

Tutorial - 4

01

Q1) Solution:- $T(n) = 3T(n/2) + n^2$ $f(n) = n^2$
 $a = 3, b = 2$

$$n^{\log_b a} = n^{\log_2 3}$$

Comparing $n^{\log_2 3}$ and n^2

$$n^{\log_2 3} < n^2 \quad (\text{Case 3})$$

\therefore according to master theorem $T(n) = \Theta(n^2)$

Q2) Solution:- $T(n) = 4T(n/2) + n^2$
 $a = 4, b = 2$

$$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n) \quad (\text{Case 2})$$

\therefore according to master theorem $T(n) = O(n^2 \log n)$

Q3) Solution:- $T(n) = T(n/2) + 2^n$
 $a = 1, b = 2$

$$n^{\log_2 1} = n^0 = 1$$

$$1 < 2^n \quad (\text{Case 3})$$

\therefore According master theorem $T(n) = O(2^n)$

Q4) Solution:- $T(n) = 2^n T(n/2) + n^2$

\therefore Master's theorem is not applicable as a is function of n .

Q5) Solution:- $T(n) = 16T(n/4) + n$
 $a = 16, b = 4, f(n) = n$

$$n^{\log_b a} = n^{\log_4 16} = n^2$$

$$n^2 > f(n) \quad (\text{Case 1})$$

$$\therefore T(n) = \Theta(n^2)$$

Q6) Solution:- $T(n) = 2T(n/2) + n \log n$
 $a = 2, b = 2, f(n) = n \log n$

$$n^{\log_b a} = n^{\log_2 2} = n$$

Now $f(n) > n$

\therefore According to masters $T(n) = \Theta(n \log n)$.

Q7/ Solution:- $T(n) = 2T(n/2) + n/\log n$

$$a=2, b=2, f(n) = \frac{n}{\log n}$$

$$n^{\log_b a} = n^{\log_2 2} = n$$

$$n > f(n)$$

\therefore According to masters theorem $T(n) = \Theta(n)$

Q8/ Solution:- $T(n) = 2T(n/4) + n^{0.51}$

$$a=2, b=4, f(n) = n^{0.51}$$

$$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$$

$$n^{0.5} < f(n)$$

\therefore According to masters Theorem $T(n) = \Theta(n^{0.51})$

Q9/ Solution:- $T(n) = 0.5 T(n/2) + 1/n$

\therefore Master's Not applicable as $a < 1$.

Q10/ Solution:- $T(n) = 16T(n/4) + n!$

$$a=16, b=4, f(n) = n!$$

$$n^{\log_b a} = n^{\log_4 16} = n^2$$

$$n^2 < n!$$

\therefore According to masters, $T(n) = \Theta(n!)$

Q11/ Solution:- $T(n) = 4T(n/4) + \log n$

$$a=4, b=2, f(n) = \log n$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$n^2 > f(n)$$

\therefore According to master's $T(n) = \Theta(n^2)$

Q12/ Solution:- $T(n) = \sqrt{n} T(n/2) + \log n$

\therefore Masters Not applicable as a is not Constant here.

Q13) Solution:- $T(n) = 3T(n/2) + n$
 $a=3, b=2, f(n)=n$
 $n^{\log_b a} = n^{\log_2 3} = n^{1.58}$
 $n^{1.58} > f(n)$

∴ According to master's theorem, $T(n) = O(n^{\log_2 3})$

Q14) Solution:- $T(n) = 3T(n/3) + \sqrt{n}$
 $a=3, b=3, f(n) = \sqrt{n}$
 $n^{\log_b a} = n^{\log_3 3} = n$
 $n > \sqrt{n}$

∴ According to master's theorem, $T(n) = O(n)$.

Q15) Solution:- $T(n) = 4T(n/2) + cn$
 $a=4, b=2, f(n) = c \cdot n$
 $n^{\log_b a} = n^{\log_2 4} = n^2$
 $n^2 > c \cdot n$

∴ According to master's theorem, $T(n) = O(n^2)$

Q16) Solution:- $T(n) = 3T(n/4) + n \log n$
 $a=3, b=4, f(n) = n \log n$
 $n^{\log_b a} = n^{\log_4 3} = n^{0.79}$
 $n^{0.79} < n \log n$

∴ According to master's theorem $T(n) = O(n \log n)$

Q17) Solution:- $T(n) = 3T(n/3) + n/2$
 $a=3, b=3, f(n) = n/2$
 $n^{\log_b a} = n^{\log_3 3} = n$
 $O(n) = O(n/2)$

∴ According to master's theorem,
 $T(n) = O(n \log n)$.

Q18) Solution:- $T(n) = 6T(n/3) + n^2 \log n$
 $a=6, b=3, f(n) = n^2 \log n$
 $n^{\log_b a} = n^{\log_3 6} = n^{1.63}$
 $n^{1.63} < n^2 \log n$

\therefore According to master's theorem $T(n) = O(n^2 \log n)$

Q19) Solution:- $T(n) = 4T(n/2) + n \log n$
 $a=4, b=2, f(n) = n \log n$
 $n^{\log_b a} = n^{\log_2 4} = n^2$
 $n^2 > n \log n$

\therefore According to master's theorem,

Q20) Solution:- $T(n) = 64T(n/8) - n^2 \log n$

Master's theorem is not applicable as $f(n)$ is not increasing function.

Q21) Solution:- $T(n) = 7T(n/3) + n^2$
 $a=7, b=3, f(n) = n^2$
 $n^{\log_b a} = n^{\log_3 7} = n^{1.7}$
 $n^{1.7} < n^2$

\therefore According to master's, $T(n) = O(n^2)$

Q22) Solution:- $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem isn't applicable since regularity condition is violated in Case 3.