1

$f_1, \ldots, f_n$ – funkcje liniowo niezależne

$g_1, \ldots, g_n$ – funkcje liniowo niezależne, t.ż. $\langle g_i,$ [illegible]

$$\langle f, g \rangle = \sum_{k=0}^{N} f(x_k)\, g(x_k)$$

$$\begin{cases} g_0 = f_0 \\ g_k = f_k - \sum_{i=0}^{k-1} \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \cdot g_i \end{cases}$$

Ortogonalizacja Grama – Schmidta

D-d indukcyjny (względem k)

Podstawa indukcji:

$k = 0 \quad g_0 = f_0$

$k = 1$:

$$g_1 = f_1 - \frac{\langle f_1, g_0 \rangle}{\langle g_0, g_0 \rangle} \cdot g_0 \qquad \langle g_0, g_1 \rangle \stackrel{?}{=} 0$$

$$\langle g_0, g_1 \rangle = \left\langle g_0, f_1 - \frac{\langle f_1, g_0 \rangle}{\langle g_0, g_0 \rangle} \cdot g_0 \right\rangle = \langle g_0, f_1 \rangle - \left\langle g_0, \frac{\langle f_1, g_0 \rangle}{\langle g_0, g_0 \rangle} \cdot g_0 \right\rangle =$$

$$= \langle g_0, f_1 \rangle - \frac{\langle f_1, g_0 \rangle}{\cancel{\langle g_0, g_0 \rangle}} \cdot \cancel{\langle g_0, g_0 \rangle} = \langle g_0, f_1 \rangle - \langle g_0, f_1 \rangle = 0$$

Krok indukcyjny :

Załóżmy, że $g_0, \dots, g_{k-1}$ ortogonalne

Pokażmy, że g_k ortogonalne z g_i, $i < k$ $\quad \langle g_k, g_i \rangle$

$$g_k = f_k - \sum_{i=0}^{k-1} \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \cdot g_i$$

$$\langle g_i, g_k \rangle = \langle g_i, f_k - \sum_{i=0}^{k-1} \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \cdot g_i \rangle =$$

$$= \langle g_i, f_k \rangle - \langle g_i, \sum_{i=0}^{k-1} \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \cdot g_i \rangle =$$

$$= \langle g_i, f_k \rangle - \sum_{i=0}^{k-1} \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \langle g_i, g_i \rangle =$$

z zał. ind. $\langle g_i, g_j \rangle = 0$ dla $i \neq j$ i $i, j < k$

$$= \langle f_k, g_i \rangle - \frac{\langle f_k, g_i \rangle}{\langle g_i, g_i \rangle} \langle g_i, g_i \rangle =$$

$$= \langle f_k, g_i \rangle - \langle f_k, g_i \rangle = 0$$

ckd