1

$$T_n(f) = \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) + \ldots + \frac{h}{2}(f_{n-1} + f_n)$$

$$m_i = \min_{[x_{i-1}, x_i]} f(x) \qquad M_i = \max_{[x_{i-1}, x_i]} f(x)$$

$$h\, m_i \leq \frac{h}{2}(f_{i-1} + f_i) \leq \frac{h}{2}(M_i + M_i) = h\, M_i$$

$$\sum_{i=1}^{n} h\, m_i \leq \sum_{i=1}^{n} \frac{h}{2}(f_{i-1} + f_i) = T_n(f) \leq \sum_{i=1}^{n} h\, M_i \qquad h = \frac{(b-a)}{n}$$

sumy dolne $\qquad n \to \infty \qquad$ sumy górne

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\int_a^b f(x)\,dx \qquad\qquad \int_a^b f(x)\,dx$$

z tw. o trzech ciągach

$$T_n(f) \to \int_a^b f(x)\,dx$$

2.

$$\max_{x \in \mathbb{R}} |f''(x)| < 2 \qquad R_n^T = -\frac{(b-a)^3 f''(\eta)}{12n^2} \qquad \eta \in (a,b), \cancel{f(\eta)} < 2$$

$$|R_n^T| = \left|\frac{(b-a)^3 f''(\eta)}{12n^2}\right| < \left|\frac{(b-a)^3 \cdot 2}{12n^2}\right| = \left|\frac{(b-a)^3}{6n^2}\right| = \frac{(b-a)^3}{6n^2} < \varepsilon$$

$$\Updownarrow$$

$$\sqrt{\frac{(b-a)^3}{6\varepsilon}} = n$$

Algorytm:

1. $n = \sqrt{\frac{(b-a)^3}{6\varepsilon}}$, $h = \left(\frac{b-a}{n}\right)$, $T = 0$
2. $T = \sum_{i=0}^{n}{}'' f(a+ih)$
3. Zwróć T

3

$$\int_{-\frac{\pi}{5}}^{\frac{\pi}{2}} \cos\left(3x - \frac{\pi}{3}\right) dx \quad , \quad R_n^S(f) = \frac{(a-b)\, h^4\, f^{(4)}(\alpha)}{180}$$

$$h = \frac{a-b}{n}$$

$$f^{(4)}(\alpha) = \underbrace{\cos\left(3x - \frac{\pi}{3}\right)}_{\leq 1} \cdot 3^4 \leq 3^4$$

$$R_n^S(f) = \frac{(a-b)^5}{180 n^4} \cdot f^{(4)}(\alpha) \leq \frac{(a-b)^5}{180 n^4} \cdot 3^4 = \frac{\left(\frac{\pi}{2} + \frac{\pi}{5}\right)^5}{180 n^4} \cdot 3^4 =$$

$$= \frac{\left(\frac{7}{10}\pi\right)^5 \cdot 9}{20 n^4} = \frac{7^5}{10^5} \cdot \frac{9 \cdot \pi^5}{20 n^4} \leq 10^{-8}$$

$$\frac{7^5 \cdot 9 \cdot \pi^5 \cdot 10^8}{20 \cdot 10^5} \leq n^4$$

$$\frac{7^5 \cdot 9 \cdot \pi^5 \cdot 10^3}{20} \leq n^4$$

$$n \geq \sqrt[4]{\frac{7^5 \cdot 9 \cdot 10^3 \cdot \pi^5}{20}}$$

$$T_n = \frac{1}{2} h_n f(a + \tfrac{0}{2} h_n) + h_n f(a + \tfrac{2}{2} h_n) + h_n f(a + \tfrac{4}{2} h_n) + \ldots + h_n f(a + \tfrac{2n-2}{2} h_n) + \frac{1}{2} h_n f(a + \tfrac{2n}{2} h_n)$$

$$M_n = h_n f(a + \tfrac{1}{2} h_n) + h_n f(a + \tfrac{3}{2} h_n) + h_n f(a + \tfrac{5}{2} h_n) + \ldots + h_n f(a + \tfrac{2n-1}{2} h_n)$$

$$T_n + M_n = \frac{1}{2} h_n f(a + \tfrac{0}{2} h_n) + h_n f(a + \tfrac{1}{2} h_n) + h_n f(a + \tfrac{2}{2} h_n) + h_n f(a + \tfrac{3}{2} h_n) + \ldots + h_n f(a + \tfrac{2n-1}{2} h_n) + \frac{1}{2} h_n f(a + \tfrac{2n}{2} h_n)$$

$$= h_n \left(\frac{1}{2} f(a) + \frac{1}{2} f(b) + \sum_{i=1}^{2n-1} f(a + \tfrac{i}{2} h_n) \right) =$$

$$= h_n \left(\frac{1}{2} f(a) + \frac{1}{2} f(b) + \sum_{i=1}^{2n-1} f(a + i h_{2n}) \right) = T_n + M_n \quad / \cdot \frac{1}{2}$$

$$\frac{1}{2} \left(T_n(f) + M_n(f) \right) = \frac{1}{2} h_n \left(\frac{1}{2} f(a) + \frac{1}{2} f(b) + \sum_{i=1}^{2n-1} f(a + i h_{2n}) \right) =$$

$$= \frac{1}{2} h_{2n} \left(f(a) + f(b) + 2 \sum_{i=1}^{2n-1} f(a + i h_{2n}) \right) = T_{2n}$$

$T_{0,k} = T_{2^k}$ $\leftarrow$ pierwsza kolumna

Resztę obliczamy ze wzoru $T_{m,k} = \dfrac{4^m T_{m-1,k+1} - T_{m-1,k}}{4^m - 1}$

Wyniki możemy nadpisywać od dołu tablicy

6

Potrzebujemy $T_1, T_2, T_4, \ldots, T_{2^{11}}$

$x_i = -1 + i h_j \quad (i = 0, \ldots, 2048)$

$h_{11} = \frac{b-a}{2048} = \frac{5}{2048}$

$h_j = \frac{5}{2^j}$

7

$T_{0,j} \equiv T_{2^{j+1}}$ $\lim_{n\to\infty} T_{0,n} = I = \int_a^b f(x)\,dx$

Baza z L13.1

$$T_{i,j} = \frac{4^i T_{i-1,j+1} - T_{i-1,j}}{4^i - 1}$$ $\lim_{j\to\infty} T_{i-1,j} = I$ zat. ind.

$$\lim_{j\to\infty} T_{i,j} = \frac{4^i \lim_{j\to\infty} T_{i-1,j+1} - \lim_{j\to\infty} T_{i-1,j}}{4^i-1} \overset{\text{z zat. ind}}{=} \frac{4^i I - I}{4^i - 1} = I$$