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We are given $f(z) = z^2 - z\bar{z}^2 - 2|z|^2$. Since $z = x + iy$, we can expand it in f to get

$$f(x, y) = -2(x^2 + y^2) + i4xy.$$

Thus $u = -2(x^2 + y^2), v = 4xy$. Then we have $u_x = -4x, u_y = -4y, v_x = 4y, v_y = 4x$. If f is holomorphic, then we must have $u_x = v_y \implies -4x = 4x \implies x = 0$. Also we must have $u_y = -v_x \implies -4y = -4y \implies y \in \mathbb{R}$. Thus f satisfies the Cauchy Riemann equations on $\{0\} \times \mathbb{R}$, which is not a domain since it is not open. Thus it is complex differentiable at each point of the type $(0, y)$ where $y \in \mathbb{R}$, but not holomorphic at any point in $\mathbb{C}$ since the points at which it satisfies the Cauchy Riemann equations is not open in $\mathbb{C}$.

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