$$V = [V_1, V_2, V_3]^T \in R^3$$

$\mathbb{R}^n$ vector space consisting of all n-dimensional real vectors.

zero ; +, −, negative. $\in \mathbb{R}^n$.

vector space with matrix $m \times n$ dimension.

vector space with polynomials: degree $\le n$ dimension $n+1$

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n.$$

$p(x) = 1 + 2x + 3x^2$ $q(x) = 2 + 3x + 4x^2$

$p(x) + q(x) = 3 + 5x + 7x^2$

degree ≤ 2

zero polynomial not form space, but zero vector is

linear independence.

$$c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$$

$$c_1 = c_2 = \cdots = c_n = 0.$$

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} C_1 + C_3 = 0 \\ C_2 + C_3 = 0 \end{cases} \qquad \begin{aligned} C_1 &= -C_3 \\ C_2 &= -C_3. \end{aligned}$$

$C_1 = C_2 = 1 \quad C_3 = -1$

linear dependent

Solving linear system.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$$\begin{cases} x_1 + 2x_2 = 5 \\ 3x_1 + 4x_2 = 11 \end{cases}$$

3×3 $\quad A = \begin{bmatrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ -11 \\ -3 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 2 & 1 & 5 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \end{array}\right] \qquad -3 + 2 \times \frac{3}{2} = 0$$

$-x_3 = 1 \quad x_3 = -1$

$\frac{1}{2}x_2 - \frac{1}{2} = 1 \quad x_2 = 3$

$2x_1 + 3 + 1 = 8 \quad x_1 = 2$

$$X = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

Finding eigenvalues / vectors.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

λ $$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)^2 - 1 = 0$$

$$\lambda_1 = 3, \ \lambda_2 = 1$$

$\lambda_1 = 3$ $$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda_2 = 1$ $$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

-1 $\quad$ 1

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$c_1 + 3c_2 = 2$$
$$2c_1 + 6c_2 = 4$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 4 & -6 & 0 & 2 \\ -2 & 7 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -8 & -2 & 0 \\ 0 & 8 & 3 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -8 & -2 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right] \qquad \begin{aligned} x_3 &= 4 \\ x_2 &= -1 \\ x_1 &= -1 \end{aligned}$$

forward elimination $\longrightarrow$ backward substitution

Gauss Transformation.

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 4 & 4 & 3 \\ 2 & 3 & 4 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}$$

$$a_{11} = 2 \qquad \left(\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 2 & 3 & 4 & 7 \end{array}\right) \quad \left(\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 3 & 6 \end{array}\right)$$

Constructing the Gauss transformation Matrix M_1

① $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

② add eliminator. $-\dfrac{a_{21}}{a_{11}} = -\dfrac{4}{2} = -2$

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

. . .

③ $M_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 4 & 4 & 3 \\ 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 2 & 3 & 4 \end{pmatrix}$

Second.

$\frac{a_{32}}{a_{22}} = -\frac{3}{2}$ $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 2 & 3 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$-\frac{a_{21}}{a_{11}} \quad \frac{a_{32}}{a_{22}} \quad M_2 \cdot (M_1 \cdot A)$

Diagonal matrix. $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ only diagonal entries $(5, 3, -2)$

det (Diagonal matrix) $= 5 \times 3 \times (-2) = -30$

upper triangular: $U = \begin{pmatrix} 4 & 2 & 1 \\ 0 & -3 & 5 \\ 0 & 0 & 6 \end{pmatrix}$

det (Diagonal matrix) $= 4 \times (-3) \times 6 = -72$

$U^{-1} \to$ upper triangular.

same as lower triangular.

orthogonal matrix. $Q^T Q = Q Q^T = I$.

$\det(\text{orthogonal matrix}) = \pm 1$

$Q^T = Q^{-1}$ $\quad\quad \|Qx\| = \|x\|$

The rows of Q are otho, $Q_i \cdot Q_j = 0$ if $i \neq j$

$Q_i \cdot Q_j = 1$

The column of Q are otho, $Q_i^T \cdot Q_j = 0$ if $i \neq j$

$Q_i^T \cdot Q_j = 1$

LU Decomposition.

$$A = \begin{pmatrix} 4 & 3 \\ 6 & 3 \end{pmatrix}$$

$$A = LU$$

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \quad\quad U = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 0 & u_{22} \end{pmatrix}$$

$$l_{21} = \frac{A_{21}}{u_{11}} = \frac{6}{4} = 1.5$$

$$L = \begin{pmatrix} 1 & 0 \\ 1.5 & 1 \end{pmatrix}$$

$$u_{22} = A_{22} - l_{21} \cdot u_{12} = 3 - (1.5 \times 3) = -1.5$$

$$U = \begin{pmatrix} 4 & 3 \\ 0 & -1.5 \end{pmatrix}$$