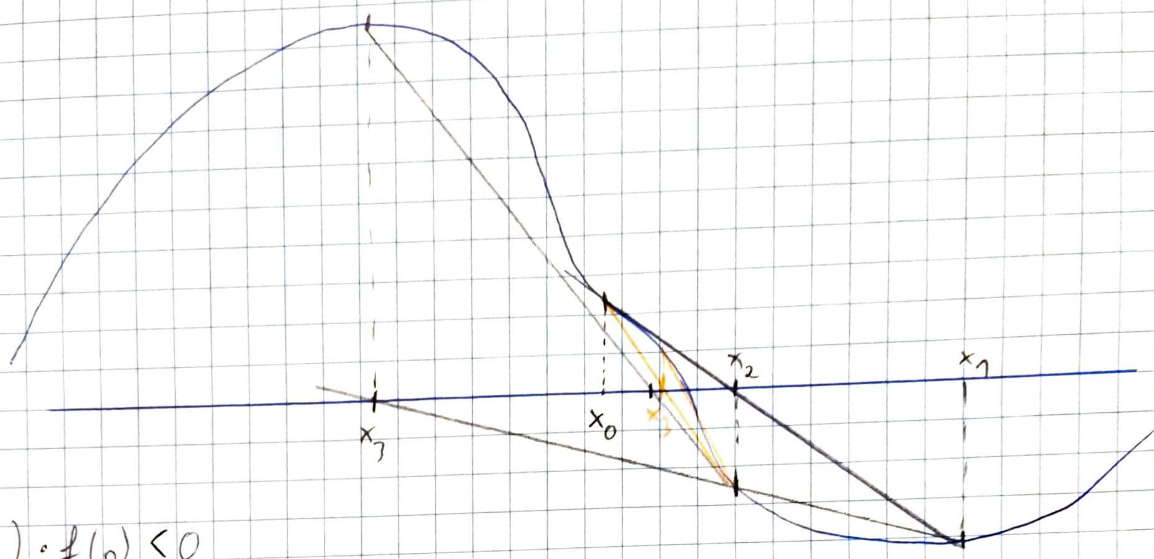
2



$f(l)\cdot f(p) < 0$

$(l_0, p_0) \supset (l_1, p_1) \supset (l_2, p_2) \supset \ldots$

1

$$x_{n+1} = x_n - f_n \frac{x_n - x_{n-1}}{f_n - f_{n-1}} =$$

$$= \frac{x_n (f_n - f_{n-1})}{f_n - f_{n-1}} - \frac{f_n (x_n - x_{n-1})}{f_n - f_{n-1}} =$$

$$= \frac{\cancel{x_n f_n} - x_n f_{n-1} - \cancel{f_n x_n} + x_{n-1} f_n}{f_n - f_{n-1}}$$

$$x_{n+1} = \frac{f_n x_{n-1} - f_{n-1} x_n}{f_n - f_{n-1}}$$

4

$f(\alpha)=0,\ f'(\alpha)\neq 0$

$$F(x) = x - \frac{f(x)}{f'(x)}$$

Aby metoda była zbieżna kwadratowo chcemy pokazać:

$F(\alpha) = \alpha$

$F'(\alpha) = 0$

$F''(\alpha) \neq 0$

$$F(\alpha) = \alpha - \frac{\overset{=0}{f(\alpha)}}{\underset{\neq 0}{f'(\alpha)}} = \alpha$$

$$F'(x) = 1 - \frac{f'(x)\cdot f'(x) - f(x)\cdot f''(x)}{(f'(x))^2} = \cancel{1-1} - \frac{f(x)\,f''(x)}{(f'(x))^2}$$

$$F'(\alpha) = -\frac{\overset{=0}{f(\alpha)}\,f''(\alpha)}{\underset{\neq 0}{(f'(\alpha))^2}} = 0$$

$$F''(x) = -\left(\frac{f(x)f''(x)}{(f'(x))^2}\right)' = \frac{(f(x)f''(x))'\cdot[f'(x)]^2 - \left([f'(x)]^2\right)'\cdot f(x)f''(x)}{(f'(x))^4} =$$

$$= \frac{\left(f'(x)f''(x) + f(x)f'''(x)\right)\cdot(f'(x))^2 - 2f(x)f'(x)f''(x)}{(f'(x))^4} = \cancel{(f'(x))^3 f''(x) + f(x)f'(x)}$$

$$= \frac{(f'(x))^3 f''(x) + f(x)(f'(x))^2 f'''(x) - 2f(x)f'(x)f''(x)}{(f'(x))^5} = \frac{(f'(x))^3 f''(x)}{(f'(x))^5} = \frac{f''(x)}{f'(x)}$$

$$F''(\alpha) = \frac{f''(\alpha)}{\underset{\neq 0}{f'(\alpha)}}$$