1

$x \in [x_{k-1}, x_k] \quad 1 \leq k \leq n$

$$s(x) = h_k^{-1}\left[\frac{1}{6}M_{k-1}(x_k - x)^3 + \frac{1}{6}M_k(x - x_{k-1})^3 + \left(y_{k-1} - \frac{1}{6}M_{k-1}h_k^2\right)(x_k + x) + \right.$$
$$\left. + \left(y_k - \frac{1}{6}M_k h_k^2\right)(x - x_{k-1})\right]$$

$$\lambda_k M_{k-1} + 2M_k + (1-\lambda_k)M_{k+1} = 6 f[x_{k-1}, x_k, x_{k+1}]$$

$$h_k = x_k - x_{k-1}, \quad \lambda_k = \frac{h_k}{h_k + h_{k+1}}$$

a) $M_0 = M_2 = 0$

0	-8		
2	8	8	
4	-8	-8	-4

$2M_1 = 6(-4) \qquad h_1 = h_2 = 2$

$M_1 = -12$

$k=1, \; x \in [0,2]$

$$s(x) = \frac{1}{2}\left[\frac{1}{6}(-12)(x-0)^3 + (-8)(2+x) + \left(8 - \frac{1}{6}(-12)2^2\right)(x-0)\right] =$$

$$= \frac{1}{2}\left[-2x^3 - 8(2+x) + 16x\right] = -x^3 - 4(2+x) + 8x = -x^3 + 4x - 8$$

$k=2, \; x \in [2,4]$

$$s(x) = \frac{1}{2}\left[\frac{1}{6}(-12)(4-x)^3 + \left(8 - \frac{1}{6}(-12)2^2\right)(4-x) + \left((-8) - \frac{1}{6}\cdot 0\right)(x-2)\right] =$$

$$= \frac{1}{2}\left[-2(4-x)^3 + 16(4-x) - 8(x-2)\right] = -(4-x)^3 + 8(4-x) - 4(x-2) =$$

$$= (x-4)^3 - 8(x-4) - 4(x-2)$$

b)

-1	4		
$-\frac{1}{2}$	2	-4	
$\frac{1}{2}$	-6	-8	$-\frac{8}{3}$
1	-24	-36.	$-\frac{56}{3}$

$h_1 = h_3 = \frac{1}{2}$

$h_2 = 1$

$M_0 = M_3 = 0$

$\lambda_1 = \frac{1}{3}$

$\lambda_2 = \frac{2}{3}$

$$\begin{cases} 2M_1 + (1-\lambda_1)M_2 = 6 \cdot \left(-\frac{8}{3}\right) \\ \lambda_2 M_1 + 2M_2 = 6 \cdot \left(-\frac{56}{3}\right) \end{cases}$$

$$\begin{cases} 2M_1 + \frac{2}{3}M_2 = -16 \\ \frac{2}{3}M_1 + 2M_2 = -112 \end{cases}$$

$M_1 = 12$

$M_2 = -60$

$k=1 \quad x \in \left[-1, -\frac{1}{2}\right]$

$$s(x) = 2\left[\frac{1}{6}12\,(x+1)^3 + (4)\left(-\frac{1}{2}-x\right) + \left(2 - \frac{1}{6}12\frac{1}{4}\right)(x+1)\right] =$$

$$= 2\left[2(x+1)^3 + 4\left(-x-\frac{1}{2}\right) + \frac{3}{2}(x+1)\right] = 4(x+1)^3 + 8\left(-x-\frac{1}{2}\right) + 3(x+1)$$

$k=2 \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$s(x) = \frac{1}{6}12\left(-\frac{1}{2}-x\right)^3 + \frac{1}{6}(-60)\left(x+\frac{1}{2}\right)^3 + \left(2 - \frac{1}{6}12\right)\left(\frac{1}{2}+x\right) + \left(-6 - \frac{1}{6}(-60)\right)\left(x+\frac{1}{2}\right) =$$

$$= 2\left(-\frac{1}{2}-x\right)^3 - 10\left(x+\frac{1}{2}\right)^3 + 4\left(x+\frac{1}{2}\right)$$

$k=3 \quad x \in \left[\frac{1}{2}, 1\right]$

$$s(x) = 2\left[\frac{1}{6}(-60)(1-x)^3 + \left(-6 - \frac{1}{6}(-60)\frac{1}{4}\right)(1+x) + \left(-24 - \frac{1}{6}\cdot 0 \cdot \frac{1}{4}\right)\left(x-\frac{1}{2}\right)\right] =$$

$$= 2\left[-10(1-x)^3 + \frac{7}{2}(1+x) - 24\left(x-\frac{1}{2}\right)\right] = -20(1-x)^3 - 7(1+x) - 48\left(x-\frac{1}{2}\right)$$

2.

$$f(x)=\begin{cases} x^3+6x^2+18x+13 & ; -2\leq x\leq -1 \quad g_1(x)\\ -5x^3-12x^2+7 & ; -1\leq x\leq 0 \quad g_2(x)\\ 5x^3-12x^2+7 & ; 0\leq x\leq 1 \quad g_3(x)\\ -x^3+6x^2-18x+13 & ; 1\leq x\leq 2 \quad g_4(x)\end{cases}$$

Warunki

4° $f''(a)=f''(b)=0$

$f''(-2)=6\cdot(-2)+12=0$

$f''(2)=-6\cdot 2+12=0$

$$f'(x)\begin{cases} g_1'(x)=3x^2+12x+18\\ g_2'(x)=-15x^2-24x\\ g_3'(x)=15x^2-24x\\ g_4'(x)=-3x^2+12x-18\end{cases}$$

3° $f, f', f'' \in C[a,b]$

Wielomiany są funkcjami ciągłymi

$$f''(x)\begin{cases} g_1''(x)=6x+12\\ g_2''(x)=-30x-24\\ g_3''(x)=30x-24\\ g_4''(x)=-6x+12\end{cases}$$

$g_1(-1)=-1+6-18+13=0$

$g_2(-1)=5-12+7=0=g_1(-1)$

$g_2(0)=7=g_3(0)$

$g_3(1)=5-12+7=0=-1+6-18+13=g_4(1)$

$g_1'(-1)=3-12+18=9=-15+24=g_2'(-1)$

$g_2'(0)=0=g_3'(0)$

$g_3'(1)=15-24=-9=-3+12-18=g_4'(1)$

$g_1''(-1)=-6+12=6=30-24=g_2''(-1)$

$g_2''(0)=-24=g_3''(0)$

$g_3''(1)=30-24=6=-6+12=g_4''(1)$

1°

$f(-2)=-8+24-36+13=-7$

$f(-1)=0$

$f(0)=7$

$f(1)=0$

$f(2)=-8+24-36+13=-7$

2° $f\Big|_{[x_{k-1},x_k]}\in\Pi_3$; $1\leq k\leq n$

oczywiste, mamy jawne wzory

3. $f(x) = \begin{cases} 2020x & ; -2 \leqslant x \leqslant -1 & g_1(x) \\ ax^3+bx^2+cx+d & -1 \leqslant x \leqslant 1 & g_2(x) \\ -2020x & 1 \leqslant x \leqslant 2 & g_3(x) \end{cases}$

1°
$f(-2) = -4040$

$f(-1) = -2020 = ax^3+bx^2+cx+d = -a+b-c+d$

$f(1) = -2020 = ax^3+bx^2+cx+d = a+b+c+d$

$f(2) = -4040$

2° Zgadza się

3° $f'(x) = \begin{cases} 2020 \\ 3ax^2+2bx+c \\ -2020 \end{cases}$ $\qquad f''(x) = \begin{cases} 0 \\ 6ax+2b \\ 0 \end{cases}$

4° Oczywiste

$\cancel{g_1(-1) = 2020 = ax^3+bx^2+cx+d = g_2(-1}$

$g_1(-1) = -2020 = -a+b+c+d = g_2(-1)$

$g_2(1) = a+b+c+d = -2020 = g_3(1)$

$g_1'(-1) = 2020 = 3a-2b+c = g_2'(-1)$

$g_2'(1) = 3a+2b+c = -2020 = g_3'(1)$

$g_1''(-1) = 0 = -6a+2b = g_2''(-1)$

$g_2''(1) = 6a+2b = 0 = g_3''(1)$

$\begin{cases} -a+b+c+d = -2020 \\ a+b+c+d = -2020 \\ 3a-2b+c = 2020 \\ 3a+2b+c = -2020 \\ -6a+2b = 0 \\ 6a+2b = 0 \end{cases}$

$\left.\begin{matrix} -3a+b=0 \\ 3a+b=0 \end{matrix}\right\} \Rightarrow a=b=0$

$-\begin{cases} -c+d = -2020 \\ c+d = -2020 \end{cases}$

$2c = 0 \Rightarrow c=0 \Rightarrow d=-2020$

$3a+2b+c = 0 = -2020$ sprzeczność

Nie istnieją stałe a, b, c, d żeby funkcja f była NIFS3

4.

$$\lambda_k M_{k-1} + 2M_k + (1-\lambda_k) M_{k+1} = d_k \qquad (k=1,2,\dots,n-1) \qquad (1)$$

$$M_0 = M_n = 0, \quad d_k = 6f[x_{k-1}, x_k, x_{k+1}], \quad \lambda_k = \frac{h_k}{h_k + h_{k+1}}, \quad h_k = x_k - x_{k-1}$$

$$\boxed{\begin{array}{l} q_0 = 0 \\ u_0 = 0 \\ \begin{cases} p_k = \lambda_k q_{k-1} + 2 \\ q_k = (\lambda_k - 1)/p_k \\ u_k = (d_k - \lambda_k u_{k-1})/p_k \end{cases} \end{array}} \qquad (k=1,2,..,n-1) \qquad (2)$$

$$\boxed{\begin{cases} M_{n-1} = u_{n-1} \\ M_k = u_k + q_k M_{k+1} \qquad (k = n-2, n-3, \dots, 2, 1) \end{cases}}$$

D-d

$k=1$

$$\begin{cases} p_1 = \lambda_1 q_0 + 2 = 2 \\ q_1 = (\lambda_1 - 1)/p_1 \\ u_1 = (d_1 - \lambda_1 u_0)/p_1 = \frac{d_1}{p_1} \end{cases} \qquad M_1 = u_1 + q_1 M_2$$

$$(1) \quad \lambda_1 \overset{=0}{M_0} + 2M_1 + (1-\lambda_1) M_2 = d_1 \quad / : p_1 \qquad p_1 = 2$$

$$M_1 + \frac{1-\lambda_1}{p_1} M_2 = \frac{d_1}{p_1}$$

$$M_1 - \frac{\lambda_1 - 1}{p_1} M_2 = \frac{d_1}{p_1}$$

$$M_1 - q_1 M_2 = u_1 \implies M_1 = u_1 + q_1 M_2$$

Załóżmy (2) dla k działa, pokażemy dla $k+1$

(1) $k+1$ $\qquad L \qquad P$

$$\lambda_{k+1} M_k + 2M_{k+1} + (1-\lambda_{k+1}) M_{k+2} = d_{k+1}$$

(2) k

$$M_k = u_k + q_k M_{k+1} \quad /\cdot \lambda_{k+1}$$

$$M_k \lambda_{k+1} = u_k \lambda_{k+1} + q_k M_{k+1} \lambda_{k+1} \quad / \text{ od lewej } -L \text{, od prawej } -P$$

$$\cancel{M_k \lambda_{k+1}} - \cancel{\lambda_{k+1} M_k} - 2M_{k+1} - (1-\lambda_{k+1}) M_{k+2} = u_k \lambda_{k+1} + q_k M_{k+1} \lambda_{k+1} - d_{k+1}$$

$$-2M_{k+1} - q_k M_{k+1} \lambda_{k+1} - (1-\lambda_{k+1}) M_{k+2} = u_k \lambda_{k+1} - d_{k+1}$$

$$2M_{k+1} + q_k M_{k+1} \lambda_{k+1} - (\lambda_{k+1} - 1) M_{k+2} = d_{k+1} - u_k \lambda_{k+1}$$

$$(2 + q_k \lambda_{k+1}) M_{k+1} - (\lambda_{k+1} - 1) M_{k+2} = d_{k+1} - u_k \lambda_{k+1}$$

$$p_{k+1} M_{k+1} - (\lambda_{k+1} - 1) M_{k+2} = d_{k+1} - u_k \lambda_{k+1} \quad /: p_{k+1}$$

$$M_{k+1} - \frac{\lambda_{k+1} - 1}{p_{k+1}} M_{k+2} = \frac{d_{k+1}}{p_{k+1}} - \frac{u_k \lambda_{k+1}}{p_{k+1}}$$

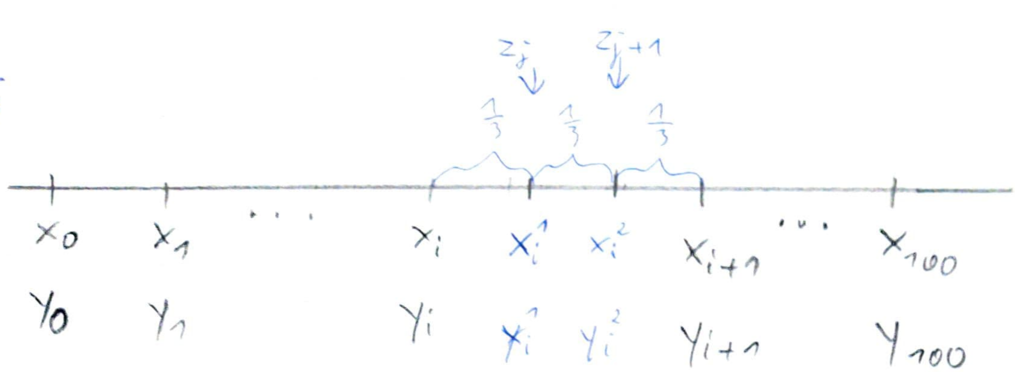
$$M_{k+1} - \frac{\lambda_{k+1} - 1}{p_{k+1}} M_{k+2} = \frac{d_{k+1} - u_k \lambda_{k+1}}{p_{k+1}}$$

$$M_{k+1} - q_{k+1} M_{k+2} = u_{k+1}$$

$$\cancel{M_{k+1} = M_{k+}}$$

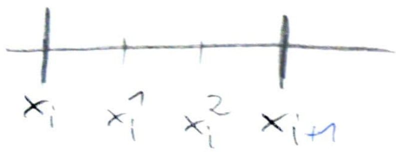
$$\boxed{M_{k+1} = u_{k+1} + q_{k+1} M_{k+2}}$$

5



Do każdego przedziału $[x_i, x_{i+1}]$ $(i=0,1,\dots,99)$ wstawiamy dwa punkty równoodległe. $(z_0, z_1, \dots z_{200})$.

Za pomocą funkcji ~~NIFS3~~ NSpline3 obliczymy wartości w $(z_0, \dots, z_{200})$.



Znamy 4 punkty więc możemy jednoznacznie wyznaczyć wielomian interpolacyjny na przedziale $[x_i ; x_{i+1}]$ trzeciego stopnia.

Znając wielomian 3go stopnia możemy łatwo wyznaczyć jego pochodną, a następnie jej miejsca zerowe.

~~Należy sprawdzić w~~

Dla tych punktów należy sprawdzić czy są one ekstremami lokalnymi licząc ich wartości

To samo należy zrobić dla granic podprzedziałów.