MTH 351 Homework 7

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1

Let $f(x) = \dfrac{1}{x+1}$. Points are evenly spaced such that $0 = x_1 < x_2 < \cdots < x_n = 2$. We can begin by taking the nth derivative of f.

$$\begin{aligned} f(x) &= (x+1)^{-1} \\ f'(x) &= (-1)(x+1)^{-2} \\ f''(x) &= (-1)(-2)(x+1)^{-3} \\ &\vdots \\ f^n(x) &= (-1)^n n!(x+1)^{-n-1} \end{aligned}$$

We write

$$|f(x) - P_n(x)| \leqslant \frac{1}{n}\left(\frac{b-a}{n-1}\right)^n max_{[a,b]}|f^n(x)|$$

This is equivalent to

$$|f(x) - P_n(x)| \leqslant \frac{1}{n}\left(\frac{2}{n-1}\right)^n max_{[0,2]}\frac{n!}{|(x+1)^{n+1}|}$$

This number is largest when $x = 0$, so we have

$$|f(x) - P_n(x)| \leqslant \frac{1}{n}\left(\frac{2}{n-1}\right)^n n!$$

To guarentee this quantity to be lower than 10^{-4} we must have $n \geqslant 31$.

2

See Matlab code turned in on canvas. Some plots are shown here, but not all.

Shown here are the two different graphs, one for P_n and one for Q_n. It appears that Q_n better approximates the function, with unevenly spaced points closer to the bounds a and b. I speculate that this is because the polynomial behavior is more extreme towards the end points, and this can be mitigated by having more data near said end points.

For the cosine function, both approximations look pretty spot on, so it becomes difficult to say that one is better than the other in this case.

3

(a)

For a given j, we write

$$s_j(x_j) = \frac{M_j - M_{j+1}}{2(x_j - x_{j+1})}(x^2 - x_j^2) + \frac{x_j M_{j+1} - x_{j+1}M_j}{x_j - x_{j+1}}(x - x_j) + y_j$$

Figure 1: P_n on the First Function

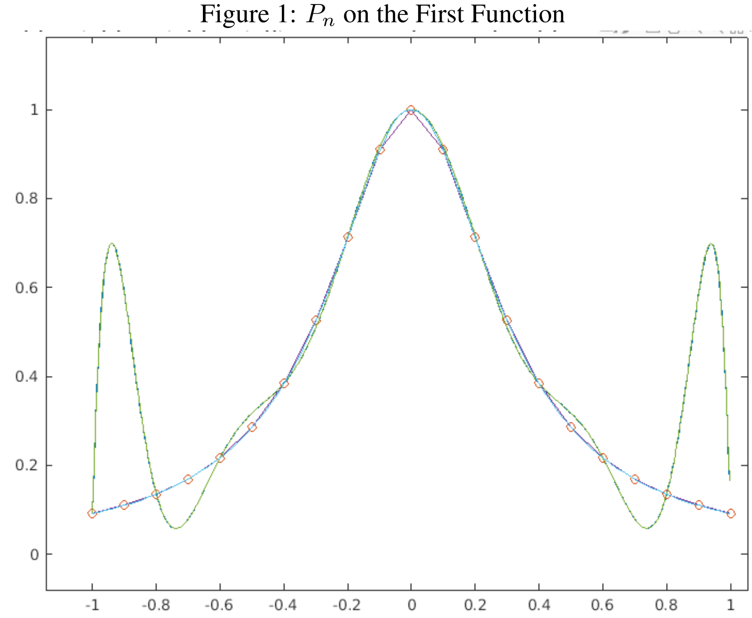


Figure 2: Q_n on the First Function

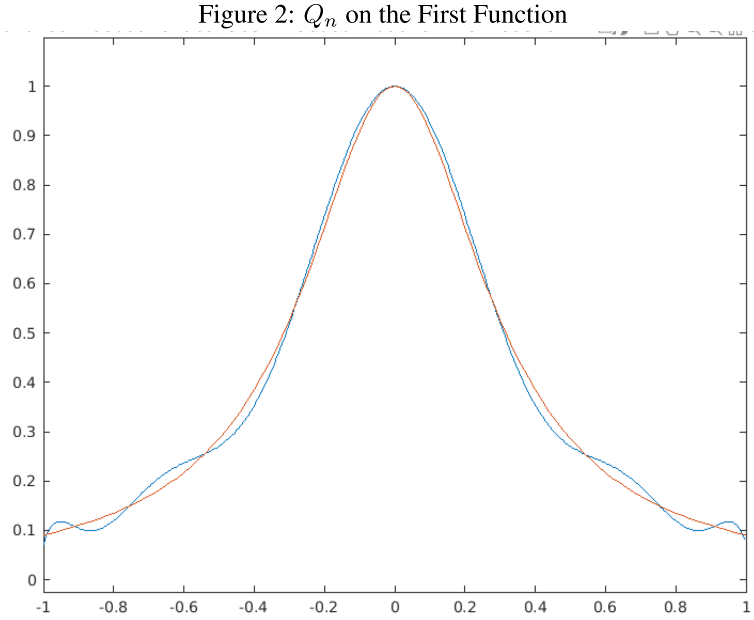


Figure 3: P_n on the Second Function

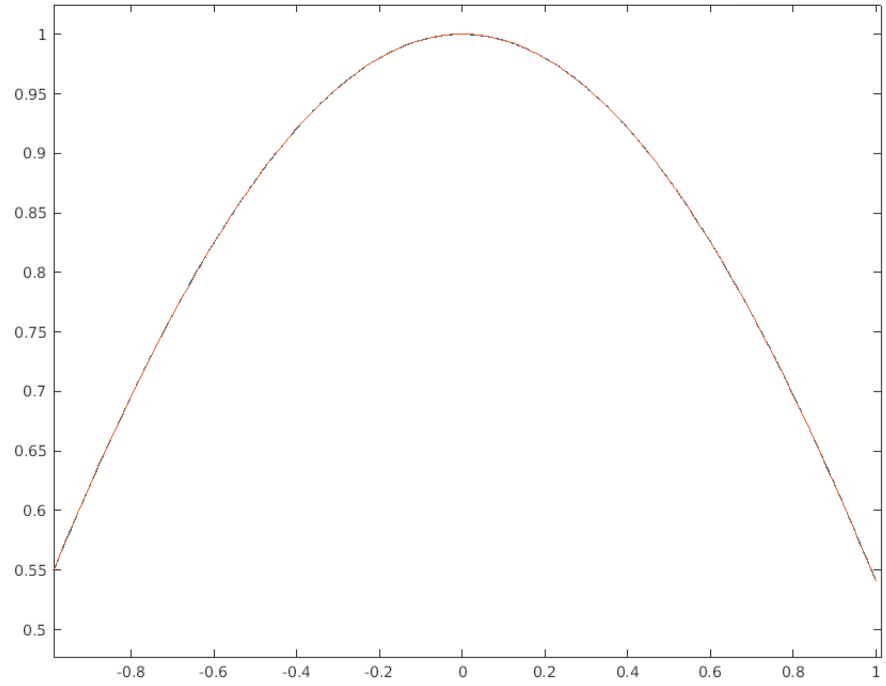


Figure 4: Q_n on the Second Function

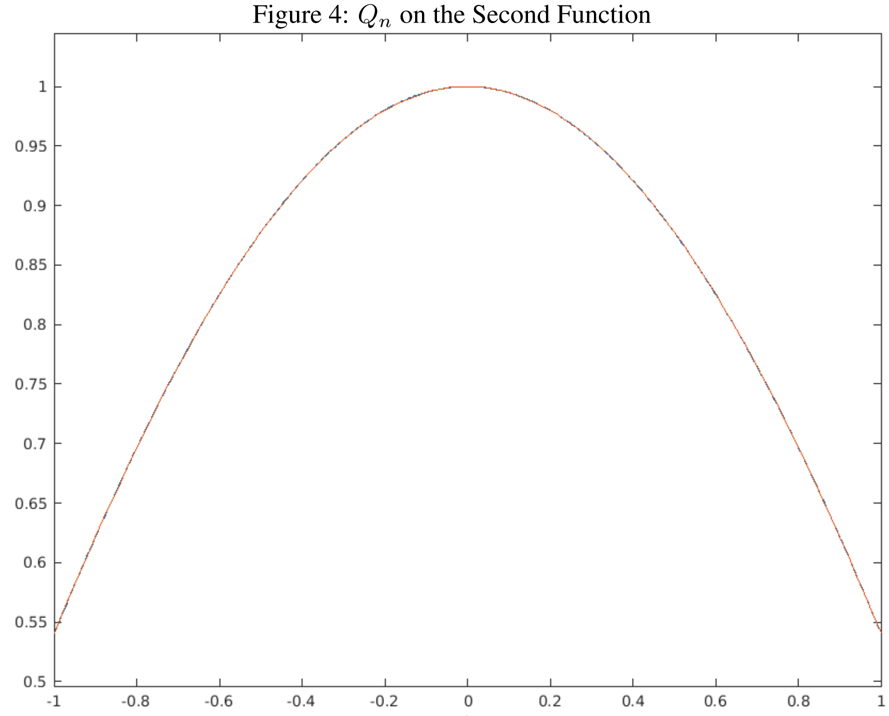
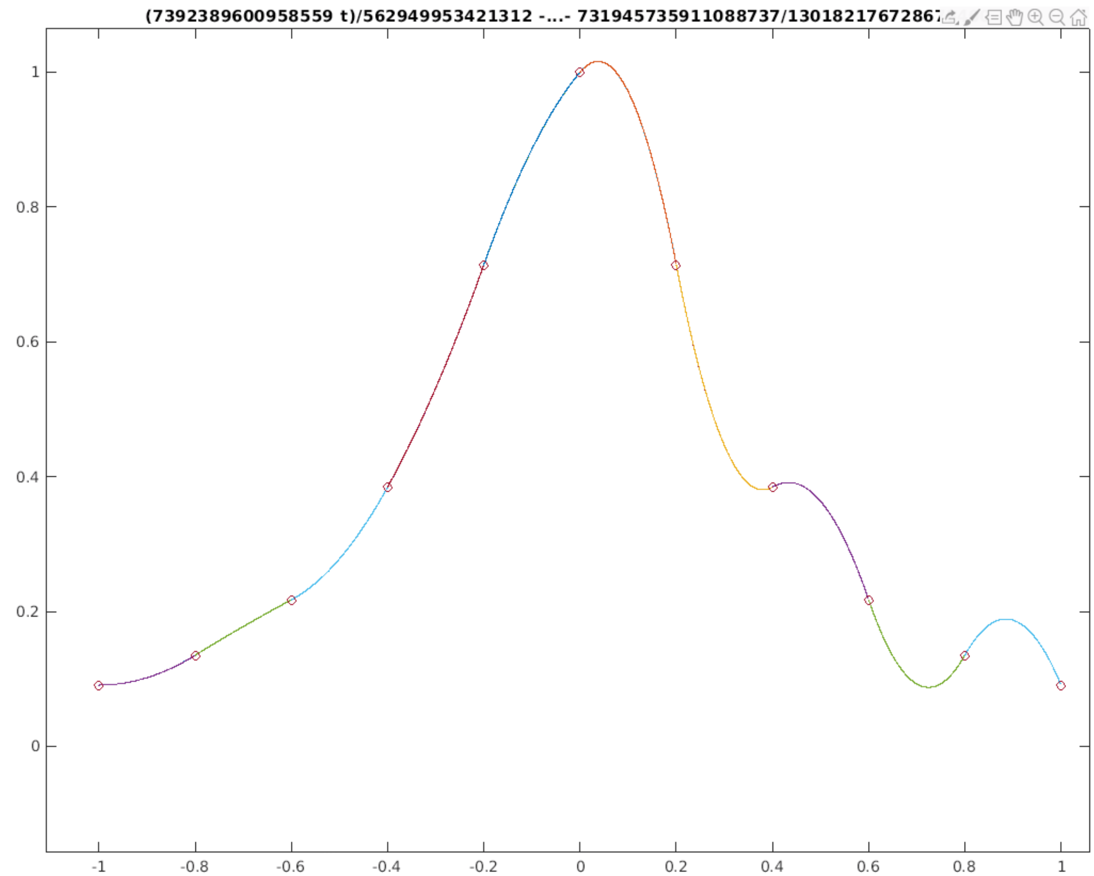


Figure 5: Quadratic Spline on the First Function



(b)

For the equation to hold, we must have that

$$(M_j + M_{j+1})\frac{x_{j+1} - x_j}{2} = y_{j+1} - y_j$$

(c)

Using Matlab, we get
$M_1 = 0$,
$M_2 = 0.4423$,
$M_3 = 0.3803$,
$M_4 = 1.2919$,
$M_5 = 2.0048$,
$M_6 = 0.8524$,
$M_7 = -3.7095$,
$M_8 = 0.4128$,
$M_9 = -2.0851$,
$M_{10} = 1.2625$,
$M_{11} = -1.7048$