Differential Geometry 1 Notes - Chapter 11

Philip Warton

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1 Differentials and Integrands

We will write differentiation as $f \longmapsto df$, where all our typical rules of derivatives apply. That is,

$$\begin{aligned} d(u^n) &= nu^{n-1}du, && \text{(power rule)} \\ d(e^u) &= e^u du, && \text{(exponential)} \\ d(\sin u) &= \cos u du, && \text{(trigonemtric derivatives)} \\ d(uv) &= udv + vdu, && \text{(product rule)} \\ d(u+cv) &= du + cdv, && \text{(distributive property)} \end{aligned}$$

Finally we write the chain rule as

$$\frac{df}{du} = \frac{df}{dq}\frac{dq}{du}$$

Alternatively

$$df = \frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial q}dq$$

2 Differential Forms

Consider $\mathbb{R}^n$ with coordinates $\{x^i\}$. Consider the formal objects $\{dx^i\}$. Let V be the almost vector space,

$$V = \langle dx^i \rangle = \{a_i dx^i\}$$