# Systems of ODE's - Homework 4

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## Problem 1

Consider the one-parameter family of linear systems given by

$$X' = \begin{bmatrix} a & \sqrt{2} + (a/2) \\ \sqrt{2} - (a/2) & 0 \end{bmatrix} X \tag{1}$$

### a)

Sketch the path traced out by this family of linear systems in the trace-determinant plane as $a$ varies.

### b)

Discuss any bifurcations that occur along this path and compute the corresponding values of $a$.

We can compute the trace and determinant of this planar system by

$$T = a + 0 \qquad D = (a \cdot 0) - (\sqrt{2} + (a/2))(\sqrt{2} - (a/2)) \tag{2}$$

$$= a - (2 - a^2/4) \tag{3}$$

Then we have $\Delta = T^2 - 4D = a^2 - 4(a - (2 - a^2/4))) = 4(2 - a)$. So we have $T$ positive if and only if $a$ is positive. Then $D$ is positive when

$$2 < a(4 + a)$$