12 E2

$$g(r,\theta) = \frac{1}{2\pi} r \cdot \exp\left(-\frac{r^2}{2}\right)$$

$$f_x(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \qquad f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)$$

$$g(x,y) = \frac{1}{2\pi} e^{\frac{-(x^2+y^2)}{2}} \qquad |J| = r$$

$$g(r,\theta) = \exp\left(-\frac{r^2\cos^2\theta + r^2\sin^2\theta}{2}\right) \cdot r = \frac{1}{2\pi} r \exp\left(-\frac{r^2}{2}\right)$$

13 E2

$$D = R^2 = X^2 + Y^2 \quad , \quad \theta = \tan^{-1}\frac{Y}{X} \Rightarrow \tan\theta\, X = Y$$

a) $X^2 + Y^2 = X^2 + X^2\tan^2\theta = X^2(1+\tan^2\theta) =$

$= X^2\left(\frac{1}{\cos^2\theta}\right) = D \Rightarrow D\cos^2\theta = X^2$

$X = \pm\cos\theta\sqrt{D} \quad Y = \pm\cos\theta\sqrt{D}\tan\theta = \sin\theta\sqrt{D}$

$$J = \begin{vmatrix} \mp\frac{1}{2}\cos\theta\frac{1}{\sqrt{D}} & \pm\sqrt{D}\sin\theta \\ \pm\frac{1}{2}\cdot\frac{1}{\sqrt{D}}\sin\theta & \pm\sqrt{D}\cos\theta \end{vmatrix} = \frac{1}{2}$$

$$f(x,y) = \frac{1}{2\pi}\exp\left(\frac{-(x^2+y^2)}{2}\right)$$

$$f(d,\theta) = \frac{1}{2\pi}\exp\left(-\frac{d}{2}\right)\frac{1}{2}$$

b) $f_1(d) = \int_0^{2\pi}\frac{1}{2}\frac{1}{2\pi}e^{-\frac{d}{2}}\,d\theta = \frac{1}{4\pi}e^{-\frac{d}{2}}\theta\Big|_0^{2\pi} = \frac{1}{2}e^{-\frac{d}{2}}$

$$f_2(\theta) = \int_0^{\infty}\frac{1}{2}\cdot\frac{1}{2\pi}e^{-\frac{d}{2}}\,dd = \frac{1}{4\pi}\int_0^{\infty}e^{-\frac{d}{2}}\,dd = \frac{1}{2\pi}$$

$f_1(d)\cdot f_2(\theta) = f(d,\theta)$ są niezależne

c) $f_1(d) = \frac{1}{2}e^{-\frac{d}{2}}$

$f = \lambda e^{-\lambda x}$ wykładnicza $\lambda = \frac{1}{2}$