Tutorial - 4

Q1) Solution :- $T(n) = 3T(n/2) + n^2$ $\quad f(n) = n^2$

$a = 3,\ b = 2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ | Case 3)

∴ according to master Theorem $T(n) = \Theta(n^2)$

Q2) Solution :- $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

∴ according to master theorem $T(n) = O(n^2 \log n)$

Q3) Solution :- $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (Case 3)

∴ According master theorem $T(n) = O(2^n)$

Q4) Solution :- $T(n) = 2^n T(n/2) + n^n$

∴ Master's theorem is not applicable as a is function of n.

Q5) Solution :- $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > F(n)$ (Case 1)

∴ $T(n) = \Theta(n^2)$

Q6) Solution :- $T(n) = 2T(n/2) + n\log n$.

$a = 2,\ b = 2,\ f(n) = n\log n$.

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

∴ According to masters $T(n) = \Theta(n \log n)$.

Q7) Solution:- $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to masters theorem $T(n) = \Theta(n)$

Q8) Solution:- $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to masters Theorem $T(n) = \Theta(n^{0.5})$

Q9) Solution:- $T(n) = 0.5\,T(n/2) + 1/n$

$\therefore$ Master's Not applicable as $a < 1$.

Q10) Solution:- $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According to masters, $T(n) = \Theta(n!)$

Q11) Solution:- $T(n) = 4T(n/4) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to master's $T(n) = \Theta(n^2)$

Q12) Solution:- $T(n) = \text{sqrt}(n)\, T\lceil n/2 \rceil + \log n$

$\therefore$ Masters not applicable as a is not constant here.

Q13> Solution:- $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to master's theorem, $T(n) = O(n^{\log_2 3})$

Q14> Solution:- $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3,\ b = 3,\ f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to master's theorem, $T(n) = \Theta(n)$.

Q15> Solution:- $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2,\ f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

$\therefore$ According to master's theorem, $T(n) = \Theta(n^2)$

Q16> Solution:- $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to master's theorem $T(n) = \Theta(n\log n)$

Q17> Solution:- $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

$\therefore$ According to master's theorem.

$T(n) = \Theta(n\log n)$.

Q18) Solution:- $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$\therefore$ According to master's theorem $T(n) = \Theta(n^2 \log n)$

Q19) solution:- $T(n) = 4T(n/2) + n \log n$

$a = 4, b = 4, f(n) = n / \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n.$

$\therefore$ According to master's theorem.

Q20) Solution:- $T(n) = 64T(n/8) - n^2 \log n$

Master's theorem is not applicable as f(n) is not increasing function.

Q21) Solution:- $T(n) = 7(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ According to master's, $T(n) = \Theta(n^2)$

Q22) Solution:- $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem isn't applicable since regularity condition is violated in Case 3.