

# PLANETARY RING DYNAMICS

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**Keywords:** celestial mechanics, disks, dust, orbits, perturbations, planets, resonances, rings, solar system dynamics

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## Summary

Planetary rings provide a natural laboratory for investigating dynamical phenomena. Thanks to their proximity to Earth, the rings surrounding the giant planets can be studied at high resolution and in great detail. Indeed, Earth-based observations and spacecraft missions have documented a diverse array of structures in planetary rings produced by both inter-particle interactions and various external perturbations. These features provide numerous opportunities to examine the detailed dynamics of particle-rich disks, and can potentially provide insights into other astrophysical disk systems like galaxies and proto-planetary disks.

This chapter provides a heuristic introduction to the dynamics of the known planetary rings. We begin by reviewing the basic architecture of the four known ring systems surrounding the giant planets. Then we turn our attention to the types of dynamical phenomena observed in the various rings. First, we consider how different forces can modify the orbital properties of individual ring particles. Next, we investigate the patterns and textures generated within a ring by the interactions among the ring particles. Finally, we discuss how these two types of processes can interact to produce structures in dense planetary rings.

## 1. Introduction to Planetary Ring Systems

Ring systems surround all four of the giant planets in the outer solar system. While all these rings consist of many small particles orbiting their respective planets, the known rings exhibit a wide diversity of structures, and occupy a broad range of dynamical environments. Furthermore, different rings exhibit patterns and features generated by such diverse processes as inter-particle interactions, gravitational perturbations from various satellites, and a number of non-gravitational forces. Hence, before we consider

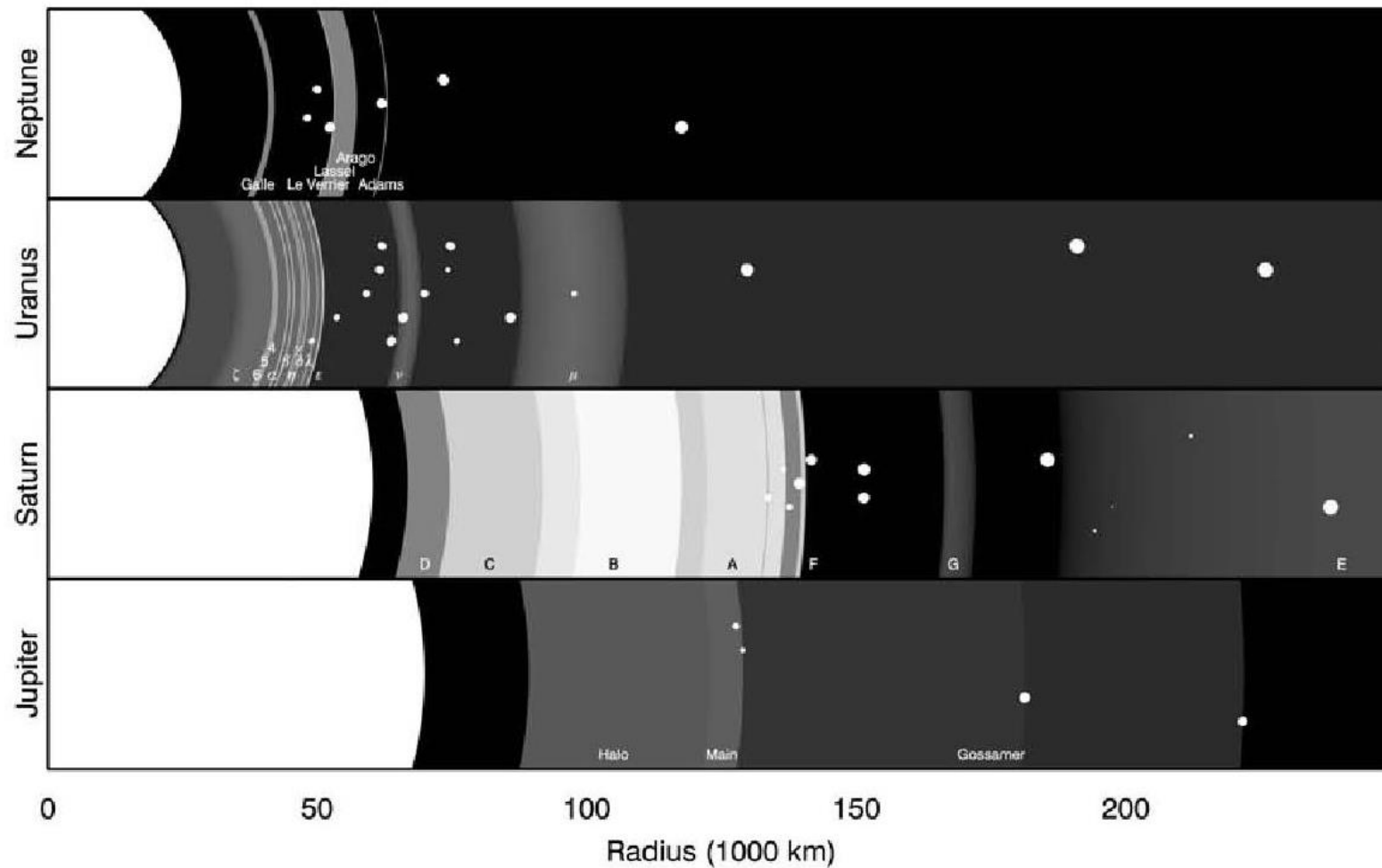


Figure 1. The ring systems of the giant planets, shown to scale. Each grey-scale level corresponds to a decade in ring optical depth. The small white dots correspond to the various moons, with the size of the dots being proportional to the logarithm of their true size. None of these moons are shown to scale with the rings.

the dynamical phenomena operating in various rings, it is useful to briefly review the properties of the known planetary rings.

As shown in Figure 1, the rings of the giant planets vary dramatically in their structure and their opacity. Opacity is a particularly useful parameter for describing and categorizing rings because it is correlated with such fundamental ring parameters as surface mass density, and because it can be directly measured by observing the amount of light transmitted through or scattered by the rings. Ring opacity is typically quantified using either a transmission coefficient  $T$  or an optical depth  $\tau = -\ln(T)$ . Such parameters depend on the exact path the light takes through the rings, but they can be used to estimate a viewer-independent parameter called the normal optical depth; the optical depth of the ring observed when the light passes perpendicularly through the rings. The normal optical depth of the known planetary rings ranges over more than eight orders of magnitude.

The most extensive and complex ring system belongs to Saturn. Furthermore, thanks to the vast amount of data returned by the Cassini spacecraft, Saturn's ring system is now the best-studied in the outer solar system. The most familiar of Saturn's rings are the so-called "Main Rings", which include the A, B and C rings, as well as the Cassini Division between the A and B rings. These are all broad rings with substantial optical depths ( $\tau$  ranging from around 0.1 to over 5) composed of millimeter-to-meter-sized ice-rich particles. Their high opacities and reflectivities make these rings the easiest ones to see with Earth-based telescopes. Still, it is important to realize that none of these rings are completely homogeneous. Instead, they possess structures on a wide range of scales, and there are even several nearly empty gaps in the A ring, the C ring, and the Cassini Division. Some of these patterns can be attributed to particle-particle interactions or various gravitational perturbations from Saturn's various moons, and therefore provide illustrative examples of the dynamical phenomena that can operate in such dense rings. However, the processes responsible for producing many other structures are still not well understood. Detailed reviews of Saturn's main rings are provided by Colwell *et al.* (2009) and Cuzzi *et al.* (2009).

In addition to these main rings, Saturn also possesses a diverse suite of (mostly) fainter rings, including the D E, F and G rings, some narrow ringlets occupying gaps in the main rings, material in the orbits of various small moons, and the enormous but tenuous disk of debris extending between the orbits of Phoebe and Iapetus. The normal optical depth of these rings ranges from 0.1 for the core of the F ring, to about  $10^{-3}$  for parts of the D ring, to  $10^{-6}$  for both the E and G rings, to as low as  $10^{-8}$  for the extensive Phoebe ring (Horanyi *et al.* 2009, Verbiscer *et al.* 2009). Unlike the dense main rings, the visible appearance of these fainter rings is dominated by dust-sized particles less than 100 microns across. Such small particles are especially sensitive to non-gravitational forces, and so the dynamics of these dusty rings are quite different from those of Saturn's main rings. Horanyi *et al.* (2009) provides a recent review of Saturn's dusty rings.

After Saturn, Uranus possesses the most substantial ring system, which is dominated by an array of dense, narrow rings. These rings are designated using either numbers (the 4, 5 and 6 rings) or Greek letters (such as the  $\alpha, \beta, \gamma, \delta, \epsilon$  and  $\eta$  rings). Most of these

rings have optical depths between 0.1 and 1.0 and are between 1 and 10 km wide. The exception is the  $\epsilon$  ring, whose width ranges between 20 and 100 km, and whose optical depth ranges between 0.5 and 2.5. These narrow, dense rings are surrounded by dusty material that has numerous fine-scale structures, including some narrow dusty ringlets like the  $\lambda$  ring. A sheet of dusty material, known as the  $\zeta$  ring, extends inwards of the dense rings, and two diffuse dusty rings, called the  $\mu$  and  $\nu$  rings, have been recently discovered further from the planet (de Pater *et al.* 2006a, b). French *et al.* (1991) and Esposito *et al.* (1991) provide the most recent comprehensive discussions of Uranus' rings, and while the data are still rather limited, several interesting dynamical phenomena have been observed in this system. For example, many of the narrow rings exhibit variations in their widths and radial locations that are due to a combination of perturbations from nearby moons and excited normal modes.

Most of Neptune's dusty rings are rather tenuous, with optical depths around  $10^{-3}$ . The exception is the Adams ring, which contains a series of longitudinally confined arcs with optical depths as high as 0.1. These arcs are of special interest here because they may be confined by a co-rotation resonance with Neptune's small moon Galatea. Porco *et al.* (1995) provide a detailed review of Neptune's ring system.

Jupiter's ring system is the most tenuous of all, and appears to be formed primarily of fine debris knocked loose from Jupiter's small moons (see Burns *et al.* 2004 for detailed discussions of this system). The so-called Main ring extends interior to the small moons Metis and Adrastea, which are likely to be important sources for the ring material. At the inner edge of the Main ring is the Ring Halo, a vertically extended distribution of debris that likely represents particles whose inclinations have been excited by their interactions with Jupiter's magnetic field. Exterior to the main ring are two extremely faint Gossamer Rings, which appear to be composed of debris knocked off from Amalthea and Thebe. Both these moons are on inclined orbits, and the vertical extent of these structures is consistent with those inclinations.

The structure and dynamics of these planetary rings are due to a diverse array of processes, including interactions among the particles within the rings and external perturbations on the orbits of individual ring particles. Furthermore, many features in the ring actually reflect interactions among multiple dynamical processes. For example, the density waves in Saturn's Main Rings are generated by resonant gravitational perturbations from Saturn's moons, but their propagation through the rings is controlled by collisions and gravitational interactions among the ring particles. Fortunately, there are also features in planetary rings that more clearly document individual dynamical phenomena. In particular, various patterns in the low optical depth rings allow us to demonstrate how external forces can perturb the orbits of individual ring particles, while the small-scale textures of the main rings provide insights into how ring particles interact with each other.

## 2. Orbital Perturbations on Individual Ring Particles

Here we consider how the orbit of an individual ring particle can be perturbed by outside forces. After reviewing the nomenclature for the orbital parameters that will be used in this chapter, we present the generic perturbation equations for these parameters.

We will then use these equations to quantify how the ring particles should respond to steady and time-variable (i.e. resonant) perturbations.

## 2.1. Orbital Elements

In the following discussions of ring dynamics, we will use both physical coordinates of ring features and orbital parameters of the component ring particles. For a planetary ring, a cylindrical coordinate system is most natural, so we will designate the location of ring features by their radius  $r$ , vertical offset  $z$ , and longitude  $\lambda$ . Note that  $\lambda$  is measured relative to a fixed direction in inertial space, so orbiting particles cycle through all possible values of  $\lambda$  once each orbit.

While cylindrical coordinates are a natural basis for describing ring features, the dynamics of these systems are best described in terms of the six classical orbital elements: the semi-major axis  $a$ , the eccentricity  $e$ , the orbital inclination  $i$ , the longitude of ascending node  $\Omega$ , the argument of pericenter  $\omega$  and the particles true anomaly  $f$  (Figure 2). The inclination of the ring particles will be measured from the planet's equatorial plane, and  $\Omega$  is measured from the same inertial direction used to define  $\lambda$ .

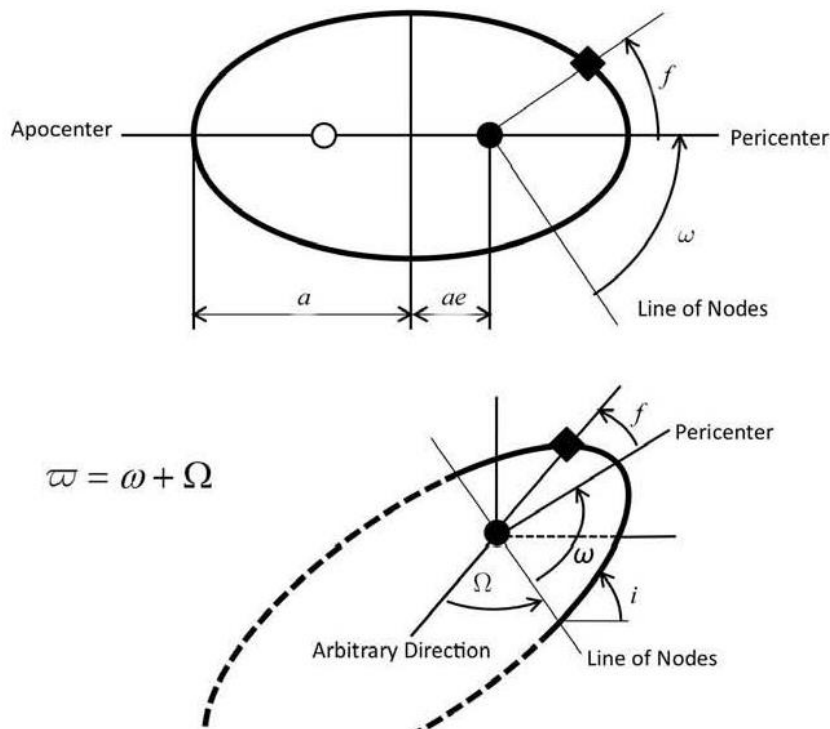


Figure 2. The orbital element nomenclature used in this chapter.

Most planetary rings are on very low inclination orbits in this coordinate system, so the longitude of pericenter  $\varpi = \Omega + \omega$  is often a more useful parameter than either  $\Omega$  or  $\omega$ . In terms of this parameter, the longitude of a ring particle can be written as  $\lambda \approx \Omega + \omega + f = \varpi + f$ . Many planetary rings also consist of particles on low-eccentricity orbits, so in the absence of other perturbations, these particles will move

around the planet at a rate  $n$  that is approximately equal to  $(GM_p/a^3)^{1/2}$  where  $G$  is the universal gravitational constant,  $M_p$  is the planet's mass and  $a$  is the orbital semi-major axis.

## 2.2. Perturbation Equations

In addition to the central force from the planet's gravity, ring particles feel various (small) perturbing forces. In general, such a force can be written as:  $\vec{F} = F_r \hat{r} + F_\lambda \hat{\lambda} + F_z \hat{z}$ , where  $\hat{r}$  is a unit vector pointing in the radial direction,  $\hat{\lambda}$  is a unit vector pointing in the azimuthal direction, and  $\hat{z}$  points normal to the orbit plane. Including such a perturbing force in the appropriate equations of motion yields a series of perturbation equations, which specify how the particles orbital elements should change over time in response to this force. A heuristic derivation of these equations can be found in Burns (1976), and we will not attempt to derive these equations here. Instead, we will simply present the equations, illustrate how they can be simplified for ring particles on nearly circular, low-inclination orbits, and briefly discuss why the resulting expressions are intuitively sensible.

First, let us consider the perturbation equation for the particles semi-major axis:

$$\frac{da}{dt} = \frac{2an}{(1-e^2)^{1/2}} \left[ \frac{F_r}{F_G} e \sin f + \frac{F_\lambda}{F_G} (1 + e \cos f) \right] \quad (1)$$

where  $F_G = GM_p m/a^2$  is the central force from the planet. The dominant term in this expression is proportional to  $F_\lambda/F_G$ . This makes sense, since a steady force applied along the direction of motion will cause the particle to accelerate, gain energy, and move away from the planet. The other terms involve  $e$ , and therefore will be small corrections for most ring particles. However, these terms can still be important in situations where the average azimuthal force applied to the particle over the course of one orbit is zero (e.g. the direction of the force is fixed in inertial space). Thus, even in the limit of a nearly circular orbit ( $e \ll 1$ ), this equation can only be slightly simplified by approximating  $(1-e^2)^{1/2}$  as simply unity:

$$\frac{da}{dt} = 2an \left[ \frac{F_r}{F_G} e \sin f + \frac{F_\lambda}{F_G} (1 + e \cos f) \right], \quad (2)$$

Next, consider the perturbation for the particles eccentricity. The general expression is:

$$\frac{de}{dt} = n(1-e^2)^{1/2} \left[ \frac{F_r}{F_G} \sin f + \frac{F_\lambda}{F_G} (\cos f + \cos \epsilon) \right], \quad (3)$$

where  $\epsilon$  is the eccentric anomaly, a quantity that is approximately equal to the true anomaly  $f$  for particles on nearly circular orbits considered here. Thus, for typical ring particles we may approximate the above expression as:

$$\frac{de}{dt} = n \left[ \frac{F_r}{F_G} \sin f + 2 \frac{F_\lambda}{F_G} \cos f \right], \quad (4)$$

In order to verify that this expression is sensible, consider the following: A force that accelerates a particle along the direction of motion near pericenter (i.e.  $F_\lambda > 0$  when  $f = 0$ ) causes the eccentricity to grow, while a similar force applied near apocenter (i.e.  $F_\lambda > 0$  when  $f = 180^\circ$ ) causes the eccentricity to shrink. Both of these results are consistent with standard orbital dynamics.

The perturbation equation for the orbital inclination is:

$$\frac{di}{dt} = n (1 - e^2)^{1/2} \frac{F_z}{F_G} \frac{\cos(\omega + f)}{1 + e \cos f}, \quad (5)$$

or, in the limit of nearly circular, low-inclination prograde orbits:

$$\frac{di}{dt} = n \left[ \frac{F_z}{F_G} \cos(\lambda - \Omega) \right], \quad (6)$$

where we have used the approximate relationship  $\lambda \simeq f + \omega + \Omega$ , which is valid for low inclinations. Again, we can verify this equation gives sensible results in various simple situations. For example, a vertical force  $F_z > 0$  applied when the particle is near its ascending node ( $\lambda \simeq \Omega$ ) and thus already moving northwards, will cause the inclination to grow, while the same force applied near the descending node ( $\lambda \simeq \Omega + 180^\circ$ ) will cause the inclination to shrink.

The perturbation equation for the longitude of ascending node is:

$$\frac{d\Omega}{dt} = \frac{n(1 - e^2)^{1/2}}{\sin i} \left[ \frac{F_z}{F_G} \frac{\sin(\omega + f)}{1 + e \cos f} \right], \quad (7)$$

For nearly circular, low inclination, prograde orbits, this expression becomes:

$$\frac{d\Omega}{dt} = \frac{n}{\sin i} \left[ \frac{F_z}{F_G} \sin(\lambda - \Omega) \right], \quad (8)$$

which is a plausible complement to the above perturbation equation for the inclination. For example, a positive vertical force applied when the particle is near its maximum vertical excursion ( $\lambda \approx \Omega + 90^\circ$ ) will delay the particles' return to the ring-plane and thus move the node longitude forward around the planet.

Finally, we have the following expression for the argument of pericenter:

$$\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} = \frac{n(1-e^2)^{1/2}}{e} \left[ -\frac{F_r}{F_G} \cos f + \frac{F_\lambda}{F_G} \sin f \frac{(2+e \cos f)}{(1+e \cos f)} \right] \quad (9)$$

For orbits with small  $e$  and  $i$ , this equation can be transformed into a simpler perturbation equation for the longitude of pericenter  $\varpi = \omega + \Omega$ :

$$\frac{d\varpi}{dt} = \frac{n}{e} \left[ -\frac{F_r}{F_G} \cos f + 2 \frac{F_\lambda}{F_G} \sin f \right] \quad (10)$$

This is a reasonable complement to the perturbation equation for the orbital eccentricity. For example, an outward radial force applied near apocenter ( $f \approx 180^\circ$ ) will delay the particles' inward motion and thus cause the pericenter to shift forward in longitude.

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### **Biographical Sketch**

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