Homework Allowance

Principles of Economics

https://jiamingmao.github.io/principles-of-economics

Each person has 1 homework allowance that allows him or her to have the score of 1 homework assignment not count toward his or her final grade.

How it works

Let J denote non-exam score.

$$J = \frac{\sum_{i=1}^{N} h_i + \text{extra credits}}{\sum_{i=1}^{N} \mathcal{H}_i} \times 100$$

, where P is the score of your final project and H is the overall score of your homework assignments:

$$H = 100 \times \sum_{i=1}^{N} h_i \Big/ \sum_{i=1}^{N} \mathcal{H}_i$$

, where N is the total number of homework assignments, h_i is how much you scored on homework i and $\mathcal{H}_i$ is the total points of homework i[1].

If you use your allowance on homework k, then

$$J = \frac{\sum_{i \neq k} h_i + \text{extra credits}}{\sum_{i \neq k} \mathcal{H}_i} \times 100$$

, i.e. h_k will not count toward H.

[1] For example, if homework 1 has 30 points and you scored 20, then $h_1 = 20$, $\mathcal{H}_1 = 30$.

Trading of Homework Allowances

We allow homework allowances to be traded. Students can buy and sell allowances using their non-exam scores as currency – which we call "J-points," or simply, "J." For example, A can buy 1 allowance from B for $10J$. This means that 10 points will be deducted from $J(A)$ and added to $J(B)$ in exchange for 1 allowance.

Allowance trading can benefit both parties. Here is an example: to simply matters, let's assume there are three homework assignments and no extra credit opportunities. Each assignment has 100 points. Two students, A and B, have the following scores: A: 20, 50, 80; B: 80, 90, 100. Thus, without using any allowances, $J(A) = 50$, $J(B) = 90$. By using 1 allowance, $J(A) = 65$, $J(B) = 95$[2]. Now, A can buy 1 allowance from B at a price of $10J$, so that A will have 2 allowances and B will have no allowances. Then after using their allowances, $J(A) = 80 - 10 = 70$ and $J(B) = 90 + 10 = 100$. Both A and B benefit from the trade in this case.

Allowance Derivatives

An option to buy

You can trade not only allowances, but derivatives based on these allowances. For example, let us again look at our hypothetical class with three homework assignments. After the first homework, A gets a score of 20 and B gets a score of 80. A is worried: "This course is difficult. Do I need more allowances? If so, should I buy more allowance now, or wait till the end of the semester? – if I buy now, what if my future homework scores become much better and I don't need them anymore? If I buy at the end of the semester, what if a lot of students want to buy then and the price becomes too high?" In this case, what A could do is to purchase a **call option** from B, which is an option that gives A the *right* to buy an allowance from B at a given price at or before the end of the semester. For example, B could agree to sell A the following call option:

[2] Assuming A and B use their allowances optimally ("rationally").

Example (Call option)**.** The right to buy 1 homework allowance from B for $10J$ at the end of the semester.

- Price: $5J$
- Payoff:
  - ▷ 1 allowance $-15J$, if A chooses to buy at the end of the semester
  - ▷ $-5J$ if A chooses not to buy

In this case, if A's homework scores at the end of the semester turn out to be 20, 50, 80, then it benefits for him to buy the allowance from B, which will allow him to raise his average score to 80 after using two allowances. After adjusting for the prices he paid for the option as well as the allowance, $J(A) = 80 - 15 = 65$[3]. On the other hand, if A's homework scores turn out to be 20, 80, 80, then it doesn't benefit him to exercise the option. In this case, after using his own allowance and paying the option price, $J(A) = 80 - 5 = 75$[4].

An option to sell

After the first homework, B gets a score of 80. Just like A is worried about the need for more allowances, B is worried that she might not need the allowance in the end: "This course looks easy. I may not need the allowance by the end of the semester. But what if other students also do well and there is no demand for allowances at the end of the semester? Shall I sell my allowance now if somebody wants it? (I heard A wants one). But if I sell it now, what if I do not do well in one of the future assignments and I do need it at the end of the semester?" In this case, what B could do is to purchase a **put option** from A, which would give B the *right* to sell A her allowance at a given price at or before the end of the semester. For example, B may purcahse the following put option from A:

[3] B gives her allowance to A and gets $15J$ in this case.
[4] B gets $5J$ in this case, while keeping her allowance.

Example (Put option)**.** The right to sell 1 homework allowance to A for $10J$ at the end of the semester.

- Price: $5J$
- Payoff:
  - ▷ -1 allowance $+5J$, if B chooses to sell at the end of the semester
  - ▷ $-5J$ if B chooses not to sell

In this case, if B's homework scores at the end of the semester turn out to be 80, 90, 100, then it benefits for her to sell the allowance to A, which will give her $J(B) = 90 + 10 - 5 = 95$[5]. On the other hand, if B's homework scores turn out to be 80, 70, 60, then it doesn't benefit her to exercise the option. In this case, after using her own allowance and paying the option price, $J(B) = 75 - 5 = 70$[6].

Protecting Your Investment

One problem in buying these derivatives is that you do not know whether the other person (called "the counterparty") is able to deliver when you exercise your option. For example, if you have purchased a call option from B and you decide to use the option at the end of the semester to buy an allowance from B, what if B has already sold her allowance to someone else and cannot sell you any allowance anymore? What if B acts like AIG and sells a bunch of options to many people and in the end cannot deliver allowances to anyone? One way to protect yourself from this possibility is to amend your contract. For example, you can add the following condition into your contract with B:

> In the event that A decides to exercise this option and B fails to deliver the allowance to A at the promised price, B agrees to give A $10J$ instead.

You can also protect the call option you bought with B by purchasing a **credit default swap** (CDS) on it from someone else. A credit default swap pays you in the event that B defaults. For example, you may purchase the following CDS from another student C:

[5] A gets 1 allowance and pays B $5J$ in this case.
[6] A gets $5J$ in this case.

Example (CDS)**.** In the event that A decides to exercise this option and B fails to deliver the allowance to A at the promised price, C would pay A $10J$.

- Price: $2J$
- Payoff:
  - ▷ $8J$ if B defaults
  - ▷ $-2J$ if B does not default

As you can see, the CDS works pretty much like an insurance. You can use it to protect your investment, or you can sell it to offer insurance to someone else.