General Topology - Notes

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We say that a topological space is some set X equipped with some topology τ such that the following are true (we call these the topological axioms).

$$(i)\ \emptyset \in \tau, X \in \tau$$

(ii) The collection is closed under any union.

(iii) The collection is closed under finite intersection.

The base of a topology is a collection of open subsets of X, such that every set in τ is a union of sets in B. Any base must have two properties. First, the base elements conver X, that is, $\bigcup B \supset X$. Second, let $B_1, B_2 \in B$. For every $x \in B_1 \cap B_2$ there is some element $B_3 \in B$ such that $B_3 \subset B_1 \cap B_2$. An order relation can be defined on a product space similarly to how we order a dictionary (expand upon this). If we have $X_1, <_1, X_2, <_2, \cdots, X_n$. Then define $<$ on $X_1 \times X_2 \times \cdots \times X_n$ by $X_{11} \times x_{21} \times \cdots \times x_{n1} < x_{12} \times x_{22} \times \cdots \times x_{n2}$. There is an interesting case of the 'order topology' for which these definitions may become quite useful. s