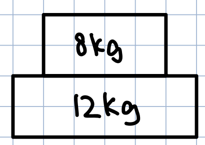
Example 1:

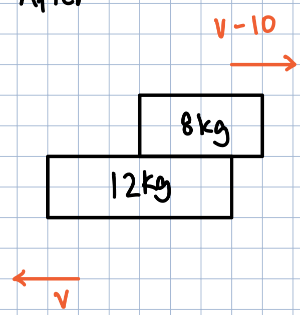
An 8 kg mass is thrown off the back of a stationary 12kg cart at a velocity of 10m/s, relative to the cart.

Assuming no friction, what is the velocity of the cart?

Before



After



m = mass

c = cart

E = Earth

$\vec{V}_{mc} = 10 m/s$ [B]

$\vec{V}_{cE} = V m/s$ [F]

$\vec{V}_{mE} = ?$

$= \vec{V}_{mc} + \vec{V}_{cE}$

$= 10$ [B] $+ V$ [F]

$= V - 10$ [F]

$\vec{P}_T = \vec{P}_T{}'$

$0 = m_{12}V_{12}' + m_8 V_8'$

$0 = 12V + 8(V-10)$

$0 = 12V + 8V - 80$

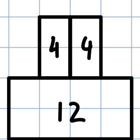
$0 = 20V - 80$

$V = 4 m/s$

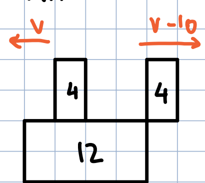
Instead of throwing the 8kg mass off the cart all at once, you throw it in 4kg chunks, each time at 10m/s with relative to cart. What is final velocity of the cart?

Throw 1

Before



After



$\vec{P}_T = P_T{}'$

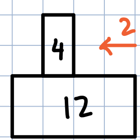
$0 = 16V + 4(V-10)$

$0 = 20V - 40$

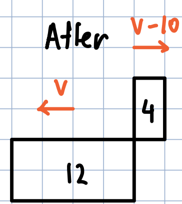
$V = 2 m/s$ [L]

Throw 2

Before



After



$P_T = P_T{}'$

$16(2) = 12V + 4(V-10)$

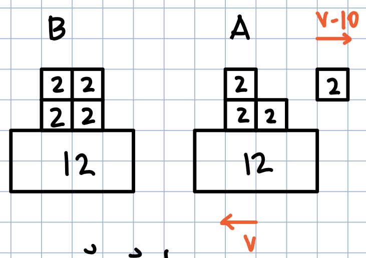
$32 = 12V + 4V - 40$

$16V = 72$

$V = 4.5 m/s$ [L]

Example 3: Repeat Example 2, but throw chunks 2kg at a time

Throw #1



$\vec{P}_T = \vec{P}_T'$

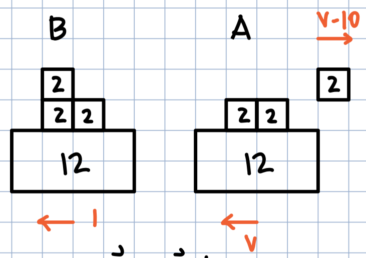
$0 = 18(v) + 2(v-10)$

$0 = 18v + 2v - 20$

$0 = 20v - 20$

$v = 1\,m/s\ [L]$

Throw #2



$\vec{P}_T = \vec{P}_T'$

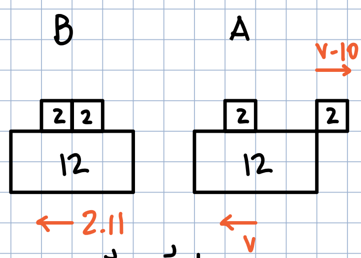
$18(1) = 16(v) + 2(v-10)$

$18 = 16v + 2v - 20$

$38 = 18v$

$v = 2.11\,m/s\ [L]$

Throw #3



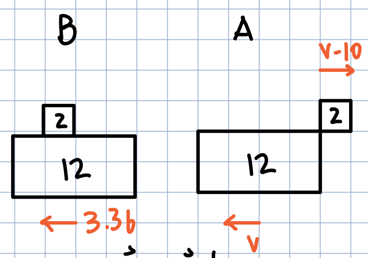
$\vec{P}_T = \vec{P}_T'$

$16(2.11) = 14v + 2(v-10)$

$33.8 = 14v + 2v - 20$

$v = 3.36\,m/s\ [L]$

Throw #4



$\vec{P}_T = \vec{P}_T'$

$14(3.36) = 12v + 2(v-10)$

$47.1 = 12v + 2v - 20$

$47.1 = 14v - 20$

$v = 4.79\,m/s\ [L]$

The smaller chunks we use, the faster the cart

Rocket Design

What is the smallest mass we can throw? 1 atom / molecule

Rocket Equation:

$$\Delta v = V_{exh} \ln\left(\frac{m_i}{m_f}\right)$$

natural logarithm

initial mass

final mass

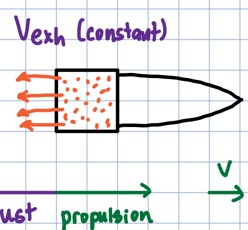
change in velocity of the rocket

Exhaust Velocity (how fast it was being released or thrown)

Apply to examples 1~3 : $\Delta v = 10 \ln\left(\frac{20}{12}\right)$

$= 5.11 m/s$ [L]

Thrust



Thrust : the push from the engine ($\vec{F}$)

$$\vec{I} = \Delta\vec{p} = m\Delta v = \vec{F}\Delta t$$

Actual $\Delta\vec{p}$ equation:

$$\Delta\vec{p} = \Delta(m\vec{v}) = \Delta m\vec{v} + m\Delta\vec{v}$$

f(x) g(x)

Rocket Thrust Equation

$$\vec{F}\Delta t = \Delta m\vec{v} + m\cancelto{0}{\Delta\vec{v}}$$

$$\vec{F}\Delta t = \Delta m\vec{v}_{exh}$$

Comes from product rule in calculus

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

derivative actual actual derivative

Engine Efficiencies

Engine efficiencies are determined by a rocket's specific impulse (ISP) which is the length of time 1kg of fuel can create 9.8N of thrust. The higher the ISP, the more efficient the engines.

Rocket type	ISP	Vexh *$\Delta m = kg$ $\vec{F} = 9.8N$
Solid Model Rocket (uses Fe_2O_3 to fuel)	80s	$\vec{F}\Delta t = \Delta m V_{exh}$ $9.8(80) = 1 V_{exh}$ $V_{exh} = 784 m/s$
"Real" Solid Rocket Motor (Ammonium perchlorate + aluminum)	230s	2254 m/s
Kerosene/ liquid oxygen (Heavy - C+H reactions)	300s	2940 m/s
Methane / liquid oxygen	330s	3234 m/s
H_2/O_2 (small particles faster chemical reaction)	450s	4410 m/s
Nuclear (subatomic reaction)	800s	7840 m/s
Ion	2000s 10000s	19600 m/s 98000 m/s

Antimatter (subatomic, even smaller particles)	100000s	980000m/s

Example 4: A 10 tonne spaceship has an engine with an ISP of 2500s and uses 1.5kg of fuel a minute.

If the rocket is propelled for 1.5h, what delta v does it experience?

$\vec{F}\Delta t = \Delta m \vec{V}_{exh}$ $\quad \Delta V = V_{exh} \ln\left(\frac{m_i}{m_f}\right)$ $\quad \vec{F}\Delta t = m\Delta\vec{V}$

$M_i = 10000\,kg$

$ISP = 2500s$ $(F = 9.8N, \Delta m = 1kg) \rightarrow V_{exh}$

$\Delta m = 1.5kg$ $\quad \Delta t = 60s \rightarrow \vec{F}$

$\Delta t = 5400s$

ISP

$\vec{F} = 9.8N$
$\Delta m = 1kg$
$\Delta t = ISP = 2500s$

$\vec{F}\Delta t = \Delta m V_{exh}$

$9.8(2500) = 1 V_{exh}$

$V_{exh} = 24500 m/s$

Thrust

$\Delta m = 1.5kg$
$\Delta t = 60s$
$V_{exh} = 24500 m/s$

$\vec{F}\Delta t = \Delta m V_{exh}$

$\vec{F}(60) = (1.5)(24500)$

$\vec{F} = 612.5N$

$\Delta\vec{V}$

$\vec{F} = 612.5N$
$\Delta t = 1.5h = 5400s$
$m = 10000\,kg$

$\vec{F}\Delta t = m\Delta V$
$612.5(5400) = 10000\,\Delta V$

$\Delta\vec{V} = 331 m/s$

Way 2:

$\frac{x}{5400} = \frac{1.5kg}{60s}$

$x = 135kg$

$m_f = 10000kg - 135kg$

$= 9865kg$

$\Delta V = V_{exh} \ln\left(\frac{m_i}{m_f}\right)$

$\Delta V = 24500 \ln\left(\frac{10000}{9865}\right)$

$= 333 m/s$