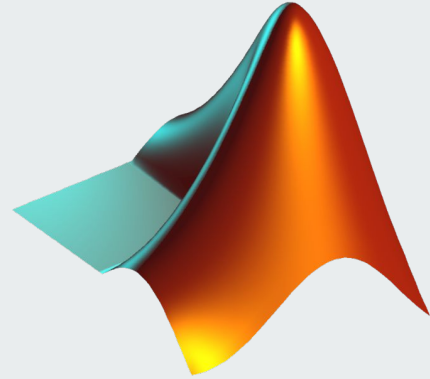


Curso de Programação em MATLAB

74 - Transformada Rápida de Fourier

Σ ExataMenteS π



Transformada Rápida de Fourier

Vamos relembrar da DTFT

$$\mathcal{O}(n^2)$$

$$\begin{bmatrix} \hat{f}_1 \\ \hat{f}_2 \\ \hat{f}_3 \\ \vdots \\ \hat{f}_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega_n & \omega_n^2 & \cdots & \omega_n^{n-1} \\ 1 & \omega_n^2 & \omega_n^4 & \cdots & \omega_n^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \cdots & \omega_n^{(n-1)^2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{bmatrix}$$



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Transformada Rápida de Fourier

A FFT mudou a história da humanidade

Discrete Fourier transform $O(N^2)$

Fast Fourier transform $O(N \log_2 N)$



Transformada Rápida de Fourier

Como?

Temos 10s de um áudio de 44KHz

$$n = 4.4 \times 10^5$$

$$\text{DTFT} \rightarrow 10^{11}$$

$$\text{FFT} \rightarrow 10^6$$

N	1000	10^6	10^9
N^2	10^6	10^{12}	10^{18}
$N \log_2 N$	10^4	20×10^6	30×10^9

$$10^{18} ns \rightarrow 31.2 \text{ years}$$

$$30 \times 10^9 ns \rightarrow 30 \text{ seconds}$$



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Transformada Rápida de Fourier

Escrever com operações matriciais mais eficientes e de menor tamanho

$\mathcal{O}(n^2)$

$$\begin{bmatrix} \hat{f}_1 \\ \hat{f}_2 \\ \hat{f}_3 \\ \vdots \\ \hat{f}_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega_n & \omega_n^2 & \cdots & \omega_n^{n-1} \\ 1 & \omega_n^2 & \omega_n^4 & \cdots & \omega_n^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \cdots & \omega_n^{(n-1)^2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{bmatrix}$$

$$= \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_n & 0 \\ 0 & F_n \end{bmatrix} P,$$

$$D = \begin{bmatrix} 1 & & & & \\ & w & & & \\ & & w^2 & & \\ & & & \ddots & \\ & & & & w^{n-1} \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$



Transformada Rápida de Fourier

Comparar a DTFT e a FFT no MATLAB

skills **tic, toc, fft**

Transformada Inversa

Reconstruir um sinal utilizando a IFFT

skills `ifft`, `fft`

