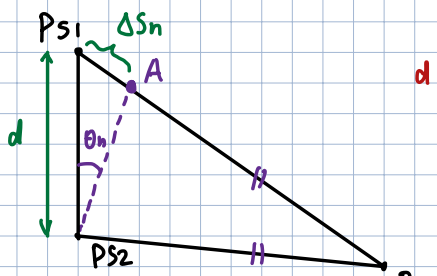
Mathematical Approximation for light

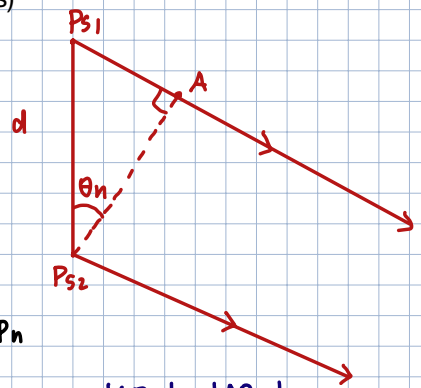
We've looked at the path difference analysis of |PnS1-PnS2|, now let's look at a mathematical approximation (Pn is far from sources)



$|P_nP_{S_1} - P_nP_{S_2}| = |AP_{S_1}|$

$\Delta S_n = |AP_{S_1}|$

$(n-0.5)\lambda = |AP_{S_1}|$



$|AP_{S_1}| = |AP_{S_1}|$

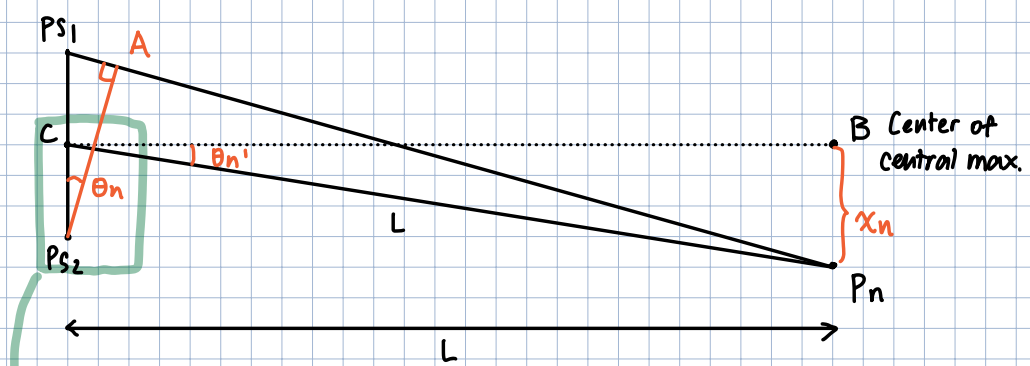
$(n-0.5)\lambda = d\sin\theta_n$

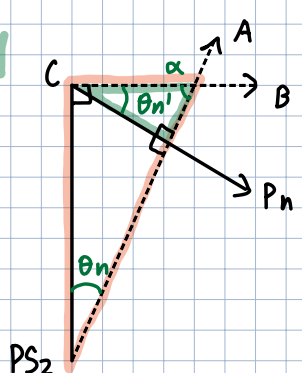
$\sin\theta_n = \frac{(n-0.5)\lambda}{d}$

$\sin\theta_n = \frac{|AP_{S_1}|}{d}$

$|AP_{S_1}| = d\sin\theta_n$

Light Approximation (if θn is too small)





$\theta_n' + 90 + \alpha = 180^\circ$

$\theta_n + 90 + \alpha = 180^\circ$

$\theta_n' = \theta_n$

$\sin\theta_n' = \sin\theta_n$

$\frac{x_n}{L} = \frac{(n-0.5)\lambda}{d}$

Example 2: Light

A 633nm laser is passed through a double slit with 100um between slits

a) If the screen is 5m away, where is the 3rd node?

$\lambda = 633 \times 10^{-9}\ m$ $\quad x_3 ?$

$d = 100 \times 10^{-6}\ m$

$L = 5m$

$n = 3$

THIS

$$\frac{x_3}{L} = \frac{(n-\frac{1}{2})\lambda}{2}$$

$$x_3 = \frac{(5)(3-\frac{1}{2})(633\times10^{-9})}{100\times10^{6}}$$

$$= 0.0791m \rightarrow 7.91cm$$

Example 1 Ripple Tank with approximation

$d = 4cm = 0.04m$

$L = 75cm = 0.75m$

$n = 3$

$x_3 = 10cm = 0.1m$

$$\frac{x_n}{L} = (n-0.5)\frac{\lambda}{d}$$

$$\lambda = \frac{x_n d}{L(n-05)}$$

$$\lambda = \frac{0.1(0.04)}{0.75(2.5)}$$

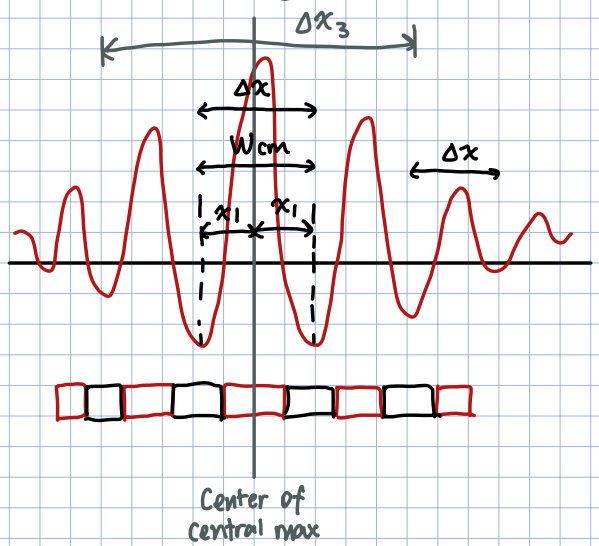
$$\lambda = 0.00213m$$
$$= 0.213cm$$

Not Best Option Here

better when

d ccccccc L

Double Slit Intensity pattern



x_n = distance from the center of central max to node n

W_{cm} = width of central band

Δx = distance from one node to the next

Δx_n = distance from one node to nth node over

$$W_{cm} = \Delta x_1 = 2x_1$$

Adjacent nodes :

$$\frac{x_n}{L} = \frac{(n-0.5)\lambda}{d}$$

$$x_n = \frac{(n-0.5)\lambda L}{d}$$

$$\Delta x = x_{n+1} - x_n$$

$$= \frac{(n+1-\frac{1}{2})\lambda L}{d} - \frac{(n-\frac{1}{2})\lambda L}{d}$$

$$= \frac{\lambda L}{d}\left(n+1-\tfrac{1}{2}-n+\tfrac{1}{2}\right)$$

$$= \frac{\lambda L}{d}$$

Nodes are z intervals apart

$$\Delta x_z = x_{n+z} - x_n$$

$$= \frac{(n+z-\frac{1}{2})\lambda L}{d} - \frac{(n-\frac{1}{2})\lambda L}{d}$$

$$= \frac{\lambda L}{d}\left(n+z-\tfrac{1}{2}-n+\tfrac{1}{2}\right)$$

$$= \frac{z\lambda L}{d}$$

Terminologies:

Light Example: When light passes through a “double slit” (two thin slits that allow light through, the rest is blocked), the two slits act like two point sources for light which behaves like a wave. The light waves diffract at slit (opening) creating circular wave patterns.

The width of each maxima (bands of light - in this case blue) are equal. The nodes are technically a thin line (white), however because the maxima weakens in intensity outward from central maxima, the nodes appear wider.

Dark fringes - nodal lines or minima (4th order dark fringe referring to 4th nodal line)

Bright fringes - maxima

Monochromatic - light with a single frequency

nth order dark angle - angle theta in formula using nth node information (for bright, you’d use the equivalent maxima equation)