Tutorial - 2

Q1 > Solution:-

```
Void fun(int n)
{ int j=1, i=0;
  while (i<n)
  { i=i+j;
    i++;
```

3 3

$j=1$, $i=0+1$

$j=2$, $i=0+1+2$

$j=3$, $i=0+1+2+3$

Loop ends when $i \geq n$

$0+1+2+3+\dots n > n$

$k=\frac{(k+1)}{2} > n$

$k^2 > n$

$k > \sqrt{n}$

$O(\sqrt{n})$

Q2 > Solution:- Recurrence Relation for Fibonocci series,

$T(n) = T(n-1) + T(n-2)$

$T(0) = T(1) = 1$

- if $T(n-1) \approx T(n-2)$

(Lower Bound)

$$
\begin{aligned}
T(n) &= 2T(n-2) \\
&= 2\{2T(n-4)\} = 4T(n-4) \\
&= 4(2T(n-6)) \\
&= 8T(n-6) \\
&= 8\{2T(n-8) \\
&= 16T(n-8) \\
&\vdots \\
T(n) &= 2^k T(n-2k)
\end{aligned}
$$

$n-2^k = 0$

$n = 2k$

$k = n/2$

$T(n) = 2^{n/2} T(0)$

$= 2^{n/2}$

$T(n) = \Omega(2^{n/2})$

- if $T(n-2) \approx T(n-1)$

$$
\begin{aligned}
T(n) &= 2T(n-1) = 2(2T(n-2)) = 4T(n-2) \\
&= 4(2T(n-3)) = 8T(n-3) \\
&= 2^k T(n-k)
\end{aligned}
$$

$n-k=0$

$k=n$

$T(n) = 2^k \times T(0) = 2^n$

$T(n) = O(2^n)$ (upper Bound)

Q3) Solution:- $O(n(\log n))$ ⇒ for (int i=0; i<n; i++)
for (int j=1; j<n; j=j*2)
// Same O(1)

- $O(n^3)$ ⇒ for (int i=0; i<n; i++)
for (int j=0; j<n; j++)
for (int k=0; k<n; k++)
// Some O(1)

- $O(\log(\log n))$ ⇒ for (int i=1; i<=n; i=i*2)
for (int j=1; j<=n; j=j*2)
// Same O(1)

Q4) Solution:- $T(n) = T(n/4) + T(n/2) + Cn^2$

- Lets assume $T(n/2) >= T(n/4)$
So, $T(n) = 2T(n/2) + Cn^2$

applying master's Theorem $(T(n) = aT(n/b) + f(n))$

$a = 2, b = 2$ $f(n) = n^2$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

Compare n^c and $f(n) = n^2$

$f(n) > n^c$ so, $T(n) = \Theta(n^2)$

Q5) Solution:- int fun (int n)
{ for (int i=1; i<=n; i++)
for (int j=1; j<n; j+=i)
// Some O(1)
}

$i=1$ $\left.\begin{matrix} j=1 \\ j=2 \\ \vdots \\ j=n \end{matrix}\right\}$ → n time

$i=2$ $\left.\begin{matrix} j=1 \\ 3 \\ 5 \\ 7 \end{matrix}\right\}$ — loop ends when $j>n$
$1+2+5+7 > n$
— n times $k > n/2$

$i=3$ — $j=1$, 4, 7 → $1+4+7 > n$
$k = n/3$

$i=4$ $k = n/4$

$i=n$ $k = n/n$

So, total Complexity $= O(n^2+n^2+n^2+\dots)$
$= O(n^2)$

Q6> Solution :- for (int i=2 ; i<=n ; i = Pow (i, k))
// Some (1)

Complexity of pow (i, k) — $O(\log N)$
$= \log(k)$

$i = 2$
$i = 2^k$
$i = 2^{k^2}$
$i = 2^{k^3}$
$i = 2^{k^4}$
⋮
$i = 2^{k^M}$

Loop Ends when $i > n$
$2^{k^n} > n$
$\log(2^{k^n}) > \log n$
$k^M \log 2 > \log n$
$k^M > \log n$
$\log(k^M) > \log(\log n)$
$M \log k > \log(\log n)$
$M > \frac{\log(\log n)}{\log(k)}$

$T(c) = O(\log(\log n))$

Q8> Solution: a> $100 < \log n < \sqrt{n} < n < \log(\log n) < n\log n < \log n! < n! <$
$n^2 < \log^2 n < 2^n < 2^{2n} < 4^n$

(b) $1 < \sqrt{\log n} < \log n < 2\log n < \log 2N < N < 2N < 4N <$
$\log(\log N) < N\log N < \log N! < N! < N^2 < 2 \times 2^N$

(c) $96 < \log_8 N < \log_2 N < n\log_6 N < n\log_2 N < \log n!$
$< N! < 5N < 8N^2 < 7N^3 < 8^{2n}$