Momentum: mass · velocity

$\vec{p} = m\vec{v}$ units: kg·m/s

Types of collisions

Type	Momentum	kinetic energy
Total Elastic collision Bounced off	Conserved	Conserved
Partially Elastic Collision Bounce off	Conserved	Some E_k is lost
Total inelastic collision Stick together	Conserved	E_k is lost

Proof of conservation of momentum

Action: Force of [A] on [B]

Reaction: Force of [B] on [A]

$$\vec{F}_A = -\vec{F}_B$$

$$m_A a_A = m_B a_B$$

$$m_A\left(\frac{V_A' - V_A}{\Delta t_A}\right) = m_B\left(\frac{V_B' - V_B}{\Delta t_B}\right) \qquad \Delta t_A = \Delta t_B$$

$$m_A V_A' - m_A V_A = -m_B V_B' + m_B V_B$$

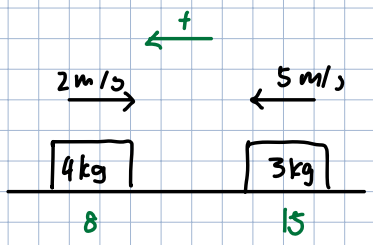
$$m_A V_A' + m_B V_B' = m_A V_A + m_B V_B$$

$$\vec{P}_A' + \vec{P}_B' = \vec{P}_A + \vec{P}_B$$

$$\vec{P}_T' = \vec{P}_T \qquad \therefore \text{Momentum is conserved}$$

TEC Example 1

Find final velocity of each cart



Conservation of momentum

$\vec{P}_T = \vec{P}_T'$

$m_4 v_4 + m_3 v_3 = m_4 v_4' + m_3 v_3'$

$4(-2) + 3(5) = 4v_4' + 3v_3'$

$7 = 4v_4' + 3v_3'$

① $v_4' = \frac{7 - 3v_3'}{4}$

sub -3m/s

$v_4' = \frac{7 - 3(-3)}{4}$

$v_4' = 4 m/s [L]$

Conservation of kinetic energy

$E_k = E_k'$

$\frac{1}{2} m_4 v_4^2 + \frac{1}{2} m_3 v_3^2 = \frac{1}{2} m_4 v_4'^2 + \frac{1}{2} m_3 v_3'^2$

$4(-2)^2 + 3(5)^2 = 4(v_4')^2 + 3(v_3')^2$

$91 = 4(v_4')^2 + 3(v_3')^2$ ② sub ①

$91 = 4\frac{(7 - 3v_3')^2}{4^2} + 3(v_3')^2$

$364 = (49 - 42v_3' + 9v_3'^2) + 12(v_3')^2$

$21v_3'^2 - 42v_3' - 315 = 0$

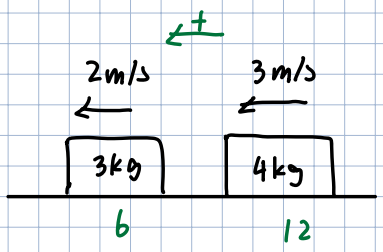
5	21	-42	-315
	↓	105	315
	21	63	0 ← Remainder is always 0

$21v_3' + 63 = 0$

$v_3' = -3 m/s [L]$

1D TEC Ex 2

Find final velocities



$\vec{P}$

$3(2)+4(3)=3v_3'+4v_4'$

$18=3v_3'+4v_4'$

$v_3'=\frac{18-4v_4'}{3}$ ①

sub v_4'

$v_3'=\frac{18-4(2.14)}{3}$

$v_3'=3.14\ m/s\ [L]$

Ek

$3(2)^2+4(3)^2=3v_3'^2+4v_4'^2$

$48=3v_3'^2+4v_4'^2$ ② sub ①

$48=3\frac{(18-4v_4')^2}{3^2}+4v_4'^2$

$144=324-144v_4'+16v_4'^2+12v_4'^2$

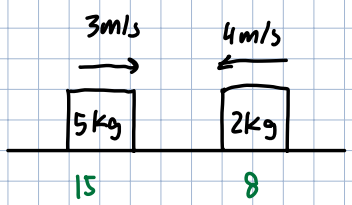
$28v_4'^2-144v_4'+180$

3	28	−144	180
	↓	84	−180
	28	−60	0

$28v_4'-60=0$

$v_4'=2.14\ m/s\ [L]$

You try:



P

$5(3)+2(-4)=5v_5'+2v_2'$

$7=5v_5'+2v_2'$

$v_5'=\frac{7-2v_2'}{5}$ sub

$v_5'=\frac{7-2(6)}{5}$

$v_5'=-1\ m/s$

Ek

$5(3)^2+2(-4)^2=5v_5'^2+2v_2'^2$

$77=5v_5'^2+2v_2'^2$

$77=5\frac{(7-2v_2')^2}{5^2}+2v_2'^2$

$385=49-28v_2'+4v_2'^2+10v_2'^2$

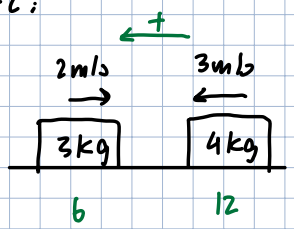
$14v_2'^2-28v_2'-336=0$

−4	14	−28	−336
	↓	−56	336
	14	−84	0

$14v_2'-84=0$

$v_2'=6\ m/s$

You try #2:



P

$3(-2)+4(3) = 3v_3' + 4v_4'$

$6 = 3v_3' + 4v_4'$

$v_3' = \frac{6-4v_4'}{3}$

$v_3' = \frac{6-4(-1.29)}{3}$

$v_3' = 3.71\ m/s$

EK

$3(-2)^2 + 4(3)^2 = 3v_3'^2 + 4v_4'^2$

$48 = 3v_3'^2 + 4v_4'^2$

$48 = 3\frac{(6-4v_4')^2}{3^2} + 4v_4'^2$

$144 = 36 - 48v_4' + 16v_4'^2 + 12v_4'^2$

$28v_4'^2 - 48v_4' - 108 = 0$

3	28	-48	-108
	↓	84	108
	28	36	0

$28v_4' + 36 = 0$

$v_4' = -1.29\ m/s$