Practice Problems

Philip Warton

November 25, 2020

3.145

If Y has a binomial distribution with n trials and probability of success p, show that teh moment-generating function for Y is

$$m(t) = (pe^t + q)^n, \quad \text{where } q = 1 - p$$

Proof. Let Y be a discrete random variable that is a binomial distribution with n trials and success probability p. Then its moment generating function is described by $m(t) = E[e^{tY}]$. This can be rewritten as $\sum_{k=0}^{n} e^{tk}\binom{n}{k}p^k q^{n-k}$. Then we wish to show that this some ends up being equal to $(pe^t + q)^n$. To show this we compute the sum as

$$m(t) = \sum_{x=0}^{n} e^{tx}\binom{n}{x}p^x q^{n-x} = \sum_{x=0}^{n}\binom{n}{x}(pe^t)^x q^{n-x} = (pe^t + q)^n$$

This last equality comes from the binomial theorem. □

3.146

Differentiate the moment-generating function in $\boxed{3.145}$ to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

We take the derivative of $m(t)$, and say that

$$m'(t) = \frac{d}{dt}(pe^t + q)^n = (pe^t + q)^{n-1}(pe^t)$$

So then our mean is equal to $m^{(1)}(0) = Thenwewishtotakethesecondderivativeofthiswhichgivesusm''(t) = \frac{d}{dt}(pe^t+q)^{n-1}(pe^t) = (pe^y + q)^{n-2}(pe^t)^2 + (pe^t + q)^{n-1}(pe^t)$