v \to \text{parent}(v) \to \text{parent}(\text{parent}(v)) \to \cdots \to s$$

The WFS algorithm marks all vertices reachable from s.

Proof. Let s be our initial point. We use induction that is based on the shortest path s to v.

$\boxed{\text{Base Case:}}$ Our vertex $v = s$, and ShortestPathLength$(v \to s) = 0$, and WFS marks it on the first iteration.

$\boxed{\text{Inductive Step:}}$ For any point v for which the shortest path $s \leftarrow v$ is smaller than $k \in \mathbb{N}$, we assume that WFS has already marked v. Let v be a point for which its minimum distance from s is k. Then let u be the neighbor of v that lies on a shortest path from v to s. Then the length of $u \to s$ is $k - 1$, and by assumption u is marked. Since v is a neighbor of u, v will be marked as well. □

What kind of data structures would be good to use for Bag? If we use a stack, then this algorithm becomes a depth first search algorithm (DFS). If we use a queue, then we have a breadth first search (BFS) algorithm. If there is a weighted graph, one can use a priority queue based on edge weight, resulting in Dijkstra's shortest path algorithm.

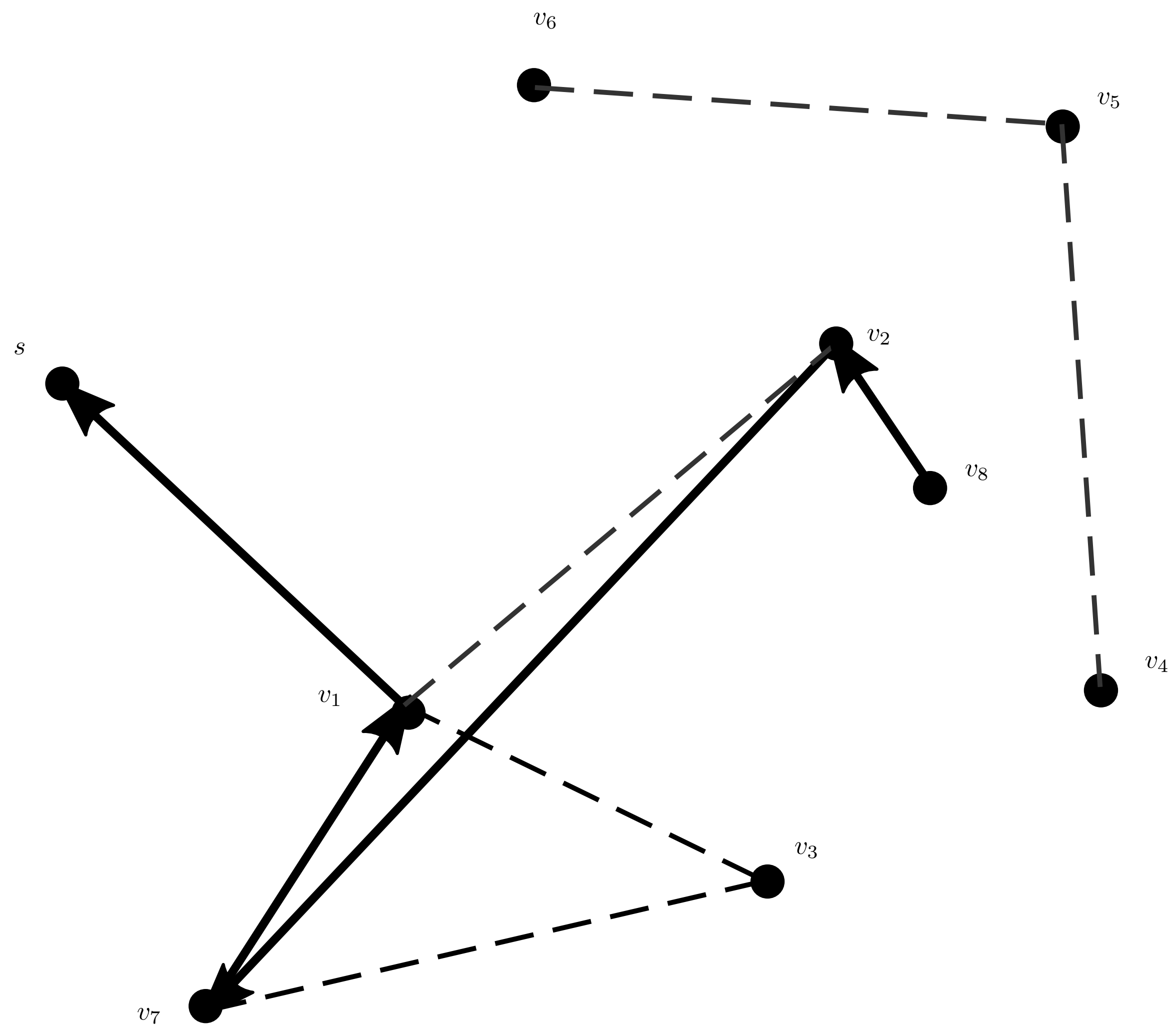

Figure 1: Whatever First Search For $v_8 \rightarrow s$