Subject "DAA"

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Tutorial - 1 (DAA)

Q1) Solution:- <u>Asymptotic Notation</u>: Asymptotic Notation are the mathematical notations used to describe the running time of an algorithm.

Different types of Asymptotic Notations:

1. <u>Big-O Notation</u> (O): It represents upper Bound of algorithm.
   $f(n) = O(g(n))$ if $f(n) \leq c * g(n)$.

2. <u>Omega Notation</u> (Ω): It represents lower bound of algorithm.
   $f(n) = \Omega(g(n))$ if $f(n) \geq c * g(n)$.

3. <u>Theta Notation</u> (θ): It represents upper and lower bound of algorithm.
   $f(n) = \theta(g(n))$ if $c_1 g(n) \leq f(n) \leq c_2 g(n)$

Q 2) Solution:-

```
for (i=1 to n)
{ i=i*2
}
```

$i=1$
$i=2$
$i=4$
$i=8$
$i=16$
$i=n$

It is forming G.P

$a_n = ar^{n-1}$

$n = ar^{k-1}$

$n = 1 \times (2)^{k-1}$

$\log n = \log 2^{k-1}$

$\log n = (k-1) \log 2$

$\boxed{k = \log n + 1}$

$\left(\begin{array}{l} a_n = n \\ r = 2 \\ a = 1 \end{array} \right)$

$O(\log n)$

Q3 Solution :- $T(n) = 3\,T(n-1)$ if $n>0$, Otherwise 1

$T(1) = 3T(0)$ $[T(0) = 1]$

$T(1) = 3\times 1$

$T(2) = 3T(1) = 3\times 3\times 1$

$T(3) = 3\times T(2) = 3\times 3\times 3$

$\vdots$

$T(n) = 3\times 3\times 3 \cdots$

$= 3^n = O(3^n)$

Q4 Solution :- $T(n) = 2T(n-1)-1$ if $n>0$, Otherwise 1

$T(0) = 1$

$T(1) = 2T(0)-1$

$T(1) = 2-1 = 1$

$T(2) = 2T(1)-1$

$T(2) = 2-1 = 1$

$T(3) = 2T(2)-1$

$= 2-1 = 1$

$\vdots$

$T(n) = 1$ $O(1)$

Q5 Solution:-
```
int i=1, s=1
while (s<=n)
{
    i++;
    s=s+i;
    Printf("#");
}
```

$i=1$ $S=1$

$i=2$ $S=1+2$

$i=3$ $S=1+2+3$

$i=4$ $S=1+2+3+4$

$\vdots$ $\vdots$

Loop Ends when $S>n$

$1+2+3+4+\cdots k > n$

$\frac{k(k+1)}{2} > n$

$k^2 > n$

$k > \sqrt{n}$

$\therefore O(\sqrt{n})$

Q6 > Solution :- Void function (int n)

```
{ int i, Count = 0;
  for ( int i=1; i*i <=n; i++)
      Count ++
}
```

$i = 1$
$i = 2$
$i = 3$
$i = 4$
⋮
$i = K$

Loop Ends when $i * i > n$

$k * k > n$

$k^2 > n$

$k > \sqrt{n}$

$O(n) = \sqrt{n}$

Q7 > Solution :- Void function (int n)

```
{ int i, j, k, Count = 0;
  for ( i = n/2 ; k = n ; i++)
     for ( j = 1 ; j <= n ; j = j*2)
        for ( k = 1 ; k <= n ; k = k*2)
            Count ++;
}
```

1st Loop : $i = n/2$ to n, i++

$= O(n/2) = O(n)$

2nd Nested loop : $j = 1$ to n $j = j * 2$

$j = 1$
$j = 2$
$j = 4$
⋮
$j = n$

$= O(\log n)$

3rd loop : $k = 1$ to n, $k = k * 2$

$k = 1$
$k = 2$
$k = 4$

$= O(\log n)$

Total Complexity $= O(n * \log n \times \log n) = O(n \log^2 n)$

Q8 > Solution :- fun (int n)

```
{ if (n==1) return;             — 1
  for (int i=1; to n)
    for (int j=1 ton)           — n2
      Print ("*");

  fun (n-3)                     — T(n-3)
}
```

$T(n) = T(n-3) + n^2$ $\because T(1) = 1$

$\rightarrow T(4) = T(4-3) + 4^2 = T(1) + (4 \times 4) = 1^2 + 4^2 =$

$\rightarrow T(7) = T(7-3) + 7^2 = 1^2 + 4^2 + 7^2$

$\rightarrow T(10) = T(10-3) + 10^3 = 1 + 4^2 + 7^2 + 10^2$

So, $T(n) = 1^2 + 4^2 + 7^2 + 10^2 \ldots n^2 = \frac{n(n+1)(2n+1)}{6} = O(n^3)$

also for terms Like $T(2), T(3), T(5)$

So, $T(n) = O(n^3)$

Q9) Solution :-

```
Void function (int n)
{
  for (int i=1 to n)  — n
      for (j=1 ; j<=n ; j=j+1)  — n
          Print ("*") ;
}
```

$i=1 \rightarrow j=1$ to n
$i=2 \rightarrow j=1$ to n
$i=3 \rightarrow j=1$ to n
$i=4 \rightarrow j=1$ to n

So, for i upto n it will take n^2

So, $T(n) = O(n^2)$

Q10) Solution :- $f_1(n) = n^k$ $\quad f_2(n) = c^n$

$k >= 1$, $c > 1$

Asymptotic relationship between f_1 & f_2 is Big O i.e $f_1(n) = O(f_2(n)) = O(c^n)$

is $n^k \leq G * c^n$ [n is some constant]