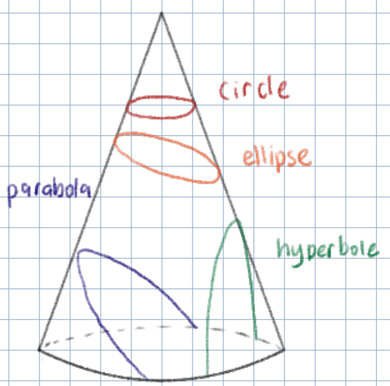
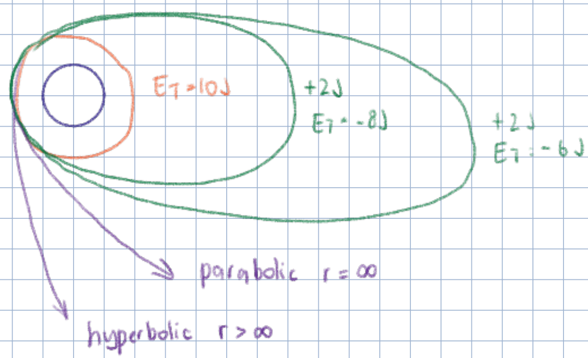
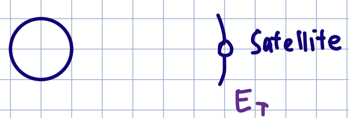




Circular Orbits

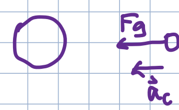
Formula Development: How much energy does an object have in a circular orbit?



$$E_T = E_g + E_k$$
$$= \frac{-GMm}{r} + \frac{1}{2}mv^2$$
$$= \frac{-GMm}{r} + \frac{1}{2}m\left(\frac{GM}{r}\right)$$
$$= \frac{GMm}{r}\left(-1+\frac{1}{2}\right)$$
$$= -\frac{1}{2}\frac{GMm}{r}$$
$$= \frac{1}{2}E_g$$

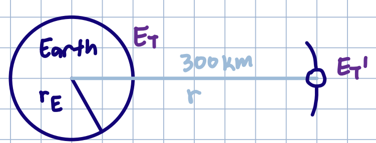
sub

v^2



$$\vec{F}_{net} = \vec{F}_g$$
$$m_s a_c = \frac{GMm}{r^2}$$
$$\frac{v^2}{r} = \frac{GM}{r^2}$$
$$v^2 = \frac{GM}{r}$$

Example 1: A 500kg satellite is to be put into a 300km high circular orbit. How much energy must be added to get it from the ground into orbit?



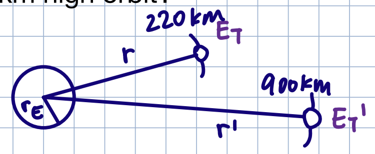
$$G = 6.67\times10^{-11}\,kgm^3/s^2$$
$$M = 5.98\times10^{24}\,kg$$
$$r_E = 6.38\times10^{6}\,m$$
$$r = r_E + 300000 = 6.68\times10^{6}\,m$$

$$E_T = E_T'$$
$$E_g + W_{in} = \frac{1}{2}E_g'$$
$$W_{in} = \frac{1}{2}E_g' - E_g$$
$$= \frac{1}{2}\left(\frac{-GMm}{r}\right) - \left(\frac{-GMm}{r_E}\right)$$
$$= GMm\left(\frac{1}{r_E} - \frac{1}{2r}\right)$$
$$= (6.67\times10^{-11})(5.98\times10^{24})(500)\left(\frac{1}{6.38\times10^{6}} - \frac{1}{2(6.68\times10^{6})}\right)$$
$$= 1.63\times10^{10}\,J$$

$= 16.3 \times 10^9 J \quad = 16.36 GJ$

$m = 500 kg$

Example 2: A 2T probe is in a 220 km high circular orbit. How much energy does it take to get it into a 900km high orbit?



$E_T = E_T'$

$\frac{1}{2}E_g + W_{in} = \frac{1}{2}E_g'$

$W_{in} = \frac{1}{2}(E_g' - E_g)$

$= \frac{1}{2}\left(\frac{-GMm}{r'} - \left(\frac{-GMm}{r}\right)\right)$

$= \frac{GMm}{2}\left(\frac{1}{r} - \frac{1}{r'}\right)$

$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(2000)}{2}\left(\frac{1}{6.6\times10^6} - \frac{1}{7.28\times10^6}\right)$

$= 5.64 GJ$

$G = 6.67\times10^{-11} m^3/kg s^2$

$M = 5.98\times10^{24} kg$

$r_E = 6.38\times10^6 m$

$r = r_E + 220000 m$

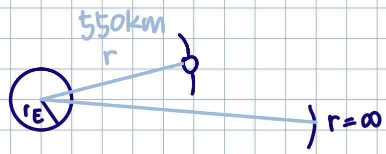
$= 6.6\times10^6 m$

$r' = r_E + 900000 m$

$= 7.28\times10^6 m$

$m = 2000 kg$

Example 3: A 1.5T probe is in a 550 km high circular orbit. How much energy is needed to get the probe to just escape the Earth?



$G = 6.67\times10^{-11}\ m^3/kgs^2$

$M_E = 5.98\times10^{24}\ kg$

$r_E = 6.38\times10^6\ m$

$r = r_E + 550000 = 6930000\ m$

$E_T = E_T'$

$\frac{1}{2}E_g + W_{in} = 0$

$\frac{1}{2}\left(\frac{-GMm}{r}\right) + W_{in} = 0$

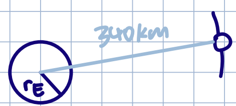
$W_{in} = -\frac{1}{2}\left(\frac{-GMm}{r}\right)$

$W_{in} = \frac{GMm}{2r}$

$W_{in} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(1500)}{2(6930000)}$

$W_{in} = 43.2\ GJ$

Example 4: A 4T spaceship is in a 340km high circular orbit. How much energy is needed so it ends up with a velocity of 2km/s in deep space?



$m = 4000\ kg$

$G = 6.67\times10^{-11}\ m^3/kgs^2$

$M_E = 5.98\times10^{24}\ kg$

$r_E = 6.38\times10^6\ m$

$r = r_E + 340000 = 6720000\ m$

$E_T = E_T'$

$\frac{1}{2}E_g + W_{in} = E_K$

$\frac{1}{2}\left(\frac{-GMm}{r}\right) + W_{in} = \frac{1}{2}mv^2$

$W_{in} = \frac{1}{2}mv^2 - \frac{1}{2}\left(\frac{-GMm}{r}\right)$

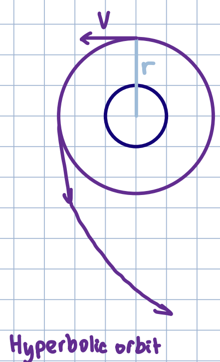
$W_{in} = \frac{1}{2}mv^2 + \frac{GMm}{2r}$

$W_{in} = \frac{1}{2}(4000)(2000)^2 + \frac{(6.67\times10^{-11})(5.98\times10^{24})(4000)}{2(6720000)}$

$W_{in} = 126.7\ GJ$

Example 5:

A 3T probe is in a 400km high circular orbit. It has a rocket engine that has an ISP of 4000 and uses 20kg of fuel a minute. How long must you fire the engine so it ends up in deep space with a velocity of 1km/s



ISP

$\Delta t = ISP = 4000s$

$\vec{F} = 9.8N$

$\Delta m = 1kg$

$\vec{F}\Delta t = \Delta m V_{exh}$

$9.8(4000) = 1 V_{exh}$

$V_{exh} = 39200 m/s$

Thrust

$V_{exh} = 39200 m/s$

$\Delta m = 20 kg$

$\Delta t = 60s$

$F\Delta t = \Delta m V_{exh}$

$F = \frac{20(39200)}{60}$

$\vec{F} = 13067N$

ΔV

$\vec{F} = 13067N$

$m = 3000kg$

$\Delta V = ?$

$\Delta t = \frac{m\Delta V}{F}$

$= \frac{3000(2323)}{13067}$

$= 740s$

① Find ΔV

$\Delta V = V_H - V_c$

② Find V_c

$\vec{F}_{net} = F_g$

$\cancel{m_p} a_c = \frac{GM\cancel{m_p}}{r^2}$

$\frac{v^2}{r} = \frac{GM}{r^2}$

$V_c = \sqrt{\frac{GM}{r}}$

$V_c = 7670 m/s$

③ Find V_H

$E_T = E_T'$

$E_g + E_k = E_k'$

$\frac{-GM\cancel{m}}{r} + \frac{1}{2}\cancel{m}V_H^2 = \frac{1}{2}\cancel{m}v'^2$

$V_H = \sqrt{v'^2 + \frac{2GM}{r}}$

$V_H = 10893 m/s$

④ Finally find ΔV

$\Delta V = V_H - V_c$

$= 10893 - 7670$

$= 3223 m/s$