Lecture

Chapter 1. The Foundations: Logic and Proofs

1.6. Rules of Inference

Basic terminologies

- **Arguments:** An argument in logic is a sequence of statements, where some statements (called premises) provide support or reasons for another statement (called the conclusion). An argument is valid if the truth of the premises guarantees the truth of the conclusion.

Structure of an Argument

- **Premises**: The statements that provide evidence or reasons.
- **Conclusion**: The statement that follows from the

Basic terminologies

- Premises: Premises are the propositions or statements in an argument that provide the foundation or support for the conclusion. They are the reasons or evidence that lead to the conclusion.
- Here's another argument:

1. **Premise 1**: If it is a cat, then it is a mammal. ($p \rightarrow q$)
2. **Premise 2**: It is a cat. (p)
3. **Conclusion**: Therefore, it is a mammal. (q)

logical Falasy

- Fallacies are errors in reasoning that lead to invalid arguments. They can occur in various forms, often due to misleading reasoning, irrelevant information, or assumptions that do not hold true.

Rules of Inference

- **Rules of inference** are logical principles that dictate the valid steps one can take to derive conclusions from premises in a logical argument.
- To deduce new statements from statements, we use rules of inference which are templates for constructing valid arguments.
- Rules of inference are our basic tools for establishing the truth of statements.

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Modus Ponens (Law of Detachment)

- **Form**: (Affirming the Antecedent)
  - If p then q ($p \rightarrow q$)
  - p (the antecedent is true)
  - Therefore, q (the consequent is true)
- **Example of Modus Ponens**

1. **Premise 1**: If it rains, the ground will be wet.
   ($p \rightarrow q$ where p is "it rains" and q is "the ground is wet.")
2. **Premise 2**: It is raining.
   (p is true.)
3. **Conclusion**: Therefore, the ground is wet.
   (q is true.)

Modus Tollens (Denying the Consequent)

- **Form:**
  - If p then q ($p \rightarrow q$)
  - $\neg$ q (the consequent is false)
  - Therefore, $\neg$ p (the antecedent is false)
- **Example :**

Premise 1: If it rains, the ground will be wet.
($p \rightarrow q$ where p is "it rains" and q is "the ground is wet.")

Premise 2: The ground is not wet.
(Not q is true.)

Conclusion: Therefore, it is not raining.
(Not p is true.)

Hypothetical Syllogism

- **Form:**
  - If p then q ($p \rightarrow q$)
  - If q then r ($q \rightarrow r$)
  - Therefore, if p then r ($p \rightarrow r$)
- **Example:**

Premise 1: If it rains, the ground will be wet.
($p \rightarrow q$ where p is "it rains" and q is "the ground is wet.")
Premise 2: If the ground is wet, the flowers will bloom.
($q \rightarrow r$)
Conclusion: Therefore, if it rains, the flowers will bloom. ($p \rightarrow r$)