MTH 483 - Cozzi, Summer 2020, Final Exam

Instructions: You must submit your solutions to this exam to Gradescope no later than **7:00pm on Saturday, August 15**. You are allowed to use one side of one standard (8.5 x 11 inch) piece of paper, and nothing else (no electronic devices, internet, books, etc.). On this piece of paper, you may write up to 15 statements. These statements may include theorems, definitions, lemmas, propositions, or corollaries (a single theorem/definition/proposition, etc., no matter how long, counts as one statement). Your piece of paper cannot contain any worked out examples, including power series for specific functions. If you don't follow these rules, you may lose points on the exam. You will submit your piece of paper with your exam on Gradescope where indicated. You are not allowed to talk to anyone else about the exam until after 7:00pm on Saturday. Do all of the problems below. You may write your solutions on your own paper. Please show your work, and write as neatly as possible. Partial credit will be given where appropriate.

Please read the **Academic Integrity Statement** on page 2. You must sign below the statement (or just sign a separate piece of paper), indicating that you have read and accept the statement. You will submit this page to Gradescope where indicated.

Problem 1 (0 points) Sign the **Academic Integrity Statement**

Integrity is a character-driven commitment to honesty, doing what is right, and guiding others to do what is right. Oregon State University students and faculty have a responsibility to act with integrity in all of our educational work, and that integrity enables this community of learners to interact in the spirit of trust, honesty, and fairness.

Academic misconduct, or violations of academic integrity, can fall into seven broad areas, including but not limited to: cheating; plagiarism; falsification; assisting; tampering; multiple submissions of work; and unauthorized recording and use.

It is important that you understand what student actions are defined as academic misconduct at Oregon State University. The OSU Libraries offer a tutorial on academic misconduct, and you can also refer to the OSU Student Code of Conduct and the Office of Student Conduct and Community Standard?s website for more information. More importantly, if you are unsure if something will violate our academic integrity policy, ask your professors, GTAs, academic advisors, or academic integrity officers.

Signature: ____________________________

The remainder of the exam is on pp. 3-4.

Problem 2 (12 points)

(a) (4 points) Write $\left(\frac{-1+i}{2\sqrt{2}}\right)^6$ in rectangular form.

(b) (8 points) Write $(1+i)^{2\pi i}$ in both polar form and rectangular form.

Problem 3 (13 points)

For each of the functions below, determine for which values of z (if any) the function is differentiable, and for which values of z (if any) the function is holomorphic. Explain your reasoning.

(a) (8 points) $f(z) = 2xy + i(x^2 + y^2)$, where $z = x + iy$

(b) (5 points) $f(z) = \frac{1}{\mathrm{Log}(z-3)}$

Problem 4 (15 points)

(a) (5 points) Evaluate $\int_\gamma (z^3 + 4z^2 + 1)\, dz$, where γ is the line segment from 1 to i.

(b) (5 points) Evaluate $\int_\gamma \frac{1}{z}\, dz$, where $\gamma(t) = e\cos t + i \sin t$ for $0 \leq t \leq \pi/2$. Express your answer in rectangular form. (Be sure to carefully justify your calculations here.)

(c) (5 points) Carefully explain why the function $f(z) = \frac{1}{z}$ has no antiderivative in $\mathbb{C}\backslash\{0\}$. (Hint: If it did have an antiderivative in $\mathbb{C}\backslash\{0\}$, then what would be true?)

Problem 5 (18 points)

Use your method of choice to compute the values of the following integrals.

(a) (6 points) $\int_\gamma \frac{\cos z}{(z^2+1)}\, dz$, where γ is the rectangle oriented counterclockwise with corners at $3+3i$, $3-3i$, $-3+3i$, and $-3-3i$.

(b) (6 points) $\int_\gamma e^{z^2}$, where γ is a triangle with vertices at -1, 1, and i, oriented counterclockwise.

(c) (6 points) $\int_{C[0,2]} \frac{e^{\pi z}}{(z-1)^{16}}\, dz$.

Problem 6 (22 points)

For parts (a), (b), and (c) use residues to evaluate the integrals.

(a) (6 points) $\int_{C[0,3]} \frac{z^4+7z+2}{z^3+z}\,dz$

(b) (5 points) $\int_{C[0,1/2]} \frac{z}{(e^z-1)^2}\,dz$.

(c) (6 points) $\int_{C[0,2]} \frac{\cos z}{\left(z^2-\frac{\pi z}{2}\right)(z-3i)}$

(d) (5 points) Suppose f is a complex function which is holomorphic everywhere on $\mathbb{C}$ except for a single point z_0, at which f has a pole of order 2. Explain why f' (which exists everywhere but at z_0), must also have a pole at z_0. What is the order of the pole of f' at z_0?

Problem 7 (15 points)

For parts (a) and (b), determine a function which has the given power series or Laurent series. In each problem, use the coefficients c_k to determine where the series converges.

(a) (5 points) $\sum_{k\geq 0} k(k-1)(z-2i)^{k-2}$

(b) (4 points) $\sum_{k\geq 0} \frac{e^i}{k!}(z-1)^{-k}$.

(c) (6 points) Find a Laurent series for $\frac{1}{2-3z+z^2}$ centered at $z=1$, and determine its annulus of convergence.

Problem 8 (5 points)

Suppose $f(z) = u(z) + iv(z)$ is an entire function which satisfies the property that u is bounded; that is, there exists $M > 0$ such that $|u(z)| \leq M$ for all $z \in \mathbb{C}$. Carefully explain why f must be a constant function. (Hint: Consider the function $\exp(f(z))$.)