MTH 483 - Cozzi, Summer 2020, Midterm

Instructions: You must submit your solutions to this exam to Gradescope no later than **8:30am on Thursday, July 23**. This exam is closed book and close notes. No technology of any kind may be used, and you are not allowed to talk to anyone else about the exam until after 8:30am on Thursday. Do all of the problems below. You may write your solutions on your own paper. Please show your work, and write as neatly as possible. Partial credit will be given where appropriate.

Please read the **Academic Integrity Statement** on page 2. You must sign below the statement, indicating that you have read and accept the statement. You will submit this page to Gradescope where indicated.

Problem 1 (0 points) Sign the **Academic Integrity Statement**

Integrity is a character-driven commitment to honesty, doing what is right, and guiding others to do what is right. Oregon State University students and faculty have a responsibility to act with integrity in all of our educational work, and that integrity enables this community of learners to interact in the spirit of trust, honesty, and fairness.

Academic misconduct, or violations of academic integrity, can fall into seven broad areas, including but not limited to: cheating; plagiarism; falsification; assisting; tampering; multiple submissions of work; and unauthorized recording and use.

It is important that you understand what student actions are defined as academic misconduct at Oregon State University. The OSU Libraries offer a tutorial on academic misconduct, and you can also refer to the OSU Student Code of Conduct and the Office of Student Conduct and Community Standard?s website for more information. More importantly, if you are unsure if something will violate our academic integrity policy, ask your professors, GTAs, academic advisors, or academic integrity officers.

Signature: ____________________________

The remainder of the exam is on pp. 3-4.

Problem 2 (20 points)

For parts (a)-(c), write the complex numbers in rectangular form.

(a) (4 points) $e^{|2-i|}$

(b) (4 points) $e^{2+\frac{9\pi}{2}i}$

(c) (4 points) $(2+2i)^{i+1}$

For parts (d) and (e), write the complex numbers in polar form.

(d) (4 points) $-4i$

(e) (4 points) $\left(\frac{1+i}{\sqrt{2}}\right)^{14}$

Problem 3 (22 points)

Sketch the sets given in (a), (b), and (c) below. Determine if each set is open, closed, or neither. **For part (b) only**, also find the isolated points and interior points of the set. Explain your reasoning (no rigorous proofs necessary).

(a) (7 points) $\{z \in \mathbb{C} : 0 < |z-1| < 2\}$

(b) (8 points) $\{z \in \mathbb{C} : |z-2| = |z-1|\}$

(c) (7 points) $\{z \in \mathbb{C} : \mathrm{Re}(z) > \mathrm{Im}(z) \textit{ and } |z-1| < 1\}$

Problem 4 (18 points)

For parts (a) and (b), determine for which values of z (if any) the function is differentiable, and for which values of z (if any) the function is holomorphic. Find the value of the function's derivative at the points where it is differentiable. Explain your reasoning.

(a) (5 points) $f(z) = z\mathrm{Log}(i)$

(b) (6 points) $f(z) = 3y + ixy$, where $z = x + iy$.

(c) (7 points) Assume $f : \mathbb{C} \to \mathbb{C}$ is a complex function, and let G denote the set of all points in $\mathbb{C}$ at which f is differentiable. Determine whether the following statements are true or false. Carefully explain your reasoning.

(i) If G is an open set, then f is holomorphic on G.

(ii) If G is not an open set, then there exists some point in G at which f is not holomorphic.

Problem 5 (10 points)

(a) (5 points) Carefully sketch the image of the line

$\{z = x + iy \in \mathbb{C} : -\infty < x < \infty,\ y = \frac{\pi}{4}\}$

under the exponential function $\exp(z)$.

(b) (5 points) Carefully sketch the image of the set

$\{z = x + iy \in \mathbb{C} : -\infty < x \leq 1,\ 0 < y < \frac{\pi}{4}\}$

under the exponential function $\exp(z)$.

Problem 6 (17 points)

Find all solutions to the following equations:

(a) (5 points) $\mathrm{Log}(z) = 7\pi i$

(b) (5 points) $\cos(z) = 0$ [Recall: $\cos(z) = \frac{1}{2}(\ \exp(iz) + \exp(-iz))$]

(c) (3 points) $\mathrm{Log}(\exp(z + \pi i)) = z$.

(d) (4 points) $\mathrm{Log}(\exp(z)) = z + \pi i$.

Problem 7 (13 points)

(a) (7 points) Using the definition of length of a piecewise smooth path γ, find the length of the top half of the circle $C[i, 12]$.

(b) (6 points) Using the definition of the integral of a complex function over a piecewise smooth path, compute

$$\int_{C[i,12]} \frac{1}{z - i}\, dz,$$

where $C[i, 12]$ is oriented counterclockwise. (Note we are integrating over the entire circle, not just the top half. Also note that you cannot just cite a HW problem. I am asking you to essentially redo the HW problem here.)