Building the Perfect Sandcastle --- Using Math!

There are few greater joys than building a big epic sandcastle, but what is always disappointing is having to watch it all get washed away! So we decided to build a mathematical model to mitigate this disappointment, creating a sandcastle foundation that will last for ages (well, maybe not ages).

Generally, we want to have a shape where its base is twice it's height. This is the case because then as the shape gets eroded from all sides, they will all shrink to a single point, rather than to a line or a plane. For this to be the case, we want the base to be double the height. If we consider only these shapes, we see that the surface area to volume ratio plays a crucial role in which shapes last the longest.

The worst of these shapes is the cuboid, which is the square extruded to half the base length. This shape will decay quickly, so don't use it if you want to make a long-lasting sand-castle. The next worst one is the square based pyramid with 45 degree angles all around the base. This shape is much better than the cuboid, but it still erodes pretty fast.

If you want the best shape, than we must optimize this surface area to volume ratio, not include the parts that touch the ground. Because of this, the best shape will be the semi-sphere! This shape is very long lasting under uniform erosion because of its very low surface area to volume ratio, and because its base is twice the length of its height. Given any shape that's twice as wide as it is tall, we can use the following line to determine how the long it will take to erode:

Time to erode = ((surface area : volume ratio) – 8.8) / (-.64)

And that is all we have to do. So go build some sandcastles based on domes, and see how long they last!