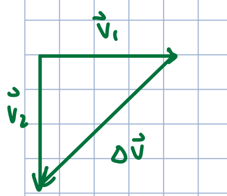
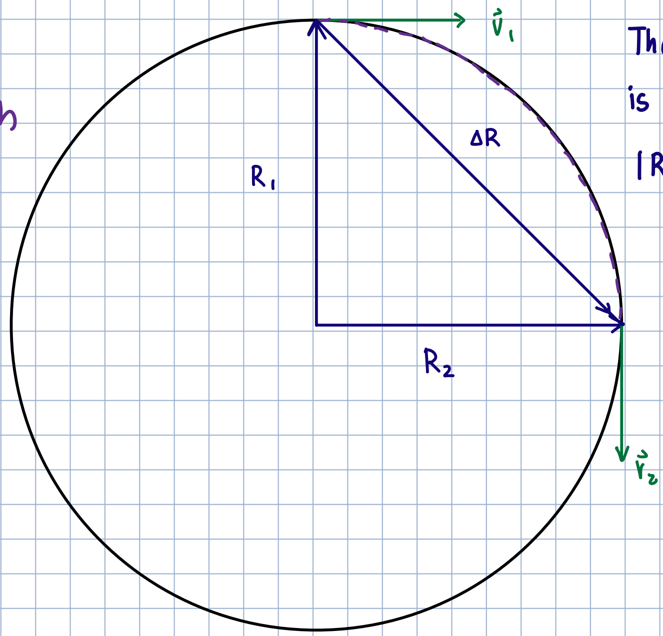
S = arc length

$\Delta R = R_2 - R_1$

$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$



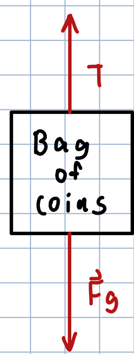


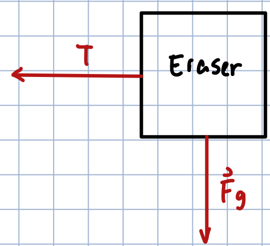
The radius of the motion is constant

$|R_1| = |R_2| = |R| = R$

The magnitude of the velocity of motion is constant

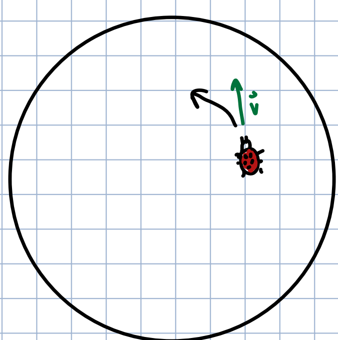
$|\vec{v}_1| = |\vec{v}_2| = |\vec{v}| = \vec{v}$

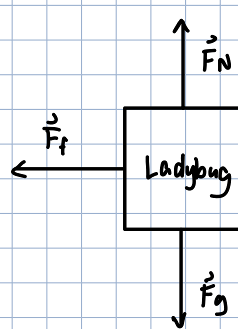




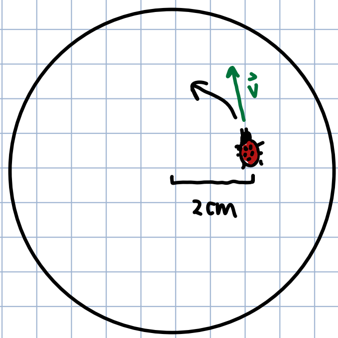
Horizontal: $\vec{F}_{Net} = ma = T$

For the eraser in circular motion, acceleration is towards the center and called centripedal acceleration (a_c)





$\vec{F}_{Net} = ma = \vec{F}_f$



Radius : 2cm

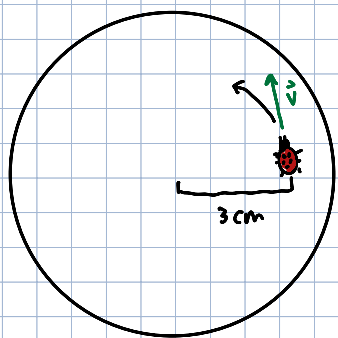
Period : 5s

Distance : $2\pi r$

$= 4\pi$ cm

Speed : $v = \frac{d}{t} = 0.8\pi$ cm/s

What happens if increase radius of motion?



Radius : 3cm

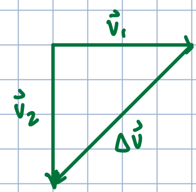
Period : 5s

Distance : $2\pi r$

$= 6\pi$ cm

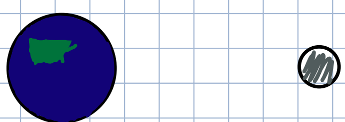
Speed : $v = \frac{d}{t} = 1.2\pi$ cm/s

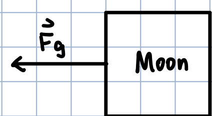
If the velocities are larger, and $\vec{a}$ is $\Delta v/t$, what do we know about the force?



ORBITS

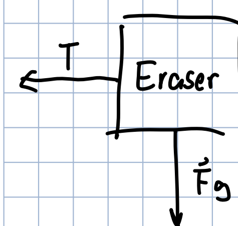
Side view

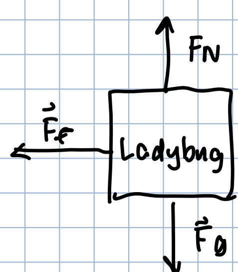


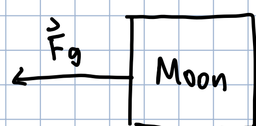


Horizontal :

$\vec{F}_{Net} = ma_c = \vec{F}_g$

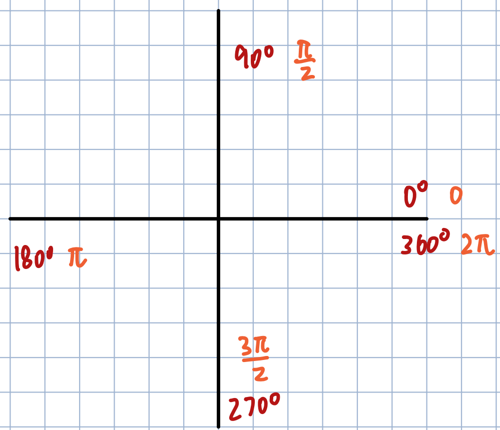






MATH REVIEW

1) Converting degree to radians



$r : d = \pi : 180$

Ex: 37° to rad

$$\frac{r}{d} = \frac{\pi}{180}$$

$$r = \frac{37\pi}{180}$$

$$= 0.646 \text{ rad}$$

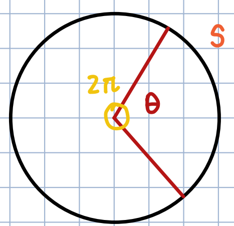
Ex: 4.35 rad to deg

$$\frac{d}{r} = \frac{180}{\pi}$$

$$d = \frac{4.35(180)}{\pi}$$

$$d = 249^\circ$$

2) Arc length



$$\frac{\theta}{2\pi} = \frac{\theta^\circ}{360} = \frac{s}{\underbrace{2\pi R}_{\text{Circumference}}} = \frac{t \rightarrow \text{Travel time}}{T \rightarrow \text{Full circle (period)}}$$

$$\frac{\theta}{2\pi} = \frac{s}{2\pi R}$$

$$\frac{\cancel{2\pi} R\theta}{\cancel{2\pi}} = s$$

$$\boxed{s = \theta r}$$ Use RADIAN

3) Limits

Ex 1)

$$\lim_{x \to 2} 3x - 5$$

i) Substitution

$$3(2) - 5$$
$$= 6 - 5$$
$$= 1$$

Ex 2)

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$

i) $\frac{2^2 + 2 - 6}{2^2 - 4}$

$$= \frac{0}{0}$$

ii)

$$= \lim_{x \to 2} \frac{(x+3)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{(2+3)}{(2+2)}$$

$$= \frac{5}{4}$$

Ex 3)

$$\lim_{x \to 2} 2x + 6$$

i) $2(2) + 6$

$= 10$

ii) Factor

$$\lim_{x \to 2} 2(x+3)$$

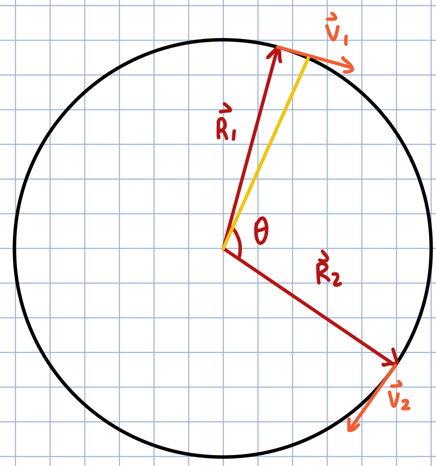
$$= 2 \lim_{x \to 2} (x+3)$$

$$= 2(2+3)$$

$$= 10$$

Acceleration Formula #1

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

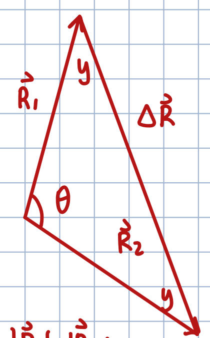


Find $\vec{a}_{inst}$ by using $\vec{a}_{avg}$ as $\Delta t \to 0$

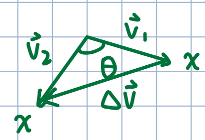
$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\frac{|R|}{|\Delta R|} = \frac{|V|}{|\Delta V|} \Rightarrow \Delta v = \frac{V \Delta R}{R}$$

sub



$|\vec{R}_1| = |\vec{R}_2|$



$|\vec{v}_2| = |\vec{v}_1|$

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\left(\frac{V \Delta R}{R}\right)}{\Delta t}$$

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{V \Delta R}{R \Delta t}$$

$$\vec{a}_{inst} = \frac{V}{R} \lim_{\Delta t \to 0} \frac{\Delta R}{\Delta t}$$

sub: $\frac{\Delta R}{\Delta t} = \frac{s}{\Delta t} = v$

$$\vec{a}_{inst} = \frac{V}{R} \lim_{\Delta t \to 0} V$$

$$\vec{a}_{inst} = \frac{V^2}{R} \cancel{\lim_{\Delta t \to 0}} 1$$

$$\boxed{\vec{a}_{inst} = \frac{V^2}{R}}$$

R

$180° = \theta + y + y$

$180 - \theta = 2y$

V

$180° = \theta + x + x$

$180 - \theta = 2x$

$2y = 2x$

$y = x$

Acceleration Formula #2

1) $V = \frac{d}{t} = \frac{2\pi R}{T}$ sub

2) $\vec{a}_{inst} = \frac{V^2}{R}$

$$\vec{a}_{inst} = \left(\frac{2\pi R}{T}\right)^2 \cdot \frac{1}{R} = \frac{2^2\pi^2 R^2}{T^2} \cdot \frac{1}{R}$$

$$\vec{a}_{inst} = \frac{4\pi^2 R}{T^2} \qquad \vec{a}_{inst} = 4\pi^2 R f^2$$

Acceleration Formula #3

1) $V = \frac{2\pi R}{T} \qquad R = \frac{VT}{2\pi}$

2) Sub 1)

$$\vec{a}_{inst} = \frac{V^2}{R} = V^2 \div \frac{VT}{2\pi} = V^2 \cdot \frac{2\pi}{VT}$$

$$\vec{a}_{inst} = \frac{2\pi V}{T}$$

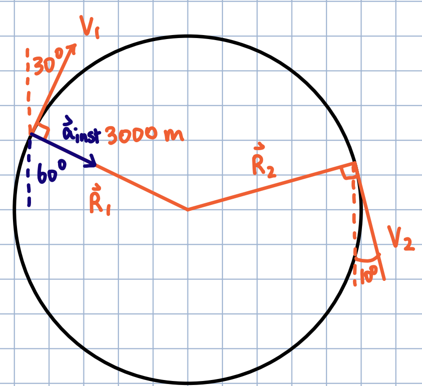
Uniform Circular Motion Example

A plane is flying at 144km/h [N30E] in a circle with a 3km radius. A little bit later it is flying at 144km/h [S10E]. Find:

40 m/s   3000 m

a) Average acceleration assuming clockwise

b) instantaneous acceleration at initial point



$V = 40\ m/s$
$R = 3000\ m$

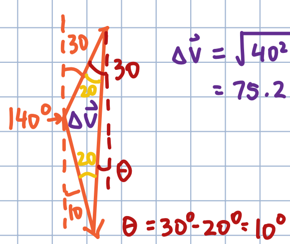
$$\vec{a}_{inst} = \frac{V^2}{R} = \frac{40^2}{3000} = 0.533\ m/s\ [E30°S]$$
[S60°E]

$$V = \frac{d}{t} = \frac{s}{t} = \frac{\theta R}{t} \qquad \frac{r}{d} = \frac{\pi}{180}$$

$$t = \frac{\theta R}{V} \qquad r = \frac{140\pi}{180}$$

$$t = \frac{\frac{140\pi}{180} \cdot 3000}{75.2}$$

$$t = 183.26s$$



$$\Delta\vec{v} = \sqrt{40^2 + 40^2 - 2 \cdot 40 \cdot 40 \cdot \cos(140)}$$
$$= 75.2\ m/s\ [S10W]$$

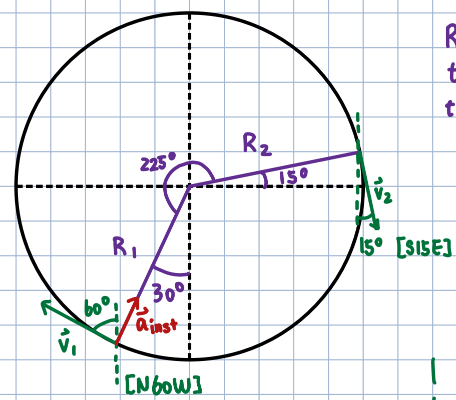
a) $a_{avg} = \frac{\Delta v}{t} = \frac{75.2}{183.26} = 0.410\ m/s^2\ [S10°W]$

Uniform Circular Motion Example #2

Moxie is flying in a circle of radius 3.5km. Initially she is [S30W] of center, and you see her 2 minutes later at [E15N]. At this point she has completed 2 full turns and then some. Find:

a) average acceleration

b) instantaneous acceleration initial



$R = 3500\,m$

$t = 120\,s$

trips = 2 + ? cycles

$t = 2\frac{225}{360}T$

$t = 2.625\,T$

$T = \frac{120}{2.625}$

$\downarrow$

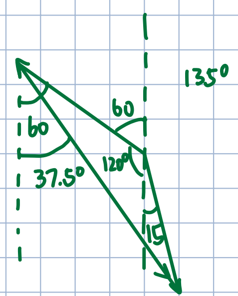
$T = 45.7\,s$

$\vec{a}_{inst} = \frac{4\pi^2 R}{T^2}$

$= \frac{4(\pi)^2(3500)}{45.7^2}$

$= 66.12\,m/s^2$ [N30E]

$V = \frac{2\pi R}{T} = \frac{2\pi(3500)}{45.7} = 481.06\,m/s$



$\Delta\vec{v} = \sqrt{481^2 + 481^2 - 2\cdot 481\cdot 481\cdot\cos(135)}$

$= 888.88\,m/s$ [S37.5E]

$\vec{a}_{avg} = \frac{\Delta v}{t} = \frac{888.88}{120}$

$= 7.41\,m/s^2$ [S37.5E]