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1. Let H be a non-abelian simple group. Consider $DH \trianglelefteq H$. Then since H is simple, we must have $DH = H$ or $DH = 0$. If $DH = 0$, then H would be abelian, which is not possible, thus H is perfect.

2. We need to calculate $D(\langle H, K\rangle)$. Any element of $\langle H, K\rangle$ can be realised as a word $w = h_1k_1 \dots h_nk_n$ for some $n \in \mathbb{N}$ and $h_1, \dots, h_n \in H, k_1, \dots, k_n \in K$. We also assume that the terms are reduced. We consider two such words w_1 and w_2, where $w_1 = h_1k_1 \dots h_nk_n, w_2 = h_1k_1 \dots h'_{n'}k'_{n'}$. Then any term in $D\langle H, K\rangle$ is of the form $w_1w_2w_1^{-1}w_2^{-1} = (h_1k_1 \dots h_nk_n)(h_1k_1 \dots h'_{n'}k'_{n'})(k_n^{-1}h_n^{-1} \dots k_1^{-1}h_1^{-1})(k_{n'}^{-1}h_{n'}^{-1} \dots k_1'^{-1}h_1'^{-1})$, which is another word in $\langle H, K\rangle$. Now take any word $w = h_1k_1 \dots h_nk_n$ in $\langle H, K\rangle$.

3. We propose that $D(g^{-1}Hg) = g^{-1}DHg$. Take any element of $D(g^{-1}Hg)$, which is of the form $(g^{-1}h_1g)(g^{-1}h_2g)(g^{-1}h_1^{-1}g)(g^{-1}h_2^{-1}g) = g^{-1}(h_1h_2h_1^{-1}h_2^{-1})g \in g^{-1}DHg$. These operations are all if and only if statements, hence $D(g^{-1}Hg) = g^{-1}DHg$. If $H = DH$, $D(g^{-1}Hg) = g^{-1}Hg$.

4. If G is simple, then either $DG = 0$ or $DG = G$. Then for abelian simple groups the maximal perfect subgroup is 0, and in the non-abelian case it is G itself. Both are clearly normal in G.

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