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Directional Derivatives and Gradients

Gradient :—

a) If 'f' is a function of 'x' and 'y' then the gradient of f is defined by

$$\nabla f(x,y) = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

b) If 'f' is a function of 'x', 'y' and 'z', then the gradient of 'f' is defined by

$$\nabla f(x,y,z) = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Q33) $z = \sin(7y^2 - 7xy)$

Here,

$$\nabla z = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

As,

$$\Rightarrow f_x = \cos(7y^2 - 7xy) \cdot (-7y)$$

$$= -7y\cos(7y^2 - 7xy)$$

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$$\Rightarrow f_y = \cos(7y^2 - 7xy) \cdot (14y - 7x)$$
$$= (14y - 7x) \cdot \cos(7y^2 - 7xy)$$

Now,

$$\nabla z = [-7y\cos(7y^2 - 7xy)]\hat{i} + [(14y - 7x)\cos(7y^2 - 7xy)]\hat{j}$$

$$\nabla z = -7y\cos(7y^2 - 7xy)\hat{i} + (14y - 7x)\cos(7y^2 - 7xy)\hat{j}$$

Q34) $z = 7\sin\left(\frac{6x}{y}\right)$

As we know that

$$\nabla z = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

Here,

$$\Rightarrow f_x = 7\cos\left(\frac{6x}{y}\right) \cdot \frac{6}{y}(1)$$
$$= \frac{42}{y}\cos\left(\frac{6x}{y}\right)$$

$$\Rightarrow f_y = 7\cos\left(\frac{6x}{y}\right) \cdot \left(\frac{-6x}{y^2}\right)$$
$$= \frac{-42x}{y^2}\cos\left(\frac{6x}{y}\right)$$

Now,

$$\nabla z = \frac{42}{y}\cos\left(\frac{6x}{y}\right)\hat{i} - \frac{42x}{y^2}\cos\left(\frac{6x}{y}\right)\hat{j}$$

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Q35) $z = \dfrac{6x+7y}{6x-7y}$

As we know that

$$\nabla z = f_x(x,y)\,\hat{i} + f_y(x,y)\,\hat{j}$$

Here,

$$\Rightarrow f_x = \frac{(6x-7y)(6) - (6x+7y)(6)}{(6x-7y)^2}$$

$$= \frac{6(\cancel{6x} - 7y - \cancel{6x} - 7y)}{(6x-7y)^2}$$

$$= \frac{6(-14y)}{(6x-7y)^2}$$

$$= \frac{-84y}{(6x-7y)^2}$$

$$\Rightarrow f_y = \frac{(6x-7y)(7) - (6x+7y)(-7)}{(6x-7y)^2}$$

$$= \frac{7(6x - \cancel{7y} + 6x + \cancel{7y})}{(6x-7y)^2}$$

$$= \frac{7(12x)}{(6x-7y)^2}$$

$$= \frac{84x}{(6x-7y)^2}$$

Now,

$$\nabla z = \frac{-84y}{(6x-7y)^2}\,\hat{i} - \frac{84x}{(6x-7y)^2}\,\hat{j}$$

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Q36) $z = \dfrac{6xe^{3y}}{x+8y}$

As we know that

$\nabla z = f_x(x,y)\,\hat{i} + f_y(x,y)\,\hat{j}$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left(\frac{6xe^{3y}}{x+8y}\right)$$

$$= 6e^{3y} \cdot \frac{\partial}{\partial x}\left(\frac{x}{x+8y}\right)$$

$$= 6e^{3y}\left(\frac{(x+8y)(1) - (x)(1)}{(x+8y)^2}\right)$$

$$= 6e^{3y}\left(\frac{x+8y-x}{(x+8y)^2}\right)$$

$$= 6e^{3y}\left(\frac{8y}{(x+8y)^2}\right)$$

$$= \frac{48ye^{3y}}{(x+8y)^2}$$

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left(\frac{6xe^{3y}}{x+8y}\right)$$

$$= 6x \cdot \frac{\partial}{\partial y}\left(\frac{e^{3y}}{x+8y}\right)$$

$$= 6x\left\{\frac{(x+8y)(3e^{3y}) - (e^{3y})(8)}{(x+8y)^2}\right\}$$

$$= 6x\left\{\frac{e^{3y}(3x+24y-8)}{(x+8y)^2}\right\}$$

$$= \frac{6xe^{3y}(3x+24y-8)}{(x+8y)^2}$$

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Thus,

$$\nabla z = \frac{48ye^{3y}}{(x+8y)^2}\hat{i} + \frac{6xe^{3y}(3x+24y-8)}{(x+8y)^2}$$

Q37) $w = -x^9 - y^3 + z^{12}$

As we know that

$$\nabla w = f_x(x,y,z)\,\hat{i} + f_y(x,y,z)\,\hat{j} + f_z(x,y,z)$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}(-x^9 - y^3 + z^{12})$$

$$= -9x^8$$

$$\Rightarrow f_y = \frac{\partial}{\partial y}(-x^9 - y^3 + z^{12})$$

$$= -3y^2$$

$$\Rightarrow f_z = \frac{\partial}{\partial z}(-x^9 - y^3 + z^{12})$$

$$= 12z^{11}$$

Thus,

$$\nabla w = -9x^8\hat{i} - 3y^2\hat{j} + 12z^{11}\hat{k}$$

Q38) $w = xe^{8y}\sin 6z$

As we know that

$$\nabla w = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}(xe^{8y}\sin 6z)$$

$$= e^{8y}\sin 6z \cdot (1)$$

$$= e^{8y}\sin 6z$$

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$$\Rightarrow f_y = \frac{\partial}{\partial y}\left(xe^{8y}\sin 6z\right)$$

$$= x\sin 6z \cdot \left(e^{8y} \cdot 8\right)$$

$$= 8xe^{8y}\sin 6z$$

$$\Rightarrow f_z = \frac{\partial}{\partial z}\left(xe^{8y}\sin 6z\right)$$

$$= xe^{8y} \cdot \cos 6z \cdot (6)$$

$$= 6xe^{8y}\cos 6z$$

Now,

$$\nabla w = e^{8y}\sin 6z\,\hat{i} + 8xe^{8y}\sin 6z\,\hat{j} + 6xe^{8y}\cos 6z\,\hat{k}$$

Q39) $w = \ln\sqrt{x^2+y^2+z^2}$

As we know that

$$\nabla w = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left(\ln\sqrt{x^2+y^2+z^2}\right)$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}} \cdot \cancel{2}x$$

$$= \frac{x}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

$$= \frac{x}{x^2+y^2+z^2}$$

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left(\ln\sqrt{x^2+y^2+z^2}\right)$$

$$= \frac{y}{x^2+y^2+z^2}$$

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$$\Rightarrow f_z = \frac{\partial}{\partial z}\left(\sqrt{x^2+y^2+z^2}\right)$$

$$= \frac{z}{x^2+y^2+z^2}$$

Now,

$$\nabla w = \frac{x}{x^2+y^2+z^2}\hat{i} + \frac{y}{x^2+y^2+z^2}\hat{j} + \frac{z}{x^2+y^2+z^2}\hat{k}$$

Q40) $w = e^{-5x}\sec x^2yz$

As we know that

$$\nabla w = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left(e^{-5x}\sec x^2yz\right)$$

$$= (e^{-5x}) \cdot \sec x^2yz \tan x^2yz \cdot (2xyz) + (\sec x^2yz)(e^{-5x} \cdot -5)$$

$$= e^{-5x}\sec x^2yz\,(2xyz\tan x^2yz - 5)$$

$$= e^{-5x}\sec(x^2yz)\left[2xyz\tan(x^2yz) - 5\right]$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left(e^{-5x}\sec x^2yz\right)$$

$$= e^{-5x} \cdot \frac{\partial}{\partial y}\left(\sec x^2yz\right)$$

$$= e^{-5x}\sec(x^2yz)\tan(x^2yz) \cdot x^2z(1)$$

$$= e^{-5x}\sec(x^2yz)\tan(x^2yz) \cdot x^2z$$

$$= x^2ze^{-5x}\sec(x^2yz)\tan(x^2yz)$$

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$$\Rightarrow f_z = \frac{\partial}{\partial z}\left(e^{-5x} \sec x^2yz\right)$$

$$= e^{-5x} \cdot \sec(x^2yz) \cdot x^2y(1) \cdot \tan(x^2yz)$$

$$= x^2ye^{-5x} \sec(x^2yz)\tan(x^2yz)$$

Now,

$$\nabla w = e^{-5x}\sec(x^2yz)\left[2xyz\tan(x^2yz) - 5\right]\hat{i} + x^2ze^{-5x}\sec(x^2yz)\tan(x^2yz)\hat{j} + x^2ye^{-5x}\sec(x^2yz)\tan(x^2yz)\hat{k}$$

Q41)

Find the gradient of f at the indicated point.

Q41) $f(x,y) = 5x^2 + y^4$; $(4,2)$

As we know that

$$\nabla f(x,y) = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}(5x^2 + y^4)$$

$$= 10x$$

$$f_x(4,2) = 10(4) = 40$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}(5x^2 + y^4)$$

$$= 4y^3$$

$$f_y(4,2) = 4(2)^3 = 32$$

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Thus,

$$\nabla f(4,2) = 40\hat{i} + 32\hat{j}$$

Q42) $f(x,y) = 5\sin x^2 + \cos 3y$; $(\sqrt{\pi/2}, 0)$

As we know that

$$\nabla f(x,y) = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}(5\sin x^2 + \cos 3y)$$

$$= 5\cos x^2 \cdot 2x$$

$$= 10x\cos x^2$$

$$f_x(\sqrt{\pi/2}, 0) = 10\left(\frac{\sqrt{\pi}}{2}\right)\cos\left(\frac{\sqrt{\pi}}{2}\right)^2$$

$$= 5\sqrt{\pi}\cos\left(\frac{\pi}{4}\right)$$

$$= 5\sqrt{\pi} \cdot \frac{1}{\sqrt{2}}$$

$$= 5\sqrt{\frac{\pi}{2}}$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}(5\sin x^2 + \cos 3y)$$

$$= 0 - \sin 3y \cdot 3$$

$$= -3\sin 3y$$

$$f_y(\sqrt{\pi/2}, 0) = -3\sin 3(0) = 0$$

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Now,

$$\nabla f(x,y) = 10x\cos(x^2)\hat{i} - 3\sin 3y\,\hat{j}$$

and

$$\nabla f(\sqrt{\pi}/2, 0) = 5\sqrt{\frac{\pi}{2}}\,\hat{i}$$

Q43) $f(x,y) = (x^2+xy)^3$; $(-1,-1)$

As we know that

$$\nabla f(x,y) = f_x(x,y)\,\hat{i} + f_y(x,y)\,\hat{j}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left[(x^2+xy)^3\right]$$

$$= 3(x^2+xy)^2 \cdot (2x+y)$$

$$= 3(2x+y)(x^2+xy)^2$$

$$f_x(-1,-1) = 3(-2-1)(1+1)^2$$

$$= 3(-3)(2)^2$$

$$= -36$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left[(x^2+xy)^3\right]$$

$$= 3(x^2+xy)^2(x)$$

$$= 3x(x^2+xy)^2$$

$$f_y(-1,-1) = 3(-1)(1+1)^2$$

$$= -3(2)^2$$

$$= -12$$

Now,

$$\nabla f(x,y) = 3(2x+y)(x^2+xy)^2\,\hat{i} + 3x(x^2+xy)^2\,\hat{j}$$

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and,

$$\nabla f(-1,-1) = -36\hat{i} - 12\hat{j}$$

Q44) $f(x,y) = (x^2+y^2)^{-1/2}$; $(3,4)$

As we know that

$$\nabla f(x,y) = f_x(x,y)\hat{i} + f_y(x,y)\hat{j}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left[(x^2+y^2)^{-1/2}\right]$$

$$= \frac{-1}{\cancel{2}}(x^2+y^2)^{-\frac{3}{2}} \cdot (\cancel{2}x)$$

$$= -x(x^2+y^2)^{-3/2}$$

$$f_x(3,4) = -3(9+16)^{-3/2}$$

$$= \frac{-3}{125}$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left[(x^2+y^2)^{-1/2}\right]$$

$$= \frac{-1}{\cancel{2}}(x^2+y^2)^{-3/2} \cdot \cancel{2}y$$

$$= -y(x^2+y^2)^{-3/2}$$

$$f_y(3,4) = -4(9+16)^{-3/2}$$

$$= \frac{-4}{125}$$

Now,

$$\nabla f(x,y) = -x(x^2+y^2)^{-3/2}\hat{i} - y(x^2+y^2)^{-3/2}\hat{j}$$

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and,

$$\nabla f(3,4) = -\frac{3}{125}\hat{i} - \frac{4}{125}\hat{j}$$

Q45) $f(x,y,z) = y\ln(x+y+z)$; $(-3,4,0)$

As we know that

$$\nabla f(x,y,z) = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}\left[y\ln(x+y+z)\right]$$

$$= y \cdot \frac{1}{x+y+z} \cdot (1+0+0)$$

$$= \frac{y}{x+y+z}$$

$$f_x(-3,4,0) = \frac{4}{-3+4+0}$$

$$= \frac{4}{1} = 4$$

$$\Rightarrow f_y = \frac{\partial}{\partial y}\left[y\ln(x+y+z)\right]$$

$$= y \cdot \frac{1}{x+y+z} \cdot (0+1+0) + \ln(x+y+z) \cdot (1)$$

$$= \frac{y}{x+y+z} + \ln(x+y+z)$$

$$f_y(-3,4,0) = \frac{4}{-3+4+0} + \ln(-3+4+0)$$

$$= \frac{4}{1} + \ln(1) = 4$$

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$$\Rightarrow \quad f_z = \frac{\partial}{\partial z}\left[y \ln(x+y+z)\right]$$

$$= y \cdot \frac{1}{x+y+z} \cdot (0+0+1)$$

$$= \frac{y}{x+y+z}$$

$$f_z(-3,4,0) = \frac{4}{-3+4+0}$$

$$= 4$$

Now,

$$\nabla f(x,y,z) = \left(\frac{y}{x+y+z}\right)\hat{i} + \left(\frac{y}{x+y+z} + \ln(x+y+z)\right)\hat{j} + \left(\frac{y}{x+y+z}\right)\hat{k}$$

and,

$$\nabla f(-3,4,0) = 4\hat{i} + 4\hat{j} + 4\hat{k}$$

Q46) $f(x,y,z) = y^2 z \tan^3 x \; ; \; \left(\frac{\pi}{4}, -3, 1\right)$

As we know that

$$\nabla f(x,y,z) = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

Here,

$$\Rightarrow f_x = \frac{\partial}{\partial x}(y^2 z \tan^3 x)$$

$$= y^2 z \cdot \frac{\partial}{\partial x}(\tan^3 x)$$

$$= y^2 z \cdot 3\tan^2 x \cdot \sec^2 x$$

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$$f_x = 3y^2 z \tan^2 x \sec^2 x$$

$$f_x\left(\frac{\pi}{4}, -3, 1\right) = 3(-3)^2(1)\tan^2\left(\frac{\pi}{4}\right)\sec^2\left(\frac{\pi}{4}\right)$$

$$= 3(9)(1)^2(\sqrt{2})^2$$

$$= 27(1)(2)$$

$$= 54$$

Also,

$$\Rightarrow f_y = \frac{\partial}{\partial y}(y^2 z \tan^3 x)$$

$$= z\tan^3 x \cdot 2y$$

$$= 2yz\tan^3 x$$

$$f_y\left(\frac{\pi}{4}, -3, 1\right) = 2(-3)(1)\tan^3\left(\frac{\pi}{4}\right)$$

$$= -6(1)^3$$

$$= -6$$

Also,

$$\Rightarrow f_z = \frac{\partial}{\partial z}(y^2 z \tan^3 x)$$

$$= y^2\tan^3 x \cdot (1)$$

$$= y^2\tan^3 x$$

$$f_z\left(\frac{\pi}{4}, -3, 1\right) = (-3)^2\tan^3\left(\frac{\pi}{4}\right)$$

$$= 9(1)^3$$

$$= 9$$

Now,

$$\nabla f(x,y,z) = (3y^2 z\tan^2 x\sec^2 x)\hat{i} + (2yz\tan^3 x)\hat{j} + (y^2\tan^3 x)\hat{k}$$

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and,

$$\nabla f\left(\frac{\pi}{4}, -3, 1\right) = 54\hat{i} - 6\hat{j} + 9\hat{k}$$

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