Probability 1 - Lecture Notes

Philip Warton

November 2, 2020

1 Markov Inequality

Suppose there is a distribution for which we don't know the probability mass function, and we do not know the variance, but we do know it's expectation, $E[x]$. What can we say about that probability? Can we bound it?

$\boxed{\text{Theorem: Markov Inequality}}$ If X is a random variable that takes only non-negative values, then for any $\alpha > 0$,

$$P(X \leq \alpha) \leqslant \frac{E[x]}{\alpha}$$

Proof.

$$P(X \geq \alpha) = \sum_{k:k\geq\alpha} p(\alpha) \leq \sum_{k:k\geq\alpha} \frac{k}{\alpha} p(k) = \frac{1}{\alpha} \sum_{k:k\geq\alpha} k \cdot p(k) \leq \frac{1}{\alpha} \sum_{k:k\geq 0} k \cdot p(k) = \frac{E[X]}{\alpha}$$

□

Note that this would likely work under integration for a continuous random variable.

$\boxed{\text{Theorem: Chebyshev Inequality}}$ If X is a random variable with a finite mean μ and variance, then for any $\kappa > 0$,

$$P(|X - \mu| \geq \kappa\sigma) \leq \frac{1}{\kappa^2}$$