MTH 351 HW 6

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a

We can set this system of equations up as

$$P(2) = a(2^3) + b(2^2) + c(2) + d = 1 \tag{1}$$
$$P(1) = a(1^3) + b(1^2) + c(1) + d = 0 \tag{2}$$
$$P(3) = a(3^3) + b(3^2) + c(3) + d = -1 \tag{3}$$
$$P(0) = d = 2 \tag{4}$$

Then we can row reduce on the matrix to get

$$rref\left(\begin{bmatrix} 8 & 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 27 & 9 & 3 & 1 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 4.5 \\ 0 & 0 & 1 & 0 & -5.5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Therefore we can write

$$P(x) = -x^3 + \frac{9}{2}x^2 - \frac{11}{2}x + 2$$

b

For the Lagrange method, we use our Matlab code. We get

$$L_1 = \frac{-(t)(t-1)(t-3)}{2}, L_2 = \frac{(t)(t-2)(t-3)}{2}, L_3 = \frac{(t)(t-1)(t-2)}{6}, L_4 = \frac{-(0.5t-1)(t-1)(t-3)}{3}$$

Then we once we simplify we get the polynomial

$$P(x) = -x^3 + \frac{9}{2}x^2 - \frac{11}{2}x + 2$$

c

Using the diagram for the Newton method, we get

$$c_0 = 1, c_1 = 1, c_2 = \frac{-3}{2}, c_3 = -1$$

Then we have

$$P(x) = 1 + 1(x-2) - \frac{3}{2}(x-2)(x-1) - (x-2)(x-1)(x-3) = -x^3 + \frac{9}{2}x^2 - \frac{11}{2}x + 2$$

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To get the divided difference, we compute

$$f[1,2,3] = \frac{f[2,3] - f[1,2]}{3-1}$$

Then we can compute the two divided differences in the numerator

$$f[2,3] = \frac{f[3] - f[2]}{3-2} = -1$$

$$f[1,2] = \frac{f[2] - f[1]}{2-1} = -2$$

Therefore plugging those back into our original divided difference computation we get

$$f[1,2,3] = \frac{-1+2}{2} = \frac{1}{2}$$

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