7.1

a)

x_k	-3	-1	0	1
y_k	-16	0	-16	32

$$\begin{array}{c|cccc}
-3 & -16 & & & \\
-1 & 0 & 8 & & \\
0 & -16 & -16 & -8 & \\
1 & 32 & 48 & 32 & 10
\end{array}$$

$$L_n(x) = -16 + 8(x+3) - 8(x+3)(x+1) + 10(x+3)(x+1)(x-0)$$

$$= -16 + 8(x+3) - 8(x+3)(x+1) + 10(x+3)(x+1)(x-0)$$

c)

x_k	-3	-1	0	1
y_k	-16	0	-16	-8

$$\begin{array}{c|cccc}
-3 & -16 & & & \\
-1 & 0 & 8 & & \\
0 & -16 & -16 & -8 & \\
1 & -8 & 8 & 12 & 5
\end{array}$$

$$L_n(x) = -16 + 8(x+3) - 8(x+3)(x+1) + 5(x+3)(x+1)(x-0)$$

L 7. 2

$$\begin{cases} f[x_i] = f(x_i) = y_i \\ f[x_i, \dots, x_j] = \dfrac{f[x_{i+1}, \dots, x_j] - f[x_i, \dots, x_{j-1}]}{x_j - x_i} \end{cases}$$

$D(n)$ - liczba dzieleń dla $f[x_i, \dots, x_j]$, $n = j - i + 1$

$D(0) = 0$

$D(1) = 1$

$D(2) = 2 \cdot D(2) + 1 = 3$

$D(n) = 2 \cdot D(n-1) + 1$

$D(n) = 2^n - 1$

Dla $n = 0$

$D(0) = 0 = 2^0 - 1$

Z $D(n) = 2^n - 1$ $\quad D(n) = 2D(n-1) + 1$

T $D(n+1) = 2^{n+1} - 1$

$D(n+1) = 2D(n) + 1 = 2(2^n - 1) + 1 = 2^{n+1} - 1$

$S(n)$ - liczba odejmowań

$S(n) = 2 \cdot D(n)$

Czyli $S_n = 2^{n+1} - 2$

$x[n+1] = \{x_0, \dots, x_n\}$

$y[n+1] = \{y_0, \dots, y_n\}$

```
for i=1, i<=n, i++:
    for j=n j>=i, j--;
        y[j] = (y[j] - y[j-1]) / (x[j] - x[j-i]);

return y[]
```

L 7.3

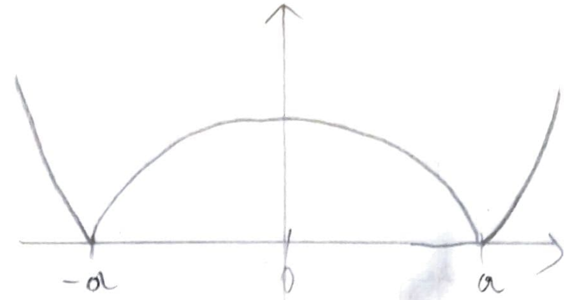
$x\in[-1,1]$ $\quad f(x) = (x-a)(x+a) = x^2 - a^2$

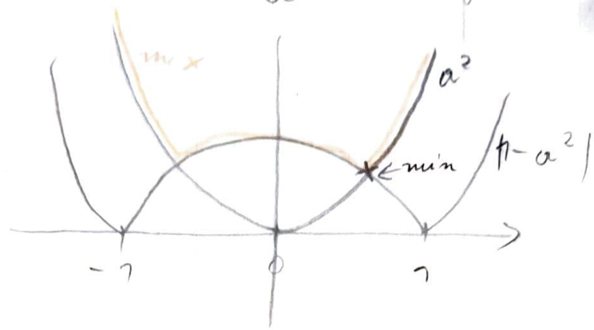
$f'(x) = 2x$

ekstrema = $\{-1, 0, 1\}$

$f(1) = f(-1) = \cancel{1^2 - 0}\ 1 - a^2$

$f(0) = a^2$





$a^2 = 1 - a^2$

$2a^2 - 1 = 0 = T_2(x)$

$\underbrace{a = \frac{1}{\sqrt{2}}}$ $\vee$ $a = -\frac{1}{\sqrt{2}}$

$f(x) = (x-b)x(x+b) = x^3 - b^2 x \qquad f'(x) = 3x^2 - b^2$

$f'(x) = 0$ wtw $x = -\frac{b}{\sqrt{3}} \vee x = \frac{b}{\sqrt{3}}$

$$\left.\begin{aligned} f(1) &= -b^2 + 1 \\ f(-1) &= b^2 - 1 \end{aligned}\right\} |f(1)| = |f(-1)|$$

$$\left.\begin{aligned} f\left(-\tfrac{b}{\sqrt{3}}\right) &= -\frac{b^3}{3\sqrt{3}} + \frac{b^3}{\sqrt{3}} = \frac{2b^3}{3\sqrt{3}} \\ f\left(\tfrac{b}{\sqrt{3}}\right) &= \frac{b^3}{3\sqrt{3}} - \frac{b^3}{\sqrt{3}} = -\frac{2b^3}{3\sqrt{3}} \end{aligned}\right\} \left|f\left(-\tfrac{b}{\sqrt{3}}\right)\right| = \left|f\left(\tfrac{b}{\sqrt{3}}\right)\right|$$

$1 - b^2 = \frac{2b^3}{3\sqrt{3}}$

$\underbrace{b = \frac{\sqrt{3}}{2}}$

L 7.6

$$f(x) = e^{\frac{x}{3}},\ f'(x) = \frac{1}{3} e^{\frac{x}{3}},\ f''(x) = \left(\frac{1}{3}\right)^2 e^{\frac{x}{3}},\ f^{(n)} = \left(\frac{1}{3}\right)^n e^{\frac{x}{3}}$$

$$f^{(n+1)} = \left(\frac{1}{3}\right)^{n+1} e^{\frac{x}{3}}$$

$$\max_{x\in[-1,1]} |f(x) - L_n(x)| \leqslant \max_{x\in[-1,1]} \frac{|f^{(n+1)}(x)|}{(n+1)!} \cdot \max |p_{n+1}(x)|$$

$$\max_{x\in[-1,1]} |p_{n+1}(x)| \leqslant \frac{1}{4} n! \cdot h^{n+1}$$

h - odległość między węzłami

$$h = \frac{2}{n}$$

$$\max_{x\in[-1,1]} \frac{\left|\left(\frac{1}{3}\right)^{n+1} \cdot e^{\frac{1}{3}}\right|}{(n+1)!} \cdot \frac{1}{4} n! \cdot \left(\frac{2}{n}\right)^{n+1} \leqslant 10^{-16}$$

$$n \geqslant 11$$

dla węzłów Czebyszewa

$$\max_{x\in[-1,1]} |p_{n+1}(x)| = \frac{1}{2^n}$$

$$\frac{\left(\frac{1}{3}\right)^{n+1} \cdot e^{\frac{1}{3}}}{(n+1)!} \frac{1}{2^n} \leqslant 10^{-16}$$

$$n \geqslant 11$$

L 7.7.

$b_k = f[x_0, x_1, \dots, x_k]$

$L_{n+1}(x) = L_n(x) + b_{n+1} \, p_{n+1}(x)$ wiemy

$L_{31}(x) = L_{30}(x) + b_{31} \, p_{31}(x)$ potrzebujemy

$f(x_{31}) = L_{31}(x_{31})$

$f(x_{31}) = L_{30}(x_{31}) + b_{31} \cdot p_{31}(x_{31})$

$$b_{31} = \frac{f(x_{31}) - L_{30}(x_{31})}{p_{31}(x_{31})}$$

$f(x_{31}) = y_{31}$ - dana

$p_{31}(x_{31}) = (x_{31} - x_0)(x_{31} - x_1) \dots (x_{31} - x_{30})$

$L_{30}(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \dots + b_{30}(x - x_0) \dots (x - x_{29})$

$L_{30}(x) = b_0 + (x - x_0)\Big(b_1 + (x - x_1)\Big(b_2 + (x - x_2)\Big(b_3 + \dots + (x - x_{29})\, b_{30}\Big)\dots\Big)$