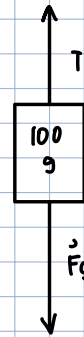


Consider a 100g stopper on a string, held up by Jeb the chihuahua

a) Find T when the stopper is stationary

$$\begin{aligned}\vec{F}_{\text{Net}} &= T + \vec{F}_g \\ m(0) &= T + m(-g) \\ T &= mg = 0.1(9.81) \\ &= 0.98 \text{ N [U]}\end{aligned}$$



b) Find T if lifting string at 2m/s

$$a = 0 \quad \text{same as a)}$$

c) Find T if moving string down at 2m/s

$$a = 0 \quad \text{same as a)}$$

d) Find T if lifting string at 2m/s²

$$\begin{aligned}a &= 2 \text{ m/s}^2 \text{ [U]} \\ \vec{F}_{\text{Net}} &= T + \vec{F}_g \\ 0.1(2) &= T + 0.1(-9.8) \\ T &= 1.18 \text{ N [U]}\end{aligned}$$

e) Find T if moving string down at 2m/s²

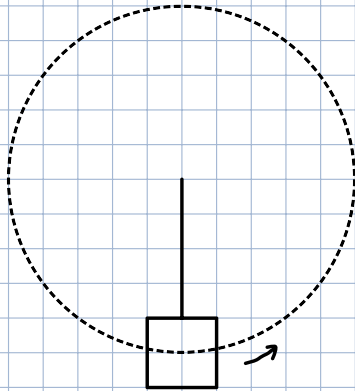
$$\begin{aligned}\vec{a} &= 2 \text{ m/s}^2 \text{ [D]} \\ \vec{F}_{\text{Net}} &= T + \vec{F}_g \\ 0.1(2) &= T + 0.1(9.81) \\ T &= 0.781 \text{ N [U]}\end{aligned}$$

f) Find T if moving string down at 15m/s²

$$\begin{aligned}\vec{a} &= 15 \text{ m/s}^2 \text{ [D]} \\ \vec{F}_{\text{Net}} &= T + \vec{F}_g \\ 0.1(15) &= T + 0.1(9.81) \\ T &= 0.519 \text{ N [D]} = T\end{aligned}$$

Pendulum: Vertical Circle

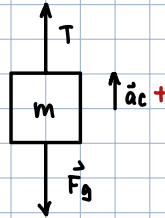
Find the tension at the bottom and top of the rotation (uniform), given the length of string and time for ten cycles



$$l = R$$

$$T = \frac{t}{10 \text{ cycles}} s$$

Bottom



$$\vec{F}_{\text{net}} = T + \vec{F}_g$$

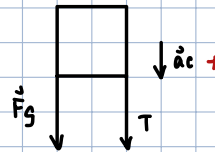
$$m\vec{a}_c = T + m(-g)$$

$$m\left(\frac{4\pi^2 R}{T^2}\right) = T - mg$$

$$m\left(\frac{4\pi^2 R}{T^2}\right) + mg = T$$

$$m\left(\frac{4\pi^2 R}{T^2} + g\right) = T$$

Top



$$\vec{F}_{\text{net}} = T + \vec{F}_g$$

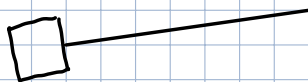
$$m\vec{a}_c = T + mg$$

$$m\vec{a}_c - mg = T$$

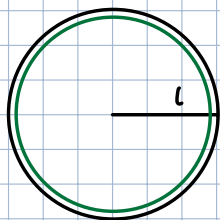
$$m(\vec{a}_c - g) = T$$

$$m\left(\frac{4\pi^2 R}{T^2} - g\right) = T$$

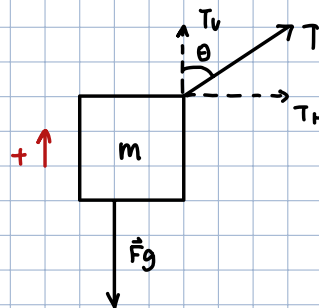
Pendulum: Horizontal circle



Bird's eye view:



Perfectly horizontal
 $l = R$
but actual
 $R < l$ because
slight drop

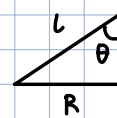


$$\vec{F}_{\text{net}} = T_v + \vec{F}_g$$

$$m(0) = T \cos \theta + m(-g)$$

$$mg = T \cos \theta$$

$$T = \frac{mg}{\cos \theta}$$



Angle from
vertical provided
(usually)

H

$$\vec{F}_{\text{net}} = T_H$$

$$m\vec{a}_c = T \sin \theta$$

$$m\vec{a}_c = \frac{mg}{\cos \theta} \sin \theta$$

$$\vec{a}_c = g \tan \theta$$

sub.

sub: $\frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$

Pendulum: Example 1

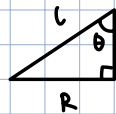
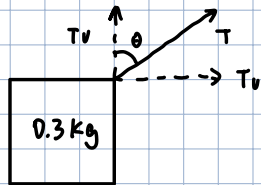
0.3 kg

4.17 m/s

A 300g pendulum is spun around in a horizontal circle. If the pendulum is moving at 15km/h and makes an angle 25 degrees from the vertical, how long is the pendulum?

θ

$L?$



$$\sin \theta = \frac{R}{L}$$

$$R = L \sin \theta$$

$$v = 4.17 \text{ m/s}$$

V

$$\vec{F}_{\text{net}} = T_v + F_g$$

$$m(0) = T \cos \theta = m(-g)$$

$$T = \frac{mg}{\cos \theta}$$

H

$$\vec{F}_{\text{net}} = T_H$$

$$m \vec{a}_c = T \sin \theta$$

$$m \vec{a}_c = \frac{mg}{\cos \theta} \sin \theta$$

$$\vec{a}_c = g \tan \theta$$

$$\frac{v^2}{R} = g \tan \theta$$

$$R = \frac{v^2}{g \tan \theta}$$

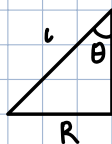
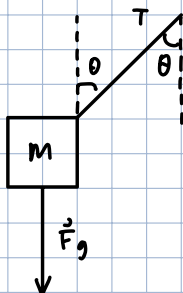
$$L \sin \theta = \frac{v^2}{g \tan \theta}$$

$$L = \frac{v^2}{g \tan \theta \sin \theta}$$

$$L = 8.98 \text{ m}$$

Example 2:

A π^R kg mass is at the end of a 3m long pendulum, moving in a horizontal circle. If the rope makes a 35 degrees angle with the vertical, how fast is the mass moving



$$\sin \theta = \frac{L}{R} \quad R = L \sin \theta$$

$$V \quad \vec{F}_{\text{net}} = T_v + \vec{F}_g$$

$$0 = T \cos \theta - mg$$

$$T \cos \theta = mg$$

$$T = \frac{mg}{\cos \theta}$$

$$H \quad \vec{F}_{\text{net}} = T_H$$

$$m \vec{a}_c = T \sin \theta$$

$$m \vec{a}_c = \frac{mg}{\cos \theta} \sin \theta$$

$$\vec{a}_c = g \tan \theta$$

$$\frac{v^2}{R} = g \tan \theta$$

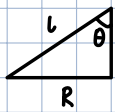
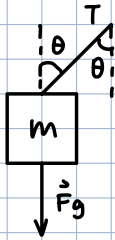
$$v = \sqrt{R g \tan \theta}$$

$$= \sqrt{L \sin \theta g \tan \theta}$$

$$= 3.44 \text{ m/s}$$

Example 3:

A 2.25m long pendulum is spun in a horizontal circle. It completes 95 turns in a minute. What angle is the rope to the vertical?



$$T = \frac{60s}{95 \text{ cycles}}$$

$$\sin \theta = \frac{R}{L}$$

$$R = L \sin \theta$$

$$\begin{aligned} \text{V} \quad \vec{F}_{\text{net}} &= T_V + \vec{F}_g \\ 0 &= T \cos \theta - mg \\ T \cos \theta &= mg \\ T &= \frac{mg}{\cos \theta} \end{aligned}$$

$$\begin{aligned} \text{H} \quad \vec{F}_{\text{net}} &= T_H \\ m \vec{a}_c &= T \sin \theta \\ m \vec{a}_c &= \frac{mg}{\cos \theta} \sin \theta \end{aligned}$$

$$\vec{a}_c = g \frac{\sin \theta}{\cos \theta}$$

$$\frac{4\pi^2 R}{T^2} = g \frac{\sin \theta}{\cos \theta}$$

$$\frac{4\pi^2 R L \sin \theta}{T^2} = g \frac{\sin \theta}{\cos \theta}$$

$$\cos \theta = \frac{g T^2}{4\pi^2 L}$$

$$\theta = \cos^{-1} \left(\frac{g T^2}{4\pi^2 L} \right)$$

$$\theta = 87.5^\circ$$