MTH 351 Homework 8

Philip Warton

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1

We want to solve the integral

$$I = \int_0^1 \frac{1}{1+4x^2} dx$$

To solve this, we use u substitution. Let $u = 2x$, then if we differentiate both sides we have $du = 2dx$. We can now rewrite the integral as

$$I = \frac{1}{2}\int_0^2 \frac{1}{1+u^2} du = \frac{1}{2}\left(\tan^{-1}(u)\Big|_0^2\right) = \frac{\tan^{-1}(2) - \tan^{-1}(0)}{2} = \frac{\tan^{-1}(2)}{2}$$

2

Let $f = \frac{1}{1+4x^2}$. We want to write the formula for the Riemann sum of I for the left, right, middle, and trapezoidal rules using sigma notation.

$$L_n = \sum_{k=0}^{n-1} \frac{f(x_k)}{n}$$

$$R_n = \sum_{k=1}^{n} \frac{f(x_k)}{n}$$

$$M_n = \sum_{k=1}^{n} \frac{f\left(\dfrac{x_k + x_{k-1}}{2}\right)}{n}$$

$$T_n = \sum_{k=1}^{n} \frac{f(x_{k-1}) + f(x_k)}{2n}$$

3

We can start by writing

$$x_0 = 0, \quad x_1 = 0.25, \quad x_2 = 0.5, \quad x_3 = 0.75, \quad x_4 = 1$$

Then we can take the image of each value x, which gives us

$$f(x_0) = 1, \quad f(x_1) = 0.8, \quad f(x_2) = 0.5, \quad f(x_3) = \frac{4}{13} = 0.3076923077, \quad f(x_4) = 0.2$$

Computing each sum we get

$$L_4 = 0.651923076$$

$$R_4 = 0.4519230769$$

$$M_4 = 0.5543935547$$

$$T_4 = 0.5519230769$$

4

See Matlab code attached on Canvas.

5

We begin by getting K and K_0. The derivatives $f'(x)$ and $f''(x)$ are

$$f'(x) = -32x^3 - 8x \qquad f''(x) = -96x^2 - 8$$

Then,

$$K = 1.3 \qquad K_0 = 8$$

With these values, our errors are bounded by

$$e_n^L, e_n^R \leqslant \frac{1.3(1)^2}{n} = \frac{1.3}{n} \qquad e_n^M \leqslant \frac{8(1)^3}{24n^2} = \frac{1}{3n^2} \qquad e_n^T \leqslant \frac{8}{12n^2} = \frac{2}{3n^2}$$

Let $\epsilon = 0.0001$. We wish to compute a value n such that the error is bounded by ϵ.

$$e_n^L, e_n^R \leqslant \epsilon \Rightarrow n > 13000, \qquad e_n^M \leqslant \epsilon \Rightarrow n > 57, \qquad e_n^T \leqslant \epsilon \Rightarrow 81$$

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%% MTH 351 HW 8 -- PHILIP WARTON

%% - Question 4

f = @(t) 1 ./ (1 + 4 .* t.^2);

n = 4;
%n = 8;
%n = 16;
%n = 32;
%n = 64;

x = [0:(1/n):1]
y = f(x)

% - L
L = 0;
for i=1:n
    L = L + (y(i) / n);
end
L

% - R
R = 0;
for i=1:n
    R = R + (y(i + 1) / n);
end
R

% - M
M = 0;
for i=1:n
    M = M + (f(((x(i) + x(i+1) / 2))) / n);
end
M

% - T
T = 0;
for i=1:n
    T = T + ((f(x(i)) + f(x(i + 1))) / (2 * n));
end
T
```