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We are given $T : \mathbb{K}^n \to \mathbb{K}^m$, where

$$(Tx)(i) = \sum_{j=1}^{n} k_{ij} x_j,$$

where $i = 1, 2, \ldots, m$. Let a_i denote the ith row of T. Then we have $\langle Tx, y\rangle = \sum_{j=1}^{m} (Tx)(i) y_j$. Expanding the entire thing, we have

$$\langle Tx, y\rangle = \sum_{1\leq i\leq m, 1\leq j\leq n} k_{ij} x_j \bar{y}_i.$$

We can write this as

$$\sum_{j=1}^{n} x_j \overline{\overline{k_{1i}} y_1 + \cdots + \overline{k_{mi}} y_m} = \langle x, \overline{T}^T y\rangle!$$

Therefore from uniqueness of adjoint we must have $T^* = \overline{T}^T$.

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