9 PIOTR GUNIA

a) Z: $f(n) = O(g(n)) \iff \exists_{c_1>0} \exists_{n_{01} \in N} \forall_{n \geq n_{01}} \; f(n) \leq c_1 g(n)$

$g(n) = O(h(n)) \iff \exists_{c_2>0} \exists_{n_{02} \in N} \forall_{n \geq n_{02}} \; g(n) \leq c_2 h(n)$

T: $f(n) = O(h(n)) \iff \exists_{c_3>0} \exists_{n_{03} \in N} \forall_{n \geq n_{03}} \; f(n) \leq c_3 h(n)$

weźmy $c_3 = c_1 \cdot c_2$ i $n_{03} = \max(n_{01}, n_{02})$

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b) $f(n) = O(g(n))$ wtw $g(n) = \Omega(f(n))$

(1) $f(n) = O(g(n)) \iff \exists_{c>0} \exists_{n_0 \in N} \forall_{n \geq n_0} \; f(n) \leq c\, g(n)$

(2) $g(n) = \Omega(f(n)) \iff \exists_{c'>0} \exists_{n_0' \in N} \forall_{n \geq n_0'} \; g(n) \geq c' \cdot f(n)$

z (1) $f(n) \leq c \cdot g(n) \overset{c>0}{\iff} \frac{1}{c} \cdot f(n) \leq g(n) \overset{c' = \frac{1}{c} > 0}{\iff} c' f(n) \leq g(n)$

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c) $f(n) = \Theta(g(n))$ wtw $g(n) = \Theta(f(n))$

$f(n) = \Theta(g(n)) \iff f(n) = \Omega(g(n)) \land f(n) = O(g(n))$

$\Updownarrow$ (b) $\qquad \Updownarrow$ (b)

$g(n) = \Theta(f(n)) \iff g(n) = O(f(n)) \land g(n) = \Omega(f(n))$