

1784

28x28

one training  
image (784 pixels)

{ There are  $m$  training images }

$$X = \begin{bmatrix} \text{---} X^{(1)} \text{---} \\ \text{---} X^{(2)} \text{---} \\ \vdots \\ \text{---} X^{(m)} \text{---} \end{bmatrix}^T$$

FORWARD PROPAGATION:

$$A^{(0)} = X \text{ (784xm)}$$

$$Z^{(1)} = W^{(1)} A^{(0)} + b^{(1)}$$

$$A^{(1)} = g(Z^{(1)}) \Rightarrow \text{ReLU}(Z^{(1)})$$

$$Z^{(2)} = W^{(2)} A^{(1)} + b^{(2)}$$

$$A^{(2)} = g(Z^{(2)}) \Rightarrow \text{Softmax}(Z^{(2)})$$

$$\{ \text{Cost function or loss} = (A^{(2)} - y)^2 \}$$

$$\frac{\delta L_0}{\delta w^1} = \frac{\delta z^1}{\delta w^1} \cdot \frac{\delta A^1}{\delta z^1} \cdot \frac{\delta z^2}{\delta A^1} \cdot \frac{\delta A^2}{\delta z^2} \cdot \frac{\delta L_0}{\delta z^2}$$

$$= \sum_{i=1}^m f'(z_1) \cdot A_i^{(0)} \cdot w_i^{(2)} \cdot 2(A_i^{(2)} - y) \cdot \frac{1}{m} \quad \text{--- (3)}$$

$$\frac{\delta L_0}{\delta b^1} = \frac{\delta z^1}{\delta b^1} \cdot \frac{\delta A^1}{\delta z^1} \cdot \frac{\delta z^2}{\delta A^1} \cdot \frac{\delta A^2}{\delta z^2} \cdot \frac{\delta L_0}{\delta z^2}$$

$$= \sum_{i=1}^m f'(z_1) \cdot w_i^{(2)} \cdot 2(A_i^{(2)} - y) \cdot \frac{1}{m} \quad \text{--- (4)}$$

$$dz_2 = 2(A^{(2)} - y) / m \quad \text{--- (A)}$$

$$dw_2 = dz_2 \cdot A^{(1)} \quad \text{--- (B)}$$

$$db_2 = \sum_{i=1}^m (dz_2) \quad \text{--- (C)}$$

$$dz_1 = w_2 \cdot dz_2 \cdot f'(z_1) \quad \text{--- (D)}$$

$$dw_1 = dz_1 \cdot x \quad \text{--- (E)}$$

$$db_1 = \sum_{i=1}^m (dz_1) \quad \text{--- (F)}$$

### Activation Functions

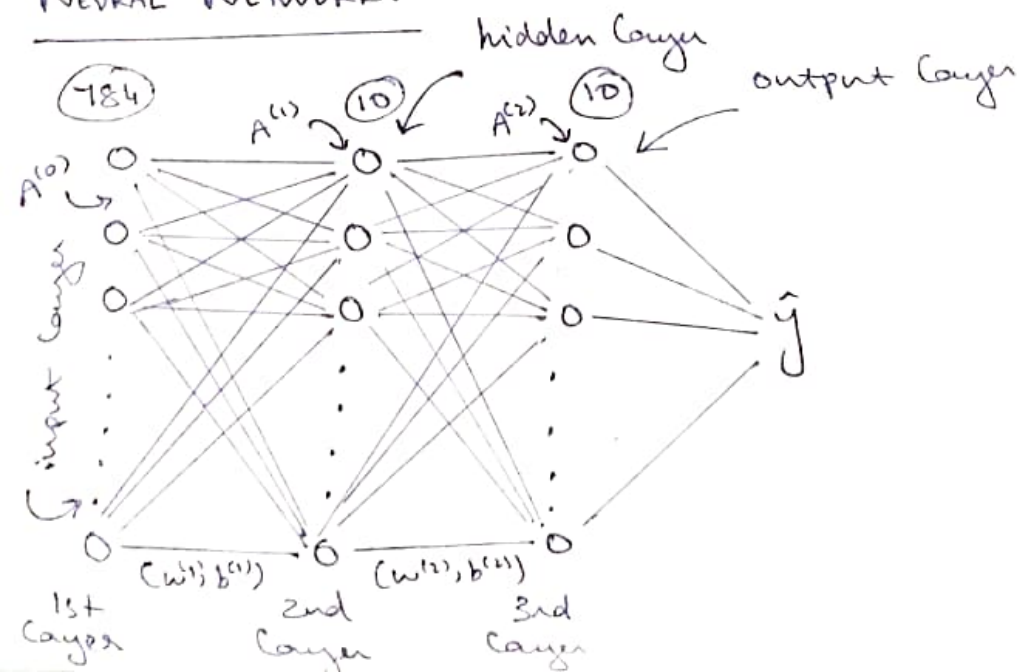
① ReLU:

$$f(x) = \max(0, x)$$

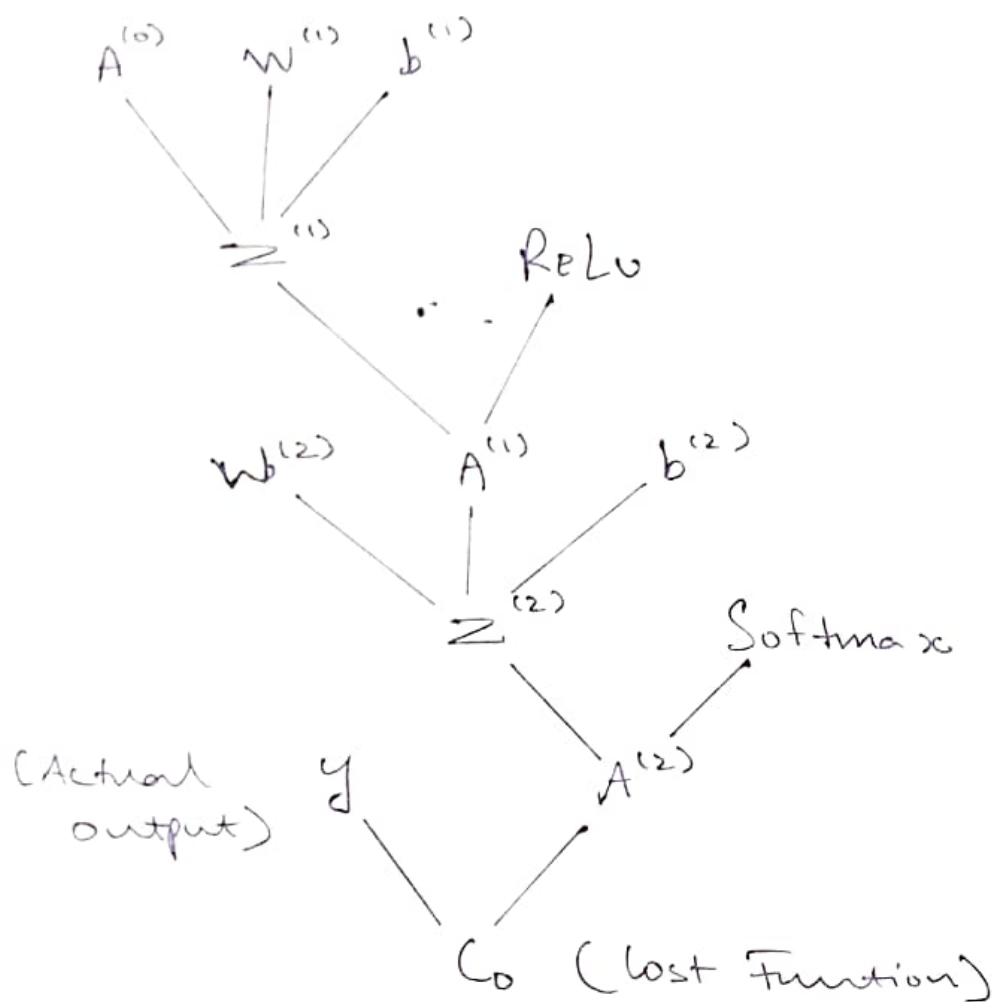
② Softmax:

$$\frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

### NEURAL NETWORK:



# Backward Propagation:



$$\begin{aligned} \frac{\partial C_0}{\partial W^{(2)}} &= \frac{\partial Z^{(2)}}{\partial W^{(2)}} \cdot \frac{\partial A^{(2)}}{\partial Z^{(2)}} \cdot \frac{\partial C_0}{\partial A^{(2)}} \\ &= A^{(1)} \cdot 2(A^{(2)} - y) \\ &\Rightarrow \sum_{i=1}^m \frac{1}{m} [A_i^{(1)} \cdot 2(A_i^{(2)} - y)] \quad \text{--- ①} \end{aligned}$$

$$\begin{aligned} \frac{\partial C_0}{\partial b^{(2)}} &= \frac{\partial Z^{(2)}}{\partial b^{(2)}} \cdot \frac{\partial A^{(2)}}{\partial Z^{(2)}} \cdot \frac{\partial C_0}{\partial A^{(2)}} \\ &= 2(A^{(2)} - y) \Rightarrow \sum_{i=1}^m \frac{1}{m} [2(A_i^{(2)} - y)] \quad \text{--- ②} \end{aligned}$$