1

$P(A \cap B) = \frac{1}{4}$

$P(A^c) = \frac{1}{3} \Rightarrow P(A) = \frac{2}{3}$

$P(B) = \frac{1}{2}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{11}{12}$$

3

$$P(Z=k) = \sum_{i=0}^{k} P(\cancel{X}=i, Y=k-i) = \sum_{i=0}^{k} P(X=i) \cdot P(Y=k-i) =$$

$$= \sum_{i=0}^{k} \binom{n_1}{i} p^i (1-p)^{n_1-i} \binom{n_2}{k-i} p^{k-i} (1-p)^{n_2-k+i} =$$

$$= \sum_{i=0}^{k} \binom{n_1}{i} \binom{n_2}{k-i} p^k (1-p)^{n_1+n_2-k} =$$

$$= p^k (1-p)^{(n_1+n_2)-k} \sum_{i=0}^{k} \binom{n_1}{i} \binom{n_2}{k-i} \overset{\text{tożsamość Cauchy'ego}}{=}$$

$$= p^k (1-p)^{(n_1+n_2)-k} \binom{n_1+n_2}{k} = B(n_1+n_2, p)$$

4

Rozkład Poissona : $f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$

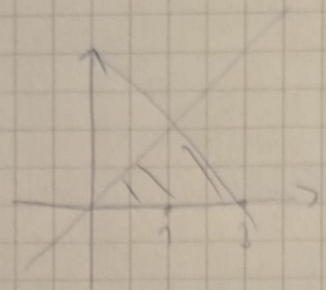
$$P(Z=k) = \sum_{i=0}^{k} P(X=i, Y=k-i) = \sum_{i=0}^{k} P(X=i) \cdot P(Y=k-i) =$$

$$= \sum_{i=0}^{k} e^{-\lambda_1} \frac{\lambda_1^i}{i!} \cdot e^{-\lambda_2} \frac{\lambda_2^{k-i}}{(k-i)!} = \sum_{i=0}^{k} e^{-(\lambda_1+\lambda_2)} \frac{\lambda_1^i}{i!} \cdot \frac{\lambda_2^{k-i}}{(k-i)!} =$$

$$= e^{-(\lambda_1+\lambda_2)} \sum_{i=0}^{k} \frac{1}{k!} \frac{k!}{i!(k-i)!} \lambda_1^i \lambda_2^{k-i} = e^{-(\lambda_1+\lambda_2)} \frac{1}{k!} \sum_{i=0}^{k} \binom{k}{i} \lambda_1^i \lambda_2^{k-i} =$$

$$= e^{-(\lambda_1+\lambda_2)} \frac{1}{k!} (\lambda_1+\lambda_2)^k = e^{-(\lambda_1+\lambda_2)} \frac{(\lambda_1+\lambda_2)^k}{k!}$$

5 $f(x,y) = 3xy$



$x: y \in [0, 1-|x-1|]$

$y: x \in [y, 2-y]$

$$f_1(x) = \int_0^{1-|x-1|} 3xy\,dy = 3x\int_0^{1-|x-1|} y\,dy = 3x\,\frac{y^2}{2}\Big|_0^{1-|x-1|} = \frac{3}{2}x\,(1-|x-1|)^2$$

$$f_2(y) = \int_y^{2-y} 3xy\,dx = 3y\int_y^{2-y} x\,dx = 3y\,\frac{x^2}{2}\Big|_y^{2-y} \quad \frac{3y\,(2-y)^2}{2} - \frac{3y^3}{2} =$$

$$= \frac{y\,(12-12y+3y^2-3y^2)}{2} = 6y-6y^2$$

6 $$EY = \int_0^1 y\,(6y-6y^2)\,dy = \int_0^1 (6y^2-6y^3)\,dy = \left(2y^3 - \frac{3y^4}{2}\right)\Big|_0^1 =$$

$$= 2-\frac{3}{2} = \frac{1}{2}$$

$f(x,y) = f_1(x)\cdot f_2(y)$ $\quad x=y=1$

$f(1,1)=3$

$f_1(1)=\frac{3}{2}$ $\quad 3 \neq \frac{3}{2}\cdot 0$

$f_2(1)=0$

7

wykonujemy $n-1$ prób aby otrzymać 2 wygrane

Prawdopodobieństwo tych dwóch wygranych liczymy

z rozkładu Bernulliego :

$$P_1 = \binom{n-1}{2} \, p^2 \, (1-p)^{n-3}$$

Prawdopodobieństwo trzeciej wygranej to też p

Więc w sumie

$$P = \binom{n-1}{2} p^2 (1-p)^{n-3} \cdot p = \underbrace{\binom{n-1}{2} p^3 (1-p)^{n-3}}$$

$$E(X) = \sum_{n=0}^{\infty} n \, (P(X=n)) = \underbrace{\sum_{n=0}^{\infty} n \binom{n-1}{2} p^3 \, (1-p)^{n-3}}$$

8

alfa : A, α

beta : B β

dzeta : Z ζ

eta : H η

lambda : Λ λ

chi : X χ

ksi : Ξ ξ

fi : Φ φ

rho : P ρ

$f(x) = \frac{1}{4}$

9 a) $X \sim U[-2, 2]$

$Y = |X|$

$F(t) = P(Y \leq t) = P(|X| \leq t) = P(-t \leq X \leq t) = \int_{-t}^{t} f(x)\,dx = \frac{1}{4} \cdot x \Big|_{-t}^{t} = \frac{1}{2} t$

$F'(t) = \left(\frac{1}{2} t\right)' = \frac{1}{2}$

b) $X \sim U[-1, 1]$ $\qquad f(x) = \frac{1}{2}$

$Y = X^3$

$F(t) = P(Y \leq t) = P(X^3 \leq t) = P(X \leq \sqrt[3]{t}) = \int_{-1}^{\sqrt[3]{t}} f(x) = \frac{1}{2}(\sqrt[3]{t} + 1)$

$F'(x) = \left[\frac{1}{2}(\sqrt[3]{t} + 1)\right]' = \frac{1}{6t}$

$Z = X^2$

$F(t) = P(Z \leq t) = P(X^2 \leq t) = P(-\sqrt{t} \leq X \leq \sqrt{t}) =$

$= \int_{-\sqrt{t}}^{\sqrt{t}} \frac{1}{2}\,dx = \frac{1}{2} 2\sqrt{t} = \sqrt{t}$

$F'(x) = [\sqrt{t}]' = \frac{1}{2} \frac{1}{t}$

11

$P(X=40) = \frac{40}{142}$ $P(Y=40) = \frac{1}{4}$

$P(X=32) = \frac{32}{142}$ $P(Y=32) = \frac{1}{4}$

$P(X=20) = \frac{20}{142}$ $P(Y=20) = \frac{1}{4}$

$P(X=50) = \frac{50}{142}$ $P(Y=50) = \frac{1}{4}$

$$E(X) = 40 \cdot \frac{40}{142} + 32 \cdot \frac{32}{142} + 20 \cdot \frac{20}{142} + 50 \cdot \frac{50}{142} = \frac{5524}{142}$$

$$E(Y) = \frac{1}{4} \cdot 40 + \frac{1}{4} \cdot 32 + \frac{1}{4} \cdot 20 + \frac{1}{4} \cdot 50 = \frac{142}{4}$$