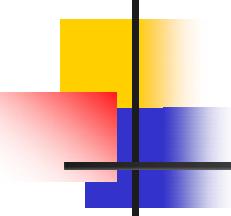


# Lecture 2

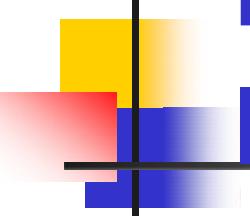
## Chapter 1. The Foundations

### 1.1 Propositional Logic



# Review: The Implication Operator

- The conditional statement (a.k.a. *implication*)  
 $p \rightarrow q$  states that  $p$  implies  $q$ .
- *I.e.*, If  $p$  is true, then  $q$  is true; but if  $p$  is not true, then  $q$  could be either true or false.
- *E.g.*, let  $p$  = “You study hard.”  
 $q$  = “You will get a good grade.”  
 $p \rightarrow q$  = “If you study hard, then you will get a good grade.” (else, it could go either way)
- $p$ : *hypothesis* or *antecedent* or *premise*
- $q$ : *conclusion* or *consequence*

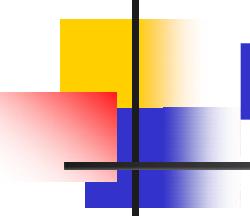


# Review: Implication Truth Table

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The only  
False case!

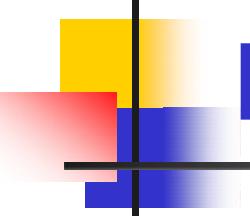
- $p \rightarrow q$  is **false** only when  $p$  is true but  $q$  is **not** true.
- $p \rightarrow q$  does **not** require that  $p$  or  $q$  are ever true!
  - E.g. “ $(1=0) \rightarrow$  pigs can fly” is TRUE!



# Examples of Implications

---

- “If this lecture ever ends, then the sun will rise tomorrow.” *True or False?* ( $T \rightarrow T$ )
- “If  $1+1=6$ , then Obama is president.”  
*True or False?* ( $F \rightarrow T$ )
- “If the moon is made of green cheese, then I am richer than Bill Gates.” *True or False?* ( $F \rightarrow F$ )
- “If Tuesday is a day of the week, then I am a penguin.” *True or False?* ( $T \rightarrow F$ )



# Examples of Implications

---

- “If this lecture ever ends, then the sun will rise tomorrow.” *True or False?* ( $T \rightarrow T$ )
- “If  $1+1=6$ , then Obama is president.” *True or False?* ( $F \rightarrow T$ )
- “If the moon is made of green cheese, then I am richer than Bill Gates.” *True or False?* ( $F \rightarrow F$ )
- “If Tuesday is a day of the week, then I am a penguin.” *True or False?* ( $T \rightarrow F$ )



# English Phrases Meaning $p \rightarrow q$

- “ $p$  implies  $q$ ”
- “if  $p$ , then  $q$ ”
- “if  $p$ ,  $q$ ”
- “when  $p$ ,  $q$ ”
- “whenever  $p$ ,  $q$ ”
- “ $q$  if  $p$ ”
- “ $q$  when  $p$ ”
- “ $q$  whenever  $p$ ”
- “ $p$  only if  $q$ ”
- “ $p$  is sufficient for  $q$ ”
- “ $q$  is necessary for  $p$ ”
- “ $q$  follows from  $p$ ”
- “ $q$  is implied by  $p$ ”

We will see some equivalent logic expressions later.

# Converse, Inverse,

## Contrapositive

Some terminology, for an implication  $p \rightarrow q$ :

- Its **converse** is:  $q \rightarrow p$ .
- Its **inverse** is:  $\neg p \rightarrow \neg q$ .
- Its **contrapositive**:  $\neg q \rightarrow \neg p$ .

$p$		$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
F	T	T		T	T
T	F	F	T	T	F
F	T	T		F	T
F	F	T		T	T

- $\neg p = \{F, F, T, T\}$   $\neg q = \{F, T, F, T\}$
- One of these three has the *same meaning* (same truth table) **Contrapositive** figure out which?



# Examples

- $p$ : Today is Easter  
 $q$ : Tomorrow is Monday
  
- $p \rightarrow q$  :  
If today is Easter then tomorrow is Monday.
  
- **Converse**:  $q \rightarrow p$   
If tomorrow is Monday then today is Easter.
  
- **Inverse**:  $\neg p \rightarrow \neg q$   
If today is not Easter then tomorrow is not Monday.
  
- **Contrapositive**:  $\neg q \rightarrow \neg p$   
If tomorrow is not Monday then today is not



# The Biconditional Operator

- The **biconditional** statement  $p \leftrightarrow q$  states that  $p$  **if and only if** (iff)  $q$ .
- $p =$  “It is below freezing.”  
 $q =$  “It is snowing.”  
 $p \leftrightarrow q =$  “It is below freezing if and only if it is snowing.”

or

= “That it is below freezing is necessary and sufficient for it to be snowing”



# Biconditional Truth Table

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- $p \leftrightarrow q$  is equivalent to  $(p \rightarrow q) \wedge (q \rightarrow p)$ .
- $p \leftrightarrow q$  means that  $p$  and  $q$  have the **same** truth value.
- $p \leftrightarrow q$  does **not** imply that  $p$  and  $q$  are true.
- Note this truth table is the exact **opposite** of  $\oplus$ 's!  
Thus,  $p \leftrightarrow q$  means  $\neg(p \oplus q)$ .



# Boolean Operations Summary

of Hawaii

- Conjunction:  $p \wedge q$ , (read  $p$  and  $q$ ), “discrete math is a required course **and** I am a computer science major”.
- Disjunction:  $, p \vee q$ , (read  $p$  or  $q$ ), “discrete math is a required course **or** I am a computer science major”.
- Exclusive or:  $p \oplus q$ , “discrete math is a required course **or** I am a computer science major **but not both**”.
- Implication:  $p \rightarrow q$ , “**if** discrete math is a required course **then** I am a computer science major”.
- Biconditional:  $p \leftrightarrow q$ , “discrete math is a required course **if and only if** I am a computer science major”.

# Boolean Operations Summary

of Hawaii

- We have seen 1 unary operator and 5 binary operators. What are they? Their truth tables are below.

$p$	$q$	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	F	T	T

- For an implication  $p \rightarrow q$
- Its **converse** is:  $q \rightarrow p$
- Its **inverse** is:  $\neg p \rightarrow \neg q$
- Its **contrapositive**:  $\neg q \rightarrow \neg p$



# Compound Propositions

- A ***propositional variable*** is a variable such as  $p$ ,  $q$ ,  $r$  (possibly subscripted, e.g.  $p_j$ ) over the Boolean domain.
- An ***atomic proposition*** is either Boolean constant or a propositional variable: e.g.  $T$ ,  $F$ ,  $p$
- A ***compound proposition*** is derived from atomic propositions by application of propositional operators: e.g.  $\neg p$ ,  $p \vee q$ ,  $(p \vee \neg q) \rightarrow q$
- Precedence of logical operators:  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$
- Precedence also can be indicated by parentheses.
  - e.g.  $\neg p \wedge q$  means  $(\neg p) \wedge q$ , not  $\neg(p \wedge q)$



# An Exercise

- Any compound proposition can be evaluated by a truth table
- $(p \vee \neg q) \rightarrow q$

$p$	$q$	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F



# Translating English

## Sentence S

Let  $p$  = “It rained last night”,

$q$  = “The sprinklers came on last night,”

$r$  = “The lawn was wet this morning.”

Translate each of the following into English:

$$\neg p =$$

“It didn’t rain last night.”

$$r \wedge \neg p$$

= “The lawn was wet this morning,  
and it didn’t rain last night.”

$$\neg r \vee p \vee q$$

= “The lawn wasn’t wet this  
morning, or it rained last night, or  
the sprinklers came on last night.”



# Another Example

- Find the converse of the following statement.
  - “Raining tomorrow is a sufficient condition for my not going to town.”
- **Step 1:** Assign propositional variables to component propositions.
  - $p$ : It will rain tomorrow
  - $q$ : I will not go to town
- **Step 2:** Symbolize the assertion:  $p \rightarrow q$
- **Step 3:** Symbolize the converse:  $q \rightarrow p$
- **Step 4:** Convert the symbols back into words.
  - “If I don’t go to town then it will rain tomorrow” or
  - “Raining tomorrow is a *necessary condition* for my not going to town.”



# Logic and Bit Operations

- A ***bit*** is a **binary** (base 2) **digit**: 0 or 1.
- Bits may be used to represent truth values.
  - By convention:  
    0 represents “False”; 1 represents “True”.
- A ***bit string of length n*** is an ordered sequence of  $n \geq 0$  bits.
- By convention, bit strings are (sometimes) written left to right:
  - e.g. the “first” bit of the bit string “1001101010” is 1.
  - What is the length of the above bit string?



# Bitwise Operations

- Boolean operations can be extended to operate on bit strings as well as single bits.
- Example:

01 1011 0110

11 0001 1101

11 1011 1111 Bit-wise OR

01 0001 0100 Bit-wise AND

10 1010 1011 Bit-wise XOR



# End of 1.1

You have learned about:

- Propositions: what they are
- Propositional logic operators'
  - symbolic notations, truth tables, English equivalents, logical meaning
- Atomic vs. compound propositions
- Bits, bit strings, and bit operations
- Next section:
  - Propositional equivalences
  - Equivalence laws
  - Proving propositional equivalences