2023HW02 逻辑代数基础*OK*!

1、一个三变量非一致判断电路，当输入的 3 个变量 A、B、C 不完全相同时，输出 F=1，否则 F=0。试列出该逻辑问题的真值表，并写出函数表达式。

参考解答：真值表如下：

A	B	C	F		A	B	C	F
0	0	0	0		1	0	0	1
0	0	1	1		1	0	1	1
0	1	0	1		1	1	0	1
0	1	1	1		1	1	1	0

$$F=(A+B+C)(\bar{A}+\bar{B}+\bar{C})$$

0r $F=\bar{A}\bar{B}C+\bar{A}B\bar{C}+\bar{A}BC+A\bar{B}\bar{C}+A\bar{B}C+AB\bar{C}$

2、直接写出下列各函数的反函数和对偶式：

(1) $F=AB+\bar{C}D+\overline{BC+\bar{D}+\bar{C}E+\overline{D+E}}$

参考解答： $F'=(A+B)(\bar{C}+D)\overline{(B+C)\bar{D}(\bar{C}+E)\overline{DE}}$

$$\bar{F}=(\bar{A}+\bar{B})(C+\bar{D})\overline{(\bar{B}+\bar{C})D(C+\bar{E})\overline{\bar{D}\bar{E}}}$$

$$\bar{F}=(\bar{A}+\bar{B})(C+\bar{D})(BC+\bar{D}+\bar{C}E+\overline{D+E})$$

$$F = AB + \bar{C}D + \overline{BC + \bar{D} + \bar{C}E + \overline{D + E}}$$

$$= AB + \bar{C}D + \overline{BC + \bar{D} + \bar{C}E + \bar{D}\bar{E}}$$

$$= AB + \bar{C}D + \overline{BC + \bar{D} + \bar{C}E}$$

注：$$= AB + \bar{C}D + (\bar{B} + \bar{C})D(C + \bar{E})$$

$$= AB + \bar{C}D + \bar{B}CD + \bar{C}D\bar{E}$$

$$= AB + \bar{C}D + \bar{B}CD$$

$$= AB + \bar{C}D + \bar{B}D$$

$$F' = (A + B)(\bar{C} + D)(\bar{B} + D)$$

$$\bar{F} = (\bar{A} + \bar{B})(C + \bar{D})(B + \bar{D})$$

(2) $$F = \overline{\overline{A + C}(\overline{BC} + D)}(B + C) + AD$$

参考解答：$$F' = (\overline{\overline{AC} + \overline{B + C}D} + BC)(A + D)$$

$$\bar{F} = (\overline{\bar{A}\bar{C} + \overline{\bar{B} + \bar{C}\bar{D}} + \bar{B}\bar{C}})(\bar{A} + \bar{D})$$

$$\bar{F} = (\overline{A + C}(\overline{BC} + D) + \overline{B + C}) \cdot \overline{AD}$$

注：

$$F=\overline{\overline{A+C}(\overline{BC}+D)}(B+C)+AD$$
$$=((A+C)+BC\bar{D})(B+C)+AD$$
$$=(A+C)(B+C)+AD$$
$$=C+AB+AD$$

(3) $F=A\cdot\overline{B+\bar{D}}+(AC+BD)E$

参考解答：$F'=(A+\overline{B\bar{D}})((A+C)(B+D)+E)$

$$\bar{F}=(\bar{A}+\overline{\bar{B}D})((\bar{A}+\bar{C})(\bar{B}+\bar{D})+\bar{E})$$

$$\bar{F}=(\bar{A}+B+\bar{D})(\overline{AC+BD}+\bar{E})$$

$$F = A\cdot\overline{B+\bar{D}}+(AC+BD)E$$

注：$= A\bar{B}D+ACE+BDE$

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3、写出逻辑函数 F 的标准与-或式：

(1) $F=\bar{B}\bar{C}+BC+A$

参考解答：$F=\bar{B}\bar{C}+BC+A$

$$=(\bar{A}+A)\bar{B}\bar{C}+(\bar{A}+A)BC+A(\bar{B}+B)(\bar{C}+C)$$

$$=\bar{A}\bar{B}\bar{C}+A\bar{B}\bar{C}+\bar{A}BC+ABC+(A\bar{B}+AB)(\bar{C}+C)$$

$$=\bar{A}\bar{B}\bar{C}+A\bar{B}\bar{C}+\bar{A}BC+ABC+(A\bar{B}+AB)\bar{C}+(A\bar{B}+AB)C$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + ABC + AB\bar{C} + A\bar{B}C$$

$$= \sum m(0,3,4,5,6,7)$$

(2) $F = BD + ACD + AB\bar{D} + A\bar{C}\bar{D}$

参考解答： $F = BD + ACD + AB\bar{D} + A\bar{C}\bar{D}$

$$= (\bar{A} + A)B(\bar{C} + C)D + A(\bar{B} + B)CD + AB(\bar{C} + C)\bar{D} + A(\bar{B} + B)\bar{C}\bar{D}$$

$$= \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD + A\bar{B}CD$$

$$+ ABCD + AB\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D}$$

$$= \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD + A\bar{B}CD$$

$$+ AB\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$= \sum m(5,7,8,11,12,13,14,15)$$

(3) $F = \overline{\bar{A}B + A\bar{B}}(AC + CD)$

参考解答： $F = \overline{\bar{A}B + A\bar{B}}(AC + CD)$

$$= \overline{A \oplus B}(AC + CD)$$

$$= (A \odot B)(AC + CD) = (AB + \bar{A}\bar{B})(AC + CD)$$

$$= ABAC + ABCD + \bar{A}\bar{B}AC + \bar{A}\bar{B}CD$$

$$= ABC + ABCD + \bar{A}\bar{B}CD$$

$$= ABC\bar{D} + ABCD + \bar{A}\bar{B}CD$$

$$= \sum m(3,14,15)$$

(4) $F = A \oplus \overline{B \oplus C}$

参考解答： $F = A \oplus \overline{B \oplus C}$

$$= \bar{A}\overline{B \oplus C} + A(B \oplus C)$$

$$= \bar{A}(B \odot C) + A(B \oplus C)$$

$$= \bar{A}(BC + \bar{B}\bar{C}) + A(\bar{B}C + B\bar{C})$$

$$= \bar{A}BC + \bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

$$= \sum m(0,3,5,6)$$

4、用真值表证明：

(1) $AB+\bar{A}C+BC=AB+\bar{A}C$

参考解答：

A	B	C	左边	右边	A	B	C	左边	右边
0	0	0	0	0	1	0	0	0	0
0	0	1	1	1	1	0	1	0	0
0	1	0	0	0	1	1	0	1	1
0	1	1	1	1	1	1	1	1	1

(2) $\bar{A}\bar{B}C+\bar{A}B\bar{C}+A\bar{B}\bar{C}+ABC=A\oplus B\oplus C$

参考解答：

A	B	C	左边	右边	A	B	C	左边	右边
0	0	0	0	0	1	0	0	1	1
0	0	1	1	1	1	0	1	0	0
0	1	0	1	1	1	1	0	0	0
0	1	1	0	0	1	1	1	1	1

5、用代数法证明：

(1) $A \oplus B \oplus C = A \odot B \odot C$

参考解答： $A \oplus B \oplus C$

$$= \bar{A}(B \oplus C) + A \cdot \overline{B \oplus C}$$

$$= \bar{A} \cdot \overline{B \odot C} + A(B \odot C)$$

$$= A \odot B \odot C$$

(2) $\bar{B}\bar{D} + BD + AB = \bar{B}\bar{D} + BD + A\bar{D}$

参考解答： $\bar{B}\bar{D} + BD + AB$

$$= A\bar{B}\bar{D} + \bar{B}\bar{D} + BD + ABD + AB\bar{D}$$

$$= A\bar{B}\bar{D} + \bar{B}\bar{D} + BD + AB\bar{D}$$

$$= \bar{B}\bar{D} + BD + A\bar{D}$$

(3) $\overline{\overline{A + B + \bar{C}} \cdot \bar{C}D} + (B + \bar{C})(A\bar{B}D + \bar{B}\bar{C}) = 1$

参考解答：$\overline{\overline{A+B+\bar{C}\cdot\bar{C}D}}+(B+\bar{C})(A\bar{B}D+\bar{B}\bar{C})$

$=(A+B+\bar{C})+(C+\bar{D})+(B+\bar{C})(A\bar{B}D+\bar{B}\bar{C})$

$=A+B+\bar{C}+C+\bar{D}+(B+\bar{C})(A\bar{B}D+\bar{B}\bar{C})$

$=A+B+1+\bar{D}+(B+\bar{C})(A\bar{B}D+\bar{B}\bar{C})=1$

(4) $(B+C)(C+D)$

$=(A+B+C)(B+C+\bar{D})(C+D+\bar{E})(C+D+E)(A+B+C+D+F)$

参考解答：$(A+B+C)(B+C+\bar{D})(C+D+\bar{E})(C+D+E)(A+B+C+D+F)$

$=(A+B+C)(B+C+\bar{D})(C+D)$

$$= (B + C + A\bar{D})(C + D)$$

$$= C + D(B + A\bar{D})$$

$$= C + BD = (B + C)(C + D)$$

6、已知$F(A,B,C,D) = \prod M(0,2,3,4,7,8,10)$，写出$F$的标准与或式以及$\bar{F}$、$F'$的所有标准式。

参考解答：$F(A,B,C,D) = \sum m(1,5,6,9,11,12,13,14,15)$

$$\bar{F}(A,B,C,D) = \sum m(0,2,3,4,7,8,10)$$

$$\bar{F}(A,B,C,D) = \prod M(1,5,6,9,11,12,13,14,15)$$

$$F'(A,B,C,D) = \sum m(5,7,8,11,12,13,15)$$

$$F'(A,B,C,D) = \prod M(0-4,6,9,10,14)$$