Date: Day:

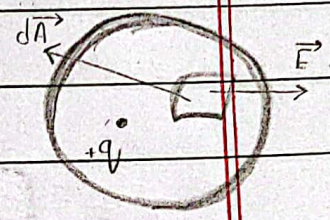
APPLIED PHYSICS

Gauss's Law

- Gauss's law states that the product $\vec{E} \cdot d\vec{A}$, where $d\vec{A}$ is an infinitesimal area of the surface, integrated over the entire surface, equals the charge enclosed by the surface Q_{encl} divided by ϵ_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Electric flux



- Gauss's law involves the concept of electric flux, which refers to the electric field passing through a given area. For a uniform ~~area~~ electric field $\vec{E}$ passing through an area A, the electric flux ϕ_E is

$$\phi_E = EA\cos\theta$$

or, $$\phi_E = \vec{E} \cdot \vec{A}$$

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

where $\oint$ sign indicates that the integral is over the value of $\vec{E}$ at

Date: Day:

every point on a closed surface

Now,

$$\phi_E = \oint \vec{E} \cdot d\vec{A} \longrightarrow (i)$$

$$\phi_E = \oint E dA \cos\theta$$

$$\phi_E = \oint E dA \cos 0^\circ$$

$$\phi_E = \oint E dA$$

$$\phi_E = E \oint dA$$

$$= E (4\pi r^2)$$

$$= \frac{kq}{\cancel{r^2}} (4\pi \cancel{r^2}) \quad \because \left(E = \frac{kq}{r^2} \right)$$

$$\phi_E = \frac{1}{\cancel{4\pi}\epsilon_0} \cdot q (\cancel{4\pi})$$

$$\phi_E = \frac{1}{\epsilon_0} \cdot q$$

$$\Rightarrow \boxed{\phi_E = \frac{Q_{encl}}{\epsilon_0}} \longrightarrow (ii)$$

Integral form

As, $\rho = \frac{dQ}{dV}$

$$\int dQ = \int \rho dV$$

$$Q = \rho \int dV \longrightarrow (iii)$$

Putting (i) and (iii) in (ii)

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \cdot \rho \int dV$$

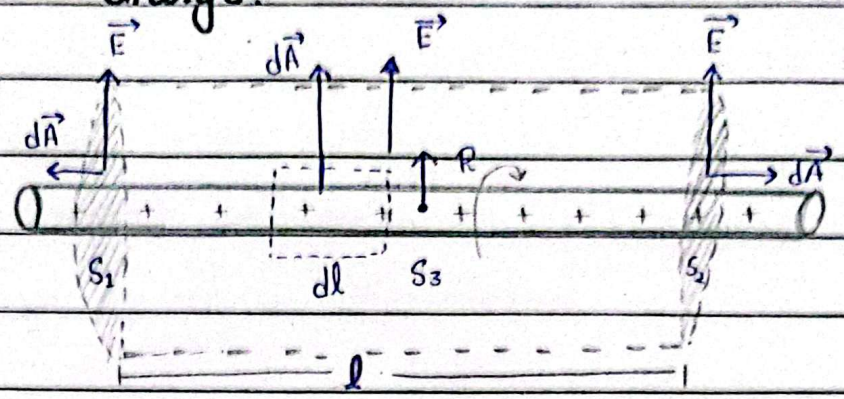
Date: Day:

$$\Rightarrow \boxed{\oint \vec{F} \cdot d\vec{A} = \frac{\rho}{\epsilon_0} \int dV}$$

* The electric flux from any closed surface is only due to the sources (positive charges) and sinks (negative) of the electric fields enclosed by the surface.
* Any charges outside the closed surface do not contribute to the electric flux.
* The net flux for the surface enclosing a charge is non-zero while for a surface which does not enclose any charge, the net flux through it is zero.
* Electric flux does not depend upon the geometry of the closed surface.

Date: Day:

1. Electric field intensity due to Long uniform line of charge.



⇒ The linear charge density of wire is

$$\lambda = \frac{dq}{dl}$$

$$\int dq = \lambda \int dl$$

$$q = \lambda l \longrightarrow (i)$$

⇒ Electric flux for surfaces S_1, S_2 and S_3 is given as,

- $\phi_1 = \int \vec{E} \cdot d\vec{A}$

$$= \int E dA \cos\theta$$

$$= \int E dA \cos 90^\circ$$

$$\phi_1 = 0$$

- $\phi_2 = \int \vec{E} \cdot d\vec{A}$

$$= \int E dA \cos 90^\circ$$

$$\phi_2 = 0$$

Date: _____ Day: _____

$$\phi_3 = \int \vec{E} \cdot d\vec{A}$$

$$= \int E\,dA \cos 0^\circ$$

$$= \int E\,dA$$

$$= E \int dA$$

$\because A = l \times w$

$A = l \times (2\pi R)$

$A = 2\pi R l$

$$\phi_3 = E\,(2\pi R)\,l$$

$\Rightarrow$ Now total flux is given

$$\phi_E = \phi_1 + \phi_2 + \phi_3$$

$$= 0 + 0 + E\,(2\pi R)\,l$$

$$\phi_E = E\,(2\pi R)\,l \longrightarrow \text{(ii)}$$

$\Rightarrow$ As we know that

$$\phi_E = \frac{1}{\epsilon_0} \cdot q$$

$$\phi_E = \frac{1}{\epsilon_0} \cdot \lambda l \longrightarrow \text{(iii)}$$

Comparing (ii) and (iii)

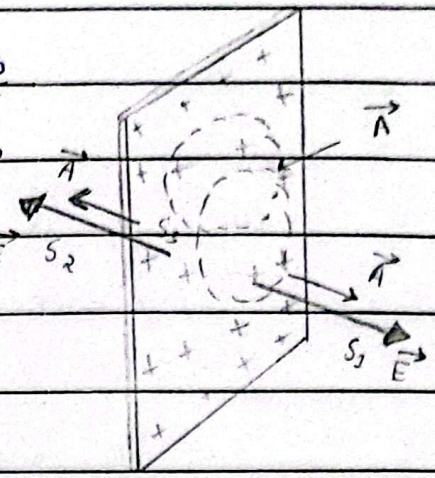
$$E\,(2\pi R)\,\cancel{l} = 1/\epsilon_0 \cdot \lambda \cancel{l}$$

$$\boxed{E = \frac{\lambda}{2\pi R \epsilon_0}}$$

or, $$\vec{E} = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{R}\,\hat{r}$$

Date: Day:

2. Electric field Intensity due to Infinite plane of charge



⇒ The surface charge density is given as

$$\sigma = \frac{dq}{dA}$$

$$dq = \sigma\, dA$$

$$\int dq = \sigma \int dA$$

$$q = \sigma A \longrightarrow (i)$$

⇒ Electric flux for surface S_1, S_2 and S_3 is given as

- $$\phi_1 = \phi_2 = \int \vec{E} \cdot d\vec{A}$$

$$= \int E\, dA \cos\theta$$

$$= \int dA\, E \cos 0^\circ$$

$$= E \int dA$$

$$\therefore \phi_1 = \phi_2 = EA$$

- For curved surface S_3

$$\phi_3 = \int E\, dA \cos 90^\circ$$

$$= \int dA\, E\, (0)$$

$$\phi_3 = 0$$

⇒ Now total flux is given as

$$\phi_E = \phi_1 + \phi_2 + \phi_3$$

Date: Day:

$$\phi_E = EA + EA + 0$$

$$\phi_E = 2EA \longrightarrow \text{(ii)}$$

$\Rightarrow$ By Gauss's law, we have

$$\phi_E = 1/\epsilon_0 \cdot q$$

$$\phi_E = 1/\epsilon_0 \cdot \sigma A \longrightarrow \text{(iii)}$$

$\Rightarrow$ Comparing equ (ii) and (iii)

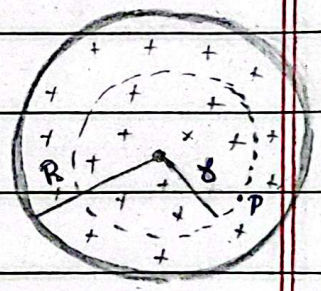
$$2E\cancel{A} = 1/\epsilon_0 \cdot \sigma \cancel{A}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

or $$\vec{F} = \frac{\sigma}{2\epsilon_0}\hat{r}$$

3. Electric field intensity due to Solid sphere of charge

Case 1 : Inside the solid sphere $(r < R)$



$\Rightarrow$ The volume charge density is given as

$$\rho = \frac{dq}{dV}$$

$$\int dq = \rho \int dV$$

$$q = \rho \cdot \frac{4\pi R^3}{3} \longrightarrow \text{(i)}$$

Date: Day:

⇒ The charge enclosed by the Gaussian surface is given as

$$q' = \rho \cdot 4/3\, \pi r^3 \longrightarrow \text{(ii)}$$

Dividing equ (ii) by (i)

$$\frac{q'}{q} = \frac{\cancel{\rho}\, \cancel{4/3}\, \pi r^3}{\cancel{\rho}\, \cancel{4/3}\, \pi R^3}$$

$$\frac{q'}{q} = \frac{r^3}{R^3}$$

$$q' = \frac{Q\, r^3}{R^3} \longrightarrow \text{(iii)}$$

⇒ Electric flux is given as

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint E\, dA \cos 0^\circ$$

$$= \oint E\, dA$$

$$= E \oint dA$$

$$\phi = E \cdot 4\pi r^2 \longrightarrow \text{(iv)}$$

⇒ By Gauss's law, we have

$$\phi = 1/\epsilon_0 \cdot q'$$

$$\phi = \frac{1}{\epsilon_0} \times \frac{r^3 Q}{R^3} \longrightarrow \text{(v)}$$

⇒ Comparing (iv) and (v)

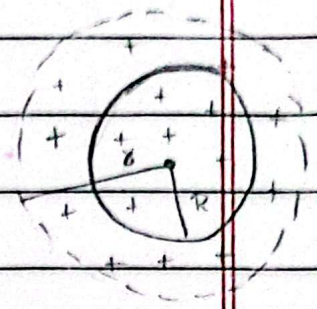
$$E \cdot 4\pi \cancel{r^2} = \frac{1}{\epsilon_0} \times \frac{\cancel{r^3} Q}{R^3}$$

Date: Day:

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q r}{R^3}}$$

$E \propto r$

Case 2: Outside the sphere $(r > R)$



⇒ As we know that

$$\varphi = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint E\,dA \cos 0^\circ$$

$$= E \oint dA$$

$$\varphi = E \cdot 4\pi r^2 \longrightarrow \text{(i)}$$

⇒ By Gauss's law, we have

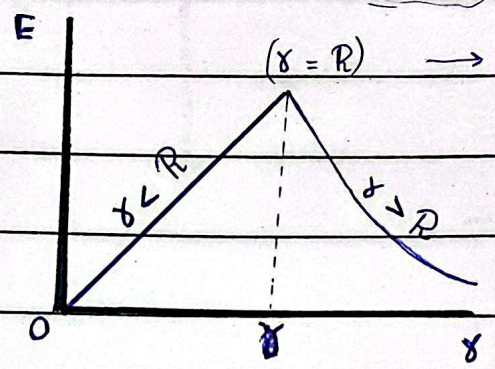
$$\varphi = \frac{1}{\epsilon_0} \cdot Q \longrightarrow \text{(ii)}$$

⇒ Comparing eq: (i) and (ii)

$$E \cdot 4\pi r^2 = 1/\epsilon_0 \cdot Q$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}} \quad (r > R)$$

or, $E \propto \frac{1}{r^2}$

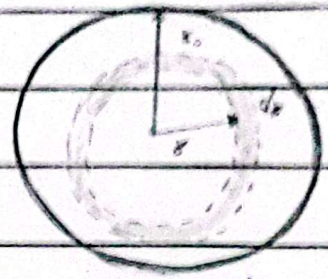


$(r = R) \longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$

Date: Day:

4. Electric field intensity due to Non-uniformly charged solid sphere

a) Find α in terms of Q and r_0, the total charge on the sphere and its radius.



⇒ The non-uniform charge density of sphere is given as

$$\rho_E = \frac{Q}{V}$$

$$Q = \rho_E V$$

so, $$Q = \int_0^{r_0} (\alpha r^2)\, dV \quad \because (\rho_E = \alpha r^2)$$ → constant

$$Q = \int_0^{r_0} (\alpha r^2)(4\pi r^2)\, dr \quad \because$$

$W = dr$

$A = 4\pi r^2$

$V = (4\pi r^2)\, dr$

$$Q = 4\pi\alpha \int_0^{r_0} r^4\, dr$$

$$Q = 4\pi\alpha \left| \frac{r_0^5}{5} \right|$$

$$Q = \frac{4\pi\alpha r_0^5}{5}$$

or, $$\boxed{\alpha = \frac{5Q}{4\pi r_0^5}}$$

Date: _____ Day: _____

b) Find the electric field E as a function of r inside the sphere.

$\Rightarrow$ By Gauss's law, we have

$$\int \vec{E} \cdot d\vec{A} = (1/\epsilon_0)(Q_{encl.})$$

$$\int E\, dA \cos\theta = (1/\epsilon_0)(Q_{encl.})$$

$$EA = (1/\epsilon_0)(Q_{encl.})$$

$$E(4\pi r^2) = 1/\epsilon_0 \, (Q_{encl.}) \longrightarrow (i)$$

Now,

$$Q_{encl.} = \int_0^r \rho_E \, dV$$

$$= \int_0^r (\alpha r^2)(4\pi r^2)\, dr$$

$$Q_{encl.} = \frac{4\pi r^5 \alpha}{5}$$

$$Q_{encl.} = \frac{\cancel{4\pi} r^5}{\cancel{5}} \cdot \left(\frac{\cancel{5} Q}{\cancel{4\pi} r_0^5}\right)$$

$$Q_{encl.} = Q\left(\frac{r^5}{r_0^5}\right) \longrightarrow (ii)$$

Now equ (i) becomes

$$E(4\pi r^2) = \frac{1}{\epsilon_0} \cdot Q\left(\frac{r^5}{r_0^5}\right)$$

$$E = \frac{1}{4\pi\epsilon_0 \cancel{r^2}} \cdot Q\left(\frac{r^{\cancel{5}}}{r_0^5}\right)$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \cdot Q\left(\frac{r^3}{r_0^5}\right)}$$

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Date: Day:

24-2: Capacitance of a parallel plate capacitor

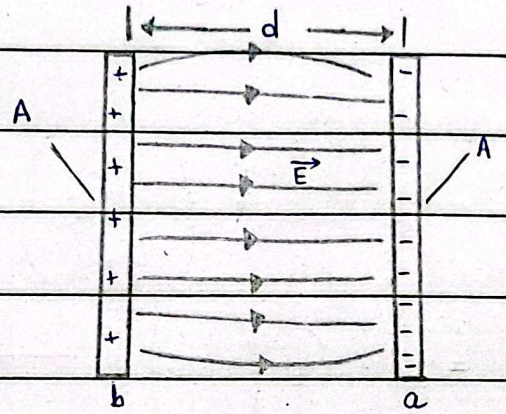
⇒ A capacitor is a device that can store electric charge and normally consists of two conducting objects placed near each other but not touching.

According to Gauss's law

$$E \cdot A = 1/\epsilon_0 \times Q_{enc.}$$

$$EA = 1/\epsilon_0 \times Q$$

$$Q = \epsilon_0 EA \longrightarrow (i)$$



Now,

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dl}$$

$$\Delta V = -\int_a^b \vec{E} \cdot \vec{dl}$$

$$\Delta V = -\int_a^b E dl \cos 180^\circ$$

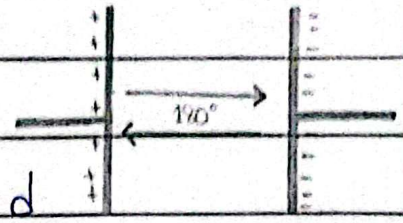
$$= \int_a^b E dl$$

$$= E \int_a^b dl$$

$$\Delta V = Ed \longrightarrow (ii)$$

Date: _____ Day: _____

Also,



$$\Delta V = Ed$$

$$\Delta V = \frac{Q}{\epsilon_0 A} \cdot d$$

$$Q = \frac{\epsilon_0 A}{d} \cdot \Delta V \longrightarrow \text{(iii)}$$

As, $Q = CV$

Using equ (iii), we get

$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 A \cancel{V}}{d} \times \frac{1}{\cancel{V}}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Case 1 : Capacitance of a Cylindrical capacitor

As we know that

$$Q = \epsilon_0 E A$$

or, $$Q = \epsilon_0 E (2\pi r l) \longrightarrow \text{(i)}$$

Date: _____ Day: _____

or, $$E = \frac{Q}{2\pi\epsilon_0 r l}$$

Now,

$$\Delta V = -\int_{R_a}^{R_b} E \cdot dr$$

$$= -\int_{R_a}^{R_b} \frac{Q}{2\pi\epsilon_0 l} \cdot \frac{dr}{r}$$

$$= -\frac{Q}{2\pi\epsilon_0 l} \int_{R_a}^{R_b} \frac{dr}{r}$$

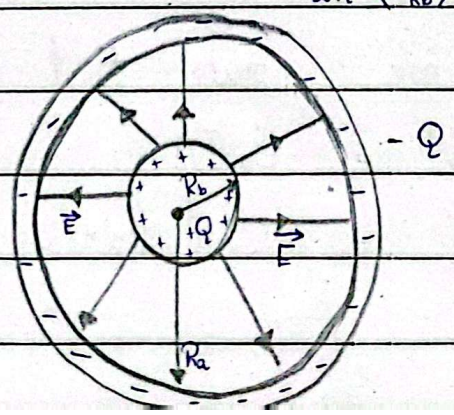
$$= -\frac{Q}{2\pi\epsilon_0 l} \ln r \Big|_{R_a}^{R_b}$$

$$= -\frac{Q}{2\pi\epsilon_0 l} \cdot \ln\left(\frac{R_b}{R_a}\right)$$

$$\Delta V = \frac{Q}{2\pi\epsilon_0 l} \ln\left(\frac{R_a}{R_b}\right)$$

or, $$\boxed{Q = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_a}{R_b}\right)} \cdot \Delta V}$$

where, $$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_a}{R_b}\right)}$$



Date: _____ Day: _____

$$Q = CV$$

or, $$C = \frac{Q}{V}$$

$$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_a}{R_b}\right)} \cdot \cancel{V} \times \frac{1}{\cancel{V}}$$

$$\boxed{C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_a}{R_b}\right)}}$$

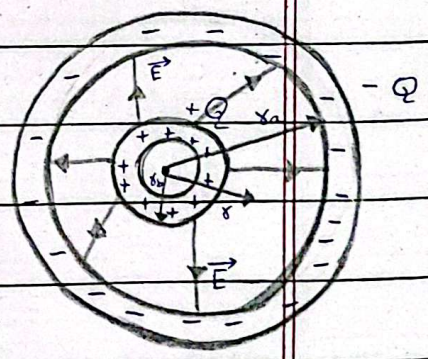
Case 2: Capacitance of a Spherical capacitor

As we know that

$$Q = \epsilon_0 EA$$

$$Q = \epsilon_0 E (4\pi r^2)$$

or $$E = \frac{Q}{4\pi r^2 \epsilon_0}$$



Now,

$$\Delta V = -\int_{R_a}^{R_b} \vec{E} \cdot d\vec{r}$$

$$\Delta V = -\int_{R_a}^{R_b} E\,dr$$

$$\Delta V = -\int_{R_a}^{R_b} \frac{Q}{4\pi\epsilon_0} \cdot \frac{dr}{r^2}$$

$$\Delta V = -\frac{Q}{4\pi\epsilon_0} \left| \frac{-1}{r} \right|_{R_a}^{R_b}$$

Date: Day:

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_b} - \frac{1}{R_a}\right)$$

$$\Delta V = \frac{Q(R_a - R_b)}{4\pi\epsilon_0 R_a R_b}$$

$$Q = \left(\frac{4\pi\epsilon_0 R_a R_b}{(R_a - R_b)}\right)\Delta V \longrightarrow \text{(i)}$$

$$Q = CV \longrightarrow \text{(ii)}$$

Comparing equ (i) and (ii)

$$C = \frac{4\pi\epsilon_0 R_a R_b}{(R_a - R_b)}$$

$$R_a \simeq \infty$$

then, $$\Delta V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_b} - \frac{1}{\infty}\right)$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0 R_b}$$

or, $$Q = (4\pi\epsilon_0 R_b)\Delta V \longrightarrow \text{(iii)}$$

$$Q = CV \longrightarrow \text{(iv)}$$

Comparing equ (iii) and (iv)

$$\boxed{C = 4\pi\epsilon_0 R_b}$$

Date: _____ Day: _____

24-4: Energy Stored by Capacitor

According to the definition of work we have

$$W = \int F\,dx$$

The work needed to store a total charge Q in a capacitor

$$W = \int_0^Q V\,dq$$

$$W = \int_0^Q \frac{q}{C}\,dq$$

$$W = \frac{1}{C}\int_0^Q q\,dq$$

$$\boxed{W = \frac{1}{2C}Q^2}$$

As, $Q = CV$

so, $W = \frac{1}{2\cancel{C}}C^{\cancel{2}}V^2$

$$\boxed{W = \frac{1}{2}CV^2}$$

Also, $C = Q/V$

so, $W = \frac{1}{2}\cdot Q\cdot V^{\cancel{2}}$

Date: Day:

$$\boxed{W = \frac{1}{2} QV}$$

Potential Energy Density

As we know that

$$W = U = \frac{1}{2} CV^2$$

As, $C = \frac{\epsilon_0 A}{d}$

and $V = -Ed$

so, $U = \frac{1}{2}\left(\frac{\epsilon_0 A}{\cancel{d}}\right)(E^2 d^{\cancel{2}})$

$$U = \frac{1}{2} \epsilon_0 E^2 A d$$

where, $V = Ad$

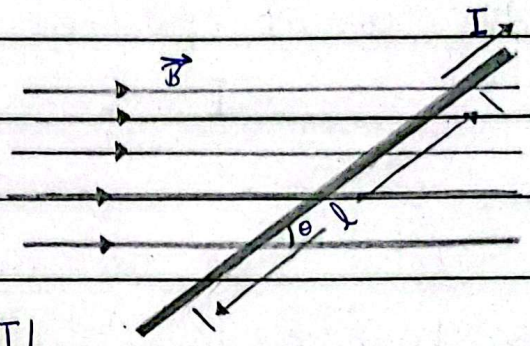
$$\frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

$$\boxed{u = \frac{1}{2} \epsilon_0 E^2}$$

Date: _____ Day: _____

27 – 3 : Force on an Electric current in a Magnetic Field

It is found experimentally that the magnitude of the force on an electric current carrying wire placed in a magnetic field is directly proportional to the current I in the wire, to the magnetic field B, and to the length of wire l exposed to the magnetic field. The force also depends on the angle θ b/w the current direction and the magnetic field being proportional to $\sin\theta$. Thus, the force is given as



$$F \propto B$$

$$F \propto l$$

$$F \propto I$$

so,

$$F \propto BIl$$

$$F \propto BIl\sin\theta$$

$$F = BIl\sin\theta \quad (1)$$

$$\boxed{F = I(l \times B)} \longrightarrow \quad (i)$$

When the current is perpendicular to the field lines ($\theta = 90^\circ$ and $\sin 90^\circ = 1$) then,

$$F_{max} = IlB$$

Date: Day:

27 - 4 : Force on an Electric Charge Moving in a Magnetic Field

We have seen that a current-carrying wire experiences a force when placed in a magnetic field. Since a current in a wire consists of moving electric charges, we can say that freely moving charges also experiences the same force.

Suppose a charged particle q passes by a given point in time t, then the current produced by it is given as

$$I = \frac{q}{t} \longrightarrow (i)$$

Let t be the time for a charge q to travel a distance l in a magnetic field B, then

$$S = l = vt \longrightarrow (ii)$$

where v is the velocity of particle.

Now, the force experienced by this particle in magnetic field is given as

$$F = Il \times B$$

Date: _____ Day: _____

$$F = \left(\frac{q}{t}\right)(vt) \times B$$

$$F = qv \times B$$

or $$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

Electron's path in a uniform magnetic field

The magnetic force acting on a charged particle in a magnetic field is given as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$F = qvB \sin 90^\circ$$

$$F_B = qvB$$

An electron travels at $1.5 \times 10^7\ ms^{-1}$ in a plane perpendicular to a uniform 0.010 T magnetic field. The electron moves at speed v in a curved path and so must experience a centripetal force $F_c = mv^2/r$.

So, $$F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

Date:_____ Day:_____

$$r = \frac{mv}{qB} \longrightarrow (i)$$

The time T required for a charged particle q moving with constant speed v to make one circular revolution in uniform magnetic field is given as

$$t = T$$

and $S = 2\pi r$ (circumference)

As $S = vt$

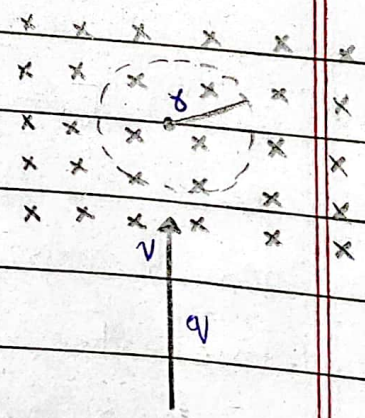
By putting values we get

$$2\pi r = vT$$

$$\frac{2\pi r}{v} = T$$

$$T = \frac{2\pi}{\cancel{v}}\left(\frac{m\cancel{v}}{qB}\right) \quad \text{using (i)}$$

$$T = \frac{2\pi m}{qB}$$



or $f = \frac{1}{T}$

$$\boxed{f = \frac{qB}{2\pi m}}$$

This is called cyclotron frequency of a particle in magnetic field because this is the frequency at which particle revolve

Date: _____ Day: _____

in a cyclotron.

27 - 8 : The Hall Effect

When a current - carrying conductor is held fixed in a magnetic field, the field exerts a sideway force on charges moving in the conductor. Example: if electrons move to the right of conductor, the inward magnetic field will exert a downward force on the electrons. $F_B = eV_d B$. Thus, the electrons will tend to move nearer to side D than C, causing P.D b/w sides C and D of conductor.

This P.D provides as electric field $\vec{E}_H$ that exerts a force $eE_H = F_e$ which is equal and opposite to the magnetic force F_B. This effect is known as <u>Hall Effect</u>

* The difference of potential produced is called Hall emf.
* The electric field due to the separation of charges is called Hall field, $\vec{E}_H$.

At Equilibrium

$$F_e = F_B$$

Date: Day:

$$\cancel{e} E_H = \cancel{e} v_d B$$

where, v_d is the drift velocity of the electrons.

$$E_H = v_d B \longrightarrow (i)$$

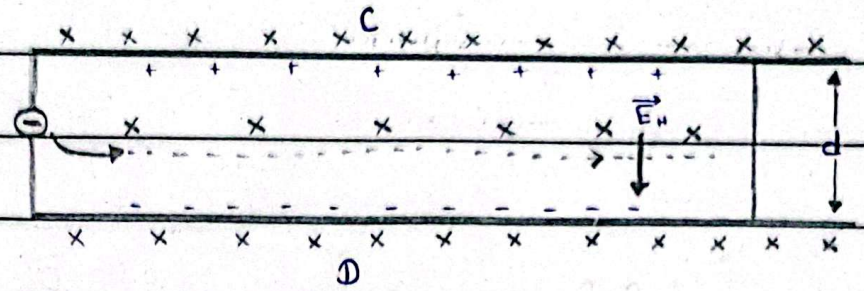
Also, the Hall emf is given as

$$\mathcal{E}_H = E_H d$$

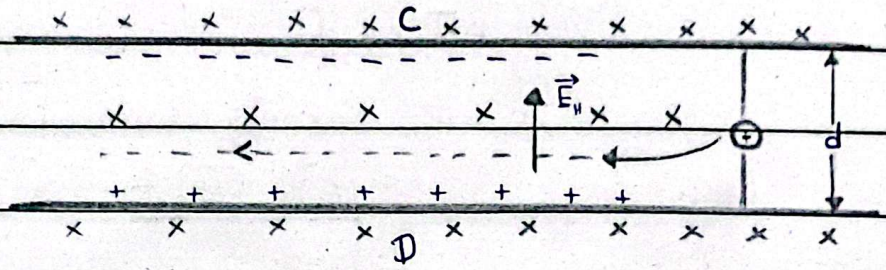
Using (i) we get

$$\boxed{\mathcal{E}_H = v_d B d}$$

where d is the width of conductor.



(a)



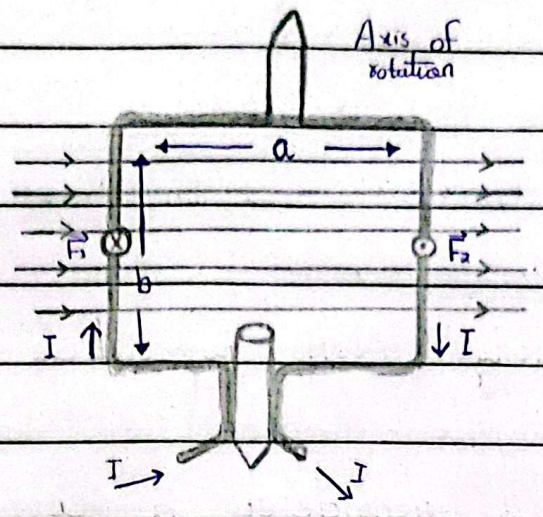
(b)

* Hall effect is used to measure the magnetic field by using a device known as <u>Hall probe</u>.

Date: Day:

27 - 5 : Torque on a Current Loop ; Magnetic Dipole Moment

When an electric current flows in a closed loop of wire placed in an external magnetic field, the magnetic force on the current can produce a torque.



The torques around this axis produced by $\vec{F_1}$ and $\vec{F_2}$ act in the same direction are given as

$$\tau_1 = r_1 \times F = a/2 \times IbB\sin\theta$$

$$\tau_2 = r_2 \times F = a/2 \times IbB\sin\theta$$

The net torque is the sum of these two torques given as

$$\tau = \tau_1 + \tau_2$$

$$\tau = \frac{a}{2} \times IbB\sin\theta + \frac{a}{2} \times IbB\sin\theta$$

Date: Day:

$$\tau = IabB\sin\theta$$

where, $ab = A$ (area)

$$\tau = IAB\sin\theta$$

For a coil having N turns, the net torque is given as

$$\tau = NIBA\sin\theta$$

or $\boxed{\tau = NIBA} \longrightarrow$ (i)

where, θ is the angle b/w $\vec{B}$ and the perpendicular to the face of the loop or coil.

⇒ The quantity NIA is called the magnetic dipole moment of the coil and is considered a vector

$$\boxed{\vec{\mu} = NI\vec{A}} \longrightarrow \text{(ii)}$$

with this definition of $\vec{\mu}$, equ (i) can be rewrite as

$$\vec{\tau} = NI\vec{A} \times \vec{B}$$

or $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}} \longrightarrow$ (iii)

⇒ As we know that, potential energy U is given as

$$U = W = \int F\,dx$$

Date: _____ Day: _____

but for rotational motion, we have

$$U = W = \int \tau \, d\theta$$

$$U = \int \mu B \sin\theta \, d\theta$$

$$U = -\mu B \cos\theta + c$$

$$\boxed{U = -\vec{\mu} \cdot \vec{B}}$$

which is the potential energy of magnetic dipole moment.

* Bar magnets, compass needles, as well as current loops are considered as magnetic dipoles.
* Electric dipole moment is

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

* Potential energy of electric dipole moment is

$$\boxed{U = -\vec{p} \cdot \vec{E}}$$

* Magnetic moment of hydrogen atom

As we know that

$$\mu = IA$$

where, $I = e/t$

$A = \pi r^2$

for a hydrogen atom

Date: Day:

$$\therefore \quad \mu = \frac{e\pi r^2}{t}$$

Now, t is the time in which e covers the distance equal to circumference $2\pi r$ with velocity v

so, $t = (2\pi r)/(v)$

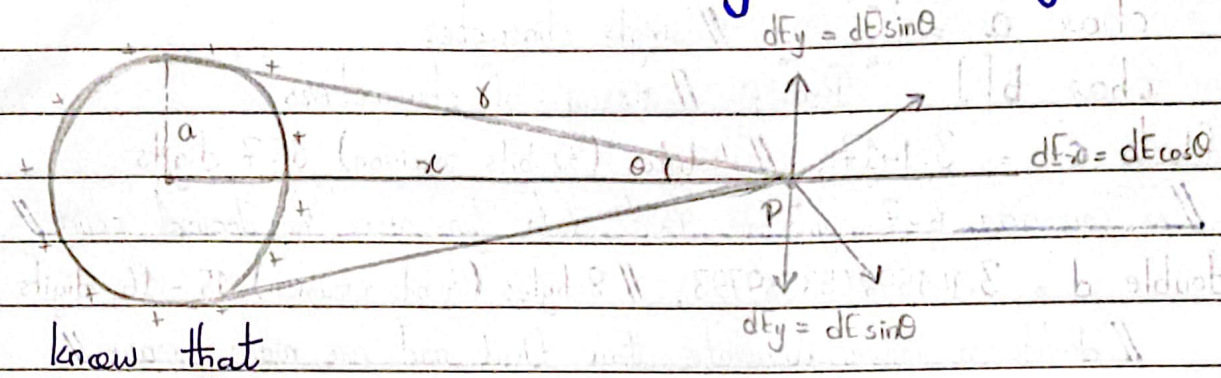
thus, $\mu = e\pi r^2 \times \frac{v}{2\pi r}$

$$\boxed{\mu = \frac{1}{2} evr}$$

Date:

Applied Physics

1) Electric Field due to a ring of Charge



As we know that

$$E = \int dE$$

$$= \int dE_x + \int dE_y$$

$$= \int dE_x + 0 \quad \because (dE_y = 0)$$

$$= \int dE \cos\theta$$

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{r^2} \cos\theta$$

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda . dl}{r^2} . \left(\frac{x}{r}\right) \quad \because \left(\lambda = \frac{dQ}{dl} \Rightarrow dQ = \lambda dl\right)$$

$$E = \frac{1}{4\pi\epsilon_0} . \frac{\lambda x}{r^3} \int dl$$

$$E = \frac{1}{4\pi\epsilon_0} . \frac{\lambda x}{r^3} . l = \frac{1}{4\pi\epsilon_0} . \frac{Qx}{r^3} \quad \because (\lambda l = Q)$$

$$E = \frac{1}{4\pi\epsilon_0} . \frac{Qx}{\sqrt{(x^2+a^2)^3}} \quad \because \left(r = \sqrt{x^2+a^2}\right)$$

$$\boxed{E = \frac{kQx}{\sqrt{(x^2+a^2)^3}}}$$

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Date:

Case (i)

when $x \gg a$

If x is very large as compared to the size of the ring (or radius) then

$$a \longrightarrow 0$$

so, $$E = \frac{kQ\cancel{x}}{x^{\cancel{3}}}$$

$$E = \frac{kQ}{x^2}$$

* In this case, the uniformly charged ring will behave as a point charge.

Case (ii)

when $x = 0$

If x is so small that it minimizes to reach at then centre of ring then

$$E \longrightarrow 0$$

so, $$E = 0$$

* In this case, E will be zero at the centre of uniformly charged ring.

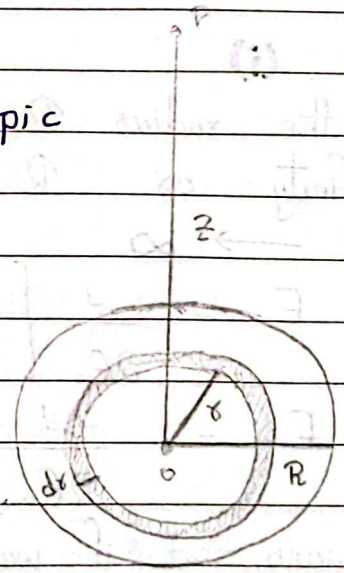
2) Electric Field due to a Disc of charge

As we know that

$$\therefore E = \int dE$$

Using the relation of previous topic

$$E = \int dE = \frac{1}{4\pi\epsilon_0}\int \frac{z\,dQ}{(z^2+r^2)^{3/2}}$$



Now,

$$\sigma = \frac{dQ}{dA}$$

$$dQ = \sigma\, dA$$

$$dQ = \sigma\, \underbrace{(2\pi r)}_{\text{circumference}}\, \underbrace{dr}_{\text{width}}$$

$\because (A = l \times b)$

Date:

Thus, $E = \int dE = \frac{1}{2\cancel{4\pi}\epsilon_0}\int \frac{z(\sigma \cancel{2\pi} r\,dr)}{(z^2+r^2)^{3/2}}$

$$E = \frac{z\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$E = \frac{z\sigma}{2\epsilon_0}\cdot\frac{1}{2}\int_0^R (z^2+r^2)^{-3/2}(2r\,dr)$$

$$E = \frac{z\sigma}{2\epsilon_0}\cdot\frac{1}{2}\left|\frac{(z^2+r^2)^{-1/2}}{-1/2}\right|_0^R$$

$$E = \frac{z\sigma}{2\epsilon_0}\left|\frac{-1}{(z^2+r^2)^{1/2}}\right|_0^R$$

$$E = \frac{z\sigma}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{(z^2+R^2)^{1/2}}\right]$$

$$E = \frac{\sigma}{2\epsilon_0}\left[\frac{z}{z} - \frac{z}{(z^2+R^2)^{1/2}}\right]$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]}$$

Case (i)

when the radius R of disc is infinity or $R \gg z$

$R \longrightarrow \infty$

so, $E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\infty}\right]$

$$E = \frac{\sigma}{2\epsilon_0}$$

* It means that if we take radius of sheet infinite then it behaves as an infinite sheet of charges whose length is infinity.

Case (ii)

when $z \ll R$

then $z \longrightarrow 0$

so,

$E = \frac{\sigma}{2\epsilon_0}\left[1 - 0\right]$

$$E = \frac{\sigma}{2\epsilon_0}$$

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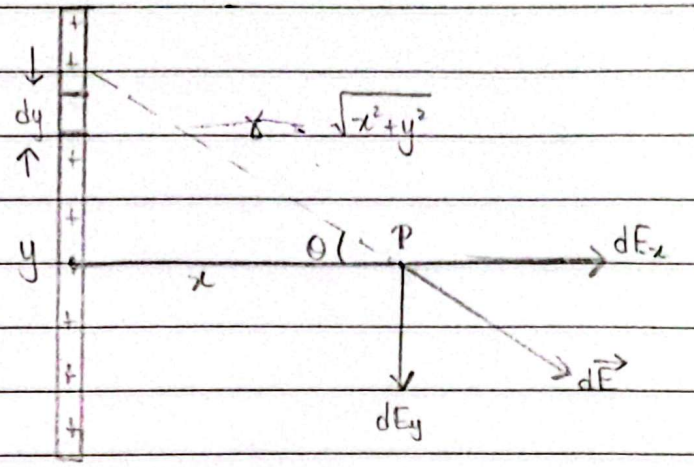
Date:

3) Electric field due to a long line of charge

As we know that

$$E = \int dE$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{r^2} \cos\theta$$



As, $E = \int dE$

$$= \int dE_x + \int dE_y$$

$$= \int dE_x + 0$$

$$= \int dE_x$$

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\, dy}{(x^2+y^2)} \cos\theta \qquad \because \left(\lambda = \frac{dQ}{dy} \Rightarrow dQ = \lambda dy \right)$$

Now,

$$\tan\theta = y/x$$

$$y = x\tan\theta$$

$$dy = x\sec^2\theta\, d\theta$$

Also, $x^2+y^2 = x^2 + x^2\tan^2\theta$

$$= x^2(1+\tan^2\theta)$$

$$= x^2\sec^2\theta$$

and $\cos\theta = x/r$

Thus,

$$E = \frac{\lambda}{4\pi\epsilon_0} \int \frac{\cancel{x\sec^2\theta}}{x^{\cancel{2}}\cancel{\sec^2\theta}}\, d\theta \cdot \cos\theta$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{x} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$E = \frac{1}{{}^{2}\cancel{4}\pi\epsilon_0} \cdot \frac{\lambda}{x} (\cancel{2}) \Rightarrow \boxed{\frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{x}}$$

Date: _____ Day: _____

WAVE MOTION

Speed of Transverse waves

The speed of transverse wave on a stretched string or cord depends on the tension in the cord, F_T, and on the mass per unit length of the cord, μ. If m is mass of a length l of cord, the $\mu = m/l$. For wave of small amplitude, the wave speed is

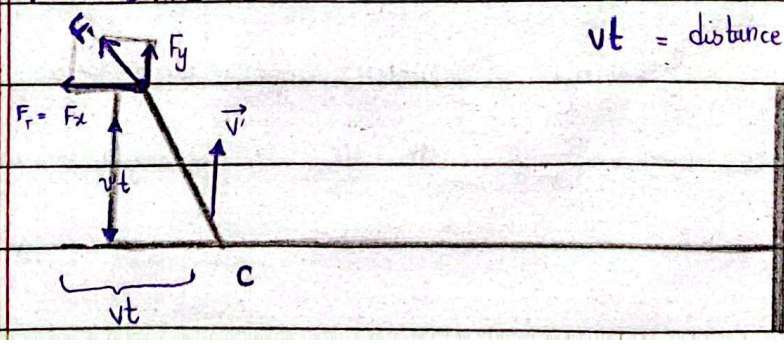
$$v = \sqrt{\frac{F_T}{\mu}}$$

Derivation:

Suppose a cord under tension F_T. The cord is pulled upward at a speed v' by force F_y. The end of cord moves upward a distane $v't$ whereas, the point C moves right a distance vt in time t.

$v't$ = distance end of cord move up

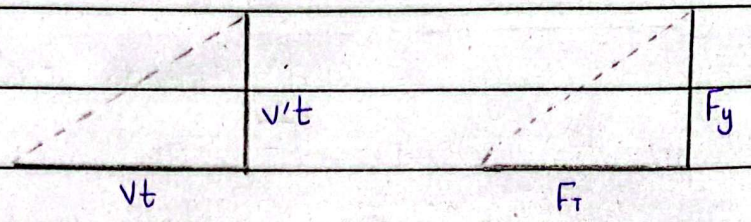
vt = distance wave moves to left



Date:_____ Day:_____

By similar triangles we have the approximate relation.

$$\frac{F_T}{F_y} = \frac{vt}{v't} = \frac{v}{v'} \longrightarrow \text{(i)}$$



The impulse given to an object is equal to its change in momentum. During the time t, the total upward impulse is

$$\Delta p = F_y t = F_T \frac{v'}{v} . t \longrightarrow \text{(ii)}$$

$$\because \left(\frac{F_T}{F_y} = \frac{v}{v'} \Rightarrow F_y = F_T \frac{v'}{v} \right.$$

$$\left. \text{for impulse : } F_y t = F_T \frac{v'}{v} . t \right)$$

Also, the Δp, is the mass of cord moving upward times its speed v'. Since the upward moving segment of cord has mass equal to m/l i.e. μ times its length i.e. vt thus we have

Date:_____ Day:_____

$$\Delta p = (\mu v t)\, v' \longrightarrow \text{(iii)}$$

Comparing equ (ii) and (iii)

$$F_T \cancel{t} \cdot \cancel{v'} = (\mu v \cancel{t})\, \cancel{v'}$$

$$v$$

$$F_T = v^2 \mu$$

or, $$v^2 = \frac{F_T}{\mu}$$

$$\boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

Wave Equation

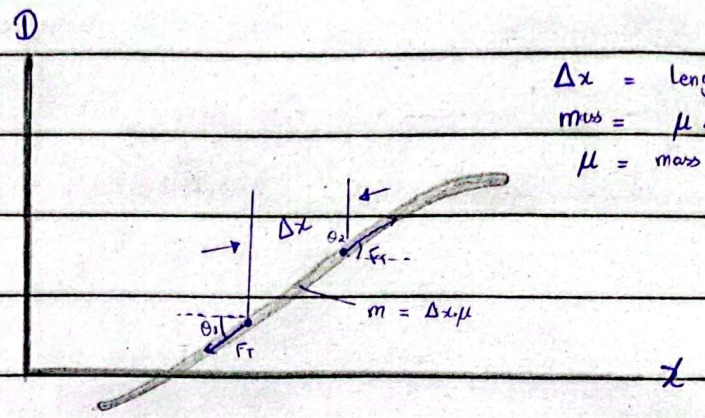
Newton's second law is used to evaluate the wave equation. We assume the amplitude of wave is small compared to the λ so each point can be assumed to move only vertically and tension in string F_T, does not vary. Thus, we now derive the wave equation for one-dimensional wave traveling.

According to Newton's 2nd law

$$F = ma$$

or, $$\Sigma F = ma \longrightarrow \text{(i)}$$

Date:_____ Day:_____



Δx = length of section

m_{us} = $\mu \Delta x$

μ = mass per unit length (P)

Here,

$$\Sigma F = F_T \sin\theta_2 - F_T \sin\theta_1$$

$$m = \mu \Delta x$$

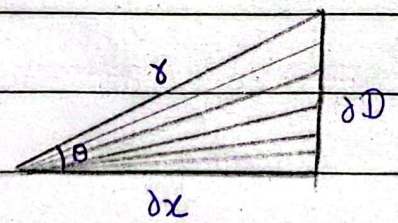
$$a = \frac{\partial^2 D_{(x,t)}}{\partial t^2}$$

Thus, equ (i) becomes

$$F_T \sin\theta_2 - F_T \sin\theta_1 = \mu \Delta x \cdot \frac{\partial^2 D_{(x,t)}}{\partial t^2}$$

$$F_T (\sin\theta_2 - \sin\theta_1) = \mu \Delta x \cdot \frac{\partial^2 D_{(x,t)}}{\partial t^2} \longrightarrow \text{(ii)}$$

According to small θ approximation



Here, $\sin\theta = \frac{y}{\gamma}$

$\tan\theta = \frac{y}{x}$

Date:_____ Day:_____

when $\theta \longrightarrow 0$, $x \longrightarrow x$

So, $\sin\theta = \tan\theta$

which is $\tan\theta = \dfrac{\partial D}{\partial x} \longrightarrow$ (iii)

or, $\tan\theta = \text{Slope} = S$

Thus, equ (ii) becomes

$$F_T(\tan\theta_2 - \tan\theta_1) = \mu \Delta x \cdot \frac{\partial^2 D}{\partial t^2}$$

$$F_T(S_2 - S_1) = \mu \Delta x \cdot \frac{\partial^2 D}{\partial t^2}$$

$$F_T(\Delta S) = \mu \Delta x \cdot \frac{\partial^2 D}{\partial t^2}$$

$$\frac{\Delta S}{\Delta x} = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2}$$

~~when~~ Taking limit as $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2}$$

$$\frac{\partial S}{\partial x} = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial D}{\partial x}\right) = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2} \quad \because (\text{from (iii)})$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2} \longrightarrow \text{(iv)}$$

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Date: Day:

As we know that

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$v^2 = \frac{F_T}{\mu}$$

or $$\frac{\mu}{F_T} = \frac{1}{v^2}$$

So, equ (iv) becomes

$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}}$$

P1: Prove that the following equation is the wave equation.

$$y(x,t) = A\cos(kx - \omega t)$$

As we know that

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

or, in this case

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \longrightarrow (i)$$

Now,

$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial y}{\partial x} = A(-\sin(kx - \omega t))\,k$$

Date:_____ Day:_____

$$\frac{\partial y}{\partial x} = -kA\sin(kx-\omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -kA\cos(kx-\omega t).\,k$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\cos(kx-\omega t)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = -k^2 y$$

Also,

$$\frac{\partial y}{\partial t} = A(-\sin(kx-\omega t)).(-\omega)$$

$$\frac{\partial y}{\partial t} = A\omega\sin(kx-\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = A\omega(\cos(kx-\omega t))(-\omega)$$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2(\cos(kx-\omega t))$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

Thus, equ (i) becomes

$$+k^2 y = 1/v^2\,(+\omega^2 y)$$

$$k^2 = 1/v^2\,(\omega^2)$$

$$\left(\frac{2\pi}{\lambda}\right)^2 = \frac{1}{v^2}(2\pi f)^2$$

k = wave number

Date: Day:

$$\frac{\cancel{(2\pi)^2}}{\lambda^2} = \frac{1}{(f\lambda)^2} = \cancel{(2\pi)^2} \cdot f^2$$

$$\frac{1}{\lambda^2} = \frac{1}{\cancel{f^2}\lambda^2} \cdot \cancel{f^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Hence Proved

Doppler's Effect

The apparent change in the frequency of sound due to the relative motion of source and observer is known as Doppler's Effect.

Case I: Observer moves towards stationary source.

The apparent frequency is given

$$f' = \frac{v'}{\lambda}$$

Here, $v' = v + v_0$

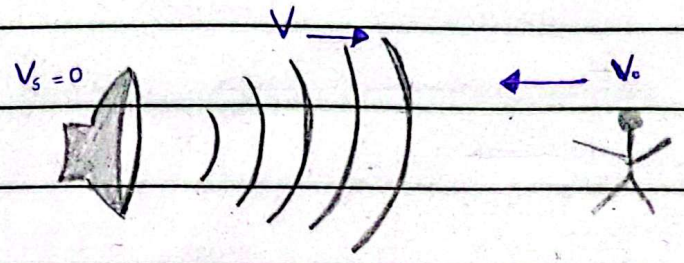
$$f' = \frac{v + v_0}{\lambda}$$

$$f' = \frac{v + v_0}{v/f}$$

Date:_____ Day:_____

$$f' = \left(\frac{v + v_o}{v}\right) f$$

$$\boxed{f' = \left(1 + \frac{V_{observer}}{V_{sound}}\right) f}$$



Case II: <u>Observer moves away from stationary source.</u>

The apparent frequency is

$$f' = \frac{v'}{\lambda}$$

Here, $v' = v - v_o$

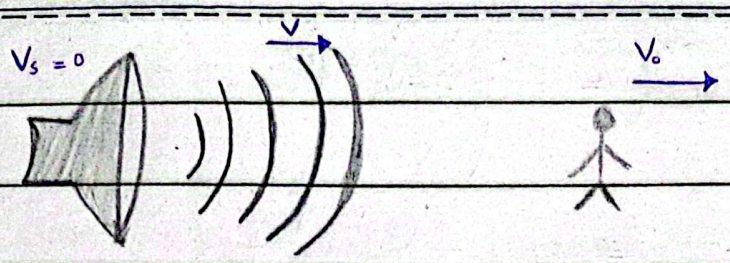
$$f' = \frac{v - v_o}{\lambda}$$

$$f' = \frac{v - v_o}{v/f}$$

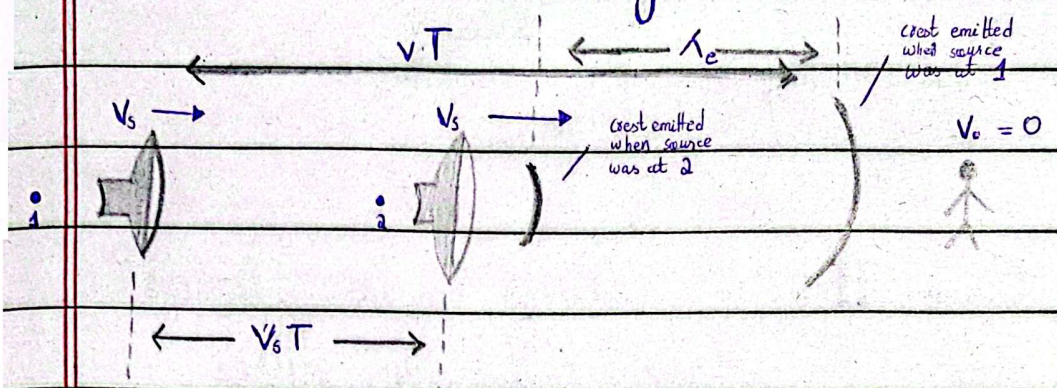
$$f' = \left(\frac{v - v_o}{v}\right) f$$

$$\boxed{f' = \left(1 - \frac{V_{observer}}{V_{sound}}\right) f}$$

Date:_____ Day:_____



Case III : Source moves towards stationary observer



The apparent frequency is

$$f' = \frac{V}{\lambda_e} \longrightarrow (i)$$

Here,

$$\lambda_e = VT - V_sT$$

$$\lambda_e = T(V - V_s)$$

$$\lambda_e = \frac{V - V_s}{f}$$

Now equ (i) becomes

$$f' = \frac{V}{\left(\frac{V - V_s}{f}\right)}$$

$$f' = \left(\frac{V}{V - V_s}\right) f$$

Date: Day:

$$f' = \left(\frac{1}{\frac{V - V_s}{V}}\right) f$$

$$\boxed{f' = \frac{f}{\left(1 - \frac{V_{source}}{V_{sound}}\right)}}$$

Case IV : Source moves away from observer

The apparent frequency is

$$f' = \frac{V}{\lambda_e} \longrightarrow (i)$$

Here,

$$\lambda_e = VT + V_sT$$

$$\lambda_e = T(V + V_s)$$

$$\lambda_e = \frac{V + V_s}{f}$$

Now equ (i) becomes

$$f' = \frac{V}{\left(\frac{V + V_s}{f}\right)}$$

$$f' = \left(\frac{V}{V + V_s}\right) f$$

$$f' = \left(\frac{1}{\frac{V + V_s}{V}}\right) f$$

$$\boxed{f' = \frac{f}{\left(1 + \frac{V_{source}}{V_{sound}}\right)}}$$