# 27.current and resistance

## Summary：

**1.** The electric current *I* in a conductor is defined as $I \equiv \frac{dQ}{dt}$

导体中的电流 I 定义为

**where *dQ* is the charge that passes through a cross sec- tion of the conductor in a time interval *dt.* The SI unit of current is the ampere (A), where 1 A = 1 C/s.**

其中 dQ 是在时间间隔 dt 内通过导体横截面的电荷，电流的 SI 单位是安培 (A)，其中 1A =1C/s

**2.** The current density *J* in a conductor is the cur- rent per unit area: $J \equiv \frac{I}{A}$

导体中的电流密度**J**是单位面积上的电流:

**3.** The resistance *R* of a conductor is defined as : $R \equiv \frac{\Delta V}{I}$

导体的电阻 R 定义为:

where *V* is the potential difference across the conductor and *I* is the current it carries. The SI unit of resistance is volts per ampere, which is defined to be 1 ohm (V); that is, 1 V = 1 V/A.

其中 DV 是导体两端的电位差，I 是它承载的电流。电阻的SI单位是伏特/安培，定义为1欧姆(V)；也就是说，1 V = 1 V/A。

**4.** The average current in a conductor is related to the motion of the charge carriers through the relationship

通过这种关系，导体中的平均电流与载流子的运动有关

$$I_{avg} = nqv_dA$$

where *n* is the density of charge carriers, *q* is the charge on each carrier, $v_d$ is the drift speed, and *A* is the cross- sectional area of the conductor.

其中**n**是载流子的密度，**q**是每个载流子上的电荷，**U**是漂移速度，**A**是导体的横截面积。

**5.** The current density in an ohmic conductor is proportional to the electric field according to the expression

根据表达式，欧姆导体中的电流密度与电场成正比

$$J = \sigma E$$

The proportionality constant s is called the conductivity of the material of which the conductor is made. The inverse of s is known as resistivity r (that is, r = 1/p). Equation

27.6 is known as Ohm's law, and a mate- rial is said to obey this law if the ratio of its current density to its applied electric field is a constant that is independent of the applied field.

比例常数**s**称为制成导体的材料的导电性。**s**的倒数称为电阻率**r(**即**r = 1/p)**。方程**27.6**被称为欧姆定律，如果一个物质的电流密度与它所施加的电场之比是一个常数，这个常数与所施加的电场无关，那么这个物质就服从这个定律。

**6.**For a uniform block of material of cross- sectional area *A* and length , the resistance over the length , is :

对于截面面积和长度均为**a**的材料块**,**电阻除以长度，为**:**

$$R = \rho \frac{\ell}{A}$$

where r is the resistivity of the material.

其中**r**为材料的电阻率。

**7.**In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the elec- trons is zero. When an electric field is applied, the electrons move (on average) with a drift velocity S$v_d$ that is opposite the electric field. The drift velocity is given by

在经典的金属导电模型中，电子被视为气体分子。在没有电场的情况下，电子的平均速度为零。当施加电场时，电子**(**平均**)**以与电场相反的漂移速度**Svd**移动。漂移速度为

$$\vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau$$

where *q* is the electron's charge, $m_e$ is the mass of the electron, and t is the average time interval between electron–atom collisions. According to this model, the resistiv- ity of the metal is

其中**q**是电子的电荷，**me**是电子的质量，**t**是电子与原子碰撞的平均时间间隔。根据该模型**,**金属的电阻率为

$$\rho = \frac{m_e}{nq^2\tau}$$

where *n* is the number of free electrons per unit volume.

**n**是单位体积的自由电子数

**8.**The resistivity of a conductor varies approximately linearly with temperature according to the expression

根据这个表达式，导体的电阻率近似地随温度线性变化

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where $r_0$ is the resistivity at some reference temperature $T_0$ and a is the temperature coefficient of resistivity.

式中$r_0$为某参考温度$T_0$下的电阻率，**a**为电阻率的温度系数。

**9.** If a potential difference $V$ is maintained across a circuit element, the power, or rate at which energy is supplied to the element, is

如果在电路元件之间保持一个电位差 **V**，则功率或能量供给元件的速率为

$$P = I\Delta V$$

Because the potential difference across a resistor is given by $V = IR$, we can express the power delivered to a resistor as

因为电阻器上的电位差是由**V =IR**给出的，我们可以将传递给电阻器的功率表示为

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

The energy delivered to a resistor by electrical transmission $T_{ET}$ appears in the form of internal energy $E_{int}$ in the resistor.

传输**T ET**传递给电阻的能量以电阻器内能**E int**的形式出现。

# HW5

**6.** A copper wire has a circular cross section with a radiuS Q/C of 1.25 mm. (a) If the wire carries a current of 3.70 A, find the drift speed of the electrons in this wire. (b) All other things being equal, what happens to the drift speed in wires made of metal having a larger number of conduction electrons per atom than copper? Explain.

**A** 铜线有一个圆形截面，半径**Q/C**为**1.25**毫米。**(a)**如果铜线携带**3.70 a**的电流，求该铜线中电子的漂移速度。

**(b)**在其他条件相同的情况下，由每个原子导电电子数比铜多的金属制成的导线的漂移速度会发生什么**?**解释

**a)** From Example 27.1 in the textbook, the density of charge carriers (electrons) in a copper wire is $n = 8.46 \times 10^{28}$ electrons/m$^3$. With $A = \pi r^2$ and $q = e$ , the drift speed of electrons in this wire is

铜线中载流子**(**电子**)**的密度为**n = 8.46 × 1028**电子**/m3**。当**A = πr2, q = e**时，该导线中电子的漂移速度为

$$v_d = \frac{I}{n|q|A} = \frac{I}{ne(\pi r^2)}$$

$$= \frac{3.70\ \text{C/s}}{(8.46\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})\pi(1.25\times10^{-3}\ \text{m})^2}$$

$$= \boxed{5.57\times10^{-5}\ \text{m/s}}$$

**b)** **The drift speed is smaller because more electrons are being conducted.**

漂移速度更小，因为更多的电子被传导。

**8.** Figure P27.8 represents a section of a conductor of nonuniform diameter carrying a current of *I* 5 5.00 A. Q/C The radius of cross-section $A_1$ is $r_1$ 5 0.400 cm. (a) What is the magnitude of the current density across $A_1$? The radius $r_2$ at $A_2$ is larger than the radius $r_1$ at $A_1$ .(b) Is the current at $A_2$ larger, smaller, or the same? (c) Is the current density at $A_2$ larger, smaller, or the same? Assume $A_2$ 5 4$A_1$. Specify the (d) radius, (e) cur- rent, and (f) current density at $A_2$.

**图P27.8表示一段直径不均匀的导体，电流为I 5.00 a。Q/C截面A1的半径为r1 5 0.400 cm。(a)流过A1的电流密度的大小是多少?A2处的半径r2大于A1处的半径r1。(b) A2处的电流是大的，小的，还是相同的?(c) A2处的电流密度是更大、更小还是相同?假设A2 5 4A1。指定(d)半径，(e)电流，(f) A2处的电流密度。**

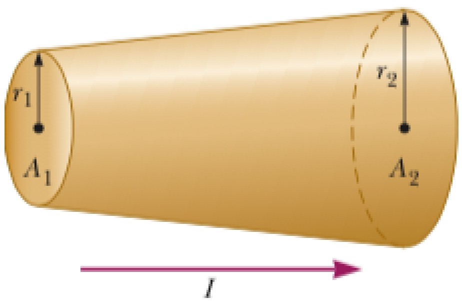


Figure P27.8

**(a)**

$$J = \frac{I}{A} = \frac{5.00\ \text{A}}{\pi\left(4.00\times10^{-3}\ \text{m}\right)^2} = \boxed{99.5\ \text{kA/m}^2}$$

**(b)** Current is the same 电流是一样的

**(c)** The cross-sectional area is greater; therefore the current density is smaller **.**截面面积较大;因此电流密度较小

**(d)**

$$A_2 = 4A_1 \quad \text{or} \quad \pi r_2^2 = 4\pi r_1^2 \qquad \text{so} \qquad r_2 = 2r_1 = \boxed{0.800\ \text{cm}}.$$

**(e)I = 5.00A**

**(f)**

$$J_2 = \frac{1}{4}J_1 = \frac{1}{4}\left(9.95\times10^4\ \text{A/m}^2\right) = \boxed{2.49\times10^4\ \text{A/m}^2}$$

**12.** An electric current in a conductor varies with time according to the expression *I*(*t*) = 100 sin (120p*t*), where *I* is in amperes and *t* is in seconds. What is the total charge passing a given point in the conductor from *t* = 0 to *t* =1/240 S?

根据**I(t) = 100sin (120pt)**的表达式，导体中的电流随时间而变化，其中**I**是安培，**t**是秒。从**t = 0**到**t =1/ 240s**，通过导体中给定点的总电荷是多少**?**

To find the total charge passing a point in a given amount of time, we use $I = dq/dt$ , from which we can write

为了求在给定时间内通过某一点的总电荷，我们 **I = dq/dt**，我们可以这样写

$$q = \int dq = \int I\,dt = \int_0^{1/240\,s} (100\ A)\sin\left(\frac{120\pi t}{s}\right) dt$$

$$q = \frac{-100\ C}{120\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = \frac{+100\ C}{120\pi} = \boxed{0.265\ C}$$

**18.** Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

发现相等长度的铝线和铜线具有相同的电阻。它们的半径之比是多少**?**

Using R= PL/A :

$$\rho_{Cu}\frac{L_{Cu}}{\pi r_{Cu}^2} = \rho_{Al}\frac{L_{Al}}{\pi r_{Al}^2} \rightarrow \frac{r_{Al}^2}{r_{Cu}^2} = \frac{\rho_{Al}}{\rho_{Cu}}$$

which yields

$$\frac{r_{Al}}{r_{Cu}} = \sqrt{\frac{\rho_{Al}}{\rho_{Cu}}} = \sqrt{\frac{2.82\times10^{-8}\ \Omega\cdot m}{1.70\times10^{-8}\ \Omega\cdot m}} = \boxed{1.29}$$

**20.** Suppose you wish to fabricate a uniform wire from a S mass *m* of a metal with density $r_m$ and resistivity r. If the wire is to have a resistance of *R* and all the metal is to be used, what must be (a) the length and (b) the diameter of this wire?

假设您想要制作统一的线从**Rm S**质量**m**的金属密度和电阻率**r**。如果导线的电阻**r**和使用所有的金属**,**必须**(a)**和**(b)**长度导线的直径**?**

(a) Given total mass $m = \rho_m V = \rho_m A\ell \quad \rightarrow \quad A = \frac{m}{\rho_m \ell}$, where

$\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{m/\rho_m\ell} = \frac{\rho\rho_m\ell^2}{m}$.

Thus, $\ell = \boxed{\sqrt{\frac{mR}{\rho\rho_m}}}$.

(b) Volume $V = \frac{m}{\rho_m}$, or

$$\frac{1}{4}\pi d^2\ell = \frac{m}{\rho_m}$$

$$d = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\frac{1}{\ell}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\sqrt{\frac{\rho\rho_m}{mR}}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\sqrt{\frac{m^2}{\rho_m^2}\frac{\rho\rho_m}{mR}}\right)^{1/2}$$

$$= \boxed{\sqrt{\frac{4}{\pi}}\left(\frac{\rho m}{\rho_m R}\right)^{1/4}}$$

**25.**If the magnitude of the drift velocity of free electrons in a copper wire is 7.84 *10-4 m/s, what is the electric field in the conductor?

如果铜线中自由电子的漂移速度大小为**7.84 *10-4 m/s**，导体中的电场是多少**?**

The resistivity and drift velocity can be related to the electric field within the copper wire:

$$\rho = \frac{m}{ne^2\tau} \rightarrow \tau = \frac{m}{\rho ne^2}$$

**and**

$$v_d = \frac{eE}{m}\tau = \frac{eE}{m}\frac{m}{\rho ne^2} = \frac{E}{\rho ne} \qquad \rightarrow \qquad E = \rho nev_d$$

where $n$ is the electron density. From Example 27.1,

$$n = \frac{N_A\rho_{Cu}}{M} = \frac{(6.02\times10^{23}\ \text{mol}^{-1})(8\,920\ \text{kg/m}^3)}{0.063\,5\ \text{kg/mol}} = 8.46\times10^{28}\ \text{m}^{-3}$$

The electric field is then

$$E = \rho nev_d$$

$$E = (1.7\times10^{-8}\ \Omega\cdot\text{m})(8.46\times10^{28}\ \text{m}^{-3})$$
$$\times(1.60\times10^{-19}\ \text{C})(7.84\times10^{-4}\ \text{m/s})$$
$$= \boxed{0.18\ \text{V/m}}$$

**27.**What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C?

If we ignore thermal expansion, the change in the material's resistivity with temperature $\rho = \rho_0 [1 + \alpha \Delta T]$ implies that the change in resistance is $R - R_0 = R_0 \alpha \Delta T$. The fractional change in resistance is defined by $f = (R - R_0)/R_0$. Therefore,

如果我们忽略热膨胀，材料电阻率随温度的变化ρ = ρ0 [1 + α ΔT]意味着电阻的变化是R−R0= R0αΔT。电阻的分数变化定义为f = (R - R0)/R0。因此

$$f = \frac{R_0 \alpha \Delta T}{R_0} = \alpha \Delta T = \left(5.00 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1}\right)(50.0^\circ\mathrm{C} - 25.0^\circ\mathrm{C}) = \boxed{0.12}$$

29.If a certain silver wire has a resistance of 6.00 V at 20.0°C, what resistance will it have at 34.0°C?

果某一银线在20.0°C时电阻为6.00 V，那么在34.0°C时电阻是多少?

$$\begin{aligned} R &= R_0\left[1 + \alpha(T - T_0)\right] \\ &= (6.00\ \Omega)\left[1 + \left(3.8 \times 10^{-3}\ ({}^\circ\mathrm{C})^{-1}\right)(34.0^\circ\mathrm{C} - 20.0^\circ\mathrm{C})\right] \\ &= \boxed{6.32\ \Omega} \end{aligned}$$

35.At what temperature will aluminum have a resistivity that is three times the resistivity copper has at room temperature?

在什么温度下, 铝的电阻率是铜在室温下电阻率的三倍**?**

Room temperature is $T_0 = 20.0°$

$$\rho_{Al} = (\rho_0)_{Al}\left[1 + \alpha_{Al}(T - T_0)\right] = 3(\rho_0)_{Cu}$$

Then, substituting numerical values from Table 27.2 gives

$$\begin{aligned} T - T_0 &= \frac{1}{\alpha_{Al}}\left[\frac{3(\rho_0)_{Cu}}{(\rho_0)_{Al}} - 1\right] \\ &= \frac{1}{3.9 \times 10^{-3}\ ({}^\circ\mathrm{C})^{-1}}\left[\frac{3(1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m})}{2.82 \times 10^{-8}\ \Omega \cdot \mathrm{m}} - 1\right] \end{aligned}$$

$T - 20.0°C = 207\ °C$

$T = 227°C$

39.A certain waffle iron is rated at 1.00 kW when con- nected to a 120-V source. (a) What current does the waffle iron carry? (b) What is its resistance?

当连接到120伏电源时，某华夫饼烙铁的额定功率为1.00 kW。(a)华夫饼电烙铁的电流是多少?(b)它的抵抗力如何?

(a) From Equation 27.21,

$$P = I\Delta V \rightarrow I = P/\Delta V = (1.00\times10^{3}\ \mathrm{W})/(120\ \mathrm{V}) = \boxed{8.33\ \mathrm{A}}$$

(b) From Equation 27.23,

$$P = \Delta V^{2}/R \rightarrow R = \Delta V^{2}/P = (120\ \mathrm{V})^{2}/(1.00\times10^{3}\ \mathrm{W}) = \boxed{14.4\ \Omega}$$

40.The potential difference across a resting neuron in the human body is about 75.0 mV and carries a current of about 0.200 mA. How much power does the neuron release?

人体静止神经元的电位差约为75毫伏，电流约为0.200毫安。神经元会释放多少能量?

$$P = I\Delta V = (0.200\times10^{-3}\ \mathrm{A})(75.0\times10^{-3}\ \mathrm{V})$$
$$= 15.0\times10^{-6}\ \mathrm{W} = \boxed{15.0\ \mu\mathrm{W}}$$

45.Batteries are rated in terms of ampere-hours (A * h). For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A * h.(a)What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A * h? (b) At $0.110 per kilowatt-hour, what is the value of the electricity produced by this battery?

电池额定的安时(A* h)。例如,一个电池可以产生电流的2.00 * 6.00 3.00 h是额定h。(A)的总能量是什么,在千瓦时,存储在一个h - v电池额定55.0 * 12.0 ?(b)按每千瓦时0.110美元计算，这种电池产生的电力价值是多少?

(a) The total energy stored in the battery is

$$\Delta U_E = q(\Delta V) = It(\Delta V)$$
$$= (55.0\ \mathrm{A\cdot h})(12.0\ \mathrm{V})\left(\frac{1\ \mathrm{C}}{1\ \mathrm{A\cdot s}}\right)\left(\frac{1\ \mathrm{J}}{1\ \mathrm{V\cdot C}}\right)\left(\frac{1\ \mathrm{W\cdot s}}{1\ \mathrm{J}}\right)$$
$$= 660\ \mathrm{W\cdot h} = \boxed{0.660\ \mathrm{kWh}}$$

(b) The value of the electricity is

$$\mathrm{Cost} = (0.660\ \mathrm{kWh})\left(\frac{\$0.110}{1\ \mathrm{kWh}}\right) = \boxed{\$0.072\ 6}$$

58.Determine the temperature at which the resistance of an aluminum wire will be twice its value at 20.0°C. Assume its coefficient of resistivity remains constant.

确定铝丝的电阻在20.0°C时是其值的两倍的温度。假设其电阻率系数保持不变。

At $T_0 = 20.0°$, $R = R_0$. Then, from Equation 27.20,

$$R = R_0\left[1+\alpha\left(T - T_0\right)\right] = 2R_0$$

Solving for the change in temperature gives

$$T - T_0 = \frac{1}{\alpha} = \frac{1}{3.9\times10^{-3}\ (°C)^{-1}}$$

$$T - 20.0°C = 256°C \quad \rightarrow \quad T = \boxed{276°C}$$

61.One wire in a high-voltage transmission line carries 1 000 A starting at 700 kV for a distance of 100 mi. If the resistance in the wire is 0.500 V/mi, what is the power loss due to the resistance of the wire?

在高压输电线路中，一根导线从700kv开始承载1000a，距离100mi，如果导线的电阻是0.500 V/mi，导线的电阻造成的功率损耗是多少?

一根导线的电阻是:

整条线路距离地电位的标称距离是700千伏，但是 它两端的电位差是:

The resistance of one wire is $\left(\frac{0.500\ \Omega}{\text{mi}}\right)(100\ \text{mi}) = 50.0\ \Omega$.

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$$IR = (1\,000\ \text{A})(50.0\ \Omega) = 50.0\ \text{kV}$$

Then it radiates as heat power

$$P = I\Delta V = (1\,000\ \text{A})(50.0\times10^3\ \text{V}) = \boxed{50.0\ \text{MW}}$$

63.A charge *Q* is placed on a capacitor of capacitance *C.* **S** The capacitor is connected into the circuit shown in Figure P27.63, with an open switch, a resistor, and an initially uncharged capacitor of capacitance 3*C*. The switch is then closed, and the circuit comes to equilib- rium. In terms of *Q* and *C*, find (a) the final poten- tial difference between the plates of each capacitor, (b) the charge on each capacitor, and (c) the final energy stored in each capacitor. (d) Find the internal energy appearing in the resistor.

将电荷Q放置在电容c的电容上。S将电容连接到如图P27.63所示的电路中，电路中有一个开路开关、一个电阻和一个初始未充电的电容3C。然后开关闭合，电路达到平衡。根据Q和C，求出(a)每个电容器的极板之间的最终电位差，(b)每个电容器上的电荷，(C)每个电容器中储存的最终能量。(d)求电阻器的内能。

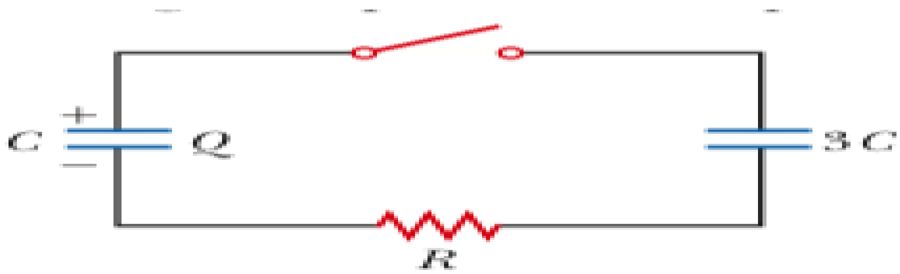


**Figure P27.63**

The original stored energy is $U_{E,i} = \frac{1}{2}Q\Delta V_i = \frac{1}{2}\frac{Q^2}{C}$.

(a) When the switch is closed, charge *Q* distributes itself over the plates of *C* and 3*C* in parallel, presenting equivalent capacitance 4*C*. Then the final potential difference is $\boxed{\Delta V_f = \frac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \frac{Q}{4C}C = \boxed{\frac{Q}{4}}$.

The larger capacitor carries charge $3C\frac{Q}{4C} = \boxed{\frac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\frac{1}{2}C(\Delta V_f)^2 = \frac{1}{2}C\left(\frac{Q}{4C}\right)^2 =$ $\boxed{\frac{Q^2}{32C}}$. The larger capacitor possesses energy $\frac{1}{2}3C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{3Q^2}{32C}}$.

(d) The total final energy is $\frac{Q^2}{32C} + \frac{3Q^2}{32C} = \frac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor:

$$\frac{Q^2}{2C} = \frac{Q^2}{8C} + \Delta E_{int} \quad \text{so} \quad \Delta E_{int} = \boxed{\frac{3Q^2}{8C}}$$