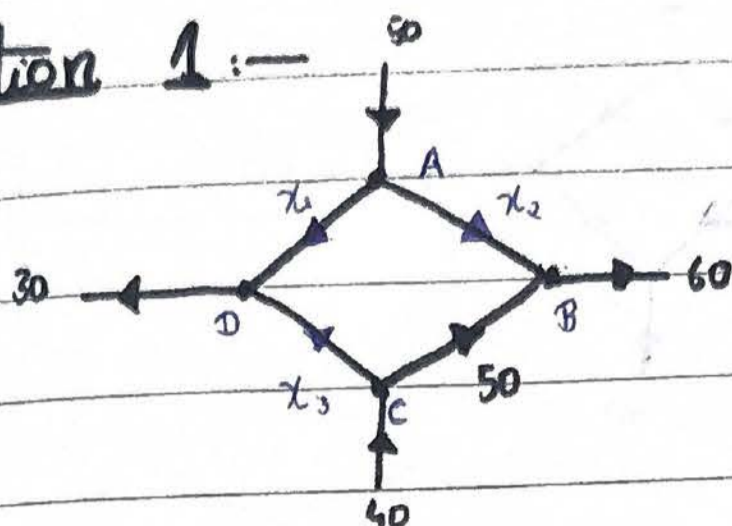


# Exercise 1.9

## Question 1:—



Network node	Flow In	Flow out
A	50	$x_1 + x_2 \rightarrow (i)$
B	$x_2 + 50$	60 $\rightarrow (ii)$
C	40	$x_3 + 50 \rightarrow (iii)$
D	$x_1 + x_3$	30 $\rightarrow (iv)$

The system can be arranged as

$$x_1 + x_2 = 50$$

$$x_2 + 50 = 60$$

$$x_3 + 50 = 40$$

$$x_1 + x_3 = 30$$

Now,

From equ (ii)

$$x_2 = +10$$

From equ (i)

$$50 = x_1 + 10$$

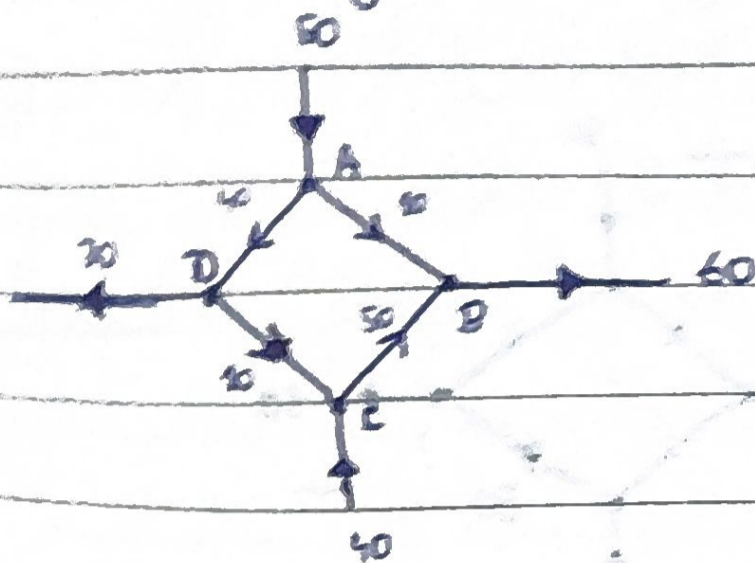
$$x_1 = 40$$

From equ (iii)

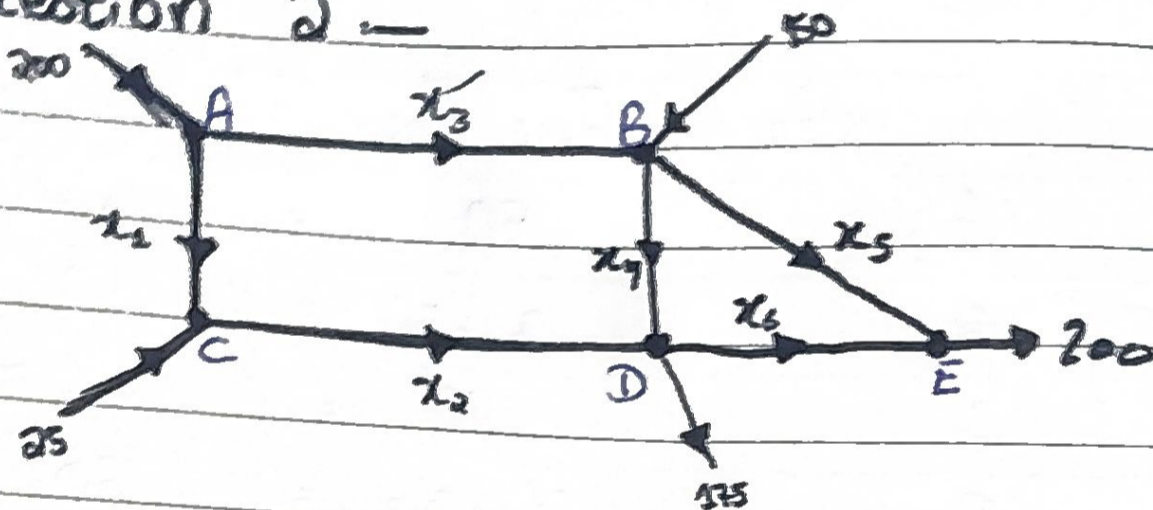
$$40 = x_3 + 50$$

$$x_3 = -10$$

Now the figure has become



Question 2 -



(a) Network	Node	Flow In	Flow out
A		200	$x_1 + x_3 \rightarrow (i)$
B		$x_3 + 150$	$x_4 + x_5 \rightarrow (ii)$
C		$x_1 + 25$	$x_2 \rightarrow (iii)$
D		$x_2 + x_4$	$x_6 + 175 \rightarrow (iv)$
E		$x_5 + x_6$	$200 \rightarrow (v)$

This system can be arranged as

$$x_1 + x_3 = 200$$

$$-x_3 + x_4 + x_5 = 150$$

$$-x_1 + x_2 = 25$$

$$x_2 + x_4 - x_6 = 175$$

$$x_5 + x_6 = 200$$

(b) The augmented matrix is given as

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & -1 & 1 & 1 & 0 & 150 \\ -1 & 1 & 0 & 0 & 0 & 0 & 25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array} \right]$$

$R_3 + R_1$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & -1 & 1 & 1 & 0 & 150 \\ 0 & 1 & 1 & 0 & 0 & 0 & 225 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array} \right]$$

$R_3 + R_2$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & -1 & 1 & 1 & 0 & 150 \\ 0 & 1 & 0 & 1 & 1 & 0 & 375 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array} \right]$$

$R_4 - R_3$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & -1 & 1 & 1 & 0 & 150 \\ 0 & 1 & 0 & 1 & 1 & 0 & 375 \\ 0 & 0 & 0 & 0 & -1 & -1 & -200 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array} \right]$$

$$R_5 + R_4 ; (-1)R_2 ; (-1)R_5$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 1 & 0 & 1 & 1 & 0 & 375 \\ 0 & 0 & 0 & 0 & +1 & +1 & +200 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

~~(-1)R\_4~~

The equations can be written as

$$x_1 + x_3 = 200$$

$$x_3 - x_4 - x_5 = -150$$

$$x_2 + x_4 + x_5 = 375$$

$$x_5 + x_6 = 200$$

Let  $x_4 = s$  and  $x_6 = t$

$$x_1 + x_3 = 200 \rightarrow (i)$$

$$x_3 - s - x_5 = -150 \rightarrow (ii)$$

$$x_2 + s + x_5 = 375 \rightarrow (iii)$$

$$x_5 + t = 200 \rightarrow (iv)$$

From (iv), we have

$$\boxed{x_5 = 200 - t}$$

Put  $x_5$  in equ (iii)

$$x_2 + s + 200 - t = 375$$

$$x_2 = 375 - 200 + t - s$$

$$\boxed{x_2 = 175 + t - s}$$

Put  $x_5$  in equ (ii)

$$x_3 + (200 - t) - s = -150$$

$$x_3 - 200 + t - s = -150$$

$$x_3 = 200 - 150 + s - t$$

$$\boxed{x_3 = 50 + s - t}$$

Put  $x_3$  in equ (i)

$$x_1 + 50 + s - t = 200$$

$$\boxed{x_1 = 150 + t - s}$$

(c) when  $x_4 = 50$  and  $x_6 = 0$

$$\Rightarrow x_4 = s = 50 \quad \text{and} \quad x_6 = t = 0$$

$$\Rightarrow x_1 = 150 + 0 - 50 = 100$$

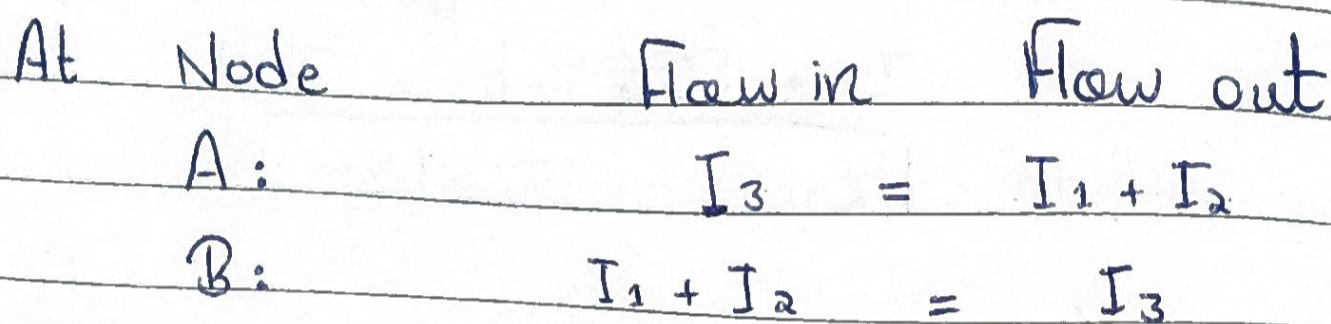
$$\Rightarrow x_2 = 175 + 0 - 50 = 125$$

$$\Rightarrow x_3 = 50 + 50 - 0 = 100$$

$$\Rightarrow x_5 = 200 - 0 = 200$$

The directions of flow agree with the arrow direction in the diagram.

(M) (T) (W) (T) (F)


$$I_3 - I_1 - I_2 = 0 \rightarrow (i)$$

	Voltage Rise	Voltage drop
Left side:	$2I_1 - 2I_2 =$	$6 + 2I_2$
Right side:	$2I_2 + 4I_3 =$	$8$

Thus, the system of equations is

$$I_1 + I_2 - I_3 = 0$$

$$2I_1 - 2I_2 = 6 + \cancel{2I_2}$$

$$2I_2 + 4I_3 = 8$$

Now, the augmented matrix is

$$\left[ \begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -2 & 0 & 6 \\ 0 & 2 & 4 & 8 \end{array} \right]$$

After solving, we get reduced echelon form as:

$$\begin{bmatrix} 1 & 0 & 0 & | & 13/5 \\ 0 & 1 & 0 & | & -2/5 \\ 0 & 0 & 1 & | & 11/5 \end{bmatrix}$$

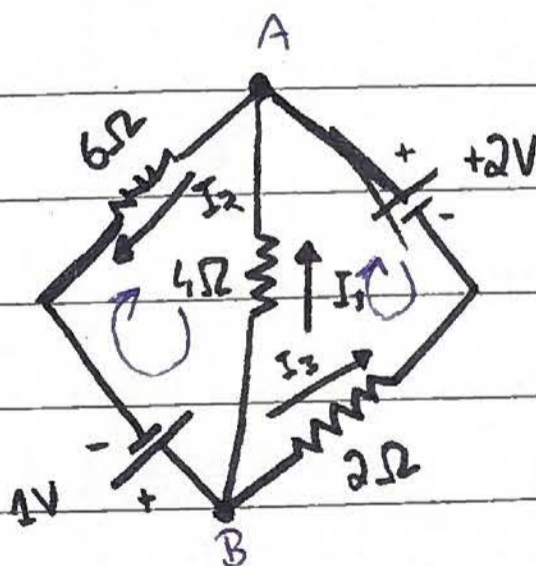
Thus,

$$I_1 = 13/5 = 2.6 \text{ A}$$

$$I_2 = -2/5 = -0.4 \text{ A}$$

$$I_3 = 11/5 = 2.2 \text{ A}$$

Question 6:—



At Node

Flow in

Flow out

A :

$$I_1 + I_3 = I_2$$

B :

$$I_2 = I_1 + I_3$$

Thus, we have

$$I_1 + I_3 - I_2 = 0 \rightarrow (i)$$

Now,

Voltage Rise

Voltage Drop

Left Side:  $6I_2 + 4I_1 = 1$

Right Side:  $2I_3 = 4I_1 + 2$

Thus, the system of equations is

$$I_1 + I_3 - I_2 = 0$$

$$4I_1 + 6I_2 = 1$$

$$2I_3 - 4I_1 = 2$$

Now, the augmented matrix is

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 6 & 0 & 1 \\ -4 & 0 & 2 & 2 \end{array} \right]$$

After solving we get the reduced echelon form as:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -5/22 \\ 0 & 1 & 0 & 7/22 \\ 0 & 0 & 1 & 6/11 \end{array} \right]$$

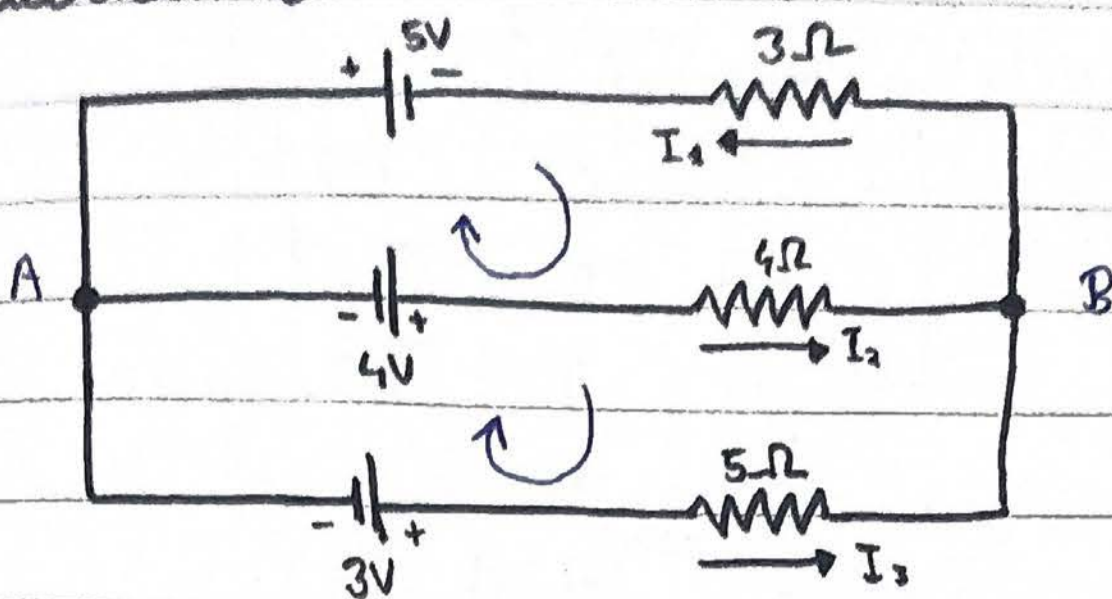
Thus,

$$I_1 = -5/22 \text{ A}$$

$$I_2 = 7/22 \text{ A}$$

$$I_3 = 6/11$$

## Question 8



At Node                      Flow in                      Flow out

$$A: \quad I_1 = I_2 + I_3$$

$$B: \quad I_2 + I_3 = I_1$$

Thus, we have

$$-I_1 + I_2 + I_3 = 0 \rightarrow (i)$$

Now,

	Voltage Rise		Voltage Drop
Left Side:	$3I_1 + 4I_2$	$=$	$5 + 4$

Right Side:	$4 + 5I_3$	$=$	$4I_2 + 3$
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Thus,

$$-I_1 + I_2 + I_3 = 0$$

$$3I_1 + 4I_2 = 9$$

$$-4I_2 + 5I_3 = -1$$

Now, the augmented matrix is

$$\left[ \begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 3 & 4 & 0 & 9 \\ 0 & -4 & 5 & -1 \end{array} \right]$$

After solving we get the reduced echelon form given as:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 77/47 \\ 0 & 1 & 0 & 48/47 \\ 0 & 0 & 1 & 29/47 \end{array} \right]$$

Thus,

$$I_1 = 77/47 A$$

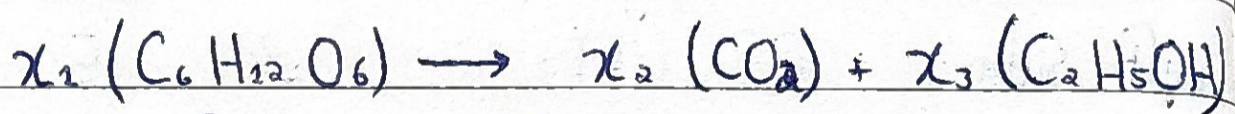
$$I_2 = 48/47 A$$

$$I_3 = 29/47 A$$

Question 10 :-



We are looking for <sup>positive values</sup>  $x_1, x_2$  and  $x_3$  such that



The no. of atoms of C, H and O on both sides must equal:

$$\begin{array}{lcl} \text{Carbon} & \text{Left Side} & \text{Right Side} \\ & 6x_1 & = x_2 + 2x_3 \end{array}$$

$$\begin{array}{lcl} \text{Hydrogen} & 12x_1 & = 6x_3 \end{array}$$

$$\begin{array}{lcl} \text{Oxygen} & 6x_1 & = 2x_2 + x_3 \end{array}$$

The linear system

$$6x_1 - x_2 - 2x_3 = 0$$

$$12x_1 - 6x_3 = 0$$

$$6x_1 - 2x_2 - x_3 = 0$$

The reduced echelon form is

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

The equations are:

$$x_1 - \frac{1}{2}x_3 = 0$$

$$x_2 - x_3 = 0$$

Let  $x_3 = t$ , we have

$$x_1 - \frac{1}{2}t = 0$$

$$x_2 - t = 0$$

$$x_3 = t$$

so,

$$x_1 = \frac{1}{2}t$$

$$x_2 = t$$

$$x_3 = t$$

The smallest positive values for unknowns occur when  $t = 2$ , thus,

$$x_1 = 1 ; x_2 = 2 ; x_3 = 2$$

Hence,

