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* Fourier Series

Representing an arbitrary periodic signal as the sum of sine function and cosine function is known as Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right]$$

$\omega_0 \rightarrow$ fundamental frequency

$2\omega_0 \rightarrow$ second harmony

$$= a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \ldots\ldots$$
$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots\ldots$$

$\boxed{\omega_0 = \frac{2\pi}{T_0}}$ (T_0 = fundamental time period of $x(t)$)

a_0 = d.c component present in $x(t)$

or

avg value of $x(t)$

$$= \boxed{\frac{1}{T_0} \int_{T_0} x(t)\,dt}$$

$$\boxed{\int_{T_0} = \int_{0}^{T_0} = \int_{T_0}^{2T_0} = \int_{-\frac{T_0}{2}}^{T_0/2}}$$

$$\boxed{\begin{aligned} a_n &= \frac{2}{T_0} \int_{T_0} x(t) \cdot \cos(n\omega_0 t)\,dt \\ b_n &= \frac{2}{T_0} \int_{T_0} x(t) \cdot \sin(n\omega_0 t)\,dt \end{aligned}}$$

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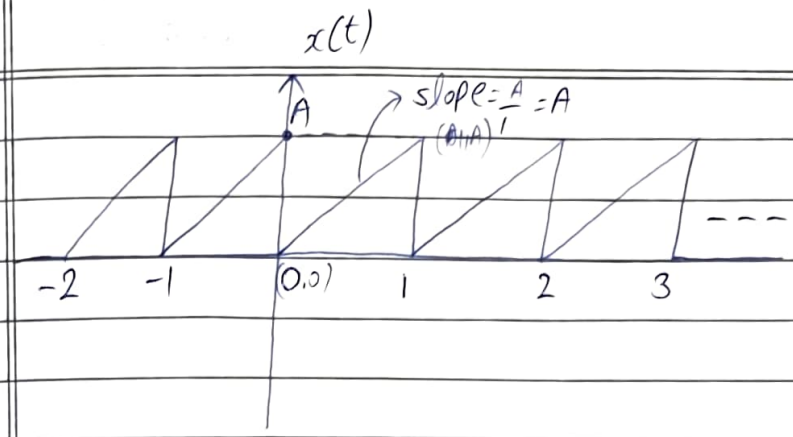
Q)



$(y-0) = A(t-0)$

$y = A.t$

$x(t) = A.t$

$T_0 = 1\,sec$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1} = 2\pi$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

$$= \frac{1}{1}\int_0^1 A t.dt = A\left(\frac{t^2}{2}\right)_0^1$$

$$= \frac{A}{2}.$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t).\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{1}\int_0^1 A.t\cos(n2\pi t)\,dt$$

$$= 2A\int_0^1 t.\cos(2n\pi t)\,dt$$

$$= 2A\left[\frac{\cos 2n\pi t + 2n\pi t\sin 2n\pi t}{4n^2\pi^2}\right]_0^1$$

$$a_n = \frac{2A}{4\pi^2 n^2}\left[(\cos 2n\pi + 2n\pi\sin 2n\pi) - (1+0)\right]$$

$$= \frac{2A}{4\pi^2 n^2}\left[(1+0) - (1+0)\right] = 0$$

$\boxed{a_n = 0}$ for odd signal

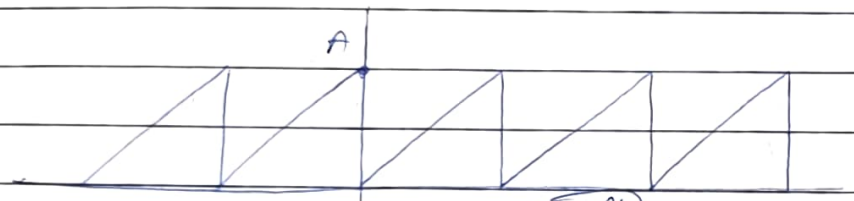
The construction of the graph doesn't require cosine wave.

For odd signal

$x(-t) = -x(t)$

For even signal

$x(-t) = x(t)$

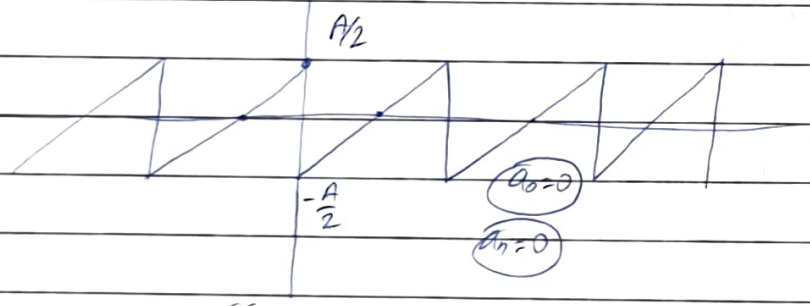


$a_0 = A/2$

$a_n = 0$

$A/2$

shift this signal downward by $A/2$



$a_0 = 0$

$a_n = 0$

there is only difference in d.c component.

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \cdot \sin(n\omega_0 t)\, dt$$

$$= \frac{2}{1} \int_0^1 A(t) \cdot \sin(2n\pi t)\, dt$$

$$= 2A \int_0^1 t \cdot \sin(2n\pi t)\, dt$$

$$= 2A \left[\frac{\sin 2n\pi t - 2n\pi t \cos 2n\pi t}{4\pi^2 n^2} \right]_0^1$$

$$= \frac{2A}{4\pi^2 n^2}\left[\left(\overset{0}{\sin 2n\pi} - 2n\pi \overset{1}{\cos 2n\pi}\right) - (0 - 0)\right]$$

$$= \frac{2A}{4\pi^2 n^2}\left[0 - 2n\pi\right]$$

$$\Rightarrow \frac{-A}{n\pi}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[\overset{0}{a_n \cos n\omega_0 t} + b_n \sin n\omega_0 t\right]$$

$$= \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin 2n\pi t$$

$$\boxed{x(t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\pi t}$$

$b_1 \sin \omega_0 t$

Power $= \frac{b_1^2}{2}$

Second harmonic power $= \frac{b_2^2}{2}$

third harmonic power $= \frac{b_3^2}{2}$

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* Trignometric Fourier Series

The trignometric Fourier series expansion of a periodic signal $x(t)$ is written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right]$$

* Exponential Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right) \right]$$

$\frac{1}{j} = -j$ $\quad j^2 = -1$

$j \cdot j = -1$

$\boxed{\Rightarrow \frac{1}{j} = -j}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) - jb_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2} \right)$$

$$= a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2} \right) e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2} \right) e^{-jn\omega_0 t}$$

Assume $a_0 = C_0$

$$\frac{a_n}{2} - \frac{jb_n}{2} = C_n$$

$$\frac{a_n}{2} + \frac{jb_n}{2} = C_{-n} \quad \text{where } \boxed{C_{-n} = C_n^*}$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + C_{-n} e^{-jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$ Exponential Fourier series representation of $x(t)$.

$a_0 = C_0$

$\frac{a_n}{2} - \frac{jb_n}{2} = C_n$

$\frac{a_n}{2} + \frac{jb_n}{2} = C_{-n}$

$$\boxed{\begin{aligned} C_0 &= a_0 \\ a_n &= C_n + C_{-n} \\ b_n &= C_n - C_{-n} \end{aligned}}$$

$$x(t) = C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_3 e^{j3\omega_0 t} + \ldots$$
$$+ C_{-1} e^{-j\omega_0 t} + C_{-2} e^{-j2\omega_0 t} + C_{-3} e^{-j3\omega_0 t} + \ldots$$

Power of $x(t) = |C_0|^2 + |C_1|^2 + |C_2|^2 + \ldots$
$+ |C_{-1}|^2 + |C_{-2}|^2 + \ldots$

$$\boxed{\text{Total power} = P_T = \sum_{n=-\infty}^{\infty} |C_n|^2}$$

E.F.S of $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$C_0 = a_0 = \frac{1}{T} \int_T x(t) \cdot dt$$

$$\boxed{C_n = \frac{1}{T} \int_T x(t) \cdot e^{-jn\omega_0 t} \cdot dt \qquad n \neq 0}$$

$$\frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}.dt = C_n$$

$$= \frac{1}{T}\int_T \left[C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_{-2} e^{-j2\omega_0 t} + \dots\right]$$

$$= \frac{1}{T}\left[\int_T C_0 \cancel{e^{-jn\omega_0 t}}^{\,0}.dt + \int_T C_1 \cancel{e^{j\omega_0 t}}^{\,0}.e^{-jn\omega_0 t}.dt + \dots\right] e^{-jn\omega_0 t}.dt$$

$$+ \int_T C_5 e^{j\omega_0 t}.e^{-j\omega_0 t} + \dots$$

$n=5$

$$= \frac{1}{\cancel{T}} C_5 \times \cancel{T}$$

$$= C_5$$

$$\Rightarrow \boxed{\frac{1}{T}\int_T x(t).e^{-jn\omega_0 t}.dt = C_n}$$ → This gives n^{th} harmonic

$$\boxed{\text{Power} = |C_n|^2}$$

Q). $x(t) = \cos\omega_0 t + \sin^2\omega_0 t$

$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1-\cos 2\omega_0 t}{2}$$

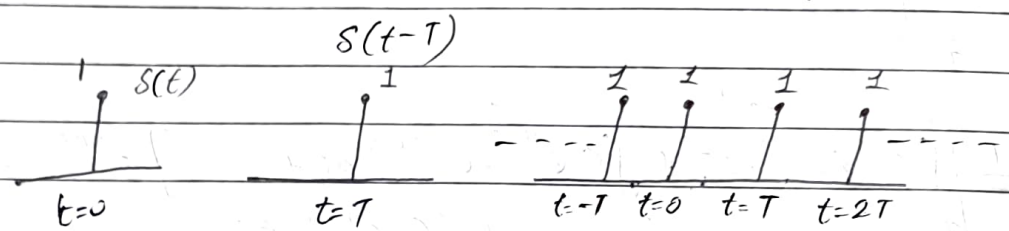
$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2} - \frac{1}{2}\left[\frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2}\right]$$

$$x(t) = \frac{1}{2} + \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} - \frac{1}{4}e^{j2\omega_0 t} - \frac{1}{4}e^{-j2\omega_0 t}$$

$C_0 = \frac{1}{2}$, $C_1 = \frac{1}{2}$, $C_{-1} = \frac{1}{2}$, $C_2 = -\frac{1}{4}$, $C_2 = -\frac{1}{4}$, $C_3 = 0$, $C_{-3} = 0$, ...

Q. Find the exponential fourier series expansion of a train of impulse signal

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$



Train of impulse

$$x(t) = \sum_{n=\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{T} x(t) \cdot e^{-jn\omega_0 t} \cdot dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \cancel{x(t)}^{1} \cdot \cancel{e^{-jn\omega_0 t}}^{1} \cdot dt$$

take one duration of time period

at $t=0$,

$$= \frac{1}{T} \times 1$$

$$\boxed{C_n = \frac{1}{T}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t}$$

$$\boxed{x(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}} \rightarrow EFS$$

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* Fourier Transform (For Non-Periodic Signal)

$$F.T\left[x(t)\right] = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}.dt$$ ω = continuous variable

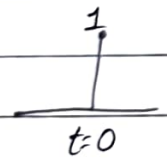
$= X(\omega) \rightarrow$ function of omega(ω)

⇒ Inverse Fourier Transform:

Frequency will be given and you have to find $x(t)$.

$$\boxed{I.F.T\left[X(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega).e^{j\omega t}.d\omega} = x(t)$$

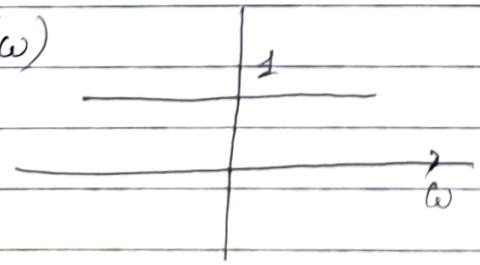
Ex 1) $x(t) = \delta(t)$



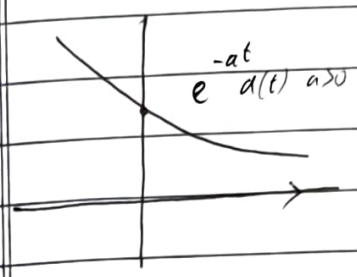
$$\left[\text{Fourier Transform } x[t]\right] = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}.dt$$

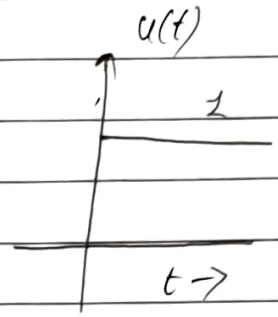
$$F.T\left[\delta(t)\right] = \int_{-\infty}^{\infty} \delta(t).e^{-j\omega t}.dt$$ at $t=0$ its value is given. So integrate it at $t=0$.

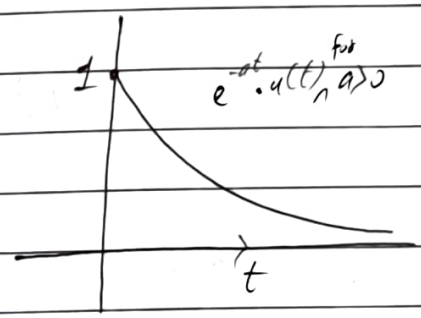
$= 1 = X(\omega)$

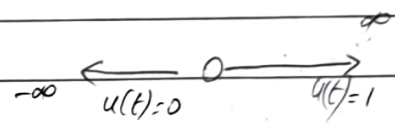


ii) $x(t) = e^{-at} \cdot u(t)$ for $a>0$









$$F.T\left[e^{-at} \cdot u(t)\right] = \int_{-\infty}^{\infty} e^{-at} \cdot u(t) \cdot e^{-j\omega t}\, dt$$

$\hookrightarrow$ between $-\infty$ to ∞ it is 1.

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt = \int_{0}^{\infty} e^{-(a+j\omega)t} \cdot dt$$

$$= \frac{-1}{a+j\omega} \left[e^{-(a+j\omega)t} \right]_{0}^{\infty}$$

$$= \frac{-1}{a+j\omega} \left[0-1 \right]$$

$$= \frac{1}{a+j\omega} \text{ (a complex function)}$$

$$= X(\omega)$$

$$X(\omega) = \frac{1}{a+j\omega}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \quad \text{(magnitude spectrum)}$$

$$\angle X(\omega) = \angle\frac{1}{a+j\omega} = \angle 1 - \angle a+j\omega \qquad \angle \frac{y}{x} = \angle y - \angle x$$

phase/argument

$$= 0 - \tan^{-1}\omega/a$$

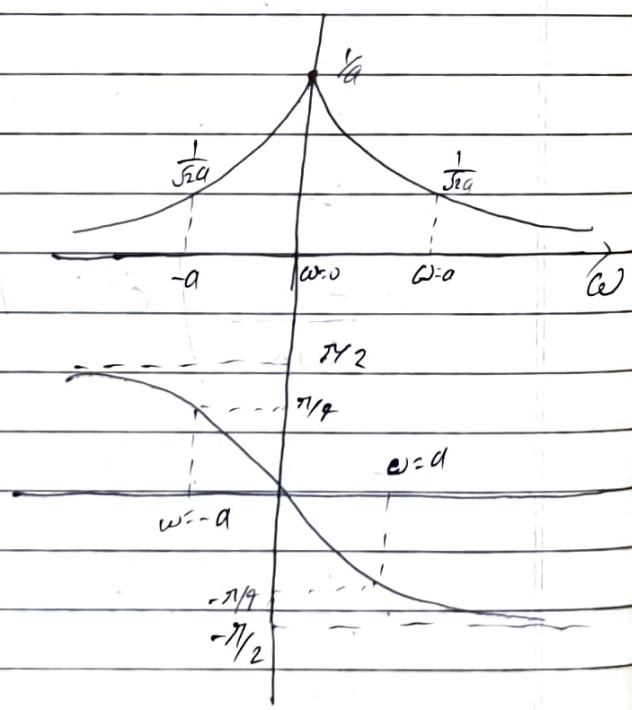
$$= -\tan^{-1}\frac{\omega}{a} \quad \text{(Phase spectrum)}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \angle X(\omega) = -\tan^{-1}(\omega/a)$$

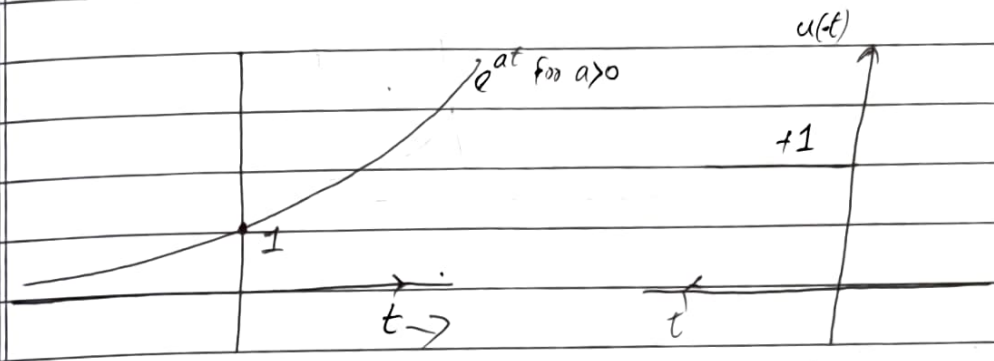
$\omega = 0$ means dc value.

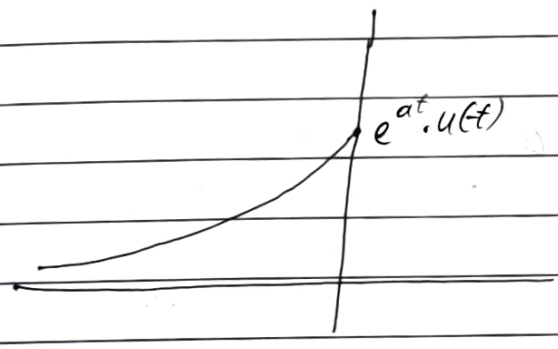
That frequency is appearing with how much phase

ω	$\lvert X(\omega)\rvert$	$\angle X(\omega)$
0	$\frac{1}{a}$	0
a	$\frac{1}{\sqrt{2}a}$	$-\pi/4$
$-a$	$\frac{1}{\sqrt{2}a}$	$\pi/4$
∞	0	$-\pi/2$
$-\infty$	0	$\pi/2$



iii) $x(t) = e^{at}\, u(-t)$ for $a>0$





$$F.T\left[e^{at}.u(-t)\right] = \int_{-\infty}^{\infty} e^{at}.\, e^{-j\omega t}.\, dt$$

$$= \int_{-\infty}^{0} e^{at}.\, e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt$$

$$= \frac{1}{a-j\omega}\left[e^{(a-j\omega)t}\right]_{-\infty}^{0}$$

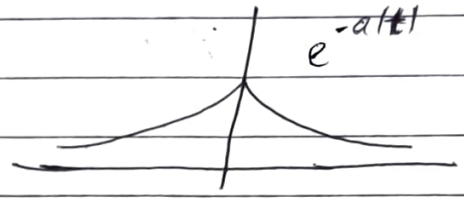
$$= \frac{1}{a-j\omega}\left[1-0\right] = \frac{1}{a-j\omega} = X(\omega)$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \angle X(\omega) = 0 - \tan^{-1}\left(\frac{-\omega}{a}\right) = \tan^{-1}\left(\frac{\omega}{a}\right)$$

$x(t) = e^{-a|t|}$ for $a>0$

$= e^{-at}$ for $t>0$

$= e^{at}$ for $t<0$



$$e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$$

$$F.T\left[e^{-a|t|}\right] = F.T\left[e^{-at}\cdot u(t)\right] + F.T\left[e^{at}\cdot u(-t)\right]$$

$$= \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2} = X(\omega)$$

Real function

POC

* Dirichlet Condition for the existence of F.S and F.T

1) Signal should be absolutely integrable from $t=-\infty$ to $t=\infty$.

2) The signal should have finite number of maximas and minimas from $t=-\infty$ to $t=\infty$.

3) The signal should have finite number of discontinuties from $t=-\infty$ to $t=\infty$.

1) $\int_{-\infty}^{\infty} |x(t)|\,dt =$ Finite value ~~the signal~~

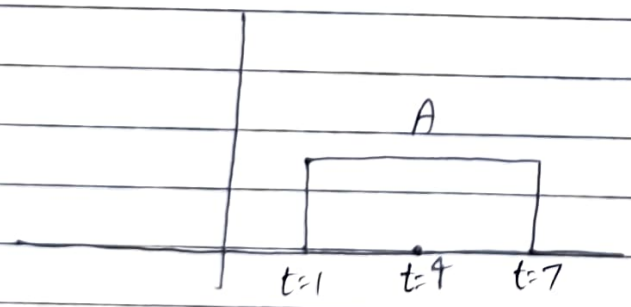
* Fourier Transform of GATE / Rectangular Signal

$A\,\mathrm{rec}\left(\frac{t-t_0}{\tau}\right)$

A = height
t_0 = center
τ = width

Ex

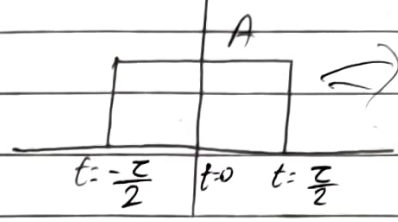
$A\,\mathrm{rec}\left(\frac{t-4}{6}\right)$



If A=0, then you are transferring bit signal for τ duration.

$A\,rec\left(\frac{t}{\tau}\right)$

We cannot find freq to we go for F.T



$\Rightarrow$ F.T

$$F.T\left[A\,rec\left(\frac{t}{\tau}\right)\right] = \int_{-\infty}^{\infty} \underbrace{A\,rec\left(t/\tau\right)}_{\text{its value is } A \text{ in } -\frac{\tau}{2} \text{ to } \frac{\tau}{2}} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\tau/2}^{\tau/2} A \cdot e^{-j\omega t} \cdot dt$$

$$= A\int_{-\tau/2}^{\tau/2} e^{-j\omega t}\,dt = \frac{A}{-j\omega}\left[e^{-j\omega t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{-A}{j\omega}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right]$$

$$= \frac{A}{j\omega}\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

$$= \frac{A}{\cancel{j}\omega} \times 2\cancel{j}\left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right]$$ 2j multiply by divide

$$= \cancel{\frac{2A}{\omega}\left[\sin\omega\tau/2\right]} \quad \frac{2A}{\omega} \times \sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{2A}{\omega} \times \frac{\omega\tau}{2} \times \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}}$$ Multiply and divide by $\frac{\omega\tau}{2}$

$$= A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$\boxed{\frac{\sin\theta}{\theta} = Sa(\theta)}$ sampling θ

$$F.T\left[A\,rec\left(\frac{t}{\tau}\right)\right] = A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$= A\tau \;;\; \text{when } \frac{\omega\tau}{2} = 0$$

When $f=0$, there is a bit whose strength is $A\tau$.

$$\frac{2\pi f\tau}{2} = 0$$

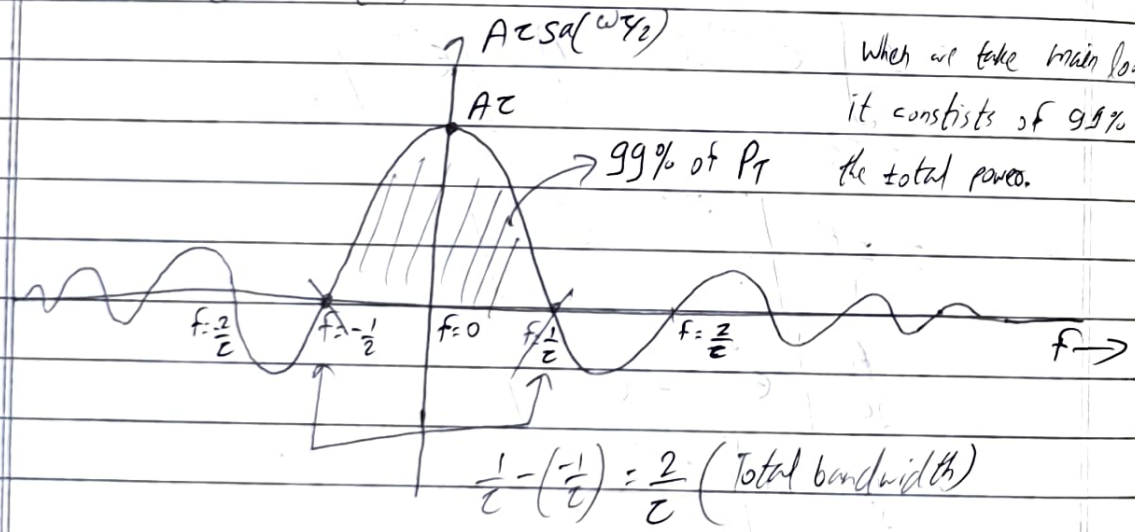
$$\Rightarrow f = 0$$

$$= 0 \;;\; \text{when } \frac{\omega\tau}{2} = n\pi$$

$$\frac{2\pi f\tau}{2} = n\pi$$

$$\boxed{f = \frac{n}{\tau}}$$

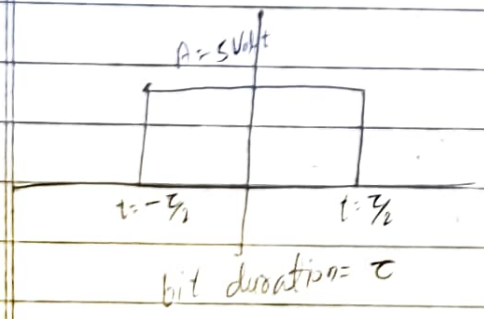
When $f = \frac{n}{\tau}$, $A\,rec(t/\tau) = 0$



When we take main loop, it constists of 99% of the total power.

$$\frac{1}{\tau} - \left(\frac{-1}{\tau}\right) = \frac{2}{\tau} \text{ (Total bandwidth)}$$

transmission bit



$A=5$ represents bit 1.

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To have more data rate, then you have to take more bandwidth for signal.

```
class Foreach2
{
  Public static void main(String args[])
  {
    int nums[] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
    int sum = 0;

    for (int x : nums)
    {
      system.out.println("Value of x is now: " +x);
      sum += x;
      system.out.println(sum);
    }
  }
}
```

nums[0], [0] = 1

nums[0], [1] = 2

nums [0], [2] = 3

PDC

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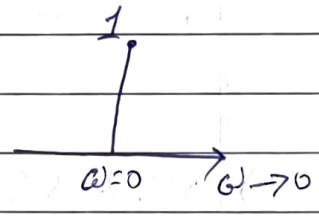
* Inverse Fourier Transform (IFT)

$$IFT[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega).e^{j\omega t}.d\omega = x(t)$$

$$X(\omega) \underset{F.T}{\overset{IFT}{\rightleftarrows}} x(t)$$

1) $X(\omega) = \delta(\omega)$

$$I.F.T[\delta(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega).e^{j\omega t}.d\omega$$



signal is 0 at all ω except ω=0, so at ω=0
$\delta(\omega)=1$, $e^{j\omega t}=1$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \cancel{\delta(\omega)}^{1}.\cancel{e^{j\omega t}}^{1}\,dt$$

$$= \frac{1}{2\pi}\cdot 1 = \frac{1}{2\pi} = x(t)$$ → constant function for all t, $x(t) = \frac{1}{2\pi}$

$$\boxed{\frac{1}{2\pi} \underset{I.F.T}{\overset{F.T}{\rightleftarrows}} \delta(\omega)}$$

$$\boxed{1 \underset{I.F.T}{\overset{F.T}{\rightleftarrows}} 2\pi\delta(\omega)}$$

$$\delta(at) = \frac{1}{a}\delta(t)$$

$$\delta(3t) = \frac{1}{3}\delta(t)$$

$$\boxed{\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)}$$

$$1 \xrightarrow{F.T} 2\pi\,\delta(\omega)$$

$$2\pi\,\delta(2\pi f)$$

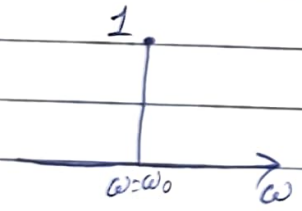
$$2\pi \frac{1}{2\pi}\,\delta(f)$$

$$= \delta(f)$$

$$1 \underset{I.F.T}{\overset{F.T}{\rightleftarrows}} 2\pi\delta(\omega)$$ → function for angular frequency

$$\boxed{1 \underset{I.F.T}{\overset{F.T}{\rightleftarrows}} \delta(f)}$$ → function for normal frequency.

ii) $X(\omega) = \delta(\omega - \omega_0)$



$$I.F.T\left[\delta(\omega-\omega_0)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\delta(\omega-\omega_0)}_{1}\, \underbrace{e^{j\omega t}}_{e^{j\omega_0 t}} \cdot d\omega$$

at $\omega = \omega_0$

$$= \frac{1}{2\pi} \cdot e^{j\omega_0 t}$$

$$\frac{1}{2\pi} e^{j\omega_0 t} \xrightarrow{F.T} \delta(\omega - \omega_0)$$

$$\boxed{e^{j\omega_0 t} \xrightarrow{F.T} 2\pi\,\delta(\omega-\omega_0) = \delta(f-f_0)}$$

$$\boxed{e^{-j\omega_0 t} \xrightarrow{F.T} 2\pi\,\delta(\omega+\omega_0) = \delta(f+f_0)}$$

$$* \quad F.T[\cos\omega_0 t] = F.T\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]$$

$$= F.T\left[\frac{1}{2}e^{j\omega_0 t}\right] + F.T\left[\frac{1}{2}e^{-j\omega_0 t}\right]$$

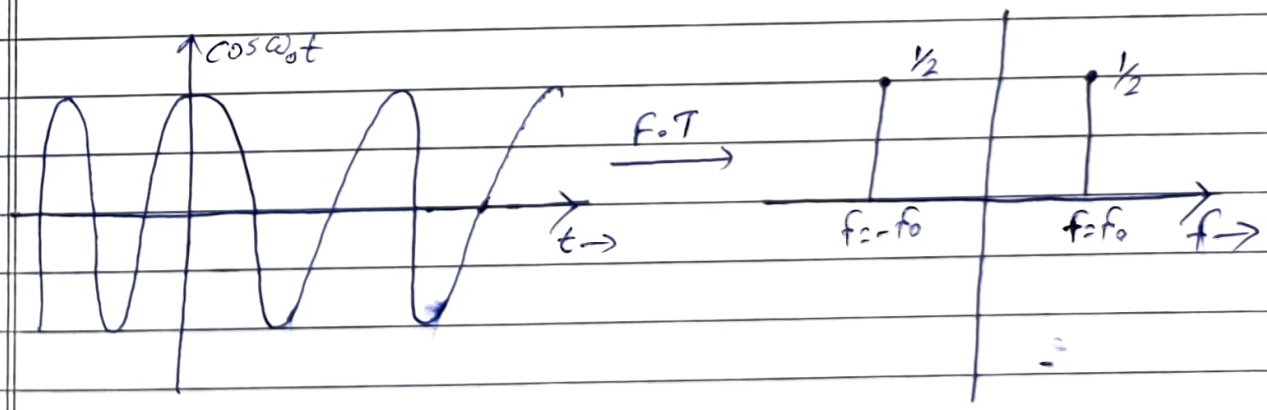
$$= \frac{1}{2}\left[F.T\left[e^{j\omega_0 t}\right] + F.T\left[e^{-j\omega_0 t}\right]\right]$$

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_0) + 2\pi\,\delta(\omega+\omega_0)\right]$$

$$= \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$$

in terms of frequency

$$= \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$



Spectrum of periodic signal is discrete.

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* $F.T[\text{periodic signal}] = F.T\left[\sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t}\right]$

$$= \sum_{n=-\infty}^{\infty} F.T\left[\underbrace{C_n}_{\text{constant}} e^{in\omega_0 t}\right]$$

$$= \sum_{n=-\infty}^{\infty} C_n \, F.T\left[e^{in\omega_0 t}\right]$$

$$= \sum_{n=-\infty}^{\infty} C_n . 2\pi \delta(\omega - n\omega_0)$$

$$= 2\pi\left[C_0 \delta(\omega_0) + C_1 \delta(\omega - \omega_0) + C_2 \delta(\omega - 2\omega_0) + \dots\right]$$

3) Frequency shifting property (Modulation Property)

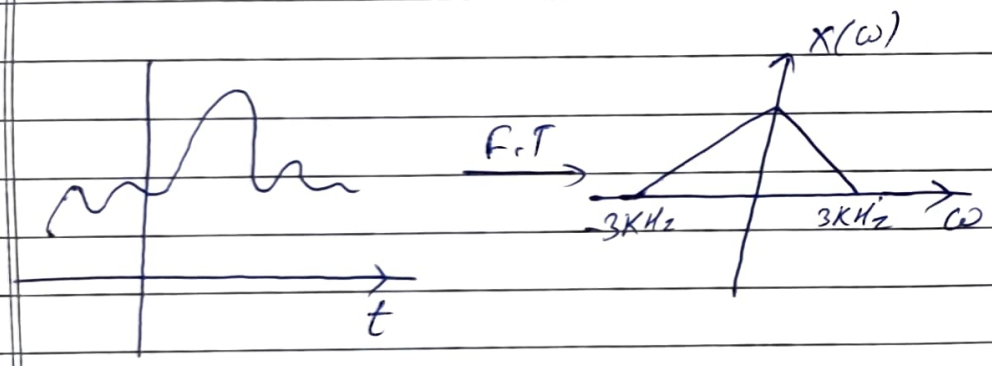
If $x(t) \xrightarrow{F.T} X(\omega)$

then

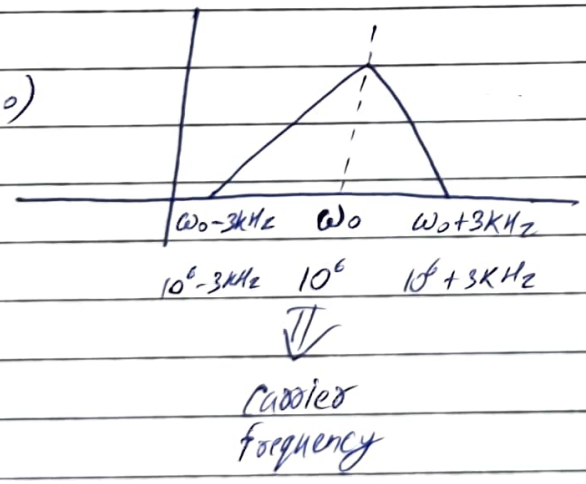
$$e^{j\omega_0 t} \cdot x(t) \xrightarrow{F.T} X(\omega-\omega_0)$$

$$\int_{-\infty}^{\infty} x(t) \cdot e^{j\omega_0 t} \cdot e^{-j\omega t} \cdot dt = \int_{-\infty}^{\infty} x(t) \cdot e^{-j(\omega-\omega_0)t} \cdot dt$$

$$= X(\omega-\omega_0)$$



$$e^{j\omega_0 t} \cdot x(t) \xrightarrow{F.T} X(\omega-\omega_0)$$



$$e^{j10^6 t} \cdot x(t) \xrightarrow{F.T} X(\omega-10^6)$$

$$e^{-j\omega_0 t} \cdot x(t) \xrightarrow{F.T} X(\omega+\omega_0)$$

$$x(t) \cdot \cos\omega_0 t \xrightarrow{F.T}$$

$$x(t) \cdot \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] \xrightarrow{FT} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

* <u>Need of modulation</u>

1) To reduce antena height

Antena height $= h \approx \frac{\lambda}{10}$

$f = 3KHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 10^5$

Antena height

$h = \frac{\lambda}{10} = 10^4$

$= 10\,km$

$X(\omega) \rightarrow 3KHz$

$X(\omega-\omega_0) = X(\omega-10^6)$

$\rightarrow 10^6 + 3KHz \rightarrow f$

$\frac{10^8}{10^6} \rightarrow \lambda$

$\approx 10^2$ $\boxed{h = 10m}$

2) To convert wideband signal to narrowband signal

Wide band signal $\Rightarrow \frac{f_H}{f_L} >> 1$

f_H → Highest frequency present

f_L → lowest frequency present

Narrow band signal $\Rightarrow \frac{f_H}{f_L} \approx 1$

Signal S1 $\longrightarrow$ 3Hz — 3KHz

$$\frac{f_H}{f_L} = \frac{3 \times 10^3}{3} = 10^3 >> 1$$

frequency translation by $\omega_0 = 10^6$

Signal S1 after translating

$\longrightarrow 10^6 + 3 \longrightarrow 10^6 + 3K$

$$\frac{10^6 + 3K}{10^6 + 3} \approx 1$$

3) To reduce effect of noise

let say noise present $10^3 - 10^5$ Hz

$3 \sin 10^3 t + 4 \sin 10^3 t$

$= 7$

4) To multiplex a no. of signals (in the same channel)

$S_0 \longrightarrow 3Hz \longrightarrow 3KHz \xrightarrow{\text{shift is } 10^6} 10^6$

$S_1 \longrightarrow 3Hz \longrightarrow 3KHz \longrightarrow 2 \times 10^6$

$\vdots \qquad \longrightarrow 3 \times 10^6$

$S_9 \longrightarrow 3Hz \longrightarrow 3KHz$

Now they are not going to mixup.

PPC

Analog Modulation

- Amplitude Modulation
- Frequency Modulation
- Phase Modulation

For any modulation, we require two signal

1) Message signal
   or
   Modulating signal
   or
   Base band signal

2) Carrier Signal
   or
   High frequency carrier signal

* Carrier Signal : $c(t) = A_c \cos 2\pi f_c t$
$= A_c \cos \omega_c t$

Carrier Signal has three parameters.

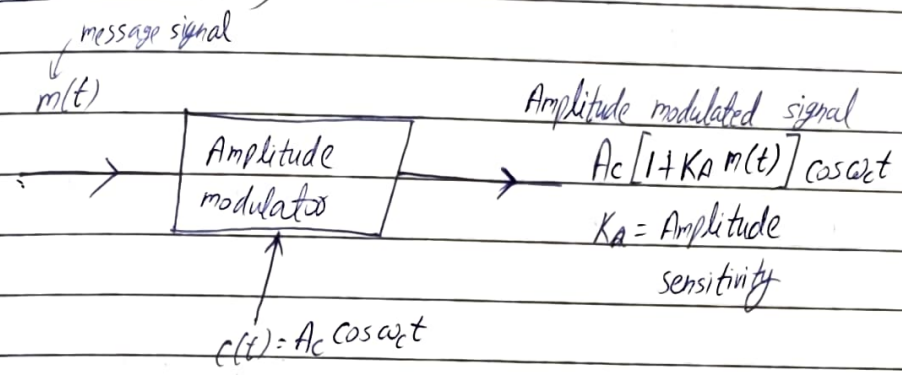
1) Amplitude
2) frequency
3) Phase

* Amplitude Modulation : It is defined as the process in which the amplitude of the high frequency carrier signal is going to vary linearly with the value of the message signal.

* Frequency Modulation : It is defined as the process in which the frequency of the carrier signal is going to vary linearly with the value of the message signal.

* **Phase Modulation:** It is defined as the process in which the phase of the carrier signal is going to vary linearly with the value of the message signal.

* **Amplitude Modulation (A.M)**

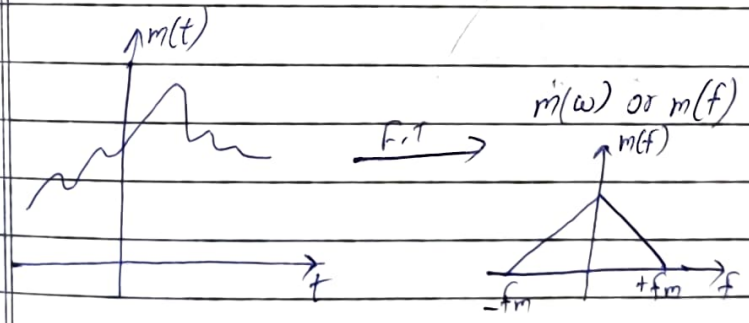


$A_c[1+K_a m(t)]\cos\omega_c t \longrightarrow$ Time domain representation of A.M signal

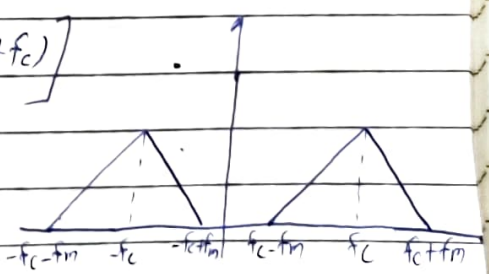
$= A_c \cos\omega_c t + A_c K_a m(t) \cos\omega_c t$

$\Downarrow$ Exclusive carrier signal

$\Downarrow$ Product of $m(t)$ and $c(t)$



$m(t) \cdot \cos\omega_c t \xrightarrow{F.T} \frac{1}{2}\left[m(f-f_c) + m(f+f_c)\right]$



Bandwidth = highest positive frequency − lowest positive frequency

$= f_c + f_m - (f_c - f_m)$

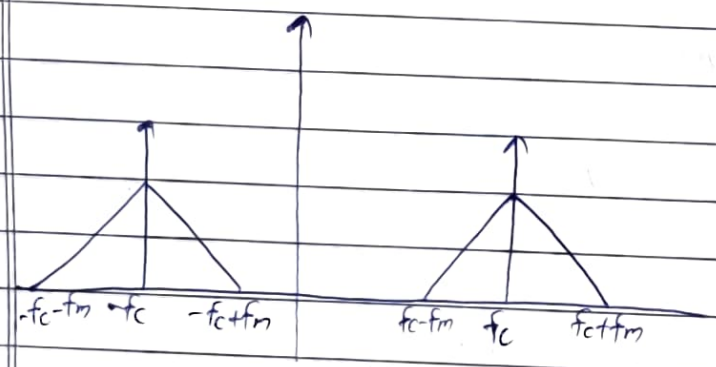
$= 2f_m$

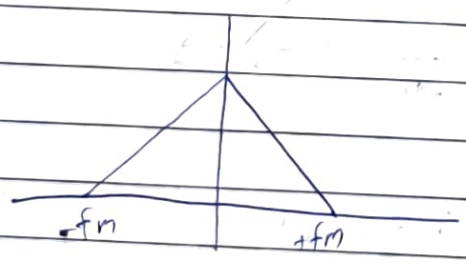
Time domain representing of A.M signal:

$$S_{AM}(t) = A_c\left[1 + K_a m(t)\right]\cos\omega_c t$$
$$= A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t$$

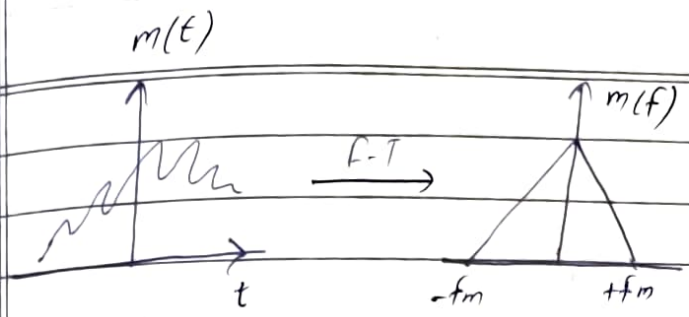
$$F.T\left[S_{AM}(t)\right] = F.T\left[A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t\right]$$

$$= \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c K_a}{2}\left[m(f-f_c) + m(f+f_c)\right]$$





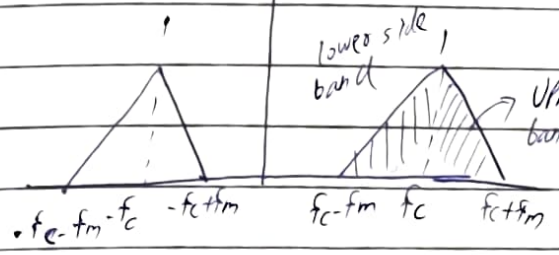
B.W = f_m



B.W of message signal
$m(t) = f_m - 0$
$= f_m$

Highest frequency present = f_m in message signal

$S_{AM}(t) \xrightarrow{F.T}$

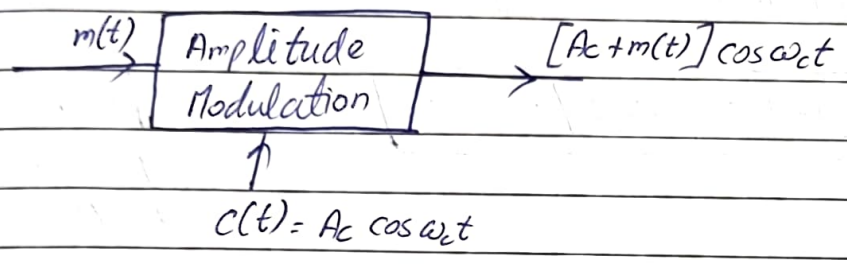


B.W. of modulated signal $= (f_c+f_m)-(f_c-f_m)$
$= 2f_m$

B.W of Amplitude modulated signal $= 2f_m = 2 \times$ B.W of message signal
$= 2 \times$ highest frequency present in the message signal.

USB $= f_c$ to f_c+f_m LSB $= f_c-f_m$ to f_c

* Amplitude Modulation



Time expression for time domain representation of Amplitude modulated signal $S_{A.M}(t)$

1) $S_{A.M}(t) = A_c[1 + k_a m(t)]\cos\omega_c t$

2) $S_{A.M}(t) = [A_c + m(t)]\cos\omega_c t$

→ Single tone message signal : $m(t) = A_m \cos 2\pi f_m t$
$= A_m \cos\omega_m t$

$$S_{A.M}(t) = [A_c + m(t)]\cos 2\pi f_c t$$
$$= [A_c + A_m \cos 2\pi f_m t]\cos 2\pi f_c t$$
$$= A_c\left[1 + \frac{A_m}{A_c}\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

$\boxed{\frac{A_m}{A_c} = \mu}$ modulation index.

$$\boxed{S_{A.M}(t) = A_c[1 + \mu\cos 2\pi f_m t]\cos 2\pi f_c t}$$

→ $S_{AM}(t) = A_c\left[1 + K_a m(t)\right]\cos 2\pi f_c t$

$= A_c\left[1 + K_a A_m \cos 2\pi f_m t\right]\cos 2\pi f_c t$

$$\boxed{S_{AM}(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right]\cos 2\pi f_c t}$$

$\boxed{\mu = K_a A_m}$

K_a = Amplitude sensitivity

A_m = Amplitude of message signal

→ $S_{AM}(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right]\cos 2\pi f_c t$

$= A_c \cos 2\pi f_c t + \cancel{A_c \mu 2\pi}\ A_c \mu \cos 2\pi f_m t \cdot \cos 2\pi f_c t$

$\Downarrow$ (exclusively carrier signal)

$\Downarrow$ $m(t).\,c(t)$ (useful power)

$= A_c \cos 2\pi f_c t + \frac{A_c.\mu}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t\right]$

$= A_c \cos 2\pi f_c t + \frac{A_c.\mu}{2}\cos 2\pi(f_c + f_m)t + \frac{A_c.\mu}{2}\cos 2\pi(f_c - f_m)t$

Total power of amplitude modulated signal $= \frac{A_c^2}{2} + \frac{\left(\frac{A_c\mu}{2}\right)^2}{2} + \frac{\left(\frac{A_c\mu}{2}\right)^2}{2}$

$= \cancel{\frac{A_c^2}{2}}\ \cancel{+}\ \frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4}$

$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$

$= P_c\left[1 + \frac{\mu^2}{2}\right]$

$a\sin\theta$

$R.M.S = \frac{a}{\sqrt{2}}$

$m.s = \left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^2}{2}$

$Power = \frac{a^2}{2}$

$\boxed{\frac{A_c^2}{2} = \text{Power of carrier signal before modulation}} = P_c$

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$$

$$= P_c + \frac{P_c\mu^2}{2}$$

P_c ⇓ waste power

$\frac{P_c\mu^2}{2}$ ⇓ useful and double sideband power

$$\boxed{\text{Efficiency of A.M } (\eta) = \frac{\text{useful power}}{\text{Total power}} = \frac{P_c\frac{\mu^2}{2}}{P_c\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2}}$$

Useful or double sideband power $= \frac{P_c\mu^2}{2}$

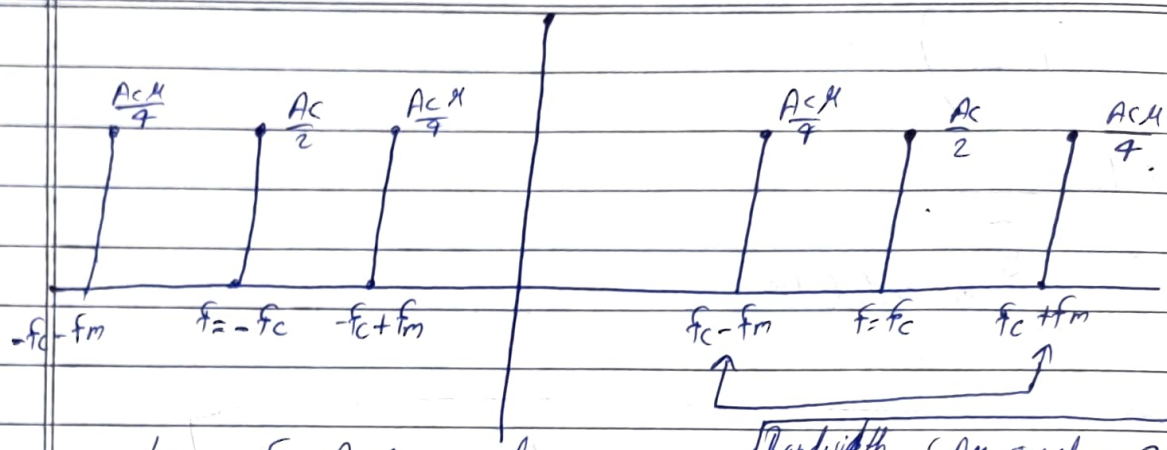
Single side band power $= \frac{P_c\mu^2}{4}$

$$S_{AM}(t) = A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c\mu}{2}\cos 2\pi(f_c-f_m)t$$

$$F.T\left[S_{AM}(t)\right] = S_{AM}(t)$$

$$= F.T\left[A_c\cos 2\pi f_c t\right] + F.T\left[\frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)t\right] + F.T\left[\frac{A_c\mu}{2}\cos 2\pi(f_c-f_m)t\right]$$

$$\boxed{\begin{aligned} S_{AM}(t) &= \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{A_c\mu}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c-f_m))\right] \\ &+ \frac{A_c\mu}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right] \end{aligned}}$$



spectrum of A.M signal

$\boxed{\text{Bandwidth of AM signal} = 2f_m}$

= 2 X message signal B.W

= 2 X highest frequency present in message signal.