Algebraic Topology — Homework 1

Philip Warton

January 10, 2022

Problem 3.1.1

Show that $\text{Ext}(H, G)$ is a contravariant functor of H for fixed G, and a covariant functor of G for fixed H.

Problem 3.1.2

Show that the maps $G \xrightarrow{n} G$ and $H \xrightarrow{n} H$ multiplying each element by the integer n induce multiplication by n in $\text{Ext}(H, G)$.

Problem 3.1.5

Regarding a cochain $\phi \in C^1(X; G)$ as a function from paths in X to G, show that if ϕ is a cocycle, then

1. $\phi(f \cdot g) = \phi(f) + \phi(g)$,
2. ϕ takes the value 0 on constant paths,
3. $\phi(f) = \phi(g)$ if $f \simeq g$,
4. ϕ is a coboundary iff $\phi(f)$ depends only on the endpoints
5. of f, for all f.

[In particular, (a) and (c) give a map $H^1(X; G) \to \text{Hom}(\pi_1(X), G)$, which the universal coefficient theorem says is an isomorphism if X is path-connected.]

Problem 3.1.6 (a)

Directly from the definitions, compute the simplicial cohomology groups of $S^1 \times S^1$ with $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients, using the Δ-complex structure given in §2.1.

Problem 3.1.8 (c)

Show that if A is a retract of X then $H^n(X; G) \approx H^n(A; G) \oplus H^n(X, A; G)$.