Exercise 1.5

Elementary Matrix

An elementary matrix is a matrix that can be obtained by performing exactly one elementary row operation on an identity matrix. There are three types of elementary row operations:

i) Swap two rows
ii) Multiply a row by a nonzero scalar
iii) Add a multiple of one row to another.

Example:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \longleftrightarrow R_3$$

$$E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Exercise 1.5

Find the row operation & corresponding elementary matrix that will restore the given elementary matrix to identity matrix.

Question 3 :-

(a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

Operation : $3R_2 + R_1$

Matrix : $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Operation : $R_1 \leftrightarrow R_3$

Matrix : $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Question 4 :—

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Operation: $(\frac{1}{3}) R_3$

Matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A.

Question 5 :—

(b) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

Operation: corresponding to E

$(-3)R_2 + R_3$

Now,

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ +1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 4 & -12 & -10 \end{bmatrix}$$

Question 6:—

(c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Operation corresponding to E

$(\frac{1}{5}) R_2$

Now,

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix}$$

In Exercises 7–8, use the following matrices and find an elementary matrix E that satisfies the stated equation.

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 8 & 1 & 5 \\ -6 & 21 & 3 \\ 3 & 4 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 8 & 1 & 5 \\ 8 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

Question 7:—

(a) $EA = B$

Here,

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$$

Let

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \longleftrightarrow R_3$$

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\because$ B was obtained from A by interchanging the first and the third row

(d) $EC = A$

Here,

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

THEOREM 1.4.5 *The matrix*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible if and only if $ad - bc \neq 0$, *in which case the inverse is given by the formula*

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{2}$$

Let,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2R_1 + R_3$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$\because$ A was obtained from C by adding 2 times the first row to third row.

Use Theorem 1.4.5 and then use the inversion algorithm to find A^{-1}:

Question 9:—

(a) $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$

(Method 1: Using Theorem 1.4.5)

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 4 \\ 2 & 7 \end{vmatrix}$$

$$= (1)(7) - (2)(4)$$

$$= -1$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$$

(Method 2 : Inversion algorithm)

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$$

$$-2R_1 + R_2$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$$(-1)R_2$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$$-4R_2 + R_1$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & 0 & -7 & 4 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

Thus,

$$A^{-1} = \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$$

Theorem 1.4.5 and Inversion Algorithm

Question 10 :—

(a) $A = \begin{bmatrix} 1 & -5 \\ 3 & -16 \end{bmatrix}$

Method 1 :—

$$|A| = \begin{vmatrix} 1 & -5 \\ 3 & -16 \end{vmatrix}$$

$$= (1)(-16) - (3)(-5)$$

$$= -16 + 15$$

$$= -1$$

Therefore, A is invertible and its inverse is given as

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} -16 & 5 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 16 & -5 \\ 3 & -1 \end{bmatrix}$$

Method 2 :—

$$A = \begin{bmatrix} 1 & -5 \\ 3 & -16 \end{bmatrix}$$

$$A/I = \begin{bmatrix} 1 & -5 \\ 3 & -16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A/I = \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 3 & -16 & 0 & 1 \end{array}\right]$$

$$-3R_1 + R_2$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$$

$$(-1)R_2$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$5R_2 + R_1$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 1 & 0 & 16 & -5 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

Hence, the inverse is

$$A^{-1} = \begin{bmatrix} 16 & -5 \\ 3 & -1 \end{bmatrix}$$

(b) $$A = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$$

<u>Method 1</u> :—

$$|A| = \begin{vmatrix} 6 & 4 \\ -3 & -2 \end{vmatrix}$$

$$= (6)(-2) - (-3)(4)$$

$$|A| = -12 + 12$$
$$= 0$$

Therefore, A is not invertible

<u>Method 2 :—</u>

$$A = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$$

$$A|I = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A|I = \left[\begin{array}{cc|cc} 6 & 4 & 1 & 0 \\ -3 & -2 & 0 & 1 \end{array}\right]$$

$$2R_2 + R_1$$

$$\underset{\sim}{R} \left[\begin{array}{cc|cc} 0 & 0 & 1 & 2 \\ -3 & -2 & 0 & 1 \end{array}\right]$$

A row of zeros was obtained on the left side, therefore the matrix is not invertible.

Inversion Algorithm

Question 12:—

(a) $\begin{bmatrix} 1/5 & 1/5 & -2/5 \\ 1/5 & 1/5 & 1/10 \\ 1/5 & -4/5 & 1/10 \end{bmatrix}$

$$\begin{bmatrix} 1/5 & 1/5 & -2/5 \\ 1/5 & 1/5 & 1/10 \\ 1/5 & -4/5 & 1/10 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 1/5 & 1/5 & 1/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$$

Multiply each row by 5

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 1 & 1 & 1/2 & 0 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 1 \end{array}\right]$$

$(-1)R_1 + R_2$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 1 \end{array}\right]$$

$(-1)R_1 + R_3$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \end{array}\right]$$

$\left(-\frac{1}{5}\right) R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 1 & 0 & -1 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \end{array}\right]$$

$\left(\frac{2}{5}\right) R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$\left(\frac{1}{2}\right) R_3 + R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$2R_3 + R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$(-1) R_2 + R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

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Hence, the inverse is

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1/5 & 1/5 & -2/5 \\ 2/5 & -3/5 & -3/10 \\ 1/5 & -4/5 & 1/10 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 2/5 & -3/5 & -3/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$$

Multiply each row by 5

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 2 & -3 & -3/2 & 0 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 5 \end{array}\right]$$

$-2R_1 + R_2$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -10 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 5 \end{array}\right]$$

$(-1)R_1 + R_3$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -10 & 5 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \end{array}\right]$$

$(-1)R_2 + R_3$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -10 & 5 & 0 \\ 0 & 0 & 0 & 5 & -5 & 5 \end{array}\right]$$

A row of zeroes was obtained on the left side, therefore the matrix is not invertible.

Question 14 :—

$$\begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} \sqrt{2} & 3\sqrt{2} & 0 & 1 & 0 & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\left(\frac{1}{\sqrt{2}}\right)R_1 \quad , \quad \left(\frac{1}{\sqrt{2}}\right)R_2$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/\sqrt{2} & 0 & 0 \\ -4 & 1 & 0 & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$4R_1 + R_2$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 13 & 0 & 2\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\left(\frac{1}{13}\right) R_2$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{2\sqrt{2}}{13} & \frac{\sqrt{2}}{26} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$-3R_2 + R_1$

$$\underset{\sim}{R} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{\sqrt{2}}{26} & \frac{-3\sqrt{2}}{26} & 0 \\ 0 & 1 & 0 & \frac{2\sqrt{2}}{13} & \frac{\sqrt{2}}{26} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Hence, the inverse is

$$\begin{bmatrix} \frac{\sqrt{2}}{26} & \frac{-3\sqrt{2}}{26} & 0 \\ \frac{2\sqrt{2}}{13} & \frac{\sqrt{2}}{26} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 16 :—

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 7 & 0 & 0 & 0 & 1 \end{array}\right]$$

$(-1)R_1 + R_2$

$(-1)R_1 + R_3$

$(-1)R_1 + R_4$

$$\overset{R}{\sim}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 0 & -1 & 0 & 1 & 0 \\ 0 & 3 & 5 & 7 & -1 & 0 & 0 & 1 \end{array}\right]$$

$(-1)R_2 + R_3 \,, \quad (-1)R_2 + R_4$

$$\overset{R}{\sim}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 5 & 7 & 0 & -1 & 0 & 1 \end{array}\right]$$

$(-1)R_3 + R_4$

$$\overset{R}{\sim}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & -1 & 1 \end{array}\right]$$

$\left(\frac{1}{3}\right) R_2 \,, \quad \left(\frac{1}{5}\right) R_3 \,, \quad \left(\frac{1}{7}\right) R_4$

$$\overset{R}{\sim}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{-1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{-1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{-1}{7} & \frac{1}{7} \end{array}\right]$$

Hence, the inverse is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/3 & 1/3 & 0 & 0 \\ 0 & -1/5 & 1/5 & 0 \\ 0 & 0 & -1/7 & 1/7 \end{bmatrix}$$

Find the inverse of each 4x4 matrices

Question 20 :—

(a) $\begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & k_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & k_2 & 0 & 0 & 1 & 0 & 0 \\ 0 & k_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ k_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_4 \qquad R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} k_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & k_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & k_2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & k_1 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1/k_4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1/k_3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/k_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/k_1 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \left(\frac{1}{k_4}\right)R_2 \\ \left(\frac{1}{k_3}\right)R_2 \\ \left(\frac{1}{k_2}\right)R_3 \\ \left(\frac{1}{k_1}\right)R_4 \end{array}$$

Hence, the inverse is

$$\begin{bmatrix} 0 & 0 & 0 & 1/k_4 \\ 0 & 0 & 1/k_3 & 0 \\ 0 & 1/k_2 & 0 & 0 \\ 1/k_1 & 0 & 0 & 0 \end{bmatrix}$$

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(b) $\begin{bmatrix} k & 0 & 0 & 0 \\ 1 & k & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & k \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} k & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & k & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & k & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & k & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply each row by $1/k$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 1/k & 1 & 0 & 0 & 0 & 1/k & 0 & 0 \\ 0 & 1/k & 1 & 0 & 0 & 0 & 1/k & 0 \\ 0 & 0 & 1/k & 1 & 0 & 0 & 0 & 1/k \end{array}\right]$$

$\left(\frac{-1}{k}\right) R_1 + R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/k^2 & 1/k & 0 & 0 \\ 0 & 1/k & 1 & 0 & 0 & 0 & 1/k & 0 \\ 0 & 0 & 1/k & 1 & 0 & 0 & 0 & 1/k \end{array}\right]$$

$\left(\frac{-1}{k}\right) R_2 + R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/k^2 & 1/k & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/k^3 & -1/k^2 & 1/k & 0 \\ 0 & 0 & 1/k & 1 & 0 & 0 & 0 & 1/k \end{array}\right]$$

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$\left(\frac{-1}{k}\right) R_3 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/k^2 & 1/k & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/k^3 & -1/k^2 & 1/k & 0 \\ 0 & 0 & 0 & 1 & -1/k^4 & 1/k^3 & -1/k^2 & 1/k \end{array}\right]$$

Hence, the inverse is

$$\begin{bmatrix} 1/k & 0 & 0 & 0 \\ -1/k^2 & 1/k & 0 & 0 \\ 1/k^3 & -1/k^2 & 1/k & 0 \\ -1/k^4 & 1/k^3 & -1/k^2 & 1/k \end{bmatrix}$$