Surface charge Density $= \sigma = \frac{Q}{A}$

Volume Charge Density $= \rho = \frac{Q}{V}$

ELECTRIC FIELD

An electric Field is the physical field that surronds electrically charged particles and exerts force on all other charge particles in field, either attracting or Repelling them.

Equation:

$$E = k\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$K = 9\times10^9 Nm^2C^{-2}$$

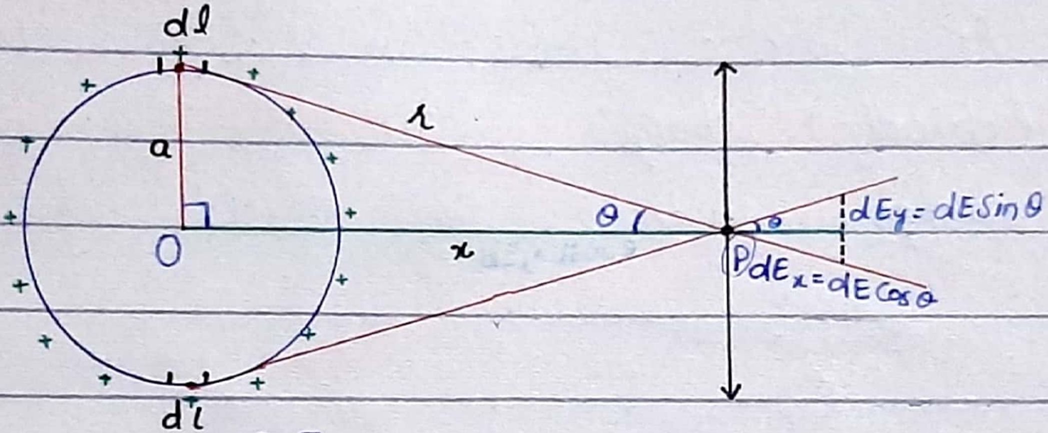
Calculation of Electric Field of Continuous Charges

Consider a Ring shaped object in which charges are distributed uniformly.

In the Ring shaped object the total distributed charge is Q. And a Point P lies at a distance

x from the centre O of the Ring. The Radius of the Ring is a.

Figure:



To calculate the electric field of the Ring we have to divide the Ring into small patches. Each patch will be equal to dl. Here r is the distance of the Point P from the patch.

Now For calculation of Electric Field on Point 'P' Resolve the point P into its components.

Total Electric Field can be given as:

$$E = \int dE$$

The Vertical Components are cancelled because they are equall in magnitude but Opposite in direction.

So

$$dE = dE_x + dE_y \qquad \lambda :$$

$$\because \; dE_y = 0$$

$$dE = dE_x$$

Hence

$$E = \int dE_x \text{ ----- (i)}$$

$$E = \int dE \cos\theta \qquad \because \; E = \frac{1}{4\pi\varepsilon_o}\frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dl}{r^2}$$

نوٹ :-

یہاں d ہمارا Derivative
کو ظاہر کرتا ہے۔
اگر کسی چیز کا Area کا ڈھونڈنا
کرنا ہو تو پہلے اس کا Derivative
لیتے ہیں پھر Integral لیتے ہیں بشرطیکہ
کہ Surface کو Small patches
میں Divide کیا گیا ہو۔

So Now

$$E = \frac{1}{4\pi\varepsilon_o}\int \frac{dQ}{r^2}\cos\theta \text{ ----- (ii)}$$

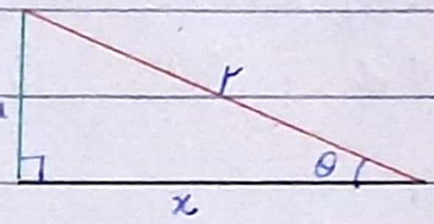
$$\because \; Q = \lambda l$$

$$E = \frac{1}{4\pi\varepsilon_o}\int \frac{d\lambda l}{r^2}\cos\theta \qquad \text{(As } \lambda \text{ is also constant)}$$

$$E = \frac{\lambda}{4\pi\varepsilon_o}\int \frac{dl}{r^2}\cos\theta \text{ ----- (iii)}$$

From Figure we get

$$\cos\theta = \frac{B}{H} = \frac{x}{r}$$



So above Equation becomes.

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2} \times \frac{x}{r}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int dl \cdot \frac{x}{r^3}$$

Here $\frac{x}{r^3}$ = Constant Because it has not 'd' of Derivative. جب l کے ساتھ d ہے۔ یہ d دراصل Derivation کا ہے۔

$$E = \frac{\lambda x}{4\pi\varepsilon_0 r^3}\int dl \text{ ----- (iv)}$$

Since

So $\quad dl = 2\pi a$

above Eq. becomes

$$E = \frac{\lambda x (2\pi a)}{4\pi\varepsilon_0 r^3}$$

$$E = \frac{x(\lambda 2\pi a)}{4\pi\varepsilon_0 r^3}$$

$\because \lambda 2\pi a$ = Total Charge inclosed Surface = Q

$$E = \frac{Qx}{4\pi\varepsilon_0 r^3} \quad \text{----(v)}$$

From Right Triangle

$$H^2 = P^2 + B^2$$

$$r^2 = a^2 + x^2$$

$$r = \sqrt{a^2 + x^2}$$

$$r = (a^2 + x^2)^{1/2}$$

Putting r in above Equation

$$\boxed{E = \frac{Qx}{4\pi\varepsilon_0 (a^2+x^2)^{3/2}}}$$

Here This is the Total Electric Field of a Ring shaped object in which charges are distributed uniformly.

SPECIAL CASES

Now Lets Discuss about some special Cases.

1: When does the electric Field in the Ring becomes Zero.

If 'x' decreases then 'E' also decrease, when x become zero; E also becomes Zero. It mean that there is no electric Field in the centre of Surface.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2+a^2)^{3/2}}$$

کیونکہ Force چاروں
اطراف سے آتی ہیں
اور opposite چارجز
والی Forces ایک
دوسرے کے Effect
کو Cancel کر دیں
گی۔

Put $x = 0$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q(0)}{((0)^2+a^2)^{3/2}}$$

$$\boxed{E = 0}$$

2. When Does the Electric Field of the Ring becomes Electric Field of a Point charge.

When the Distance of Point 'P' 'x' becomes equal to the Distance of Patch 'r' from Point 'P' Then Electric Field of Ring becomes the electric field of Point Charge.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \cdot \frac{x}{r}$$

Put $x = r$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \cdot \cancel{\frac{r}{r}}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

3. If $x >> a$ than $a \simeq 0$

(a can be neglected)

If Point 'P' situated as the infinity Distance of 'x' Hence 'x' is too large that in front of 'x' we can neglect the Radius 'a' Because in this case 'a' does not have a significant impact on Electric Field.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2+a^2)^{3/2}}$$

Put $a \simeq 0$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2+0)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{x^3}$$

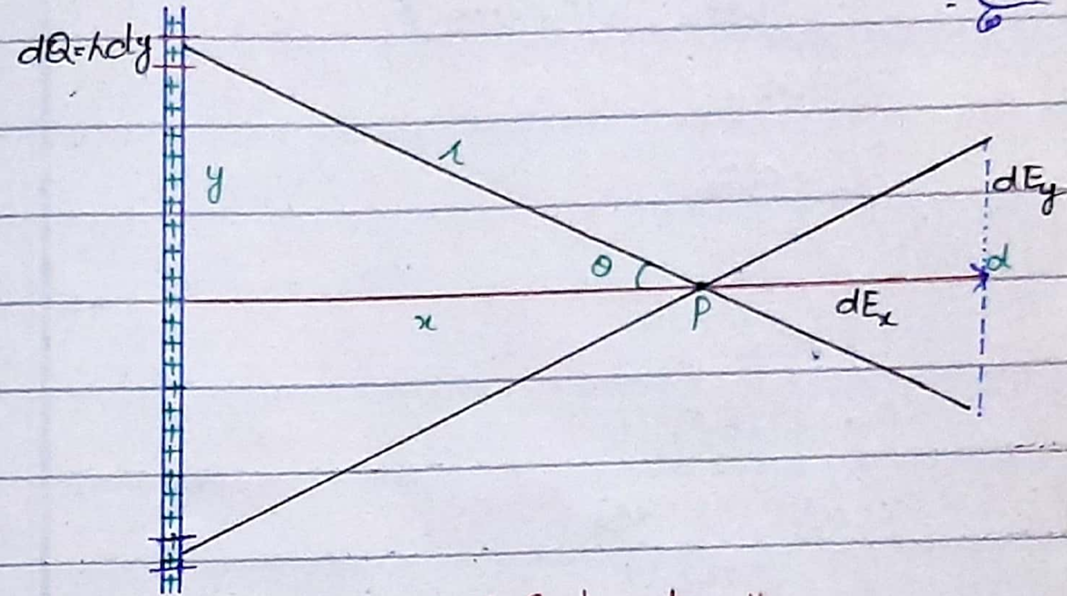
$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 x^2}}$$

اگر y کو کم کرنے سے θ میں بھی تبدیلی آئے تو نیچے والی Topic

In the case of a Rod

Take a Rod at which uniformly charges are distributed. Consider a point 'P' lies at a distance of x. To calculate the electric field on the Rod we have to divide the rod into small patches.

Figure: Rod کے ہر Patch کیلئے θ کی قیمت بدل جاتی ہے۔



Long Rod of infinite length

To Calculate the electric Field we have to dissolve the Point 'p' into its components dE_x and dE_y.
Here dE_y is zero because they are same in magnitude but opposite direction.

So

$$dE = dE_x + dE_y \ \ldots\ \text{i}$$

$$\because dE_y = 0$$

$$dE = dE_x$$

Now

$$E = \int dE_x$$

$$E = \int dE \cos\theta$$

$$\because dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$$

So

$$E = \int \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta \ \ldots\ \text{ii}$$

Here $\frac{1}{4\pi\varepsilon_0}$ is constant so

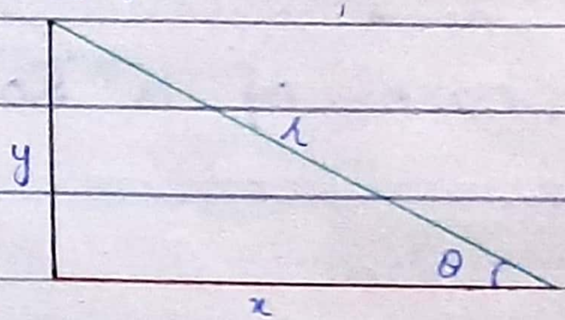
$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r^2}\cos\theta \ \ldots\ \text{iii}$$

$$\because dQ = \lambda dl$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\cos\theta$$

As λ is constant

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{r^2}\cos\theta \ \ldots\ \text{iv}$$



$$r^2 = x^2 + y^2$$

So above equation becomes

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{x^2+y^2}\cos\theta \ \ldots\ \text{v}$$

Here

From above Figure:

Eq^n

$\tan\theta = \frac{P}{B} = \frac{y}{x} = \frac{x}{y}$

$\tan\theta = \frac{y}{x}$

$y = x\tan\theta$

$dy = d(x\tan\theta)$

$dy = x\sec^2\theta\, d\theta$

From above equation V

$x^2 + y^2 = x^2 + x^2\tan^2\theta$

$= x^2(1+\tan^2\theta)$

$x^2 + y^2 = x^2\sec^2\theta$

Now Putting these Values

$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{\cancel{x}\,\cancel{\sec^2\theta}\cos\theta\, d\theta}{\cancel{x^2}\,\cancel{\sec^2\theta}}$

$E\ \frac{\lambda}{4\pi\varepsilon_0}\int \frac{\cos\theta\, d\theta}{x}$

As $\frac{1}{x}$ also Constant

So

$E = \frac{\lambda}{4\pi\varepsilon_0 x}\int \cos\theta\, d\theta$

By Applying Limit

$E = \frac{\lambda}{4\pi\varepsilon_0 x}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$

$= \frac{\lambda}{4\pi\varepsilon_0 x}\left|\sin\theta\right|_{-\pi/2}^{\pi/2}$ $\quad \because \int \cos\theta = \sin\theta$

$= \frac{\lambda}{4\pi\varepsilon_0 x}\left\{\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right\}$

$= \frac{\lambda}{4\pi\varepsilon_0 x}\{1-(-1)\}$

$= \frac{\lambda}{4\pi\varepsilon_0 x}(2)$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 x}}$$

The above equation tells the electric field of an object of which angle changes while we change the length of an object.

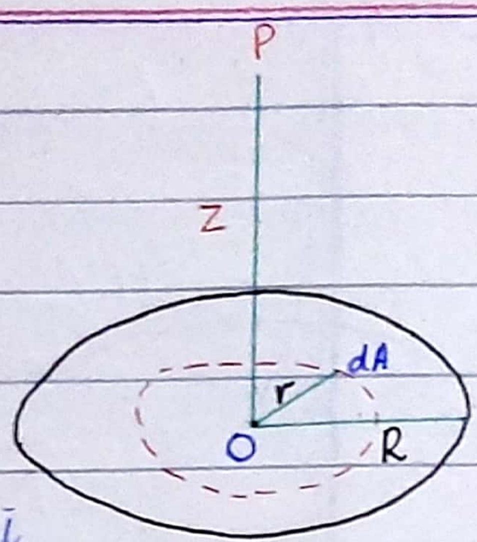
Calculation Of Electric Field Due to Surface Charge Distribution

Consider a plate object in which charges are distributed uniformly. The total radius of the object is R, and a Point P lies at a distance of z from the centre of the plate.

To calculate the Electric Field of the object, first we will convert the object into small equal patches. Each patch has Area of dA and the radius of each is r

(ہم نے اتنے چھوٹے Patches میں تقسیم کرنا ہے جنکی Electric Field اور Area ہمیں معلوم ہو).

Figure:



As we know that

Area = length x width

So the Area of the patch is

$A = 2\pi r\, dr$

Surface charge Density can be given as:

$$\sigma = \frac{Q}{A}$$

As we are considering a small patch so the surface charge density of this patch becomes

$$\sigma = \frac{dQ}{dA}$$

As

$$dQ = \sigma\, dA$$

As we know that the electric Field due to ring is:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2+a^2)^{3/2}}$$

From the Figure the above equation becomes:

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{z\,dQ}{(z^2+r^2)^{3/2}}$$

To calculate the electric field,

$$E = \int dE$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{z\,dQ}{(z^2+r^2)^{3/2}}$$

$\because$ $dQ = \sigma\,dA$

$\because$ $dA = 2\pi r\,dr$

$\therefore$ $dQ = \sigma(2\pi r\,dr)$

Hence

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{z\sigma(2\pi r)dr}{(z^2+r^2)^{3/2}}$$

Since $z\sigma 2\pi$ is constant so

$$= \frac{z\sigma(\cancel{2}\cancel{\pi})}{\cancel{4}\cancel{\pi}\varepsilon_0} \int \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$E = \frac{z\sigma}{2\varepsilon_0} \int \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

Applying Limit $0 \to R$

So we have

$$E = \frac{z\sigma}{2\varepsilon_0} \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

Now Using

$$= \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

X-ing And Dividing by 2.

$$= \frac{1}{2} \int_0^R (z^2+r^2)^{-3/2}\, 2(r\,dr)$$

(Power Rule)

$$= \frac{1}{2} \left| \frac{(z^2+r^2)^{-3/2+1}}{\frac{-3}{2}+1} \right|_0^R$$

$$= \frac{1}{\cancel{2}} \left| \frac{(z^2+r^2)^{-1/2}}{-1/\cancel{2}} \right|_0^R$$

$$E = -\left| \frac{1}{(z^2 + r^2)^{1/2}} \right|_0^R$$

(یہاں r کی جگہ Limit کی ویلیو پٹ کرنی ہے

$$= -\left\{ \frac{1}{(z^2 + 0^2)^{1/2}} - \frac{1}{(z^2 + R^2)^{1/2}} \right\}$$

$$= -\left\{ \frac{1}{(z^2)^{1/2}} - \frac{1}{(z^2 + R^2)^{1/2}} \right\}$$

$$= -\left\{ \frac{1}{z} - \frac{1}{(z^2 + R^2)^{1/2}} \right\}$$

Now Put this Value at the place of rdr. So, we have

$$E = \frac{z\sigma}{2\varepsilon_0} \left[\frac{1}{z} - \frac{1}{(z^2 + R^2)^{1/2}} \right]$$

$$E = \frac{\sigma}{2\varepsilon_0} \left[\frac{z}{z} - \frac{z}{(z^2 + R^2)^{1/2}} \right]$$

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{(z^2 + R^2)^{1/2}} \right]$$

This is the total electric Field of a surface at which charges are distributed uniformly.

Special Case:

When Will be Electric Field Maximum?

If the Point 'P' lies on the centre and the distance of Point 'P' becomes zero from the centre. i.e $(z = 0)$ then Maximum electric field will produced.

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]$$

As $z = 0$

So

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{0}{(0^2+R^2)^{1/2}}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}(1)$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

We can also explain the same case in this manner. that if the Radius 'R' of the object becomes too much greater that in front of 'R' Z becomes so small so that we can neglect the z.

$R >> Z$ then

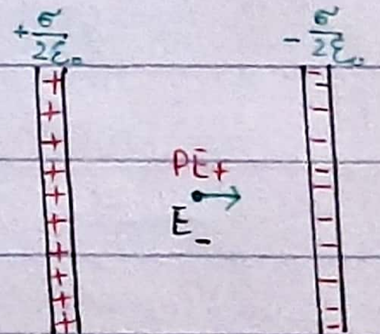
$z \simeq 0$

Then Maximum electric Field will produced.

In Cases Of Point Charge

Case-I

Consider a point charge q
(پوائنٹ چارج ہمیشہ ہی +ve ہوتا ہے)
that is placed between two charged capacitor plates. As



Here we can see that the positive capacitor plate is repelling the point charge towards negative charge plate and negative charge plate already attracting the point charge towards itself. So, At centre of capacitor plates the electric field can be given as:

نوٹ:: $\left(-\frac{\sigma}{2\varepsilon_0}\right)$ میں (-) صرف E_- کو ظاہر کرتا ہے اس کو فارمولے میں استعمال نہیں کرتے۔

$$E_{TOTAL} = E_+ + E_-$$

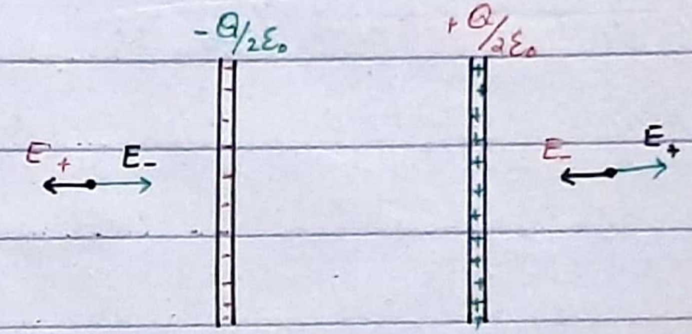
$$E_{total} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$\boxed{E_{total} = \frac{\sigma}{\varepsilon_0}}$$

This is maximum electric field that is present in the centre of plates.

Case-II

If the point charge lies at Left or right side of a charged plate of capacitor. For Example:



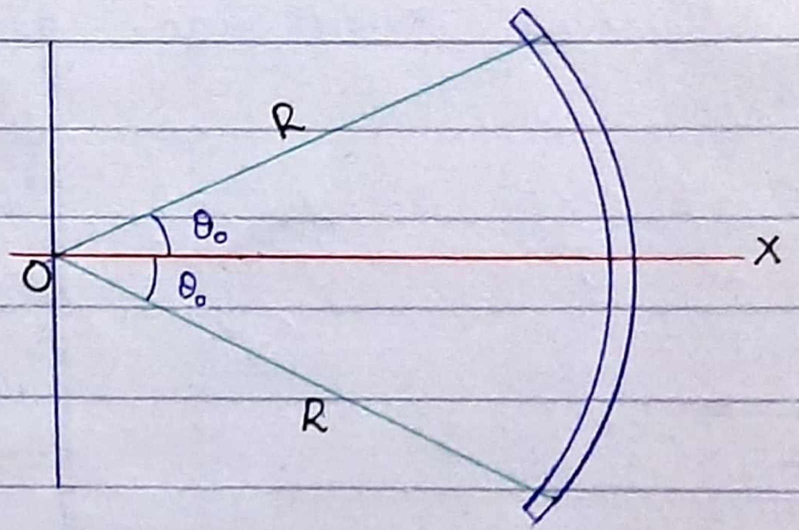
Here in such case the electriy field intensity is given as:

$$E_{TOTAL} = E_+ = E_-$$

$$\boxed{E = \sigma/2\varepsilon_0}$$

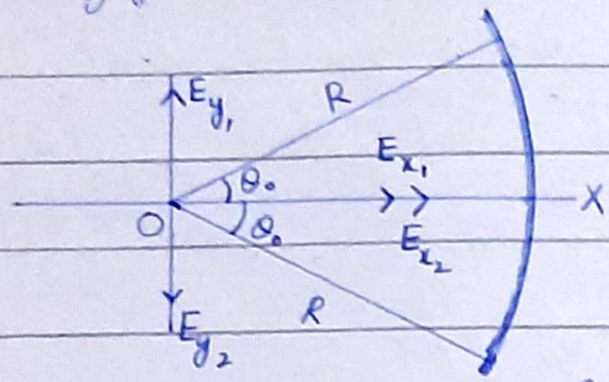
Problems

1: A thin rod is bent into the shape of an arc of a circle, of radius R carries a uniform charge per unit length λ. The arc subtends a total angle $2\theta_0$. Symmetric about the x-axis as shown in figure.



Determine the electric field E at point O.

By using the figure:



By Resolving the axis into their components. So we have two components of x-axis which E_{x_1} and E_{x_2} which lies on the same axis and in same direction. but in case of y-axis y-axis also have two components E_{y_1} and E_{y_2}. But they will be cancel each other effect because they have same magnitude but opposite direction. So here the total Electric Field becomes

$$E = E_x + E_y$$

$$\because E_y = 0$$

So

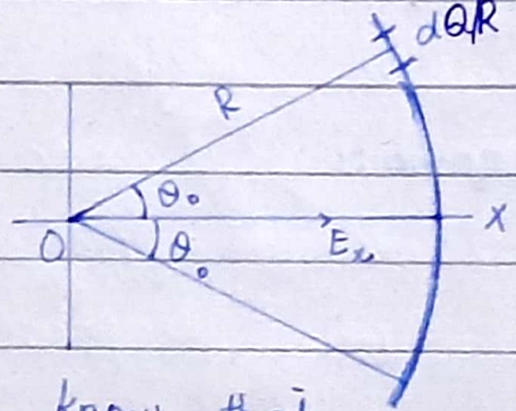
$$E = E_x$$

So

In order to calculate the calculate the electric Field, we use E_x. Here we will consider a small patch of the curve. As we know that

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

From Figure



As we know that

$$\lambda = \frac{Q}{l}$$

This is the Linear charge density for a small patch it will become

$$\lambda = \frac{dQ}{R} \quad \because l = R$$

$$dQ = \lambda R$$

Now using

$$E = \int dE_x$$

$$\because E = \int \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

In case of E_x the above eq. becomes

$$E = 2\int \frac{1}{4\pi\varepsilon_0} \frac{\lambda R}{R^2} \cos(\theta_0) d\theta$$

$\because dQ = \lambda R$

$\because r = R \quad \theta = 2\theta_0$

$$= \frac{\lambda R}{4\pi\varepsilon_0 R^2} (2) \int \cos(\theta_0) d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} (2) \int \cos\theta_0 \, d\theta$$

$$= \frac{\cancel{2}\lambda}{\cancel{4}\pi\varepsilon_0 R} \int \cos\theta_0 \, d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0 R} \int \cos\theta_0 \, d\theta$$

From statement it is clear that θ changes from 0 to θ_0 so in above equation we can apply the limit as:

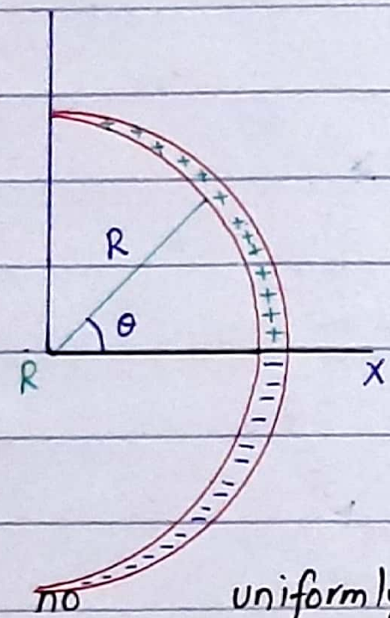
$$E = \frac{\lambda}{2\pi \varepsilon_0 R} \int_0^{\theta_0} \cos\theta_0 \, d\theta$$

$$= \frac{\lambda}{2\pi \varepsilon_0 R} \Big| \sin\theta \Big|_0^{\theta_0}$$

$$= \frac{\lambda}{2\pi \varepsilon_0 R} \Big| \sin(\theta_0) - \sin(0) \Big|$$

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 R} \sin\theta_0}$$

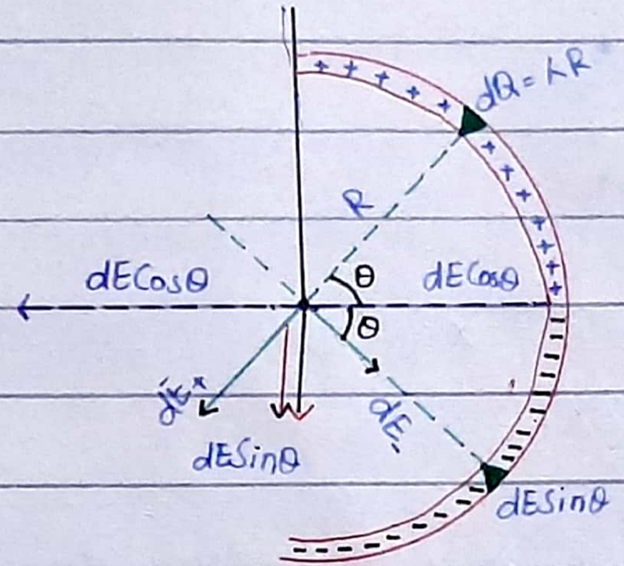
2- A thin glass rod is a semicircle of radius R as show below.



A charge is ~~no~~ uniformly distributed along the rod with a linear charge density given by $\lambda = \lambda_0 \sin\theta$ where λ_0 is a positive constant and point P lies at the centre of the ring.

Here

From the figure we can understand that there is a point charge which contain positive charge. The point charge is repelling by the positive charges and attracting by the negative charges. Here We Can see the figure:



Here in the figure we can see that the point charge is moving downward towards negative Charges. Means when we will resolve the axis into their components then the x-axis's components due to same magnitude but opposite direction will cancel each other effect. Hence, we will have

$$dE = dE_+ = dE_-$$

As

$$\because dE_{netx} = 0$$

So we have

$$dE_{nety} = 2dE\sin\theta$$

(2 یہاں اس لیے استعمال ہوا ہے کہ ایک اوپر والا y-axis اور دوسرا نیچے والا y-axis.)

So we know

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

But In case of dE_{nety} we know

$$\lambda = \frac{dQ}{R} \Rightarrow dQ = \lambda R$$

$$r = R$$

So, by Putting the Values, we have

$$E = \int dE_{nety}$$

$$= \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{R^2} 2\sin\theta$$

$$= \frac{\cancel{2}\, 1}{4\pi\varepsilon_0} \frac{\cancel{R}}{\cancel{R^2}} \int \lambda \sin\theta \, d\theta$$

$$= \frac{1}{2\pi\varepsilon_0 R} \int \lambda \sin\theta \, d\theta$$

$$\because \lambda = \lambda_0 \sin\theta$$

$$= \frac{1}{2\pi\varepsilon_0 R} \int (\lambda_0 \sin\theta)(\sin\theta) \, d\theta$$

$$= \frac{1}{2\pi\varepsilon_0 R} \int \lambda_0 \sin^2\theta \, d\theta$$

$$= \frac{1 \, (\lambda_0)}{2\pi\varepsilon_0 R} \int \sin^2\theta \, d\theta$$

یہاں ہم (0 سے $\pi/2$) تک کی Limit کو Apply کریں گے۔

$$= \frac{\lambda_0}{2\pi\varepsilon_0 R} \int_0^{\pi/2} \sin^2\theta \, d\theta$$

$$= \frac{\lambda_0}{2\pi \varepsilon_0 R} \int_0^{\pi/2} \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= \frac{\lambda_0}{2\pi \varepsilon_0 R} \left| \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right|_0^{\pi/2}$$

$$= \frac{\lambda_0}{2\pi \varepsilon_0 R} \left\{ \left(\frac{\pi/2}{2} - \frac{\sin 2(\pi/2)}{4} \right) - \left(\frac{0}{2} - \frac{\sin(2(0))}{4} \right) \right\}$$

$$= \frac{\lambda_0}{2\pi \varepsilon_0 R} \left\{ \frac{\pi}{4} - \frac{\sin(\pi)}{4} - (0) \right\}$$

$$= \frac{\lambda_0}{2\pi \varepsilon_0 R} \left\{ \frac{\pi}{4} - 0 \right\}$$

$$= \frac{\lambda_0}{2\cancel{\pi} \varepsilon_0 R} \left\{ \frac{\cancel{\pi}}{4} \right\}$$

$$\boxed{E = -\frac{\lambda_0}{8 \varepsilon_0 R}}$$

Since negative sign indicates the direction of point charge to the negative charge capacitor.

Chapter 22

Gauss's Law

Electric Flux

Gauss's Law involves the conncept of electric flux. That can be defines as:

"Total number of electric fields passing normally through a certain area is called electric flux."

→ Electric Flux is represented by ϕ_e.

Formula:

$$\Phi_e = \vec{E} \cdot \vec{A}$$
$$= EA\cos\theta$$

The electric flux depends upon

- Electric intensity
- Surface Area
- Orientation of Surface

FLUX means flow.

- It is a scalar quantity. (Have no direction)
- Its SI Unit is Nm^2C^{-1}.

Cases

Now lets discuss some case in which we will see that whenever the electric flux will maximum and whenever the electric flux will minimum.

Case-I: Maximum Flux:

When the direction of vector Area is held parallel to the field lines. Then the electric flux will be maximum.

$$\phi_e = EA\cos\theta$$

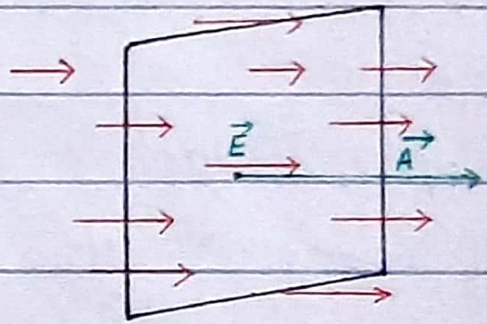
$$\because \theta = 0$$

$$\phi_e = EA\cos(0)$$

$$\because \cos(0) = 1$$

$$\phi_e = EA(1)$$

$$\phi_e = EA$$



Case-II: Minimum Flux:

When the direction of vector Area is held perpendicular to the electric field lines. Then then electric flux will be minimum. As:

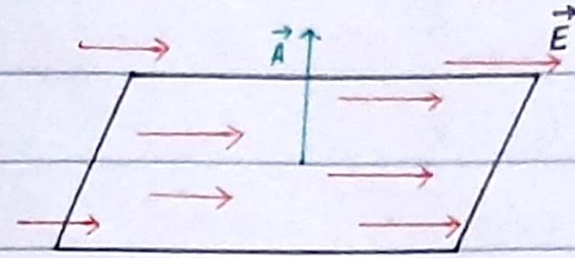
$$\phi_e = EA\cos\theta$$

$$\because \theta = 90^\circ$$

$$\phi_e = EA\cos(90)$$

$$\because \cos 90 = 0$$

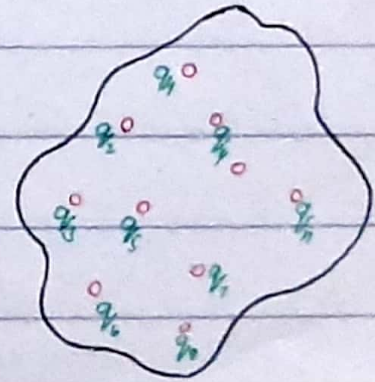
$$\phi_e = EA(0)$$



$$\phi_e = 0$$

ELECTRIC FLUX THROUGH A SURFACE ENCLOSING CHARGE

Consider an irregular surface in which Point charges $q_1, q_2, q_3, \ldots, q_n$ are distributed arbitrarily. Each charge will act as an independent source of electric flux. The flux through the Surface due to point charge q_1 is $\phi_1 = \frac{q_1}{\varepsilon_0}$



Similarly flux through other charges can be given as:

$$\phi_2 = \frac{q_2}{\varepsilon_0}$$

$$\phi_3 = \frac{q_3}{\varepsilon_0}$$

$$\vdots \qquad \vdots$$

$$\phi_n = \frac{q_n}{\varepsilon_0}$$

The total electric flux passing through the whole closed surface will be

$$\phi_1 + \phi_2 + \phi_3 + \dots + \phi_n = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \dots + \frac{q_n}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + \dots + q_n)$$

$$= \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i$$

$$\because \sum_{i=1}^{n} q_i = Q$$

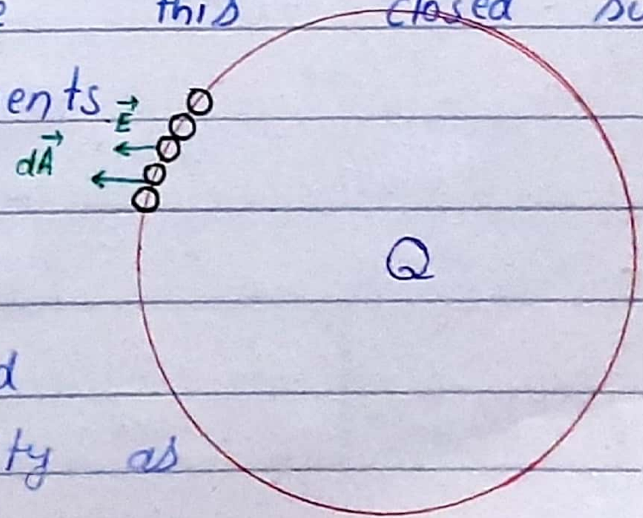
$$\phi_e = \vec{E} \cdot \vec{A} = \frac{1}{\varepsilon_0} Q$$

Here Q is the total charged enclosed by the surface

ELECTRIC FLUX THROUGH A CLOSED Surface

Consider a Sphere of radius r due to a point charge q at its centre.

In order to calculate the electric flux through whole surface of sphere, we divide this closed surface into n number of small elements. The electric flux through each element. The Area of each element is given as $d\vec{A}$ and the electric field lines intensity as



E.

So the flux through an element can be given as:

$$\vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} Q$$

$$E.A = \oint E.\,dA$$
$$= \oint E\,dA$$
$$= E\oint dA$$
$$= E(A)$$

The Area $A = 4\pi r^2$

So

$$= E(4\pi r^2)$$

Now

$$E(4\pi r^2) = \frac{1}{\varepsilon_0} Q$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

$\Rightarrow$ Columb's Law.

Cases

Now Lets disscuss the case in which we will find that when will be the electric flux through the surface zero.

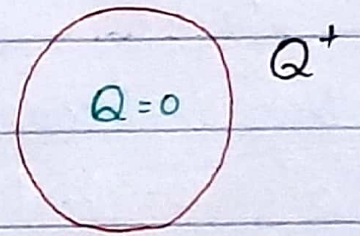
In the above figure if the charge enclosed by the surface is zero. Means charges lies outside the surface.

Inside $Q = 0$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

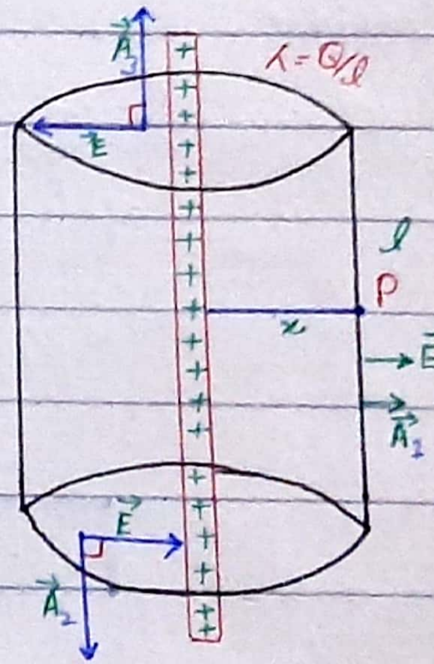
$$= \frac{0}{4\pi\varepsilon_0 r^2}$$

$$\boxed{E = 0}$$



ELECTRICAL FLUX THROUGH Cylinder

Consider a Cylinder in which a wire is attached. to produce electric field Lines. We can find out the direction of electric field Lines by using Right Hand Rule.



In Order to calculate the electric flux through the cylinder we take a point 'P' at a distance of 'x' from the wire.

If we want to calculate the electric flux through the cylinder then, we have to calculate the flux through three sides of the cylinder.

As we know

$$\vec{E}\cdot\vec{A} = \frac{1}{\varepsilon_0} Q_{enclosed}$$

As

$$\lambda = \frac{Q}{l}$$

And

$$Q_{enclosed} = \lambda l$$

The electric Flux through the curved surface is

$\phi_1 = \vec{E}\cdot\vec{A}_1$ ---- (i) (Here $\cos\theta$ ($\because \theta = 0$) So Maximum output).

Similarly electric Flux through upper flat surface will be

$\phi_3 = \vec{E}\cdot\vec{A}_3 \cos 90^\circ$ (Because $\vec{E}$ and $\vec{A}$ are $\perp$)

$\phi_3 = 0$ ---- (ii)

Similarly in the lower flat surface the electric flux will be

$\phi_2 = \vec{E}\cdot\vec{A}_2 \cos 90^\circ$

$\phi_2 = 0$ ---- (iii)

By adding the all equations we have

$$\vec{E}.\vec{A_1} + \vec{E}.\vec{A_2} + \vec{E}.\vec{A_3} = \frac{1}{\varepsilon_o} Q_{enclosed}$$

$\because Q_{enclosed} = \lambda l$

$$\vec{E}.\vec{A_2} = \frac{1}{\varepsilon_o} \lambda l$$

As

Area = length x width = $l \times 2\pi r = 2\pi r l$

Here $\qquad$ Here $\because r = x$

$$\vec{E}(2\pi x l) = \frac{1}{\varepsilon_o} \lambda l$$

$$\vec{E} = \frac{\lambda \cancel{l}}{2\pi \varepsilon_o x \cancel{l}}$$

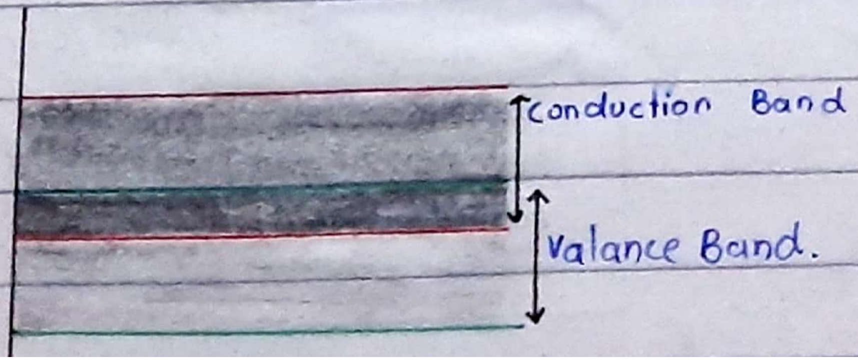
$$\boxed{\vec{E} = \frac{\lambda}{2\pi \varepsilon_o x}}$$

Some Definations about Practicle No. 02:

Conductors: These material which conduct electricity easily, called conductor. e.g copper, Silver. Their conductivities are of the order of $10^7 (\Omega m)^{-1}$

Conductors are those materials which have large number of free electrons for electrical Conduction In term of Energy Band the valane and Conduction bands are overlapped. i.e. there is no distinction between these two bands.

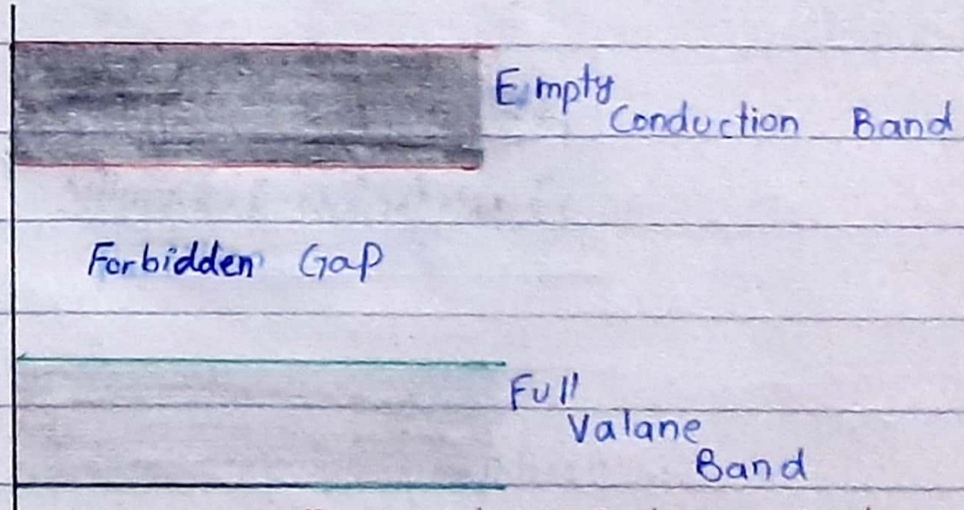
Energy Band:



Insulators:

In insulators the electrons are tightly bound in their valance shell and they are not free to move. In these materials the Valance band is Completely filled and the Conduction band is Empty. So there is a Large Energy Gap between the conduction band and Valance band. And due to this difference they cannot move from Valance band to the conduction band.

Energy Band:



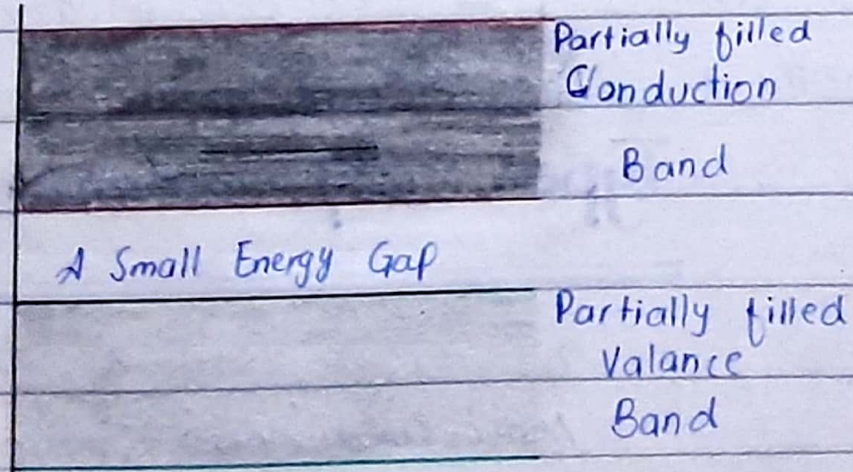
∴ A Large Difference between Conduction Band and Valance Band.

Semi-Conductors:

They are those materials which have Partially Filled Valance band and Partially filled Conduction band. A very narrow energy gap between the two bands exists and normally order of 1eV.

These materials are not conductors not insulators but it depends on condition that they behave like conductors or insulators.

Energy Gap:



: A Small Gap between Valance & Conduction Gap

A Small energy gap means by providing a small amount of energy the electrons can jump from Valance band to Conduction band easily.

Now there is a question that Why we use semi-conductors in our mostly experiments instead of conductors while conductors conducts electricity easily on low temperature?

In such cases, we use semi-conductors because we can control the conductivity of semi-conductors. But how we Control the conductivity of semi-conductors? we can control the conductivity of semi-conductors by a method called Dopping

Types Of Semi-Conductors

There are two types of semi-conductors which are:

Extrinsic Semiconductors:

The impure semiconductor (in which impurity is added) is called extrinsic semi-conductors.

Intrinsic Semiconductors:

A semiconductor in its pure form is called intrinsic Semiconductors.

The electrical behaviour behaviour of semiconductor purely depends on the purity of semiconductors.

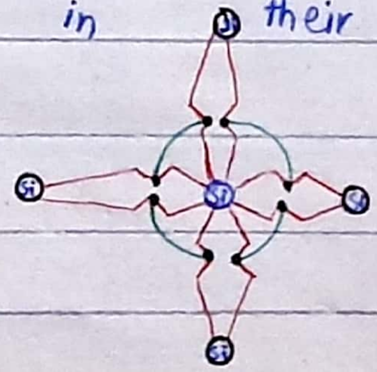
Example:

- Silicon
- Germinium

Dopping:

The process of adding selected impurity in pure semiconductors is called dopping.

- The impurity atoms are added in 1 to 10^6 ratio.
- Due to addition of impurity always extrinsic semiconductors generate.
- Since semiconductors belong to the 4th group of Periodic Table. Means they have 4 electrons in their valance shell and form four bonds with their neighbour atoms. This effectively allocates 8 electrons to the outermost orbit of each atom which is a stable state. Due to these covalent bonds the electrons are bound in their respective shells.



N-Type Material: (Previous Fig: is its example).

When a silicon material is dopped with pentavalent impurity i.e. atom from the fifth group like

⇒ Arsenic

⇒ Antimany

⇒ Phosphorus etc

four electrons of the impurity atom make four covalent bonds with the silicon atoms and the fifth electron remain unbounded.

This become the free electron in the crystal. This type of dopped or extrinsic semiconductors is called N-Type Semiconductor.

Donor The impurity atom is called donor atom because it donates a free electron. This electron is thermally excited into the conduction band.

P-Type Semiconductor:

When a silicon crystal is doped with a Trivalent impurity. i.e.

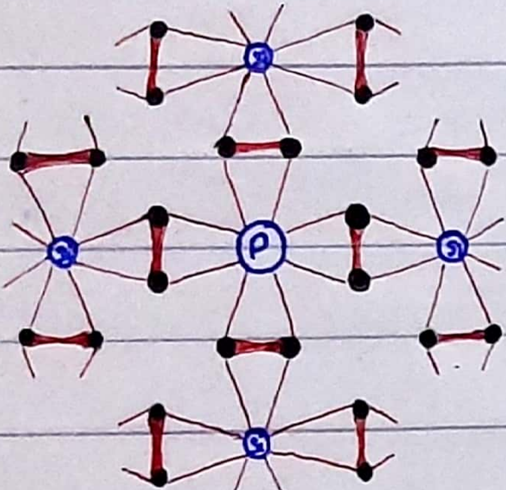
- ⇒ Aluminum
- ⇒ Boron
- ⇒ Indium
- ⇒ Gallium

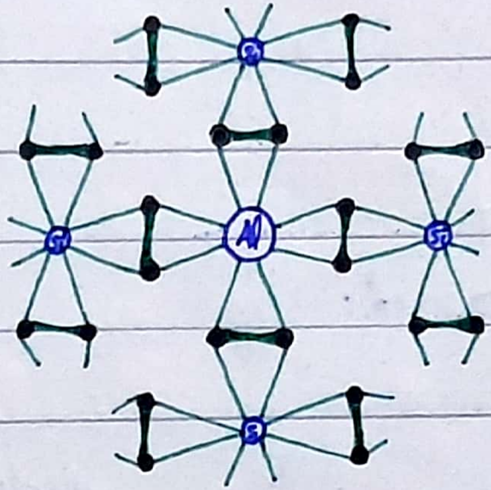
Three Valence electrons of the impurity atom three covalent bonds with the silicon atoms, there is a missing electron to form the forth covalent bond with the forth neighbor silicon atom.

In other words, there is a Vacancy in that bond This vacancy is called the hole and is considered to have a positive charge. This Vacancy can accommodate an electron. This type of semiconductor is called

P-type Semiconductor.

Acceptor: The impurity atom in this case is called the acceptor impurity because it can accept an electron from a nearby silicon atom thus by creating a hole in the Valence band.





States:

Electrons in single isolated atom have discrete energies and are bound to their nuclei. In solid, when large number of atoms, say N come close enough to eachother to form solid, each energy level splits up into N sub levels, called states. This action is due to the action of the forces exerted due to other atoms in solid state. These energy state are discrete but so close to eachother they look like Continuous Enery Band.

Bands

Now Let's discuss about bands. Basically there are three bands of Materials.

- Valence Band
- Conduction Band
- Forbidden Band

Valence Band:

The electrons in the outermost shell are called Valence electrons and the energy band formed by these electrons is called Valence Band. This band may be either completely filled or partially filled but cannot be empty.

Conduction Band:

The band above the Valence band is called condudion band. In this band electrons move freely and conduct electric current. The electrons in this band are called conduction electrons or free electrons. This band may be completely filled or partially filled.

Forbidden Energy Band:

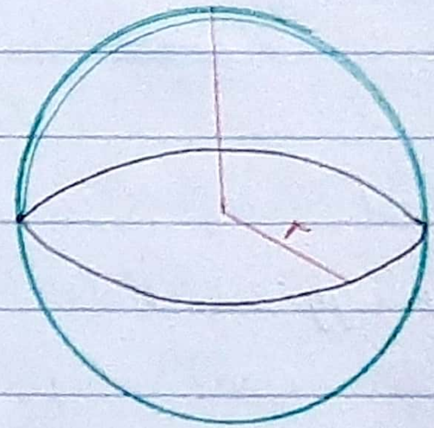
There is a Range of Energy states which cannot be occupied by electrons. These are called forbidden energy states and its range is called forbidden energy Gap.

Gauss's Law Applications

Sphere:

A Sphere is a three-dimensional geometrical object that is analogue to two dimensional circle.

A Sphere is a set of points that are all at the same distance r from a given point in three-dimensional space. That given point is the centre of the sphere, and r is the sphere's radius.



Volume $\frac{4}{3}\pi r^3$

Surface Area $4\pi r^2$

Euler char 2

Symmetry Group O(3)

When we study a sphere in physic, we study about two types of sphere:

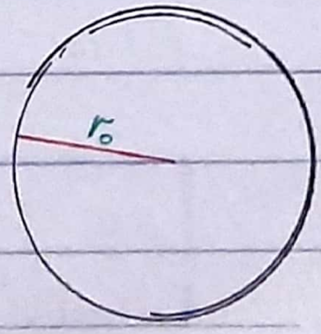
- Hollow Sphere
- Solid Sphere

And Solid Sphere is classified into two more classes which are

- Conductive Sphere
- Non- Conductive Sphere

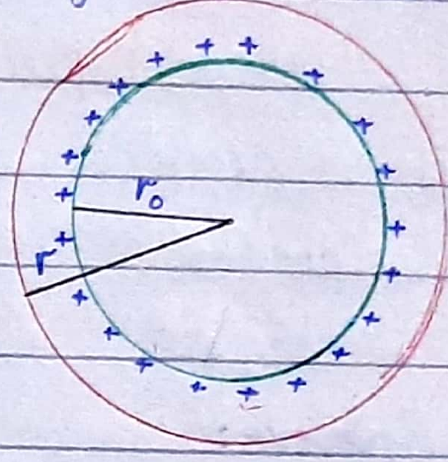
Hollow Sphere:

Let take a hollow sphere. Since the sphere is hollow means there is no charge lies at centre of sphere, when we will apply electric Field around it all charges will be distributed on the surface of the sphere. As:



This is a hollow sphere of radius 'r' when we apply electric field around it all charges will come

On the surface of Sphere. To calculate the Electric Field, we imagine a guassian surface of radius r outside the sphere.



The total charge enclosed by the surface is Q. In order to calculate the Electric Field the electric flux we have

$$E \cdot dA = \frac{1}{\varepsilon_0} Q_{en}$$

$$E(A) = \frac{1}{\varepsilon_0}(Q)$$

$$\because A = 4\pi r^2$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

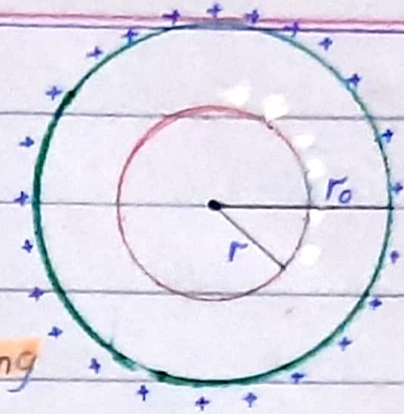
جو Gaussian Surface جو بھی
کسی بھی Sphere کے باہر
Inside of the Sphere
چاہے وہ Sphere جو ہے
ہو یا وہ Solid ہو۔ سارا
Gaussian Surface کے اندر آئے گا
کیونکہ $(r > r_0)$

We have the above electric Field if the Radius of Gaussian surface is greater than the Radius of sphere $(r > r_0)$

NOTE

Gaussian Surface جو کہ Sphere سے بڑی ہے مطلب
سارا چارج Gaussian Surface کے اندر ہی ہے close ہو
چاہے چارج پھیلا ہو یا ایک Point پر جمع ہو فرق نہیں پڑتا۔

If $r < r_0$



It means if we Consider a Gaussian surface which has smaller radius than the radius of sphere Means we are considering the surface inside sphere.

If we do this and Now want to calculate the electric field of sphere. we have

$$E.dA = \frac{1}{\varepsilon_0} Q_{en}$$

$\because Q = 0$ (inside the hollow sphere)

So

$$\oint \vec{E}.d\vec{A} = \frac{1}{\varepsilon_0} Q_{en}$$

$$E(A) = \frac{1}{\varepsilon_0}(0)$$

$$E(4\pi r^2) = 0$$

$$\boxed{E = 0}$$

Due to Absence of charge the electric field insid the sphere is Zero.

→ It is a point to be noted that there is no difference between Hollow Sphere and Solid Conductive Sphere. In both terms charge store at outside the sphere. Means, inside the sphere the charge is zero for both hollow and solid conductive sphere.

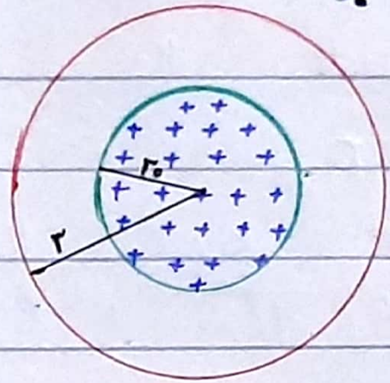
Non-conductive Solid Sphere

In case of Non-conductive solid sphere, if we want to calculate the electric field of solid sphere, first of all we must be know that either the charge is uniformly distributed or Non-uniformly distributed.

Uniformly Charge Distribution

Let's Consider a Non-Conductive solid sphere in which charges are distributed uniformly distributed. In Order to Calculate the electric field of the sphere we study three Cases which are given below:

Case-I:



In case-I consider a guassian surface out side the sphere. Means

$r > r_o$

Hence the charge enclosed by the guassian surface is Q_{en} As we know

$$\oint \vec{E}.d\vec{A} = \frac{1}{\varepsilon_o} Q_{en}$$

$$E\,(A) = \frac{1}{\varepsilon_o} Q_{en}$$

$$\because A = 4\pi r^2$$

$$E\,(4\pi r^2) = \frac{1}{\varepsilon_o} Q_{en}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_o}\frac{Q_{en}}{r^2}}$$

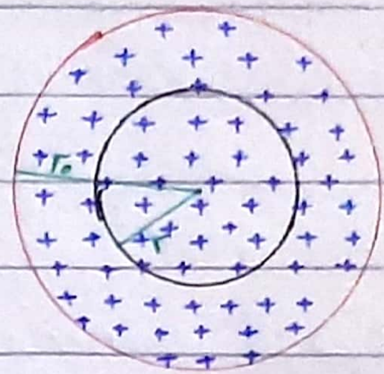
In the above equation we can see that the all factors are constant except r^2. Mean electric field intensity can be changed by changing the Value of r^2. Here

$$E \propto \frac{1}{r^2}$$

Case-II

In this case consider a guassian suface inside the sphere. Means

$r < r_0$



Hence, we know that

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{en} \quad \text{.....(i)}$$

Here To calculate the electric fied first of all we have to calculate the value of Q_{en}.

As we know that the charge Density of the sphere can be given as

$$\rho_E = \frac{Q}{V}$$

As we know that

Volume of sphere $= \frac{4}{3}\pi r^3$

So we have

$$\rho_E = \frac{Q}{\frac{4}{3}\pi r_0^3}$$

- r_0 (radius of Sphere)
- Q (Total charge enclosed by Sphere.)

This is the charge density for the whole sphere. If we want to calculate the charge density of Gaussian surface. Since the charge is uniformly distributed so we have:

r (radius of Gaussian Surface)

$$\rho_E = \frac{Q_{en}}{\frac{4}{3}\pi r^3}$$

Q_{en} (charge enclosed by Gaussian Surface)

Since charge is uniformly distributed so we can write

Density of sphere = Density of Gaussian Surface

$$\frac{Q}{\cancel{\frac{4}{3}\pi} r_0^3} = \frac{Q_{en}}{\cancel{\frac{4}{3}\pi} r^3}$$

$$Q_{en} = \frac{Q \times r^3}{r_0^3}$$

Put this value in eq. i

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{en}$$

$$E(A) = \frac{1}{\varepsilon_0} \frac{Q}{r_0^3} r^3$$

$\because \quad A = 4\pi r^2$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0} \frac{r^3 Q}{r_0^3}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{r\,Q}{r_0^3}}$$

In this case we can see that all the factors are constant except Radius of Gaussian surface 'r'. Mean Electric Field depends on 'r'

$$E \propto r$$

Case-III

In this case we will use the above case, as we said the Electric Field depends upon radius of Gaussian surface.

$$E \propto r$$

If we started to increase the radius of gaussian surface and at one stage r becomes equal to r_0. Then the electric field can be.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{r\,Q}{r_0^3}$$

As $r = r_0$

So

$$E = \frac{1}{4\pi\varepsilon_0} \left(\frac{\cancel{r_0}}{\cancel{r_0^3}}\right) Q$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_0^2}}$$

This is the electric Field.

Graph:

We can also express these cases in term of graph. As, we have seen in case # 01

$$E \propto \frac{1}{r^2}$$

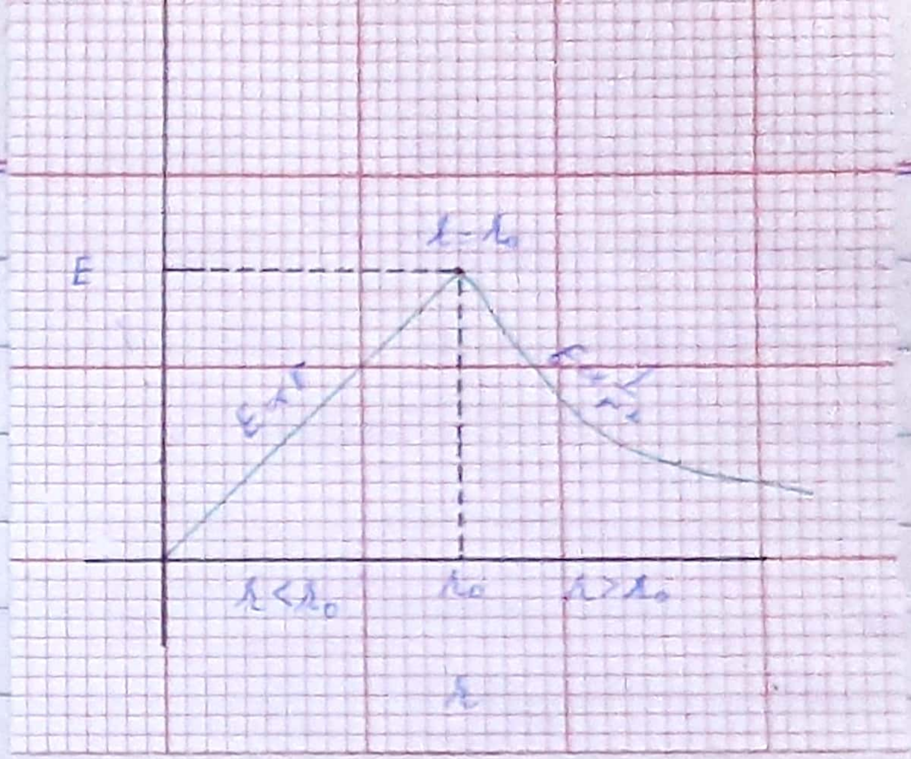
And in Case # 02

$E \propto r$

And in Case # 03

$r = r_0$

So there graph can be



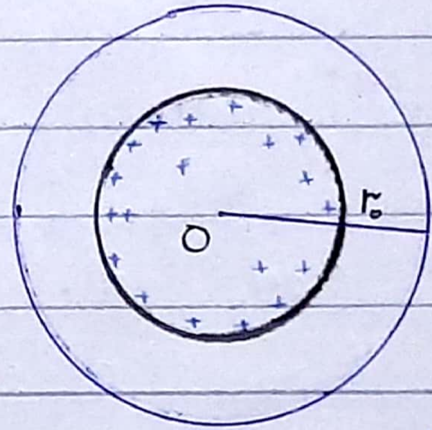
Non-Uniformly Charge Distribution

we have talked about the Non-conductive Solid Sphere and have learnt that in these spheres charges may be distributed uniformly or non-uniformly. Now we are going to talk about Non-Conductive Solid sphere in which charges are distributed Non-uniformly.

Consider a Non-conductive solid sphere having the Radius of r_o. The charge density of the sphere can be given as:

$$\rho = \alpha r^2$$

The above equation tells that if the radius of the sphere increases then the charge density of the sphere also increases and vice-versa.



If the radius of the sphere is r_o and the charge enclosed by the sphere is Q. In order to Calculate the Electric Field Intensity of the sphere, we will consider a gussian surface. Here if we consider the gussian surface outside the sphere, It Means all the charges are enclosed in a sphere. Means Result will be same as result was for the conductive Solid or Hollow sphere $(r > r_0)$

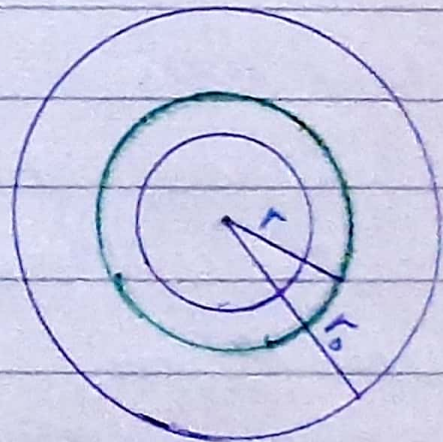
But Here charge is non-uniformly distributed so we have to divide the sphere into small patches Make sure, Each patch of sphere has a thickness of dr

From above equation

$$\rho_E = \alpha r^2$$

In Order to find the Electric Field Intensity I have ~~to~~ find the Value of

- α
- E



Here from the Figure:

we have

$$\rho_E = \frac{Q}{V}$$

$$Q = \rho_E V$$

By Applying integration

$$Q = \int_0^{r_0} \rho_E \, dV$$

we have

$$dV = 4\pi r^2 dr$$

$$\rho_E = \alpha r^2$$

so

$$Q = \int_0^{r_0} \alpha r^2 4\pi r^2 dr$$

$$= \int_0^{r_0} \alpha 4\pi r^4 dr$$

$$= \alpha 4\pi \int_0^{r_0} r^4 dr$$

$$Q = \alpha 4\pi \left| \frac{r^5}{5} \right|_0^{r_0}$$

$$Q = \alpha 4\pi \left\{ \frac{r_0^5}{5} - \frac{0}{5} \right\}$$

$$Q = \frac{4\pi\alpha r_0^5}{5}$$

$$\boxed{\alpha = \frac{5Q}{4\pi r_0^5}}$$

Since, the charge is non-uniformly distributed but Depends on the Radius of Sphere patches for Each patch.

Now we are going to calculate the total charge enclosed by the gussian surface.

I have

$$Q_{en} = \int_0^{r} \rho_E \, dV$$

Similarly Now we have

$$Q_{en} = \int_0^r \alpha r^2 4\pi r^2 dr$$

$$= \int_0^r 4\pi\alpha r^4 dr$$

$$= 4\pi\alpha \int_0^r r^4 dr$$

$$= 4\pi\alpha \left| \frac{r^5}{5} \right|_0^r$$

$$= 4\pi\alpha \left\{ \frac{r^5}{5} - \frac{0}{5} \right\}$$

$$Q_{en} = 4\pi\alpha \frac{r^5}{5}$$

$$\because \alpha = \frac{5Q}{4\pi r_o^5}$$

$$Q_{en} = \cancel{4\pi} \left(\frac{\cancel{5}Q}{\cancel{4\pi} r_o^5} \right) \frac{r^5}{\cancel{5}}$$

$$\boxed{Q_{en} = \frac{r^5 Q}{r_o^5}}$$

According to Gauss's Law

$$\oint E.dA = \frac{1}{\varepsilon_o} Q_{en}$$

$$\because E\,A = \frac{1}{\varepsilon_o} Q_{en}$$

$$\because A = 4\pi r^2$$

$$\because Q_{en} = \frac{r^5 Q}{r_o^5}$$

By putting

$$E(4\pi r^2) = \frac{1}{\varepsilon_o}\left(\frac{r^5}{r_o^5}\right)Q$$

$$E = \frac{1}{\varepsilon_o}\left(\frac{\cancel{r^5}}{r_o^5}\right)Q \times \frac{1}{4\pi \cancel{r^2}}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{r^3 Q}{r_0^5}}$$

This is the total electric charge intensity of the Non-Conductive Solid sphere in which charges are non-uniformly distributed.

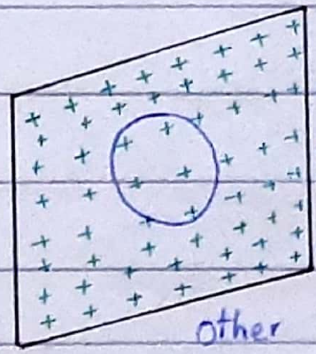
Some other Applications

1: If I have a surface Area on which charge distribution uniformly. In order to calculate the electric field intensity we have the formula:

$$\int E\,dA = \frac{1}{\varepsilon_0} Q_{en}$$

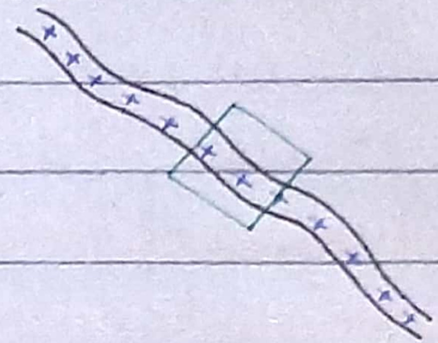
$$2EA = \frac{1}{\varepsilon_0}(\sigma A)$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



This Gaussian surface have two Areas one front and other from back side.

2. If we have a non-uniformly shaped object but charges are distributed uniformly. By imagine a gussian surface we can calculate the Electric Field Intensity as:



$$\int E.dA = \frac{1}{\varepsilon_o} Q$$

$$EA = \frac{1}{\varepsilon_o} \sigma A$$

$$E = \frac{\sigma}{\varepsilon_o}$$

Chapter 24

Capacitance, Dielectrics, Electrical Energy Storage

Capacitor:

A Capacitor is a device that can store electric charges.

Normally, A capacitor conists of two conducting objects (usually plates or sheets) placed near each other but not touching. Capacitors are widely used in electronic circuits.

Capacitance:

The ability of a capacitor to store charges is called Capacitence of a capacitor.

- It is Represented by C.
- It can be given as:

$$Q = CV$$

$$C = \frac{Q}{V}$$

- It's SI Unit is Columb per Volt called Farred.

Potential Gradiant

The quantity $\frac{\Delta V}{d}$ is the rate of of change of maximum value of potential with distance and termed as Potential Gradient.

$$\Delta V = -Ed$$

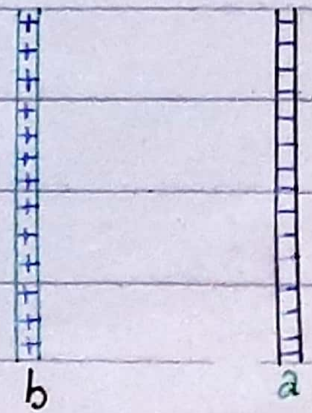
- Negative sign shows that the direction of E is along the decreasing potential. (Move from a point of low potential to a point of high potential)

Determination Of Capacitence

usually we talk about three types of capacitors which are

- Parallel plate capacitors
- Spherical plate capacitors
- Cylindrical capacitors

• Parallel Plate Capacitors



Consider two parallel pates one of which is positively charged while other is negatively charged. In order to calculate the capacitence of these parallel plates a non-conductive material is placed between the these two capacitor plates.

According to Gauss's Law we have

$$\oint E.dA = \frac{1}{\varepsilon_0} Q_{en}$$

$$V_b - V_a = -\int_a^b \vec{E}.\vec{dl}$$

$$\Delta V = -\int_a^b E\,dl \cos\theta$$

$\theta = 180^\circ$ Means $\cos\theta = \cos(180^\circ) = -1$

$$\Delta V = -\int_a^b E\,dl\,(-1)$$

$$\Delta V = \int_a^b E\,dl$$

(l is distance between two plates)

Potential Gradient (-ve)

NOTE:

/* +ve charges are present in large amount but -ve charges are in small amount That's the Reason we apply Limit from Min R_b to Max R_a. $(R_a - R_b)$ */

we have

$$\boxed{\Delta V = Ed}$$

As we know

$$EA = \frac{1}{\varepsilon_0} Q$$

$$E = \frac{Q}{\varepsilon_0 A}$$

→ Putting in above Equation

$$\Delta V = \left(\frac{Q}{\varepsilon_0 A}\right) d$$

$$\frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

$$\because \frac{Q}{V} = C$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

This is the capacitence of parallel plate capacitors.

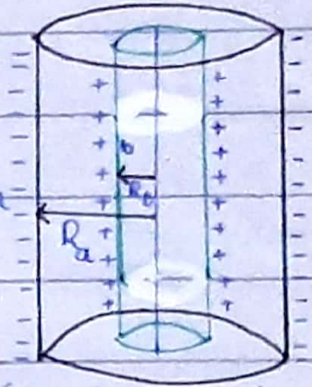
• Cylindrical Capacitor

Consider a Cylindrical Capacitor, in this way the internal capacitor is +vely charged and external cylindrical capacitor is negatively charge. The Radius of +ve charged capacitor is R_b and of -ve charged capacitor is R_a. In order to calculate the capacitence of this cylindrical capacitor is:

جب یہاں پہ ہم Capacitor کو Cylinder کی طرح لیتا ہے Electric Field صرف Curved Surface کے Along ہوگا۔

(Gauss's Law)

$$\oint E \cdot dA = \frac{1}{\varepsilon_0} Q_{en} \quad \text{....(i)}$$



جب یہ Outer Surface سے Centre کی طرف آتے ہیں تو Radius کم ہوتا ہے۔ اس لیے ہم کو ظاہر کرنے کے لیے (-) کا استعمال کیا جاتا ہے۔

And Potential Gradient

$$V_b - V_a = -\int_{R_a}^{R_b} E \cdot dr \quad \text{....(ii)}$$

Now using (i)

$$\oint E \cdot dA = \frac{1}{\varepsilon_0} Q_{en}$$

$$EA = \frac{1}{\varepsilon_0} Q_{en}$$

$\because$ Surface Area $= 2\pi r l$

$$E(2\pi r l) = \frac{1}{\varepsilon_0} Q_{en}$$

$$E = \frac{1}{2\pi \varepsilon_0 r l} Q_{en} \quad \text{.....(iii)}$$

Putting equation (iii) in (ii)

$$\Delta V = -\int_{R_a}^{R_b} \frac{1}{2\pi \varepsilon_0 r l} Q_{en} \cdot dr$$

$$= -\frac{Q_{en}}{2\pi \varepsilon_0 l} \int_{R_a}^{R_b} \frac{dr}{r}$$

$$\because \int \frac{dx}{x} = \ln x$$

$$= \frac{-Q}{2\pi l \varepsilon_0} \Big| \ln r \Big|_{R_a}^{R_b}$$

$$= -\frac{Q}{2\pi l \varepsilon_0} \left\{ \ln(R_b) - \ln(R_a) \right\}$$

$$= -\frac{Q}{2\pi l \varepsilon_0} \ln(R_b) + \frac{Q_{en}}{2\pi l \varepsilon_0} \ln(R_a)$$

$$= \frac{Q}{2\pi l \varepsilon_0} \left\{ \ln(R_a) - \ln(R_b) \right\}$$

$\because \ln A - \ln B = \ln A/B$

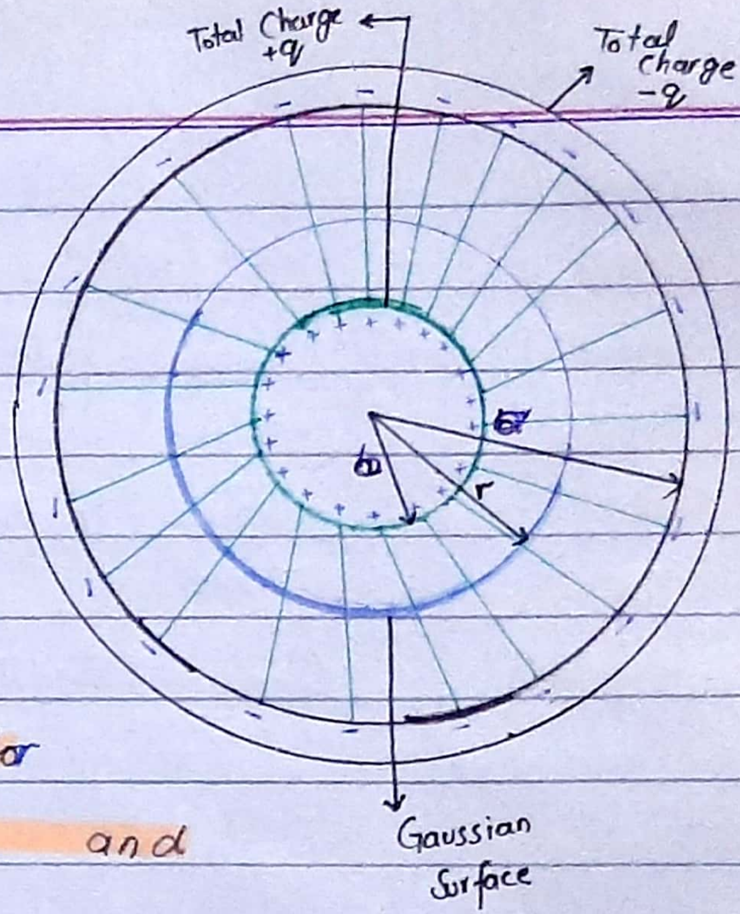
$$\Delta V = \frac{Q}{2\pi l \varepsilon_0} \ln\left(\frac{R_a}{R_b}\right)$$

$$\frac{Q}{\Delta V} = \frac{2\pi l \varepsilon_0}{\ln(R_a/R_b)}$$

→ This is the capacitence of cylindrical capacitor.

$$\boxed{C = \frac{2\pi l \varepsilon_0}{\ln(R_a/R_b)}}$$

Spherical Capacitor



Consider a spherical plate capacitor in which similarly like above capacitors negative and positivly charged capacitors are given. To calculate the capacitence of the capacitor we will use Gauss's Law and potential Gradient.

We have

$$\oint E.dA = \frac{1}{\varepsilon_o} Q_{en}$$

$$EA = \frac{1}{\varepsilon_o} Q$$

$$\because A = 4\pi r^2$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_o} Q$$

$$\boxed{E = \frac{1}{4\pi \varepsilon_o r^2} Q}$$

Now potential Gradient

$$V_b - V_a = -\int_{R_a}^{R_b} E \cdot dr$$

Putting Value of E

$$V = -\int_{R_a}^{R_b} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} dr$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \int_{R_a}^{R_b} \frac{dr}{r^2}$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \int_{R_a}^{R_b} r^{-2} dr$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \left| \frac{r^{-2+1}}{-2+1} \right|_{R_a}^{R_b}$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \left| \frac{-1}{r} \right|_{R_a}^{R_b}$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left| \frac{1}{r} \right|_{R_a}^{R_b}$$

⇐ (-) کو (-) سے Multiply کروا دیا۔ پھر Limit کی ویلیو Apply کی۔

$$= \frac{1}{4\pi\varepsilon_0} Q \left\{ \frac{1}{R_b} - \frac{1}{R_a} \right\}$$

$$V = \frac{1}{4\pi\varepsilon_0} Q \left\{ \frac{R_a - R_b}{R_a R_b} \right\}$$

$$\frac{Q}{V} = \frac{4\pi\varepsilon_0 R_a R_b}{R_a - R_b}$$

$$\boxed{C = \frac{4\pi\varepsilon_0 R_a R_b}{R_a - R_b}}$$

→ This is the Capacitence of Spherical plate Capacitors.

Chapter 27

MAGNETISM

Magnetic Field:

The space or a region around a magnet where the effect of magnetism, such as the deflection of a compass needle, can be detected is called Magnetic Field.

Magnetic Line of forces:

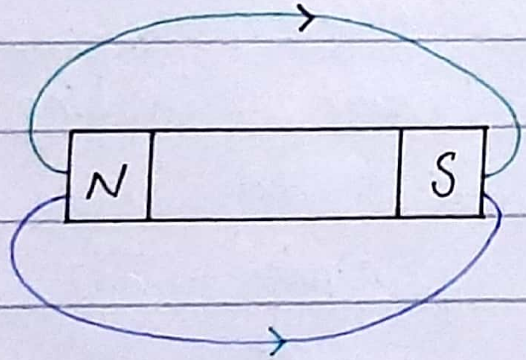
The path of an isolated

north magnetic pole in a magnetic field represents a magnetic line of force.

A Single pole of a Magnet cannot be obtained.

جب بھی کسی بڑے Magnet کو توڑا جاتا ہے تو بھی اس کے چھوٹے چھوٹے حصوں کے نارتھ اور ساؤتھ پول ہوتے ہیں۔ کسی بھی مقناطیس کے دو پول کی بجائے ایک سنگل پول حاصل کرنا Impossible ہے۔

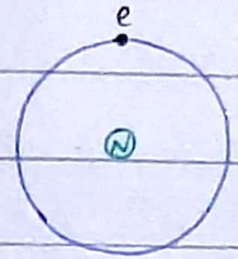
→ The Direction of Magnetic Field Lines is from North pole to South pole. As:



How does Magnetic Field Produce?

Magnetic Field is produce due to change in Flux.

If I have an electron in a orbit, with a nucleour N. as:



Here we can see that the nucleous is attracting the electron towards it self and electron is also repelling by outer electrons, so it will not stay at a constant place, it start revolving, Due to which Flux is produced and due to revolution electron is changing it's position, means change in flux is occurring, As a result, a Magnetic Field Produced.

نوٹ :-

← اگر کسی Material میں Even نمبر الیکٹرونز ہیں تو ان میں سے آدھے clockwise گھومتے ہیں
اور آدھے Anti clockwise گھومیں گے

Physics for Scientists & Engineers

(DOUGLAS C. Giancoli)

After Mid Term

CHAPTER # 15

WAVES

What is wave?

The disturbance which travel from one point to another point with or without any medium is called a wave.

Types Of Wave

As we know that this world is full of waves. But two main types of wave are given below:

- Mechanical Waves
- Electromagnetic Waves

ELECTROMAGNETIC WAVES

Electromagnetic waves are those waves which required no any medium for their propagation.

Example:

The examples of electromagnetic waves are

- Visible Light
- Radio waves
- Television Signals

Why do they don't need any Medium for propagation?

Electromagnetic waves don't need any medium for their propagation because in these waves Electric Field and Magnetic Field both are Perpendicular To eachother, as a result, the resultant Field is zero.

NOTE :-

The Speed of Machanical waves is very less as compared to the Electromagnetic waves

MECHANICAL WAVES

Mechanical Waves are those waves which required a Medium for their propagation.

Example:

The examples of Mechanical waves are given below:

- Throw a pebble in water, water elements disturb.
- Sound waves
- Jump Rope waves.

NOTE:

A Mechanical wave is a wave that is not capable of transmitting its energy through a vacuum.

Types Of Mechenical Waves

There are two types of Mechanical waves.

- Longitudunal Waves
- Transverse waves
- Circular Transverse Waves

Longitudunal waves:

The waves in which Practicles of Medium moves parallel to the wave motion or to extent of wave.

Example:

An example of longitudnal wave is compression moving along a slinky.

Transverse Waves:

The wave in which particles of Medium move perpendicular to the motion of wave are called Transverse Waves.

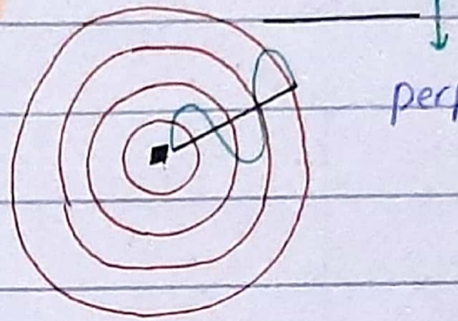
Examples:

- Radio waves
- Sound waves
- Water waves

How does the above waves are Tranverse waves?

The above waves are called Transverse waves Let's check Through an example of water waves.

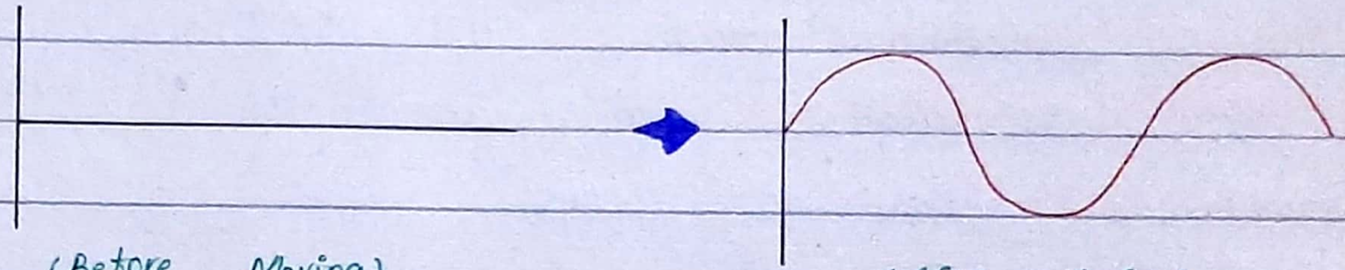
When we threw an pebble into water and see that waves produced, and how these waves are transerve waves. Because, when Pebble produce waves we can see the angle between particles of Medium and wave motion is 90° Means both are perpendicular. As, shown in figure:



THE WAVE EQUATION

Many types of waves satisfy an important general equation thus it is the equivalent to Newton's second law of motion for particles. This equation of motion for a wave is called the wave equation.

Let's take a rope and apply force on it in order to create a wave in the string. As



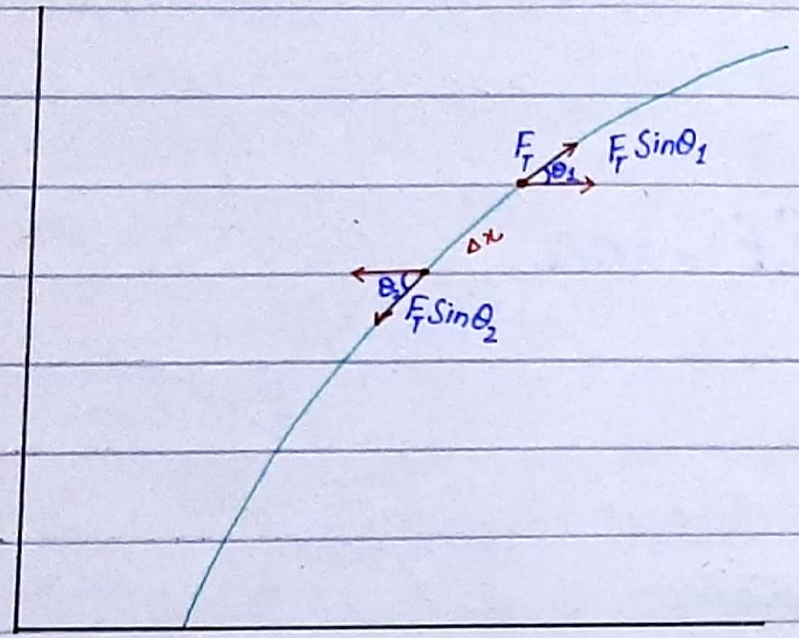
(Before Moving) (After Moving)

Here, we see that when we have applied force on the string, an acceleration has been produced in the direction of motion of string. And we know that rope has some mass also. So According To Newton's 2nd Law of Motion.

$$F = ma \quad \text{.....} \quad (i$$

In order to calculate the wave equation, Let take a small part of wave and resolve it into its components.

Figure:



(T for Tension)

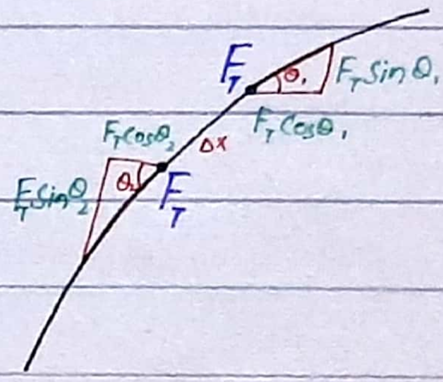
We assume the amplitude of wave is small as compared to the wavelength, so that each point on the string can be assumed to move only vertically, and the Tension in the string F_T does not varry during vibration.

Hence By Apply Newton's 2nd Law of Motion, we get the equation.

$$F_y = ma_y \quad \text{------ (ii)}$$

As, we we know at a time, not only one force is working on y-axis. So, we can write the above equation as:

$$\sum F_y = ma_y \quad \text{---- (iii)}$$



From the figure, we can write the above equation in this manner

$$F_T \sin\theta_2 - F_T \sin\theta_1 = ma_y \quad \text{--- (iv)}$$

Since we know that, we are considering a small patch of string and this patch of string will also have some mass. So we can find the mass of this special patch from Linear mass density which can be given as:

$$\mu = \frac{m}{l}$$

Since from figure, we have

$l = \Delta x$ (The difference b/w Two forces)

so

$$\mu = \frac{m}{\Delta x}$$

and

$$m = \mu \Delta x \quad \text{.....(v}$$

By putting the equation (v) in eq. (iv)

$$F_T \sin\theta_2 - F_T \sin\theta_1 = \mu \Delta x a_y \quad \text{---- (vi}$$

Since we know that the acceleration is the double derivative of displacement. As. By taking first derivative of displacement we get velocity and by taking second derivative. the velocity turns into acceleration.

And we have the acceleration $a_y = \frac{\partial^2 D}{\partial t^2}$ $\Rightarrow \left(V = \frac{S}{t} \right)$

$\Rightarrow \left(a = \frac{V}{t} \right)$

Since the motion is only vertically and we use the partial derivative notation because the displacement D is a function of both x and t.

Now

Put the value of acceleration in equation (vi)

$$F_T \sin\theta_2 - F_T \sin\theta_1 = \mu \Delta x \frac{\partial^2 D}{\partial t^2} \quad \text{.....(vii)}$$

As the angles θ_1 and θ_2 are assumed small. So $\sin\theta \approx \tan\theta$ and $\tan\theta$ is equal to the slope of string at each point.

$$\sin\theta \approx \tan\theta \approx \frac{\partial D}{\partial x} = \text{Slope}$$

By using equation (vii)

$$F_T \sin\theta_2 - F_T \sin\theta_1 = \mu \Delta x \frac{\partial^2 D}{\partial t^2}$$

$$F_T (\sin\theta_2 - \sin\theta_1) = \mu \Delta x \frac{\partial^2 D}{\partial t^2}$$

From above relation $\sin\theta \simeq \tan\theta$ So

$$F_T(\tan\theta_2 - \tan\theta_1) = \mu \Delta x \frac{\partial^2 D}{\partial t^2}$$

Since

$\tan\theta = \text{Slope} = S$ (So above equation becomes)

$$F_T(S_2 - S_1) = \mu \Delta x \frac{\partial^2 D}{\partial t^2}$$

$$F_T(\partial S) = \mu \Delta x \frac{\partial^2 D}{\partial t^2}$$

$$F_T \frac{\partial S}{\partial x} = \mu \frac{\partial^2 D}{\partial t^2} \quad \text{------ (viii}$$

Since

$$S = \frac{\partial D}{\partial x} \rightarrow S = \frac{\partial D}{\partial x} \rightarrow \text{If } \Delta x \simeq 0$$

$S = \frac{\partial D}{\partial x}$ (also a function of time)

Put this in equation (viii)

$$F_T \frac{\partial}{\partial x}\left(\frac{\partial D}{\partial x}\right) = \mu \frac{\partial^2 D}{\partial t^2}$$

$$\frac{F_T}{\mu}\frac{\partial^2 D}{\partial x^2} = \frac{\partial^2 D}{\partial t^2} \quad \text{......} (ix$$

As we know velocity of Transerve wave

$$V = \sqrt{\frac{F_T}{\mu}}$$

Taking square

$$V^2 = \frac{F_T}{\mu} \quad \text{......} (x)$$

Put in eq. (ix).

$$V^2 \frac{\partial^2 D}{\partial x^2} = \frac{\partial^2 D}{\partial t^2}$$

$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 D}{\partial t^2}}$$

This is called one-dimensional wave Equation. It can describe not only small amplitude wave on a stretched string, but also small amplitude longitudinal waves (such as sound waves) in gases, Liquid and elastic solid.

Noise & Music

Noise:

The sound which has unpleasent effect on air and has no periodicity is called Noise.

→ Noise is produced by horns of Vehicles, crackers, machines etc.

Music:

The sound which has pleasent effect on air and has periodicity is called Music.

➜ Music is produced by musical instruments like piano, Guitar, Flute etc.

Note:

It is not important that every musical instrument cannot produce Noise.

Periodicity

Periodicity means that if elements are arranged in an order, the properties of elements would repeal some period.

Periodicity Makes your sound Noise or Music.

If the Periodicity is controlled, it will produce Music and if the Periodicity is not controlled then Noise produces.

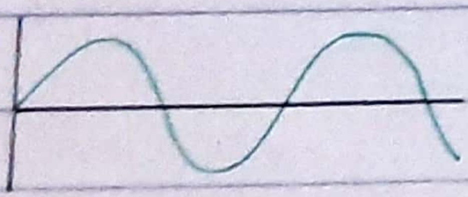
PITCH:

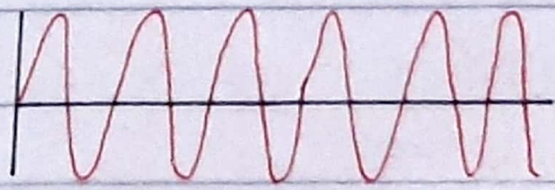
{Position of a single sound in the complete range of Sound}

The pitch of sounds is determined by the frequency of vibration of the sound waves that produce them.

We can differentiate between different sounds on the basis of pitch.

- Sounds having high-frequency produced high-pitched noises.
- Sound waves having low-frequency produce low-pitched sounds.





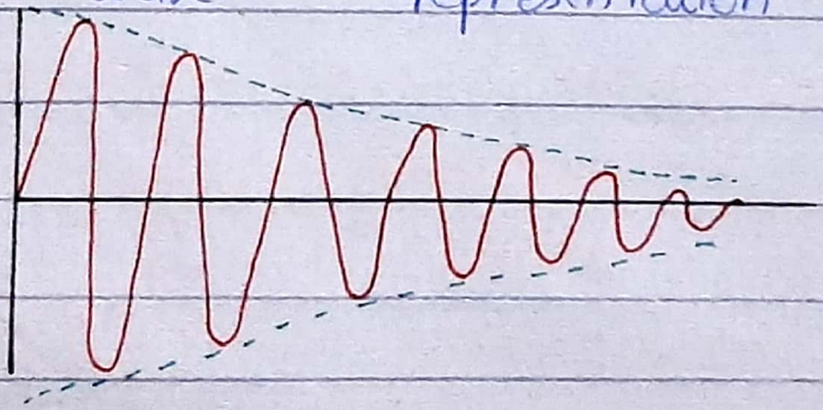
Frequency:

A frequency is how frequent sound waves pass a certain point in a second.

Harmonic Oscillator:

It is called Harmonic. Because the solution of Newton's second Law (a second order differential equation that determines the motion of the object) are sines and cosines of time with the particular frequency.

It's wave representation is given below:



Intensity:

It is the Power delivered per unit area.
Intensity of wave is proportional to the square of amplitude of the wave.
Usually, we measure the intensity in Bell or decibell.

For Sound Waves:

$I_o = 10^6$

$I = 10^{12}$

$\log \frac{I}{I_o} = \log \frac{10^{12}}{10^6} = \log 10^{12-6} = \log 10^6 = $ "6dB"

$$\boxed{\text{Intensity of Sound waves} = \frac{E}{A \cdot t}}$$

Standing/Stationary Waves

The waves travelling in same medium but coming from opposite direction super impose.

Example:

Waves produced by stringed musical instruments.

Node:

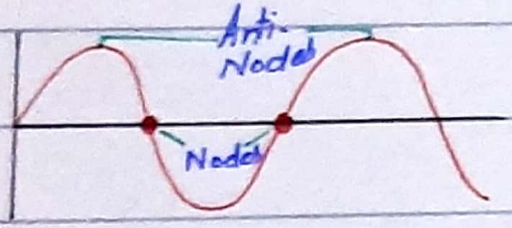
A node is a point along a standing wave where the wave has minimum amplitude.

Anti-Node:

An anti-node is a point along a standing wave where the wave has maximum amplitude.

Stationary Waves In Air Column

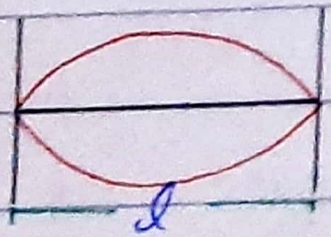
Graphically Representation:



Consider we have a wave and let's check the Relation of frequency with wavelength and increasing the number of cycles.

First Half Cycle:

Let consider the half cycle of wave



As we know

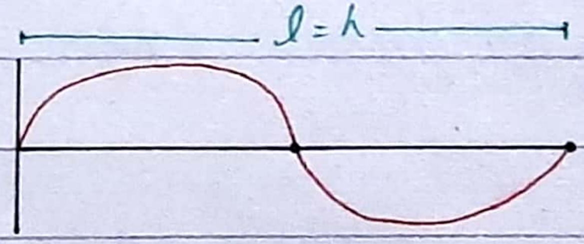
$v = f\lambda$

$f_1 = \frac{v}{\lambda_1}$

$\boxed{f_1 = \frac{v}{2l}}$

For 1-Complete Cycle:

Now let's check for one complete cycle.



As we know the wavelength is the distance from the crest of one wave to crest of next wave.

Now we have

$$\Rightarrow l = \lambda$$

$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = \frac{v}{l} \quad (\times\text{-ing } + \div\text{-ing by } 2)$$

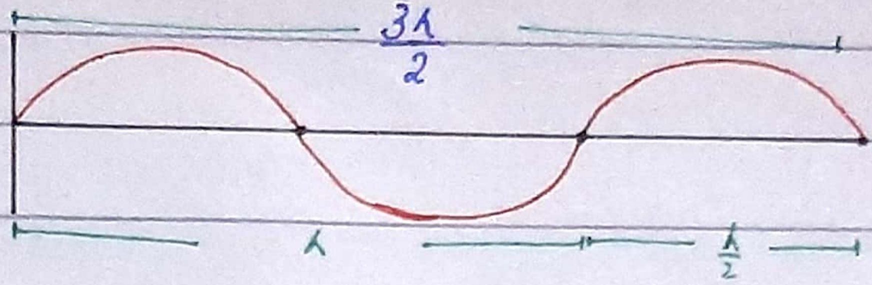
$$f_2 = 2\left(\frac{v}{2l}\right)$$

$$\because \frac{v}{2l} = f_1$$

$$\boxed{f_2 = 2f_1}$$

Now For 1.5 Cycles:

Now If we cosider one complete and one half cycle And we have one length of wave $\frac{3\lambda}{2}$ As:



$\Rightarrow l = \frac{3\lambda}{2}$

Now

$\Rightarrow \lambda_3 = \frac{2l}{3}$

$f_3 = \frac{v}{\lambda_3}$

$f_3 = \frac{v}{2l/3}$

$f_3 = 3\left(\frac{v}{2l}\right) \qquad \because \frac{v}{2l} = f_1$

$\boxed{f_3 = 3 f_1}$

For n-cycles:

From above relations we can say that, if we have n-number of cycles then, I can find a relation such as:

$$\boxed{f_n = n f_1}$$

From Relation we can say that

As the wavelength decreases frequency increases

TRASERVE WAVE FREQUENCY

The frequency of a transerve wave is found by determining the number of high points (crests) that go by a point in one second.

A wave with high frequency has many crests that pass by the point.

FORMULA

$$V = \sqrt{\frac{F_T}{\mu}}$$

As

$$V = f\lambda$$

$$f = \frac{V}{\lambda}$$

$f \propto V$ and $f \propto \frac{1}{\lambda}$

اگر Musical Instrument جن میں String ہوتی ہیں ان Strings میں اگر Distance
ایک جیسا ہوتا ہے اگر تمام Strings کو ایک ہی Position سے ٹچ کیا جائے تو پیدا ہونے والی

Sounds مختلف ہوں گی کیونکہ ان strings کی Speed مختلف ہو گی اور Speed کی وجہ سے
frequency مختلف ہو گی اور پیدا ہونے والی Sound بھی مختلف ہو گی کیونکہ

$$v = \sqrt{\frac{F_T}{\mu}}$$

کیونکہ چاروں strings کی μ کی ویلیو مختلف ہوتی ہے۔ جب آپ string کی کسی بھی جگہ پر ٹچ کریں تو ان کی μ کی ویلیو تو Same ہو گی لیکن Different Positions پر ٹچ کرنے سے λ کی ویلیو ضرور تبدیل ہوں گی جس کی وجہ سے Different جگہوں پر ٹچ کرنے سے مختلف Sounds پیدا ہوں گی۔ اگر ان میں Periodicity ہو گی تو یہ Music کہلائے گا اور اگر Periodicity نہ ہو تو یہ Noise کہلائے گی۔

IMP. Points:

From the above topic we have learnt that

- Loundness of a sound depends on intensity.
  - Higher Intensity the sound will be louder.
  - Lower Intensity the sound will be lower.

- The Frequency produce Sounds.
- If we combine different frequencies sounds It has different effect on human's ear
  - If there is a Periodicity in sound and having pleasent effect on human's ear then it will called Music.
  - If there is no Periodicity in sound and having unpleasent effect on human's ear then it will called Noise.

Since we know that frequency can be given as:

$$f = \frac{v}{\lambda}$$

In Case of Transerve waves the velocity can be given as:

$$v = \sqrt{\frac{F_T}{\mu}}$$

- V depends on μ.
- For each sound the value of μ is different.
- Due to different velocities the frequency of sound changes and New Sounds produce. Because

$$f \propto v$$

⇒ Higher the frequency, higher will be velocity.

⇒ Lower the velocity, lower will be frequency.

- Music has everlasting effect on human's ear.