To investigate the relation between spacing and falling time of a domino series

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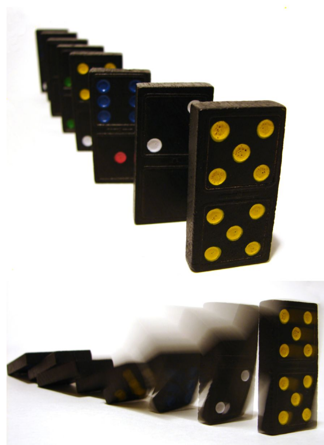
Pushing down the first domino, then all the dominos will follow. The domino effect is an amazing illustration of the irresistible chain reaction. Falling time is an important factor of dominos, since controlling of domino waves, in the fields of engineering, requires the precise value of the falling time. Through observation, it is apparent that the falling time of dominos will increase while spacing decrease. However, it is still unclear for me whether the relation between spacing and falling time of dominos. I am curious of the method to theoretically and empirically determine the relation. Thus, the research question is: **What is the relation between spacing and falling time of a domino series?**

1. Introduction



Graph 1: Real domino effect.

The dominos fall down due to gravity and the contact force exerted by the prior domino. However, for dominos in different positions, the magnitude of the force exerted on it is different. The first domino experienced only gravity while the last domino experienced force from all the prior dominos. The purpose of the IA is to find the equation governs the falling time of dominos in relation with the spacing of dominos.

2. Background Research

According to the dimension analysis by B.G. McLachlan, G. Beaupre, A.B. Cox, and L. Gore, $v = \sqrt{gl}G(\frac{d}{l})$ [1],

where v is the velocity of the domino wave, g is the gravitational acceleration, l is the height of the domino, d is the spacing between dominos, G is an undetermined function that has parameter $\frac{d}{l}$ (13).

In addition, according to the thesis paper Domino Waves by C.J. Efthimiou and M.D. Johnson, $G(\frac{d}{l}) \approx \frac{l}{d}$ [2] (8).

The velocity of the domino wave could be expressed as $v = \frac{nd}{t}$ [3], where n is the number of spaces between dominos.

Substitute [2] and [3] into [1],

$$\frac{nd}{t} = \sqrt{gl}\,\frac{l}{d}$$

$$= t = \frac{n}{\sqrt{gll}} d^2 \text{ [4].}$$

Hence, the falling time of constant number of dominos is proportional to the square of the spacing of dominos, under the circumstance that the height and number of dominos are constant.

In this experiment, it will first be confirmed the proportional relation, and the percentage discrepancy of the value of $\frac{n}{\sqrt{gll}}$ will be computed to determine the accuracy of the experiment.

3. Methodology

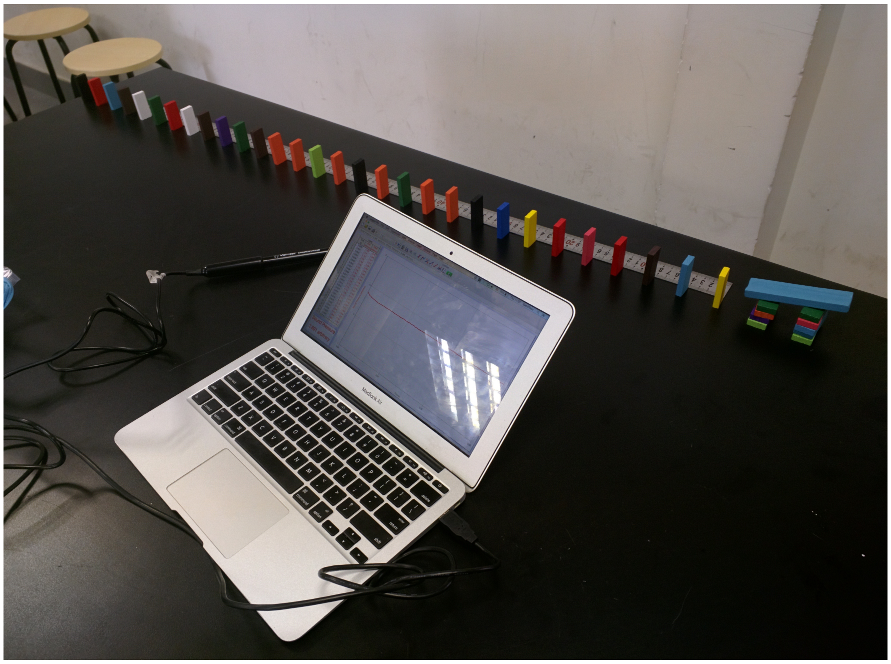
3.1. Instruments

Table 1: Instruments used

Instrument	Precision
One-meter ruler	0.001m
Sound Recorder (recording time)	0.001s

The one-meter ruler is used to assist arranging dominos and to measure the spacing between dominos. A sound recorder, placed beside the dominos, is used to indirectly record the falling time of dominos.

A marble sphere on an inclined rail with constant angle against the horizontal is used to push the dominos. The marble sphere is placed on the same position to ensure the initial momentum of the dominos is the same for each trial.



Graph 2: Example of setting up experiment.

3.2. Controlling Variables

The only changing independent variable is the spacing between dominos. According to the theory, the dependent variable would be the falling time of a domino series.

Variables to be controlled in the experiment are the height and the number of spacings, which are 4.3cm and 29 respectively, all the other quantities of any single domino, and the initial momentum of the first domino. Magnifying the height of dominos would make the dependent variable smaller. Magnifying the number of dominos would make the dependent variable greater. Thus, the range of the experiment is from 4.0 cm to 1.5 cm.

3.3. Reducing Error

In order to minimize the error placing the dominos, a one-meter ruler is placed next to the line of dominos. All the dominos are aligned to the position on the ruler where the reading is a multiple of 0.5 centimeters.

In order to reduce the random error, each trial is repeated three times and 7 trials would be conducted to reach a conclusion.

3.4. Manipulating And Recording Variables

The only independent variable is the spacing between dominos. The falling time is measured by the sound recorder after a line of dominos is arranged with a ruler.

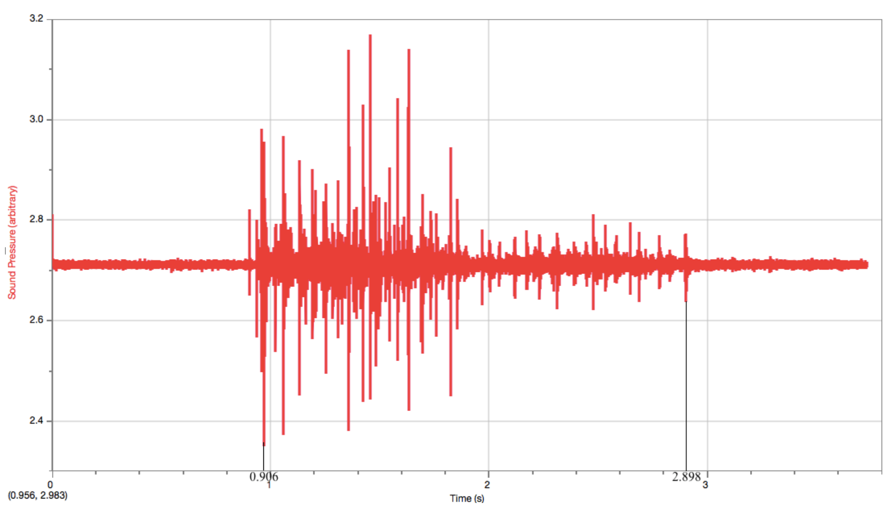
For convenience, the space between dominos is often set to be multiple of 0.5 centimeters, with one exception of 3.7 centimeters. The height of dominos is 4.3 centimeters; therefore, the maximum spacing chosen should be 4 centimeters because of both feasibility and convenience of the experiment. The minimum spacing chosen should be 1.5 centimeters, because dominos could not collapse with a push when spacing between dominos was smaller than or equal to 1.0 centimeter. The one-meter ruler is used to assist arranging dominos. Each domino is placed beside the ruler to ensure that the dominos are in a straight line with constant spacing. In addition, the ruler also measure the spacing between dominos with uncertainty ±0.0005m.

Since it is expensive and difficult to determine the wave velocity from the high-speed camera, a sound recorder is chosen. While conducting the experiment, the surrounding is kept silent to obtain only the sound of domino's collision. The falling time is calculated from the difference of the first crust and last crust of the longitudinal sound wave with uncertainty ±0.002s.

3.5. Safety And Environmental Issues

The experiment does not require the use of water and animals. The electricity from the laptop is sufficient to perform the experiment with Logger Pro and the sound recorder.

4. Experimental Data



Graph 3: Example of sound recorder data.

The first crust or the last crust, marked with a black line, is defined to be the crust that has a significantly greater value than other crust. Its corresponding time is determined from the graph, as the figures on the left corder in the button illustrate. All the graphs are included in the Appendix I on page 12.

trial	t_1 (s)	t_2 (s)	Δt (s)
1-1	0.906	2.898	1.992
1-2	2.105	4.162	2.057
1-3	0.922	3.113	2.191
2-1	1.453	3.319	1.866
2-2	1.087	3.018	1.931
2-3	0.961	2.678	1.717
3-1	2.841	4.392	1.551
3-2	1.033	2.672	1.639
3-3	1.160	2.709	1.549
4-1	0.996	2.055	1.059
4-2	1.017	2.198	1.181
4-3	1.066	2.195	1.129
5-1	1.124	1.957	0.833
5-2	0.815	1.713	0.898
5-3	0.913	1.895	0.982
6-1	0.977	1.670	0.693
6-2	0.932	1.660	0.728
6-3	1.555	2.232	0.677
7-1	1.125	1.682	0.557
7-2	2.364	2.835	0.471
7-3	0.860	1.398	0.538

Graph 4: Processing Raw Data

The graph above showed how the raw data is processed. The falling time is calculated from the difference of the corresponding time of the first crust and the last crust. Since it involve the subtraction of values, the uncertainty = 0.001 + 0.001 = 0.002s. Thus, the uncertainty of the falling time is ±0.002s.

Table 1

Trial	Spacing Between Dominos d /m ± 0.0005m	Average Falling Time of Dominos $\bar{t}$ /s ± 0.006s
1	0.0400	2.080
2	0.0370	1.838
3	0.0350	1.580
4	0.0300	1.123
5	0.0250	0.904
6	0.0200	0.699
7	0.0150	0.522

Table 3: Processed And Linearized Data

Trial	Spacing^2 Between Dominos d^2 /m ± 0.00004m	Average Falling Time of Dominos $\bar{t}$ /s ± 0.006s
1	0.00160	2.080
2	0.00137	1.838
3	0.00123	1.580
4	0.00090	1.123
5	0.00063	0.904
6	0.00040	0.699
7	0.00023	0.522

Sample of calculation:

For the first trial in Table 2, $\bar{t} = \dfrac{0.906 + 2.105 + 0.922}{3} = 1.922s$.

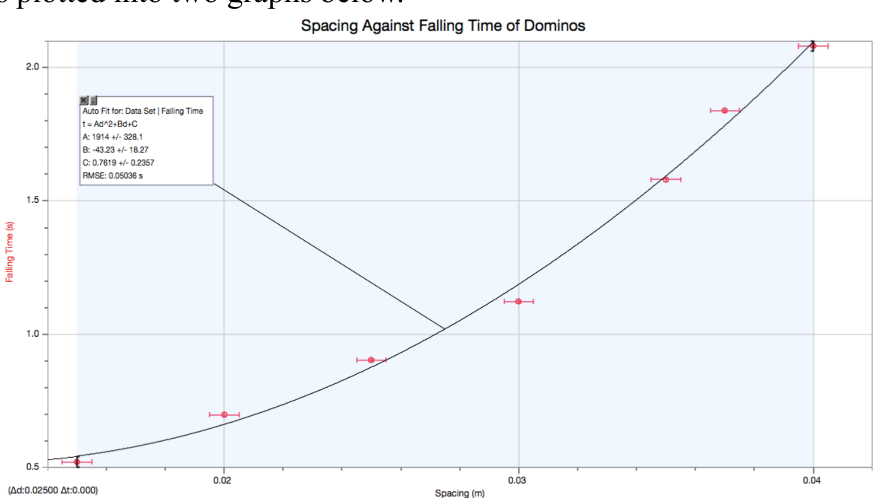
The uncertainty for the average falling time $\Delta\bar{t} = 0.002 + 0.002 + 0.002 = 0.006s$.

For the first Trial in Table 3, $d^2 = 0.0400^2 = 0.00160m^2$.

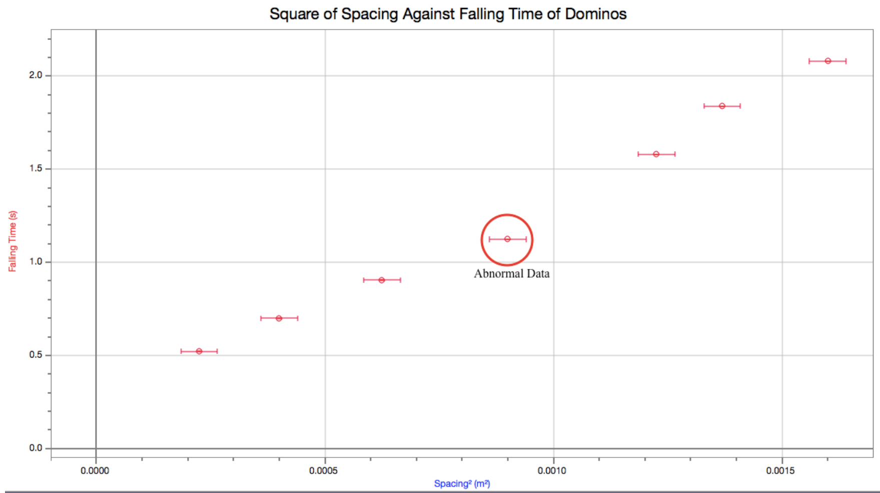
The uncertainty for the square of spacing $\Delta d^2 = 0.0016 \times \left(2 \times \dfrac{0.0005}{0.0400}\right) = 0.00004m^2$.

The uncertainty of the square the spacing between dominos is the largest uncertainty of the column, which is ±0.00004m^2.

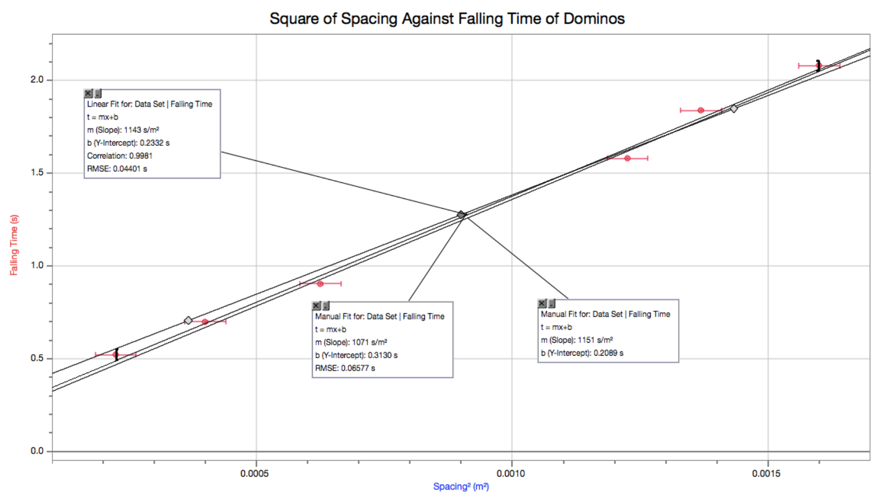
The data is plotted into two graphs below.



Graph 5: Relation between spacing and falling time.



Graph 6-1: Linearized Data with one circled abnormal data, which was deleted in Graph 6-2.



Graph 6-2: Relation between square of spacing and falling time. One abnormal data was deleted.

From the graph, the relation that falling time is proportional to the square of the spacing of the dominos ($t \propto d^2$) is obtained. Since the best fit line can pass through all of the lines, except the deleted fourth one, which was caused by the sound that interfered the experiment, the theoretical equation could be considered verified.

Maximum Gradient = 1151 sm^{-2},

Minimum Gradient = 1071 sm^{-2},

Uncertainty of the gradient = $\dfrac{1151-1071}{2} = 40\,sm^{-2}$.

Thus, the gradient ($\dfrac{n}{\sqrt{gll}}$) = 1140 ± 40 sm^{-2}.

Percentage Difference = $\frac{40}{1140} = 3.5\%$.

Since the theoretical value of the constant $\frac{n}{\sqrt{gll}}$ is 1038 sm^{-2} , (g = 9.81 ms^{-2}, l = 0.043m, and n = 29), Percentage Discrepancy = $\frac{1140-1038}{1038} = 9.8\%$. Y-intercept was is 0.3130s.

5. Conclusion

Since the experiment has a small percentage difference of 3.5% and a moderate percentage discrepancy of 9.8%, the result is acceptable; thus, the equation $t = \frac{n}{\sqrt{gll}} d^2$ is verified, which implied that the falling time increases rapidly when spacing increases, under the condition that all the other variables remain constant. However, systematic error in the experiment is significant. Various factors, listed in Table 4 below, contributed to the deviation and vibration of data.

Table 4: Errors

Type of Error	Error Source	Effect
Systematic Error	There is friction between dominos and the table.	It will slow down the falling of dominos; thus, the gradient would be greater than the theoretical value. It will also result in a non-zero y-intercept.
	There is air resistance throughout the experiment.	
	The sound of collision dissipated some potential energy of dominos.	
	The dominos, which are considered as a one-dimension object in theory, have considerable amount of width.	
Random Error	Insufficient number of experiments are conducted.	The data will vibrate around the gradient $\frac{n}{\sqrt{gll}}$, with the uncertainty 40 ms^{-2}.
	Dominos are not arranged in an exact straight line.	
	The initial force exerted on the dominos could not be kept constantly exactly.	

Since the systematic errors are due to environmental constraints, there is little space for improvement of the accuracy of the experiment. However, several methods could be applied to enhance the precision. Firstly, more repeats should be conducted to minimize the effect of random error. Secondly, another ruler should be used to further ensure that dominos are in a straight line throughout the experiment. Thirdly, a spring could be used to push the domino, making the value of the initial momentum of the first domino constant. In addition, in order to reach a more comprehensive conclusion, wider interval of spacing of dominos, from 1.5 cm to 6 cm should be used, which could be achieved by using taller dominos.

Despite the systematic errors and random errors, the theoretical deduction and verification from the experiment both suggest that the equation $t = \frac{n}{\sqrt{gll}} d^2$ is true for the domino effect. Further investigation could focus on the relation between the width of dominos and the their falling time.

Bibliography

Efthimiou, C. J., and M. D. Johnson. "Domino Waves." SIAM Review 49.1 (2007): 8. Web. 25 Apr. 2015.

McLachlan, B. G., G. Beaupre, A. B. Cox, and L. Gore. "Falling Dominoes(D. E. Daykin)." 25.3 (2002): 13. Web of Science. Web. 25 Apr. 2015.

Appendix I: All the raw data from graph