Orbits are ellipses

Ellipse defined by focii

Amount of flattening : eccentricity

The longest axis : major axis

shortest axis : minor axis

1: Each planet's orbit about the sun is an ellipse

The sun's center is always located at one focus of the orbital ellipse

2: The imaginary line joining a planet and the Sun sweeps equal areas of space

during equal time intervals as the planet orbits.

Perihelion Aphelion

3: The squares of the orbital periods of the planets are directly proportional to the cubes

of the semi-major axes of their orbits.

Kepler's Third Law

$T^2 \propto R^3$

$\underline{T^2 = KR^3}$

T: Orbital period

R: Orbital radius

K is the Kepler's Constant which is the same for all sattelites of a central body

Ex: K is the same for all planets in the Solar System because they all orbit the sun → central mass.

Example 1:

Mercury orbits the Sun in 88 days.

Find orbital radius

Earth: T_E = 1 year

R_E = 1 AU

$K = \frac{T^2}{R^3} = \frac{1yr^2}{1AU^3}$

Mercury: T_m = 88 days = 0.241 y

$K = \frac{1yr^2}{1AU^3}$

$R_m = \sqrt[3]{\frac{T_m^2}{K}} = \sqrt[3]{\frac{0.241^2}{1}} \rightarrow \sqrt[3]{\frac{\cancel{yr^2}}{\cancel{yr^2}/AU^3}}$

$= 0.387$ AU

Ex 2:

Saturn has orbital radius of 9AU.

Find one trip around the sun

Ts? $R_s = 9AU$

$T = \sqrt{KR_s^3} = \sqrt{1(9)^3}$

$= 27\ yr \times \frac{365\ d}{yr} = 9855$ days

Newton proves Kepler's Laws

What factors affect the force of gravity?

$F_g \propto \frac{mM}{R^2}$

$F_g \propto \frac{GmM}{R^2}$

$G = \frac{F_g R^2}{mm} = \frac{(N)(m^2)}{(kg)^2}$

M = central mass

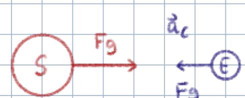
m = smaller mass

R = orbital radius

G = universal gravitational constant

$6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Planetary Motion



$\vec{F}_{Net} = \vec{F}_g$

$m_E \vec{a}_c = \frac{Gm_E M_s}{R^2}$

$\frac{4\pi^2 R}{T^2} = \frac{GM_s}{R^2}$

$T^2 = \frac{4\pi^2 R^3}{GM_s}$ K Kepler's constant

g = 9.81 (earth)

$F_g = \frac{Gm}{R^2} m$

Kepler's Law Example 3

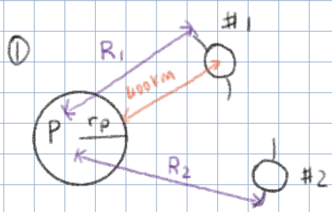
A planet has a circumference of 24000km. A moon is orbiting at an altitude of 400km and completes 2 revolutions in 3 hours. A 2nd moon orbits 2 times in a week.

Find:

a) The closest the two moons can get

b) The density of the planet

①



$C = 2\pi r_p = 24000 km$

$r_p = 3820 km$

#1

$T_1 = \frac{3hr}{2rev} = 1.5 hr/rev$

$R_1 = r_p + alt = 3820 + 400$

$= 4220 km$

$K = \frac{T_1^2}{R_1^3} = \frac{1.5^2}{4220^3}$

$= 2.99 \times 10^{-11} \left(\frac{hr^2}{km^3}\right)$

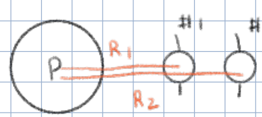
#2

$T_2 = \frac{7 days}{2 rev} \times \frac{24h}{1 day}$

$= 84 hr/rev$

$R_2 = \sqrt[3]{\frac{T_2^2}{K}} = \sqrt[3]{\frac{(84)^2}{2.99 \times 10^{-11}}}$

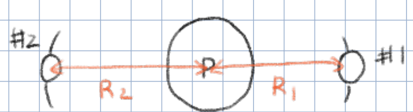
$= 61769 km$



Closest distance : $R_2 - R_1 = 61769 - 4220$

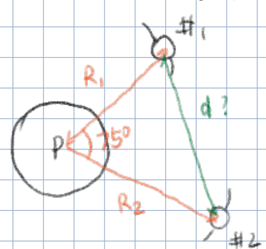
$= 57549 km$

Variation to a) #1 : Furthest distance



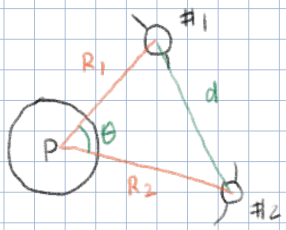
$dist = R_1 + R_2$

Variation to a) #2 : if 75°,



$d = \sqrt{R_1^2 + R_2^2 - 2R_1R_2\cos\theta}$

Variation to a) #3: if they are 59000km apart, θ?



$$\cos\theta = \frac{d^2 - R_1^2 - R_2^2}{-2R_1R_2}$$

b) Density of planet

$$D = \frac{M_p}{V_p}$$

$$= \frac{1.53\times10^{24}}{2.33\times10^{20}}$$

$$= 6567\ \frac{kg}{m^3}$$

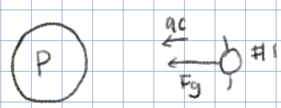
$r_p = 3820km$

$$V_p = \frac{4}{3}\pi r_p^3$$

$$= \frac{4}{3}(\pi)(3820)^2$$

$$= 2.33\times10^{11}\ km^3$$

$$= 2.33\times10^{20}\ m^3$$



$$\vec{F}_{Net} = \vec{F}_g$$

$$m_1\vec{a}_c = \frac{G m_1 M_p}{R_1^2}$$

$$\frac{4\pi^2R}{T_1^2} = \frac{GM_p}{R_1^2}$$

$$M_p = \frac{4\pi^2R_1^3}{G\,T_1^2}$$

$$M_p = \frac{4\pi^2(4220000)^3}{6.67\times10^{-11}(5400)^2}$$

$$M_p = 1.53\times10^{24}\ kg$$

Show Work

$R_1 = 4220000m$

$T_1 = 1.5h \times \frac{3600}{h}$

$= 5400s$