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To prove the result, we will decompose an arbitrary permutation $\sigma \in S_n$ into transpositions. We will then show that each transposition can be written as a product of transpositions of the form $(ii+1)$. We have $\sigma = \tau_1 \dots \tau_r$, where τ_i is some transposition of the form $(k_1 k + 1 + k_2)_i$. Note that we can assume the first element of τ_i is strictly lesser than the second since if it weren't we could just invert the order without any loss of generality. We state that $(k_1 k_1 + k_2)$ can written as a product of finite transpositions of the form $(tt+1)$. See that $(k_1 k_1 + k_2 - 1) = (k_1 + k_2 - 1 k_1 + k_2)(k_1 k_1 + k_2)(k_1 + k_2 - 1 k_1 + k_2)$. Since we have $(k_1 k_1 + k_2 - 1)$, we have lowered the second entry of the transposition by one. In $k_2 - 1$ steps, we will get $(k_1 k_1 + 1) = (k_1 + 1 k_1 + 2)(k_1 k_1 + 2)(k_1 + 1 k_1 + 2)$, which means that we can stop. We have τ_i as a product of $2(k_2 - 1) + 1$ transpositions of the desired type. We can do this for all transpositions to get our result. Thus we can generate any permutation in S_n by exchanging adjacent elements. The bubble sort algorithm works this way too, which accepts a permutation then returns the list of n numbers. This is essentially the same problem.

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