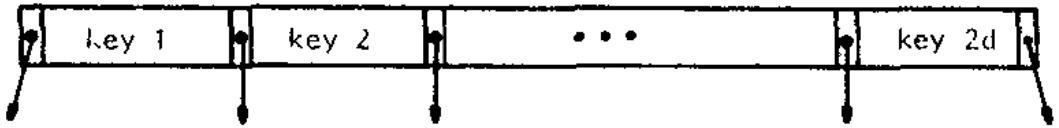


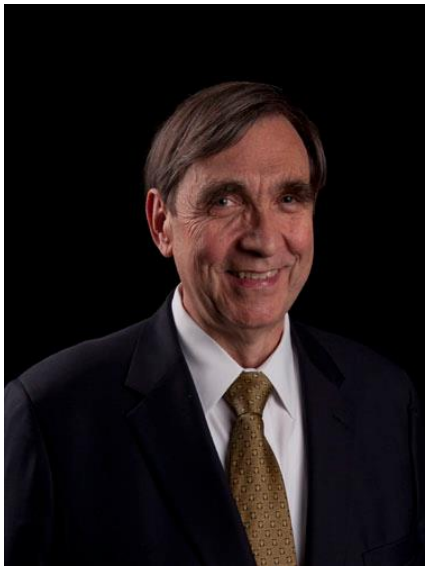

FIGURE 4. A node in a B-tree of order $d$ with $2d$ keys and $2d + 1$ pointers.

# Conception Avancée de Bases de Données

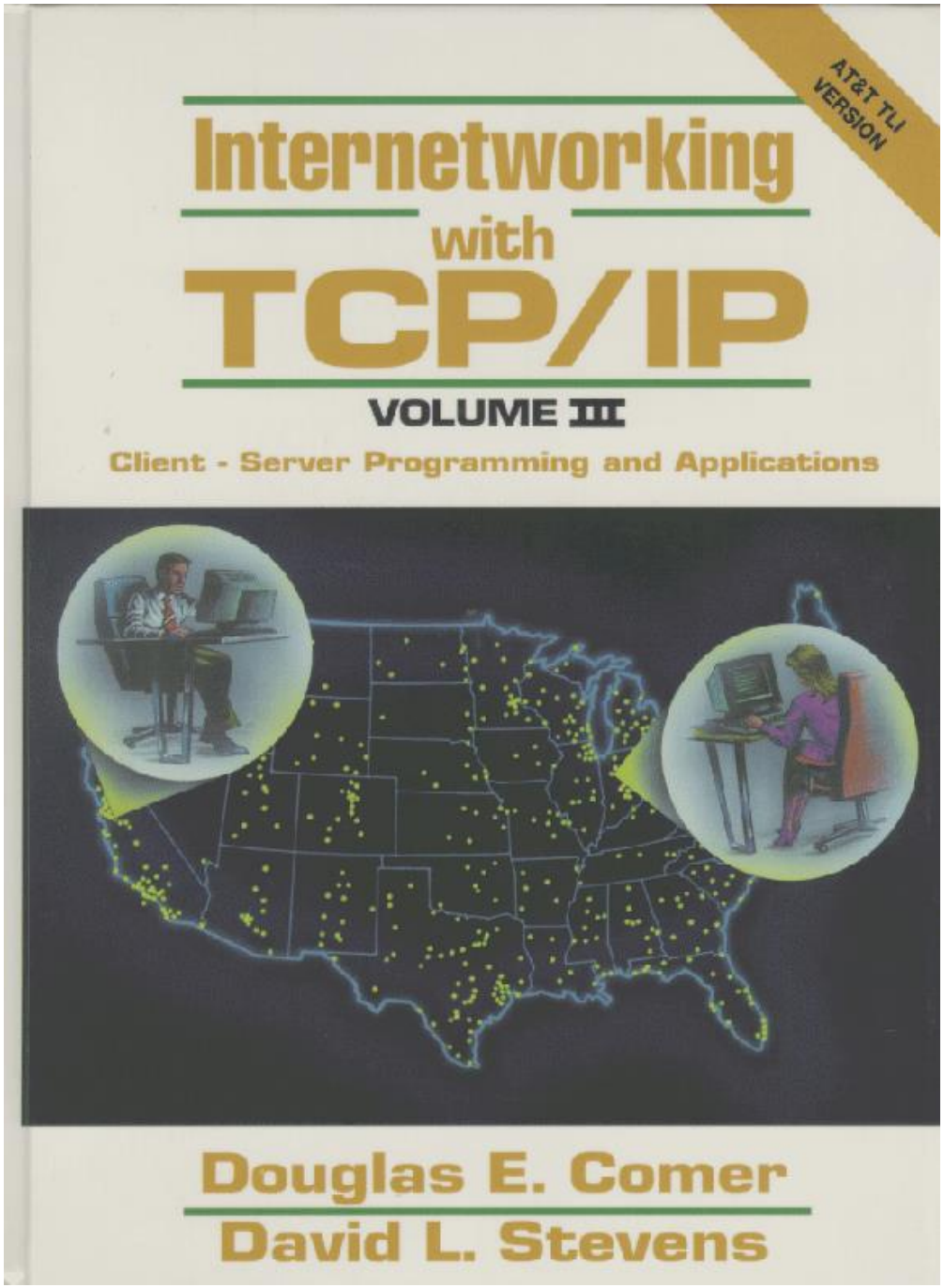
B+Tree

**Traduction en cours**

# Seminal Article



- **The Ubiquitous B-Tree**
- DOUGLAS COMER
- *Computer Science Department, Purdue Untverstty, West Lafayette, Indiana 47907*


AT&T TLI VERSION
Internetworking with TCP/IP
VOLUME III
Client - Server Programming and Applications
Douglas E. Comer
David L. Stevens

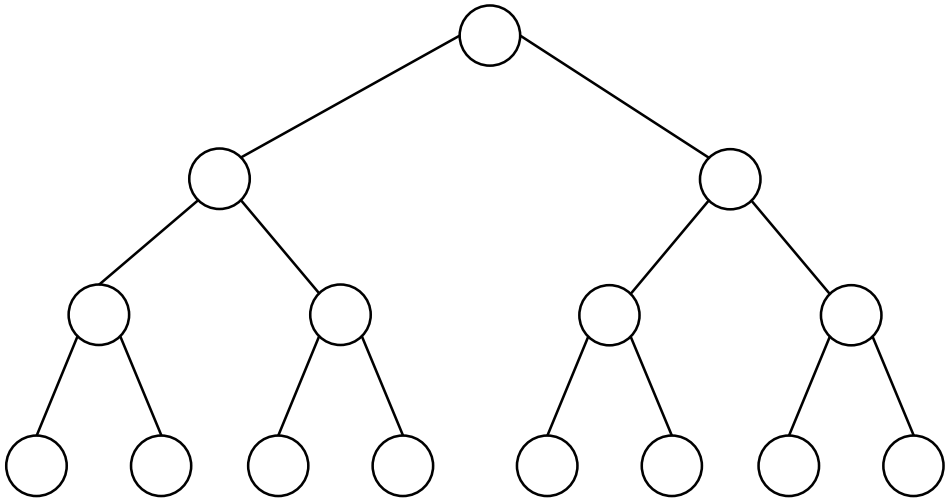
# Trees (dads : Dictionary of Algorithms and Data Structures)

- Binary Tree
  - A tree with at most two children for each node.
- Binary Search Tree
  - A binary tree where every node's left subtree has keys less than the node's key, and every right subtree has keys greater than the node's key.
- B-Tree
  - A BST in which every node has between m and 2m children, where m is a fixed integer.

NIST
National Institute of Standards and Technology

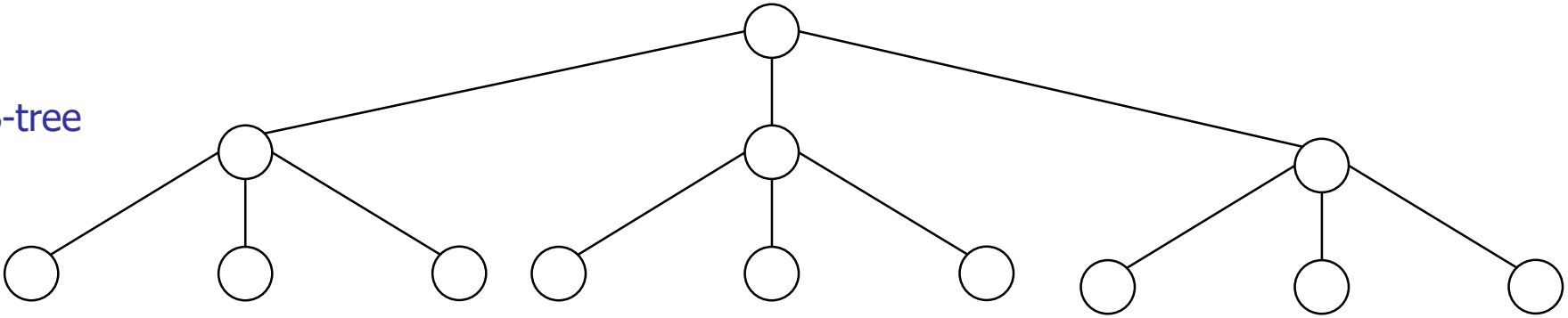
**http://xlinux.nist.gov/dads/HTML/**

# BST B-tree

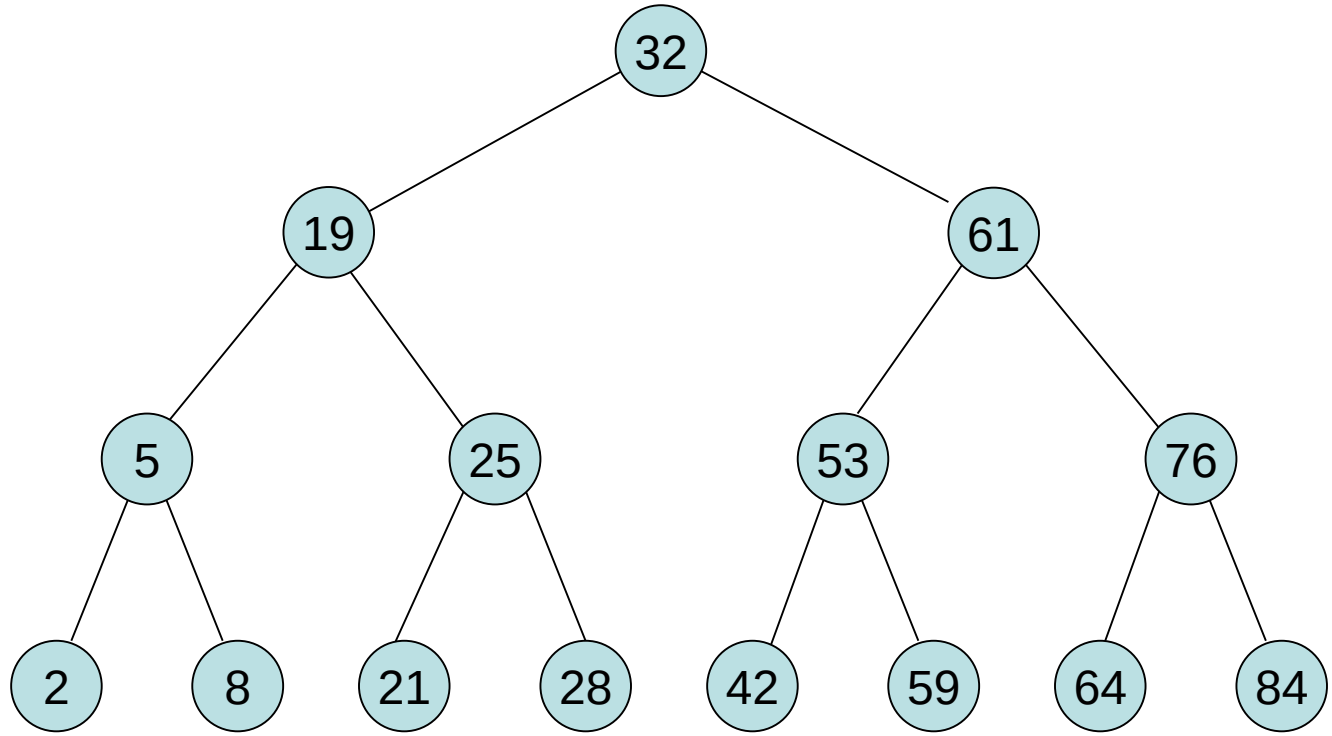


BST

B-tree

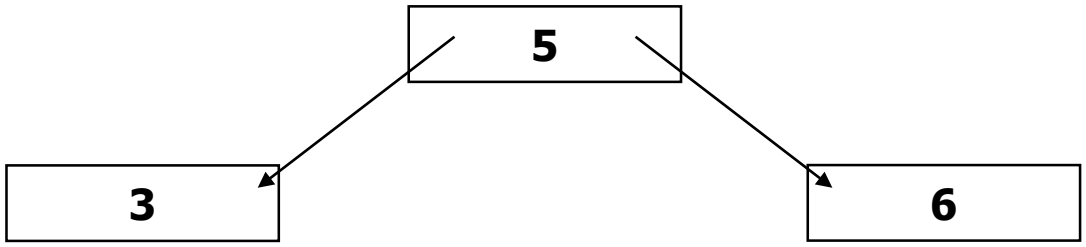


# BST example



# Arbre binaire de recherche (BST)

- Un arbre binaire de recherche est un arbre binaire dans lequel :
  - Chaque noeud possède une clé,
  - Telle que chaque nœud du sous-arbre gauche ait une clé inférieure ou égale à celle du nœud considéré,
  - Et chaque noeud du sous-arbre droit possède une clé supérieure ou égale à celle-ci

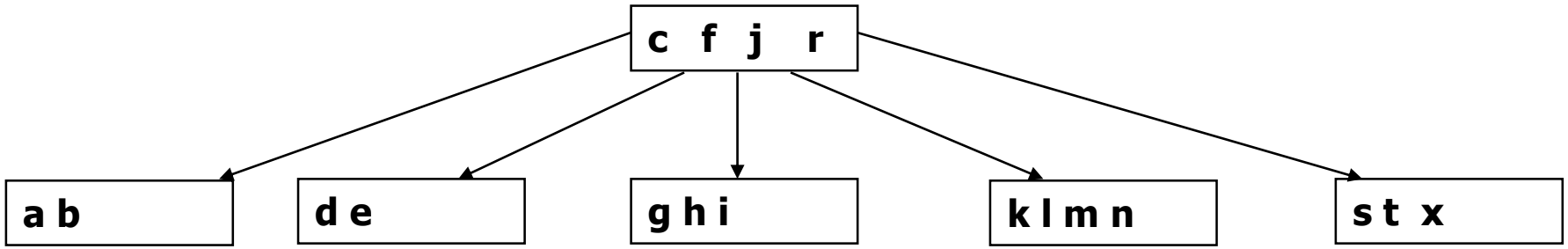


# Arbre équilibré (B tree)

- Structure arborescente dans laquelle tous les chemins de la racine aux feuilles ont même longueur.
  - Tous les noeuds feuilles sont au même niveau
  - La hiérarchie de l'arbre grossit par la racine

- Par opposition aux arbres dégénérés (arbres peignes).
  - Opération de recherche de complexité logarithmique au lieu de complexité linéaire.

# Arbre B (Btree) d'ordre 2

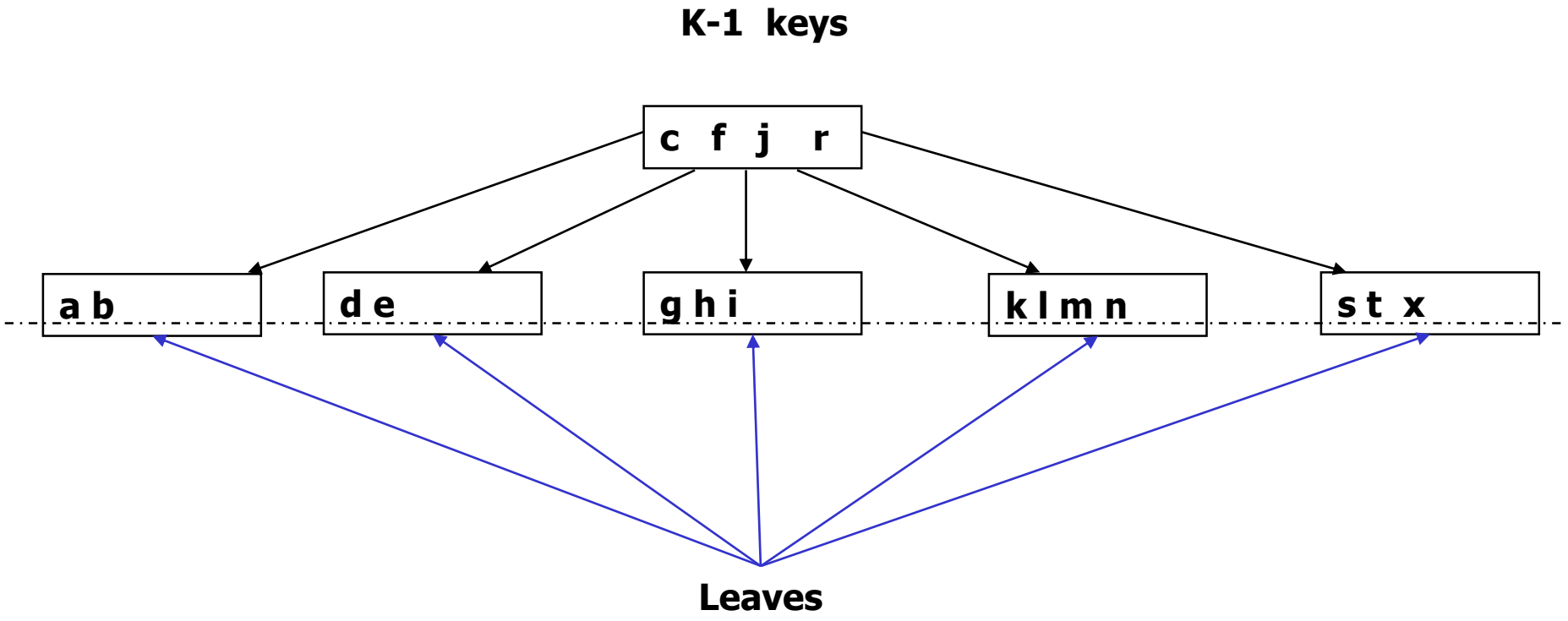



Un arbre-B d' ordre M Chaque noeud contient au moins M et au plus 2M éléments

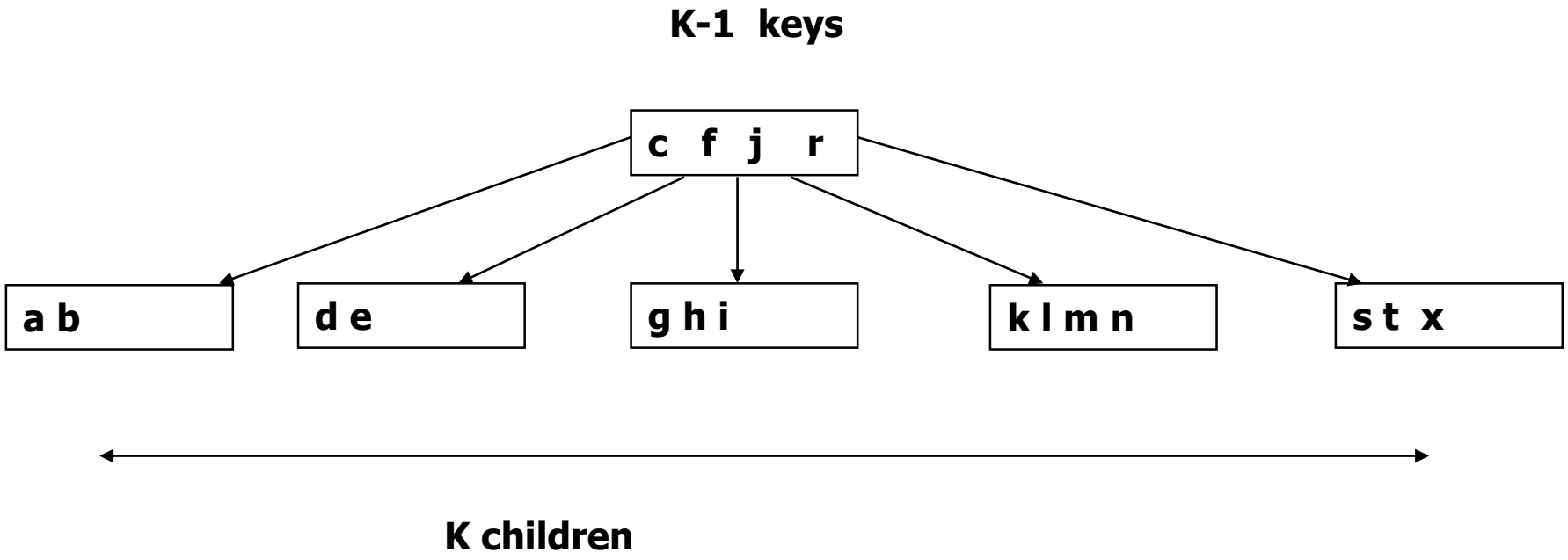
# M order B-tree

- A B-tree of order m satisfies:
  1. Every node has at most 2m children.
  2. Every node has at least m children.
  3. The root has at least two children.
  4. All leaves appear in the same level
  5. A non-leaf node with k children contains k−1 keys.

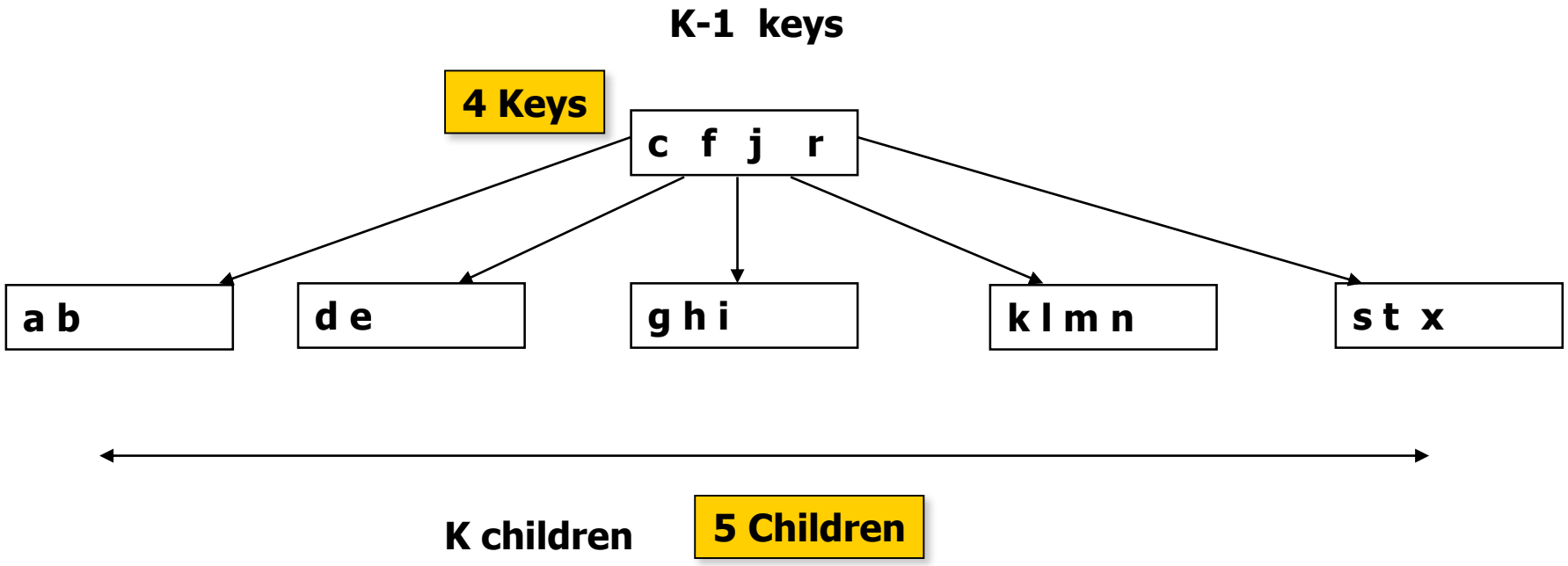
# All leaves appear in the same level



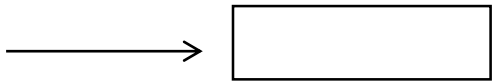
# A non-leaf node with k children contains k−1 keys.



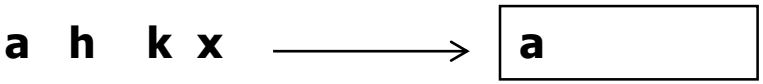
# A non-leaf node with k children contains k−1 keys.



# Arbre B (ordre 2) création

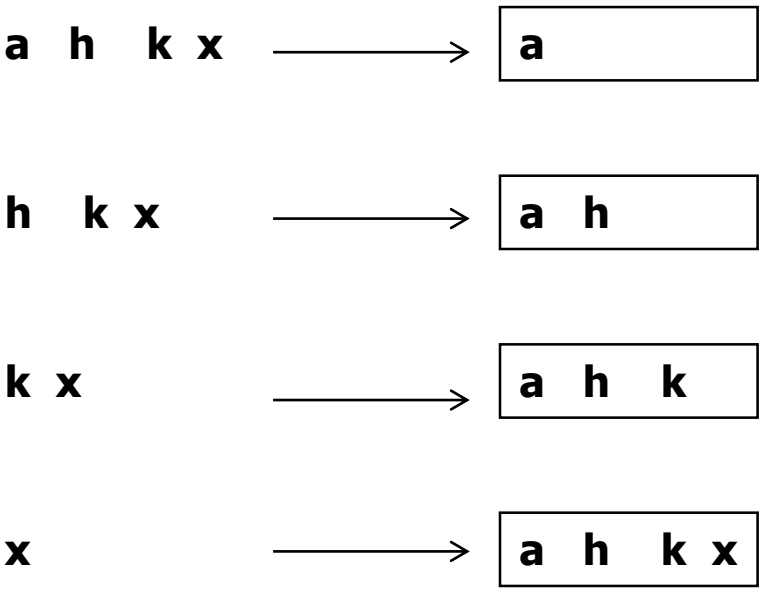
**a h k x**  **Noeud Racine (Root)**

# Arbre B (ordre 2) création




**Seule la racine peut avoir un nombre d'élements inférieure à m**

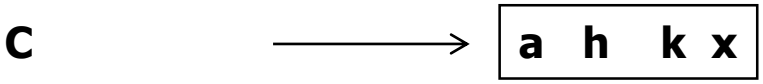
# Arbre B (ordre 2) création



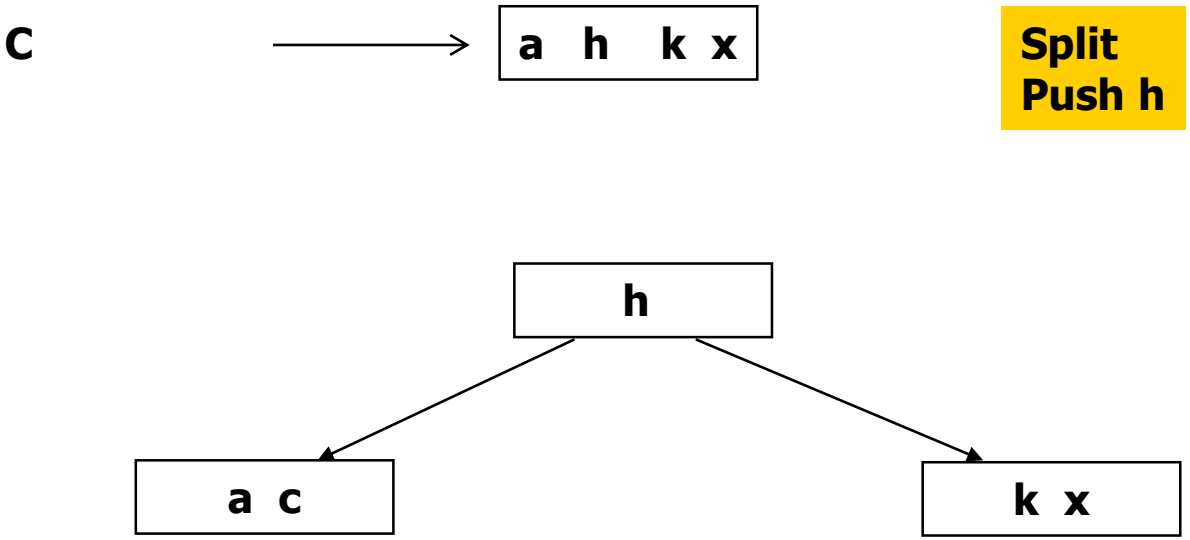

**Noeud Racine (Root)**

**Root Full**

# Arbre B création



# Arbre B création



# Algorithme : Insert(key)

Find(key) to locate new position

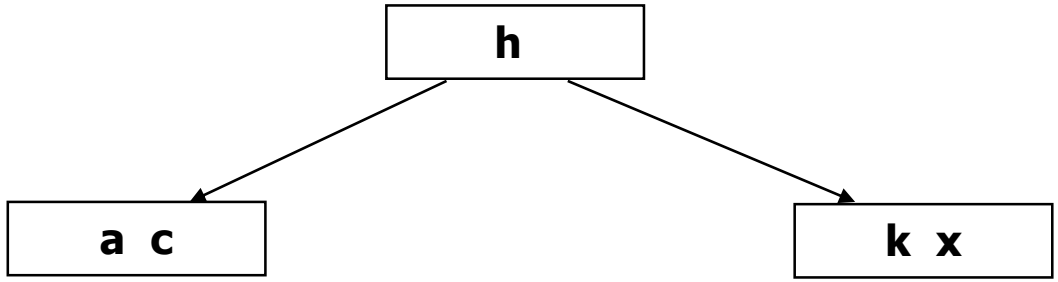
move right neighbors within node

if overflow split into 2 nodes

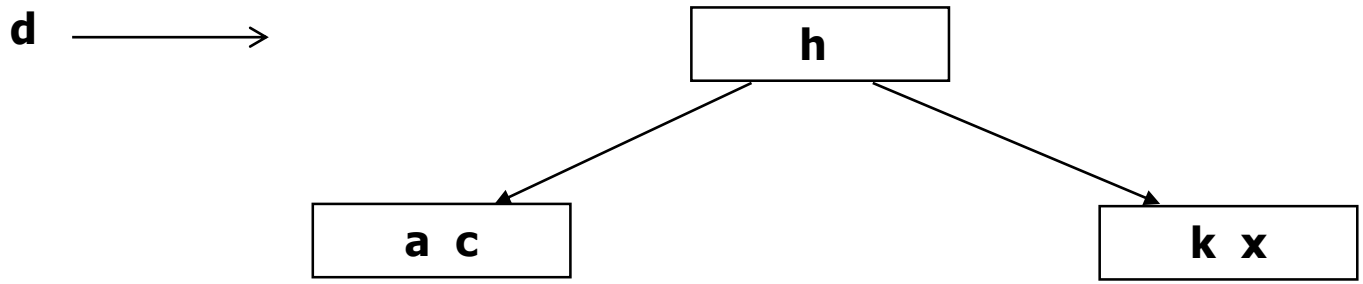
    promote middle key to parent node

    if parent overflows, repeat

# Arbre B insertion



# Arbre B insertion



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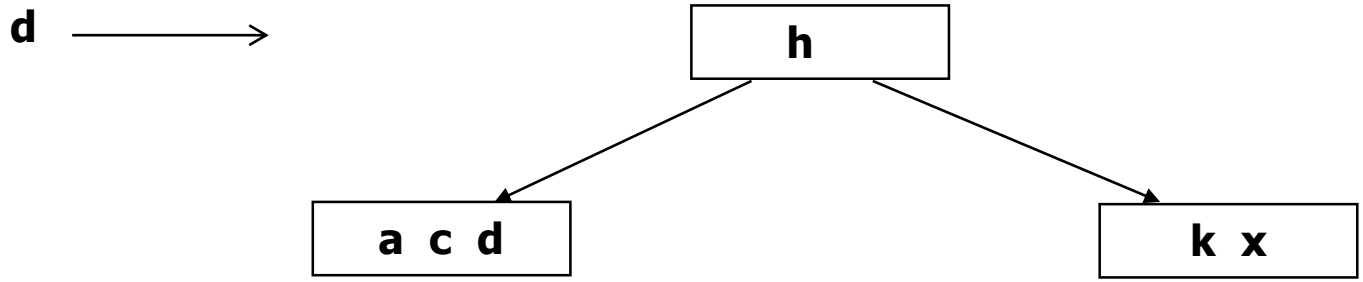

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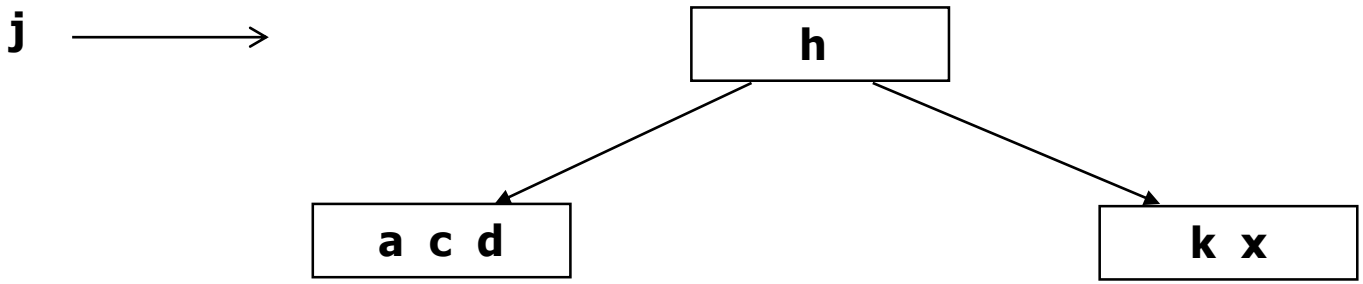

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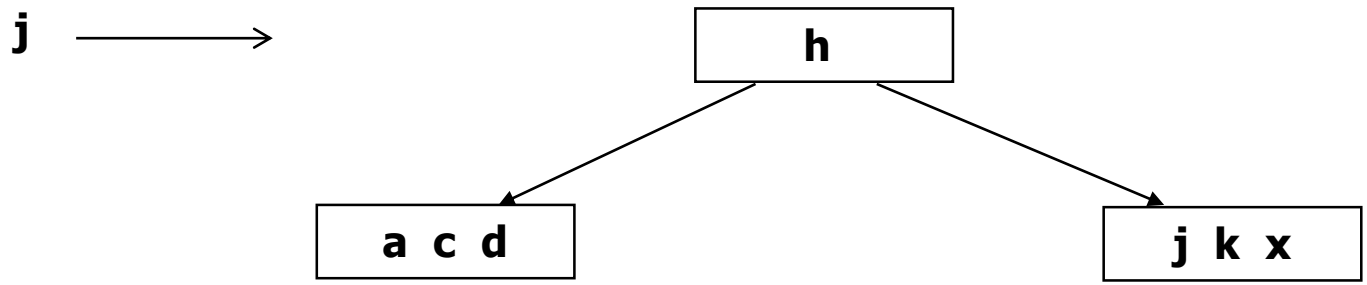

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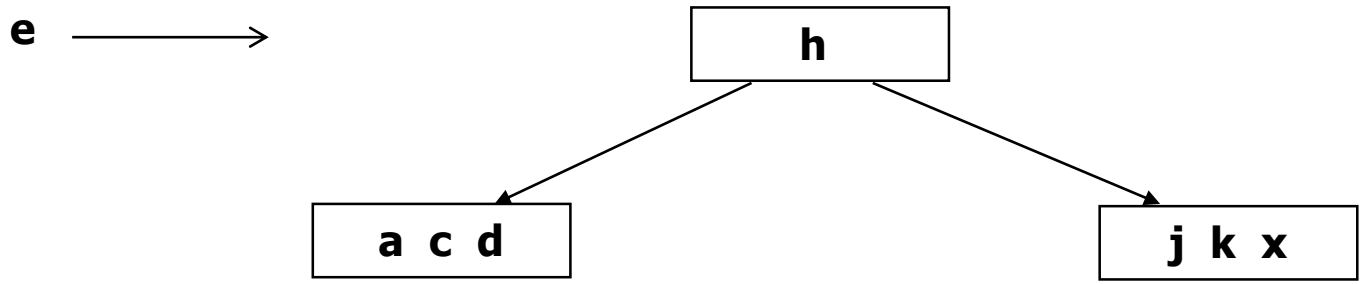

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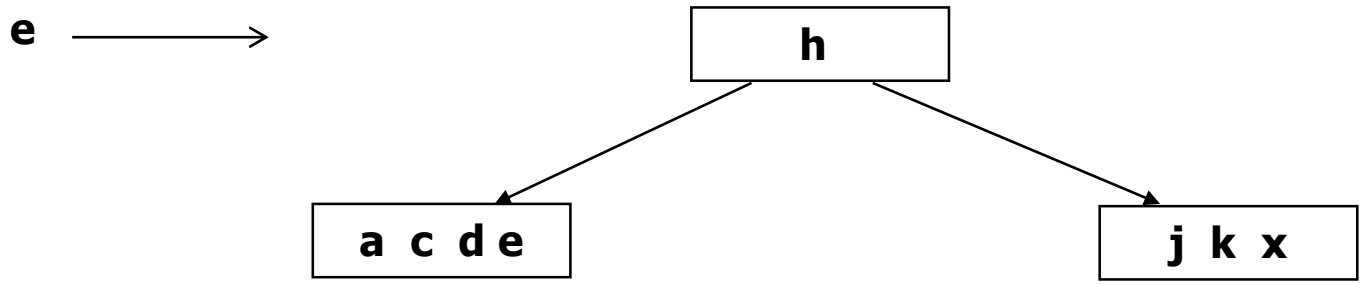

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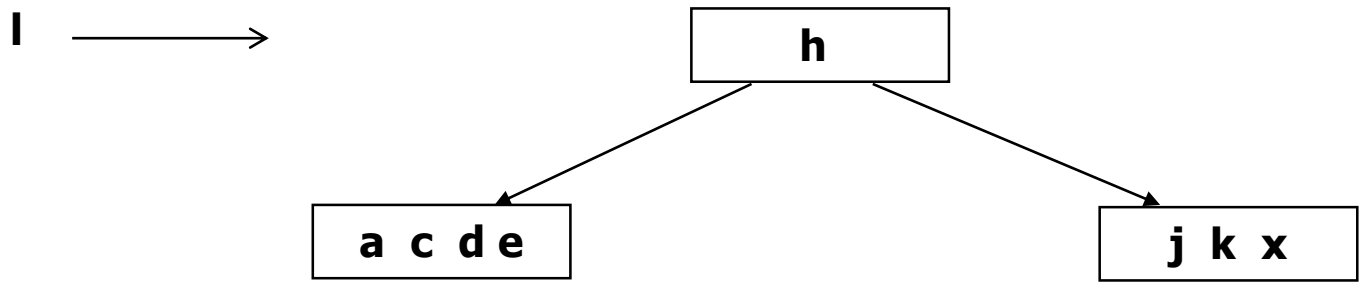

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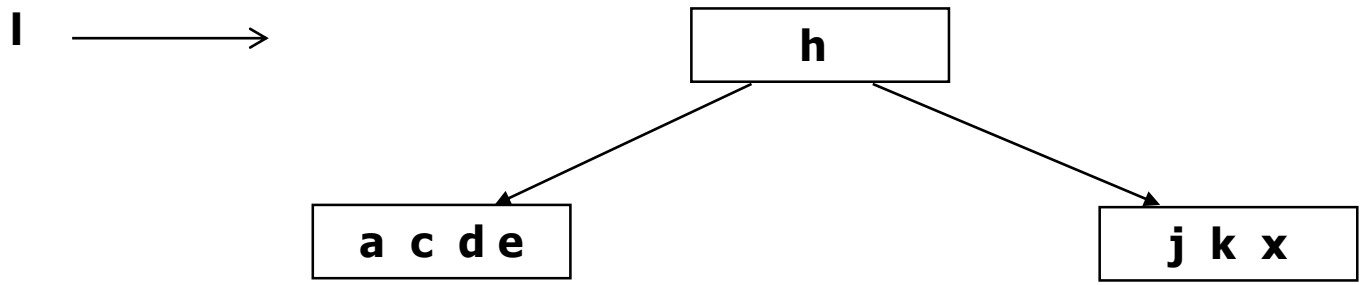

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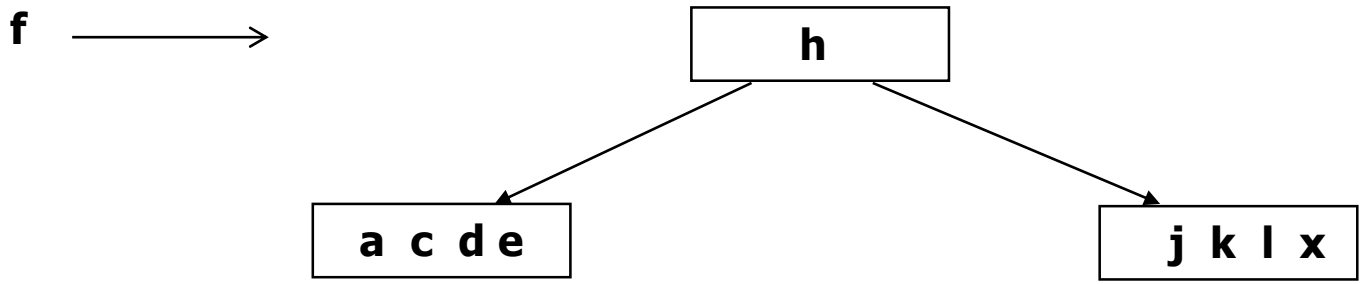

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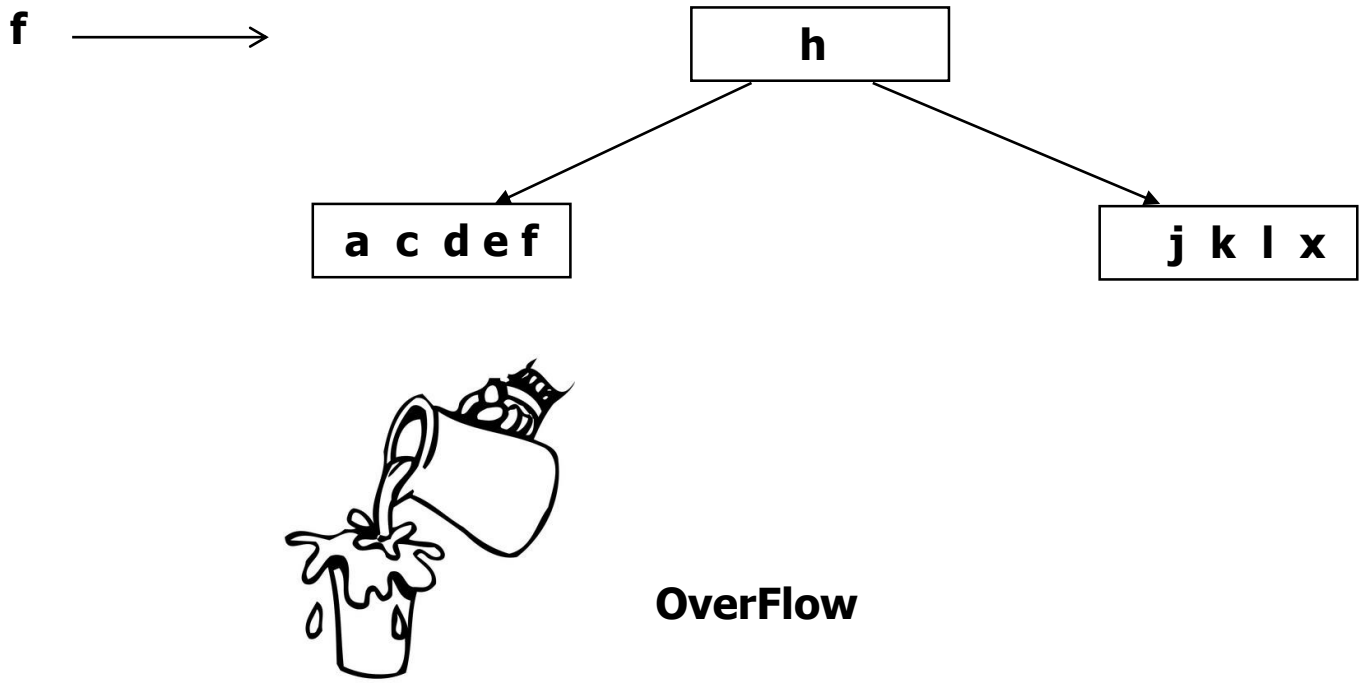

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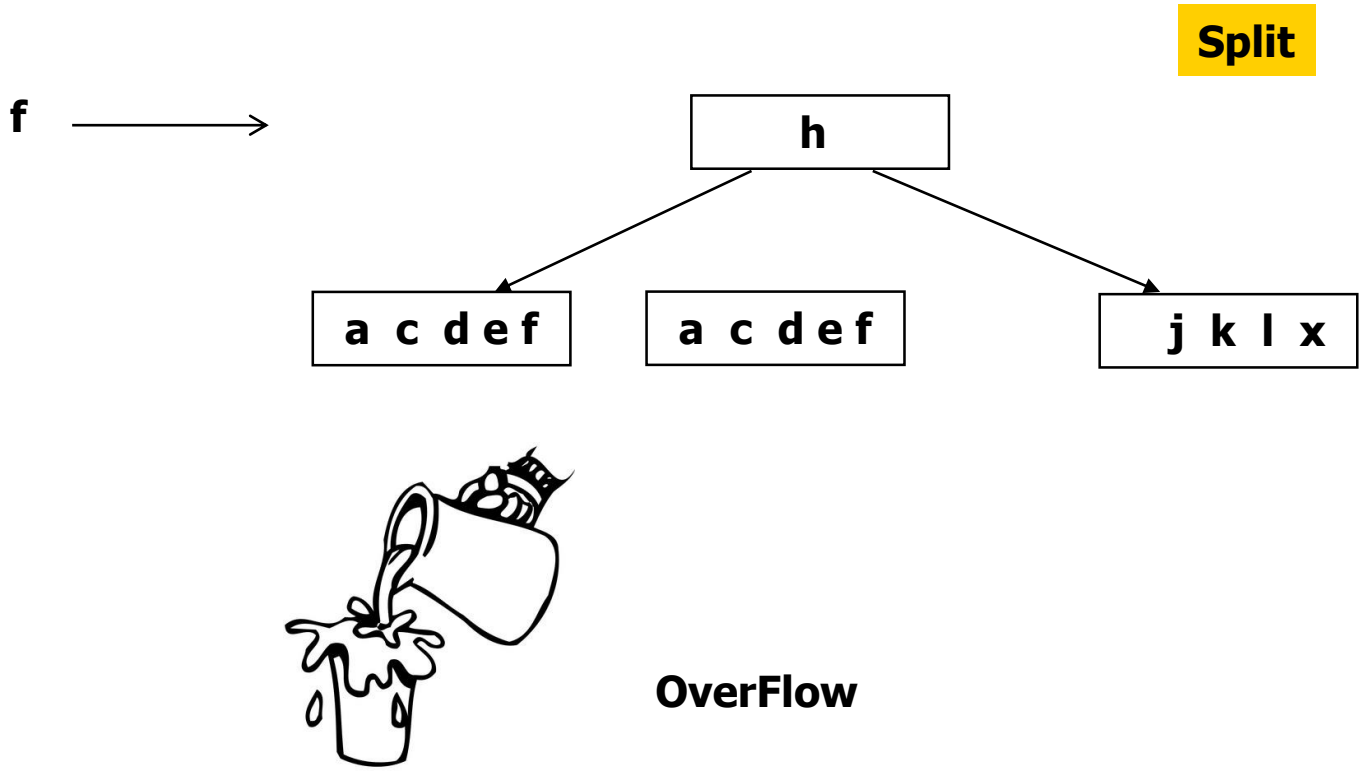

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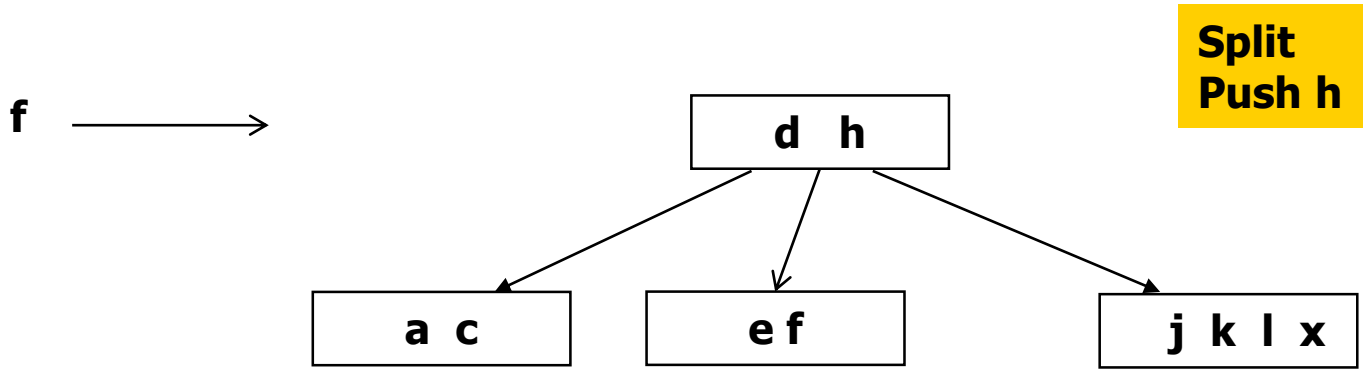
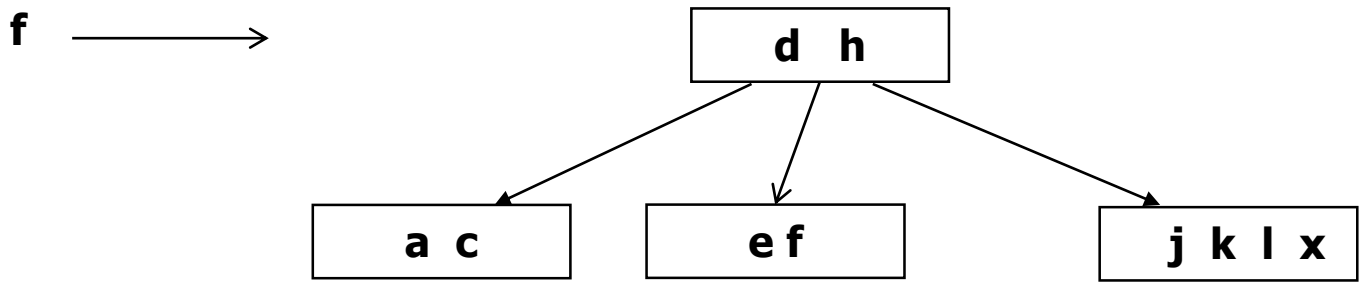



**Split**
**Push h**

**OverFlow**

**Promote middle key**

# Arbre B insertion



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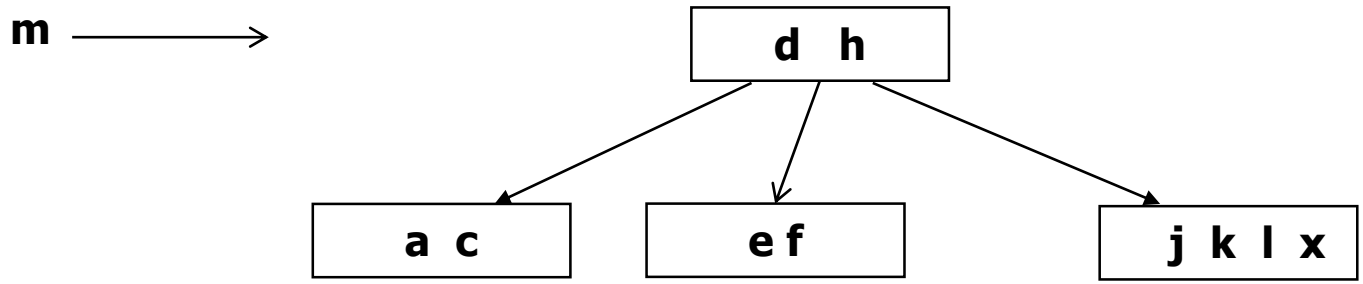

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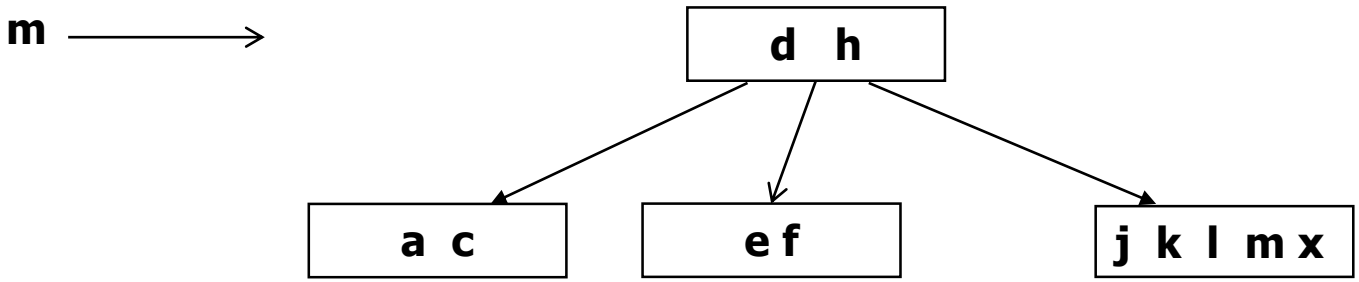


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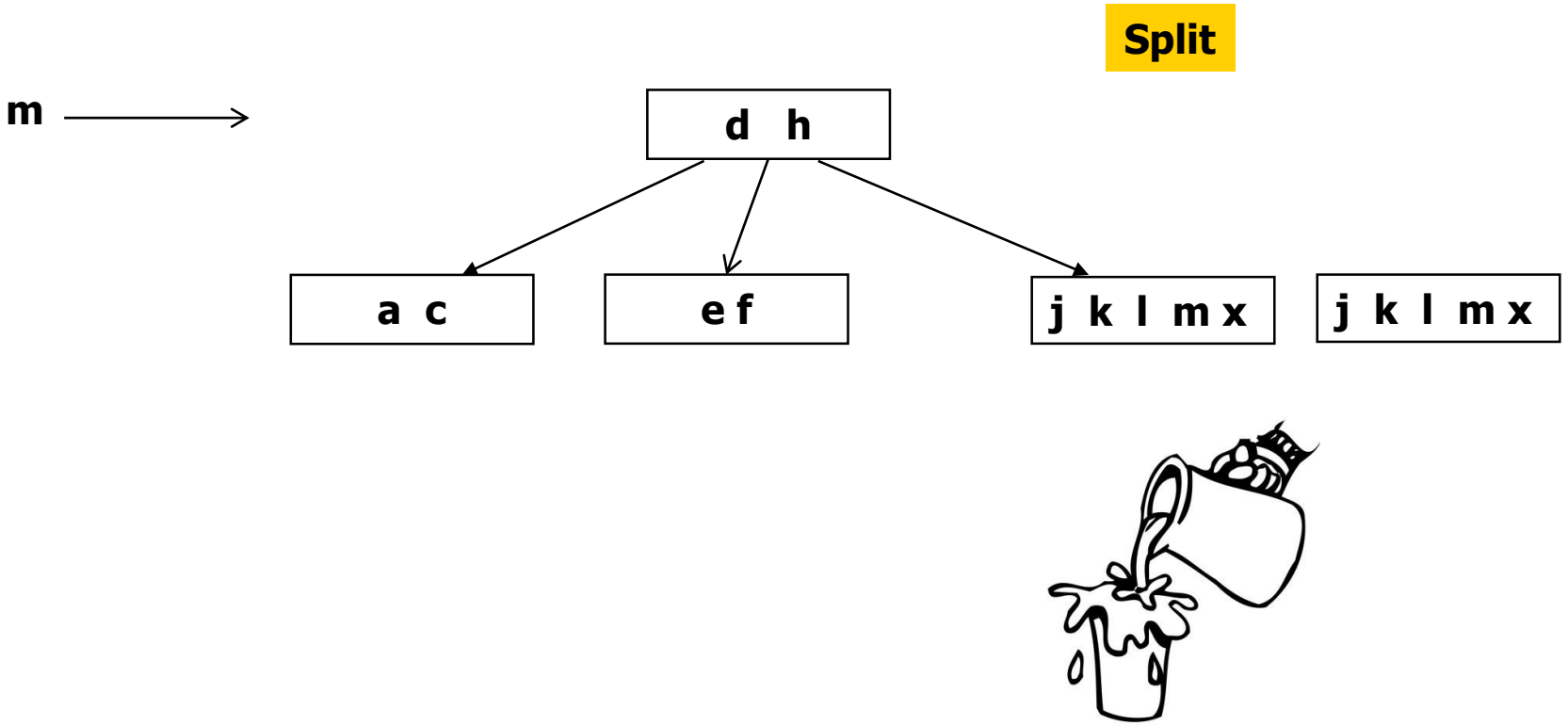
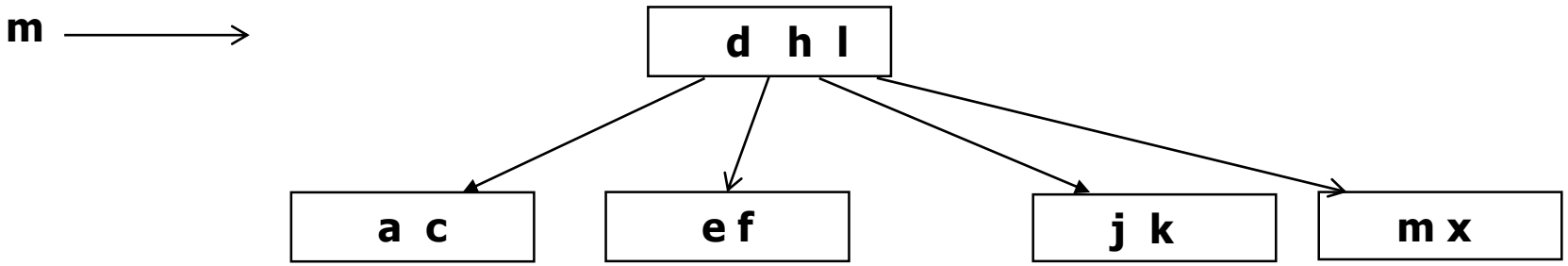


# Arbre B insertion

**Split**
**Push l**

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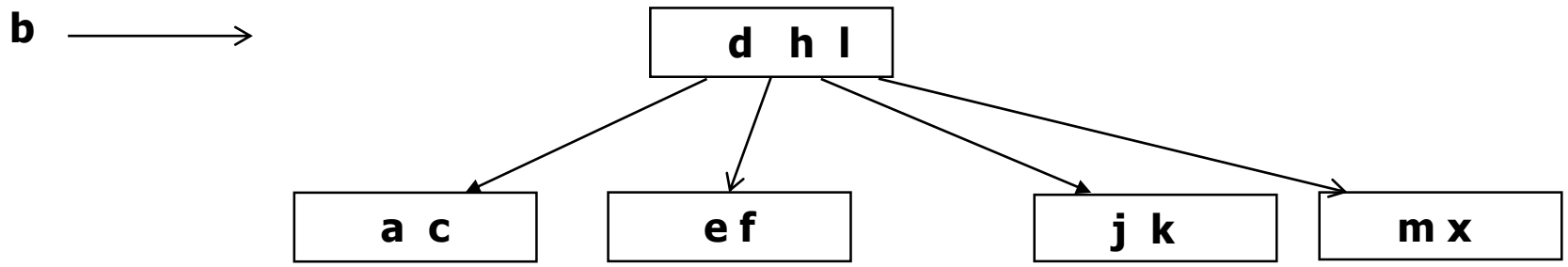

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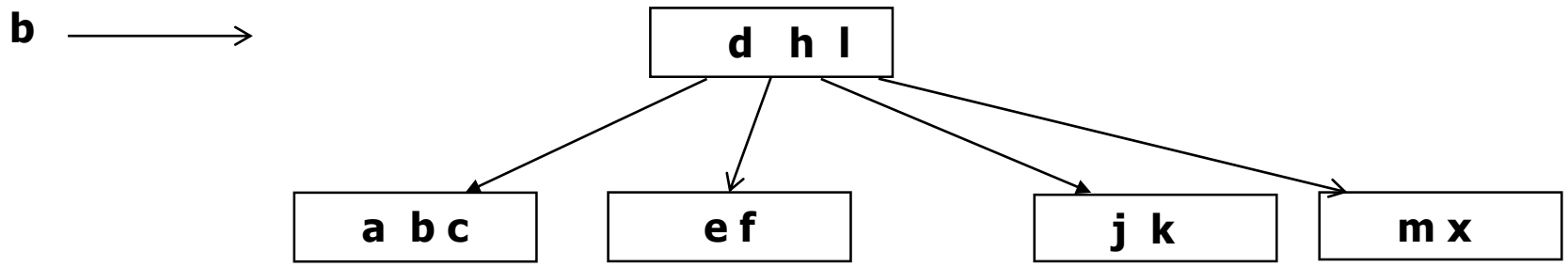

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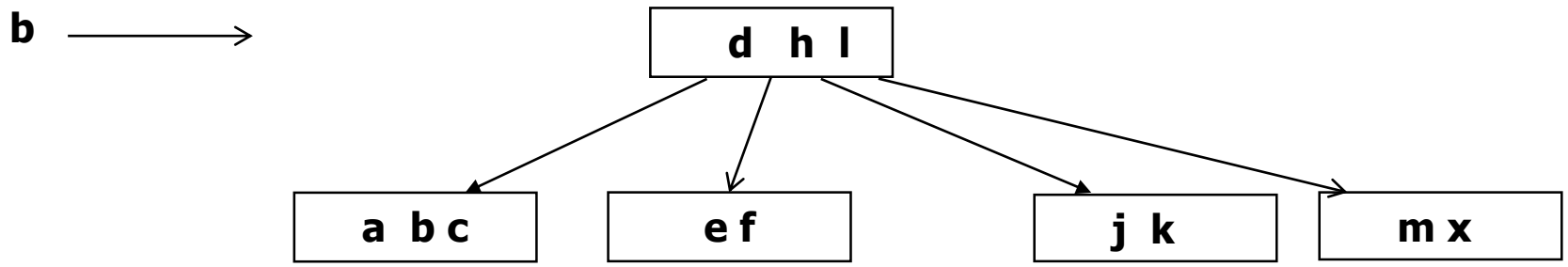

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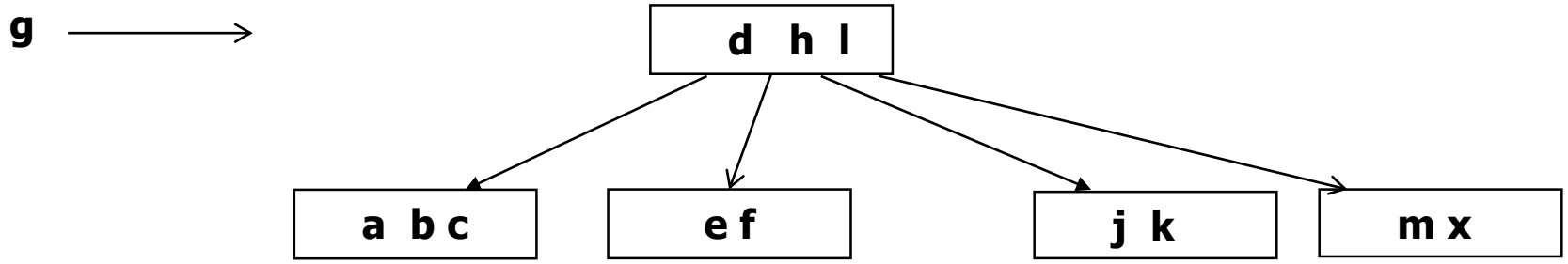

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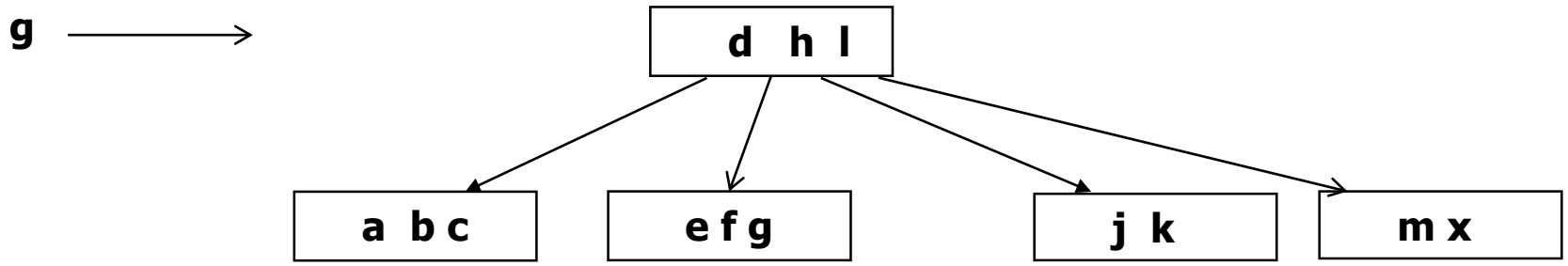

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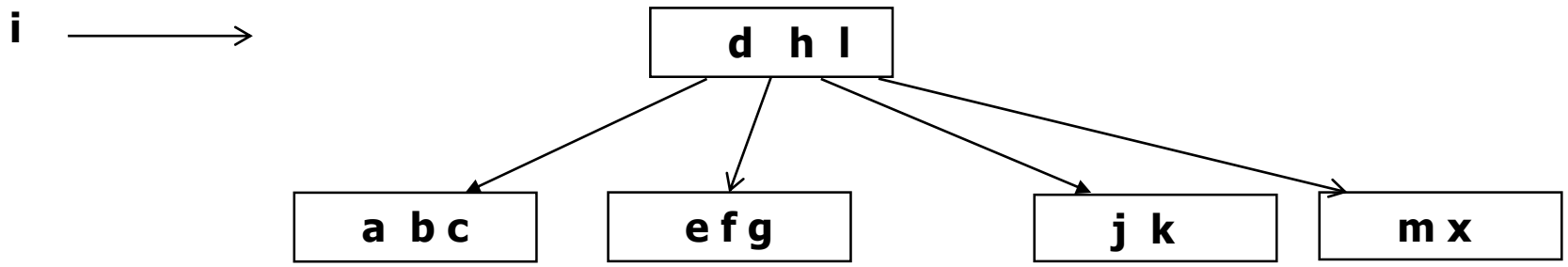

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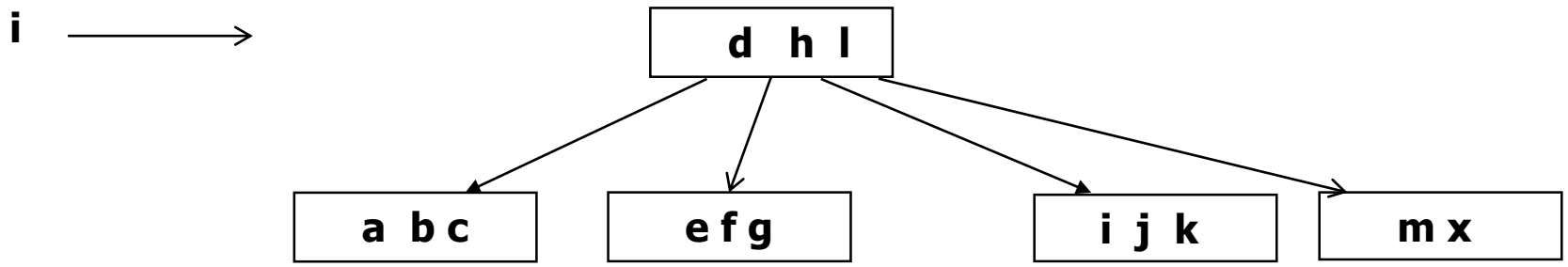

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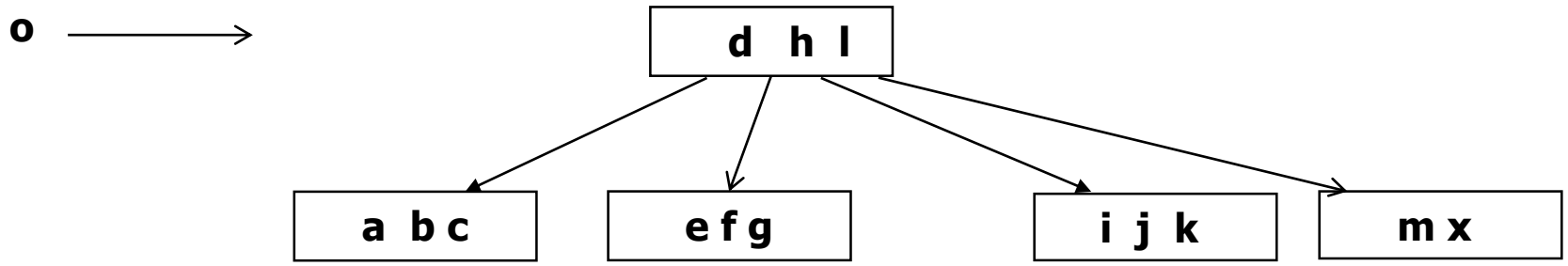

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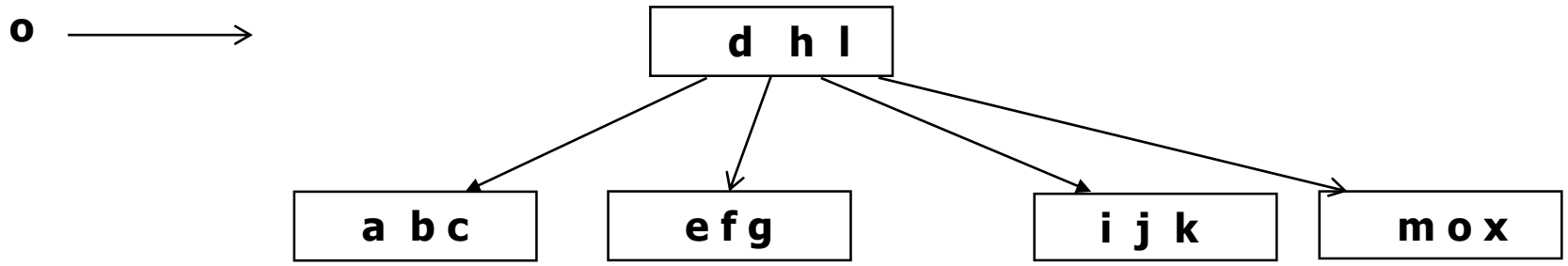
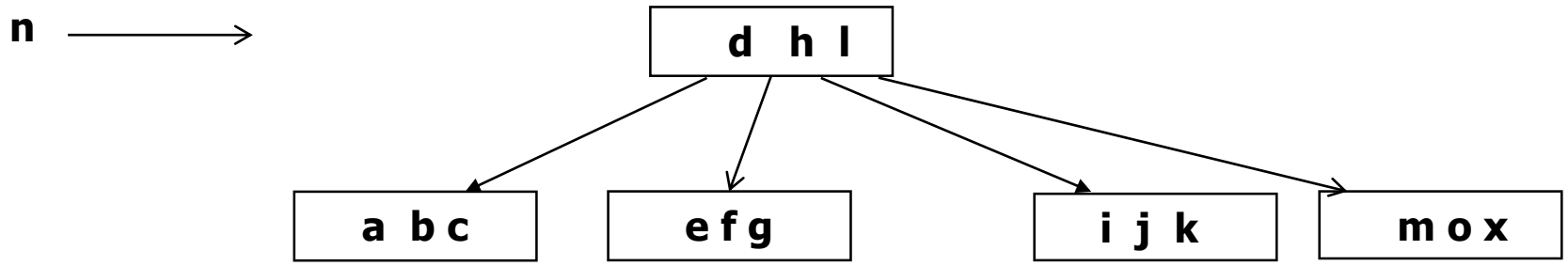
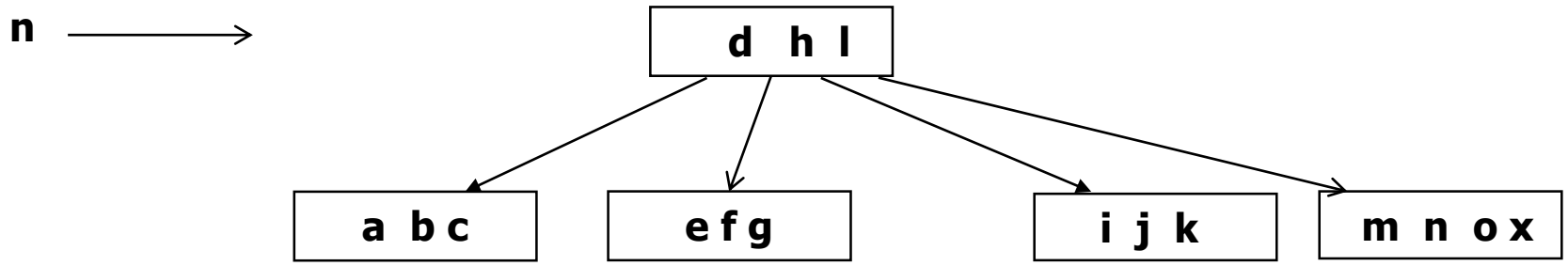


# Arbre B insertion



# Arbre B insertion



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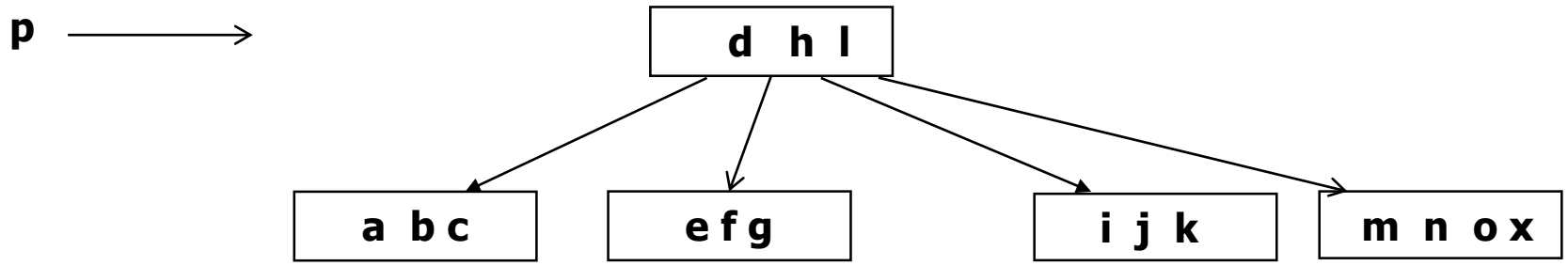
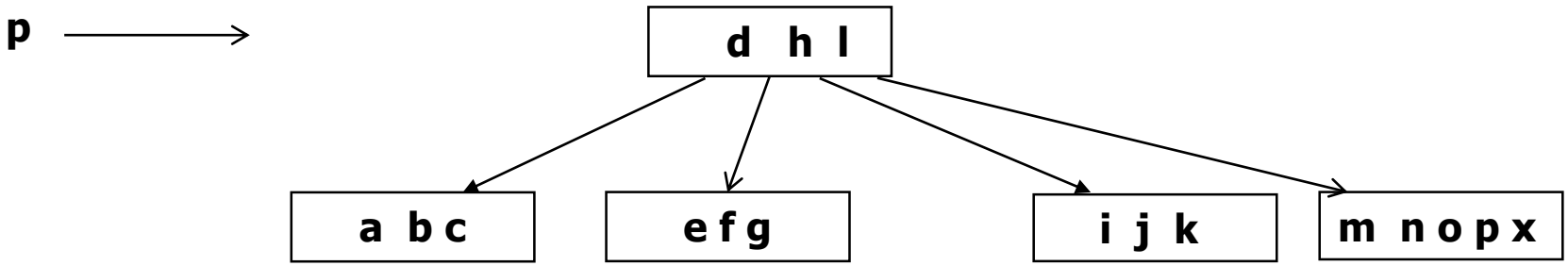
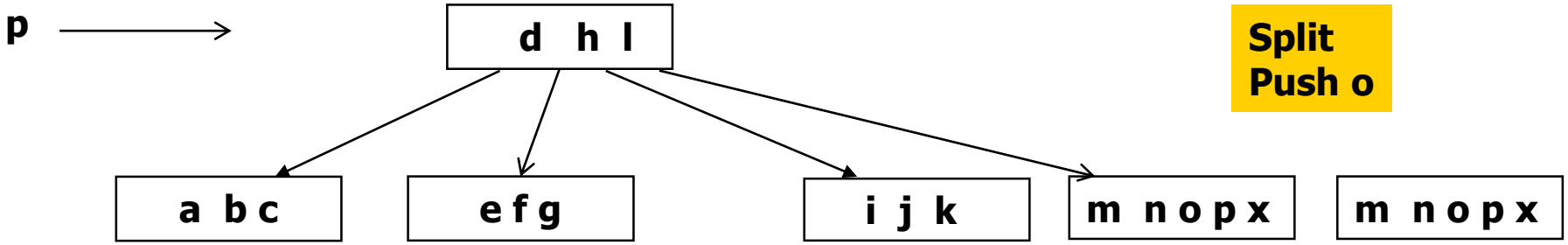


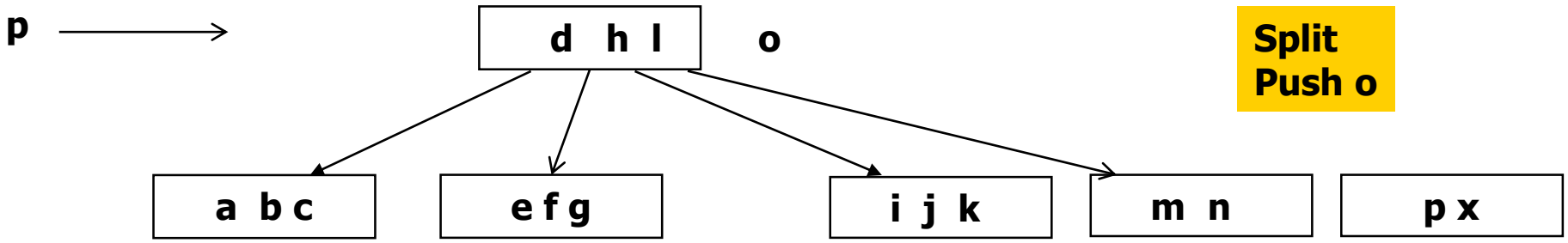
# Arbre B insertion




# Arbre B insertion

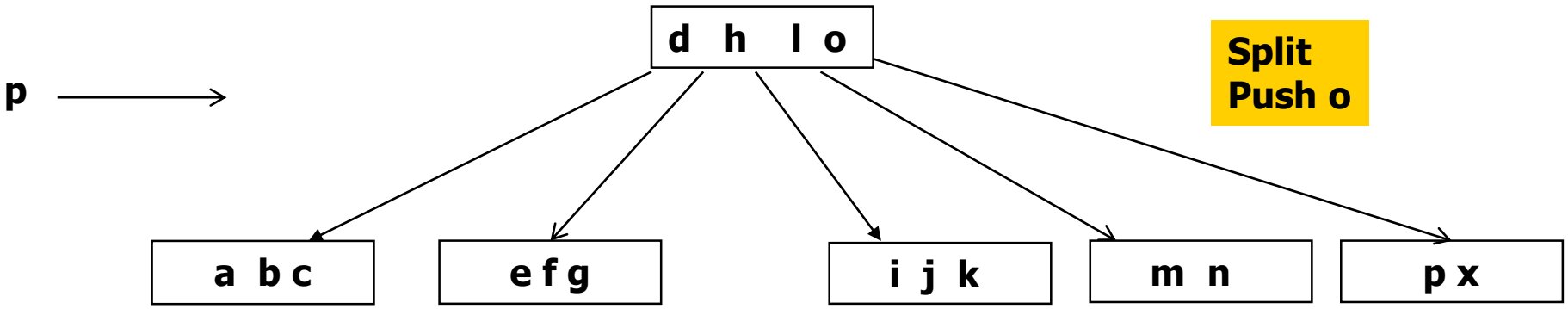



# Arbre B insertion



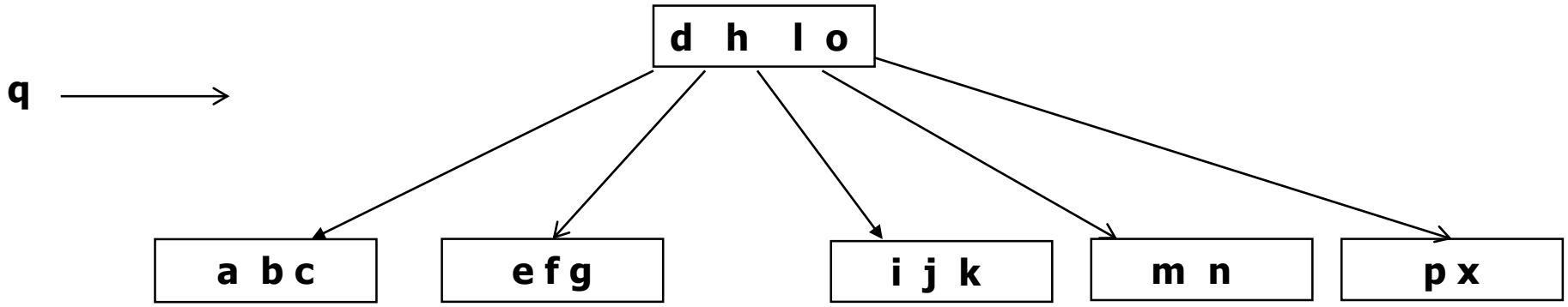

**Split**
**Push o**

# Arbre B insertion

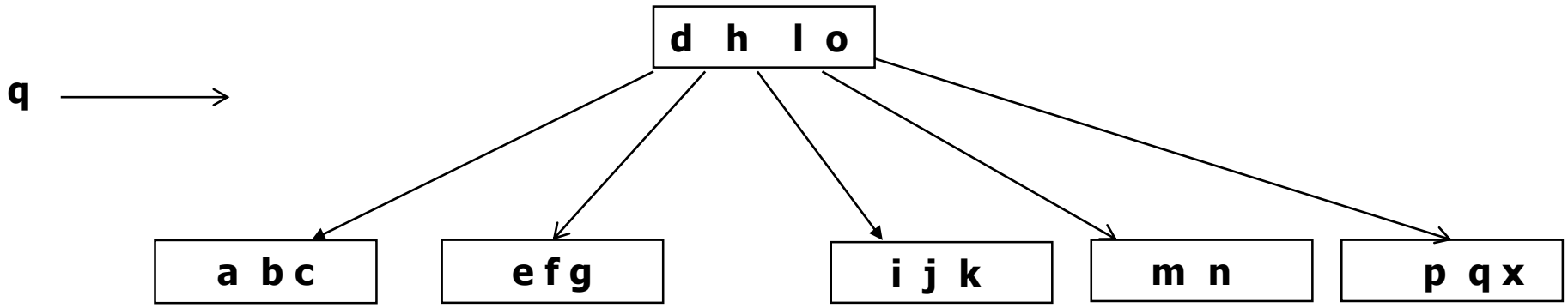



**Split**
**Push o**

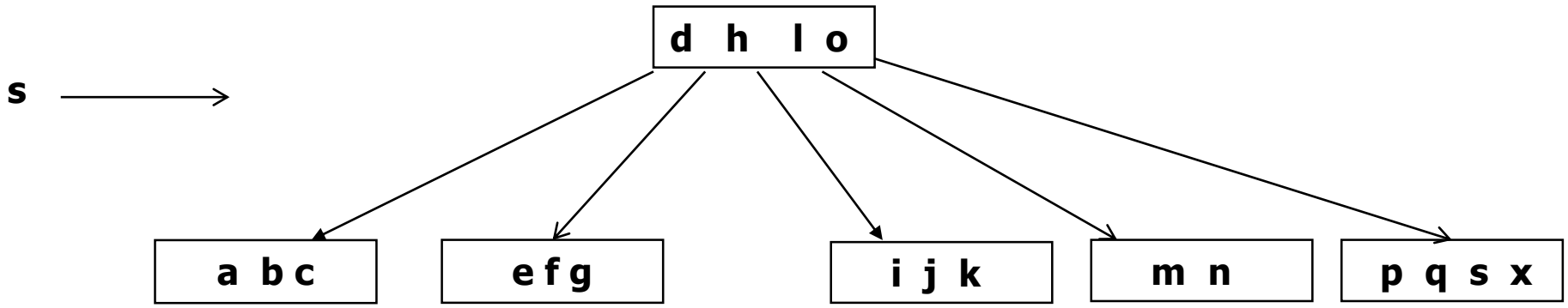
# Arbre B insertion



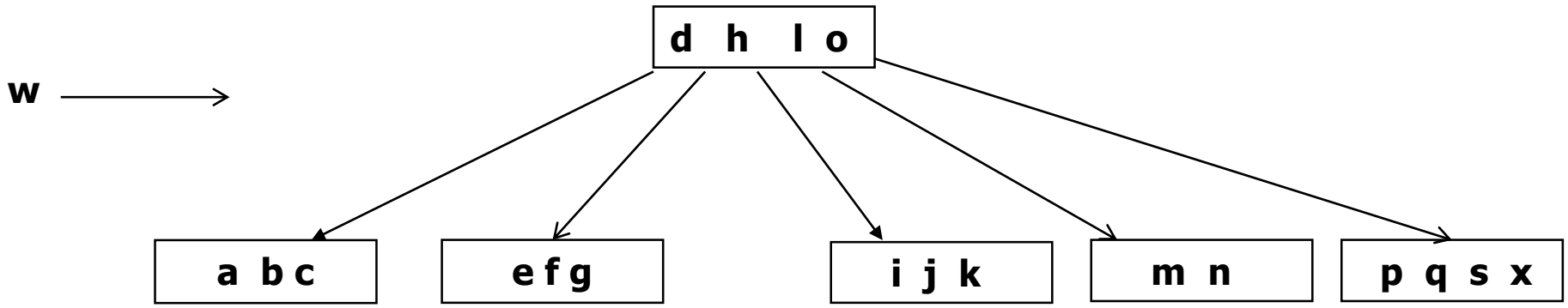
# Arbre B insertion



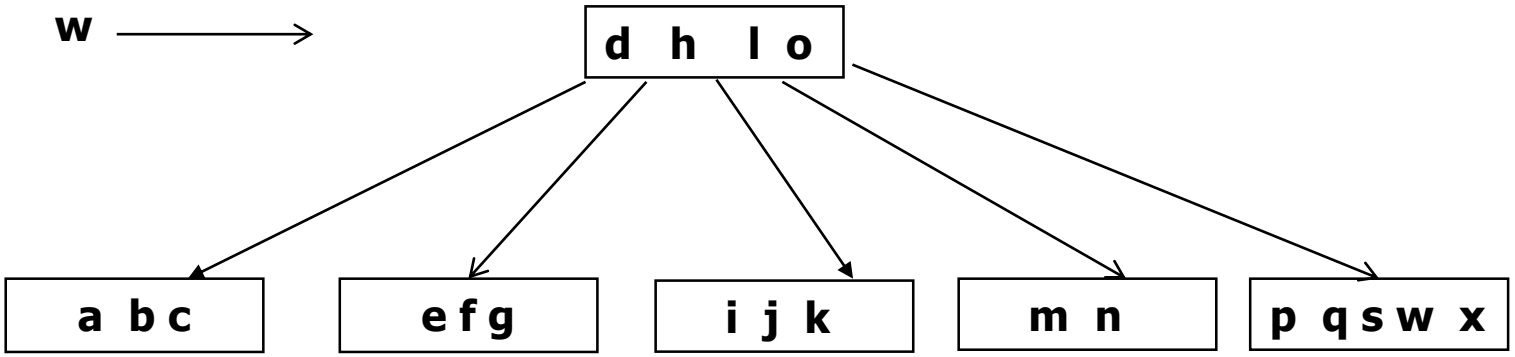
# Arbre B insertion



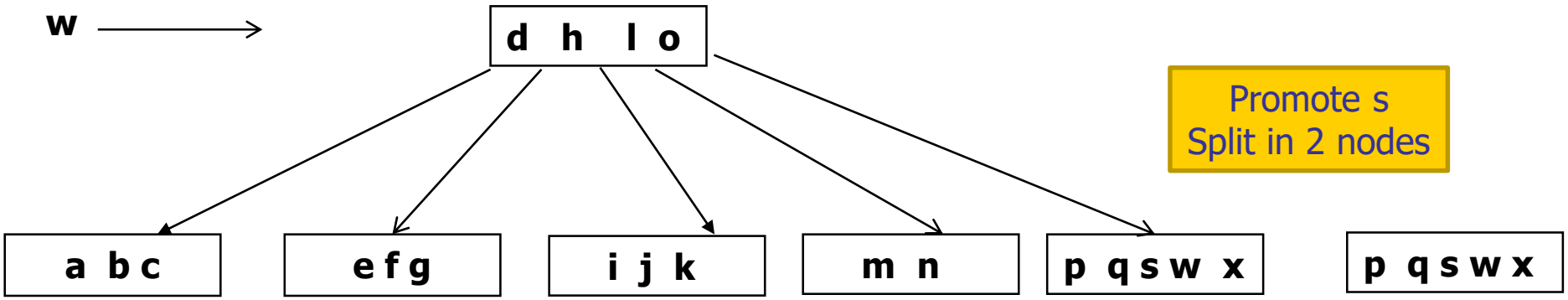
# Arbre B insertion



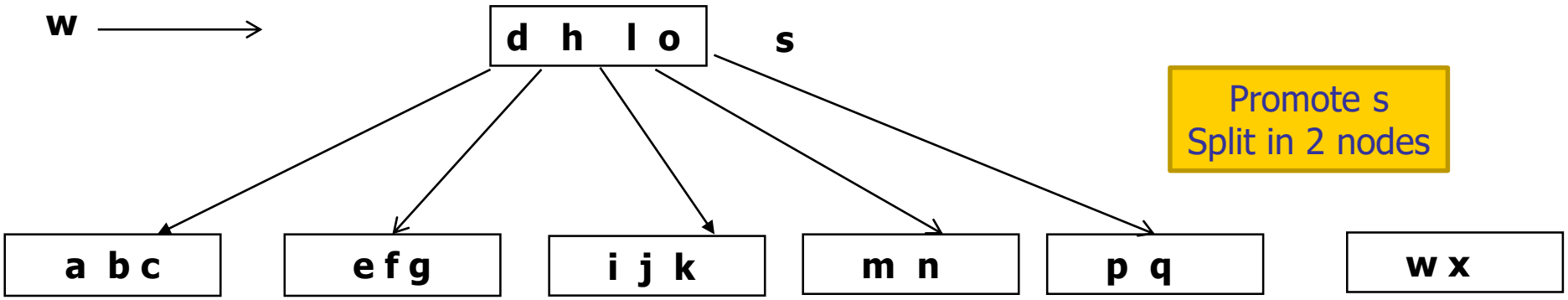
# Arbre B insertion




# Arbre B insertion

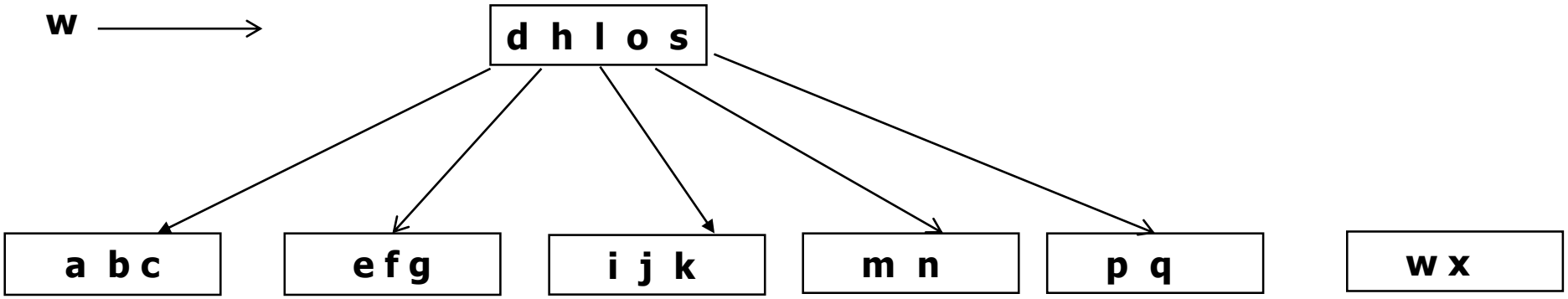



# Arbre B insertion




Promote s
Split in 2 nodes

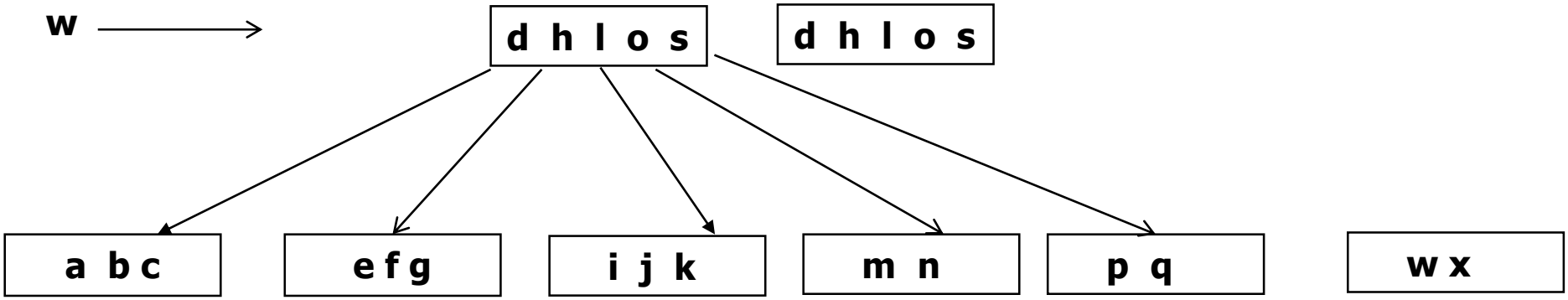
# Arbre B insertion




# Arbre B insertion

Promote l
Split in 2 nodes

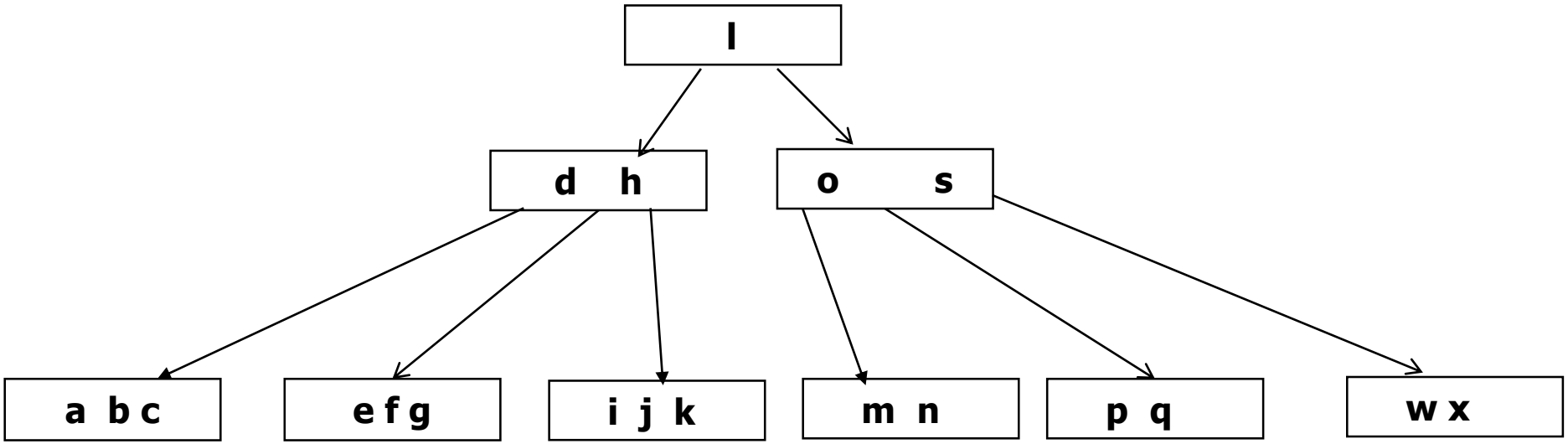
w →



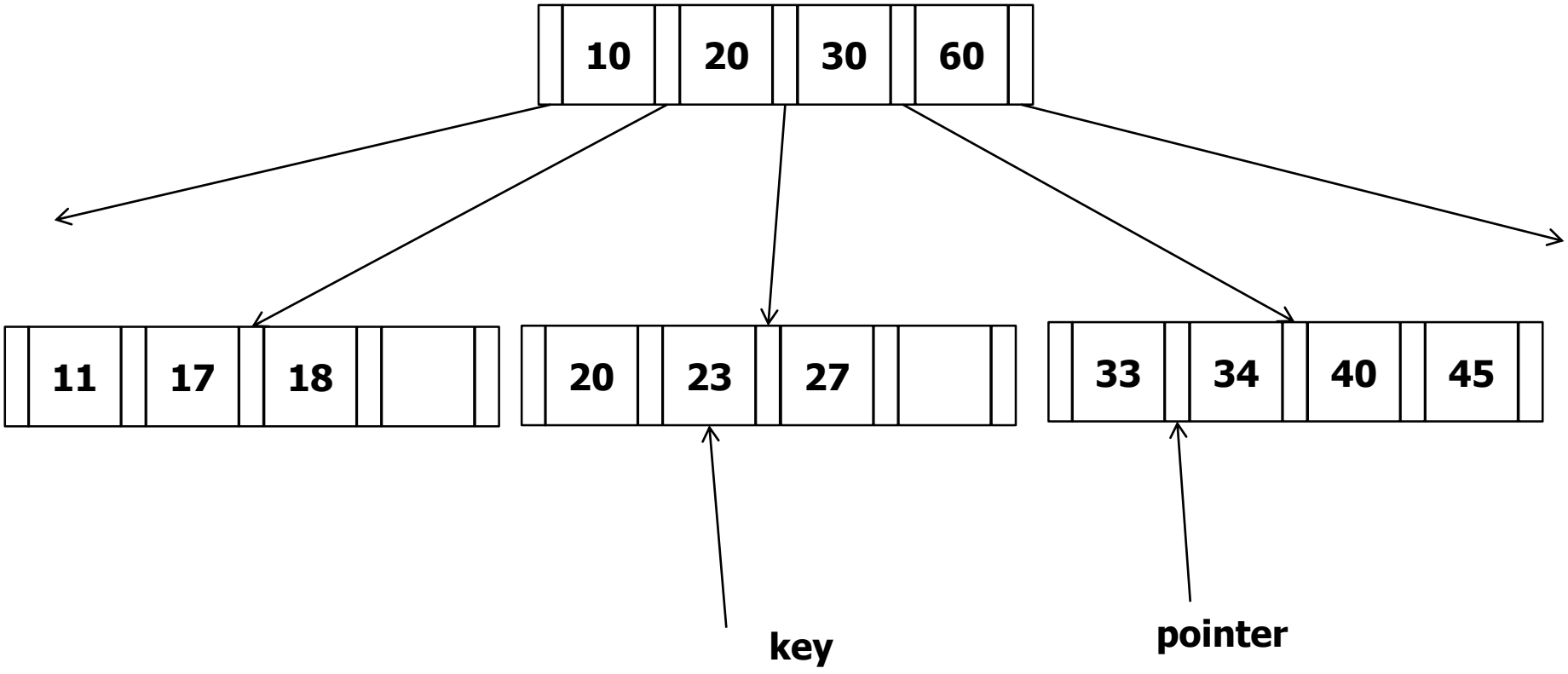

# Arbre B insertion




# Arbre B insertion



# B-tree representation



# Comer Article Representation

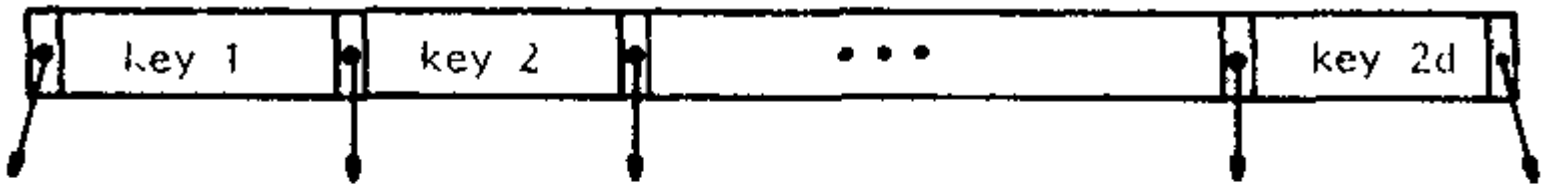
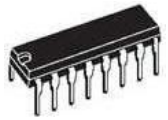


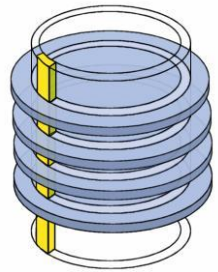

FIGURE 4. A node in a B-tree of order $d$ with $2d$ keys and $2d + 1$ pointers.

# BST vs B-tree

- BST : Binary Seach Tree
  - BST nodes contain only one key.
  - BST nodes have only Two children
  - In memory tree



- B-tree :
  - B-tree may have a variable number of keys and children
  - Multiway tree
  - Disk tree

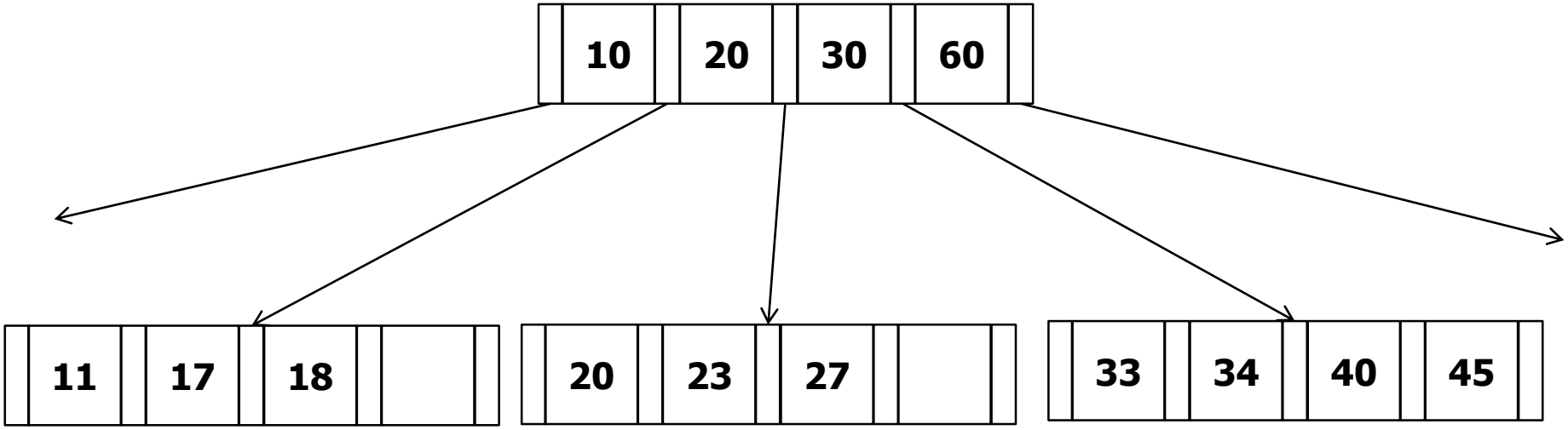


# Height

- Tree search time is proportional to the height of the tree.
- BST height :
  - BST nodes contain only one key.
  - $log_2\ n$
    - Where *n* is the number of nodes in the tree.
- B-tree height
  - B-tree contains a lot of keys in each node so that the height of the tree keep small.
  - worst case : $O(log_b\ n)$.
    - Where b base of the logarithm depend on the Btree degree.

# B-tree example

**K children in the node.**
**k – 1 keys in the node.**

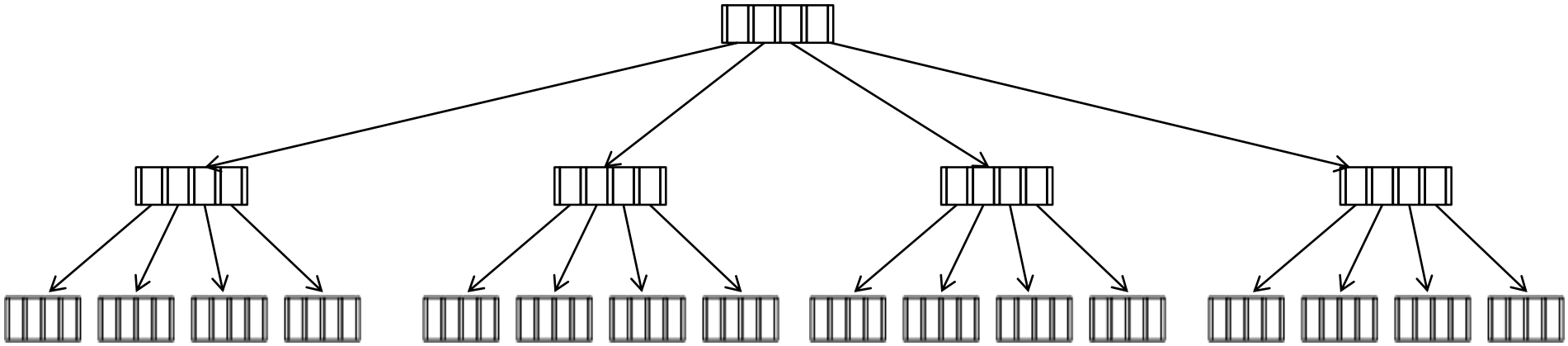


# B-tree Height and Key Numbers

**1** ⟶ **4** **m = 2**

**2** ⟶ **4 x 4 = 16**

**3** ⟶ **16 x 4 = 64**



# B-tree Height and Key Numbers m = 2

| Height | Keys # |
|---|---|
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |
| 4 | 256 |
| 5 | 1024 |
| 6 | 4096 |
| 7 | 16384 |
| 8 | 65536 |
| 9 | 262144 |
| 10 | 1048576 |

# B-tree Height and Key Numbers and m

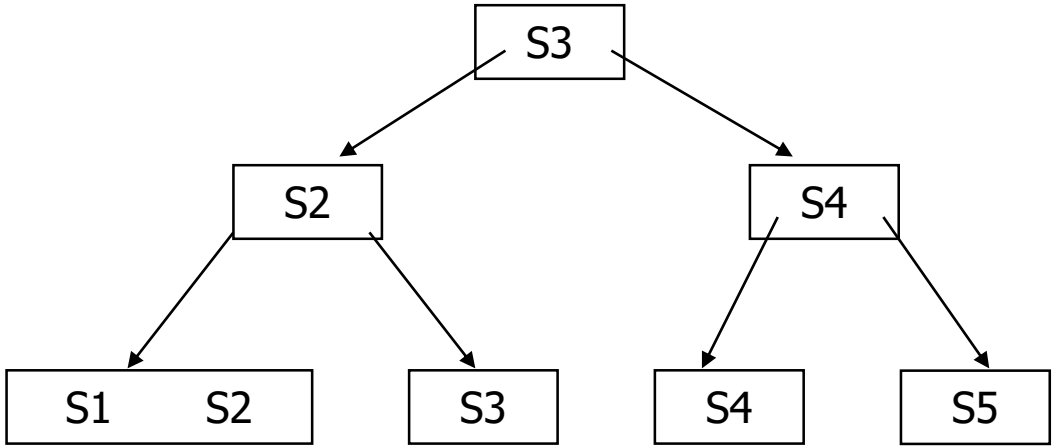
| | 2*m | | | | |
|---|---|---|---|---|---|
| | 2 | 4 | 6 | 8 | 10 |
| Height | Keys # | Keys # | Keys # | Keys # | Keys # |
| 1 | 2 | 4 | 6 | 8 | 10 |
| 2 | 4 | 16 | 36 | 64 | 100 |
| 3 | 8 | 64 | 216 | 512 | 1000 |
| 4 | 16 | 256 | 1296 | 4096 | 10000 |
| 5 | 32 | 1024 | 7776 | 32768 | 100000 |
| 6 | 64 | 4096 | 46656 | 262144 | 1000000 |
| 7 | 128 | 16384 | 279936 | 2097152 | 10000000 |
| 8 | 256 | 65536 | 1679616 | 16777216 | 100000000 |
| 9 | 512 | 262144 | 10077696 | 134217728 | 1000000000 |
| 10 | 1024 | 1048576 | 60466176 | 1073741824 | 10000000000 |

# B-tree Height and Key Numbers and m

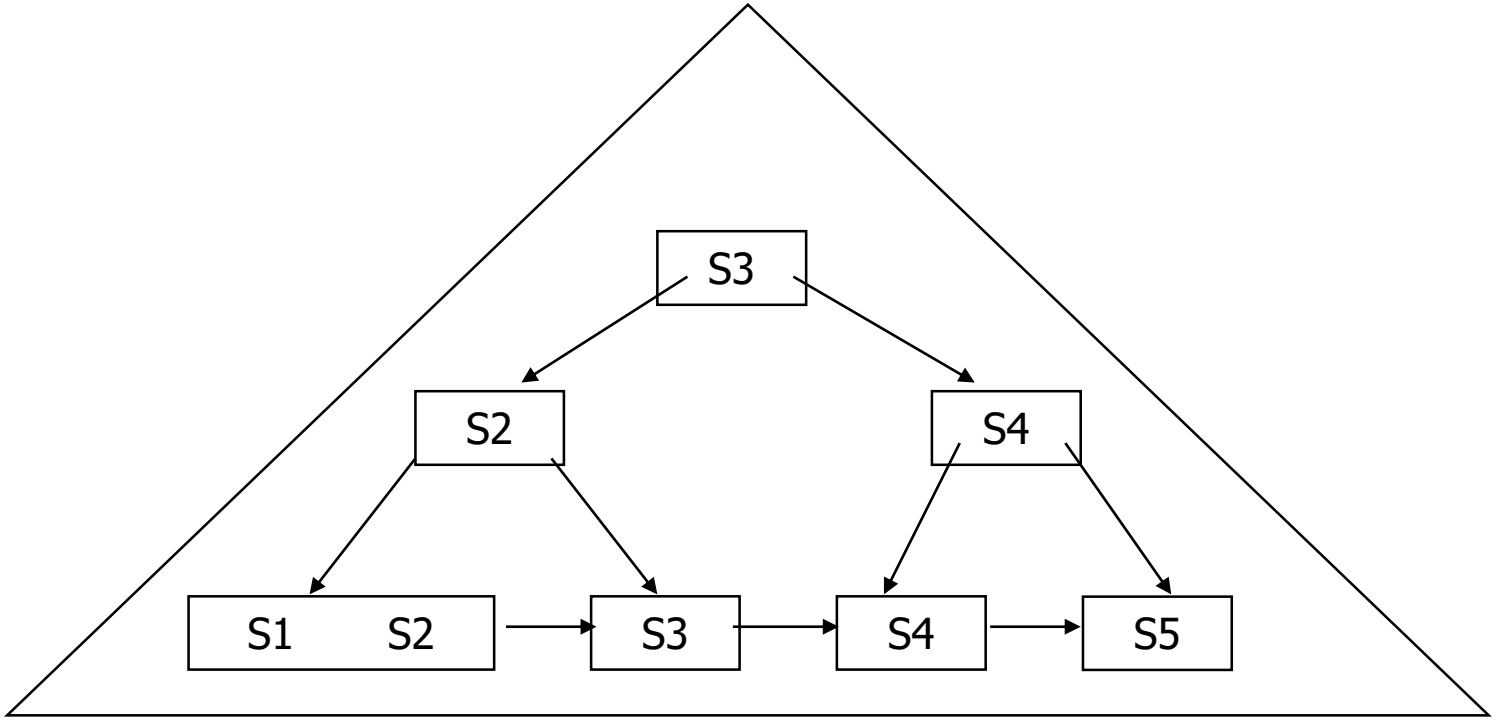
- Keys Number :: Tree Height at the power of 2 x m.
- Tree Height :: $\log_{(2 \times m)}(\#\ \text{Keys})$

**Order of Magnitude**

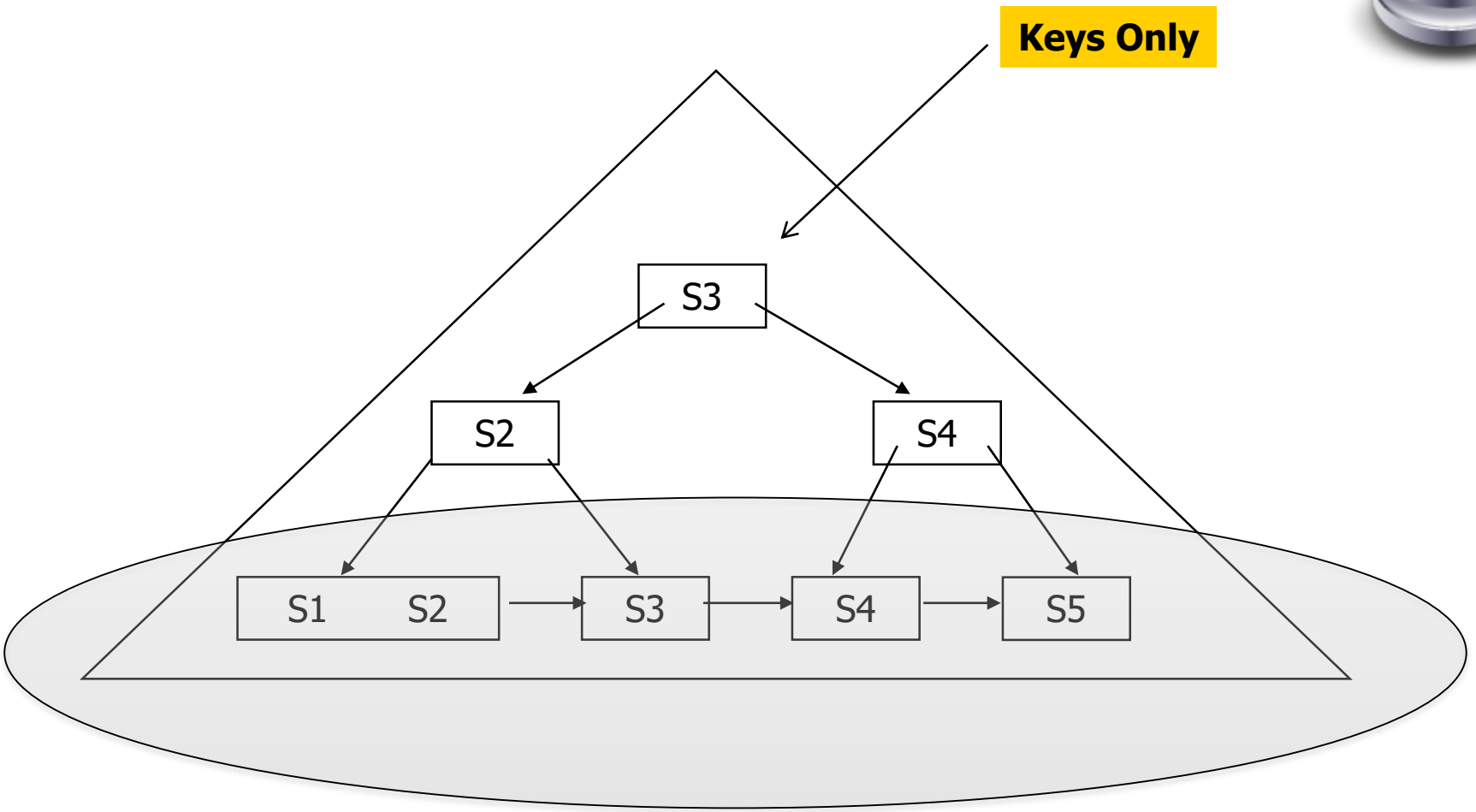
# B-tree Index



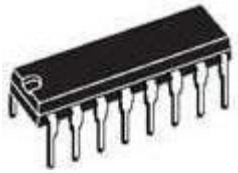
# B+tree



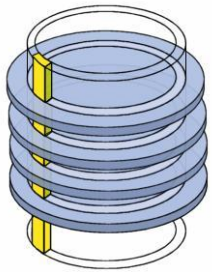
# B+tree



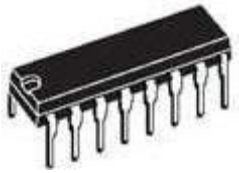
# B+tree Index Disk and Memory





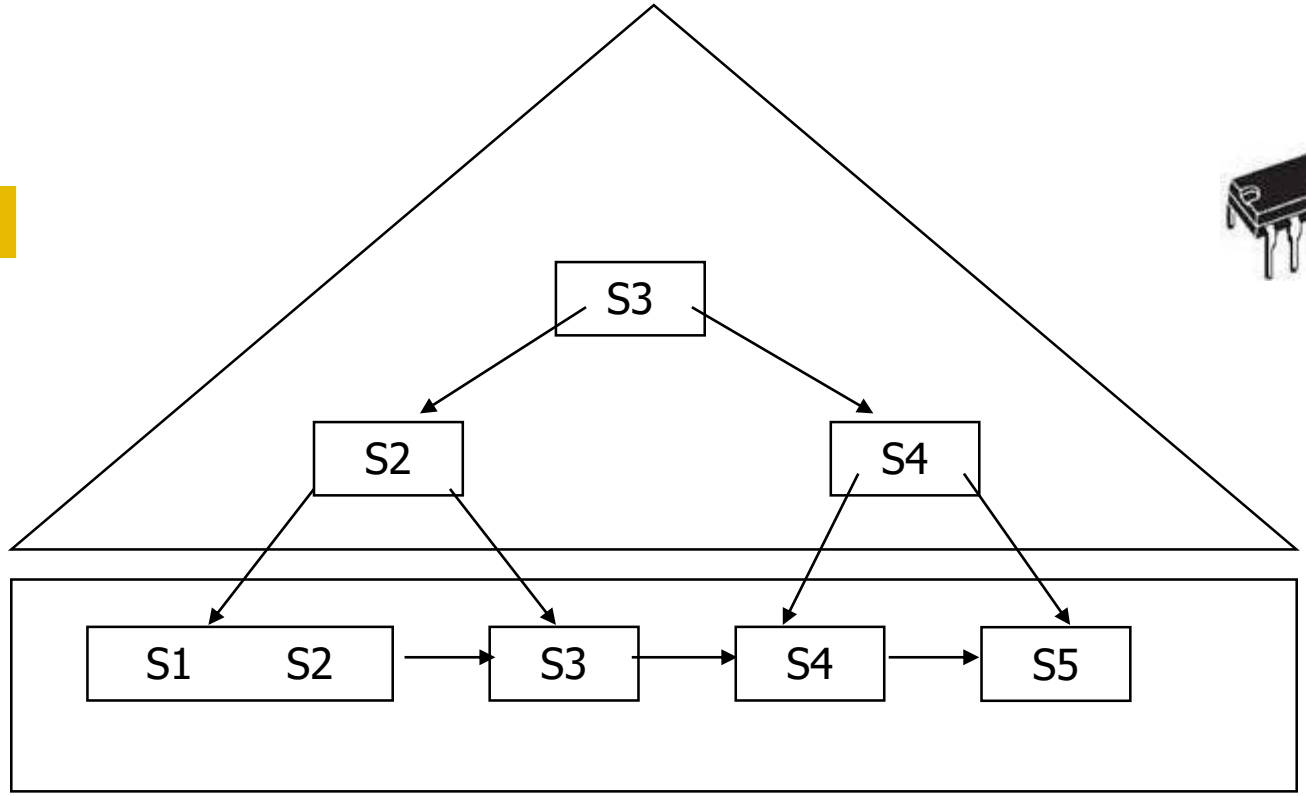



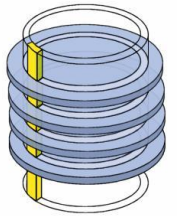
# B+tree Index Disk and Memory



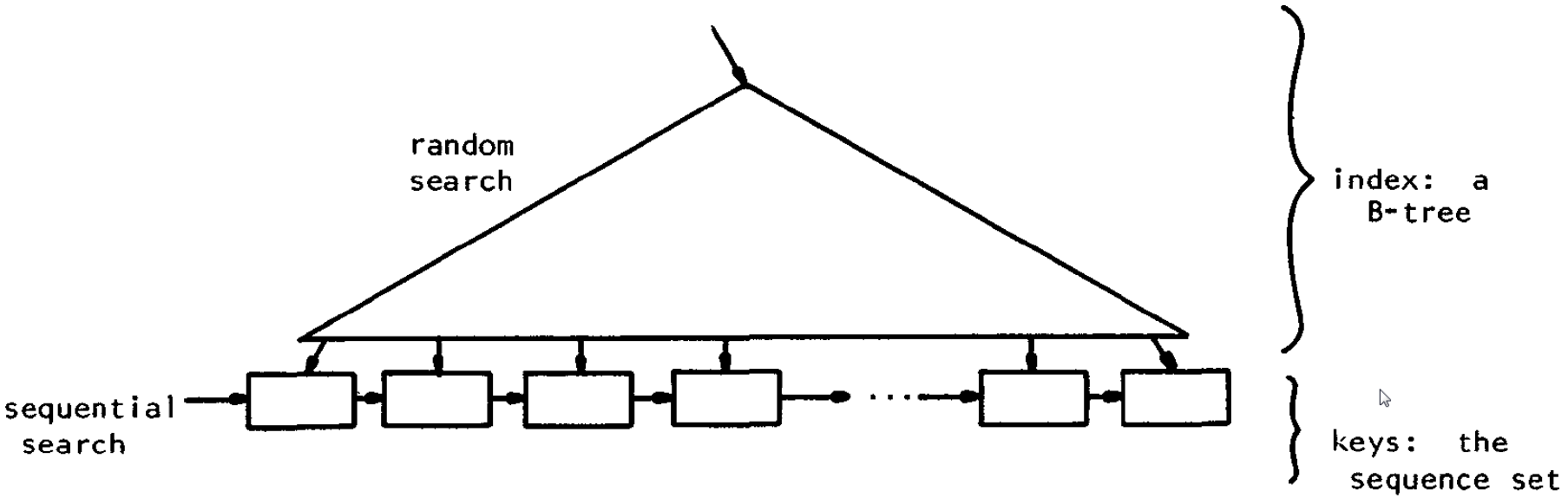
**Index : B tree**

**Leafs : Data**

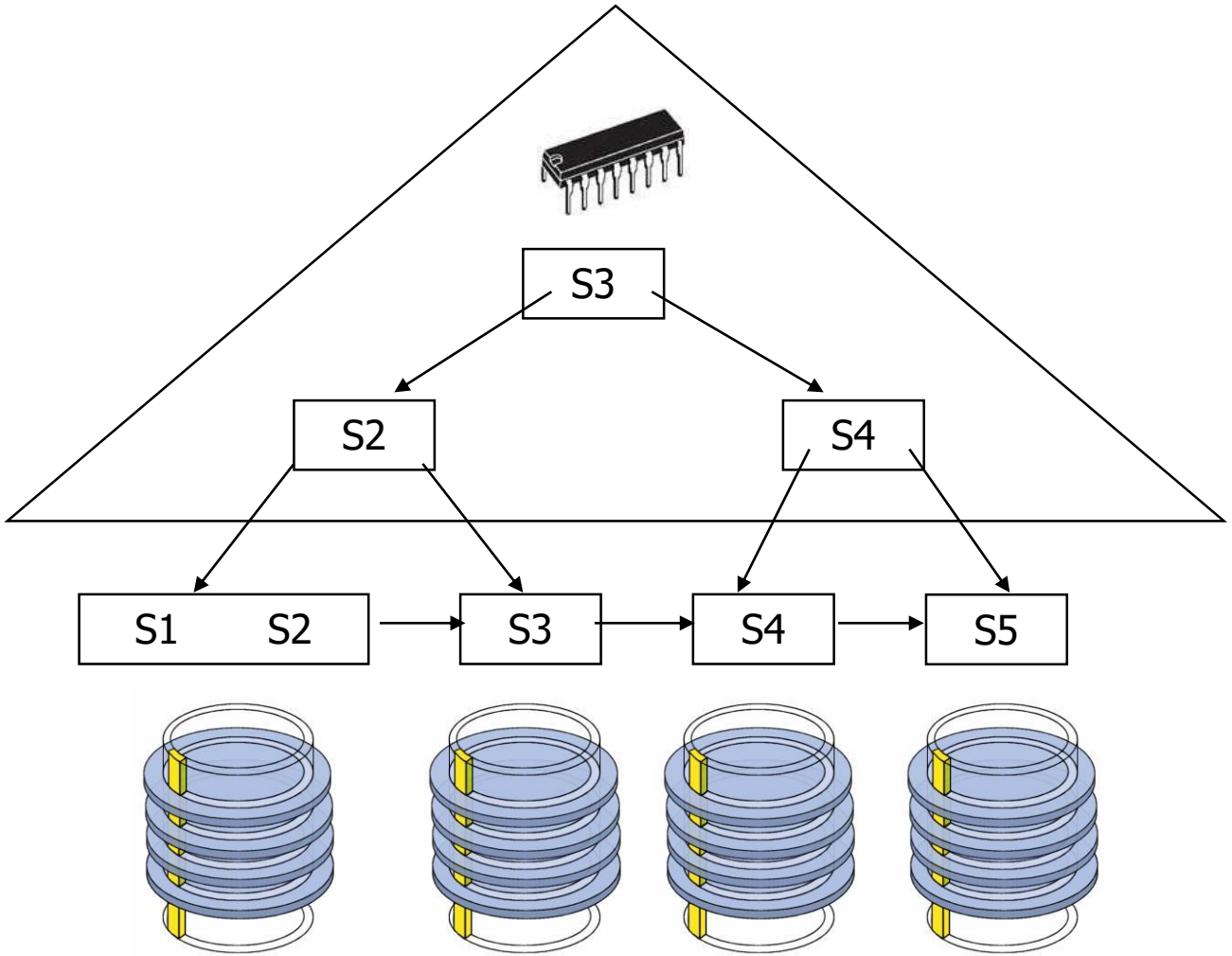


# Comer Paper Representation



# B+tree Index Disk and Memory



# Memory VS disk