Algebra 2 Homework 5

February 17, 2024

Solution of problem 1*:* □

Solution of problem 2*:* We have a quadratic equation in x^2, which gives us $x^2 = \pm\omega$. Solving this further, we get $x^4 + x^2 + 1 = (x - \omega)(x + \omega)(x - i\omega)(x + i\omega)$. Then clearly $\mathbb{Q}(i, \omega)$ contains the splitting field. Also, adjoining all the roots to $\mathbb{Q}$ gives us $\mathbb{Q}(\omega, -\omega, i\omega, -i\omega)$ which certainly contains $\mathbb{Q}(i, \omega)$. Thus the splitting field is $\mathbb{Q}(i, \omega)$. □

Solution of problem 3*:* The polynomial $x^6 - 4$ splits into linear factors in $\mathbb{C}$, where the roots $\pm\zeta_3\alpha$, where $\zeta_3 \in \{1, \omega, \omega^2\}$, and $\alpha = \sqrt[3]{2}$. We propose that the splitting field is $\mathbb{Q}(\alpha, \omega)$. This clearly contains all the roots of this polynomial, thus it contains the splitting field. Also, we get the splitting field by adjoining all the roots to $\mathbb{Q}$, which clearly contains $\mathbb{Q}(\alpha, \omega)$, thus it is the splitting field. □

Solution of problem 4*:* $\circlearrowright$ □