DEFINITION 1 If A is an $n \times n$ matrix, then a nonzero vector $\mathbf{x}$ in R^n is called an ***eigenvector*** of A (or of the matrix operator T_A) if $A\mathbf{x}$ is a scalar multiple of $\mathbf{x}$; that is,

$$A\mathbf{x} = \lambda\mathbf{x}$$

for some scalar λ. The scalar λ is called an ***eigenvalue*** of A (or of T_A), and $\mathbf{x}$ is said to be an ***eigenvector corresponding to*** $\boldsymbol{\lambda}$.

THEOREM 5.1.1 *If A is an $n \times n$ matrix, then λ is an eigenvalue of A if and only if it satisfies the equation*

$$\det(\lambda I - A) = 0 \tag{1}$$

This is called the ***characteristic equation*** *of A.*

THEOREM 5.1.2 *If A is an $n \times n$ triangular matrix (upper triangular, lower triangular, or diagonal), then the eigenvalues of A are the entries on the main diagonal of A.*

THEOREM 5.1.3 *If A is an $n \times n$ matrix, the following statements are equivalent.*

(*a*) λ *is an eigenvalue of* A.

(*b*) λ *is a solution of the characteristic equation* $\det(\lambda I - A) = 0$.

(*c*) *The system of equations* $(\lambda I - A)\mathbf{x} = \mathbf{0}$ *has nontrivial solutions.*

(*d*) *There is a nonzero vector* $\mathbf{x}$ *such that* $A\mathbf{x} = \lambda\mathbf{x}$.

Exercise 5.1

Question 1 :—

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad ; \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

To check if x is an eigenvector, we must calculate the product Ax

$$Ax = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Ax = \begin{bmatrix} (1)(1) + (2)(-1) \\ (3)(1) + (2)(-1) \end{bmatrix}$$

$$Ax = \begin{bmatrix} 1-2 \\ 3-2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

An eigenvalue must satisfy the equation $Ax = \lambda x$, when λ is the eigenvalue.

Compare Ax with x to find eigenvalue.

$$Ax = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1x$$

- Since Ax is a scalar multiple of x, x is confirmed to be an eigenvector.
- The corresponding eigenvalue is $\lambda = -1$.

Question 4 :-

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} ; \quad x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

To check if x is an eigenvector, we must calculate the product Ax

$$Ax = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Ax = \begin{bmatrix} (2)(1) + (-1)(1) + (-1)(1) \\ (-1)(1) + (2)(+1) + (-1)(1) \\ (-1)(1) + (-1)(1) + (2)(1) \end{bmatrix}$$

$$Ax = \begin{bmatrix} 2 - 1 - 1 \\ -1 + 2 - 1 \\ -1 - 1 + 2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 2 - 2 \\ -2 + 2 \\ -2 + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

An eigenvalue must satisfy the equation $Ax = \lambda x$, where λ is the eigenvalue.

Compare Ax with x to find eigenvalue.

$$Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0x$$

⇒ Since Ax is a scalar multiple of x, x is confirmed to be an eigenvector.

⇒ The corresponding eigenvalue is $\lambda = 0$.

Question 5 :—

To solve these, we follow three steps for each matrix:

1. **Characteristic Equation:** Find $\det(\lambda I - A) = 0$.
2. **Eigenvalues:** Solve the equation for λ.

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

⇒ <u>Characteristic Equation</u>

The given equation is

$$\det(\lambda I - A) = 0$$

Here,

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & 0 - 4 \\ 0 - 2 & \lambda - 3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{bmatrix}$$

Now,

$$\det \begin{bmatrix} \lambda-1 & -4 \\ -2 & \lambda-3 \end{bmatrix} = 0$$

$$\begin{vmatrix} \lambda-1 & -4 \\ -2 & \lambda-3 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda-3) - (-2)(-4) = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

⇒ <u>Eigenvalues</u>:

$$\lambda^2 - 4\lambda - 5 = 0$$

$$\lambda^2 - 5\lambda + \lambda - 5 = 0$$

$$\lambda(\lambda-5) + 1(\lambda-5) = 0$$

$$(\lambda-5)(\lambda+1) = 0$$

$$\lambda - 5 = 0 \qquad \lambda + 1 = 0$$

$$\lambda = 5 \qquad \lambda = -1$$

⇒ Bases for Eigenspaces:

→ For $\lambda = 5$; solve $5I - A$

$$5I - A = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$$

The row reduced echelon form

of $(5I - A)$ is given as

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

The general solution of $(5I - A)x = 0$ can be found as

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - x_2 \\ 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In equation form, we have

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

Let $x_2 = t$, we have

$$x_1 = t$$

$$x_2 = t$$

In vector form, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Hence,

<u>Basis:</u> $\{(1, 1)\}$

→ For $\lambda = -1$; solve $-1I - A$

$$-1I - A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 \\ -2 & -4 \end{bmatrix}$$

The row reduced echelon form is

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

The general solution of $(-1I - A)x = 0$ is

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_1 + 2x_2 \\ 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In equation form, we have

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

Let $x_2 = t$, we have

$$x_1 = -2t$$

$$x_2 = t$$

In vector form, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2t \\ t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

• <u>Basis</u> : $\{(-2, 1)\}$

(b) $\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

⇒ <u>Characteristic Equation:</u>

The given equation is

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda+2 & 0+7 \\ 0-1 & \lambda-2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda+2 & 7 \\ -1 & \lambda-2 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} (\lambda+2) & 7 \\ -1 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda+2)(\lambda-2) - (-1)(7) = 0$$

$$\lambda^2 + 2\lambda - 2\lambda - 4 + 7 = 0$$

$$\lambda^2 + 3 = 0$$

Thus, we have no real eigenvalues.

Question 6:—

(b) $\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$

→ Characteristic Equation:

The given equation is

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-2 & 3 \\ 0 & \lambda-2 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda-2 & 3 \\ 0 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda-2)(\lambda-2) - (0)(3) = 0$$

$$(\lambda-2)^2 = 0$$

→ Eigenvalues:

$$(\lambda-2)^2 = 0$$

$$\sqrt{(\lambda-2)^2} = \sqrt{0}$$

$$\lambda - 2 = 0$$

$$\lambda = 2$$

→ Bases for Eigenspaces:

→ For $\lambda = 2$; solve $2I - A$

$$2I - A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$$

$$2I - A = \begin{bmatrix} 2-2 & 0+3 \\ 0-0 & 2-2 \end{bmatrix}$$

$$2I - A = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$$

The row reduced echelon form is

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The general solution of

$(2I - A)x = 0$ can be found as

$$(2I - A)x = 0$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Here,

$$x_2 = 0$$

Let $x_1 = t$

In vector form, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Hence,

<u>Basis</u>: $\{(1, 0)\}$

(d) $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$

⇒ <u>Characteristic Equation:</u>

The given equation is

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & -2 \\ 2 & \lambda+1 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda-1 & -2 \\ 2 & \lambda+1 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda+1) - (2)(-2) = 0$$

$$\lambda^2 - 1 + 4 = 0$$

$$\lambda^2 + 3 = 0$$

Thus, we have no real eigenvalues

Question 7 :-

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

→ Characteristic Equation

The given equation is

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{vmatrix} = 0$$

Expand along the second column

$$(\lambda-1)\begin{vmatrix} \lambda-4 & -1 \\ 2 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-1)\left[(\lambda-4)(\lambda-1) - (2)(-1)\right] = 0$$

$$(\lambda-1)\left[\lambda^2 - \lambda - 4\lambda + 4 + 2\right] = 0$$

$$(\lambda-1)(\lambda^2 - 5\lambda + 6) = 0$$

$$(\lambda-1)(\lambda-2)(\lambda-3) = 0$$

⇒ <u>Eigenvalues:</u>

$$(\lambda-1)\cdot(\lambda-2)(\lambda-3) = 0$$

so, we have

$$\lambda - 1 = 0 \Rightarrow \lambda = 1$$

$$\lambda - 2 = 0 \Rightarrow \lambda = 2$$

$$\lambda - 3 = 0 \Rightarrow \lambda = 3$$

⇒ <u>Bases for Eigenspaces:</u>

→ For $\lambda = 1$; solve $(1I - A)x = 0$

$$(1I - A)x = 0$$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 1-4 & 0-0 & 0-1 \\ 0+2 & 1-1 & 0-0 \\ 0+2 & 0-0 & 1-1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

The row reduced echelon form is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_1 \\ x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The general solution is given as

$x_1 = 0$; $x_3 0$

Let $x_2 = t$

In vector form, we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

→ Basis : $\{(0, 1, 0)\}$

Same for $\lambda = 2$ and $\lambda = 3$.

Question 11 —

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

⇒ Characteristic Equation:

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-4 & 0 & 1 \\ 0 & \lambda-3 & 0 \\ -1 & 0 & \lambda-2 \end{bmatrix}\right) = 0$$

Expand along the second column

$$(\lambda-3)\begin{vmatrix} \lambda-4 & 1 \\ -1 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\left[(\lambda-4)(\lambda-2) - (-1)(1)\right] = 0$$

$$(\lambda-3)(\lambda^2-4\lambda-2\lambda+8+1) = 0$$

$$(\lambda-3)(\lambda^2-6\lambda+9) = 0$$

$$(\lambda-3)^3$$

→ <u>Eigenvalue</u>:

$$(\lambda-3)^3 = 0$$

$$\sqrt[3]{(\lambda-3)^{\cancel{3}}} = \sqrt[3]{0}$$

$$\lambda-3 = 0$$

$$\lambda = 3$$

→ <u>Bases for Eigenspaces</u>:

→ For $\lambda = 3$; solve $(3I - A)x = 0$

$$3I - A$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

The row reduced echelon form is

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now,

$$(3I - A)x = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_1 - x_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The equation form is given as

$$x_1 - x_3 = 0$$

$$x_1 = x_3$$

Let $x_3 = t$ and $x_2 = s$

so, we have

$$x_1 = t \; ; \; x_2 = s \; ; \; x_3 \; t$$

In vector form, we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

- <u>Basis</u> : $\{(0,1,0), (1,0,1)\}$

Question 12 :—

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

⇒ <u>Characteristic Equation</u> :

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & 3 & -3 \\ -3 & \lambda+5 & -3 \\ -6 & 6 & \lambda-4 \end{bmatrix}\right) = 0$$

$$\det\begin{bmatrix} \lambda-1 & 3 & -3 \\ -3 & \lambda+5 & -3 \\ -6 & 6 & \lambda-4 \end{bmatrix} = 0$$

$$C_2 \longrightarrow C_2 + C_1$$

$$\begin{vmatrix} \lambda-1 & \lambda+2 & -3 \\ -3 & \lambda+2 & -3 \\ -6 & 0 & \lambda-4 \end{vmatrix} = 0$$

$$(\lambda+2)\begin{vmatrix} \lambda-1 & 1 & -3 \\ -3 & 1 & -3 \\ -6 & 0 & \lambda-4 \end{vmatrix} = 0$$

$$R_1 \longrightarrow R_1 - R_2$$

$$(\lambda+2)\begin{vmatrix} \lambda+2 & 0 & 0 \\ -3 & 1 & -3 \\ -6 & 0 & \lambda-4 \end{vmatrix} = 0$$

Expand by first row:

$$(\lambda+2)\cdot(\lambda+2)\begin{vmatrix} 1 & -3 \\ 0 & \lambda-4 \end{vmatrix} = 0$$

$$(\lambda+2)^2(\lambda-4) = 0$$

→ Eigenvalues:

$$(\lambda+2)^2(\lambda-4) = 0$$

$(\lambda+2)^2 = 0$ | $(\lambda-4) = 0$

$\lambda = -2$ | $\lambda = 4$

→ Bases for Eigenspaces:

→ For $\lambda = -2$; solve $(-2I - A)x = 0$

$-2I - A$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 & -3 \\ -3 & 3 & -3 \\ -6 & 6 & -6 \end{bmatrix}$$

Now,

$$\begin{bmatrix} -3 & 3 & -3 \\ -3 & 3 & -3 \\ -6 & 6 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -3x_1 + 3x_2 - 3x_3 \\ -3x_1 + 3x_2 - 3x_3 \\ -6x_1 + 6x_2 - 6x_3 \end{bmatrix} = 0$$

In general, the solution is

$$x_1 - x_2 + x_3 = 0$$

and,

$$x_2 = x_1 + x_3$$

Thus, in vector form we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + x_3 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Basis : $\{(1,1,0), (0,1,1)\}$

$\because$ Same for $\lambda = 4$.