8 PIOTR GUNIA

a) $n^2 \overset{?}{\in} O(n^3)$ Tak, z wykładu $\forall_{\alpha,\beta}\ \alpha \leq \beta \quad n^\alpha \in O(n^\beta)$

b) $n^3 \overset{?}{\in} O(n^{2,99})$ Nie

$$\lim_{n\to\infty} \frac{n^3}{n^{2,99}} = \lim_{n\to\infty} n^{\frac{1}{100}} = \infty$$

c) $2^{n+1} \overset{?}{\in} O(2^n)$ Tak, weźmy $c=2$, $n_0 = 1$

wtedy $2^{n+1} \leq 2 \cdot 2^n = 2^{n+1}$ dla każdego $n \geq n_0$

d) $(n+1)! \overset{?}{\in} O(n!)$ Nie

$$\lim_{n\to\infty} \frac{(n+1)!}{n!} = \lim_{n\to\infty} \frac{n!(n+1)}{n!} = \lim_{n\to\infty} n+1 = \infty$$

e) $\log_2 n \in O(\sqrt{n})$ TAK

$\forall_{\alpha > 0}\ (\ln n)^c = O(n^\alpha)$

$$\log_2 n = \cancel{\frac{\log x}{\log 2}}\ \frac{\ln n}{\ln 2} = \ln n \cdot \underbrace{\frac{1}{\ln 2}}_{= \text{const.}}$$

z wykładu $\ln n = O(n^\alpha)$

f) $\sqrt{n} \in O(\log_2 n)$ Nie

$$\lim_{n\to\infty} \frac{\overset{\to\infty}{\sqrt{n}}}{\underset{\to\infty}{\log_2 n}} \overset{H}{=} \lim_{n\to\infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n\ln 2}} = \lim_{n\to\infty} \frac{n \ln 2}{2\sqrt{n}} = \lim_{n\to\infty} \frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}} \frac{\sqrt{n}\ln 2}{2} = \infty$$