Algebra Homework 4

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September 28, 2023

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1. Let R be a UFD. Suppose $I \subseteq R[x]$ is an ideal containing two nonzero elements f, g having no common factor. Prove that I contains a nonzero constant, i.e., and element of $R\backslash\{0\}\}$.

2. Let R be a PID and $f, g \in R[x]$ be 2 nonzero polyomials having no common factor. Prove that there are only finitely many prime ideals in $R[x]$ containing f and g. Moreover, any such prime ideal is maximal and is of the type $(p, h(x))$, where p is a nonzero prime element of R and $\overline{h(x)} \in \frac{R}{pR}[x]$ is irreducible.

3. Let R be a PID. Prove that any prime ideal $\mathfrak{p}$ in $R[x]$ is in one of the two following forms:

   (a) $\mathfrak{p} = (0)$,

   (b) $\mathfrak{p} = (f(x))$, for some irreducible polynomial f,

   (c) $\mathfrak{p} = (p, \underline{h(x)})$, where $p \in R\backslash\{0\}$ is a nonzero prime element of R and R and $\overline{h(x)}$ is irreducible modulo p.

   Moreover the primes in (iii) are maximal.

Solution:

1.