1 $\quad X = \{x_0, x_1, \dots x_n\} \quad p(x) > 0$ dla $x \in X$

$$\|f\| = \sqrt{\sum_{k=0}^{N} p(x_k) f(x_k)^2}$$

$\cancel{1^\circ} \quad \cancel{\sqrt{}}$

$1^\circ \quad \|f\| \geq 0$

$$\sqrt{\sum_{k=0}^{N} \underbrace{p(x_k)}_{>0} \underbrace{f(x_k)^2}_{\geq 0}}$$

$2^\circ \quad \|\alpha f\| = |\alpha| \cdot \|f\|$

$$\|\alpha f\| = \sqrt{\sum_{k=0}^{N} p(x_k) \cdot (\alpha f(x_k))^2} = \sqrt{\alpha^2} \sqrt{\sum_{k=0}^{N} p(x_k) \cdot (f(x_k))^2} = |\alpha| \, \|f\|$$

$3^\circ \quad \|f+g\| \leq \|f\| + \|g\|$

$$\|f+g\| = \sqrt{\sum_{k=0}^{N} p(x_k) \, (f(x_k) + g(x_k))^2} \leq \sqrt{\sum_{k=0}^{N} p(x_k) f(x_k)^2} + \sqrt{\sum_{k=0}^{N} p(x_k) g(x_k)^2}$$

$$\Updownarrow$$

$$\sum_{k=0}^{N} p(x_k) \left(f(x_k)^2 + g(x_k)^2 + 2 f(x_k) g(x_k) \right) \leq \sum_{k=0}^{N} p(x_k) f(x_k)^2 + \sum_{k=0}^{N} p(x_k) g(x_k)^2 + 2 \sqrt{\sum_{k=0}^{N} p(x_k) f(x_k)^2} \sqrt{\sum_{k=0}^{N} p(x_k) g(x_k)^2}$$

$$\cancel{\text{tzn} \sum_{k=0}^{N} p(x_k) f(x_k)^2 + \sum_{k=0}^{N} p(x_k) g(x_k)^2 + 2 \sum p(x_k) f(x_k) g(x_k) \leq \sum_k p(x_k) f(x_k)^2 + \sum_k p(x_k) g(x_k)^2 + 2 \sqrt{\sum_k p(x_k) f(x_k)^2} \sqrt{\sum_k p(x_k) (g(x_k))^2}}$$

$$\left(\sum_{k=0}^{N} p(x_k) \, g(x_k) \right)^2 \leq \sum_{k=0}^{N} p(x_k) (f(x_k))^2 \; \sum_{k=0}^{N} p(x_k) (g(x_k))^2$$

Nierówność Cauchy'ego - Schwarza

$$\left(\sum_{i=0}^{N} x_i y_i \right)^2 \leq \left(\sum_{i=0}^{N} x_i^2 \right) \left(\sum_{i=0}^{N} y_i^2 \right)$$

$$x_i = \sqrt{p(x_k)} f(x_k) \quad y_i = \sqrt{p(x_k)} \, g(x_k)$$

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$$\left(\sum_{i=0}^{N} x_i y_i \right)^2 \leqslant \left(\sum_{i=0}^{N} x_i^2 \right) \left(\sum_{i=0}^{N} y_i^2 \right)$$

$$\sum_{i=0}^{n} \left(\sum_{j=0}^{n} (x_i y_j - x_j y_i)^2 \right) \geqslant 0$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} (x_i y_j - x_j y_i)^2 = \sum_{i=0}^{n} \sum_{j=0}^{n} (x_i^2 y_j^2 - 2 x_i y_j x_j y_i + x_j^2 y_i^2) =$$

$$\sum_i \sum_j x_i^2 y_j^2 - 2 \sum_i \sum_j x_i y_j x_j y_i + \sum_i \sum_j x_j^2 y_i^2 =$$

$$\sum_i x_i^2 \sum_j y_j^2 + \sum_i y_i^2 \sum_j x_j^2 - 2 \sum_i x_i y_i \sum_j x_j y_j$$

$$2 \left(\sum_i x_i^2 \right) \left(\sum_i y_i^2 \right) - 2 \left(\sum_i x_i y_i \right)^2 \geqslant 0$$

$$\left(\sum_{i=0}^{n} x_i x_i \right)^2 \leqslant \cancel{2} \left(\sum_{i=0}^{n} x_i^2 \right) \left(\sum_{i=0}^{n} y_i^2 \right)$$

ckd

2 $$\|f - w^*\| = \min_{a \in \mathbb{R}} \sqrt{\sum_{k=0}^{N} \left(f(x_k) - \underset{\overset{\shortparallel}{a x (2021x - 2020) + 1977}}{w(x_k)} \right)^2}$$

$$E(a) = \sum_{k=0}^{N} \left(f(x_k) - a\, x_k (2021 x_k - 2020) - 1977 \right)^2$$

$$E'(a) = \sum_k 2 \left(f(x_k) - a x_k (2021 x_k - 2020) - 1977 \right) \cdot \left(-x_k (2021 x_k - 2020) \right) = 0$$

$$\sum_k \left(f(x_k) - a x_k (2021 x_k - 2020) - 1977 \right) \left(x_k (2021 x_k - 2020) \right) = 0$$

$$\sum_k \left(f(x_k) - 1977 \right) \left(x_k (2021 x_k - 2020) \right) - a x_k (2021 x_k - 2020) \left(x_k (2021 x_k - 2020) \right) = 0$$

$$\sum_k \left(f(x_k) - 1977 \right) \left(x_k (2021 x_k - 2020) \right) - a \sum_k x_k^2 (2021 x_k - 2020)^2 = 0$$

$$a = \frac{\sum_k \left(f(x_k) - 1977 \right) \left(x_k (2021 x_k - 2020) \right)}{\sum_k \left(x_k (2021 x_k - 2020) \right)^2} \cancel{=} \cancel{\sum_{k=0}^{N} (y_k - 1977) \cdot y'(x_k)}$$

3 $$\sum_{k=0}^{r} \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \left[y_k - a\left(\cos(2x_k+2020)+x_k^3\right)\right]^2$$

$$b_k = \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \qquad c_k = \cos(2x_k+2020)+x_k^3$$

$$E(a) \quad \sum_{k=0}^{r} b_k (y_k - a c_k)^2$$

$$E'(a) = \sum_{k=0}^{r} b_k 2 (y_k - a c_k)(-c_k) = 0$$

$$\sum_{k=0}^{r} b_k (y_k - a c_k)(c_k) = 0$$

$$\sum_k \left(b_k y_k c_k - a b_k c_k^2\right) = 0$$

$$\sum_k b_k y_k c_k - a \sum_k b_k c_k^2 = 0$$

$$a = \frac{\sum_k b_k y_k c_k}{\sum_k b_k c_k^2}$$

$$a = \frac{\sum_{k=0}^{r} \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \left(\cos(2x_k+2020)+x_k^3\right) y_k}{\sum_{k=0}^{r} \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \left(\cos(2x_k+2020)+x_k^3\right)^2}$$

5 Z wykładu mamy:

$$a = \frac{(N+1)\, S_4 - S_1 S_3}{(N+1)\, S_2 - S_1^2}, \quad b = \frac{S_2 S_3 - S_1 S_4}{(N+1)\, S_2 - S_1^2}$$

$N+1 = 8$

$$S_1 = \sum_{k=0}^{N} x_k$$

$$S_2 = \sum_{k=0}^{N} x_k^2$$

$$S_3 = \sum_{k=0}^{N} f(x_k)$$

$$S_4 = \sum_{k=0}^{N} x_k f(x_k)$$

Zatem

$$S_1 = \sum_{k=0}^{7} x_k = 365$$

$$S_2 = \sum_{k=0}^{7} x_k^2 = 26525$$

$$S_3 = \sum_{k=0}^{7} f(x_k) = 514{,}5$$

$$S_4 = \sum_{k=0}^{7} x_k f(x_k) = 22685$$

Z tego:

$a = -0{,}0799$

$b = 67{,}9593$

$$y \approx e^{ax+b}$$

$$\ln y \approx ax+b$$

$$g = ax+b$$

$$y = e^{g}$$

$$\|f-g\|_2 = \sqrt{\sum_{k=0}^{r}\left[f(x_k)-g(x_k)\right]^2} = \sqrt{\sum_{k=0}^{r}\left[f(x_k) - \cancel{g(}\, ax_k+b\right]^2}$$

Z wykładu wiemy:

$$a = \frac{(r+1)\,s_4 - s_1 s_3}{(r+1)\,s_2 - s_1^2} \qquad b = \frac{s_2 s_3 - s_1 s_4}{(r+1)\,s_2 - s_1^2} \qquad s_{1,2,3,4} \text{ jak w L10.5}$$

$$y = e^{\frac{(r+1)\,s_4 - s_1 s_3}{(r+1)\,s_2 - s_1^2}x + \frac{s_2 s_3 - s_1 s_4}{(r+1)\,s_2 - s_1^2}}$$