Exercise 1.1

Question 11

(a) $3x - 2y = 4 \rightarrow$ (i)

$6x - 4y = 9 \rightarrow$ (ii)

Multiplying equ (i) by -2

$-6x + 4y = -8 \rightarrow$ (iii)

Adding equ (ii) and (iii)

$$\begin{array}{r} -6x + 4y = -8 \\ 6x - 4y = 9 \\ \hline 0 = 1 \end{array}$$

Hence, the final equations are

$$3x - 2y = 4$$

$$0 = 1$$

Since the second equation is contradictory so the original system has no solutions.

(b) $2x - 4y = 1 \rightarrow$ (i)

$4x - 8y = 2 \rightarrow$ (ii)

Multiplying equ (i) by -2

$-4x + 8y = -2 \rightarrow$ (iii)

Adding equ (ii) and (iii)

$$\begin{array}{r} -4x + 8y = -2 \\ 4x - 8y = 2 \\ \hline 0 = 0 \end{array}$$

$\Rightarrow$

Hence, the final equations are

$$2x - 4y = 1$$

$$0 = 0$$

The second equation does not impose any restriction on x and y therefore we can omit it. Also,

$$2x - 4y = 1$$

$$2x = 4y + 1$$

$$x = 2y + \frac{1}{2}$$

or, $x = 2t + \frac{1}{2}$, $y = t$

where, t is an arbitrary real number.

(c) $x - 2y = 0 \longrightarrow$ (i)

$x - 4y = 8 \longrightarrow$ (ii)

Subtracting equ (i) from (ii)

$$x - 4y = 8$$

$$\underline{-x + 2y = 0}$$

$$-2y = 8$$

$$\boxed{y = -4}$$

Putting $y = -4$ in equ (i)

$$x - 2(-4) = 0$$

$$x + 8 = 0$$

$$\boxed{x = -8}$$

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Question 12 :—

$$2x - 3y = a \longrightarrow (i)$$

$$4x - 6y = b \longrightarrow (ii)$$

Multiplying equ (i) by -2

$$-4x + 6y = -2a \longrightarrow (iii)$$

Adding equ (ii) and (iii)

$$\cancel{4x} - \cancel{6y} = b$$

$$\underline{\cancel{-4x} + \cancel{6y} = -2a}$$

$$0 = b - 2a$$

Thus, the final equations are

$$2x - 3y = a$$

$$0 = b - 2a$$

⇒ If $b - 2a = 0$ $(b = 2a)$ then the system has infinitely many solutions.

⇒ If $b - 2a \neq 0$ $(b \neq 2a)$ then the system has no solutions.

⇒ There are no values of a and b for which the system has one solution.

Question 13 :—

(c) $3v - 8w + 2x - y + 4z = 0$

Solving the equation for v, we obtain

$$3v = 8w - 2x + y - 4z$$

$$v = \frac{8}{3}w - \frac{2}{3}x + \frac{y}{3} - \frac{4}{3}z$$

and,

$$v = \frac{8}{3} t_1 - \frac{2}{3} t_2 + \frac{1}{3} t_3 - \frac{4}{3} t_4$$

$$w = t_1, \quad y = t_3, \quad x = t_2, \quad z = t_4$$

Question 14 :—

(c) $4x_1 + 2x_2 + 3x_3 + x_4 = 20$

Solving the equation for x_1, we get

$$4x_1 = -2x_2 + 3x_3 - x_4 + 20$$

$$x_1 = -\frac{2}{4} x_2 - \frac{3}{4} x_3 - \frac{x_4}{4} + \frac{20}{4}$$

$$x_1 = 5 - \frac{1}{2} x_2 - \frac{3}{4} x_3 - \frac{x_4}{4}$$

and,

$$x_1 = 5 - \frac{1}{2}\gamma - \frac{3}{4}\delta - \frac{1}{4} t$$

$$x_2 = \gamma, \quad x_3 = \delta, \quad x_4 = t$$