Where we are in space

Math Review

slope $= \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$ tangent : $\frac{\text{instantaneous}}{\text{derivative}}$ (velocity) from d-t graph

Derivatives

a) $y = 3x^2$

$\frac{dy}{dx} = 6x$

b) $y = 3x^{-101}$

$\frac{dy}{dx} = -303x^{-102}$

Integrals

$\frac{dy}{dx} \rightarrow y'$

a) $y' = x^4$

$y = \frac{1}{5}x^5 + C$

b) $y' = e^2 x^{10}$

$y = e^2 \frac{1}{11}x^{11}$

$y = \frac{e^2}{11}x^{11} + C$

c) $y' = \frac{7}{x^8} = 7x^{-8}$

$y = -x^{-7}$

d)

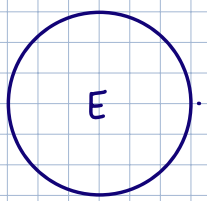
$E'(r) = \frac{7G\pi}{r^2} = 7G\pi r^{-2}$

$E = -7G\pi r^{-1}$

Gravitational Potential Calculations

Example: Surface of the Earth

How much energy does it take to lift a 20kg mass 100m?



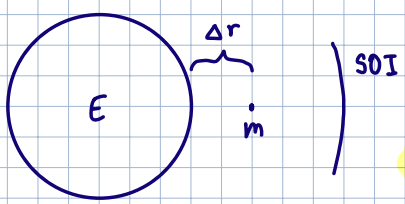
$E_g = mgh$

$= 20(9.81)(100)$

$= 19600\ J$

Example: Into Space

How much energy does it take to lift a 20kg mass to a height of 100km?



$\Delta E_g = \vec{F}_g \cdot d \cos\theta$ (1)

$\Delta E_g = \frac{GMm}{r^2}\Delta r$

$\frac{dE}{dr} = \frac{\Delta E_g}{\Delta r} = \frac{GMm}{r^2}$

$E_g = -GMmr^{-1}$

$E_g = \frac{-GMm}{r}$

$G = 6.67\times10^{-11}\ kgm^3/s^2$

$M = 5.98\times10^{24}\ kg$

$m = 20\ kg$

$alt = 100000\ m$

$r_E = 6.38\times10^6\ m$

$r = r_E + alt$

$= 6.48\times10^6\ m$

$\Delta E = E_g' - E_g$

$= \frac{-GMm}{r} - \left(\frac{-GMm}{r_E}\right)$

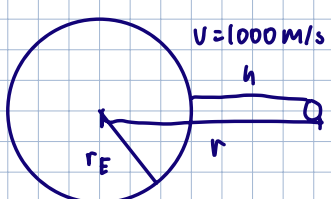
$= GMm\left(\frac{1}{r_E} - \frac{1}{r}\right)$

$= 6.67\times10^{-11}(5.88\times10^{24})(20)\left(\frac{1}{6.38\times10^6} - \frac{1}{6.48\times10^6}\right)$

$= 1.93\times10^7\ J$

Example 2: Within Sphere of Influence

How high will a ball go if you throw it up at 1km/s?



$G = 6.67 \times 10^{-11}\ kg\,m^3/s^2$

$M = 5.98 \times 10^{24}\ kg$

$m = ?$

$r_E = 6.38 \times 10^6\ m$

$h = r - r_E$

$= 6.43 \times 10^6 - 6.38 \times 10^6$

$= 51437\ m$

$$E_T = E_T'$$

$$E_g + E_K = E_g'$$

$$\left[\frac{-GMm}{r_E} + \frac{1}{2}mv^2 = \frac{-GMm}{r}\right] \div -GMm$$

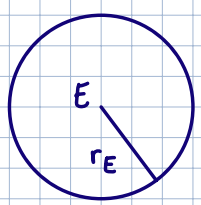
$$\frac{1}{r_E} - \frac{v^2}{2GM} = \frac{1}{r}$$

$$\frac{1}{6.38 \times 10^6} - \frac{1000^2}{2(6.67 \times 10^{-11})(5.98 \times 10^{24})} = \frac{1}{r}$$

$$r = 6.43 \times 10^6\ m$$

Example 4: Escape Velocity (Escape Earth's gravity/SOI)

How fast can you throw a ball so it just leaves Earth?



$G = 6.67 \times 10^{-11}\ kg\,m^3/s^2$

$M = 5.98 \times 10^{24}\ kg$

$m = ?$

$r_E = 6.38 \times 10^6\ m$

$r = \infty$

$$E_T = E_T'$$

$$E_g + E_K = \cancel{E_g}^{\,0} + \cancel{E_K}^{\,0}$$

$$\frac{-GMm}{r_E} + \frac{1}{2}mv^2 = 0$$

$$\frac{v^2}{2} = \frac{GM}{r_E}$$

$$v = \sqrt{\frac{2GM}{r_E}}$$

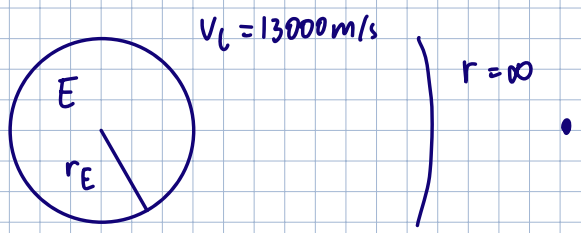
$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}}$$

$$= 11182\ m/s$$

$$= 11.2\ km/s$$

Example 5: Deep Space

A ball is thrown at 13km/s. How fast would it travel once it reached deep space?



$G = 6.67 \times 10^{-11}\ kg\,m^3/s^2$

$M = 5.98 \times 10^{24}\ kg$

$r_E = 6.38 \times 10^6\ m$

$m = ?$

$r > \infty$

$$E_T = E_T'$$

$$E_g + E_k = E_k'$$

$$\frac{-GMm}{r_E} + \frac{1}{2}mv^2 = \frac{1}{2}mv'^2$$

$$\frac{-2GM}{r_E} + v^2 = v'^2$$

$$v' = \sqrt{\frac{-2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.35 \times 10^6} + 13000^2}$$

$$v' = 6585.8\ m/s$$